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Analysis of the *U B V* Light Curves of TT Hydrae by Kopal's Frequency Domain Method

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Abstract. The light curves of the totally eclipsing system TT Hya in *UBV* colours observed by Kulkarni and Abhyankar during 1973–77 have been analysed by Kopal's frequency domain method with slight modification. We find r_s (primary) = 0·104 ± 0·005, r_g (secondary) = 0·215 ± 0·008 and i = 89° ± 1°. The value of r_g obtained in this study is smaller than that determined earlier by Kulkarni and Abhyankar by the method of Russell and Merrill; this confirms the undersized nature of the secondary component. The ultraviolet colour excess of the secondary is also confirmed.

Key words: close binaries—absolute dimensions

1. Introduction

The Algol type eclipsing binary TT Hya shows a deep total eclipse and an almost inconspicuous secondary eclipse. From an analysis of the *UBV* light curves obtained at Japal Rangapur Observatory, Kulkarni and Abhyankar (1978, 1980) have derived the eclipse elements of the system by the Russell-Merrill method. On combining with Popper's radial–velocity data they (Kulkarni and Abhyankar 1981) have obtained the absolute dimensions of the components and a model of the system in which the secondary is found to be undersized and the ultraviolet (UV) excess during secondary eclipse is attributed to circumstellar matter around the primary. Since the evolutionary status of undersized secondaries is controversial (Hall 1974) we have re-analysed the same data by Kopal's frequency domain technique which gives the solution in closed form. The general method outlined by Kopal (1979) for occultation eclipses is modified to some extent for convenience of numerical calculations.

2. Evaluation of generalised moments

The first step is to evaluate the generalised moments $\bar{A}_{2m}$ defined by

$$\bar{A}_{2m} = \int_0^{\pi/2} \{l(\pi/2) - l(\psi)\}\, d(\sin^{2m}\psi), \tag{1}$$

m = 1, 2, 3 *etc.* As the light curve usually shows some asymmetry we reflect the observed points for negatives ψ's on to the positive side and obtain a symmetric curve for ψ=0° to 180°. If the observed (or normal) points are numerous the integrals of Equation (1) can be evaluated numerically by trapezoidal rule. But since the observations of Kulkarni and Abhyankar (1978) did not cover all the phases at quadrature the estimation of the generalised moments by this procedure becomes erroneous especially for higher order moments that depend largely on the intensities near ψ= 90°. We have, therefore, followed a slightly different procedure as described below.

The $\bar{A}_{2m}$ s actually consist of three parts:

$$\bar{A}_{2m} = A_{2m} + A_{2m}^{\text{prox}} + \mathfrak{P}_{2m} \tag{2}$$

where A_{2m} are the moments for spherical stars, A_{2m}^{prox} include the proximity effects such as reflection and rotational-tidal distortion, and $\mathfrak{P}_{2m}$ are photometric perturbations within eclipses due to differential limb-darkening and gravity brightening caused by the distortions of both components. At first we neglect the photometric perturbations and write

$$A'_{2m} = \bar{A}_{2m} - A_{2m}^{\text{prox}}, \tag{3}$$

where A'_{2m} are the rectified moments. Now representing the light outside eclipse by

$$l(\psi) = 1 + c_0 + \sum_{j=1}^{n} c_j \cos^j \psi, \tag{4}$$

we can evaluate the rectification coefficients c_j by

$$c_j = \int_{-a}^{+a} \{l(\pi/2) - l(\psi)\}\, \mathscr{P}_j^{(a,n)}(x)\, dx \tag{5}$$

where χ= cos ψ and a = cos ψ_e, ψ_e being the angle of external tangency; n is usually taken to be 4. The functions $\mathscr{P}_j$ will be discussed in the next section. Then, A_{2m}^{prox} are given by

$$A_{2m}^{\text{prox}} = -\int_0^{\pi/2} \left\{\sum_{j=1}^{n} c_j \cos^j \psi\right\} d(\sin^{2m}\psi), \tag{6}$$

and the rectified moments A'_{2m}, by Equation (3). However,as explained above, A'_{2m} obtained in this way would be inaccurate due to the errors in $\bar{A}_{2m}$ caused by the paucity of observed points near $\psi = 90°$. But since

$$l(\pi/2) = 1 + c_0 = l(\psi) - \sum_{j=1}^{n} c_j \cos^j \psi, \tag{7}$$

we can evaluate $l(\pi/2) = 1 + c_0$ by averaging the right-hand side of Equation (7) over all points outside the eclipse. We then have

$$A'_{2m} = \int_0^{\psi_e} \left\{1 + c_0 - l(\psi) + \sum_{j=1}^{n} c_j \cos^j \psi\right\} d(\sin^{2m} \psi) \tag{8}$$

because the integrand can be equated to zero for $\psi > \psi_e$. This amounts to rectifying the light curve before evaluating the moments. Thus, we are able to avoid the use of points near $\psi = 90°$ for evaluating the moments. Further, we also obtain $A_0 = L_1 = 1 + c_0 - \langle l(\text{totality})\rangle$

3. General expressions for $\mathcal{P}_j^{(a,n)}(x)$

Kopal (1979) has given expressions for $\mathcal{P}_j^{(a,4)}(x)$for three values of $\psi_e = 30°$, 45° and 60°. However, in order to enhance the accuracy of c_js it is desirable to keep $a = \cos \psi_e$ as a free parameter to be chosen appropriately for any particular case. Then we obtain the following expressions for $\mathcal{P}_j$'s:

$$\mathcal{P}_1^{(a,4)}\left(\frac{x}{a}\right) = \frac{15}{8}\left(\frac{7x^3}{a^5} - \frac{5x}{a^3}\right),$$

$$\mathcal{P}_2^{(a,4)}\left(\frac{x}{a}\right) = \frac{105}{32}\left(\frac{45x^4}{2a^7} - \frac{21x^2}{a^5} + \frac{5}{2a^3}\right),$$

$$\mathcal{P}_3^{(a,4)}\left(\frac{x}{a}\right) = -\frac{35}{8}\left(\frac{5x^3}{a^7} - \frac{3x}{a^5}\right),$$

$$\mathcal{P}_4^{(a,4)}\left(\frac{x}{a}\right) = -\frac{315}{64}\left(\frac{35x^4}{2a^9} - \frac{15x^2}{a^7} + \frac{3}{2a^5}\right). \tag{9}$$

In the case of TT Hya we have taken $\psi_e = 19°$ which is close to its value of 18·55 found by Kulkarni and Abhyankar (1980) as well as the final value obtained in this study. The c_j 's and the rectified moments for *UBV* light curves obtained from the normal points tabulated by Kulkarni (1979) are given in Table 1.

Table 1. c_j's and rectified moments for TT Hya.

Parameter	*V*	*B*	*U*
c_1	−0·0081985	−0·0073175	−0·0136457
c_2	−0·0212155	+0·0010891	+0·0365864
c_3	−0·0073379	−0·0065493	−0·0022387
c_4	−0·0510867	−0·0331143	−0·0482243
A_0	0·7131	0·8561	0·8952
A'_2	0·0325181	0·0395723	0·0418011
A'_4	0·0017927	0·0022144	0·0023926
A'_6	0·0001127	0·0001412	0·0001575
A'_8	0·0000078	0·0000098	0·0000114

4. Preliminary elements

Taking $A_{2m} = A'_{2m}$ we can derive the preliminary elements by the following standard method. From

$$A_2 = L_1 \bar{C}_3,$$

$$A_4 = L_1 (\bar{C}_3^2 + \bar{C}_2^2),$$

$$A_6 = L_1 (\bar{C}_3^3 + 3\,\bar{C}_2^2 \bar{C}_3 + \bar{C}_1 \bar{C}_2^2), \tag{10}$$

we get $\bar{C}_3$, $\bar{C}_2$ and $\bar{C}_1$, and from

$$A_8 = L_1 (\bar{C}_3^4 + 4\,\bar{C}_1 \bar{C}_2^2 \bar{C}_3 + 6\,\bar{C}_2^2 \bar{C}_3^2 + a\,\bar{C}_2^4 + b\,\bar{C}_1^2 \bar{C}_2^2),$$

where

$$a = \frac{30\,(3 - u_1)(35 - 19u_1)}{7(15 - 7u_1)^2}$$

and

$$b = \frac{7\,(15 - 7u_1)(315 - 187u_1)}{27\,(35 - 19u_1)}, \tag{11}$$

we can find u_1. The inaccuracy of A_8 did not allow us to determine a unique value for u_1 Hence we have adopted u_1 = 0·520, 0·680 and 0·420 for *V, B* and *U* respectively, as given by Kulkarni and Abhyankar (1980), Then

$$C_3 = \bar{C}_3,$$

$$C_2 = \sqrt{\frac{5\,(3 - u_1)}{(15 - 7u_1)}}\,\bar{C}_2,$$

$$C_1 = \frac{7\,(15 - 7u_1)}{3\,(35 - 19u_1)}\,\bar{C}_1, \tag{12}$$

give C_1, C_2, C_3 from which we get the elements by

$$r_{1,2}^2 = \frac{C_{1,2}^2}{(1 - C_3)\, C_2 + C_2^2}$$

and

$$\sin^2 i = \frac{C_1}{(1 - C_3)\, C_1 + C_2^2}. \tag{13}$$

The various constants and preliminary elements obtained in this way are given in Table 2.

5. Photometric perturbations and improvement of elements

The preliminary elements are derived by neglecting the photometric perturbations. We can now improve the elements by allowing for the photometric perturbations evaluated on the basis of the preliminary elements. The complete formulae are given by Livaniou (1977). They involve two additional quantities: the mass ratio m_2/m_1 for which we have taken a value of 0·269 found from Popper's radial velocity data, and the gravity-brightening coefficient τ_1 which is taken as unity. We then obtain

$$A_{2m} = A'_{2m} - \mathfrak{P}_{2m} \tag{14}$$

and follow the procedure of Section 4 to derive a set of improved elements. The iterative procedure can be repeated until convergence is achieved. We required 6 iterations for *V*, 11 for *B* and 9 for *U* light curves. The starting, intermediate and final values for various parameters are given in Tables 3, 4 and 5 for *V*, *B* and *U*, respectively. The final definitive elements obtained by averaging the results for the three colours are given in Table 6 which also gives for comparison the elements obtained by Kulkarni and Abhyankar.

Table 2. Preliminary elements for TT Hya.

Parameter	V	B	U
$\bar{C}_1$	0·0086911	0·0083840	0·0104844
$\bar{C}_2$	0·0208446	0·0212124	0·0221879
$\bar{C}_3$	0·0456010	0·0462239	0·0466947
u_1	0·520	0·680	0·420
C_1	0·0091709	0·0090725	0·0109190
C_2	0·0217779	0·0225771	0·0229476
C_3	0·0456010	0·0462239	0·0466947
r_1	0·0954735	0·0947791	0·1044142
r_2	0·2267179	0·2358583	0·2194387
i	85°·528	84°·301	87°·760

Table 3. Corrected moments, elements and photometric perturbations for *V* light curve of TT Hya.

	Initial value	Third iteration	Final (mean of two)
r_s	0·0954735	0·1014171	0·1008162
r_g	0·2267179	0·2135840	0·2148870
i	85°·5	90°·0	90°·0
$\mathfrak{P}_2$	−0·0010592	−0·0008109	−0·0008326
$\mathfrak{P}_4$	−0·0000849	−0·0000714	−0·0000742
$\mathfrak{P}_6$	−0·0000068	−0·0000061	−0·0000063
A_2	0·0335773	0·0333290	0·0333507
A_4	0·0018776	0·0018641	0·0018669
A_6	0·0001195	0·0001188	0·0001190

Table 4. Corrected moments, elements and photometric perturbations for *B* light curve of TT Hya.

	Initial value	Third iteration	Final (mean of two)
r_s	0·0947791	0·1001969	0·1002206
r_g	0·2358583	0·2218484	0·2232298
i	84°·3	87°·8	87°·3
$\mathfrak{P}_2$	−0·0015010	−0·0011347	−0·0011688
$\mathfrak{P}_4$	−0·0001170	−0·0001038	−0·0001057
$\mathfrak{P}_6$	−0·0000088	−0·0000089	−0·0000091
A_2	0·0410733	0·0407070	0·0407411
A_4	0·0023314	0·0023183	0·0023201
A_6	0·0001500	0·0001501	0·0001503

Table 5. Corrected moments, elements and photometric perturbations for *U* light curve of TT Hya.

	Initial value	Fifth iteration	Final (mean of two)
r_s	0·1044142	0·1097163	0·1097275
r_g	0·2194387	0·2076176	0·2076721
i	87°·8	90°·0	90°·0
$\mathfrak{P}_2$	−0·0011813	−0·0009304	−0·0009316
$\mathfrak{P}_4$	−0·0001033	−0·0000777	−0·0000779
$\mathfrak{P}_6$	−0·0000090	−0·0000065	−0·0000066
A_2	0·0429824	0·0427315	0·0427327
A_4	0·0024958	0·0024703	0·0024705
A_6	0·0001665	0·0001640	0·0001641

Table 6. Final eclipse elements for TT Hya.

		Shapley (1927)	Kulkarni and Abhyankar (1981) (Russell and Merrill method)	Koul and Abhyankar (Kopal's frequency domain method)
r_s		0·072	0·0929	0·104 ± 0·005
r_g		0·240	0·2438	0·215 ± 0·008
i		82°·6	83°·64	89°·0 ± 1°
L_s	(*V*)		0·7682	0·7131
L_g	(*V*)		0·2318	0·2869
L_s	(*B*)		0·8858	0·8561
L_g	(*B*)		0·1142	0·1439
L_s	(*U*)		0·9100	0·8952
L_g	(*U*)		0·0900	0·1048

Table 7. Colours and spectral types for components of TT Hya.

	Kulkarni and Abhyankar (1981) (Russell and Merrill method)	Koul and Abhyankar (Kopal's frequency domain method)
(*V*) Primary	7·526	7·624
(*V*) Secondary	8·903	8·612
(*B*−*V*) Primary	+0·027	−0·023
(*B*−*V*) Secondary	+0·970	+0·925
(*U*−*B*) Primary	−0·019	−0·040
(*U*−*B*) Secondary	+0·283	+0·353
(Sp) Primary	A1 V	A0 V
(Sp) Secondary	K1 III	G8 III
UV colour excess of secondary	0·5–0·6 mag	0·35 mag

6. Conclusions

From Table 6 we note that the radius of the primary is 12 per cent larger and that of the secondary 12 per cent smaller as compared to the values obtained by Kulkarni and Abhyankar (1981). In particular the radius of the secondary has now turned out to be 0·215 ± 0·008 which is much smaller than the Roche lobe radius of 0·264 for m_2/m_1 = 0·269 according to Plavec and Kratochvil (1964). We thus confirm the undersized nature of the secondary in TT Hya.

In Table 7 we give the colours and spectral types of the two components obtained from the present study. The UV excess is present in secondary, but its value is reduced from 0·60 to 0·35 magnitude because the spectral type of that star is found to be G8 III instead of Kl III.

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