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Intensity and Polarization Line Profiles in a Semi-Infinite Rayleigh-Scattering Planetary Atmosphere. III. Variation of Polarization Profiles and the Stokes Parameter Q over the Disk

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Abstract. The variation of the polarization profiles, the Stokes parameters Q and *U*, and the angle defining the plane of polarization along the intensity equator and along the mirror meridian, on which $\mu = \mu_0$, in a Rayleigh-scattering atmosphere is studied. It is found that these variations are more complex than thought hitherto, particularly at large phase angles.

Key words: Rayleigh scattering – polarization profiles – planetary atmospheres.

1. Introduction

In Paper I (Bhatia & Abhyankar 1982), we had presented results for the variation of polarization line profiles with phase angle in the integrated light of the planet. While this approach, at times, enables us to compare theoretical results with observational values, some of the finer details are lost in such a computation. For example, we had mentioned that for zero phase angle there is an increase in polarization as we go from the line centre outwards at some points on the disk and a decrease at some other points while there is an increase in the integrated flux. Further, $U = 0$ in the integrated flux. Therefore, if we are to isolate and explain some of the effects of multiple scattering, computations at different points on the disk at various phase angles have to be performed. Although at present we do not have many observations of polarization profiles, it is hoped that improved instrumentation in the future will enable us to obtain not only the polarization profiles and the Stokes parameter Q, but also the plane of polarization, defined by the angle χ.

As explained, in Paper II (Bhatia & Abhyankar 1983), the Stokes parameters I_l^e, I_r^e

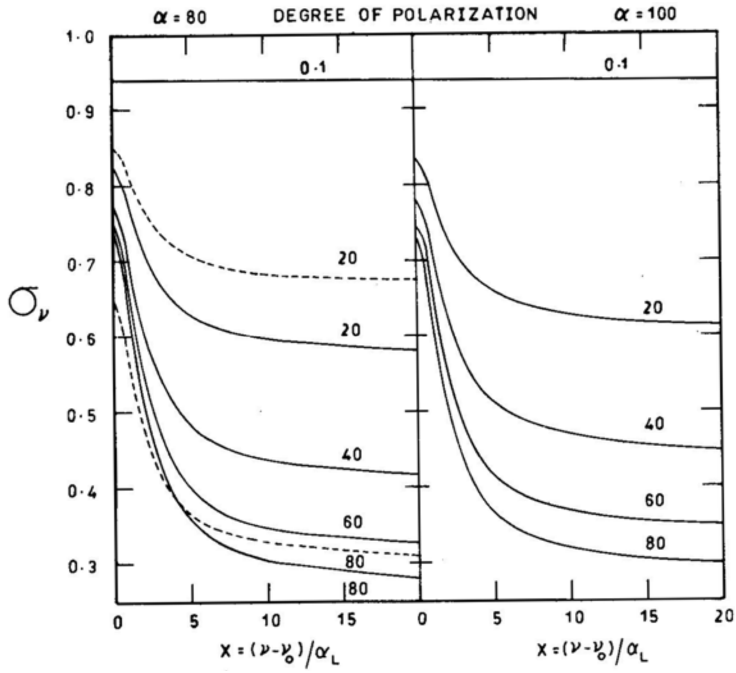

Figure 1. Polarization profiles for α = 80 and 100 at various colatitudes on the mirror meridian. Dashed curves (left) are for a longitude of 20 degrees. Numbers on the curves denote colatitude β along the mirror meridian.

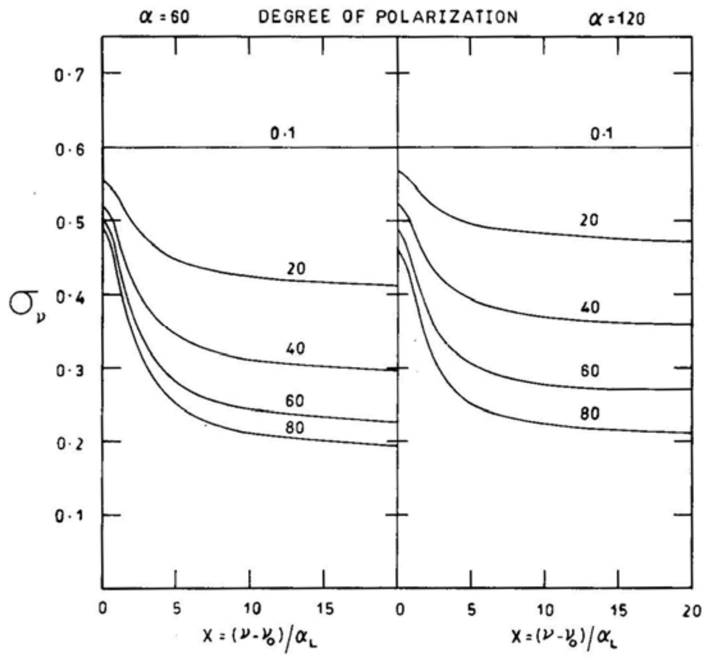

Figure 2. Polarization profiles for α = 60 and 120, at various colatitudes on the mirror meridian.

and U are calculated and tabulated by Bhatia (1982) for points along the intensity equator and along the mirror meridian, on which $\mu = \mu_0$, for various phase angles. They were used for calculating the total intensity $I = I_l^e + I_r^e$, $Q = I_l^e + I_r^e$ $I_P = (Q^2 + U^2)^{1/2}$, degree of polarization $\sigma = I_P/I$, and $\chi = ½ \tan (U/Q)$ measured from east .towards the respective poles. All polarization profiles were obtained for a line of intermediate strength (ϖ_0 = 0.6) which is fairly representative, and ϖ_c was taken to be 0.997. The results are displayed in Fig 1 to 8 and discussed below.

2. Variation along the mirror meridian

2.1 *Polarization Profiles*

2.1.1 *General remarks*

A polarization profile is the plot of degree of polarization σ, *i.e.*

$$I_p/I = (Q^2 + U^2)^{1/2}/(I_l^e + I_r^e)$$

against frequency. Whether this quantity shows an increase or decrease from the line centre outwards depends on whether the ratio I_P/I increases or decreases, *i.e.* whether the intensity of polarized light increases faster or slower than that of the total light. Further, since we expect a smooth variation of the total intensity with frequency, any details in the polarization profile can be ascribed to the behaviour of either Q or U or both.

At colatitude $\beta = 0.1$, we are looking at a point very near the pole at grazing incidence, as can be seen from the values of μ in Table 1 of Paper II. Consequently, at all albedos we see singly scattered light, the contributions of the higher orders of scattering being negligible. Now, for each of the Stokes parameters, $I_i (\varpi_2) = (\varpi_2/\omega_1)\, I_i\, (\varpi_1)$ for single scattering. In fact our calculations show that the ratio of the intensities for the Stokes components I_l^e , I_l^e and for the total intensity I for $\varpi = 0.997$ and for $\varpi = 0.1$ at β = 0.1 is almost 9.97, proving that the emergent photons have undergone essentially single scattering. Further, computations for $a = 90$ at $\beta = 0.1$ confirms this: for $\varpi = 0.1$, the degree of polarization is 99.98 per cent while for $\varpi = 0.997$ it is 99.56 per cent; in both cases the polarization is very close to 100 per cent as expected for single scattering at $a = 90$. Therefore, since $U \approx 0$, the degree of polarization can be written as $(I_l^e - I_r^e)/(I_l^e + I_r^e)$ which is evidently independent of albedo for single scattering. This is seen at all the phase angles (*cf.* Figs 1 to 4).

2.1.2 *Intermediate phase angles*

The polarization profiles for intermediate phase angles (α = 40 to 140) have been studied previously by Lenoble (1970) and Molenkamp (1974) for Rayleigh phase matrix and by Fymat (1974) for Rayleigh-Cabannes phase matrix for a few points on the disk. Since all of them showed a general decrease in polarization from the line centre outwards, further studies at other phase angles were considered unnecessary and that multiple scattering always decreased the degree of polarization became an accepted fact. As discussed below, our calculations confirm this trend for intermediate phase angles, but there are marked differences at extreme phase angles which begin to show up in some profiles for phase angles 40 and 140 also.

At phase angles 60 to 120 (Figs 1 and 2), the curves for $\beta = 0.1$ show the maximum polarization and do not exhibit any appreciable change along the line for reasons men-

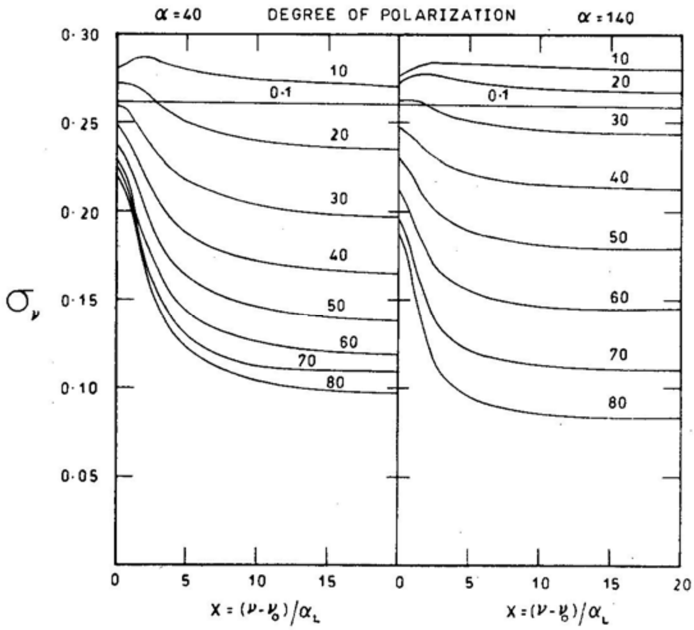

Figure 3. Polarization profiles for α = 40 and 140, at various colatitudes on the mirror meridian.

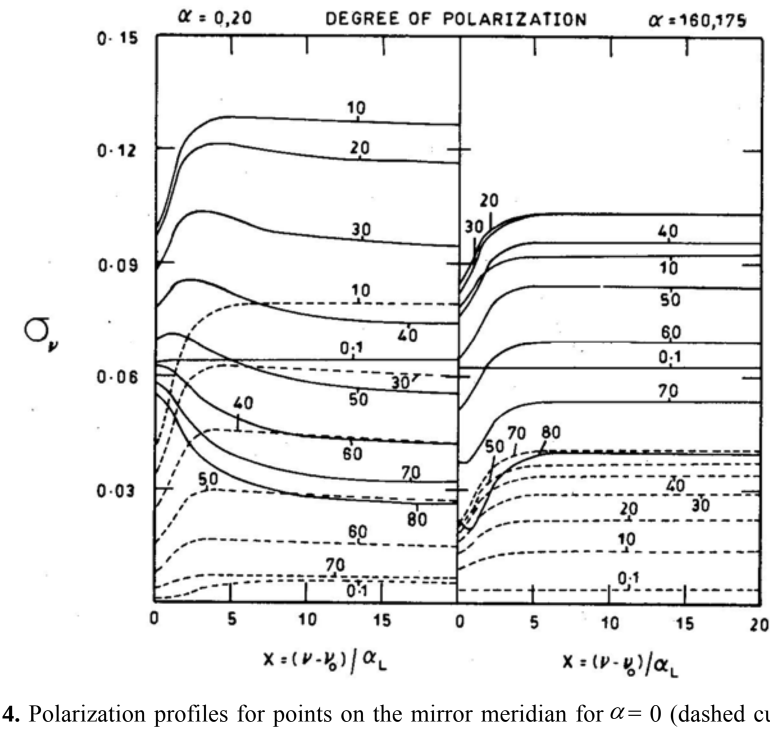

Figure 4. Polarization profiles for points on the mirror meridian for α = 0 (dashed curves) and α = 20 (solid curves) on the left, and for α = 160 (solid curves) and α = 175 (dashed curves) on the right. Numbers on the profiles indicate colatitude β. For α = 175, the polarization profiles for β = 60 and 80 are not shown, because they are very close to those for β = 50 and 70, respectively.

tioned earlier. At all other colatitudes the polarization decreases along the line due to increasing multiple scattering towards the wings. It also decreases with increasing colatitude, where again the effective number of scatterings is large. Also shown in Fig. 1 are the polarization profiles at α = 80 for longitude 20 at colatitudes 20 and 80 (dashed curves). At colatitude 20, the dashed curve is above that for longitudes 40 (mirror meridian) while at colatitude 80, the inner portion of the dashed curve is lower and the wings are higher.

At phase angles 40 and 140, the two effects mentioned in the preceding paragraph, *viz.* the decrease of polarization along the line and also with latitude, are valid in general (Fig. 3). However, two additional features are seen: (i) there is an initial increase in polarization from β = 0.1 to β = 10, and (ii) the profile for β = 10 for phase angle 40 and the profiles for β = 10 and 20 for phase angle 140 show an initial increase in polarization from the line centre outwards and then a decrease.

It will be seen from the above that there is some similarity between the variation of polarization along the line and along the meridian. At phase angles 60 to 120 the polarization decreases along the line, and if we confine ourselves to a particular albedo, it decreases with colatitude. In both cases there is an increase in the number of scatterings, in the first case because of the higher value of the albedo in the wings and in the second case because of the geometry. For phase angles 40 and 140 the behaviour of the polarization profile at say β = 10 is similar to that for the variation along the meridian *viz.* an initial increase followed by a decrease. The factor common to both these variations is the increase in the number of scatterings.

2.1.3 *Extreme phase angles*

Polarization profiles for phase angles 0, 20,160 and 175 are shown in Fig. 4. The solid lines refer to phase angles 20 and 160 and the dashed lines to 0 and 175. For phase angles 0 and 175 there is a general increase of polarization towards the wings at all colatitudes, which is also seen for most of the colatitudes for phase angle 160 and for some of the colatitudes for phase angle 20. We have mentioned in a preceding section that the degree of polarization will decrease or increase along the line depending on whether the intensity of polarized light increases faster or slower than that of the total light. Here we are seeing a general increase of the intensity of polarized light at a rate faster than that of the total light. In other words as opposed to the trend for intermediate phase angles, the degree of polarization increases with multiple scattering; we have already mentioned this in Paper I where we had noted that van de Hulst's (1980) calcu lations show that the polarization carried by the 4th order of scattering was the highest.

We shall now discuss each phase angle separately:

(1) For phase angle 0, there is a sharp increase in the degree of polarization between β = 0.1 and 10, the latter having the maximum polarization among all the colatitudes and also the maximum increase along the line. This indicates that at β = 10 there is a sharp increase in the intensity of polarized light compared to that of the total light which we can ascribe to multiple scattering. Further, there is a continuous decrease in polarization (at all points of the line) towards the equator although along the line there is always an increase of polarization with a slight decrease in the wings.

(2) For phase angle 175, although there is an increase in polarization at β = 10 compared with that at β = 0.1, it is not as sharp as that for phase angle 0; further, there is a

continuous increase in polarization all along the line as we go towards the equator, with none of the profiles showing a decrease in the wings as was the case for phase angle 0. However, it is to be noted that the profiles shown are for ϖ = 0.6. It is found that for albedo ω = 0.1, the polarization increases till β = 30 and then decreases towards the equator while for ϖ = 0.4 the decrease occurs at ,β = 85. Confining our discussion to ϖ = 0.1, the variation of polarization with colatitude shows a behaviour similar to that for the variation of polarization along the line at phase angle 0 where there was an initial increase in polarization followed by a decrease in the wings. Here again we see the similarity between the variation with albedo and with geometry, which we have already noted before.

(3) For phase angle 20, the polarization curves show an interesting phenomenon similar to what we have seen for some colatitudes for phase angles 40 and 140: leaving aside the curve for β = 0.1 which shows an almost constant polarization along the line for the reason mentioned earlier, we find that for β = 10 there is an increase in polarization with albedo with very little fall in the wings. As we go towards the equator, the region of the line over which the polarization increases with albedo shrinks, till at β = 60 we have the familiar curve showing a decrease in polarization all along the line. This happens for the following reason: at β = 10 the ratio of the total intensity for ϖ = 0.997 and = 0.1 is 2.3 while at β = 60 it is 4.8, showing that the total intensity increases considerably in the wings at β = 60 than at β = 10. This increase takes place gradually as we go towards the equator giving rise to the above phenomenon. Further, there is a significant increase in polarization between β = 0.1 and 10; calculations for β = 5 show that for this colatitude the polarization curve is nearer to that for β = 10 than for ,β = 0.1.

(4) For phase angle 160 there is a sharp increase at β = 10 compared with β = 0.1. For other colatitudes up to β = 60, there is an increase of polarization along the line with a very slight drop in the wings as was seen for phase angle 0. At β = 70 and 80 we first see a small decrease of polarization and then an increase, which places this trend in a new category. Till now we had three types of polarization profiles: the usual decrease of polarization along the line (Lenoble 1970, alsei see below and Paper I); an increase in polarization along the line with a decrease in the wings; and an increase in polarization along the line with no decrease in the wings. One fact worth keeping in mind is that in all the cases discussed above, the degree of polarization is small, the maximum being about 13 per cent at β = 10 for phase angle 20. One can, therefore, conclude that in genewithral the degree of polarization increases with multiple scattering when the degree of polarization carried by the initial orders of scattering is low and that it decreases if the initial polarization carried is high.

2.2 *Variation of* Q, *U and* χ

(i) Q = $I_l^e - I_r^e$ profiles are shown in Fig. 5 on the left for phase angles 0, 80 and 160 for colatitude 50; also shown is the profile for β = 80 for phase angle 160. In the first three cases | Q | increases with increasing albedo towards the wings. Among these, the | Q | values are highest for phase angle 80 and in fact we expect them to be maximum near a phase angle of 90, where the difference between the values of the phase function for I_l^e and I_r^e is the greatest. The increase of | Q | with albedo is evidently due to multiple scattering. Furthermore, it is seen that | Q | increases first with colatitude, reaches a maximum, and then shows a decrease. The colatitude at which this occurs varies with the albedo and also the phase angle. For example, for ϖ = 0.9, | Q | reaches a maximum value at colatitudes 20, 30, 40, 50, 40, 30, 30, 20 and 20 for phase angles 0, 20, 40, 60, 80,

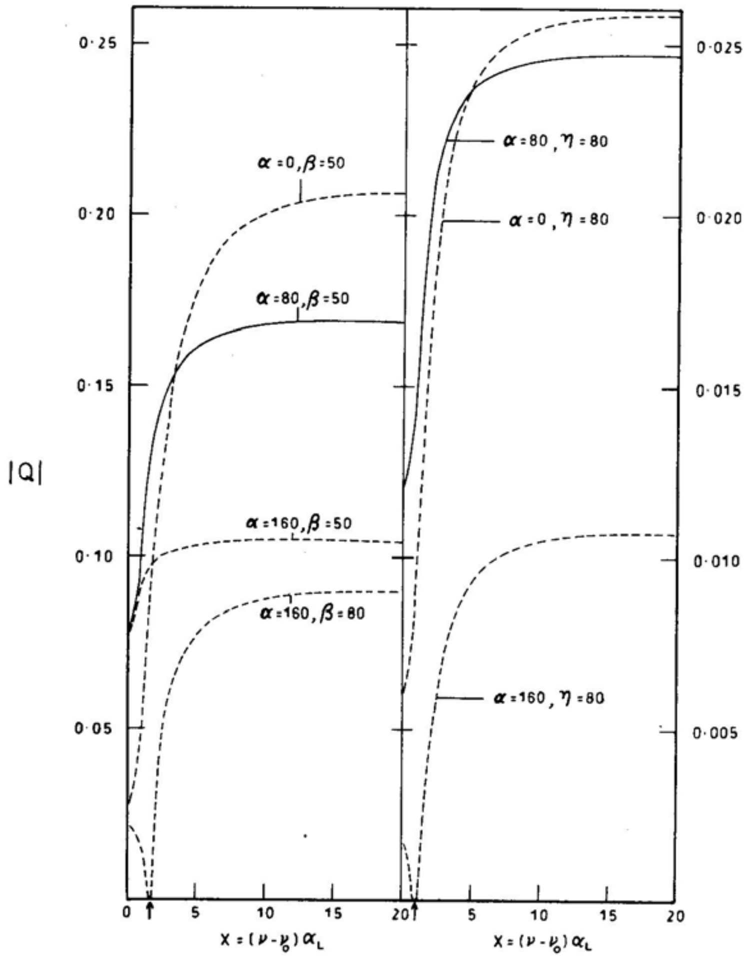

Figure 5. Q profiles for points on the mirror meridian on the left and along the intensity equator on the right. For dashed curves refer to the scale at right.

100, 120, 140, 160 and 175, respectively, while for ϖ = 0.1 the corresponding colatitudes are 10, 20, 30, 50, 85, 50, 30, 20, 20 and 20. Note that there appears to be a symmetry in the colatitude around a phase angle of about 90.

This variation of | Q | with colatitude is similar to that of the polarization with albedo for phase angle 0; Q I evidently first increases with multiple scattering because at β = 0.1 we have essentially single scattering, with the number of scatterings increaseing towards the equator. However, as the number of scatterings becomes very large, | Q | decreases again.

Q is negative and its absolute value increases with albedo at most of the colatitudes. One exception at β = 80 for phase angle 160 is shown in Fig. 5. Here, Q is negative near the line centre but its absolute value first decreases outwards, reaches zero (this point is indicated by an arrow) and becomes more and more positive before levelling off in the wings. A similar behaviour is seen in the polarization curve at the same colatitude (*cf.* Fig. 4) which is evidently due to this variation of | Q | .

An explanation for this behaviour of | Q | again requires calculations according to successive orders of scattering method. Qualitatively, one can say that the value of I_l^e is less than that of I_r^e initially; however, as the number of scatterings increase with increasing albedo, the rate at which I_l^e increases becomes greater than the rate for I_r^e, and eventually I_l^e becomes more than I_r^e. We are here dealing with small values of I_l^e and I_r^e and with grazing incidence, but at $\alpha = 0$ and 175, where the same holds, no such behaviour is seen.

(ii) The value of the Stokes parameter U for a particular colatitude and albedo is maximum at phase angle 100, decreasing towards large and small phase angles. Further, its variation with colatitude displays a behaviour similar to that for Q: its absolute value increases with colatitude, reaches a maximum and then decreases, the colatitude at which this occurs, again, being a function of the albedo and phase angle although now the colatitude at which this occurs for $\varpi = 0.9$ is 50 for phase angles 0 to 60, and 60 for other phase angles. Again, as for Q, U is negative at most of the phase angles, except at middle and higher colatitudes for phase angles 20 and 40.

(iii) The angle given by $\tan 2\chi = U/Q$, defines the plane of polarization. Evidently, this plane rotates with multiple scattering, usually through a small angle. The manner in which the plane rotates with albedo and the geometry of the point, *i.e.* multiple scattering, can be deduced from the value of χ. For example at all phase angles and colatitudes, this angle increases with increasing albedo. Further, since multiple scattering increases towards the equator, we expect the value of χ also to increase; however, similar to the behaviour of Q and U noted above, the value of χ generally increases from 90

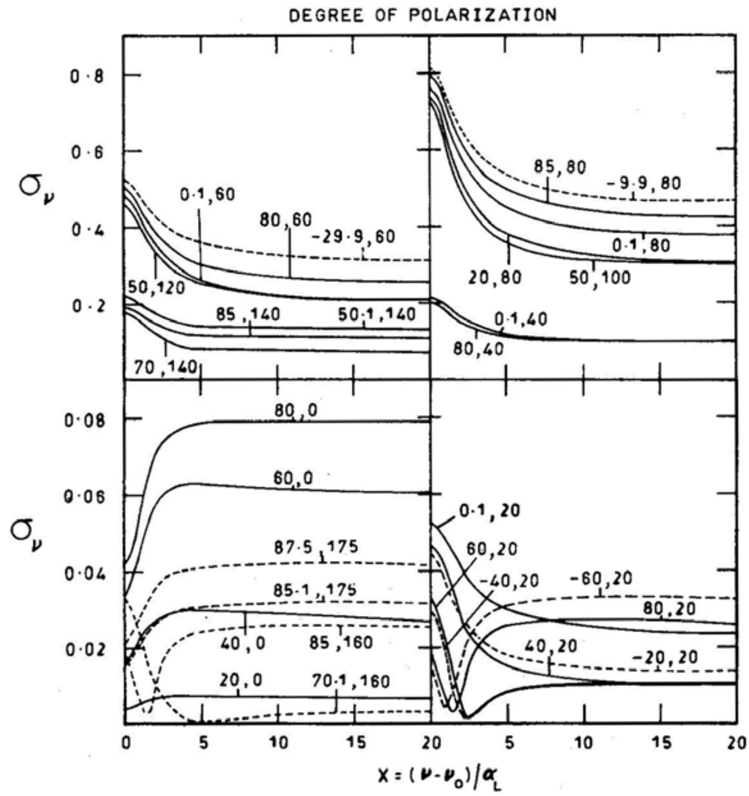

Figure 6. Polarization profiles for different longitudes (first number) and phase angles (second number) along the intensity equator.

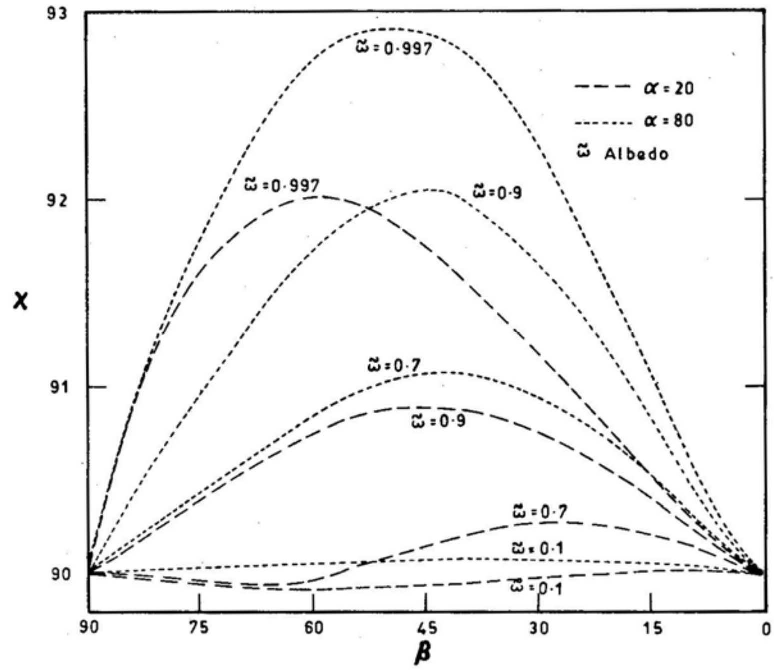

Figure 7. Variation, of χ with colatitude for α = 20 and 80.

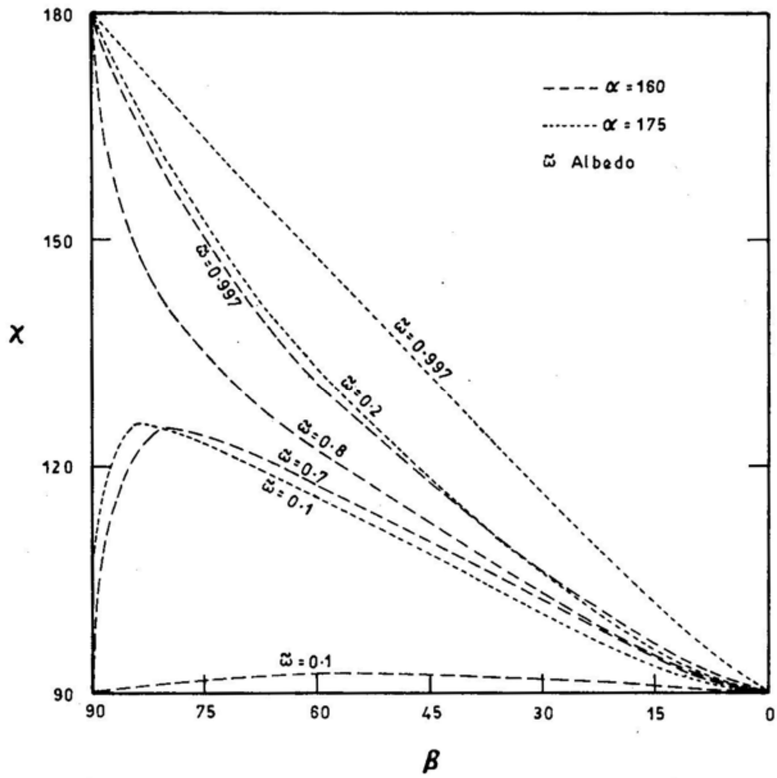

Figure 8. Variation of χ with colatitude for α= 160 and 175.

near the pole till a certain colatitude, which is a function of phase angle, and then decreases again towards the equator, reaching a value of 90. This is what we find for phase angles 60 to 140 for all values of the albedo, as illustrated in Fig. 7 for α = 80. The same thing is true for α = 160 when $\tilde{\omega} \lesssim 0.7$ and for α = 175 when $\omega \lesssim 0.1$; however, for higher values of the albedo, χ increases continously to 180 at the equator as can be seen from Fig. 8. In the other cases where there is an increase of χ followed by a decrease, the maximum value of χ increases with increasing phase angle and the colatitude at which χ reaches maximum also shifts towards the equator. Consequently, the drop of χ towards the equator also becomes sharper with increasing phase angle.

For phase angles 20 and 40 a complex behaviour is seen. After an initial increase up to about β = 10 for α = 20 (see Fig. 7) and up to about β = 30 for α = 40, for some values of the albedo, χ decreases, reaches a minimum below 90 and then its value increases again to 90.

3. Variation along the equator

3.1 *Polarization Profiles*

3.1.1 *Intermediate phase angles*

For phase angles 60 to 120 there is the usual decrease in polarization along the line (see Fig. 6). As for the variation towards the limb, for α = 60, a broad trend is seen: an initial decrease towards mid-longitudes followed by an increase thereafter towards the limb. However, for small values of the albedo, there is an initial increase, before the above-mentioned decrease sets in. Towards the terminator there is a continuous increase for all the albedos. For α = 80, there is a decrease till η = 30.0, followed by an increase right till the limb. There is an increase towards the terminator also. For α = 100, the decrease lasts till η = 50.0 after which there is an increase, while for α = 120 the decrease lasts till η = 60.0.

3.1.2 *Extreme phase angles*

These curves are also shown in Fig. 6. For phase angle 0, at all the longitudes, the degree of polarization increases as we go from the line centre outwards and decreases in the extreme wings, a fact noted and discussed before. Further, the degree of polarization increases towards the limb; this was seen to be the case in the variation along the mirror meridian except for β = 0.1 which is a special case anyway because it involves, essentially, single scattering only.

The situation for α = 20 is more complex. At longitudes η = 0.1, 20.1 (the curve for the latter is not shown separately because it is very close to the one for 0.1), 40 and 20, the degree of polarization decreases from the line centre outwards. At η = 60, however, the polarization first decreases quite sharply and then increases by a small but significant amount; an almost similar curve is seen for η = – 40. For η = 80 and – 60 the initial drop is smaller compared with the subsequent rise. Similar curves were seen for two colatitudes of 70 and 80 for phase angle 160 (*cf.* Fig. 4) although the dip there was not so pronounced.

The variation of the degree of polarization from the centre of the disk towards the limb is complex. For all the albedos, there is a slight increase of polarization between η = 0.1 and 10.0. Thereafter, for smaller values of the albedo there is a continuous small decrease towards the limb up to about η = 80.0, with a slight increase near η = 85.0. For larger values of the albedo, there is an initial decrease after η = 10 which continues up to η = 40, after which there is an increase till about η = 85.0 when there is a slight decrease again. For the intermediate albedos, after the initial increase at η = 10, there is a decrease followed by an increase at mid-longitudes, and final decrease near η = 85; however, for ϖ = 0.8, there is no increase near η – 85. A similar complex behaviour is seen towards the terminator: for smaller values of albedo, there is a continuous, small decrease with a slight increase near the terminator (η = – 69.9). For intermediate and large values of the albedo, there is an initial decrease followed by a rise. Here for ϖ = 0.8 a minimum value of .1E – 4 is reached near η = 50.0, after which there is an increase. Some of the features discussed here can be seen in Fig. 6. An explanation for all these features clearly requires computations of the polarization carried by the successive orders of scattering.

The curves for phase angle 40 are relatively uninteresting because they show the usual decrease of polarization along the line. As for the variation towards the limb, for small and intermediate values of the albedo, there is an initial small increase till about η = 30.0, thereafter a decrease till about η = 80.0 and a small increase toward the limb near η = 85.0. For higher values of the albedo, there is a continuous decrease till mid-longitudes after which there is an initial small decrease and then a rise for all values of the albedo. For α = 140, the curves show a decrease of polarization along the line and also with longitude up to about η = 80.0 when there is an increase.

For α = 160, the two curves for η = 70.1 and 85.0 are shown in Fig. 6. While at η = 70.1 there is a decrease in polarization towards the wing with a small increase in the extreme wings, at η = 85.0, there is an initial sharp decrease followed by a small but significant increase, akin to what we have seen for some curves for phase angle 20. This also underscores the fact that for smaller albedos the polarization decreases towards the limb while for higher albedos it increases. For phase angle 175 there is a straight-forward increase in polarization along the line with no decrease in the wings for longitudes 85.1 and 87.5. Further, there is an increase towards the limb at all albedos.

3.2 *Variation of* Q

Since U = 0 for all the cases, χ = 90 or 0. Consequently we shall discuss the variation of Q only for ϖ = 0.9. Here again we shall discuss the variation of | Q |.

| Q | profiles for three phase angles of 0, 80 and 160 for the same longitude are shown in Fig. 5 on the right. In two cases (α = 0 and 80) | Q | increases monotonically before levelling off in the wings, it being always positive for α = 0 and always negative for α = 80. However, for α = 160, Q is negative at first, its absolute values decreases, reaches zero and increases again before levelling off in the wings; the point at which it changes sign is shown by an arrow. A comparison with the polarization profile in Fig. 6 for the same longitude shows that the polarization profile has that shape because of this behaviour of Q.

For phase angle 0, Q is positive for all the longitudes, with the value increasing between η = 0.1 and η = 70.0; thereafter, there is a decrease towards the limb. For phase angle 20, Q is negative, shows an initial increase in the absolute value up to about η = 10 and decreases thereafter till about η = 70.0, where it changes its sign for some albedos. Thereafter, there is an increase till about η = 80.0, and then a slight decrease. As we go towards the terminator, Q decreases till about η = – 40.0, increases at η = – 50.0 and – 60.0 and again decreases sharply near the terminator.

For phase angle 40, Q is always negative, increases till η = 50, decreases at longitudes 60, 70 and 80, and increases again near η = 85.0; towards the terminator again there is a continuous decrease. Again at phase angles 80, 100, 120 and 140 there is a continuous decrease in the value of Q towards the limb. At phase angle 160, Q changes sign at η = 80.0 and increases towards the limb while for phase angle 175, Q is positive throughout and increases towards the limb.

The manner in which Q changes with geometry is indeed very complex. The explanation for the polarization profiles will clearly require the explanation for the behaviour of Q. This will require the calculations by the method of successive orders of scattering.

3.3 *Some Comments on the Variation of Polarization*

In all the cases considered so far, the variation along the equator has been much smaller compared with the variation along the mirror meridian. This is so because of the difference in geometries: in the latter case, both μ, μ_0 and $\varphi - \varphi_0$ considerably vary simultaneously, while in the former case only one of the angles reaches an extreme value at a particular point; further, the value of $\varphi - \varphi_0$ is either 0, or π along the equator, giving $U = 0$.

4. Conclusions

It will be clear from the above discussions that variation of polarization and other parameters like Q and χ are much more complex for Rayleigh scattering than thought hitherto. The simple picture of multiple scattering decreasing the degree of polarization will have to be modified to include more complex behaviour, especially at extreme phase angles. A fuller understanding of the phenomena will require calculations using the method of successive orders. A few results have already been reported by van de Hulst (1980). Further, the variation with albedo for Mie scattering remains to be seen. Lastly, as has been noted at a few places, there exists a correlation between the variation with colatitude and with albedo; both involve either an increase or decrease in the number of scatterings.

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