ELSEVIER

Available online at www.sciencedirect.com

SCIENCE DIRECT®

Int. J. Miner. Process. 72 (2003) 417–427

INTERNATIONAL JOURNAL OF MINERAL PROCESSING

www.elsevier.com/locate/ijminpro

Balling and granulation kinetics revisited

P.C. Kapur*, V. Runkana

Tata Research Development and Design Centre, 54-B, Hadapsar Industrial Estate, Pune 411 013, India

Received 20 June 2002; received in revised form 10 September 2002; accepted 1 July 2003

Abstract

Balling of finely comminuted solids by random coalescence and granulation of iron ore fines and other minerals by autolayering are two major size enlargement processes.

The existing kinetic model for random coalescence does not take into account the strong dependence of coordination number on the size distribution of agglomerating entities. We present a coordination number based coalescence model, which mimics the underlying physical process more realistically. Simulations show that in spite of highly diverse model structures, random and coordination coalescence models give remarkably similar results.

Only static models of autolayering are available presently. These map the input size distribution of feed solids into steady state or terminal size distribution of granules, with little or no information on the path traversed by the process. We propose a continuous-time dynamic model of autolayering within the population balance framework. The model, which is based on the proportionate growth postulate of autolayering, agrees reasonably well with experimental data.

Keywords: balling and granulation; modeling of kinetics by coalescence and autolayering; population balance equations

1. Introduction

Based on carefully designed and demonstrably reproducible experiments on a laboratory scale balling drum, Kapur and Fuerstenau (1964) proposed a suite of elementary growth mechanisms for a class of size enlargement processes, known variously as agglomeration, balling, granulation or wet pelletization. These mechanisms are designated as random coalescence, nonrandom or preferential coalescence, crushing and layering, snowballing or layering, and abrasion transfer. Later, Kapur et al. (1993) employed the term autolayering for a special case of the layering mechanism, which plays a dominant role in granulation of iron ore fines. The overall growth of pellets in an agglomerating device takes place either by a single elementary growth mechanism or by coupling of two or more mechanisms. Moreover, the pattern of growth may switch from one mechanism to another as pellets grow in size along with compaction, that is, reduction in the porosity. In many instances, it is sufficient as well as convenient to assume that growth is primarily driven by a single elementary mechanism, at least over some defined region or regions of growth. Thus in the balling of finely comminuted solids, random coalescence dominates in the nuclei growth and the transition regions (Kapur and Fuerstenau, 1964, 1969), while nonrandom coalescence is the principal mechanism in

* Corresponding author. R-485 New Rajinder Nagar, New Delhi 110 060, India.

E-mail address: pck@vsnl.com (P.C. Kapur).

0301-7516/$ - see front matter © 2003 Elsevier B.V. All rights reserved.
doi:10.1016/S0301-7516(03)00116-9

the ball growth region (Kapur, 1972; Ouchiyama and Tanaka, 1974; Pulvermacher and Ruckenstein, 1975). Again, granulation of iron ore fines occurs essentially by coating of fines present in the feed onto the coarse size fractions which act as seeds or nuclei, that is, by the autolayering mechanism (Kapur et al., 1993).

Many attempts have been made to describe the kinetics of balling and granulation by various elementary growth mechanisms (Sastry and Fuerstenau, 1970, 1977; Kapur, 1971, 1972, 1978; Ouchiyama and Tanaka, 1982; Litster et al., 1986; Adetayo and Ennis, 1997). The particle population balance based kinetic models are useful for analyzing the agglomeration systems, especially the continuous industrial circuits. Moreover, the specific growth rate constant in these phenomenological models provides a uniform and consistent basis for comparing the "ballability" of a particulate material as a function of the agglomeration machine and the feed characteristics, such as fineness, moisture content, etc. (Kapur, 1978). In most industrial applications of balling and granulation, the mean pellet size and dispersion or spread in size are of utmost importance. The kinetic models track the evolution of pellet size spectrum and provide quantitative information on its statistical parameters. Finally, these models can be embedded in appropriate strategies for model-based, extended-horizon on-line control of the size enlargement processes.

In this communication, we propose to reexamine the existing kinetic models for two of the most important agglomeration processes, namely, balling by random coalescence and granulation by autolayering. The objective in first case is to update the mathematical model in order to mimic the underlying physical process in a more realistic manner than what has been possible hitherto. In the second case, a dynamic model is proposed in place of the currently available models, which are only static in nature.

2. Coalescence mechanism

2.1. Random coalescence model

The mathematical description of the kinetics of coalescence between pellets is somewhat similar to that of the coagulation phenomenon in colloidal suspensions. In latter case, the rate of coagulation between particles (or coagula) of two discrete sizes, say i and j, is proportional to the product of their number concentrations, $n_i \times n_j$. On the other hand, Kapur and Fuerstenau (1969) argued that an agglomerating charge has the character of a loosely packed bed of particles (or granules) where each pellet at any instant of time is surrounded by a cage of neighbors. Therefore, the rate of coalescence in this so-called restricted-in-space (Sastry and Fuerstenau, 1970) environment should be proportional to number of pellets of size i and *number fraction* of pellets of size j, or vice versa. Moreover, in random coalescence, specific rate constant or coalescence kernel k is by definition independent of the size of the interacting pellets. Under these stipulations, the balling kinetics may be formulated in the framework of particle population balance (Kapur and Fuerstenau, 1969; Kapur, 1978).

$$\frac{\mathrm{d}n_i(t)}{\mathrm{d}t} = -kn_i(t)\sum_{j=1}^{\infty}\frac{n_j(t)}{N(t)} + \frac{k}{2}\sum_{j=1}^{i-1} n_{i-j}(t)\frac{n_j(t)}{N(t)};$$
$$i = 1, 2, 3 \ldots \tag{1}$$

where $n_i(t)$ is the number of pellets of size index i at balling time t. Ignoring reduction in pellet porosity as a second-order effect, the volume size v_i associated with the i-th size index is i times v_1, the initial nuclei volume size at $t=0$. Since the total number of pellets at any instant $N(t)$ is

$$N(t) = \sum_{i=1}^{\infty} n_i(t) \tag{2}$$

Eq. (1) simplifies to:

$$\frac{\mathrm{d}n_i(t)}{\mathrm{d}t} = -kn_i(t) + \frac{k}{2N(t)}\sum_{j=1}^{i-1} n_{i-j}(t)n_j(t);$$
$$i = 1, 2, 3 \ldots \tag{3}$$

Moreover, it is readily shown by summing Eq. (3) over all sizes that the rate of depletion of the agglomerating species conforms to the first order decay kinetics.

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = -\frac{k}{2}N(t) \tag{4}$$

Starting with single size nuclei of volume v_1 and employing generating functions (or z-transform),

Kapur and Fuerstenau (1969) solved simultaneously the set of infinite number of equations in Eq. (3).

$$n_i(t) = N(0)(-1)^{i+1}\left(\exp\left[-\frac{kt}{2}\right]-1\right)^{i-1}\exp[-kt] \tag{5}$$

Moreover, they showed that the mean volume size grows exponentially with time

$$\bar{v}(t) = v_1\exp\left[\frac{kt}{2}\right] \tag{6}$$

Even though the random coalescence model is apparently in agreement with many experimental data (Kapur, 1978), it is not entirely satisfactory at the conceptual level. This is because the size and number of nearest neighbor pellets in a cage are strong functions of the instantaneous pellet size distribution of the agglomerating charge. Consequently, the assumption of a size-independent kernel in the random coalescence model is seemingly quite drastic and the agreement between the model and the balling data is somewhat inexplicable and indeed intriguing. In what follows, we propose a modified coalescence model that explicitly incorporates the size-dependent coordination number of pellets in the balling charge, and compare the results with the random coalescence model. Our objective is to ascertain if the more realistic coordination number-based coalescence model (or coordination coalescence model) would lead to results that are similar to the random coalescence model.

2.2. Coordination coalescence model

In a series of papers Suzuki and coworkers (Suzuki et al., 1981; Suzuki and Oshima, 1983, 1985) have shown that in a packed bed of size distributed particles (or pellets), the number of particles of diameter j coordinated around a particle of diameter i is given by

$$K_{j,i} = S_j K_{j,i}(b) \tag{7}$$

where the fraction of surface area associated with particles of size j is

$$S_j = \frac{n_j(t)v_j^{2/3}}{\sum_{k=1}^{\infty} n_k(t)v_k^{2/3}} \tag{8}$$

and coordination number in a binary bed of i and j size particles only is given by

$$K_{j,i}(b) = 0.134K(\phi) \times \left(\frac{\left(\frac{v_i}{v_j}\right)^{1/3}+1}{1+\left(\frac{v_i}{v_j}\right)^{1/3}-\sqrt{\left(\frac{v_i}{v_j}\right)^{1/3}\left(\left(\frac{v_i}{v_j}\right)^{1/3}+2\right)}}\right) \tag{9}$$

where $K(\phi)$ is coordination number in a bed of single size particles packed to solid fraction ϕ.

Even though the composition of the cage is distributed and it fluctuates continuously in a dynamic environment, for our purpose it is sufficient to consider the average indices of the agglomerating bed. Accordingly, the total number of pellets surrounding a pellet of size i is given by

$$K_i = \sum_{j=1}^{\infty} K_{j,i} \tag{10}$$

The rate of coalescence between pellets of size i and j is

$$\text{Rate}(j,i) = kn_i(t)\frac{K_{j,i}}{K_i} \tag{11}$$

Hence, the equation for coordination coalescence model becomes

$$\frac{dn_i(t)}{dt} = -kn_i(t)\sum_{j=1}^{\infty}\frac{K_{j,i}}{K_i} + \frac{k}{2}\sum_{j=1}^{i-1} n_{i-j}(t)\frac{K_{j,i-j}}{K_{i-j}}; \quad i = 1,2,3\ldots \tag{12}$$

where the collision frequency of pellet of size i with its neighbors and the probability of coalescence, given a collision, both assumed independent of size, are embedded in the rate constant k. Because of the asymmetric nature of the coordination number, the number of j size pellets coordinated around the i size pellet is not same as the number of i size pellets coordinated around the j size pellet. While both kinds of combinations are enumerated in the appearance or gain term (second term in right hand side of the equation above), only one kind of combination is counted in the

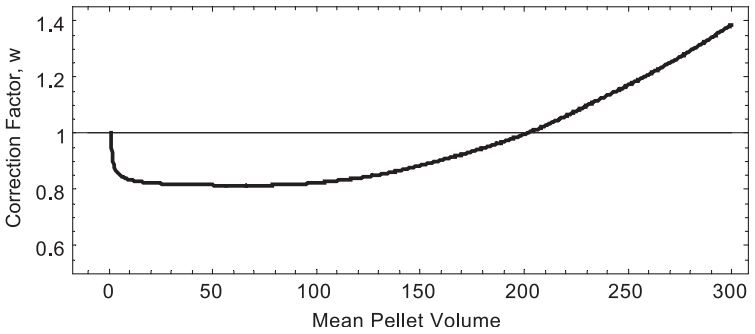

Fig. 1. Correction factor in coordination coalescence model as a function of mean pellet volume.

disappearance or the loss term (first term in right hand side). A consequence of this lack of symmetry is that the total volume of the solid may not be conserved. In order to ensure that the volume loss and volume gain rates are always equal, we introduce a correction factor w in the disappearance term and rewrite Eq. (12) in a simplified form as

$$\frac{\mathrm{d}n_i(t)}{\mathrm{d}t} = -kw\, n_i(t) + \frac{k}{2}\sum_{j=1}^{i-1} n_{i-j}(t)\frac{K_{j,i-j}}{K_{i-j}};$$

$$i = 1, 2, 3 \ldots \tag{13}$$

The correction factor is computed by invoking the time invariance of total volume

$$\frac{\mathrm{d}\sum_{i=1}^{\infty} v_i n_i(t)}{\mathrm{d}t} = 0 \tag{14}$$

Recalling $v_i = iv_1$, Eqs. (13) and (14) combine to yield

$$w = \frac{\sum_{i=1}^{\infty} i\sum_{j=1}^{i-1} n_{i-j}(t)\frac{K_{j,i-j}}{K_{i-j}}}{2\sum_{i=1}^{\infty} i n_i(t)} \tag{15}$$

A set comprising 1000 equations of the kind seen in Eq. (13) were solved simultaneously by the Runge–Kutta method. The initial condition used was 10^5 nuclei of volume size one. The results were compared with the analytical solutions to the random coalescence model given in Eqs. (4)–(6). For a comparison that is as fair as possible, we chose not to use the techniques of breaking up the size scale in geometrically increasing segments, proposed by Hounslow et al. (1988) and others for reducing the number of equations and increasing the size increment ratio.

Fig. 1 shows variation of the correction factor as a function of pellet mean volume. Note that $w = 1$ in random coalescence model. It would seem that a relatively small correction, ranging from unity to about 0.85, is needed to conserve the material volume in coordination coalescence model. The sharp increase in the correction factor when mean volume exceeds about 250 units (i.e. 250 times the initial volume) is due to the truncation error that begins to manifest because of the limited number of 1000 discrete sizes employed rather than the infinite number required by the model. Fig. 2 compares the total number of pellets remaining as a function of balling time in random and coordination coalescence models. In the former case, granules are depleted exponentially, as evident from Eq. (4). The specific rate constant k in the latter case is adjusted

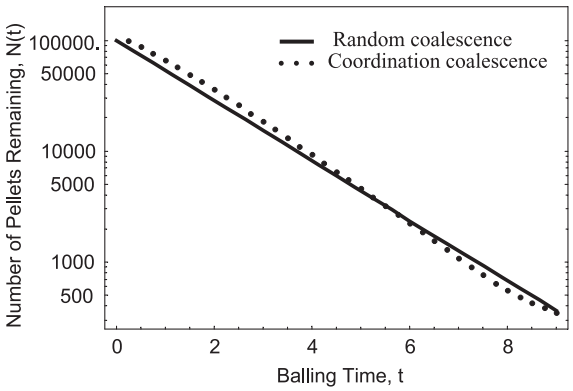

Fig. 2. Number of pellets remaining in agglomerating charge with balling time in random and coordination coalescence models. k(random)/k(coordination) ratio is adjusted to 1.2 for best agreement.

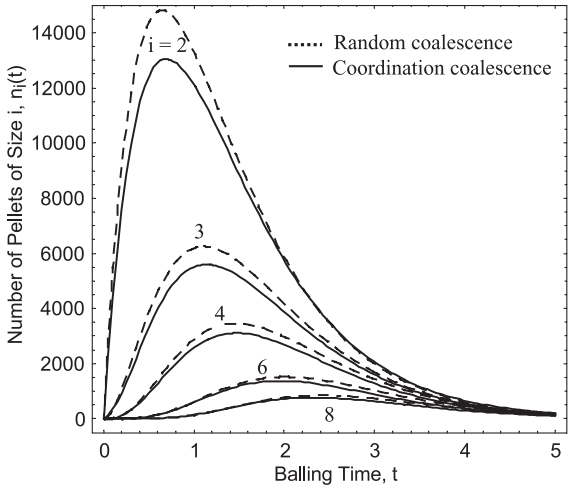

Fig. 3. Variation of pellet population with balling time in discrete sizes 2, 3, 4, 6 and 8 in random and coordination coalescence models.

in order to obtain a population drop, which follows the random coalescence kinetics as closely as possible, as shown in the figure. The ratio of specific rate constants k(random)/k(coalescence) in two cases turns out to be 1.25. Figs. 3–5 compare the growth and decay curves of pellet population in various discrete size intervals, as generated by Eq. (5) for the random coalescence model and by numerical solution of the coordination

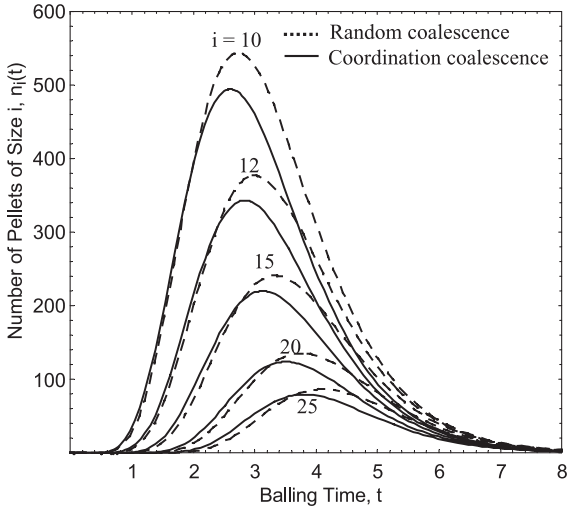

Fig. 4. Variation of pellet population with balling time in discrete sizes 10, 12, 15, 20 and 25 in random and coordination coalescence models.

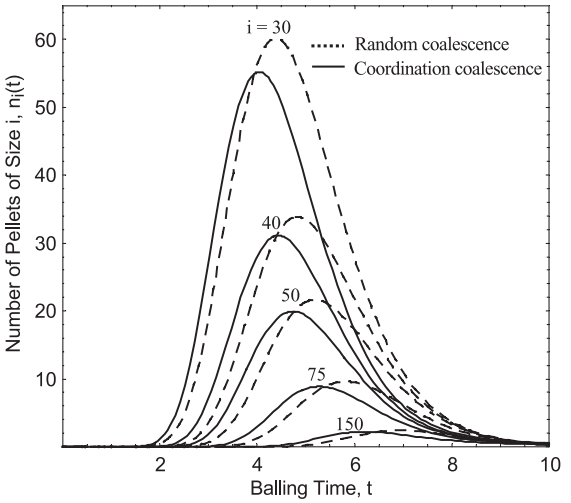

Fig. 5. Variation of pellet population with balling time in discrete sizes 30, 40, 50, 75 and 150 in random and coordination coalescence models.

coalescence model. Even though the divergence between the two sets of curves increases with size, the overall trends including position and height of the peaks are quite similar. Fig. 6 compares the evolution of mean pellet size in the two models for different rate constants while maintaining the rate ratio at 1.25 in all cases. The agreement is quite good. In practice, balling rates can be altered over a broad range by controlling the moisture content.

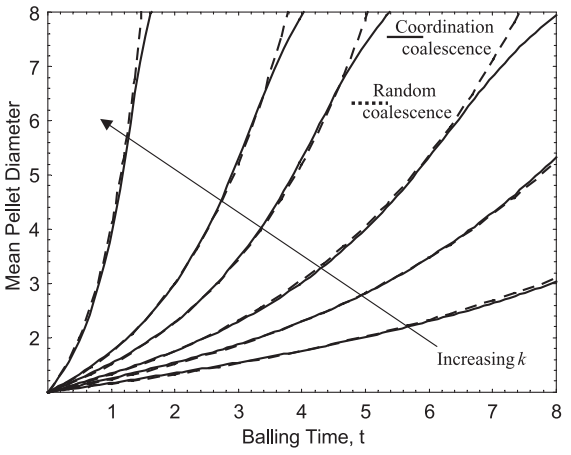

Fig. 6. Growth of mean pellet diameter with balling time in random and coordination coalescence models.

3. Autolayering mechanism

3.1. Static modeling of autolayering

Millions of tons of iron ore fines and other minerals are granulated every year as sinter feed by the autolayering mechanism. The raw feed is characterized by a rather broad size distribution, ranging from about 10 mm to finer than 0.063 mm. It is customary to classify the feed particles as seeds, intermediates and fines (Litster et al., 1986; Kapur et al., 1993). The presence of fines in the sinter bed is highly detrimental for the permeability of the bed and hence for the sintering process. The objective of granulation is to eliminate fines by simple expedient of layering these onto coarse particles of the feed, which act as seeds or nuclei. Depending primarily upon the moisture content of the granulating charge and the relative proportion of seeds and fines, the intermediate size particles can act as either nuclei or layering fines or both, or seemingly take no part in the granulation (Furui et al., 1977; Nagano et al., 1985; Litster et al., 1986; Peters et al., 1989). The classification of feed particles may be generalized by introducing a partition function (Litster et al., 1986), defined as fraction of particles of a given size that act as seeds. Thus, in the limits, partition function is unity when all particles are embedded as nuclei in the granules and it is zero when all particles are consumed by layering.

The autolayering process has been described in terms of two idealized postulates (Kapur, 1995). In the *t*-postulate, the rate of pick up of fine particles is proportional to surface area of the rolling granule and a layer is formed whose thickness t is the same irrespective of the seed size (Peters et al., 1989). In the more general *p*-postulate, the rate of layering is proportional to volume of the rolling granule, and consequently, the granule size is proportional to the seed size. In the mixed postulate, layering occurs initially by a coating of fixed thickness, which is followed by proportionate growth of granules (Litster and Waters, 1988).

Only static modeling of autolayering has been attempted so far, presumably because of the highly nonlinear nature of the process. These models map the input size distribution of feed into steady state or terminal granule size distribution, with little or no information on the path traversed by the process. Litster et al. (1986) invoke mass balance to calculate the final granule size distribution, starting with a priori knowledge of the partition function. On the other hand, Kapur et al. (1993) incorporate a known value of thickness t or of proportionate growth parameter p into an interval-by-interval marching algorithm, which essentially mimics the process in a virtual domain. The partition function and the size distribution of fines in the deposited layer are generated concurrently in the course of the computations. Unfortunately, neither approach is suitable for continuous-time dynamic simulation of granulation by autolayering. In what follows, we employ the particle population balance to construct an explicitly dynamic model of autolayering.

3.2. Dynamic model of autolayering

The absolute number–diameter distribution of feed $n(x)$, $0 < x < \infty$ is divided into two components, layering fines and coarse seeds. For sake of clarity, separate notations are assigned to fines $0 < x_f < x_c$, and seeds $x_c \le x_s < \infty$, where x_c is a cutoff size which separates the components. Fig. 7 illustrates a granule of instantaneous size $y(t)$ having a nucleus of size x_s, which is layered by size distributed fines x_f. It is postulated that

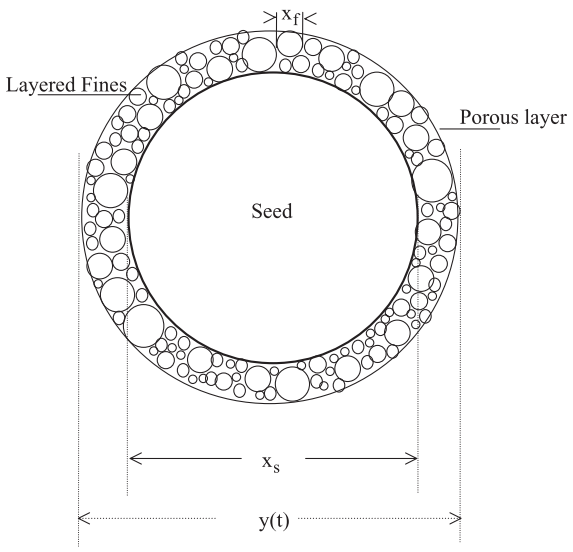

Fig. 7. Notation used to represent a seed of size x_s layered with fines of distributed size x_f to give a granule of size y at time t.

the rate at which fines layer onto the surface of a granule of size $y(t)$ is given by

$$\text{Rate}(y, x_f) = k y^b(t) n(x_f, t) \tag{16}$$

where k is a specific rate constant and b is an unspecified exponent. Because of continuous depletion of fines by layering, the absolute number–diameter distribution of fines $n(x_f,t)$ varies with time in following manner

$$\frac{dn(x_f,t)}{dt} = -kn(x_f,t)\int_{y_c(t)}^{\infty} y^b(t)n'(y)dy(t); \quad 0 < x_f < x_c \tag{17}$$

where $y_c(t)$ is size of the granule with smallest seed of size x_c and $n'(y)$ or $n'(y(t))$ is absolute number–diameter distribution of granules, which is an implicit function of time. It is readily shown that the rate of increase of a granule volume $v(t)$ is

$$\frac{dv(t)}{dt} = \frac{\pi k}{6(1-\varepsilon)} y^b(t)\int_0^{x_c} x_f^3 n(x_f,t)dx_f \tag{18}$$

where ε is fractional porosity of the deposited layer. Converting granule volume into diameter $y(t)$ yields

$$\frac{dy(t)}{dt} = \frac{k}{3(1-\varepsilon)} y^{b-2}(t)\int_0^{x_c} x_f^3 n(x_f,t)dx_f \tag{19}$$

with the initial condition

$$y(0) = x_s; \quad x_c \le x_s < \infty \tag{20}$$

Next, we set $b=3$ in Eq. (19) and integrate

$$y(t) = x_s \exp\left[\frac{k}{3(1-\varepsilon)}\int_0^t\int_0^{x_c} x_f^3 n(x_f,t')dx_f dt'\right] \tag{21}$$

Or, in terms of a lumped parameter p

$$y(t) = y(0)p(k,\varepsilon,x_c,t) \tag{22}$$

which is the expression for proportionate growth of granules stated above in the p-postulate and, as such, justifies the specialization of exponent $b=3$. Moreover, given that there is only one seed per granule, conservation of total number of seeds in the granulating charge requires that

$$n'(y)dy = n(x_s)dx_s \tag{23}$$

In other words, Eq. (17) can be rewritten as

$$\frac{dn(x_f,t)}{dt} = -kn(x_f,t)\int_{x_c}^{\infty} y^3(t)n(x_s)dx_s \tag{24}$$

Simultaneous solution of Eqs. (19) and (24) results in a dynamic model of granulation by autolayering. For the purpose of computations, it is convenient to discretize the size such that the layering fines lie in first g intervals and the seeds reside in intervals $g+1$ and above. If $\tilde{y}_i(t)$ is the mean granule size in i-th interval and $\tilde{x}_j$ is the mean layering fine in j-th interval, Eqs. (19) and (24) can be written as

$$\frac{dn_j(t)}{dt} = -kn_j(t)\sum_{i=g+1}^{u} \tilde{y}_i^3(t)n_i; \quad j = 1\ldots g \tag{25}$$

with the initial condition $n_j(0)=n_j$, the distribution of feed in discrete size intervals at $t=0$, and

$$\frac{d\tilde{y}_i(t)}{dt} = \frac{k}{3(1-\varepsilon)}\tilde{y}_i(t)\sum_{j=1}^{g} \tilde{x}_j^3 n_j(t); \quad i = g+1\ldots u \tag{26}$$

with the initial condition $\tilde{y}_i(0)=\tilde{x}_i$ in the seed intervals. Obviously, the computational accuracy would depend on the number of size intervals chosen and the interval width employed, which need not be identical.

For demonstration, the proposed model was employed to simulate the granulation of iron ore fines in a laboratory batch drum. The experimental details are given elsewhere (Venkataramana et al., 1997). The feed, which had 20.17% minus 100-mesh (0.15 mm) fines and 5.61% moisture, was divided into 11 size intervals. The first six intervals were assigned to layering fines and remaining five intervals were allotted to seeds. The cutoff size was 1 mm. The resulting 11 simultaneous equations were solved numerically using the *Mathematica*® package. The porosity of the layer was assumed 50%. The specific rate constant k was estimated by

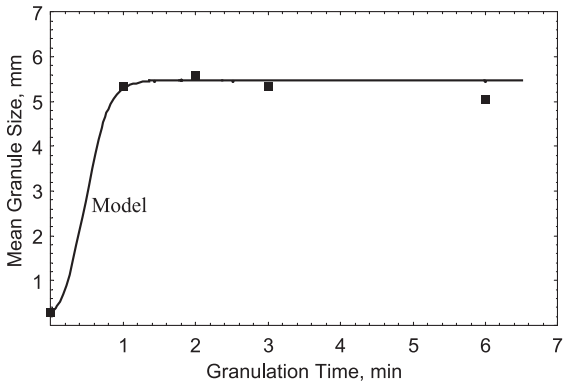

Fig. 8. Comparison of measured mean granule size and autolayering model as a function of granulation time. Best possible fit is obtained by adjusting the value of the specific rate constant k.

matching the simulated overall surface-volume mean granule diameter with experimental data, as shown in Fig. 8. The best agreement was obtained with $k = 0.11$, which was used in all simulations. It will be seen that granulation occurs in two stages. Initially in the transient autolayering regime, mean granule size increases steeply and then growth virtually stops or undergoes a very slow rate of growth/decay, presumably by the abrasion transfer mechanism. For all practical purpose, the terminal size distribution of granules is attained in less than a minute of granulation time in this system. Fig. 9 shows the simulated growth of mean granule size in each of the five seed intervals, while Fig. 10 tracks the depletion of layering fines in the six intervals.

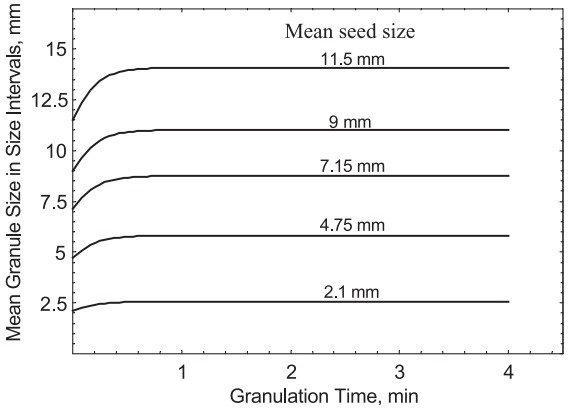

Fig. 9. Simulation of granule growth in individual seed size intervals with granulation time.

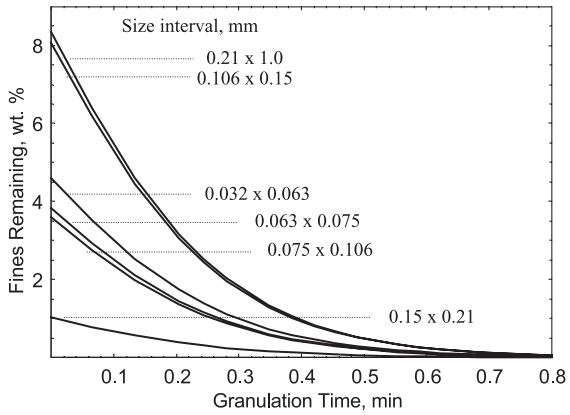

Fig. 10. Simulation of fines remaining in size intervals as a function of granulation time.

The model cumulative finer distributions are compared in Fig. 11 with the experimental data at various granulation times. The feed size distribution is also included in the figure. Unfortunately, it was not possible to acquire reliable experimental data in the transient regime. This is because significant time lags occur due to inertia and friction within the sticky charge when the granulation drum is started and stopped. These end effects can introduce considerable noise in measurements at short time durations. Nevertheless, the overall agreement in the figure is quite satisfactory. Finally, Fig. 12 shows that the growth ratio p (granule diameter/seed diameter) calculated from Fig. 9 is 1.224. Although the consistency of p is a consequence of the choice of exponent $b = 3$, it also suggests that the discre-

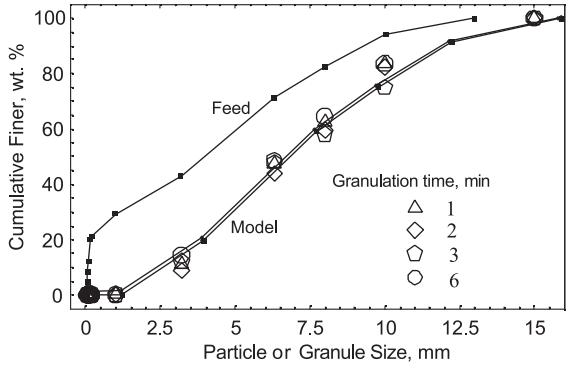

Fig. 11. Model granule size distributions compared with measured distributions at different granulation times.

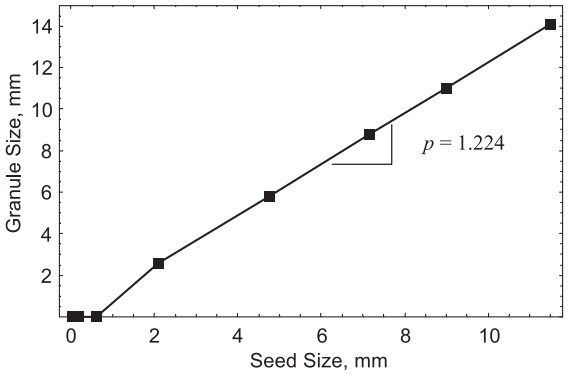

Fig. 12. Ratio of granule size and seed size, showing the operation of p-postulate in autolayering.

tization procedure, leading to Eqs. (25) and (26), does not introduce any significant distortion at the implementation stage of the model.

4. Concluding remarks

The total volume may not be conserved in coordination coalescence model because of two reasons. One, volume gain or loss due to asymmetric nature of coordination number, and two, volume loss due to limited number of size classes chosen for numerical computation. It is, however, not difficult to distinguish between the two causes. In the latter case, volume loss is discernible only when a significant number of pellets begin to form whose sizes are larger than the maximum size employed in the model. Thus, in the simulation carried out here with 1000 discrete size classes, the volume loss begins to overwhelm the volume gain when mean pellet volume exceeds about 250 units. Evidently, the correction factor w represents a composite correction for both the underlying causes. It is remarkable that in spite of their different structures, random and coordination coalescence models should lead to similar results. Consequently, notwithstanding its somewhat unrealistic basis, random coalescence model provides a satisfactory representation of the balling process, apart from the fact that it is much more convenient to compute and implement.

Since it cannot be calculated from first principles, specific rate constant k in the coalescence models as well as the dynamic autolayering model must be estimated from experimental data. In addition, for implementing the autolayering model it is necessary to provide an estimate of either the cutoff size x_c or the proportionate growth parameter p from experimental data. Kapur (1995) had earlier shown that, in theory at least, p could be calculated by exploiting the water balance in the granulating material

$$p = \left[\frac{\alpha\varepsilon\rho_l + W\varepsilon\rho_s}{\alpha\varepsilon\rho_l + W\varepsilon\rho_s + W\rho_s}\right]^{1/3} \tag{27}$$

where $\alpha \leq 1$ is a small correction for any trapped air bubbles in the layer and, due to curvature of the liquid–air interface at granule surface, W is water per unit weight of solid feed, and ρ_l and ρ_s are liquid and solid densities, respectively. This relationship is strictly valid if feed particles are nonporous and wetting, and both particles and granules are exactly spherical. Moreover, some uncertainty is invariably associated with porosity ε of the deposited layer, since it depends on the extent of packing of size distributed particles in a relatively thin layer. Further investigation and model testing will be needed before one can take advantage of the water balance for modeling autolayering kinetics.

Notation

Coalescence mechanism

$K(\phi)$	coordination number in a bed of single size pellets (or particles) packed to solid fraction ϕ
K_i	total number of pellets surrounding a pellet of size i
$K_{j,i}$	number of pellets of size j coordinated around a pellets of size i
$K_{j,i}(b)$	number of pellets of size j coordinated around a pellets of size i in a bed of i and j size pellets only
k	random coalescence kernel or specific rate constant
N	total number of pellets or granules
n_i	number of pellets of size index i
S_j	fraction of surface area associated with pellets of size j
t	balling time

v_1 initial nuclei volume size
v_i volume size associated with the i-th size index
$\bar{v}$ mean volume size
w correction factor
ϕ volume fraction of solids

Autolayering Mechanism

b exponent in the rate equation for layering
g size index or class of the largest size fines
k specific rate constant of layering
$n(x)$ absolute number–diameter distribution of feed
$n'(y)$ absolute number–diameter distribution of granules
W water content per unit solid mass of the agglomerating charge
x size of feed particles
x_c cutoff size for layering
x_f mean size of fines
x_s mean size of seeds
$\tilde{x}$ mean feed size
y granule size
$\tilde{y}$ mean granule size
y_c size of the granule with the smallest seed of size x_c
p model parameter or constant of proportionality in p-postulate
u size index or class of the largest size nuclei
v granule volume
α correction factor for layer of fines
ε fractional porosity of the layer
ρ_l, ρ_s liquid and solid densities

Acknowledgements

PCK acknowledges Professor D.W. Fuerstenau for introducing him to balling and granulation. Authors highly appreciate the encouragement and management support extended by Professor Mathai Joseph of Tata Research Development and Design Center.

References

Adetayo, A.A., Ennis, B.J., 1997. Unifying approach to modelling granule coalescence mechanisms. AIChE Journal 43 (4), 927–934.

Furui, T., Kawazy, M., Sugawara, K., Fujiwara, T., Kagawa, M., Sawamura, A., Uno, S., 1977. Technology of preparation of raw materials to be sintered. Nippon Steel Technical Report. Overseas 10, 36–46.

Hounslow, M.J., Ryall, R.L., Marshall, V.R., 1988. A discretized population balance for nucleation, growth and aggregation. AIChE Journal 34 (11), 1821–1832.

Kapur, P.C., 1971. The crushing and layering mechanism of granule growth. Chemical Engineering Science 26, 1093–1099.

Kapur, P.C., 1972. Kinetics of granulation by non-random coalescence mechanism. Chemical Engineering Science 27, 1863–1869.

Kapur, P.C., 1978. Balling and granulation. In: Drew, T.B., et al., (Eds.), Advances in Chemical Engineering, vol. 10. Academic Press, New York, pp. 55–123.

Kapur, P.C., 1995. Particle population balance in granulation of iron ores by an auto-layering mode. In: Mehrotra, S.P., Shekhar, R. (Eds.), Mineral Processing: Recent Advances and Future Trends. Allied Publishers, New Delhi, pp. 703–717.

Kapur, P.C., Fuerstenau, D.W., 1964. Kinetics of green pelletisation. Transactions of AIME 229, 348–355.

Kapur, P.C., Fuerstenau, D.W., 1969. A coalescence model for granulation. Industrial & Engineering Chemistry Process Design and Development 8, 56–62.

Kapur, P.C., Kapur, P., Fuerstenau, D.W., 1993. An auto-layering model for the granulation of iron ore fines. International Journal of Mineral Processing 39, 239–250.

Litster, J.D., Waters, A.G., 1988. Influence of the material properties of iron ore sinter feed on granulation effectiveness. Powder Technology 55, 141–151.

Litster, J.D., Waters, A.G., Nicol, S.K., 1986. A model for predicting the size distribution of product from a granulating drum. Transactions of the Iron and Steel Institute of Japan 26, 1036–1044.

Nagano, S., Noda, H., Yanaka, H., Shiobara, K., Yamaoka, Y., 1985. Characteristics of sinter feed and its influence on sintering. In: Capes, C.E. (Ed.), 4th International Symposium on Agglomeration, Toronto, Canada. Iron and Steel Society of AIME, Warrendale, PA, USA, pp. 191–202.

Ouchiyama, N., Tanaka, T., 1974. Mathematical model in the kinetics of granulation. Industrial & Engineering Chemistry Process Design and Development 13, 383–389.

Ouchiyama, N., Tanaka, T., 1982. Kinetic analysis and simulation of batch granulation. Industrial & Engineering Chemistry Process Design and Development 21, 29–35.

Peters, K.H., Beer, H., Kropla, H.W., Rinne, K., 1989. Granulation behavior of sinter mix components, with particular respect to coke breeze particle size and its influence on the iron ore sintering process. In: Cross, M., Oliver, R. (Eds.), 5th International Symposium on Agglomeration. Inst. of Chem. Eng., Rugby, UK, pp. 51–70.

Pulvermacher, B., Ruckenstein, E., 1975. Time evolution of the size spectrum in granulation. The Chemical Engineering Journal 9, 21–29.

Sastry, K.V.S., Fuerstenau, D.W., 1970. Size distribution of agglomerates in coalescing disperse phase systems. Industrial and Engineering Chemistry Fundamentals 9, 145–149.

Sastry, K.V.S., Fuerstenau, D.W., 1977. Kinetics of green pellet

growth by the layering mechanism. Transactions of AIME 262, 43–47.

Suzuki, M., Oshima, T., 1983. Estimation of the coordination number in a multicomponent mixture of spheres. Powder Technology 35, 159–166.

Suzuki, M., Oshima, T., 1985. Coordination number of a multicomponent randomly packed bed of spheres with size distribution. Powder Technology 44, 213–218.

Suzuki, M., Makino, K., Yamada, M., Iinoya, K., 1981. A study on the coordination number in a system of randomly packed, uniform-sized spherical particles. International Chemical Engineering 21, 482–488.

Venkataramana, R., Gupta, S.S., Kapur, P.C., Ramachandran, N., 1997. Modelling of iron ore sinter feed granulation. Tata Search, 66–72.