Fluid simulation studies of the dynamical behaviour of one dimensional relativistic electromagnetic solitons

Vikrant Saxena, Amita Das, Abhijit Sen, Predhiman Kaw
Institute for Plasma Research, Bhat, Gandhinagar - 382428, India

Abstract

A numerical fluid simulation investigation of the temporal evolution of a special class of traveling wave solutions of the one dimensional relativistic cold plasma model is reported. The solutions consist of coupled electromagnetic and plasma waves in a solitary pulse shape (*Phys. Rev. Lett.* **68**, *3172 (1992)*; *Phys. Plasmas* **9**, *1820 (2002)*). Issues pertaining to their stability, mutual collisional interactions and propagation in an inhomogeneous plasma medium are addressed. It is found that solitary pulses that consist of a single light peak trapped in a modulated density structure are long lived whereas structures with multiple peaks of trapped light develop an instability at the trailing edge. The interaction properties of two single peak structures show interesting dependencies on their relative amplitudes and propagation speeds and can be understood in terms of their propagation characteristics in an inhomogeneous plasma medium.

I. INTRODUCTION

The propagation and interaction of ultra-high intensity lasers ($I \geq 10^{18} W/cm^2$) in plasmas display a rich variety of nonlinear physics associated with the relativistic motion of the electron fluid in the electric fields of these lasers. An important early work in this area is the pioneering study by Akhiezer and Polovin [1] in 1956. They found exact nonlinear wave solutions for relativistically intense electromagnetic waves in a plasma that was modelled by coupled Maxwell and relativistic electron fluid equations. These early theoretical results obtained well before the discovery of lasers have now become very relevant due to the experimental realization of high intensity electric fields from modern powerful laser systems. This has promoted a resurgence of activities in this field. A variety of other nonlinear solutions, in the form of one dimensional nonlinear propagating solitonic structures have been obtained and discussed by various authors over the past several years [2, 3, 4, 5, 6, 7, 8]. One particular class of relativistic solitons, in the form of modulated light pulses coupled to plasma waves, has received special attention in the past decade in a number of analytic and simulation studies [3, 4, 5, 6, 7, 8] and continues to be of current interest [9, 10, 13]. These solitary pulses are exact traveling wave solutions of the relativistic hydrodynamic equations and provide a useful paradigm for the investigation of various phenomena associated with intense laser plasma interactions that occur in applications like laser fusion, plasma based particle accelerators, photon acceleration schemes etc. In a strictly mathemetical sense these are not true soliton solutions, except in some simplifying limits when they obey canonical integrable model equations like the nonlinear Schrodinger equation. From a practical point of view it is therefore relevant to investigate such dynamical issues as the accessibility of these solutions, the stability and lifetime of these pulses, their mutual collision properties, their propagation characteristics in an inhomogeneous medium etc. Many of these issues are open questions today and are difficult to address analytically. In this paper we address some of these issues with the help of numerical simulation studies of the full set of relativistic one dimensional cold electron fluid equations along with the Maxwell's equations for the electromagnetic field evolution. The question of accessibility of these solitons was partially addressed in [11, 12, 13] where the interaction of an externally imposed circularly polarized intense electromagnetic wave with a plasma was studied numerically by particle as well as fluid simulations; and some part of of the laser energy was found to be trapped in nonpropagating soliton like pulse structures. In our simulations we do not consider the vacuum plasma interface problem but confine ourselves to the dynamics of soliton solutions within the plasma. For this purpose we use the traveling wave solutions of [4] as initial conditions and investigate their time evolution under various conditions. In general we find that these solitary pulses have a fairly long life time in that they continue to propagate stably through the plasma medium over several plasma periods. Single hump structures are found to have longer life times than the multi-hump ones. The latter develop a distinct instability at the rear portion of the pulse and thereafter begin to radiate away. It is also observed that pure solitonic traits of preserving individual identities through a mutual collision process are displayed only by small amplitude solutions obtained in the simplifying weak density approximation. The general large amplitude solutions do not possess these solitonic traits. When these structures propagate in an inhomogeneous medium they gain (lose) speed while traveling down (up) the density gradient and can be reflected back if the plasma density is large enough. In our present studies we have ignored ion motions and thermal effects. The influence of these effects will be studied in a future work.

The paper is organized as follows. The governing evolution equations are presented in the next section. We also briefly recapitulate here the solitonic traveling wave solutions of the relativistic cold fluid model as discussed in [3, 4, 5, 7]. In section III we present results of our evolution studies for solitons propagating in a homogeneous plasma. We show that solitonic structures (both having single and multiple peaks of light waves) survive with no distortion for several plasma periods. The single hump solutions are extremely robust and have been observed to move undistorted for the entire simulation duration which was of the order of a few thousand plasma periods. The multiple peak solutions in contrast get distorted and develop an instability at their rear end after around $30-40$ plasma periods.

The interaction of two oppositely propagating structures are also considered in section III. In section IV we investigate the evolution of these solitonic structures in a plasma which is inhomogeneous i.e. for which the background plasma density varies spatially. Section V contains a summary and a discussion of our results.

II. GOVERNING EQUATIONS AND STATIONARY SOLUTIONS

The governing equations are the relativistic set of fluid evolution equations for a cold plasma in one dimension. We consider spatial variations to exist only along x, the direction of propagation, and consider the ions to be stationary. The relevant set of fluid and field equations are then,

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0. \tag{1}$$

$$(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x})(\gamma u) = \frac{\partial \phi}{\partial x} - \frac{1}{2\gamma}\frac{\partial A_\perp^2}{\partial x} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n - n_0(x) \tag{3}$$

$$\frac{\partial^2 \vec{A}_\perp}{\partial x^2} - \frac{\partial^2 \vec{A}_\perp}{\partial t^2} = \frac{n\vec{A}_\perp}{\gamma} \tag{4}$$

where (1) is the electron continuity equation, (2) is the parallel electron momentum equation, (3) is the Poisson's equation for the electrostatic potential ϕ, (4) is the wave equation for the vector potential $\vec{A}_\perp$ and other notations are standard. The perpendicular electron momentum equation has been integrated exactly to obtain the conservation of the transverse canonical momenta (sum of particle and the field momenta) as $u_\perp - \vec{A}_\perp/\gamma = 0$ and used to eliminate $u_\perp$ in the above equations. Here γ is the relativistic factor

$$\gamma = \sqrt{\frac{1 + A_\perp^2}{1 - u^2}}$$

In writing the above equations we have chosen to normalize the density by some appropriate density n_{00}. The length is normalized by the corresponding skin depth c/ω_{pe0} (where $\omega_{pe0} = \sqrt{4\pi n_{00} e^2/m_e}$) and time by the inverse of the plasma frequency ω_{pe0}^{-1}. The scalar and vector potentials are normalized by mc^2/e. In Poisson's equation $n_0(x)$ corresponds to the background ion density normalized by n_{00}.

The coupled set of nonlinear equations(1- 4) permit a variety of coherent solutions. A class of one dimensional propagating solutions with modulated envelope structure of the above set have been obtained by using the coordinate transformation $\xi = x - \beta t$ and $\tau = t$ (where β represents the group velocity of the structure). The vector potential is assumed to be circularly polarized and has a sinusoidal phase variation of the form $\vec{A} = (a(\xi)/2)[\{\hat{y} + i\hat{z}\}\exp(-i\lambda\tau) + c.c.]$. The plasma oscillations associated with the envelope structure are assumed to have no dependence on τ. The above transformations convert

Eqs.(1,2) into ordinary differential equations which can be integrated to give $n(\beta - u) = \beta$ and $\gamma(1 - \beta u) - \phi = 1$, where one assumes that at the boundaries $u = 0$, $\phi = 0$ and $n = 1$. One eliminates n to write Poisson's equation (Eq.(3)) as

$$\phi'' = \frac{u}{(\beta - u)} \tag{5}$$

Here $'$ denotes derivative with respect to ξ. Writing $a(\xi) = Rexp(i\theta)$, the wave equation (Eq.(4)) can be written as

$$R'' + \frac{R}{1-\beta^2}\left[\left(\lambda^2 - \frac{M^2}{R^4}\right)\frac{1}{1-\beta^2} - \frac{\beta}{\beta - u}\frac{1-\beta u}{1+\phi}\right] = 0 \tag{6}$$

Here $M = R^2[(1-\beta^2)\theta' - \lambda\beta]$ is a constant of integration and $R^2 = A_y^2 + A_z^2$. Eqs.(5,6) form a coupled set of second order differential equations in two fields ϕ and R respectively. The longitudinal velocity u appearing in the two equations can be expressed entirely in terms of R and ϕ as

$$u = \frac{\beta(1+R^2) - (1+\phi)[(1+\phi)^2 - (1-\beta^2)(1+R^2)]^{1/2}}{(1+\phi)^2 + \beta^2(1+R^2)} \tag{7}$$

Eqs.(5,6) have been solved by Kaw *et al.* [4] and others [7] for $M = 0$. The analytical solutions for the above equations were obtained only in the weak density response limit where the approximation $\phi \approx (1+R^2)^{1/2} - 1$ holds and the coupled set of Eqs.(5,6) can be reduced to a single equation in R. The amplitude of the solutions in this case are small and retaining terms upto the cubic order in (R) the reduced equation becomes the nonlinear Schrodinger equation, viz.

$$R'' - cR + dR^3 = 0 \tag{8}$$

where $c = (1-\beta^2-\lambda^2)/(1-\beta^2)^2$ and $d = [4\lambda^2 - (1-\beta^2)(3+\beta^2)]/[2(1-\beta^2)^3]$. Equation (8) can be integrated exactly to obtain a soliton solution of the sech form

$$R = R_m \mathrm{sech}\,(\sqrt{c}\xi) \tag{9}$$

where

$$R_m = \sqrt{\left(\frac{2c}{d}\right)} = 2\left[\frac{(1-\beta^2)(1-\beta^2-\lambda^2)}{4\lambda^2 - (1-\beta^2)(3+\beta^2)}\right]^{1/2} \tag{10}$$

The analytic form of the potential is then, $\phi \approx (1+R^2)^{1/2} - 1 \approx R^2/2 = (R_m^2/2)\mathrm{sech}^2(\sqrt{(c)}\xi)$. It is thus clear that for low amplitude ($R_m << 1$) solutions, ϕ is invariably much smaller than R.

In the absence of any such simplifying assumption the equations (Eqs.(5,6)) cannot be solved analytically. However, for the general case several varieties of numerical solutions have been obtained. The simplest variety is that where the modulated structure involves only a single peaked structure for both ϕ and R. Such a solution is displayed in the top subplot of Fig.1. Another class of solutions involves multiple peaks for the radiation field R trapped within a modulated envelope of scalar potential ϕ. For these structures the amplitude of scalar potential is very high compared to that of the radiation field R. We show one such solution in the bottom subplot of Fig.1 for the parameters $\lambda = 0.44466066$ and $\beta = 0.5$. These two classes of solutions have different spectral properties. The single hump solitons have been found to have a continous spectrum in the λ and β parameter space whereas the multi-hump solitons exist at discrete values of λ for a given value of β. The spectral domains of these solutions have been numerically obtained in [7] and we reproduce the $\lambda - \beta$ diagram of that paper as Fig.2 here. The parameter p refers to the number of extrema in the radiation field (for instance multihump structure in Fig.1 would correspond to $p = 9$). The uppermost curve in Fig.2 corresponds to the weak density response limit where the analytical sech solutions given by Eq.(9) hold. The shaded region shows the continuum values of λ and β for which general single hump ($p = 1$)solutions have been obtained. As one goes down in λ in the shaded region from the upper curve one moves away from the weak density response analytical solutions to the large amplitude numerical solutions of Fig.1 displayed in the upper subplot. The various lines in Fig.2 for $p > 1$ correspond to the discrete spectrum of multipeak solutions.

The set of equations (viz. Eq.(1- 4)) are numerically solved to study the time dependent problem. Equations (1) and (2) are evolved using the Flux Corrected Transport (FCT) algorithm [14]. The vector potential as per the solutions discussed above is chosen to be circularly polarized. The wave equation (Eq.(4)), which is a second order equation, is first written in terms of two coupled first order convective equations which are then evolved by the FCT [14] scheme. We choose periodic boundary conditions along x. The box length is taken large enough to ensure that the excitations do not re-enter from the other boundary due to periodicity.

In the next section we investigate the dynamical properties of the various solitonic structures identified in the spectral diagram of Fig.2 obtained by Poornkala *et al.* [7] in a homogeneous plasma background. For this we choose $n_0(x) = 1$. In section IV we consider an inhomogeneous background plasma density with a tanh density profile.

III. SOLITON EVOLUTION STUDIES IN A HOMOGENEOUS PLASMA

We first evolve the coupled set of equations Eq.(1 - 4) for an initial condition corresponding to the small amplitude analytic sech soliton solutions obtained by Kaw *et al.* [4] in the limit of weak density response. In this limit and under the assumption of stationarity in a frame moving with velocity β the coupled set of equations described in the last section, reduce to a cubic nonlinear Schrodinger equation which is known to admit exact soliton solutions. These solitonic structures are stable and due to the infinite number of invariants supported by the equations preserve their identity even under intense nonlinear interactions suffered during collisions with oppositely propagating solitons. These features have been reproduced in our simulations (see Fig.3) and serve to validate the correctness and accuracy of the code.

We next carry out simulations for those solutions for which no analytic expression is available and whose forms have been obtained numerically. These structures are the general solutions with no restrictive assumptions on the magnitude of the amplitude. As described in the last section, these general nonlinear solutions have been well categorized with respect to their eigen - spectrum, group speed and number of wave cycles of the vector potential structure in an earlier work [7]. However, several questions pertaining to their dynamical properties remain open for investigation. Specifically the issue of stability of these structures as well as the question of whether or not these structures possess solitonic traits of preserving individual identities in a collision process remain undetermined. To date there has been no theoretical investigation seeking an answer to these questions. Our approach in this paper has been to carry out a numerical study to shed light on this problem. We find clear evidence that these structures are stable for a very long time interval. However, the stability properties are different for the two classes of structures (viz. single and multiple peak structures). The stability seems to depend crucially on the number of peaks of the light wave field supported by the structure. The single peak solutions are very robust and have been observed to propagate undistorted for the entire duration of the simulation lasting for a few thousand plasma periods. However these robust single peak solutions do not preserve their identity under collisions - the characteristic feature displayed by pure solitons. This shows that these solutions are distinct from pure solitons and at best can be regarded as solitary wave solutions. However, as has been the practice in the literature on laser plasma interaction, we will continue to refer to these structures as solitons. The distinction with exact solitons will be made by refering to the value of their amplitudes (small for exact NLS solitons and large for solitary waves solutions) wherever necessary.

We first choose a particular single peak high amplitude nonlinear solution depicted in the plot of Fig.4 corresponding to the following values of the parameters viz. $\lambda = 0.92$ and $\beta = 0.05$. Clearly for these values of λ and β the solution lies in the shaded area of the $\lambda - \beta$ plane in Fig.2. For this solution the weak density approximation is not valid and hence it does not satisfy the cubic nonlinear Schrodinger equation (NLS). In Fig.4 we display the difference in the structure of the exact solution with the sech expression (Eq.8 of Kaw *et al.* [4]) obtained under the approximation of weak density response for the same values of parameters viz. $\lambda = 0.92$ and $\beta = 0.05$. It is clear from the figure that the difference between the two is quite significant and thereby it demonstrates the fact that these structures do not represent solutions in the weak density response regime.

We show in Fig.5 that this high amplitude solution propagates with a group velocity of $\beta = 0.05$ and survives for more than 1800 plasma periods, (we have gone even beyond this time in our simulations and have seen no perceivable distortion in its structure). This is a positive indication of the stability of this solution. The evolution of a number of other large amplitude numerically obtained single peak structures were investigated. They all propagate undistorted during the entire simulation time which ranges upto a few thousands of plasma periods.

We now investigate the stability of other variety of structures namely those having multiple light wave peaks. We show in Fig.6 the amplitudes of the fields ϕ and R and in Fig.7 the magnitude of the density at four different times in the evolution of one such multiple peak solution with $\lambda = 0.44466066$ and $\beta = 0.5$. The structure does propagate undistorted for several (almost 30) plasma periods. However, it is not stable as one can see from the plots at later times where an additional structure appears to emerge from the trailing end. Basically, the multi peak structures are not stable and emit radiation from their tail end. It should be noted here that the observed instability occuring at the tail end can not be interpreted as though a multiple peak solutions is trying to break up into stable single peak solutions. This is because the amplitude of the exact single peak solitonic solutions does not exceed the value of $R_m = \sqrt{3}$. (determined from the restriction of positivity of density for $\beta = 0$ [5], for finite β the amplitude is always less than this value, where it gets restricted by the lower curve of the shaded region in Fig.2).

The very fact that the single peak solutions are stable and the mutiple peak solutions are not could be the reason that in several PIC simulations of the propagation of intense relativistic laser beam in plasma one only sees the formation of single peak solitonic structures [15], and there are no reports on observations of multipeak structures.

The next issue of interest is to observe whether these solutions preserve their identity as and when they undergo collisions. We observe that the unstable multi peak structures cannot withstand any collisional interaction. Our numerical studies of mutual collisions between these solitons show that following a collision there remains no semblance to localized structures of any kind. Hence for the study of collisional interaction we present results obtained for the robust single peak solutions only.

To investigate this we first choose two oppositely propagating large amplitude identical structures i.e. both having same $\lambda = 0.92$, but the group speed $\beta = 0.05$ for one and $\beta = -0.05$ for the other. In Fig.8 we depict the process of collisions between these two oppositely propagating structures. We observe that after the collision (see subplot for $t = 1800$) the emerging structures are identical to the incident ones. The only difference (which is more clear in the animated visualization) with Fig.3 (which depicts a collision between small amplitude solitons) is that in Fig.3 there occurs a time (e.g. $t = 1750$) of intense interaction where any semblance with the two impinging structure has dissappeared. The two structures basically spatially overlap. No such feature is observed in the collision depicted in Fig.8. As the structures approach each other, their group speeds are observed to diminish and seem to vanish once the structures touch each other. At all times, for this particular case of collision, amidst single peak, equal and large amplitude structures, the two structures continue to be identified separately. The structures then appear to reflect from each other. In this case, thus , even though the identity of the two structures after collision, seems to be preserved, however, the collisional interaction seems to be perceptibly different from that exhibited by pure solitons. However, we also observe that as the value of $\mid \beta \mid$ for the two oppositely propagating equal amplitude structures is increased the collision starts resembling that of low amplitude NLS solitons as can be seen from Fig.9, which shows the collisional interaction of a large amplitude solution with $\lambda = 0.9345$ and $\beta = 0.3$. This suggests that for low group speeds the collisional interaction seems to reflect the two structures, whereas at higher group speeds the two structures pass through each other undistorted. The two distinct varieties of collisions and their dependence of β and amplitude would be discussed in detail in the next section where we try to understand them by interpreting the collisions as the propagation of one soliton on the inhomogeneous density of the other soliton.

However, on the basis of these studies alone one cannot arrive at any general conclusion on whether these large amplitude structures have solitonic traits or not. It is quite likely that the symmetry of the two underlying structures may have inhibited any exchange of energy between the two colliding structures, due to which they might have retained their identities even after they encounter each other. To pin this down we also carried out a study of collision amidst two large but different amplitude structures; their parameters being (e.g. one with $\lambda = 0.92$, $\beta = 0.05$ and other with $\lambda = 0.87$ and $\beta = -0.01$). The study of collision between these structures has been illustrated in Fig.10. In this case we observe that the identity of the two colliding structures gets lost after the interaction. There is a clear exchange of energy amidst the two entities, as a result of which the emerging structures after collision comprises of two peaked patterns, propagating in opposite directions but having very different amplitudes than the original structures. In fact it appears as though the intensity of the larger amplitude structure has increased and that of the smaller one has diminished as they cross each other after colliding. We observe that the larger amplitude structure formed after collision remains almost static. These structures separately cannot be another set of solutions of the system from the same argument of maximum amplitude being restricted to $R_m = \sqrt{3}$ [5] which is clearly exceeded by the larger amplitude structure of Fig.10 at $t = 2600$. There is also a significant amount of radiation (more apparant from the density plots of Fig.10) which shows that not all the energy is localized in these structures.

¿From the above studies it becomes clear that the destabilization (either inherent as in the multipeak structure or aided by the collisional interaction amongst two dissimilar single peak solitonic structures) predominantly leads to formation of long lived slowly varying single peak structures which are either static or move with low propagation speeds. The amplitudes of these observed structures are invariably higher than the value demanded by the exact nonlinear solutions, hence they tend to radiate slowly. However, their survival for a considerably long time makes them an interesting set of transient structures.

IV. SOLITON EVOLUTION STUDIES IN AN INHOMOGENEOUS PLASMA

In most experimental situations the background plasma medium has a density inhomogeneity. It is therefore a matter of practical interest to investigate the propagation of these large amplitude structures in an inhomogeneous medium. Here too we concentrate on studying the evolution of single peak structures through the inhomogeneous medium as they exhibit excellent stability characteristics. The inhomogeneity of the background plasma has been chosen to be of a simple tangent hyperbolic form.

We find that as the single peak structure approaches a rarer plasma background its speed increases. On the other hand if the background plasma density increases the group speed is observed to decrease. We also observe that if the structures encounter a significant increase in the background plasma density, then the structures may even get reflected from a certain critical background density point. This has been shown in Fig.11 where we have plotted the amplitude R of the structure at various times with solid lines. The background plasma density has been taken to vary weakly with a tanh profile and the dashed lines in the figure represent the background plasma density through which the structures propagate. The five subplots in the left column show the reflection of a large amplitude solution. For comparison we have also shown in the subplots of the right column the reflection of a small amplitude solution (which is an exact NLS soliton). There is no qualitative difference in the propagation features of large amplitude solutions and the small amplitude solutions of the NLS soliton variety, through the inhomogeneous plasma media.

A detailed investigation of the dependence of critical background density n_{ref} at which the structures reflect on parameters such as β, λ and the amplitude R_m has been made. This dependence has been depicted in the plot of Fig.12. We observe that n_{ref} depends very weakly on R_m and λ but varies strongly with the group velocity β of the structures. In fact it can be seen from the plot of n_{ref} vs. β shown in Fig.13 that for any value of R_m (and/or λ) the points fit the curve $n_{ref} = 1/(1-\beta^2)$ closely. The dependence $n_{ref} = 1/(1-\beta^2)$ corresponds to the dependence of critical density for reflection on the group speed β of linear wave packets. Basically one has from the linear dispersion relation $\beta = (1-\omega_p^2/\omega^2)^{1/2} = (1-n_{00}/n_{ref})^{1/2}$ (here $n_{00} = 1$). The fact that this relationship is obeyed closely by these nonlinear structures is extremely interesting. It should be realised that the amplitude of these structures are high only in a small central region. Though the relativistic factor γ at the maximum intensity point for some of these solutions are around $1.3-1.4$ and the electron density gets evacuated from unity to around 0.8 to 0.9 causing the ratio to be as small as $n/\gamma \sim 0.6$ at the central region. Although this suggests an effective reduction of plasma frequency from unity to 0.7746, but such a reduction happens only at the core region of the structure. At the edges, in fact an accumulation and hence enhancement of n occurs and the intensity of the radiation field is also weak there. Clearly, if one employs the simplistic explanation of reflection being governed by local effective value of the plasma frequency, the edge of the soliton which is the first to encounter the inhomogeneous rise of density would reflect according to the linear relationship. This suggests that the nonlinear structure moves like a single coherent robust entity in the presence of weak inhomogeneity of infinite extent (e.g. such as the chosen tanh form) and instead of distorting or breaking merely follows the dynamics dictated by its edge which obeys the linear reflection relationship. It would be of interest to study whether the coherence of the nonlinear structure survives when it encounters a sudden significant rise in density for finite spatial region. In fact this question has been answered in our collision studies indirectly which we discuss below.

We first discuss the two distinct collisional behaviour observed in the context of equal amplitude solutions described in the last section in the light of above studies on reflection through inhomogeneous background plasma density. For $\beta = 0.05$ the critical density point for reflection would be $n_{ref} = 1.0025$ from the relationship $n_{ref} = 1/(1-\beta^2)$. For the collision amidst two low amplitude NLS soliton moving with this group speed depicted in Fig.3 one can see that the value of maximum density of the structures are around $n_{max} = 1.001 < 1.0025$. Thus the density perturbation due to one soliton is insufficient to reflect the other. This results in strong overlapping of the two structures after which they emerge again moving in opposite directions. On the other hand it can be seen from Fig.8, where the large amplitude soliton moving with $\beta = 0.05$ is depicted, the value of maximum density due to each of the structure is around $n_{max} = 1.0479 > 1.0025$. Thus the individual structures get reflected as they encounter the density inhomogeneity due to the other structure. In the previous section we had also seen (Fig.9) that collision amidst two large amplitude faster moving $\mid \beta \mid = 0.3$ solitons, show traits similar to the low amplitude NLS soliton. This can be explained as follows, here $n_{ref} = 1/(1-\beta^2) = 1.0989$, whereas the n_{max} of individual soliton (see $t = 0$) is much less than 1.01 and even during intense interaction the perturbed density takes up a maximum value of about 1.0721. Both these values are less than n_{ref}, hence in this case instead of reflection a strong overlapping occurs and the two structures seem to pass through each other.

In these solitonic structures the electron density accumulation takes place only at the edge region (See Fig.1). The central region in fact gets evacuated and has lesser density. Thus the collision between two solitonic structures in some sense mimics the propagation of one structure through a localized density inhomogeneity. We take a relook at some of the collisions studied in the previous section with this perspective and try to identify differences if any in the propagation through these two distinct classes of inhomogeneities (viz. sudden localized density inhomogeneity vs. slowly increasing inhomogeneity). The collision between unequal amplitude structures depicted in Fig.10 points at an interesting difference. The energy exchange occuring in this case may be viewed as the smaller of the two solitonic structures depicted in Fig.10 (coming from left) being unable to suffer a total reflection with its identity preserved (unlike what is observed of tanh inhomogeneity) as it encounters a localized density inhomogeneity of the larger amplitude soliton coming from the right. After collision a very small amplitude remnant structure observed in the right side is the part which gets transmitted through the localized density inhomogeneity. A complete parametrization of reflection and transmission properties of these solitonic structures through localized inhomogeneities is in progress and would be presented in a subsequent publication soon.

V. SUMMARY AND DISCUSSION

The present work deals with the study of dynamical properties of one dimensional relativistic solitons in the form of modulated light pulses coupled to plasma waves. Earlier studies have focussed primarily on obtaining various traveling wave solutions and their categorization in terms of their group speeds, frequency, and the form of solution (e.g. number of light wave peaks in the solution etc.). It was also shown earlier that the small amplitude solutions obtained under the approximation of weak density response were identical to the NLS solitons. However, nothing conclusive as regards the stability and possibility of exhibiting any kind of solitonic traits was known about the large amplitude solutions. We have used the combined fluid-Maxwell set of equations to evolve such large amplitude solutions in time to ascertain some of these properties.

Our simulations show that the large amplitude solutions in which there is only one peak of trapped light wave are very robust and survive for the entire (several thousands of plasma periods) simulation duration. The structures which have multiple peaks of light waves enclosed within the modulated structure are seen to survive in some cases only upto a few plasma periods (in the example shown it is upto 30 plasma periods). However, these structures are found to be susceptible to an instability which develops at their trailing edge. As a result of this instability these structures are extremely fragile.

The single peak large amplitude structures though extremely stable are in some sense distinct from true solitons. They do not preserve their identities upon collisions. Only under the special case when the oppositely propagating structures are identical that they preserve their identity after collisional interaction. In all other collisional encounters there is always an exchange of energy amidst the two colliding structures and also a significant transfer of energy into radiation. Hence these structures can at best be regarded as solitary waves and are not true solitons which are constrained by infinite number of conservation laws.

When the colliding structures are identical they display two distinct variety of interactions depending on the group speed β and the amplitude of the structures. In one case the structures merely reflect from each other (the overlapping is weak and in fact negligible), while in the other case the structures approach each other, overlap and seem to pass through each other. These features and their dependence on β as well as on their amplitude can be understood by realising that the solitons as they approach each other view the density enhancement at the edge of the other structure as an inhomogeneity.

The propagation of these structures through background inhomogeneous plasma density has also been studied. These studies show qualitative resemblance with the features displayed by exact NLS solitons as they pass through density inhomogeneity. For instance the group speed of these structures decreases (increases) as the background density increases (decreases). They also suffer total reflection from weakly varying density inhomogeneity as an exact NLS soliton would do. We have studied in detail parametric dependence of the critical density for reflection n_{ref}. The studies reveals interestingly a very weak dependence on the amplitude R_m and the eigen value λ of the solitonic structure. However, a strong dependence on β of the form $n_{ref} = 1/(1-\beta^2)$ has been observed. This dependence on group speed β is identical to what a linear wave packet does in the presence of density inhomogeneity. This is interesting as it shows that even though the structure is strongly nonlinear, there is negligible manifestation of this nonlinearity in the reflection property.

The quantitative dependence of the form $n_{ref} = 1/(1-\beta^2)$ helps in identifying the parameter regime for the two varieties of collisions occuring for equal amplitude solitonic structures described in the paragraph above. Reflection occurs whenever $1/(1-\beta^2) < n_{max}$ (here $\mid \beta \mid$ is the group speed and n_{max} is the maximum amplitude of density of each of the identical colliding structures), and a strong overlapping takes place when the inequality is reverse.

Our studies on collision between unequal amplitude structures also indicates that propagation through localized inhomogeneity may show distinctive features. A part of soliton can get transmitted through a localized inhomogeneity provided the width is small and amplitude is also appropriate. A detailed investigation is underway and would be presented elsewhere soon.

The present work has thus shown conclusively that while most of the evolution characteristics of single peak large amplitude structure resemble soliton solutions, they still do not qualify to be exact solitons in a strict mathematical sense. However, their robustness and their close resemblance to solitonic properties can make them quite useful in practical applications such as stable transfer of energy packets deep into laser fusion targets or coherent propagation in plasma based accleration devices.

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FIGURE CAPTIONS

FIG. 1. The spatial profile of R (solid lines), ϕ (dotted lines) and n (dashed lines) for single peak (upper subplot) and multiple peak (lower subplot) nonlinear travelling wave solutions for the coupled laser plasma system. The upper subplot is for $\lambda = 0.92$ and $\beta = 0.05$. The multiple peak structure corresponds to $\lambda = 0.44466066$ and $\beta = 0.5$.

FIG. 2. The $\lambda - \beta$ values for which single peak (shaded region) and multiple peaks (the lines with various p values; with p indicating the number of extrema in R) are possible.

FIG. 3. Shows the collisional interaction amongst two small amplitude single peak solitons. The parameters corresponding to the two structures are $\beta = \pm 0.05$ and $\lambda = 0.944$. The plot of R, ϕ and n representing vector potential, scalar potential and electron density respectively at three different times are shown. The $t = 0$ plot shows the initial condition, at $t = 1750$ the interaction is maximum and $t = 3600$ shows the configuration after collision. Here time and lengths are normalized to ω_p^{-1} and $c\omega_p^{-1}$ respectively.

FIG. 4. Comparison of high amplitude solution obtained numerically (dashed line) for $\beta = 0.05$ and $\lambda = 0.92$ with sech profile for these parameters (solid line) obtained from Eq.8 of Kaw *et al.* [4]. The difference clearly shows that for these solutions the approximation of weak density response is not valid.

FIG. 5. Shows the time evolution of a high amplitude single peak solutions having $\beta = 0.05$ and $\lambda = 0.92$. The solid, dashed and dot dashed plots represent the structure at $t = 0$, $t = 900$ and $t = 1800$ respectively. The figure clearly shows that these solutions are stable.

FIG. 6. Shows plots of R (solid line) and ϕ (dashed line) profiles for a multipeak structure at four different times, here $\beta = 0.5$ and $\lambda = 0.44466066$. The structure retains its identity upto about 30 plasma periods after which it starts shedding radiation from the tail end.

FIG. 7. The four subplots show the profile of the electron density corresponding to the data and subplots of Fig.6.

FIG. 8. Shows the results of collisional interaction amidst two oppositely propagating high amplitude single peak identical solutions. The parameters for the two are $\beta = \pm 0.05$ and $\lambda = 0.92$. The two structures seem to reflect.

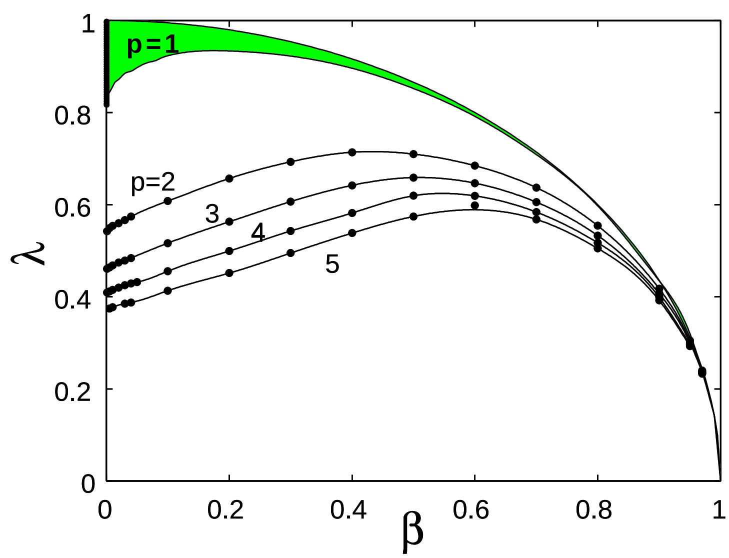
FIG. 9. Shows the results of collisional interaction amidst two oppositely propagating high amplitude single peak identical solutions. The parameters for the two are $\beta = \pm 0.3$ and $\lambda = 0.9345$. Here the group speed is much higher than that of Fig.8. The structures show strong overlapping and seem to pass through each other.

FIG. 10. Collisional interaction amidst two high amplitude single peak structures which are not identical. One (at $t = 0$ on left) has $\beta = 0.05$ and $\lambda = 0.92$ and other (at $t = 0$ on right) with $\beta = -0.01$ and $\lambda = 0.87$. It is clear (compare plots at $t = 0$ with $t = 2600$) that the structures do not retain their identity after collision.

FIG. 11. Shows the comparison of reflection from inhomogeneous density of the high amplitude single peak solutions (subplots in left column) with that of a small amplitude NLS soliton (subplots in right column).The background density has been illustrated by dashed lines and its values are depicted by the vertical axis on the right side of each subplots.

FIG. 12. Plot showing the parametric dependence of n_{ref} (the value of background density at which reflection occurs) on the soliton amplitude λ (upper subplot) and R_m (lower subplot) for various β. The plots suggests that the dependence on R_m and λ is weak. In the inset we have shown this weak dependence by magnifying the axis for $\beta = 0.1$.

FIG. 13. Variation of the critical background density n_{ref} at which reflection occurs with the group speed for the entire range of R_m and λ depicted in the data of Fig.12. The circles are obtained from simulation and the dashed curve shows the fit to $1/(1 - \beta^2)$.



p=1
p=2
3
4
5
λ
β
0
0.2
0.4
0.6
0.8
1