RADIO-EMITTING ELECTRONS AND COSMIC RAY CONFINEMENT

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Abstract. The propagation of cosmic ray electrons in the framework of the Disk-Halo diffusion model in which the diffusion coefficient $D \propto z^{\delta} E^{\mu}$ (where z is the distance from the galactic plane and E is the energy), and the magnetic field $H \alpha z^{-\xi}$ has been examined by making use of the recently available radio data up to 8 GHz toward the Anticenter (A) and Halo Minimum (M). The following inferences are then made. From the difference in the frequency at which steepening occurs in the radio spectra toward A and H, it is found that the observations are consistent with the magnetic field decreasing with z such that $\xi = 0.24$–0.37. An electron injection spectrum with a single power law down to energies well below 1 GeV cannot explain satisfactorily the observed radio spectra. All observations, however, can be understood in a self consistent way if the observed steepening of the radio spectra, and hence the interstellar electron spectrum, is due partly to the deviation in the power law electron injection spectrum below a few GeV and partly to the first break arising from electron energy losses occurring in the same energy region. In this case, using the value of ξ obtained above and a value of $\mu = 0.3$–0.6, it is found that the spectral index γ_0 of the injected electrons above a few GeV has a value between 1.9 and 2.3 and the index δ a value between 0.5 and 1. Further, if the electrons and protons have the same spectral shape at injection, then $\gamma_0 = 2.1$–2.3.

1. Introduction

It is now generally believed that cosmic rays we sample near the Earth are of galactic origin and are well confined within the Galaxy at least up to energies of $\sim 10^{15}$ eV. During recent years there have been suggestions, with reasonable basis, that even within the Galaxy, cosmic rays are produced and confined inside the galactic disk; the propagation of the particles within the Disk is then treated in the 'Homogeneous Model' in which there is spatial homogeneity of particles in the confinement volume. Nevertheless, very recent studies relating to the nonthermal radio continuum from the Galaxy are providing increasingly reliable evidence for the existence of a radio halo (Webster, 1975; Baldwin, 1976) and thereby emphasize the need for considering seriously the 'Diffusion Model' for the propagation of cosmic rays. In this model also cosmic ray sources are assumed to be uniformly distributed within the Disk but the particles are able to diffuse freely out of the Disk and be confined within an extended region of galactic space – the Halo. Such a model has been investigated theoretically in some detail during recent years (Bulanov and Dogel, 1974; Ptuskin, 1974; Bulanov and Dogel, 1975; Ginzburg and Ptuskin, 1975). In particular, Bulanov and Dogel (1975) have considered for cosmic ray electrons a generalized treatment in

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which the leakage lifetime has a power law dependence on the energy of the particle while the diffusion coefficient and mean magnetic field have power law dependence on z, the distance from the galactic plane.

Further, background radio observations have now become available at frequencies in excess of 1 GHz in the direction of the Anticenter (A) and the Halo Minimum (H) (Webster, 1974, 1976) to enable one to examine such models reliably. In the present investigation, we have first made use of the evidence from the radio data for the steepening, at different frequencies, of the observed spectra in the direction of the Anticenter and the Halo (Webster, 1976) to deduce quantitative information on the variation of the magnetic field with z. Second, we have made use of the calculations of Bulanov and Dogel (1975) and the radio data to make interesting deductions regarding the injection spectrum of cosmic ray electrons and their propagation in the Galaxy.

2. Data Used in the Present Analysis

2.1. THE RADIO DATA

In Figure 1 are summarized the radio data we have used in the present analysis. The data points associated with the directions A and H are from a compilation due to Webster (1976) in the frequency range of 10–8000 MHz; the data points given in arbitrary units by Webster have been normalized by us using the radio survey at 81.5 MHz (Purton, 1966). It is interesting to note that in both directions there is spectral steepening and we have been able to fit well with power law spectra of the type $I(\nu)\alpha\nu^{-\alpha}$ before and after the steepening; the spectral indices thus obtained and the errors on them are summarized in Table I. There is also good evidence that the steepening occurs at different frequencies in the two directions, namely at about 200 MHz in the Halo Minimum direction and at about 330 MHz in the case of the Anticenter. The isotropic metagalactic component of the background radio continuum is known to be considerably weaker than the galactic component with a spectral shape steeper than the galactic emission in the frequency range below a few hundred MHz. Even so, its effect will be to make the true galactic component somewhat flatter at low frequencies compared to that given in column 3 of Table I. In order to correct for this effect we have made use of the data on the metagalactic component (line M of Figure 1) compiled by Daniel and Stephens (1970) and determined the spectral shapes and indices applicable to the galactic component. These are included in Figure 1 as curves A' and H', and in Table I, from which it is found that the corrected radio spectra A' and H' at low frequencies also have the same spectral shape.

2.2. THEORETICAL RELATIONS FROM THE DIFFUSION MODEL

In our analysis in Section 4 we follow the treatment of the diffusion model developed by Bulanov and Dogel (1974, 1975). In what follows we present the essentials of their treatment (Bulanov and Dogel, 1975). In view of the complexity of the problems the authors make the following simplified assumptions.

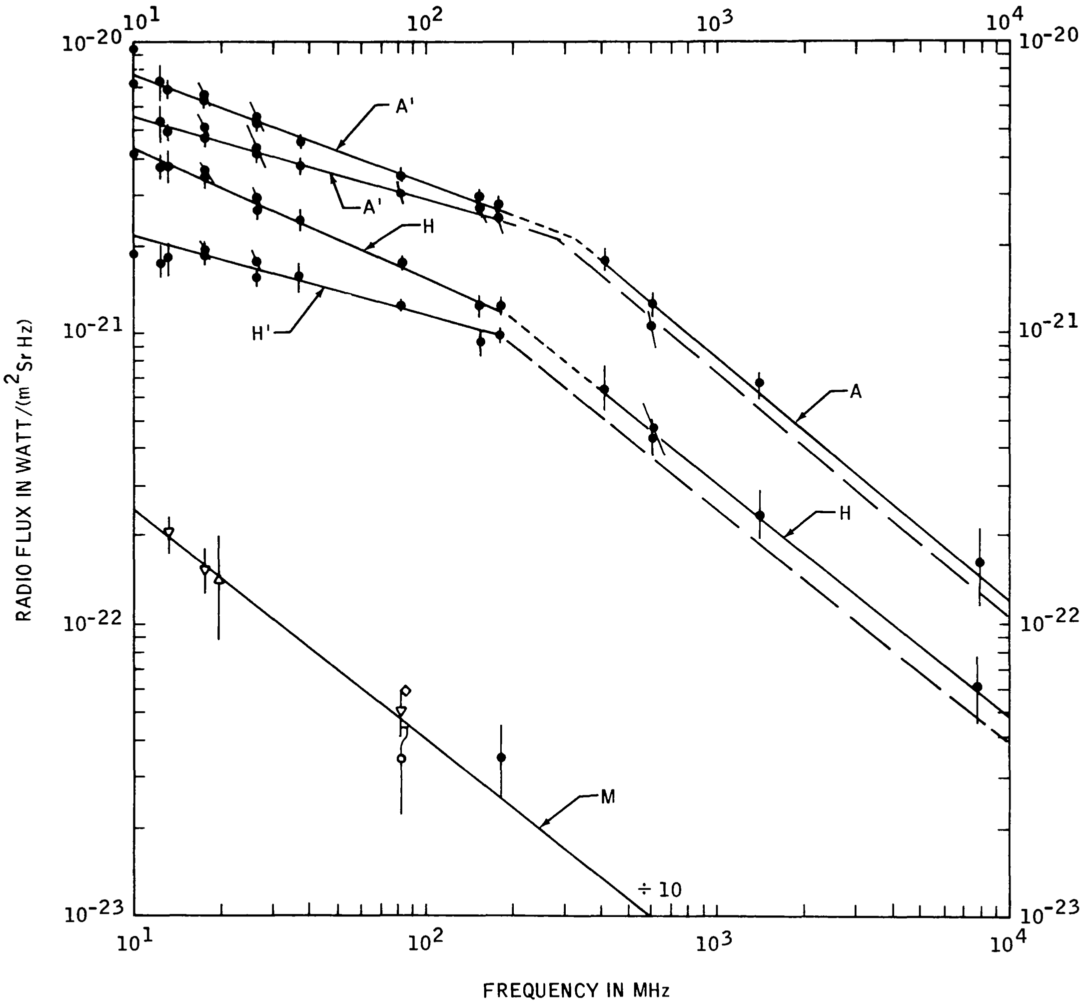

Fig. 1. Observational data on the background radio continuum. The data points for the directions towards the Anticenter (A) and Halo Minimum (H) compiled by Webster (1976) have been normalized using the survey by Purton (1966) at 81.5 MHz; straight-line fits have been made by us for frequencies $\leqslant 200$ MHz and $\geqslant 400$ MHz. A power law spectrum has also been fitted to the data on the metagalactic component (M) compiled by Daniel and Stephens (1970). Lines A' and H' have been obtained by subtracting the contribution of M from A and H.

(i) The sources of cosmic rays are uniformly distributed within the disk of the Galaxy with a half-thickness b; and the electrons injected into the disk have a single power law energy spectrum of the type

$$J(E) = AE^{-\gamma_0} \tag{1}$$

where A and γ_0 are constants.

(ii) The region of residence and propagation of cosmic rays is a cylinder with a half thickness d which is larger than b; the particle density beyond the boundary at d is zero.

(iii) The diffusion coefficient D is a function of energy and z and is represented as

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TABLE I

Spectral indices fitted before and after the radio steepening in the direction of the anticenter and halo minimum

	Frequency range in MHz	Spectral index α	
		Without correction for metagalactic component	After correction for metagalactic component
Anticenter	10–200	0.35 ± 0.02	0.27 ± 0.02^{a}
(A)	400–8000	0.79 ± 0.08	0.79 ± 0.08
Halo Minimum	10–200	0.45 ± 0.02	0.27 ± 0.02^{a}
(H)	400–8000	0.79 ± 0.10	0.79 ± 0.10

[a] These errors do not include the uncertainties in the metagalactic component.

$$D(E, z) = D_0 \left(\frac{z}{b}\right)^{\delta} \left(\frac{E}{E_0}\right)^{\mu}, \tag{2}$$

where D_0, E_0, δ and μ are constants.

(iv) The magnetic field within the confinement space varies as

$$H(z) = H_0 \left(\frac{z}{b}\right)^{-\xi}, \tag{3}$$

where H_0 and ξ are constants.

(v) A variable $\lambda(E)$ which represents the mean distance traversed by the electrons when they lose about half their energy is also introduced such that

$$\lambda(E) \sim \left[\frac{(1-\mu)[2(1-\xi)-\delta]^2 D_0 E^{\mu-1}}{b^{\delta+2\xi} E_0^{\mu} \beta}\right]^{1/[2(1-\xi)-\delta]}. \tag{4}$$

TABLE II

The electron spectral index γ and the radio spectral indices α_A and α_H as calculated by Bulanov and Dogel (1975)

$\lambda(E)$	$\lambda(E) > d$	$b < \lambda(E) < d$		$\lambda(E) < b$
		Synchrotron losses ≫ inverse Compton losses	Inverse Compton losses ≫ synchrotron losses	
(1)	(2)	(3a)	(3b)	(4)
γ	$\gamma_0 + \mu$	$(\gamma_0+1) - \frac{(1-\mu)(1-2\xi)}{(1-2\xi)+(1-\delta)}$	$(\gamma_0+1) - \frac{1-\mu}{2-\delta}$	$\gamma_0 + 1$
α_A	$\frac{\gamma_0+\mu-1}{2}$	$\frac{1}{2}\left[\gamma_0 - \frac{(1-\mu)(1-2\xi)}{(1-2\xi)+(1-\delta)}\right]$	$\frac{1}{2}\left[\gamma_0 - \frac{1-\mu}{2-\delta}\right]$	$\frac{\gamma_0}{2}$
α_H	$\frac{\gamma_0+\mu-1}{2}$	$\frac{1}{2}\left[\gamma_0 - \xi\,\frac{\{\gamma_0-2(1-\delta)\}(1-\mu)}{(1-2\xi)+(1-\delta)}\right]$	$\frac{1}{2}\left[\gamma_0 - \frac{\xi(\gamma_0+2)(1-\mu)}{2+\xi(1-\mu)+2(1-\delta)}\right]$	$\frac{\gamma_0}{2}$

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Here β is the factor in the energy loss term for electrons represented as $-(\mathrm{d}E/\mathrm{d}t)=\beta E^2$ and includes both synchrotron and inverse Compton losses. The diffusion equation is then solved to obtain the electron spectrum and its spatial dependence and thereby permit the derivation of relations connecting the radio spectral indices α_A and α_H and the quantities γ_0, μ, δ and ξ for three energy ranges (and hence frequency ranges) corresponding to $\lambda(E)>d$, $b<\lambda(E)<d$, and $\lambda(E)<b$. In these calculations it is assumed that $\delta\leqslant 1$, $\mu\leqslant 1$ and $\xi\leqslant 0.5$ in order to ensure physically meaningful conditions for the diffusion of particles in the model. Bulanov and Dogel (1975) derive two sets of relations under the assumption that (i) the inverse Compton losses are important compared to synchrotron losses (Disk situation) and (ii) the opposite is the case (Halo situation). These relations are summarized in Table II from which it is evident that the calculations predict two breaks in the radio spectra arising from the breaks in the electron energy spectrum at two energies determined by the conditions $\lambda(E\ )\sim d$ and $\lambda(E_2)\sim b$.

3. Magnetic Field Variation in the Halo with *z*

The observed steepening of the radio spectrum in the direction of *A* at a frequency significantly higher than that in the direction of *H* (Section 2.1) is a clear evidence for a decrease in the magnetic field in the direction of *H* and a strong support for the

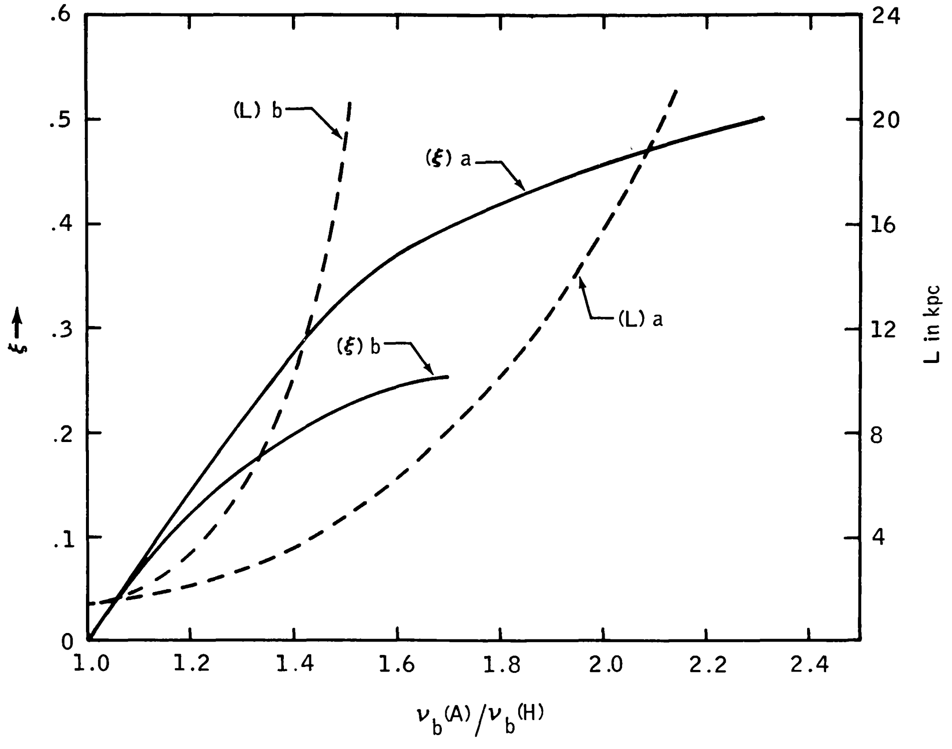

Fig. 2. The dependence of ξ the index of the power law dependence of the magnetic field on z, and L the radiating distance toward H, on the ratio $\nu_b(A)/\nu_b(H)$ of the break frequencies in the radio spectra toward A and H. Solid curves $\xi(a)$ and $\xi(b)$ refer to ξ for assumptions a and b described in the text. Similarly dashed curves $L(a)$ and $L(b)$ refer to L.

Diffusion Model. We take advantage of this to make an estimate of the parameter ξ in Equation (3) without any detailed assumption regarding the model. In order to do this we approximated the electron spectrum needed in the Disk to account for the shape and intensity of the radio spectrum measured in the direction of A (Figure 1) by two suitable power laws. We then calculated the radio spectrum in the direction of H for varying values of ξ; in this the integration was carried out up to a distance L such that the calculated spectrum became the same as the observed one in the direction of H. From this break frequency $\nu_b(H)$ at which the Halo spectrum breaks was determined. This in turn permitted us to obtain the ratio of break frequencies $\nu_b(A)/\nu_b(H)$ for two possible situations: (a) the electron intensity $J(E)$ is uniform in the Halo, i.e., independent of z; and (b) the cosmic ray energy density is proportional to the magnetic energy density, i.e., $J(E, z) \propto z^{-2}$. We note here that the z dependence predicted on the basis of the model of Bulanov and Dogel (1975) involves ξ and δ but lies between the above two possibilities. We would also like to point out that the calculated break frequencies in the radio spectra are well defined and their ratios are reasonably insensitive to the spectral indices used for the electrons. In Figure 2 are plotted ξ and L as a function of $\nu_b(A)/\nu_b(H)$ for assumptions a and b.

We find from Figure 1 that the observed value of $\nu_b(A)/\nu_b(H) \approx 1.6$ which from Figure 2 corresponds to $\xi \approx 0.37$ and $L \approx 6$ kpc in the case where the electron density is independent of z and $\xi \approx 0.24$ and $L \sim 50$ kpc in the case where $J(E, z) \propto z^{-2}$. In the latter case it is also noted that 90% of the observed radio flux is emitted from a region only about 25 kpc. This demonstrates that while the value of L is very sensitive on the assumed z dependence of electron intensity and magnetic field, ξ varies little. If the cosmic ray intensity does in fact decrease with z, as it is likely to be, the value of $\xi \approx 0.24$ and L is larger than 6 kpc. It is interesting to point out here that the self consistent model for the hydrostatic equilibrium of the Disk and the radio emission toward the Halo (Badhwar and Stephens, 1977) also gives $\xi \approx 0.26$ for $0.3 \text{ kpc} \lesssim z \lesssim 3$ kpc.

4. Interpretation of the Radio Data

Observations on the galactic radio continuum summarized in Table I require an interstellar electron spectrum which is a power law above several GeV and at least up to about 25 GeV with an index $\gamma = 2.58 \pm 0.16$; it is expected to be considerably flatter below these energies though the exact shape is difficult to define because of the uncertainty in the contribution from the metagalactic component below about 400 MHz. Observations on cosmic ray electrons made near the Earth are also generally consistent with this though in terms of details there is uncertainty arising from measurement errors (Anand *et al.*, 1975; Freier *et al.*, 1975; Badhwar *et al.*, 1977). Such an electron spectral shape has in the past been interpreted as intrinsic to electrons injected into interstellar space from source regions. (For example, Higdon, 1975; Apparao and Daniel, 1977.) In this section, we take the alternate point of view that the steepening of the electron spectrum is due to energy losses during their propagation in the Galaxy and examine its validity and implications within the framework of the diffusion model.

An examination of Table II reveals that the calculated radio spectrum (and hence the electron spectrum) in the directions of A and H are expected to have two breaks but the spectral index before the first and after the second breaks should have the same value. On the other hand observational data presently available on the radio and cosmic ray electron spectra give evidence for only one steepening. We will, therefore, examine whether the observed steepening can be understood as consistent with one of the calculated breaks. Towards this end, we have in each case made use of the observed radio spectral indices before and after the steepening (Table I) and the appropriate relations from Table II.

Alternative 1 – The observed radio spectral steepening is the calculated second break: On first inspection of Table I the equality of $\alpha_A = \alpha_H = 0.79 \pm 0.10$ at frequencies above a few hundred MHz, makes it attractive to suggest that what is observed in the radio spectrum is the second break and that the index γ_0 of the injection electron spectrum has steepened to $\gamma_0 + 1$. An important consequence of this is that the equilibrium spectrum of cosmic ray protons with a power law index of 2.75 ± 0.03 in the energy range of 50–1000 GeV (Ryan *et al.*, 1972) should lead to an equilibrium secondary positron spectrum with an index of 3.75 ± 0.03 at energies $\gtrsim 5$ GeV. But the only available experimental measurement on high energy positrons in the range 5–30 GeV yields a value of 2.3 ± 0.5 (Buffington *et al.*, 1975). In spite of the large errors associated with this measurement, it is difficult to see how the true value can be as large as 3.75.

We have also estimated the values of μ, δ, and ξ which will satisfy the appropriate relations in Table II for the spectral indices in Table I. This was carried out for all possible combinations, with and without corrections for the metagalactic component in the radio data and by allowing for the associated errors. It turns out then that a physically meaningful set of values are obtained only when no correction is made for the metagalactic component and when synchrotron energy losses dominate over inverse Compton losses. The values thus obtained are: $\mu \leqslant 0.12$; $\delta \approx 0.93$–0.99; and $\xi \approx 0.25$–0.5. However, we know from experiments so far carried out on the energy spectra of cosmic ray heavy nuclei that μ should have a value between 0.3 and 0.6. (Smith *et al.*, 1973; Webber *et al.*, 1973; Juliusson, 1974; Daniel and Stephens, 1975 for a summary.) The upper limit of 0.12 required for μ for this alternative is therefore exceedingly difficult to compromise with the measured values.

For these reasons it seems unlikely that the observed steepening in the radio spectrum corresponds to the calculated second break in the electron spectrum.

Alternative 2 – The observed spectral steepening is the calculated first break: Bulanov and Dogel (1975) have also examined this alternative using the radio data below 800 MHz and obtained values of $\delta \leqslant 0.4$ and $\xi \leqslant 0.2$ by taking for γ_0 a value of 2.0 (Dogel *et al.*, 1975) and for μ a value of 0.2. However, it will be shown in what follows that the inferences made by Bulanov and Dogel (1975) are in conflict with those of the present investigation primarily because their analysis was based on the limited observational data then available.

We expect from the relations in Table II that in this alternative the radio spectral index before the break should have the same value of $(\gamma_0+\mu-1)/2$ in the direction of A and H but be different after the break with $\alpha_H>\alpha_A$. However, an inspection of Table I makes it evident that because of the small errors associated with the measured indices, such a condition can be attained before the break only if the corrections applied for the metagalactic component in Table I are nearly correct. Though such a correction will still leave the indices after the break unaffected, the larger errors associated with them are not inconsistent with the true value in the direction of H to be larger than that for A by as much as 0.1–0.2 as would be required for Table II. Notwithstanding this, it can be easily seen that these values of the spectral indices before and after steepening, typically of 0.27 and 0.70 respectively in the case of A and 0.27 and 0.90 in the case of H would necessarily imply that $\gamma_0 \lesssim 1.5$ and the electron spectrum suffers a first break of $\gtrsim 1$. On the basis of these considerations it does not seem possible to accept this alternative as well.

Alternative 3 – The injection spectrum is not a single power law: We recall at this stage that the calculations of Bulanov and Dogel (1975) have been made with the specific assumption that the injection electron spectrum is a single power law right down to low energies. If, however, this is not so and the injection spectrum has an intrinsic flattening which also occurs at energies below several GeV thereby contributing partly to the observed flattening of the radio spectrum at frequencies below a few hundred MHz, one can examine further what useful inference can be made. Let us therefore assume that the injection electron spectrum can be represented by two power laws with an index of γ_0' at low energies and γ_0'' at high energies such that the radio index observed at high frequencies is the asymptotic value after the first break; and the second break has not yet appeared. In this case it is clear that because of the fact that effects due to corrections for the metagalactic radio component and solar modulation also occur in the same energy domain below a few GeV, no reliable interpretation is possible below a few hundred MHz except to say that the electron spectral index $\gamma_0' \lesssim 1.54$. On the other hand regarding the observations beyond the steepening, one can still use the relations under columns 3a and 3b of Table II to deduce information on γ_0'', μ, δ, and ξ. It may also be noted that since the first break is greatly influenced by particles diffusing into the Halo, inverse Compton scattering is likely to be the dominant energy loss process and hence we first use the relations under column 3b of Table II to examine the implication of this alternative. At the very outset it is evident that there are far more variables to solve than the number of equations. We therefore proceed as follows. We allow at high frequencies α_A to take values between 0.7 and 0.9 and α_H between 0.75 and 0.95 such that they are consistent with the data values and errors summarized in Table I and at the same time permit $\alpha_H>\alpha_A$ as needed if the region we are concerned with here is $b<\lambda(E)<d$. Since we know that $2\alpha_A \leqslant \gamma_0'' \leqslant 2\alpha_A+1-\mu$, it implies that γ_0'' has limiting values of 1.4 and 2.8. Furthermore,

if one uses the value of μ to be between 0.3 and 0.6 and examines all possible combinations of values of α_A and α_H, it is found that if $\delta \leqslant 1$ and $\xi \leqslant 0.5$, then $\gamma_0 \approx 1.9$–2.3 and $\mu \approx 0.3$–0.5. These deductions are also well satisfied if one uses the relations under column 3a of Table II when synchrotron losses dominate over inverse Compton losses. Though from these considerations we are unable to assign any definitive values for δ and ξ, if one makes use of the value of ξ deduced in Section 3, one finds the δ should take values between 0.5 and 1. Thus, in summary the radio data can be satisfactorily and better understood within the framework of the diffusion model if the injection electron spectrum has an intrinsic flattening below a few GeV and the observed flattening of the radio spectrum is consequently due only partly to energy losses suffered by the radio electrons.

We have already pointed out that the electron energy spectrum in the range of 5–25 GeV should have an index of 2.58 ± 0.16. Measurements of the proton spectrum in the same energy interval (Smith *et al.*, 1973; Simpson, 1971) show that the index is between 2.5 and 2.6. Thus there exists the clear possibility that the injection spectrum of protons and electrons is the same. In such a case $\gamma_0 \approx 2.1$–2.3 and $\mu \approx 0.4$. As shown earlier these values are quite consistent with the radio data as well.

Alternative 4 – There is still one other possibility which deserves mention here. We have noted (Table I) that after correcting for the metagalactic component, the radio spectra in the directions of A and H are also quite consistent, within experimental errors, with the same spectral shape composed of two power laws with an exponent of ≈ 0.27 before the bend and ≈ 0.79 after. The possibility, therefore, cannot be ruled out that what we observe corresponds to the electron injection spectrum steepened only because of μ; no steepening due to energy loss processes have set in up to at least about 25 GeV. In this case it seems evident that the lifetime of cosmic rays will be significantly shorter than 10^8 yr.

5. Conclusions

In the present investigation we have made use of the latest available observational radio data compiled by Webster (1976) in the direction of the Anticenter and Halo Minimum and the treatment of the Disk-Halo diffusion model for cosmic ray propagation by Bulanov and Dogel (1975), and deduced interesting information on the radio spectra and the cosmic ray electrons responsible for them. From these we are able to make the following inferences:

(i) Observational evidence indicates that the steepening of the radio spectrum in the direction of H is centered about 200 MHz and at a higher frequency of about 330 MHz in the direction of A; the spectral indices at frequencies beyond the steepening in the two directions have the same value within errors. The former observation provides evidence for a lower mean magnetic field in the direction of H compared to that of A and thereby lend support for the existence of a radio halo and the adoption of the diffusion model for cosmic ray propagation. It has also been used to estimate the value of ξ as 0.24–0.37; the radiating distance L needed in the direction of the Halo Minimum is $\gtrsim 6$ kpc. It is important to mention here that the method of estimating ξ and L is not dependent on any detailed assumption other than the need for a Disk-Halo confinement volume for cosmic rays.

(ii) An injection electron spectrum with a single power law up to the lowest energies will not be able to explain the radio observations satisfactorily within the diffusion model.

(iii) The observed steepening in the radio spectra, and hence in the interstellar electron spectrum, can be satisfactorily understood as due partly to the deviation from a power law injection spectrum below a few GeV, and partly to the first break arising from energy losses suffered by electrons at about the same energy region. The most plausible choice of parameters in this case will be $\gamma_0 \approx 1.9$–2.3; $\mu = 0.3$–0.5; $\delta \approx 0.5$–1; and $\xi \simeq 0.24$–0.37. Furthermore, if electrons and protons have the same injection spectrum above a few GeV, the value of the index can be further narrowed to 2.1–2.3. In this case, one can very well explain the radio observations as well as the cosmic ray proton and electron data.

It is evident from the foregoing that more reliable and accurate measurements on cosmic rays negatrons and positrons, the radio spectra from other galactic directions and the value of μ from nucleonic components will permit more definitive conclusions.

Acknowledgement

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Note added in Proof: Dogel *et al.* (private communication, 1977) have now corrected the expression for α_H given in Table II, column 3a. The new expression should read

$$\alpha_H = \frac{1}{2}\left[\gamma_0 - \xi \frac{(\gamma_0 + 2)(1-\mu)}{2(1-2\xi) + \xi(1-\mu) + 2(1-\delta)}\right].$$

However, this does not affect any of our conclusions.

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