THE DIFFRACTION OF LIGHT BY SUPERSONIC WAVES.

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1. *Introduction.*

An elementary theory of the diffraction of light by supersonic waves in liquids was put forward by Sir C. V. Raman and myself[1] to explain many of the important features of the phenomenon observed by Bär and others. The theory was developed under the restriction that a light beam undergoes no amplitude changes on its wavefront during its passage through the supersonic field. This restriction enabled us to get closed expressions for the intensities of the diffraction orders. Later, experimental conditions actually satisfying the above restriction were realised by Bär and Sanders who have reported reasonable agreement with our calculations. We have also developed the exact theory[2] without the above restriction. Though it explains certain of the features when the restriction is not satisfied experimentally, yet it is far from being satisfactory, as the expressions are too complicated. The exact theory developed by Extermann and Wannier[3] suffers also from the same defect. They have only plotted the intensity diagrams of the diffraction orders for certain values of the parameters entering the theory. The purpose of this paper is to point out an extreme case where one can get closed expressions for the intensities of the diffraction orders. While the elementary theory is valid in the low frequency region (5 to 6 megacycles per second or less), the theory contained in this paper is valid in any frequency region so long as the supersonic field is not strong enough to excite the second and other higher orders. It is also found that in the very high frequency region, the first orders would be dominant over the higher ones.

Let us assume that the sound field creates a periodic fluctuation in the refractive index of the medium and consider the case when the direction of the incident light is normal to the direction of the sound waves. The amplitudes of the diffraction orders satisfy the equations[2]

$$2\frac{d\Phi_r}{d\xi} - \Phi_{r-1} + \Phi_{r+1} = i\rho r^2\Phi_r, \qquad (1)$$

$$r = -\infty, \ldots, -1, 0, 1, \ldots, \infty,$$

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where

$i = \sqrt{-1}$,

$\rho = \lambda^2/\mu_0 \mu \lambda^{*2}$,

$\xi = 2\pi\mu z/\lambda$,

μ_0 = the refractive index of the medium,

μ = the amplitude of the fluctuation of the index,

λ = the wave-length of the incident light,

λ^* = the wave-length of the sound waves,

z = the width of the sound field along the direction of the incident light.

The boundary conditions to be satisfied by Φ's are

$$\Phi_0(0) = 1 \quad \text{and} \quad \Phi_r(0) = 0, \qquad r \neq 0, \tag{2}$$

which mean that the intensity of the incident light is unity. If ρ is zero in (1), the solutions of (1) satisfying (2) are given by $\Phi_r(\xi) = J_r(\xi)$ where J_r is the Bessel function of the rth order. This corresponds to our elementary theory.

If ρ is *very large*, then the equation (1) can only be satisfied if

$$2\frac{d\Phi_r}{d\xi} \approx ir\rho^2\Phi_r, \qquad r \neq 0 \tag{3}$$

which means that

$$\Phi_r \approx A_r \exp(i r \rho^2 \xi/2), \qquad r \neq 0. \tag{4}$$

But A_r must nearly be zero by virtue of (2). If $r = 0$

$$\frac{d\Phi_0}{d\xi} \approx 0 \tag{5}$$

and hence $\Phi_0 = 1$ by virtue of (2). This means that when ρ is very large, the diffraction effect will not be prominent as is otherwise the case when ρ is nearly zero.

Let us assume that ρ is so large that the second and all other higher orders have vanishing amplitudes. In this case the equations are

$$\begin{aligned} \frac{d\Phi_0}{d\xi} + \Phi_1 &= 0, \\ 2\frac{d\Phi_1}{d\xi} - \Phi_0 &= i\rho\Phi_1, \end{aligned} \tag{6}$$

as one can easily see that $\Phi_1 = -\Phi_{-1}$. If we put $\Phi_0 = A_0 \exp(i\lambda\xi)$ and $\Phi_1 = A_1 \exp(i\lambda\xi)$, it is easy to find out that λ satisfies the equation

$$2\lambda^2 - \lambda\rho - 1 = 0, \tag{7}$$

the roots of which are

$$\lambda_1, \lambda_2 = \frac{\rho \pm \sqrt{\rho^2 + 8}}{4}.$$

Thus the amplitudes can be written as

$$\begin{aligned}\Phi_0 &= A_0 \exp(i\lambda_1\xi) + A_1 \exp(i\lambda_2\xi)\\ \Phi_1 &= A_1 \exp(i\lambda_1\xi) + B_1 \exp(i\lambda_2\xi).\end{aligned} \tag{8}$$

By virtue of (2) and (6) we must have

$$\begin{aligned}A_0 + B_0 &= 1,\\ A_1 + B_1 &= 0,\\ A_1 + i A_0 \lambda_1 &= 0,\\ B_1 + i B_0 \lambda_2 &= 0.\end{aligned} \tag{9}$$

Solving (9) we can write the solutions for Φ_0 and Φ_1 as

$$\begin{aligned}\Phi_0 &= \tfrac{1}{2}\left(1 - \frac{\rho}{\sqrt{\rho^2+8}}\right)\exp\left\{i\left[\frac{\rho+\sqrt{\rho^2+8}}{4}\right]\xi\right\}\\ &\quad + \frac{1}{2}\left(1 + \frac{\rho}{\sqrt{\rho^2+8}}\right)\exp\left\{i\left[\frac{\rho-\sqrt{\rho^2+8}}{4}\right]\xi\right\}\\ \Phi_1 &= \frac{-i}{\sqrt{\rho^2+8}}\left[\exp\left\{i\left[\frac{\rho+\sqrt{\rho^2+8}}{4}\right]\xi\right\} - \exp\left\{i\left[\frac{\rho-\sqrt{\rho^2+8}}{4}\right]\xi\right\}\right.\end{aligned} \tag{10}$$

which may also be written as

$$\begin{aligned}\Phi_0 &= \left[1 - \frac{8}{\rho^2+8}\sin^2\left\{\frac{\sqrt{\rho^2+8}}{4}\xi\right\}\right]^{\frac{1}{2}} \times\\ &\quad \exp\left\{i\left[\frac{\rho\xi}{4} - \tan^{-1}\left(\frac{\rho}{\sqrt{\rho^2+8}}\tan\left(\frac{\sqrt{\rho^2+8}}{4}\xi\right)\right)\right]\right\}\\ \Phi_1 &= \frac{2}{\sqrt{\rho^2+8}}\sin\left\{\frac{\sqrt{\rho^2+8}}{4}\xi\right\}\exp\{i\rho\xi/4\}.\end{aligned} \tag{11}$$

It may be seen from (12) that the phase of the central order depends on the index fluctuation while that of the first order does not depend on it, as $\rho\xi$ is independent of μ. The intensities of the diffraction orders are given by

$$\begin{aligned}I_0 &= 1 - \frac{8}{\rho^2+8}\sin^2\left\{\frac{\sqrt{\rho^2+8}}{4}\xi\right\}\\ I_1 &= I_{-1} = \frac{4}{\rho^2+8}\sin^2\left\{\frac{\sqrt{\rho^2+8}}{4}\xi\right\}.\end{aligned} \tag{12}$$

If ρ becomes very large $I_0 \approx 1$ and $I_1 \approx 0$ in conformity with our earlier conclusion. If ρ is very small, $I_0 \approx 1 - \sin^2(\xi/\sqrt{2})$ and $I_1 \approx \frac{1}{2}\sin^2(\xi/\sqrt{2})$. As the second orders are not observed in this region only when ξ is small, the above expressions may be easily seen to be the approximations of $J_0^2(\xi)$ and $J_1^2(\xi)$.

Leaving these two extreme cases, it may be seen from (12) that when

$$\frac{\sqrt{\rho^2+8}}{4}\,\xi = \pi s$$

or $$z = \frac{s}{\sqrt{\left(\frac{\lambda}{\mu_0 \lambda^{*2}}\right)^2 + 8\left(\frac{\mu}{\lambda}\right)^2}}\,, \quad \text{where } s \text{ a positive integer,} \tag{13}$$

the intensity of the first order is zero which means the absence of the diffraction effect.

We have assumed here that ρ is so large that the second and other higher orders have vanishing intensity. We should thus expect reasonable agreement between (12) and one of the diagrams in Extermann and Wannier's paper[3] in which the second order has a small intensity while all the higher orders are absent. This diagram corresponds to $\rho = 1$ and we should thus expect the validity of (13) when $\rho > 1$. (Extermann's θ and D are $\frac{1}{\rho}$ and $\rho\,\xi/2$ respectively in our notation.) In the following diagram, the dotted

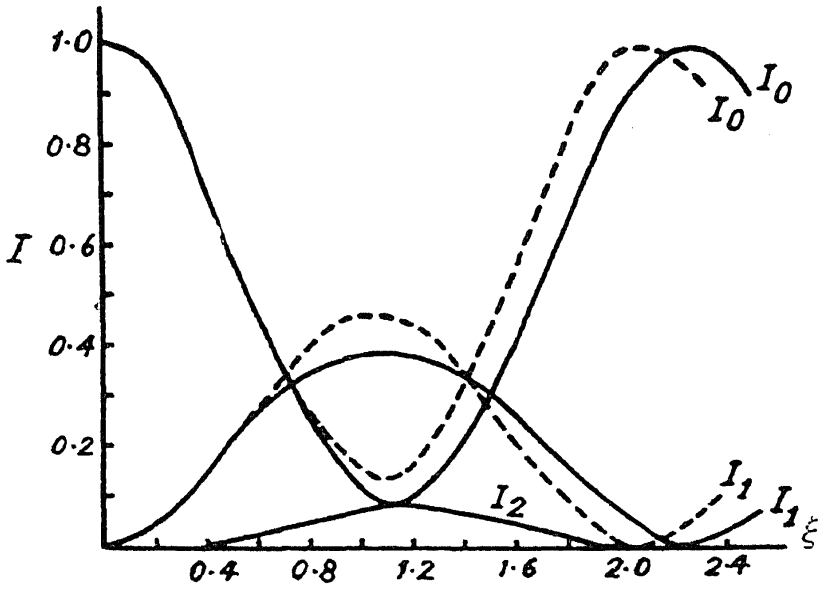

FIG. 1.

The dotted curves practically coincide with the continuous ones of Extermann and Wannier for low values of ξ.

curves are according to (12), while the continuous ones are taken from the paper of Extermann and Wannier. Complete agreement cannot be expected as our expressions (12) are valid only when the second and other higher orders are not present, which is not exactly the case as can be seen in the curve for the intensity of the second order which is not entirely negligible. If we remember that Extermann's curves were plotted from extensive

calculations, the usefulness of (12) when $\rho \geqslant 1$ may not be without significance, as he has only plotted three intensity diagrams two of which correspond to $\rho < 1$ and the remaining to $\rho = 1$.

Before concluding we may point out the significance of (11) in the application of the theory to the diffraction of light by supersonic waves in solids. If we consider a longitudinal sound wave, it creates an index fluctuation given by an ellipsoid, two of whose axes lie in a plane perpendicular to the direction of the incident light and one of which lies along the direction of the sound wave. If the incident light is linearly polarised, we will have to decompose the incident light into two components, the polarisations of which lie along the above axes and consider the propagation of each independent of the other. It may be seen from (11) that the phases of the two components of the central order would be different as they depend on the principal index fluctuations which are different. Thus the resultant of the two components of the central order should be elliptically polarised. The first order would be linearly polarised as can be seen from (11), in the limits of the validity of the theory. In general, one should expect the diffraction orders to be elliptically polarised. But when $\rho \backsimeq 0$, the phases of the diffraction orders do not depend on the index fluctuations and we should expect, as has been pointed out by Mueller and myself,[4] that the diffraction orders would be linearly polarised.

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