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Directional dependence of spin currents induced by Aharonov-Casher phase

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Abstract

We have calculated the persistent spin current of an open ring induced by the Aharonov-Casher phase. For unpolarized electrons there exist no persistent charge currents, but persistent spin currents. We show that, in general, the magnitude of the persistent spin current in a ring depends on the direction of the direct current flow from one reservoir to another. The persistent spin current is modulated by the cosine function of the spin precession angle. The nonadiabatic Aharonov-Casher phase gives anomalous behaviors. The Aharonov-Anandan phase is determined by the solid angle of spin precession. When the nonadiabatic Aharonov-Anandan phase approaches a constant value with the increase of the electric field, the periodic behavior of the spin persistent current occurs in an adiabatic limit. In this limit the periodic behavior of the persistent spin current could be understood by the effective spin-dependent Aharonov-Bohm flux.

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The electric and magnetic properties of mesoscopic systems have recently received much attention in the light of several experimental observations [1–4]. Mesoscopic physics deals with the structure made of metallic or semiconducting material on a nanometer scale. The length scale associated with the dimensions in these systems are much smaller than the inelastic mean free path or phase breaking length. In this regime, an electron maintains phase coherence across the entire sample. In general, a system with a large degree of freedom is called mesoscopic if the length up to which the wave function retains phase coherence exceeding the size of the system. The main characteristics of mesoscopic systems is the quantum coherence. These systems, which are now accessible experimentally, provide an ideal test ground for the quantum mechanical models beyond the atomic realm. These systems have revealed, several interesting and previously unexpected quantum effects at low temperatures [1,4–6], which are associated with the quantum interference of electron waves, quantization of energy levels, and discreteness of electron charge. Persistent currents in mesoscopic normal metal rings are purely mesoscopic effects in the sense that they are strongly suppressed when the ring size exceeds the characteristic dephasing length of the electrons or the inelastic mean free path [7,8]. Studies have been extended to include multichannel rings, spin-orbit coupling, disorder, electron-electron interaction effects, etc. [1,9,10].

Theoretical treatments up to date have been mostly concentrated on isolated rings. Persistent current occur not only in isolated rings but also in the rings connected via leads to electron reservoirs, namely open systems [11–13]. In a recent experiment Maily et al. have measured the persistent currents in both closed and open rings [8]. Recently Jayannavar et al. noted the several novel effects related to persistent currents can arise in open systems, which have no analogue in closed or isolated systems [14–17]. Especially the directional dependence of persistent current in open system can be useful for separating the persistent current from noises.

In 1984, Aharonov and Casher (AC) [18] noticed the possibility of the dual effect of the AB phase and discovered the AC phase for a neutral magnetic moment encircling a charged line. In a fundamental generalization of Berry's idea [19], Aharonov and Anandan

(AA) removed the adiabatic restriction and studied the geometric phase for the nonadiabatic cyclic evolution [20]. By removing the dynamical part, Aharonov and Anandan defined the nonadiabatic geometric phase for the cyclic evolution called the AA phase. Qian and Su [21] has demonstrated the existence of the AA phase in the AC effect. In the adiabatic limit this AA phase becomes the spin-orbit Berry phase introduced by Aronov and Lyanda-Geller [22]. Loss, Goldbart, and Balatsky discovered that Berry phase can induce persistent spin currents [23]. And Balatsky and Altshuler noticed spin-orbit interaction produces persistent spin and mass currents [24]. Along this line of study of the spin phase effects on the electron transport problem, Ryu [25] has shown that various spin motive forces [26] can be described in a unified fashion based on the Goldhabor-Anandan [27] gauge theory for a low energy spin particle. The persistent current induced by the Aharonov-Casher (AC) phase is much smaller than the persistent current induced by the Aharonov-Bohm (AB) phase, so the directional dependence will be extremely useful for the detection of that current. The transport behavior induced by the AC phase is recently studied [28,29].

In our present treatment we consider a one-dimensional metal loop of length L coupled to two electron reservoirs as shown in Fig. 1. In the ring there is a cylindrically symmetric electric field to produce a spin-orbit interaction. This spin-orbit interaction gives the AC phase with cyclic evolution. This idealization to one-dimension corresponds experimentally to a network of high-mobility quantum wires with narrow width such that only the lower subband is filled. Our calculations are for noninteracting systems of electrons. In such a geometry the AC effect manifests itself not only in a transport phenomenon but also in a persistent current. The left and right reservoirs are characterized by chemical potentials μ_1 and μ_2, respectively. We have introduced a δ-function impurity of strength V at a length $L_d(= 2L)$ to the right of the metal loop (marked by $\times$ in Fig. 1). The presence of the impurity breaks the spatial symmetry of the system. We also restrict to the case of $L_1 = L_2$, to avoid the additional contribution arising due to the difference in transport current across upper and lower arms. If $\mu_1 > \mu_2$ the net current flows from the left to the right and vice versa, if $\mu_1 < \mu_2$. The scattering of the electronic wave function occurs at the junctions J_1, J_2

and at the impurity site I. In our model we have complete spatial separation between elastic processes in the loop and the inelastic processes in the reservoirs. The inelastic processes in the reservoir are essential to obtain a finite conductance.

When $\mu_1 > \mu_2$, the steady flux of electrons with an energy E is injected from the reservoir 1. These electrons moving to the right are first scattered at the junction J_1 and subsequently at J_2 and I (together with multiple reflections at J_1, J_2, and I). The electrons emitted by the reservoir 2 are first scattered at I and subsequently at J_2 and J_1. Since there is no spatial symmetry, for these two different cases the electron wave function (scattering states) has a different complex amplitude at J_1 and J_2. The persistent current in a metallic loop is sensitive to the boundary condition, and hence we observe that the magnitude of the persistent current depends on the direction of the current flow. Obviously the conductance of an entire network (calculated via the quantum transmission coefficient) does not depend on the direction of the current flow. This implies that there is no simple scaling relation between the persistent currents and the conductance of the entire network.

First we consider the situation wherein the direct current flows from the left reservoir to the right reservoir. In the presence of cylindrically symmetric electric fields $\mathbf{E}$, the one-particle Hamiltonian for non-interacting electrons is given by

$$H = \frac{1}{2m_e}(\mathbf{p} - \frac{\mu}{c}\boldsymbol{\sigma} \times \mathbf{E})^2, \tag{1}$$

where $\boldsymbol{\sigma} \times \frac{\mathbf{E}}{2}$ represents a spin-orbit coupling and σ^α with $\alpha = 1, 2, 3$ are Pauli matrices. Adopting a cylindrical coordinate system and the electric field $\mathbf{E} = E(\cos\chi\hat{r} - \sin\chi\hat{z})$ we have the following Hamiltonian in a closed ring

$$H = \frac{\hbar^2}{2m_e a^2}\left(-i\partial_\phi - \frac{\mu E a}{2\hbar c}(\sin\chi\cos\phi\sigma_x + \sin\chi\sin\phi\sigma_y + \cos\chi\sigma_z)\right)^2, \tag{2}$$

where a is the radius of the ring. The eigenfunctions $\Psi_{n,\pm}$ and eigenvalues $E_{n,\pm}$ of Hamiltonian (2) in a closed ring are obtained as [30]

$$\Psi_{n,\pm} = \frac{1}{\sqrt{2\pi}}e^{in\phi}\begin{pmatrix} \cos\frac{\beta_\pm}{2} \\ \pm e^{i\phi}\sin\frac{\beta_\pm}{2} \end{pmatrix},$$

$$E_{n,\pm} = \frac{\hbar^2}{2ma^2}\left(n - \frac{\Phi^{\pm}_{\rm AC}}{2\pi}\right)^2 \ , \tag{3}$$

and $$\Phi^{\pm}_{\rm AC} = -\pi(1-\lambda_{\pm}) \ ,$$

where $\lambda_{\pm} \equiv \pm\sqrt{\omega_1^2 + (\omega_3+1)^2}$ are eigenvalues of $\omega_1\sigma^1 + (\omega_3+1)\sigma^3$, and the angle $\beta_{\pm}$ are defined by $\tan\beta_+ \equiv \omega_1/(\omega_3+1)$, and $\beta_- = \pi - \beta_+$. Here ω_1 and ω_3 are denoted by $\omega_1 \equiv \frac{\mu E a}{\hbar c}\sin\chi$ and $\omega_3 \equiv \frac{\mu E a}{\hbar c}\cos\chi$ and $\mu = e\hbar/2m_e c$ is the Bohr magneton. The evolution of a spin state in the presence of the electric field is determined by the following parallel transporter [30].

$$\Omega(\phi) = P\exp\left[i\frac{\mu E a}{2\hbar c}\int_0^{\phi}(\sin\chi\cos\phi'\sigma^1 + \sin\chi\sin\phi'\sigma^2 + \cos\chi\sigma^3)d\phi'\right], \tag{4}$$

where P is the path ordering operator. It relates the wave function $\Psi(\phi)$ to $\Psi(0)$. In general, the spin state that has been parallel transported around the ring does not return to the initial spin state. However, for the special initial spin state, the spin state after a parallel transport around the ring returns to the initial state except the phase factor as $\Psi(2\pi) = \exp[i\Phi^{(\pm)}_{\rm AC}]\Psi(0)$. This spin state is the eigenstate of $\omega_1\sigma^1 + (\omega_3+1)\sigma^3$ [30]. Then the spin state at ϕ is obtained as

$$\Psi^{(\pm)}(\phi) = e^{i(1-\lambda_{\pm})\phi/2}\begin{pmatrix} \cos\frac{\beta_{\pm}}{2} \\ \pm e^{i\phi}\sin\frac{\beta_{\pm}}{2} \end{pmatrix} . \tag{5}$$

After a cyclic evolution this spin state returns to the initial state apart from the AC phase.

To derive an expression for the persistent current and the transmission coefficient, we apply the one-dimensional quantum waveguide theory developed in Ref. [31]. We use the local coordinate system for each circuit such that the x coordinate is taken along the electron current flow. The origin of each local coordinate is taken at each junction. At each junction charge density and current are conserved, and electron spins are matched. We assume that an electron spin is not changed while electron passes a junction and neglect the spin-flip process as in Ref. [22]. Since the two reservoirs are mutually phase incoherent, we have to solve the problem separately for the electrons emitted from the left and the right reservoirs. First we consider the case wherein electrons are emitted from the left reservoirs. The reservoirs emit electron carriers with the Fermi distribution $f(E) = (\exp[(E-\mu_1)/k_B T]+1)^{-1}$. This results in a current flowing from the left to the right.

The textured electric field can be made by putting the extra charge in the center of the ring together with a circular gate along the ring. Then except for the point I (where we have introduced a δ-function potential), in the input and output leads, there is no normal electric field, and the Hamiltonian (1) becomes that for the free particle

$$H = \frac{1}{2m_e} p_x^2, \tag{6}$$

since $p_y = 0$ and $E_z = 0$ in the leads. Thus the incident and reflected spin states acquire only the phases of ikx and $-ikx$, respectively. Since it is always possible to use the eigenstates of the ring at $\phi = 0$ (J_1) as the basis of general incident spin states, and there is no spin flip process, we can treat the eigenstates of the ring at $\phi = 0$ as the incident spin states separately. This spin state changes its direction during the movement along the ring as described in Eq. (5).

When an electron is transported from the input junction in the clockwise direction along the upper loop, it picks up a phase $\gamma = 1/2\Phi_{\mathrm{AC}}^{\pm}$ at the output junction. And when the electron is transported in the counter clockwise direction along the lower loop, the electron acquires the phase $\delta = -1/2\Phi_{\mathrm{AC}}^{\pm}$. Thus the total phase around the loop becomes $(\gamma - \delta) = \Phi_{\mathrm{AC}}^{\pm}$. Since the effect of the electric field on the above spin state brings the phase shift of wave function, the energy of the electron in the loop $E = \hbar^2[k_1^{\pm} - \Phi_{\mathrm{AC}}^{\pm}/2\pi r]^2/2m$ should be equal to the energy of the injected electron $\hbar^2 k^2/2m$. Thus we take the wave vector $k_1^{\pm} = k + \Phi_{\mathrm{AC}}^{\pm}/2\pi r$ for the electron moving along the clockwise direction, and $k_2^{\pm} = k - \frac{\Phi_{\mathrm{AC}}^{\pm}}{2\pi r}$ for the electron moving in the opposite direction. Let the spin state $\left(\cos\beta_{\pm}/2\ ,\ \ \pm e^{i\phi}\sin\beta_{\pm}/2\right)^t$ be $\mathcal{X}_{\phi}^{\pm}$, where t means the transpose of the vector. Then the wave functions in the circuits can be written as

$$
\begin{aligned}
\Psi_1^{\pm} &= (e^{ikx} + a^{\pm} e^{-ikx})\mathcal{X}_0^{\pm}, \\
\Psi_2^{\pm} &= (c_1^{\pm} e^{ik_1^{\pm} x} + c_2^{\pm} e^{-ik_2^{\pm} x})\mathcal{X}_{\phi}^{\pm}, \\
\Psi_3^{\pm} &= (d_1^{\pm} e^{ik_2^{\pm} x} + d_2^{\pm} e^{-ik_1^{\pm} x})\mathcal{X}_{\phi'}^{\pm}, \qquad (7) \\
\Psi_4^{\pm} &= (f_1^{\pm} e^{ik_2^{\pm} x} + f_2^{\pm} e^{-ik_1^{\pm} x})\mathcal{X}_{\pi}^{\pm}, \\
\Psi_5^{\pm} &= g^{\pm} e^{ikx} \mathcal{X}_{\pi}^{\pm},
\end{aligned}
$$

where the wavefunctions Ψ_{1-5} are for the following regions, input lead, $J_1 - J_2$ upper arm, $J_1 - J_2$ lower arm, $J_2 - I$ and output lead, respectively. We use the Griffith boundary conditions [32–34] at the junctions. We have obtained analytical expressions for the persistent currents. However, here we present our results graphically since the analytical expression is too lengthy.

In the open system, the transport current is symmetric with respect to the AC flux. Hence the persistent current is defined as the antisymmetric part of the ring current with respect to the AC flux. As is well known, the Hamiltonian considered has the time-reversal symmetry. Because of this time reversal symmetry the persistent charge currents for the unpolarized incident electrons always vanish. In the presence of the net spin polarization the AC effect leads to charge currents proportional to $n_{\uparrow} - n_{\downarrow}$, where $n_{\uparrow}$ and $n_{\downarrow}$ are the number of spin up electrons and down electrons, respectively. In Fig. 2 we have plotted the persistent charge currents for the incident spin-up eigenstate of the ring at $\phi = 0$ in the dimensionless unit J/k as a function of the normalized field strength η for tilt angles $\chi = 0$(A), $\pi/2$(B), $3\pi/4$(C), and π(D), the dimensionless momentum $kL = 7$, and the impurity strength $VL = 10$. In Fig. 2 the solid and dashed curves represent the magnitudes of the persistent charge currents flowing in the loop ${J_C^+}^L/k$, and ${J_C^+}^R/k$, respectively for the spin-up eigenstate of the ring. Where the subscript L and R represent when the dc current flows in the left and right directions, respectively. One can readily notice from Fig. 2 the difference between the values of the persistent charge current for the electron emitted from the left reservoir (solid line) and that for the electron emitted from the right reservoir (dashed line). This shows clearly that the persistent charge currents in a metal loop connected to two reservoirs depend on the direction of direct current flow from one reservoir to the other. For $V = 0$ we can recover the symmetric case, so there is no directional dependence on the persistent charge currents. We can also see the anomalous behaviors in the persistent charge current when the normalized electric field η is small and $\chi \neq 0, \pi$. This anomalous behavior comes from the nonadiabatic AC phase. The persistent charge currents for the spin-down eigenstate of the ring is exactly opposite to the spin-up persistent charge currents because of the time reversal symmetry.

The persistent spin current J_S^a is defined as the antisymmetric part of $< \Psi_r|(\mathbf{p} - \mu r/c \cdot \sigma \times \mathbf{E})_\phi \sigma^a/\hbar|\Psi_r >$. Where $a = 1$, 2 and 3 is the spin indices and $|\Psi_r >$ is the state on the ring. It has the additional contribution from σ^a operator to the persistent charge current. Because of the cylindrical symmetry, the persistent spin current with the x and y direction vanishes. From $< \Psi_{n,\alpha}|\sigma^3|\Psi_{n,\alpha} >= \cos\beta_\alpha$, and $\cos_{\beta_-} = -\cos\beta_+$, the persistent spin current $< J_S^3 >$ of spin-down eigenstate of the ring is the same as that of spin-up eigenstate. This implies that the persistent spin current would be independent of spin polarization. It should be noticed that the magnetic field necessary for spin polarization is not required to observe the persistent spin current, different from the persistent charge current. Fig. 3 shows the persistent spin current as a function of normalized electric field strength η for the same values as that of the persistent charge current. In the case of $\chi \neq 0, \pi$, the anomalous behavior is more definite than that of the persistent charge current, because of the additional contribution from the modulation $\cos\beta_+$.

We give a simple picture to understand the anomalous behavior of persistent spin current intuitively. We consider the spin up eigenstate only in the following since SO interaction term is time-reversal invariant. It is also the eigenstate of $\omega_1\sigma^1 + (\omega_3 + 1)\sigma^3$ with spin up. In a ring the system has a cylindrical symmetry, so the spin direction at ϕ has the polar angle β_+ and the azimuthal angle ϕ. It means that the spin precesses about $\hat{z}$ direction with an angle β_+ during the cyclic evolution. From the similarity of the mathematical structure of the AC effect with the AB effect we can rewrite the spin-orbit coupling term as the effective spin dependent gauge field $\frac{e}{c}\mathbf{A}_{\text{eff}}$, with $\mathbf{A}_{\text{eff}} = \frac{\mu}{\hbar e}(\mathbf{S}\times\mathbf{E})$. Where $\mathbf{S}$ is the spin operator. In the semi-classical approach a spin is a three-dimensional vector with a certain direction. The $\frac{\mu}{\hbar e}(\mathbf{S}\times\mathbf{E})$ is calculated as $\frac{\mu E}{2e}\cos(\beta_+ - \chi)\hat{\phi}$. This is constant during the motion as far as the field strength E and the tilt angle χ is fixed. Hence this $\frac{\mu}{\hbar e}(\mathbf{S}\times\mathbf{E})$ is described as $\mathbf{A}_{\text{eff}} = \frac{\Phi}{2\pi a}\hat{\phi}$. Where $\Phi = (\nabla\times\mathbf{A}_{\text{eff}})\cdot\mathbf{F}$ is the magnetic flux through the ring section area $F(=\pi a^2)$. The phase acquired from this effective AB situation - we call this $\Phi_{\text{AB}}^{\text{eff}}$ - is

$$\Phi_{\text{AB}}^{\text{eff}} = \frac{\pi e a^2 E}{2m_e c^2}\cos(\beta_+ - \chi). \tag{8}$$

This effective spin dependent AB phase acquired by a charge e around a flux $\Phi = (\nabla\times\mathbf{A}_{\text{eff}})\cdot\mathbf{F}$ turns out to be the same as the dynamical phase acquired by a spin due to the SO interaction [21,30]. In the AB situation a charge does not precess, but in the AC situation the spin precesses during the cyclic evolution, bringing an additional effect. The difference between the effective AB phase for a charge and the AC phase for a spin becomes the AA phase, which comes from the extra spin degrees of freedom.

The AA phase is associated with the spin precession. To get this phase we parametrize the path of the spin by the azimuthal angle ϕ. The spin state $|\mathbf{S}\cdot\hat{n};+>$ satisfies

$$\mathbf{S}\cdot\hat{n}(\phi)|\mathbf{S}\cdot\hat{n};+> \;=\; \frac{\hbar}{2}|\mathbf{S}\cdot\hat{n};+> \;, \tag{9}$$

where $\hat{n}(\phi)$ is the unit vector with polar angle β_+ and azimuthal angle ϕ. And the spin state $|\mathbf{S}\cdot\hat{n};+>$ becomes $\mathcal{X}_{\phi}^{+}$. Since this spin $\mathbf{S}$ remains parallel to $\hat{n}(\phi)$ during the rotation, formally this is identical to the problem considered by Berry for a spin $\mathbf{S}$ in an adiabatically changing magnetic field $\mathbf{B}(t)$.

$$g\mathbf{S}\cdot\mathbf{B}(t)|\mathbf{B}(t),\ m_s> \;=\; E|\mathbf{B}(t),\ m_s> \tag{10}$$

where g is related to the gyromagnetic ratio and m_s is the component of the spin along the direction of $\mathbf{B}(t)$. Berry showed that $\gamma(C) = -m_s\Omega(C)$, where $\gamma(C)$ is Berry's phase and $\Omega(C)$ is the solid angle subtended by the curve C with respect to the origin $\mathbf{B} = 0$. In our case, the phase accumulated is

$$\gamma(C) \;=\; -\frac{1}{2}\Omega(C)\;, \tag{11}$$

where $\Omega(C)$ is the solid angle subtended by the loop C with respect to $\mathbf{n} \;=\; 0$. In this case C is a circle and $\Omega(C) \;=\; 2\pi(1-\cos\beta_+)$. This geometric phase $\gamma(C)$ is the AA phase, and thus the AA phase becomes $-\pi(1-\cos\beta_+)$.

From the above intuitive picture, the anomalous behavior is understandable by the precession of the spin. The AA phase is determined by the solid angle of spin precession and the dynamical phase is the effective AB phase induced by the spin dependent AB flux. Let us first consider the adiabatic approximation of the spin evolution. The condition for the adiabatic limit is $\eta \gg 1$. In this case the spin state is an eigenstate of the parallel transporter. The dynamical phase of the adiabatic solution is given by $\Phi^{\pm}_{\rm dyn} \approx \pm\sqrt{\omega_1^2+\omega_3^2}$ from Eq. (8). Also the adiabatic approximation of the AA phase is the Berry phase, $\Phi^{\pm}_{\rm Berry} = -\pi(1\mp\cos\chi)$. These phases are equal in Ref. [22] for proper parameter transformation. In this limit, the spin precession angle β_+ becomes the fixed tilt angle χ. The AA phase gives a constant shift to AC phase. That is, the effective spin dependent Φ^{eff}_{AB} determines the periodic behaviors. For $\chi = 0$, the AC phase consists of the dynamical phase only and persistent spin current oscillates periodically. It is clear that the anomalous behaviors of the persistent spin currents come from the change of the spin precession angle with varying electric field. For a fixed tilt angle the precession angle β_+ of spin-up depends only on the field strength η as

$$\cos\beta_+ = \frac{\eta\cos\chi+1}{\sqrt{\eta^2\sin\chi^2+(\eta\cos\chi+1)^2}}.$$

For $\eta > 0$, this can be negative for $\chi > \pi/2$. We can see this change of sign of the persistent spin current in Fig 3 (C) and (D) in comparison with the persistent charge current.

In Fig. 4 we have plotted the persistent spin currents J_S^{+R}/k and J_S^{+L}/k as a function of dimensionless impurity potential VL, for a fixed value of $kL = 7$, for $\eta = 6$ and for tilt angle $\pi/2$. In this case the modulation $\cos\beta_+$ has a fixed value 0.16. The magnitude J_S^{+R}/k decreases monotonically to zero as $VL \to \infty$. This is due to the fact that in this limit electrons emitted by the right reservoir do not enter the loop and cannot contribute to the persistent spin currents. The absolute magnitude of J_C^{+L}/k saturates to a value in the same limit. This corresponds to a situation where the loop is connected to a single reservoir μ_L, where the connection is truncated at the point I (the impurity state). In Fig. 5 we have plotted the dimensionless conductance T ($|g^+|^2$) as a function of kL, for $\eta = 6$, for $VL = 10$ and for $\chi = \pi/2$. The electrical conductance exhibits a peak for certain values of kL. These peaks occur due to the resonance of the incident electron energy coincides with one of the eigenenergies of the ring or with one of the bound state energies of the stub $J_2 - I$. But the peaks do not appear at the exactly same energy as the eigenenergies since the multiple scatterings at junctions shift the energy levels. And we can see the effect of the bound state of the stub $J_2 - I$ will decrease as kL becomes much higher than VL.

In conclusion, we have shown that the magnitude of the persistent spin current induced by the Aharonov-Casher phase in a normal metal loop connected to two reservoirs depends on the direction of the direct current flow, which should be an experimentally verifiable feature. In the presence of the AC flux, the Hamiltonian has the time reversal symmetry, so the persistent charge current always vanishes for unpolarized incident electrons. But the persistent spin current still exists. That is, if the spin-up electrons of the ring circles counter-clockwise, then the spin-down electrons evolves clockwise and vise versa. Since the conductance of the entire network does not depend on the direction of the direct current flow, there is no simple scale relationship between the persistent spin currents and the conductance of the entire network. The anomalous behaviors appear when the spin precession angle changes as a function of the field strength. The periodic behavior in the adiabatic limit can be understood as the effect of spin dependent AB flux. In this case the spin precession angle does not change so that the AA phase is constant. The difference between the magnitude of the persistent spin currents (on the direction of the current flow) can be made significant by adjusting the impurity potential. This can be achieved experimentally by having a gate in one of the leads connected to the reservoirs and by appropriately varying the gate voltage. Such an experiment can also be useful for separating the persistent spin currents from other parasitical currents (or signals) associated with measurements. When there are time reversal symmetry breaking terms in the Hamiltonian, we expect that the persistent charge current will also appear even for unpolarized incident electrons. The natural terms are the Zeeman coupling and the AB flux of the localized magnetic field. The directional dependence of spin and charge currents in the presence of the Zeeman coupling, AB flux and AC flux is currently under study by us.

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FIGURES

FIG. 1. An open metallic loop connected to two electron reservoirs. There exist a cylindrically symmetric electric field which gives the AC flux.

FIG. 2. The persistent charge current as a function of the normalized electric field η for a fixed value of $kL = 7$, $VL = 10$, tilt angles (A) $\chi = 0$, (B) $\pi/2$, (C) $3\pi/4$, and (D) π. The solid line represents persistent charge current for ${J_C^+}^L/k$ and the dashed curve represents ${J_C^+}^R/k$.

FIG. 3. The persistent spin currents vs η for same values in Fig. 2. The solid line represents ${J_S^+}^L/k$ and dashed curve represents ${J_S^+}^R/k$. And the dotted line represents the modulation function as an envelope.

FIG. 4. The persistent spin currents vs the strength of impurity potential for fixed values of $kL = 7$, $\eta = 6$ and $\chi = \pi/2$. The solid line represents ${J_S^+}^L/k$ and dashed curve represents ${J_S^+}^R/k$.

FIG. 5. Conductance oscillations vs kL for $\eta = 6$, $VL = 10$ and χ $\pi/2$.

Fig. 1

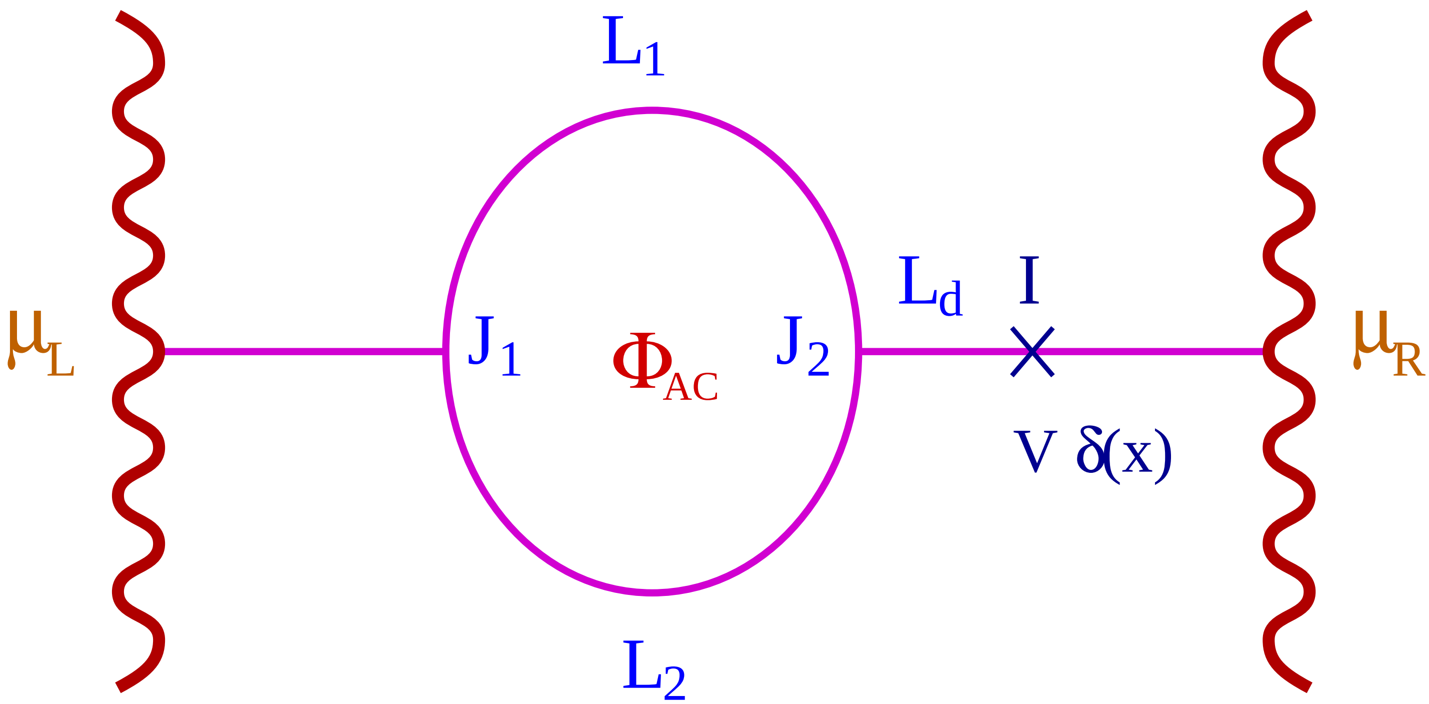

Fig. 2

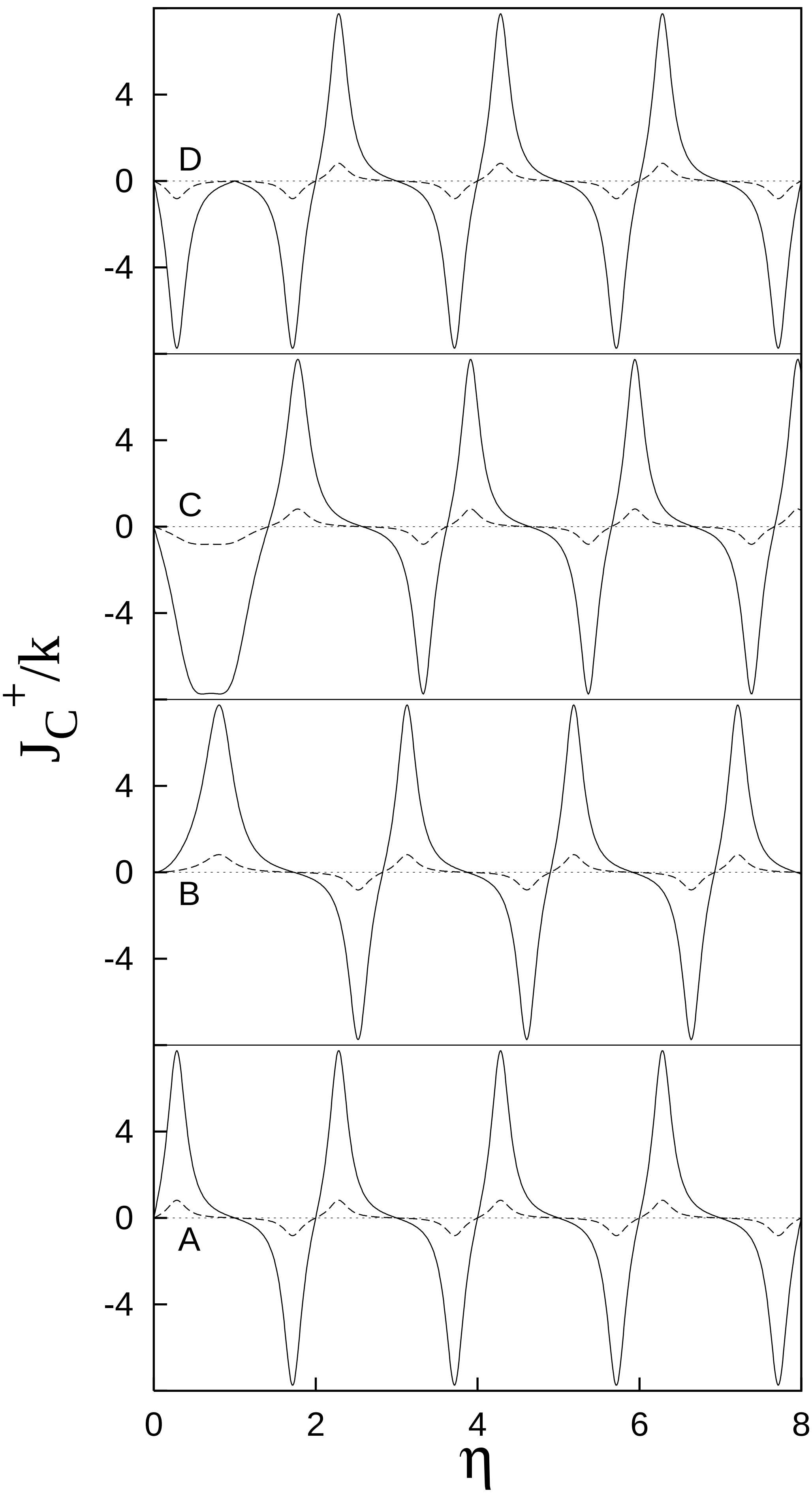

Fig. 3

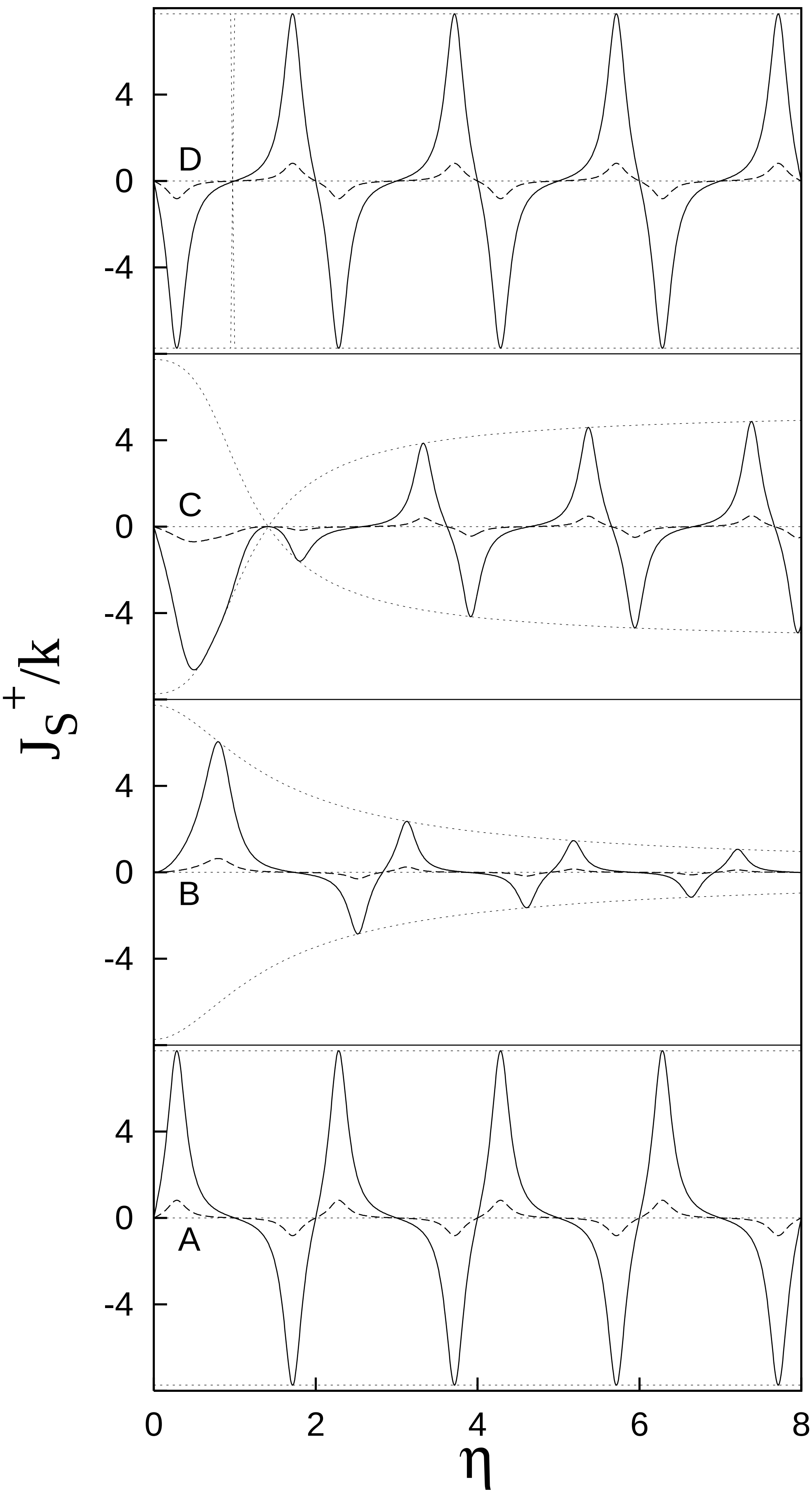

Fig. 4

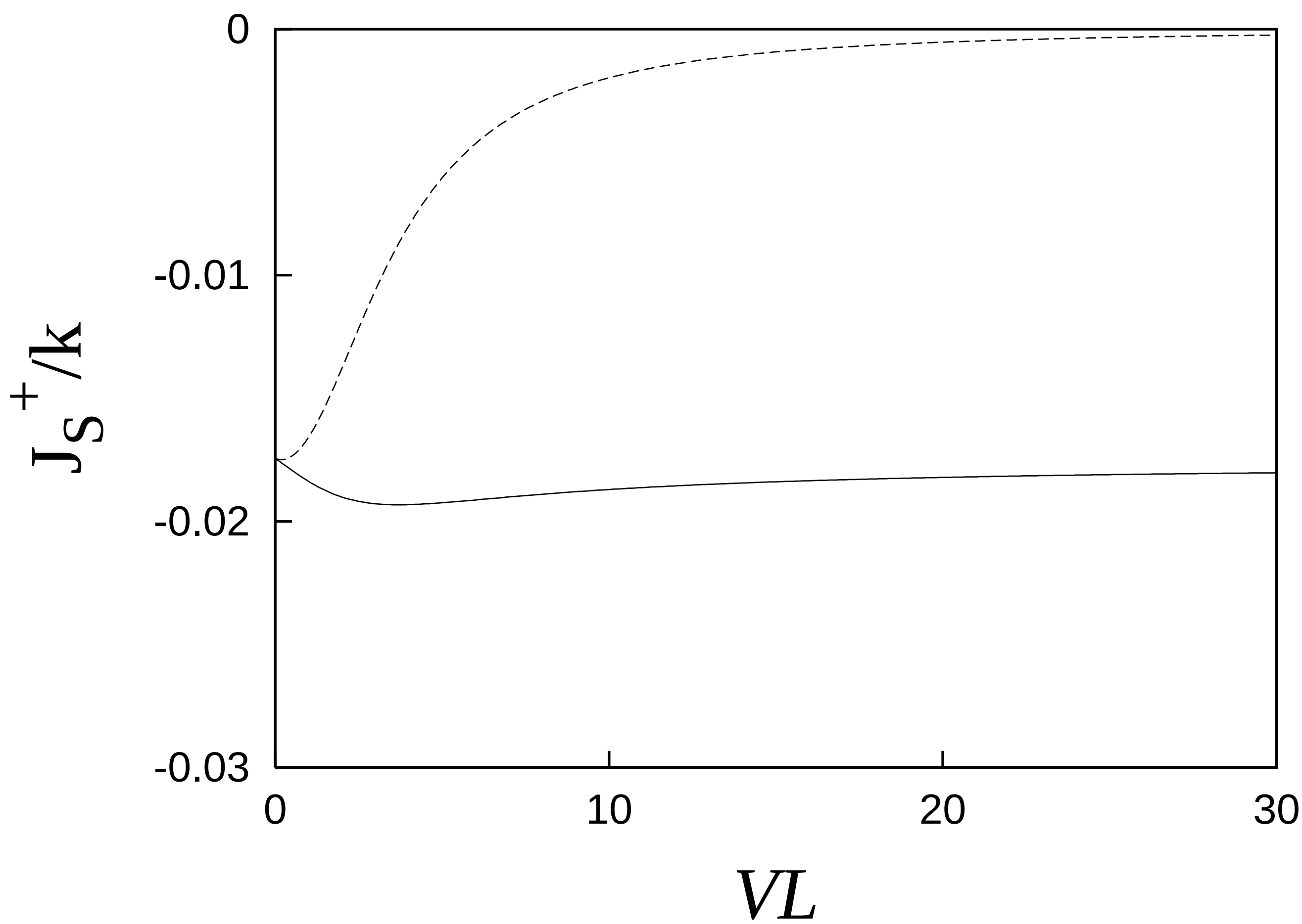

Fig. 5