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HIDDEN SYMMETRIES OF M THEORY

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Abstract

A worldvolume action for membrane is considered to study the target space local symmetries. We introduce a set of generators of canonical transformations to exhibit the target space symmetries such as the general coordinate transformation and the gauge transformation of antisymmetric tensor field. Similar results are derived for type IIB string with manifestly S-duality-invariant worldsheet action.

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There has been considerable progress in our understanding of nonperturbative aspects of string theory [1,2]. It is recognized that extended objects, such as the p-branes and D-branes, have played a key role in these developments [3]. They appear as nonperturbative solutions of the low energy string effective action and they have been instrumental for our understanding of duality symmetry conjectures in string theory and in providing insights into string dynamics in various dimensions. It is now accepted that there are intimate connections amongst the five string theories and that there is an underlying fundamental theory, M-theory or F-theory, and that the five different string theories are manifestations of various phases of that theory [4,5]. It is also believed that the low energy effective action of the M-theory can be identified with that of $D = 11$ supergravity. There is mounting evidence that M-theory encompasses and unifies string theories and string dynamics in diverse dimensions. The low energy effective action of this theory contains the antisymmetric tensor, besides the graviton, in its bosonic sector. Therefore, the membrane that couples to the three index antisymmetric tensor field is expected to be the natural extended fundamental object [6] in eleven dimensions, with five-brane as its solitonic counterpart; consequently, a lot of attention has been focused on the study of the branes in $D = 11$ theory and their implications [7].

An interesting and important question, which begs an answer, is whether the fundamental (super)membrane can provide the degrees of freedoms of M-theory. It is not clear at this moment whether the quantum mechanical (super)membrane theory is a consistent one [8]. Thus, if the quantum theory is inconsistent, the answer to the above question is negative. We may mention that not too much is definitively known about the consistency or inconsistency of the membrane theory since, as is well known, this issue is very intimately connected with large the N behaviour of the $U(N)$ matrix model [9]. In the recent past the proposal of the M(atrix) model [10] has led to very interesting developments [11]. The M(atrix) theory reveals various salient features of the M-theory. According to the basic postulate of the M(atrix) theory, the dynamics of the eleven dimensional M-theory finds its description in the many body quantum mechanics of N $D0$-branes of the type IIA theory in the limit $N \to \infty$. The compactified M(atrix) theory has close connections with super Yang-Mills theories through dualities. Furthermore, it provides a theoretical basis to the understanding of the microscopic dynamics of M-theory and holds the promise of exploring various aspects of string theories nonperturbatively [12].

Recently, there have been attempts to study the supermembrane action in curved backgrounds with target space tensor fields [13] since, in spite of the above mentioned shortcomings, the membrane theory does provide an intricate relation with M-theory. In this context, there have been attempts to unravel how much the world volume theories know about the spacetime [14].

The purpose of the present investigation is to study properties of the bosonic membrane worldvolume theory and try to expose how the theory is encoded with target space local symmetries, which are associated with general coordinate transformation (CGT) and the gauge symmetries of the three index tensor field. We adopt a procedure similar to the one proposed by Veneziano [15], in the context of string theory, to derive gravitational Ward identities, and by Veneziano and the author [16,17] to derive Ward identities for various massless excitations of strings: both compactified and noncompactified [18]. To briefly recall the formalism, a Hamiltonian phase space framework is adopted and the Hamiltonian form of action is derived. Then, a set of generating functionals associated with the local (target space) symmetries of the theory are introduced. Next, it is shown that the variation of the action, under these canonical transformations, can be compensated by suitable (gauge) transformation of the massless backgrounds. Finally, it is argued that the phase space path integral measure remains invariant, at least classically; the desired Ward identities follow as immediate consequences. One of the fascinating, aspects of our works was that the canonical transformations implemented on the worldsheet action, indeed revealed the local symmetries of the theory in the target space. We have adopted a similar approach, and we will show that it is possible to introduce canonical transformations associated with general coordinate transformation and gauge transformation in the target space of the M-theory. Our results are to be understood as classical one in view of the preceding remarks regarding the quantum theory of membranes.

The bosonic membrane action in curved space in the presence of antisymmetric tensor field has the following form:

$$S_M = T\int d^3\xi \Big\{\sqrt{g} - \frac{1}{6}\epsilon^{ijk}A_{ijk}\Big\}, \tag{1}$$

where g_{ij} and A_{ijk} are pullbacks to the world volume of the spacetime metric and the antisymmetric tensor field of eleven dimensional supergravity; although we focus our attention on the bosonic theory we often refer to it as $D = 11$ supergravity and T stands for the constant tension. When writtten explicitly, with $g = \det\, g_{ij}$:

$$g_{ij} = \partial_i X^M \partial_j X^N G_{MN} \tag{2}$$

$$A_{ijk} = A_{MNP}\partial_i X^M \partial_j X^N \partial_k X^P. \tag{3}$$

Here, and everywhere, lower-case latin letters $i, j, k, ..$ etc., and upper-case letters $M, N, P, ..$etc., refer to the worldvolume and target space indices, respectively.

It is useful to deal with a different form of action, introduced by Bergshoeff, London and Townsend [19]. In this reformulation of the action, the tension of the membrane could be generated through the introduction of a worldvolume two-form potential.

$$S = \int d^3\xi \frac{1}{\lambda}\Big\{\det\, g + (*\mathcal{G})^2\Big\}, \tag{4}$$

where, λ is a Lagrangian multiplier field.

$$\mathcal{G}_{ijk} = \partial_i U_{jk} + \partial_k U_{ij} + \partial_j U_{ki} - A_{ijk} \tag{5}$$

U_{ij} is the worldvolume antisymmetric tensor field, which has no local degrees of freedom, g_{ij} and A_{ijk} are defined above, and the dual, $*\mathcal{G} = \frac{1}{3}\epsilon^{ijk}\mathcal{G}_{ijk}$.

The equation of motion for U, in form notation, is $\mathrm{d}(\frac{*\mathcal{G}}{\lambda}) = 0$, and one solves

$$*\mathcal{G} = T\lambda, \tag{6}$$

T being a constant, to be identified as the tension. Now, if one writes down the rest of the field equations and makes the above substitution (6), then the bosonic membrane equations of motion are recovered. If we want to substitute (6) into the action (4) and derive the field equations of bosonic membrane, then it is necessary to add a surface term to the action. Once the extra term is added, one gets the same result as substituting $*\mathcal{G}$ into the field equations derived from (4).

The canonical momenta P_S associated with the coordinates X^S are given by

$$P_S = \frac{1}{2\lambda}\frac{\partial g}{\partial \dot{X}^S} - \frac{*\mathcal{G}}{\lambda}A_{SMN}\partial_i X^M \partial_j X^N \epsilon^{0ij}. \tag{7}$$

We denote the worldvolume coordinates as τ, ξ and σ, and $\dot{X}^S$ denotes derivative with respect to τ. The derivatives with respect to the other two worldvolume coordinates will be denoted, sometimes, as $X^S{}_{,i}$ and $i = \xi, \sigma$ as an economy in notation. A lengthy, but straightforward computation reveals that the following two relations are satisfied:

$$P_S X^S{}_{,i} = 0 \tag{8}$$

and it is easy to see that these two are analogous to the constraint $P \cdot X' = 0$ in string theory (in that case prime was derivative with respect to σ). The canonical momentum for the world volume gauge field is

$$\mathcal{P}_{mn} = \frac{*\mathcal{G}\epsilon_{0mn}}{\lambda}. \tag{9}$$

Furthermore,

$$\mathcal{H} = (P_S + \mathcal{P}^{mn}\mathcal{A}_{Smn})^2 + \{(X,{}_{\xi}^{M})^2(X,{}_{\sigma}^{N})^2 - (X,{}_{\xi} . X,{}_{\sigma})^2\}(\mathcal{P}^{mn}\epsilon_{0mn})^2 = 0. \tag{10}$$

We identify this with the Hamiltonian density and $\mathcal{A}_{Smn} = A_{SMN}\partial_m X^M \partial_n X^N$. The tensor $\mathcal{P}_{mn}$ is antisymmetric and note that conjugate momentum of U_{0n}, $\mathcal{P}_{0n} = 0$, just as in the case of Maxwell electrodynamics, the conjugate momentum of A_0 vanishes. Then, if one adopts the constrained Hamiltonian approach, the Gauss law $\partial_i E^i = 0$ appears as a secondary constraint. For the case at hand, the corresponding procedure yields the constraint $\partial_m \mathcal{P}^{mn} = 0$ for the worldvolume index antisymmetric gauge field. The antisymmetry property of $\mathcal{P}_{mn}$, together with the constraint, implies that it is proportional to ϵ_{mn}.

Let us introduce a generator of infinitesimal canonical transformation, in order to expose that the theory is encoded with information on general coordinate invariance in target space

$$\Phi_G = \int d\xi d\sigma P_L \Lambda^L(X), \tag{11}$$

$\Lambda^L(X)$ being the infinitesimal parameter. The coordinate and conjugate momenta transform as follows under Φ_G (recall that variation of a function of phase space variables, $F(X,P)$, is $\delta F = [F, \Phi_G]_{PB}$):

$$\delta_\Phi X^S = \Lambda^S(X), \qquad \delta_\Phi P_L = -P_N \Lambda^N{}_{,L}(X). \tag{12}$$

Indeed, under Φ_G the coordinate X^S is shifted infinitesimally, as is the case under general coordinate transformation with $\Lambda^S(X)$ as the parameter. The metric and the tensor fields are functions of the coordinates $\{X^N\}$ and their variations are

$$\delta_\Phi G_{MN} = \frac{\delta G_{MN}}{\delta X^L}\delta_\Phi X^L, \quad \delta_\Phi A_{MNP} = \frac{\delta A_{MNP}}{\delta X^L}\delta_\Phi X^L, \tag{13}$$

due to the shift in X^S. The variation of the Hamiltonian action $S_H = \int d^3\xi\{P_S.\dot{X}^S - H\}$ can be carried out with the above transformation rules for the coordinates, canonical momenta and the backgrounds. Next, we consider the transformation of the backgrounds under general coordinate transformations (GCT) with the following rules (treating them as tensors in the target space):

$$\delta_{GCT} G_{ST} = -G_{SR} \Lambda^R{}_{,T} - G_{RT} \Lambda^R{}_{,S} - G_{ST,R} \Lambda^R \tag{14}$$

and

$$\delta_{GCT} A_{TUV} = -A_{RUV} \Lambda^R{}_{,T} - A_{TRV} \Lambda^R{}_{,U} - A_{TUR} \Lambda^R{}_{,V} - A_{TUV,R} \Lambda^R. \tag{15}$$

Finally, it can be shown that the following relation

$$\delta_\Phi S_H = -\delta_{GCT} S_H \tag{16}$$

hold for the Hamiltonian action. Thus if we, formally, define

$$\mathbf{Z}[G, A] = \int [\text{phase space}, ...] \ \exp(-iS_H). \tag{17}$$

Then we can argue that under the canonical transformation the phase space measure remains invariant (at least classically) and the variation of the Hamiltonian action under the canonical transformation Φ_G can be compensated by the general coordinate transformation of the backgrounds. Therefore, $\mathbf{Z}$ satisfies the following relation

$$\mathbf{Z}[G, A] = \mathbf{Z}[G + \delta_{GCT} G, A + \delta_{GCT} A] \tag{18}$$

leading to the equation

$$\int dx^M \left\{ \frac{\delta \mathbf{Z}}{\delta G_{NP}(x)} \delta_{GCT} G_{NP}(x) + \frac{\delta \mathbf{Z}}{\delta A_{NPQ}(x)} \delta_{GCT} A_{NPQ}(x) \right\} = 0. \tag{19}$$

Since the infinitesimal parameter $\Lambda^L(Y)$ is arbitrary, we can functionally differentiate the above equation with repect $\Lambda^L(Y)$ to arrive at

$$\begin{aligned} \int d^M x \Bigg\{ \frac{\delta \mathbf{Z}}{\delta G_{MN}(x)} & [G_{ML}(x) \partial_N \delta(x - Y) + G_{LN}(x) \partial_M \delta(x - Y) \\ + \partial_L G_{MN}(x) \delta(x - Y)] + \frac{\delta \mathbf{Z}}{\delta A_{MNP}(x)} & [A_{LNP} \partial_M \delta(x - Y) + A_{MLP} \partial_N \delta(x - Y) \\ & + A_{MNL}(x) \partial_P \delta(x - Y) + \partial_L AMNP \delta(x - Y)] \Bigg\} = 0. \end{aligned} \tag{20}$$

Thus we see that **Z** exhibits invariance under general coordinate transformations, a result similar to the property of the generating functional considered by us in the case of string theory by us [16,17].

A few remarks are in order at this stage. The action, S_H, appearing in eq.(17) is the Hamiltonian action and it contains the other pieces such as the ghost part (in case of covariant quantization), a piece taking into account the constraints, etc. Explicit checks show that taking constraints (8) into account does not change the relation (16). We are fully aware that constrained Hamiltonian [20] BRST quantization of higher dimensional extended objects, along the line one adopts for string theory, is difficult to accomplish because of various technical hudles that one encounters [21]. However, if the additional pieces in the action do not depend on the phase space variables $\{X^L, P_L\}$, then under the canonical transformation (12) which is induced by the generator Φ_G, the relation (16) will be satisfied since it is a functional of only $\{X, P\}$. However, it is not necessary to work in the frame work of covariant formulation to derive the Ward identities. One could make a suitable gauge choice, as discusses in [13] following the work of [22] and one will be able to discuss the local symmetries following the approach of Veneziano [15] as he obtained the Ward identities without resorting to BRST formalism. It is worth mentioning that in the Hamiltonian phase space approach the results are derived elegantly. We might repeat that our purpose here is to explore how much information about the local target space symmetries can be extracted from point of view of the worldvolume action (4) of the membrane.

In order to explore the gauge symmetry associated with the three index antisymmetric tensor field, let us introduce another generator for the canonical transformation:

$$\Gamma_A = \int d\xi d\sigma \mathcal{P}^{mn} \Psi_{AB}(X) \partial_m X^A \partial_n X^B \tag{21}$$

Here $m, n = \sigma, \xi$ components tensor. We find that $\delta_\Gamma X^A = 0$, and

$$\delta_\Gamma P_A = -\mathcal{P}^{mn} \Big[\partial_A \Psi_{BC} \partial_m X^B \partial_n X^C + \partial_C \Psi_{AB} \partial_m X^C \partial_n X^B + \partial_B \Psi_{CA} \partial_m X^C \partial_n X^B \Big] . \tag{22}$$

We can thus compute the variation of S_H from the above transformation rules. Note that $\delta_\Gamma G_{MN} = 0$ and $\delta_\Gamma A_{MNP} = 0$ since X^A's have vanishing shift under Γ_A.

The variation of the antisymmetric tensor field, under the local gauge transformation, is given by

$$\delta_{gauge} A(X)_{BCD} = \partial_B \Psi(X)_{CD} + \partial_D \Psi(X)_{BC} + \partial(X)_C \Psi_{DB}. \tag{23}$$

After some long, but straightforward calculations, we find that

$$\delta_\Gamma S_H = -\delta_{gauge} S_H \tag{24}$$

is satisfied. If we adopt arguments similar to the one for deriving the invariance properties of the generating functional under GCT, the corresponding relation is

$$\int d^M x \frac{\delta \mathbf{Z}}{\delta A(x)_{BCD}} \delta_{gauge} A(x)_{BCD} = 0. \tag{25}$$

Recall that $\delta_{gauge} A_{BCD}$, given by (23), involves the gauge parameters Ψ_{AB}. Therefore, if we functionally differentiate eq. (25) with respect to $\Psi_{AB}(Y)$, the final expression is

$$\int d^M x \frac{\delta \mathbf{Z}}{\delta A_{BCD}} \left[\partial_B \delta(x-Y) \delta_C^N \delta_D^P + \partial_D \delta(x-Y) \delta_B^N \delta_C^P + \partial_C \delta(x-Y) \delta_D^N \delta_B^P \right] = 0. \tag{26}$$

This is the gauge invariance property of the generating functional $\mathbf{Z}$ and a similar relation was derived in the context of string theory [16,17] to obtain the gauge Ward identities associated with the two form potential. A natural next step would have been to take a string functional derivative of eq. (20) and eq. (26) with respect to the background fields $G(Y_i)_{MN}$ and $A(Y_i)_{MNP}$. Notice that the right-hand sides of both these equations will still be zero. In the case of string theory, it was possible to test such Ward identities (modulo anomalies) for choice of simple background configurations. However, on this occasion, although we demonstrate the invariance properties of the generating functional, we are not in a position to carry out explicit computations. It is worth while to mention that NS-NS branes and D-branes, in the context of type IIA and IIB theories, appear as extended objects and it might be possible to check our results, for the corresponding antisymmetric fields, through explicit calculations. This might be achieved by adopting a more powerful technique as was utilized to explore gauge symmetries [23] associated with the excited levels of string states explored earlier [24]. We hope to report our results in these directions in the future.

We present below some results on type the IIB string action and study a few interesting properties of the action in the light of our investigations of the worldvolume action. This is intimately connected with the works of Townsend and of Cederwall and Townsend [25].

In order to construct a manifestly S-dual type IIB superstring action, let us recall that type IIB string is endowed with a pair of two form tensor fields coming from the NS-NS and R-R sectors. Therefore, a pair of worldsheet gauge potentials, denoted by V_i and $\tilde{V}_j$, are introduced, and the corresonding modified field strengths are

$$F_{ij} = \partial_i V_j - \partial_j V_i - B_{ij}, \quad \tilde{F}_{ij} = \partial_i \tilde{V}_j - \partial_j \tilde{V}_j - \tilde{B}_{ij}, \tag{27}$$

where $B_{ij} = B_{MN}\partial_i X^M \partial_j X^N$ and $\tilde{B}_{ij} = \tilde{B}_{MN}\partial_i X^M \partial_j X^N$ are the pullbacks of the two antisymmetric tensor fields coming from the NS-NS and the R-R sectors respectively. We note that scalar dilaton, ϕ, and pseudoscalar axion, χ, coming from the NS-NS and the R-R sectors, are also present in the massless spectrum of type IIB theory. The manifest $SL(2,Z)$ invariant action is

$$S = \frac{1}{2}\int d^2\xi \lambda \Big\{ g + e^{-\phi}(*F)^2 + e^{\phi}\left[*(\tilde{F} - \chi F)^2 \right] \Big\}, \tag{28}$$

where λ is again the Lagrange multiplier field, $*F = \epsilon^{ij}F_{ij}$ and $*\tilde{F} = \epsilon^{ij}\tilde{F}_{ij}$ are the worldsheet scalar densities, $g_{ij} = g_{MN}\partial_i X^M \partial_j X^N$ is the pullback of the 'Einstein frame' metric and $g = \det\, g_{ij}$. Notice that the dilaton and axion can be combined to form the $SL(2,R)$ matrix and B_{MN} and $\tilde{B}_{MN}$ identified as a doublet as follows:

$$\mathcal{S} \equiv \begin{pmatrix} \chi^2 e^{\phi} + e^{-\phi} & \chi e^{\phi} \\ \chi e^{\phi} & e^{\phi} \end{pmatrix}, \qquad \hat{\mathbf{B}}_{MN} \equiv \begin{pmatrix} B_{MN} \\ \tilde{B}_{MN} \end{pmatrix}. \tag{29}$$

Under $SL(2,Z)$, $\mathcal{S} \to \mathcal{U}\mathcal{S}\mathcal{U}^T$ and $\hat{\mathbf{B}} \to (\mathcal{U}^T)^{-1}\hat{\mathbf{B}}$, $\mathcal{U} \in SL(2,Z)$. The worldsheet gauge fields V_i and $\tilde{V}_j$ can be paired in the same way as the 2-form potentials and their transformation laws are required to be exactly the same as those of the two-form potentials to ensure $SL(2,Z)$ invariance of the action. The equations of motion of $\tilde{V}$ lead to the relation:

$$* \tilde{F} = e^{-\phi}\lambda T_s, \tag{30}$$

T_s is a constant and is identified to be the tension. It is easy to derive the Hamiltonian constraint

$$H = (P_M + \tilde{E}\tilde{B}_M + EB_M)^2 + (X')^2 \left[e^{\phi}(E + \chi\tilde{E})^2 + e^{-\phi}\tilde{E}^2\right] \tag{31}$$

and the one generating the σ reparametrization corresponds to $P_M X'^M$; here τ, σ denote the worldsheet coordinates; P_M is conjugate momentum of X^M; E and $\tilde{E}$, are momenta conjugate to σ components of potentials V and $\tilde{V}$ respectively and the prime denotes derivative with respect to σ. For brevity of notation,

$$B_M = X'^N B_{MN}, \qquad \tilde{B}_M = X'^N \tilde{B}_{MN}. \tag{32}$$

As is well known, in constraint analysis, we shall end up with the Gauss law conditions $\partial_\sigma E = 0$ and $\partial_\sigma \tilde{E} = 0$, leading to the conclusion that E and $\tilde{E}$ are independent of σ. Furthermore, the field equations for V_σ and $\tilde{V}_\sigma$ components imply that E and $\tilde{E}$ do not vary with time and these electric fields take integer values in appropriate units. Thus the relevant Hamiltonian action is

$$S_H = \int d^2\xi \left[\dot{X}^M P_M - \frac{1}{2}\lambda\left\{(P_M + m\tilde{B}_M + nB_M)^2 + X'^2\left[e^{\phi}(m + n\chi)^2 + e^{-\phi}n^2\right]\right\}\right]. \tag{33}$$

Here *dot* stands for τ derivative and it is understood that we also add the constraint $X' \cdot P$ with its multiplier to the above equation.

We want to demonstrate how the gauge invariances of this theory, associated with the pair of two-form potentials, appears in the approach we have been persuing. The generator associated with the NS-NS, B-field canonical transformation is

$$\Phi_B = E \int d\sigma \Psi_M(X) X'^M; \tag{34}$$

with the other one, it is

$$\Phi_{\tilde{B}} = \tilde{E} \int d\sigma \tilde{\Psi}_M(X) X'^M. \tag{35}$$

Note the appearance of constant electric fields (which are quantized) explicitly in the above formulae. As before, one can compute the variations of all the phase space variables under Φ_B or $\Phi_{\tilde{B}}$ and, if so desired, one can compute the variations when both the canonical transformations are implemented. Also note that the variation of the two backgrounds, under the Abelian gauge transformations, are

$$\delta_{gauge} B = \partial_M \Psi_N - \partial_N \Psi_M, \quad \delta_{gauge'} \tilde{B} = \partial_M \tilde{\Psi}_N - \partial_N \tilde{\Psi}_M. \tag{36}$$

Then, one arrives at the following result

$$\delta_{\Phi_B + \Phi_{\tilde{B}}} S_H = (\delta_{gauge} + \delta_{gauge'}) S_H, \tag{37}$$

which eventually leads to the Ward identities derived by us [16,17] in the context of bosonic strings (both compact and noncompact). Neeedless to say that, with a generating functional $\int d\sigma P_M \Lambda^M(X)$, we can derive the gravitational Ward identities in a straightforward manner.

It has been argued that the type IIB action also provides glimpses of the 12-dimensional world, once the coordinates are identified as $\mathbf{X} = (X^M, V_1, \tilde{V}_1)$, the corresponding canonical momenta as $\mathbf{P} = (P_M, E, \tilde{E})$, and then the Hamiltonian action could be written in a compact form. Although the constraints are not invariant under 12 dimensional Lorentz transformation, there is invariance under ten dimensional Lorentz transformation and the $SL(2,R)$. We would like to present an interesting form for the Hamiltonian, which is strikingly similar to the one derived by us for strings in the presence of backgrounds and there is a matrix which is very much like the **M**-matrix used in the context of $O(d,d)$ symmetry. The Hamiltonian can be written as

$$\mathcal{H} = \frac{1}{2}\mathcal{P}^T \mathcal{M} \mathcal{P} \tag{38}$$

in a compact matrix notation, with the following definitions:

$$\mathcal{P} = \begin{pmatrix} \mathbf{P}_M & \mathbf{X}'^M \end{pmatrix} \tag{39}$$

$$\mathcal{M} = \begin{pmatrix} g^{MN} & g^{MP}(EB_{PN} + \tilde{E}\tilde{B}_{PN}) \\ -(EB_{MP} + \tilde{E}\tilde{B}_{MP})g^{PN} & \mathcal{W}^2 g_{MN} + \left[2E\tilde{E}Bg^{-1}\tilde{B} + E^2 Bg^{-1}B + \tilde{E}^2\tilde{B}g^{-1}\tilde{B}\right]_{MN} \end{pmatrix} \tag{40}$$

and

$$\mathcal{W}^2 = e^{\phi}(E + \chi\tilde{E})^2 + e^{-\phi}\tilde{E}^2. \tag{41}$$

The structure of $\mathcal{W}$ is quite interesting: from the S-duality attributes of the type IIB theory, the combination of dilaton and axion tells us about the tension. If we go to the string frame with the definition $g_{string} = e^{\phi}$, tension is given by $T_s = \sqrt{\frac{n^2}{g^2_{string}} + (m + n\chi)^2}$, in agreement with the results of Schwarz [26]. On the other hand, when we envisage IIB theory from the F-theory perspective [27], ϕ and χ are the modular parameters of the torus and are geometrical objects from the point of view of a 12-dimensional theory. Furthermore, the pair of integers (m,n) have a simple interpretation as quantized momenta.

In summary, we considered the membrane action [19] in curved space in the presence of the antisymmetric tensor field. The action has a novel property that a worldvolume gauge field and a Lagrange multiplier field are introduced; when we solve the equations of motion of the gauge field on the world volume the tension appears (see eq. (6)). We introduced generators of canonical transformations associated with general coordinate transformations and the gauge transformation of the three index antisymmetric tensor field. Then using the techniques introduced by Veneziano and by Veneziano and me, the invariance properties of the generating function are derived. These results are to be envisaged as classical ones, since anomalies might afflict these results owing to quantum effects in some cases. Next, we considered a manifestly S-duality invariant action for type IIB theory and showed how the target space local symmetries can be exhibited by introducing generators of canonical transformation leading to invariance of the generating function. We point out how the Hamiltonian of the type IIB action, in the presence of the worldsheet gauge field resembles the well known M-matrix (which very often appears in the context of $O(d,d)$ symmetry) when we perceive the Hamiltonian from the F theory perspective. Our approach might be useful to provide further understanding of the properties of the worldvolume theory of D-branes.

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