PHOTO-ELASTIC EFFECT IN CRYSTALS

A New Result for the T_h Class

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1. INTRODUCTION

ONE of us (Bhagavantam, 1942) derived by a group theoretical method the number of independent constants needed to describe the photo-elastic behaviour of each of the 32 crystal classes. It was found that the numbers are at variance with those given by Pockels (1889, 1906) and current in literature, for certain classes, namely, C_4, S_4 and C_{4h} of the tetragonal system C_3, S_6, C_{3h}, C_6, and C_{6h} of the trigonal system and T and T_h of the cubic system. The non-vanishing stress-optical coefficients were also worked out directly for all the 32 classes and the findings of the group theoretical method were confirmed. On the other hand, the same method, when applied to the elastic and optical coefficients and later to derive the number of independent constants needed to describe the piezo-electric and electro-optical Kerr effects, optical activity and pyro-electricity in crystals (Saksena, 1944; Suryanarayana, 1945, 1946) gave results which agreed with the known numbers for the respective phenomena.

The present experimental investigation relating to the photo-elastic behaviour of certain crystals, is undertaken to clear the existing discrepancy. The results obtained are in entire agreement with the findings of the group theoretical method.

2. CHOICE OF THE MATERIAL

As mentioned above, the T_h class of the cubic system is a case in which, while Pockels gave three constants in common with the other classes of this system, the theory developed by Bhagavantam requires four constants. Of the cubic crystals with which Pockels himself had worked, only two, namely potassium alum and ammonium alum, belong to the T_h class (Wyckoff, 1931). But Pockels had not determined the constants completely, on account of inhomogeneity of the material he worked with (*vide* Pockels, *Lehrbuch der Kristalloptik*, 1906, p. 480, foot-note 2). The original paper of Pockels (1892) dealing with the alums has not been available to the authors, and we are unable to find out if he has himself noticed any discrepancies with the theory. Single crystals of potassium alum of optical quality could be grown to the necessary sizes and as the substance is quite stable under normal atmospheric conditions and in view of the fact that Pockels had worked with it, potassium alum is chosen for our investigations.

3. Theoretical Considerations

The non-vanishing stress-optical coefficients for T and T_h classes are given by the equations (1),

$$\begin{aligned} B_{11} - B &= -(q_{11} P_{xx} + q_{12} P_{yy} + q_{13} P_{zz}); \quad B_{23} = -q_{44} P_{yz}; \\ B_{22} - B &= -(q_{13} P_{xx} + q_{11} P_{yy} + q_{12} P_{zz}); \quad B_{31} = -q_{44} P_{zx}; \\ B_{33} - B &= -(q_{12} P_{xx} + q_{13} P_{yy} + q_{11} P_{zz}); \quad B_{12} = -q_{44} P_{xy}; \end{aligned} \tag{1}$$

consisting of four independent constants, while according to Pockels $q_{12} = q_{13}$ leaving only three constants. Here, $B = \frac{1}{n^2}$, $B_{11} = \frac{1}{n_{11}^2}$, etc., where n is the refractive index of the undeformed cubic crystal, n_{11} etc., are the components describing the Fresnel ellipsoid in the deformed condition, $P_{xx}, \ldots . P_{xy}$ are the components of stress tensor and q_{11}, q_{12}, q_{13} and q_{44} are the stress-optical coefficients, all referred to the crystallographic axial system.

One direct and striking result of the non-equivalence of q_{12} and q_{13} is that the crystal becomes biaxial by a simple compression along a cube axis, say Z-axis, instead of becoming uniaxial as in the case of the classes T_d, O and O_h, where $q_{12} = q_{13}$. This point could be settled by a direct observation of the double refractions, or more precisely, the path differences per unit pressure and unit length of the light beam in the crystal, between two beams, one vibrating along and the other perpendicular to, the direction of the pressure for light propagated along the other two cube axes, say X and Y, *i.e.*, when the observations are made along these axes. These two path differences should be different if the crystal has four independent constants. Again, for the same direction of pressure, the path differences should be identical when observations are made parallel to [110] and [$\bar{1}$10] directions.

The expressions for the path differences for the various cases are given in Table I. The derivations follow well-known principles, employed earlier by Pockels.

If $q_{12} = q_{13}$, expressions in column 4 become identical with those given in column 5. An examination of the table shows that two rectangular parallelopipeds with their lengths parallel to a cube axis, say [001], and the other two edges parallel (1) to the cube axes [100] and [010], and (2) to the [110] and [$\bar{1}$10] directions, will provide evidence in favour of the one or the other scheme with internal checks also. There is no difference between

TABLE I

No.	Direction of pressure	Direction of observation	Expression for path-difference for T and T_h classes	Expression according to Pockels' scheme
1	[001]	[100]	$\frac{2}{n^3}\ (q_{11}-q_{12})$	$\frac{2}{n^3}\ (q_{11}-q_{12})$
2	[001]	[010]	$\frac{2}{n^3}\ (q_{11}-q_{13})$	,,
3	[001]	[110]	$\frac{1}{n^3}\ (2q_{11}-q_{12}-q_{13})$	,,
4	[001]	[$\bar{1}$10]	,,	,,
5	[111]	[$\bar{2}$11]	$\frac{2}{n^3}\ q_{44}$	$\frac{2}{n^2}\ q_{44}$
6	[111]	[01$\bar{1}$]	,,	,,

the two schemes in case pressure is along the cube diagonal, *i.e.*, [111] direction.

4. EXPERIMENTAL ARRANGEMENTS

All the prisms needed were cut and polished from crystals grown in this laboratory by slow evaporation over a period of three months, of aqueous solutions of B. D. H. Analar potassium alum. Half a dozen crops were raised and the material was chosen from the best crop. Only the most perfect single crystals without flaws were selected, regard being shown for flawlessness and not so much for large size. Almost all the specimens chosen had very well-developed cube and dodecahedral faces in addition to the usual octahedral faces. All the prisms of lengths from 8 to 12 mm. and lateral dimensions 3 to 4 mm., have at least one of their ends terminated on a natural face. The four longer faces are ground parallel to natural edges along the required directions. The orientations are accurate to within 1°. None of the prisms show any double refraction under crossed Nicols in an unstrained state.

Light from a sodium vapour lamp is condensed by a lens on the slit of a collimator. The emergent parallel beam of light passes through a Nicol mounted on a circular scale with vernier, the vibration direction of the Nicol being maintained at + 45° or − 45° to the vertical. The beam then passes through the crystal prism into a Babinet compensator, the principal axes of which are vertical and horizontal. The light after passing through the compensator is observed through an eye-piece containing a Nicol crossed with the polarizer and the usual Babinet fringes are obtained. When the crystal prism is compressed, the fringes shift one way or the other and the magnitude of the shift is a measure of the path difference produced between the horizontally and vertically vibrating beams of light passing through the compressed crystal.

Compression is produced by a lever arrangement. Four fine v-grooves perpendicular to the length of the bar are marked on a cold steel bar 17″ × ½″ × ½″, two on the upper and two on the lower side. In the upper groove A near one end sits a knife-edge fixed in a vice firmly clamped to a massive table, the groove B at the other end, distant 16″ from the first groove, being used for the knife-edge carrying weights. Two grooves on the lower side of the bar, at distances 2″ and 4″ from the groove A, to take the knife-edge on the crystal prism, give a mechanical advantage of about 8 and 4 as the case may be. The knife-edge which rests on the prism has a lead plate of 2 mm. thickness cemented to its base, of dimensions 1·2 × 1 cm., to help distribute the load uniformly. The lower end of the prism rests on a horizontal solid metal support on which a small rubber washer 1 mm. thick is placed. The knife-edges are of spindle steel.

The knife-edge is placed symmetrically over the prism held vertical, with the required edge parallel to the light beam, on the metal support and the lever placed horizontally such that the two knife-edges are engaged in the grooves. A black paper slit stuck on to the metal support by soft wax just allows only the light coming through the prism.

With one of the prisms, measurements of the fringe shift for stresses about 0·25 kg./mm.^{2} and 0·5 kg./mm.^{2} are made and there was no appreciable departure from linearity. So, stresses upto about 0·5 kg./mm.^{2} only are used in all cases, except in the [111] direction. In this case, to obtain a displacement of the Babinet fringes which can be measured reasonably accurately, stresses upto 0·9 kg./mm.^{2} are employed. The displacement of the fringes is measured at the middle of the length of the prism, at three places along its breadth, namely near the left edge, the middle and the right edge. Before each set of observations, a preliminary measurement of the fringe displacement at the three positions is made and only when the distribution of the load is fairly uniform, the final measurements are made. In case the distribution is non-uniform, the position of the prism is adjusted until the distribution is fairly uniform. In each of the three positions, the polarizing Nicol and the analysing Nicol of the compensator, are turned through 90°, such that in the two positions, the incident light is polarized at + 45° and − 45° to the vertical. Thus in all, six measurements of the fringe displacement are made. For each of the six measurements, the initial position (without load) of the central Babinet fringe is taken twice before placing the load and twice after removing the load. The final position of the Babinet fringe (on load) is obtained by four settings of the crosswire on the edge of the fringe. Thus each of the six measurements is the mean difference of four settings of the initial and final positions of the fringe. In the case of the [111] prism, each of the six measurements is the mean difference of six settings of the initial and final positions of the fringe. In none of the prisms was there any noticeable consistent residual double refraction after unloading (as deduced from the shift of the initial position).

The prism is then turned through 90° and the measurements are repeated similarly with identical load. Specimen observations for one of the prisms are given later in the paper.

To determine the absolute value of the stress-optical constants, a measure of the absolute change in the refractive index for vertically or horizontally polarized beam of light is needed. The method adopted by Pockels for other crystals was to measure the absolute path retardation by an interferometric method using two equal plane parallel prisms of the crystal in the two beams of the interferometer. When one of the prisms is compressed, the shifts in the fringes of the interferometer for light vibrating vertically and horizontally give a measure of the absolute changes in the refractive index for the two beams, when account is taken of the increase in the thickness of the prism by compression. In the absence of plane parallel prisms, the crystal itself can be used as an interferometer by forming localised interference fringes in the crystal itself, as has been done by Ramachandran (1947) recently in his determination of the photo-elastic constants of diamond.

The arrangement adopted is to work a pair of surfaces of the prism to get reasonably widely separated localised fringes, *i.e.*, curves of equal thickness, with reflected light. If light is reflected at the usual 45° angle at a glass plate, the observation of the shift of the fringes is not possible when the light is vibrating horizontally, because of the very low intensity. So, light is reflected at an angle of about 10° on to a pair of faces of the prism and the localised fringes formed by reflection of the beam at the two surfaces, are observed through a microscope. In obtaining the fringes, thicknesses of the crystal which give rise to destructive interference between the D_1 and D_2 lines are to be avoided. The microscope is focussed on a reference mark on the crystal itself and the movement of the fringes on loading is to be judged with reference to this mark. The shifting of the fringes is the combined result of the change in the thickness of the crystal and the change in the refractive index. Because of the low value of the elastic constants, a shift of one complete fringe could be produced in alum by stresses of the order of 0·3 kg./mm.[2] The procedure adopted is to note the load for a shift of a complete fringe for light beams polarized vertically and horizontally. The sign of the shift, namely, whether the order of interference is decreasing or increasing at the point under observation, settles the sign of the stress-optical constants unambiguously.

In order to provide checks as well as to study any possible effect of the ratio of the length to the lateral dimensions, on the observed stress-optical coefficients, two prisms of different dimensions are taken for each orientation for pressures in the [001] direction.

A description of the five prisms employed in the investigation is given in Table II.

TABLE II

Description of the prisms employed

No. of prism	Length, l		Breadth, b		Thickness, d	
	cm.	$\|^l$ to	cm.	$\|^l$ to	cm.	$\|^l$ to
I ..	1·17	[111]	0·408	[$\bar{2}$11]	0·360	[01$\bar{1}$]
II ..	0·78	[001]	0·293	[100]	0·294	[010]
III ..	0·86	[001]	0·409	[100]	0·407	[010]
IV ..	1·26	[001]	0·412	[110]	0·348	[$\bar{1}$10]
V ..	0·78	[001]	0·403	[110]	0·291	[$\bar{1}$10]

5. RESULTS

(a) Determination of $(q_{11}-q_{12})$, $(q_{11}-q_{13})$ *and* q_{44}.

Complete observations with the Babinet compensator for Prism II are given as an example in Table III. The results on all five prisms are given in detail in Table IV.

In prism III, observations along [100] and [010] with a load of 1 kg. were made first and then the observations with 2 kg. It may be noted that the shifts for 2 kg. are very slightly less than twice the shifts for 1 kg., suggesting a small residual double refraction. The difference however, is of the same order as the experimental errors themselves, so that we can assume that upto stresses of the order of 0·5 kg./mm.[2] the condition of linearity is satisfied.

The fringe width of the Babinet compensator is 144·6 divisions, which corresponds to a path retardation of one wavelength of sodium light. The

TABLE III

Observations on Prism II

Load = 1 kg. ; mechanical advantage = 3·973

The readings are head scale divisions of the Babinet compensator. 50 head scale divisions equal 1 main scale division

Observation along [010], 0·294 cm.

	Middle				Left				Right				
	+ 45°		− 45°		− 45°		+ 45°		+ 45°		− 45°		
	Initial	Final	I	F	I	F	I	F	I	F	I	F	
1	15 27·	15 01·5	15 30	15 07	15 29·5	15 04	15 30	15 05·5	15 32	15 08	15 32	15 07	
2	28·5	03	30	07	28	03·5	30·5	05	30	11	32·5	09·5	
3	29	01·5	28	07	30·5	03	28	05·5	31	10	31·5	09	
4	26	02	30·5	07	30	03·5	23·5	05	31	10	30	10	
Mean	27·62	02	29·62	07	29·5	03·5	29·25	05·25	31	09·75	31·5	08·88	
Diff.	25·62		22·62		26·0		24·0		21·25		22·62		
Mean	24·12				25·0				21·94				Mean = 23·69

Observation along [100], 0·293 cm.

	Middle				Left				Right				
	− 45°		+ 45°		+ 45°		− 45°		− 45°		+ 45°		
	I	F	I	F	I	F	I	F	I	F	I	F	
1	15 25·5	14 46	15 25·5	14 48	15 23·5	14 48	15 25·5	14 48	15 26·5	15 02	15 30	14 47·5	
2	24·5	15 00	25·5	46	25·5	15 00	25·5	15 01	26·5	03	28	15 02	
3	25·5	14 48	26·5	15 00	24·0	00	23	14 47	25·5	02	29·5	00·5	
4	25·5	48	25·5	14 46	25·5	14 47	26	49	24	02	27·5	02·5	
Mean	15 25·25	14 48	15 25·75	14 47·5	15 24·62	14 48·75	15·25	14 48·75	15 25·62	15 02·25	15 28·75	15 00·62	
Diff.	27·25		28·25		25·87		26·25		23·37		28·13		
Mean	27·75				26·06				25·75				Mean = 26·52

TABLE IV

Photo-elastic constants of potassium alum: observations with compensator

Prism No.	Observation: Parallel to	Observation: cm.	Load in kg.	Mechanical advantage of lever	Shift of the Babinet fringe in divisions of head scale: Left + 45°	Left − 45°	Middle + 45°	Middle − 45°	Right + 45°	Right − 45°	Mean	Stress-optical constants in units of 10^{-5}	Expression for the constent × $\frac{n^3}{2}$
I	[$\bar{2}$11]	0·408	1·6	7·90	7·9	8·3	7·9	8·0	6·6	10	8·12	0·63	(q_{44})
	[01$\bar{1}$]	0·360			4·9	6·2	8·5	9·0	8·2	7·9	7·45	0·65	,,
II	[010]	0·294	1·0	3·973	24·0	26·0	25·62	22·62	21·25	22·62	23·69	4·79	$(q_{11}-q_{13})$
	[100]	0·293			25·87	26·25	28·25	27·25	28·13	23·37	26·52	5·38	$(q_{11}-q_{12})$
III	[010]	0·407	1·0	,,	14·75	15·45	18·35	16·2	17·4	15·9	16·34	4·58	$(q_{11}-q_{13})$
	[010]		2·0	,,	33·00	33·75	31·63	34·12	30·13	30·88	32·26		
	[100]	0·409	1·0	,,	18·25	17·88	17·50	19·00	19·22	19·25	18·52	5·17	$(q_{11}-q_{12})$
	[100]		2·0		37·4	35·1	35·25	37·65	36·37	37·15	36·49		
IV	[110]	0·412	1·6		33·75	35·12	36·0	33·87	29·5	28·63	32·81	4·93	$(2q_{11}-q_{12}-q_{13})/2$
	[110]			,,	30·38	34·25	35·13	35·00	31·63	31·62	33·01	4·96	
	[110]	0·348			29·88	30·87	28·37	28·63	26·13	26·75	28·44	5·06	
V	[110]	0·403	1·4		31·0	31·37	37·25	32·88	34·13	36·75	33·90	4·86	,,
	[110]	0·291		,,	24·37	22·87	21·87	23·76	24·75	25·88	23·92	4·75	

stress-optical constants are calculated as follows. From the mean shift of the Babinet fringe, the path difference δ corresponding to unit stress (1 kg./mm.²) and unit length (1 cm.) of light beam in the stressed crystal is calculated, noting that the path difference is $5 \cdot 893 \times 10^{-5}$ cm. for a shift of 144·6 divisions. The stress-optical constant in the corresponding direction of observation is $= \frac{2}{n^3}\,\delta$, where δ is the path difference $(n_x - n_z)$ between the horizontally and vertically vibrating light beams. In the above experiments, the shift was always in the same direction as for common glass, *i.e.*, the crystal becomes negative uniaxial when compressed along the trigonal axis. Hence, the sign of the constants q_{44}, $(q_{11} - q_{12})$ and $(q_{11} - q_{13})$ is negative.

An examination of Table IV reveals the following facts:

(1) In prism II, stress-optical constants in the X and Y directions differ from each other by 12% which is much more than any possible experimental error.

(2) The two values bracketed together in prism IV, are the values of one and the same constant when observed along two directions and differ by 2% which is the order of the experimental error. The mean of these values, namely, 5·01 differs by 1·5% from the mean (5·085) of the two constants from prism II, which difference is again of the order of the experimental error.

(3) In prism III, while the two constants differ from each other again by about 12%, the absolute values of the constants for this prism are smaller by 4% than the corresponding values for prism II.

(4) In prism V, the two values differ by about 2%, which is, as mentioned above, within the experimental error and the mean of these values, namely, 4·805 differs by 1·5% from the mean (4·875) of the constants for prism III. Again, the absolute value of the mean value for $(2q_{11} - q_{12} - q_{13})$ from this prism is smaller by 4% than the corresponding value from prism IV.

(5) The first two rows under prism IV, show the consistency with which the constants are obtained for two independent observations, the first row being obtained when the distribution of stress is more non-uniform than in the case of the second row.

In view of the above, it can be stated with confidence, that the difference between the constants $(q_{11} - q_{12})$ and $(q_{11} - q_{13})$ in this case is about 12% and quite real. Hence the description of the photo-elastic behaviour of potassium alum needs four independent constants and not three because $q_{12} \neq q_{13}$.

The consistent differences between the values for prisms II and III and the corresponding values for prisms IV and V, have to be attributed to the effect of the relative dimensions of the prisms. We can see that the conditions for de Saint-Venant's principle to apply are better satisfied in the case of prisms II and IV than in prisms III and V. Hence, the values from prisms II and IV are adopted as the better approximations to the actual values. (This principle of de Saint-Venant states that, at a sufficient distance from the point of application of a load, the stress system depends only upon the statical resultant of this load and not upon its particular mode of application.)

Giving a weightage of 2 to the mean value of $(2q_{11}-q_{12}-q_{13})$ from prism IV and unit weight to the values of $(q_{11}-q_{12})$ and $(q_{11}-q_{13})$ from prism II, the most probable values of $(q_{11}-q_{12})$ and $(q_{11}-q_{13})$ are deduced by forming normal equations from the above observational equations. They are $(q_{11}-q_{12})=-5\cdot32\times10^{-5}$ and $(q_{11}-q_{13})=-4\cdot73\times10^{-5}$. It is from these values, that $(p_{11}-p_{12})$ and $(p_{11}-p_{13})$ are calculated in a later para. The values from prisms III and V will be 4% smaller but they also point out, as mentioned above, that the stress-optical constants are four and not three. The mean of q_{44} is $-0\cdot64\times10^{-5}$.

(b) Determination of the absolute stress-optical coefficients q_{11}, q_{12} *and* q_{13}.

For this purpose, prism II is used. The side which was 0.294 cm. long was reduced to 0·277 cm. and a set of distorted elliptical fringes were obtained in that direction. The other dimension 0·293 was reduced to 0·289. The crystal was acting like a double convex lens of a very large focal length in the direction of observation. Two suitable marks near the middle of the vertical height were chosen and the loads for a shift of a complete fringe at each of those marks, for light vibrating vertically and horizontally are noted. The spacing of the fringes is actually about 1/5 mm. and the dark bands themselves were, for well-known reasons, not very sharp. On this account, a change of about 50 gm. in a load of 800 gms. will just escape notice. With these limitations, at both the points, the loads for a shift of one complete fringe for vertically and horizontally vibrating beams of light, are 800 gm. and 600 gm. respectively, when the mechanical advantage of loading is 3·793. Any slight non-uniformity of stress existing escapes notice. The shift was away from the centre of the fringe system and hence, at the point of observation, the order of interference was increasing.

The absolute values of the constants are calculated from the equations

$$n - n_z = \frac{\lambda}{2t_2}\left[\frac{2n}{\lambda}(t_2 - t_1) - \delta n\right] \text{ and}$$

$$q_{11} = (n - n_z)\frac{2}{n^3 P_{zz}}.$$

Here, n is the refractive index of the undeformed crystal, n_z the refractive index in the stressed crystal for light with vibration direction vertical, $(t_2 - t_1)$ the increase in thickness of the crystal along the direction of observation for the corresponding load, λ is the wavelength of the light, δn is the increase in the order of interference at the point of observation and P_{zz} is the stress producing the shift. Here, δn is $+1$ and n, the refractive index is taken as 1·456 (Landolt Bornstein Tables, Zweiter Erganzungsband, Zweiter Teil, p. 917) for sodium light. Similar equations which hold for light vibrating horizontally are

$$n - n_x = \frac{\lambda}{2t_3}\left[\frac{2n}{\lambda}(t_3 - t_1) - \delta n\right] \text{ and}$$

$$q_{13} = (n - n_x)\frac{2}{n^3 P_{zz}}.$$

The value of P_{zz} here is different from the P_{zz} given earlier.

The elastic constants of potassium alum have been determined by Voigt (1919) and recently in this laboratory by Sundara Rao (1947). The necessary constants are given in Table V.

TABLE V

Unit	10^{11} dynes/cm.^{2}			10^{-12} cm.^{2}/dyne	–
Constant	C_{11}	C_{12}	C_{44}	S_{11}	σ
Voigt ..	2·43	1·009	0·843	5·441	0·2933
Sundara Rao ..	2·56	1·07	0·86	5·182	0·2947

The mean of these two sets has been adopted in this paper, after converting them into units of kg./mm.^{2}, for determining the change in thickness of the prism, as well as for evaluating the strain-optical coefficients.

Employing the data given above, we get

$$q_{11} = +3{\cdot}0 \times 10^{-5} \text{ and } q_{13} = +8{\cdot}9 \times 10^{-5},$$

giving a difference $(q_{11} - q_{13})$ of $-5{\cdot}9 \times 10^{-5}$ against the most probable value of $-4{\cdot}73 \times 10^{-5}$ given earlier. It must be pointed out that a value of 750 gm., for example, instead of the 800 gm. for the load used, brings q_{11} to $+4\cdot1\times10^{-5}$ from $3\cdot0\times10^{-5}$. Hence, in view of the facts stated earlier about the fringe displacements, the values of q_{11} and q_{13} can be regarded as giving the order of magnitude only. Provisionally the values may be taken as

$$q_{11}=3\cdot6\times10^{-5},\ q_{13}=8\cdot3\times10^{-5} \text{ and } q_{12}=8\cdot9\times10^{-5}$$

as obtained by forming normal equations after giving any large weight for the Babinet observations compared to the observations on absolute values.

(*c*) *Evaluation of the strain-optical coefficients.*

The four strain-optical coefficients are calculated from the equations

$$p_{11}=q_{11}c_{11}+(q_{12}+q_{13})\,c_{12};\qquad p_{44}=q_{44}c_{44};$$

$$p_{12}=q_{12}c_{11}+(q_{11}+q_{13})\,c_{12};\qquad (p_{11}-p_{12}=(q_{11}-q_{12})\,(c_{11}-c_{12});$$

$$p_{13}=q_{13}c_{11}+(q_{11}+q_{12})\,c_{12};\qquad (p_{11}-p_{13})=(q_{11}-q_{13})\,(c_{11}-c_{12});$$

adopting the mean values

$$c_{11}=2\cdot55\times10^{3}\ \text{kg./mm.}^{2},\ c_{12}=1\cdot06\times10^{3}\ \text{kg./mm.}^{2}$$

and $c_{44}=0\cdot866\times10^{3}$ kg./mm.^{2}.

The values are

$$(p_{11}-p_{12})=\ -0\cdot0792\qquad p_{11}=+0\cdot27$$

$$(p_{11}-p_{13})=\ -0\cdot0704\qquad p_{12}=+0\cdot35$$

$$p_{44}=\ -0\cdot0056\qquad p_{13}=+0\cdot34$$

6. Discussion of the Results

The values of Pockels may be compared with the above values. His q_1 which is $(q_{11}-q_{12})$ in our notation, is given as $-4\cdot30$ (Pockels, 1906; Szivessy, 1929). As mentioned above, the original paper of Pockels was not available to the authors, but from his other papers on rocksalt and fluorite, we infer that it may be the mean of the two constants $(q_{11}-q_{12})$ and $(q_{11}-q_{13})$ in our notation. The value is considerably smaller in magnitude than our value $-5\cdot02$. His value of q_{44} is $-0\cdot455$ to be compared with our $-0\cdot64$ which is also higher. The amount of double refraction produced when the crystal is compressed along a cube axis is about 8 times as large as that produced when it is compressed along the cube diagonal. Wertheim also has measured the double refraction in alum (Coker and Filon, 1931) earlier than Pockels but neither the direction of pressure nor that of observation with reference to the crystallographic axes was given.

It may also be noted that the absolute stress-optical constants q_{11}, q_{12} and q_{13} for alum are the largest known of all cubic crystals so far studied.

In conclusion, it may be pointed out that four independent constants are required for describing the photo-elastic behaviour of potassium alum which belongs to the class T_h, on account of the fact that in this class the cube axes are only digonal and not tetragonal. Apparently, it is this point that Pockels missed. Now, it can be stated as a general rule applicable to cubic crystals of all the 5 classes, that a pressure along any axis of trigonal or tetragonal symmetry makes the crystal optically uniaxial and a pressure along any diagonal axis, or a general direction, makes the crystal biaxial. This can easily be understood if it is remembered that the symmetry properties of the optical ellipsoid follow the symmetry of all the other physical properties, according to Neumann's principle.

7. Summary

To test an earlier prediction by Bhagavantam based on a group theoretical method, that the T and T_h classes of the cubic system need four independent constants instead of three, for describing their photo-elastic properties, crystals of potassium alum belonging to the T_h class are studied completely. It has been found that the crystal needs four independent constants and all the four stress-optical constants of alum are determined for the first time. For sodium D lines, they are $(q_{11}-q_{12}) = -5.32$, $(q_{11}-q_{13}) = -4\cdot73$, $q_{44} = -0\cdot64$, $q_{11} = +3\cdot6$, $q_{12} = +8\cdot9$, $q_{13} = +8\cdot3$, all in units of 10^{-5} when stress is expressed as kg./mm.^{2} and the path difference in cm. The values of the strain-optical coefficients are $(p_{11}-p_{12}) = -0\cdot0792$, $(p_{11}-p_{13}) = -0\cdot0704$, $p_{44} = -0\cdot0056$, $p_{11} = +0\cdot27$, $p_{12} = +0\cdot35$, $p_{13} = +0\cdot34$.

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