Enhanced thermodynamic efficiency in time asymmetric ratchets

Raishma Krishnan,[1,*] Soumen Roy,[1,†] and A. M. Jayannavar[1,‡]

[1] *Institute of Physics, Sachivalaya Marg, Bhubaneswar 751005, India*

Abstract: The energetic efficiency of an overdamped Brownian particle in a saw tooth potential in the presence of time asymmetric forcing is studied in the adiabatic limit. An error made in the earlier work on the same problem in literature is corrected. We find that asymmetry in potential together with temporal asymmetry in forcing leads to much enhanced efficiency without fine tuning of parameters. The origin of this is traced to the suppression of backward current. We also present a comparative study between the role of continuous and discontinuous ratchet forces on these measurable quantities. We find that the thermal fluctuations can optimize the energy transduction, the range of parameters, however, being very small. This ratchet model also displays current reversals on tuning of parameters even in the adiabatic regime. The possible relationships between nature of currents, entropy production and input energy are also addressed.

PACS numbers: 05.40.-a, 05.60.Cd, 02.50.Ey.

I. INTRODUCTION

The study of the nature of directed motion induced by random noise in periodic systems in the absence of a bias has attracted wide interest. By now, the rectification of thermal fluctuations have become a major area of research in nonequilibrium statistical mechanics. The presence of unbiased nonequilibrium perturbations, either stochastic or deterministic, together with a broken spatial or temporal asymmetry play a key role in obtaining directed motion without violating the second law of thermodynamics. Such systems or ratchets convert nonequilibrium fluctuations into useful work in the presence of load. Moreover, in these systems, noise play a constructive role (i.e., transformation of noise in spatially periodic systems into directed current). A large family of models of Brownian ratchets [1, 2, 3, 4] have been introduced to obtain insight into the basic mechanism of noise rectification. Some of them are flashing ratchets, rocking ratchets, time asymmetric ratchets, frictional ratchets etc [2]. Numerous studies have been carried out to understand the nature of currents, their possible reversals and also the efficiency of energy transduction. The results obtained are utilized to develop proper models that efficiently separate particles of micro and nano sizes and also for the development of machines at nano scales [4]. Such models are also the prototype to understand the basic mechanism of operation of molecular motors or protein molecules in our cells that transfer cargo and organelles very efficiently in a very noisy environment. It also has extensions in game theory under the name of Parrondo's paradox [5]. These are basically counter intuitive games based on translation of the dynamics of Brownian particle in a flashing ratchet to gambling games. Here, two losing games (or strategies), when alternated randomly or periodically give rise to a winning game. These paradoxes have a profound role in several multidisciplinary areas.

With the emergence of a separate subfield called stochastic energetics [6, 7] it is possible to establish the compatibility between Langevin or Fokker-Planck formalism, which describes stochastic dynamics, and the laws of thermodymanics. Using this framework one can calculate various physical quantities such as thermodynamic efficiency of energy transduction [8], energy dissipation (hysteresis loss), entropy (entropy production) [9] etc., thereby providing a new tool to study systems far from equilibrium.

The intrinsic irreversibility associated with ratchet operation makes the ratchet to be less efficient. For example, the attained value of efficiency in flashing and rocking ratchet were found to be below the subpercentage regime. However, it has been shown that at very low temperatures fine tuning of parameters could lead to a larger efficiency, the regime of parameters being very narrow [10]. Optimization of energetic efficiency of the sawtooth ratchet in presence of spatial symmetry but in presence of time symmetric rocking has been worked out in detail in [10]. Moreover, protocols to optimize the efficiency is given in [10, 11].

Recently Makhnovskii et al. [12] constructed a special type of flashing ratchet with two asymmetric double-well periodic-potential- states displaced by half a period. Such flashing ratchet models were found to be highly efficient with efficiency an order of magnitude higher than in earlier models [6, 7, 8, 13]. The basic idea behind this enhanced efficiency is that even for diffusive Brownian motion the choice of appropriate potential profile ensures suppression of backward motion and hence reduction in the accompanying dissipation. Similar to the case of flashing ratchets [12], we had earlier [14] stud-

*Electronic address: raishma@iopb.res.in
†Electronic address: sroy@iopb.res.in
‡Electronic address: jayan@iopb.res.in

ied the motion of a particle in a rocking ratchet by applying a temporally asymmetric but unbiased periodic forcings [15, 16, 17, 18] in the presence of a sinusoidal potential. The efficiency obtained was very high, much above the subpercentage level (about $\sim 30-40\%$ without fine tuning) in the presence of temporal asymmetry alone.

In the present work we study the same problem but in a saw tooth potential and make a comparison so as to elucidate the sensitivity of these physical quantities on the smoothness or regularity of the underlying ratchet potential. The important underlying factor is the temporal asymmetry [15, 16, 17, 18] in the external forcing which leads to noise induced currents in the absence of external bias even for the case of spatially symmetric potential. In this adiabatically rocked time asymmetric correlation ratchet, a larger force field is applied for a short time interval of period in one direction as compared to a smaller force for a longer time interval in the other direction, see Fig. 1. Some qualitative differences between the smooth and piecewise linear ratchet potential which are observed is discussed. The surprisingly sensitive dependence of the physical quantities such as unidirectional current on the degree of regularity or smoothness of the ratchets (continuous and discontinuous forces) has been demonstrated by Doering et al [19].

Ai et al. [18] have also studied the same problem of a Brownian particle moving in a periodic saw tooth potential subjected to a temporally asymmetric periodic rocking. However, there is an error in the expression for the energy per unit time that a ratchet gets from the external force or in other words, the input energy [14]. In this work we take into account this correction and have calculated the efficiency and other physical quantities and presented our results. We find that the temporal asymmetry in driving enhances the efficiency in a very significant manner even for a spatially symmetric potential. Also, in the presence of spatial asymmetry in potential, the efficiency is found to be almost 90% at low temperature. Current reversals are also observed in the parameter space of operation even in the adiabatic regime.

We also present our analysis of the behaviour of entropy production, current and input energy with temperature in this ratchet system. In the absence of any bias the noise induced currents show a peak with temperature. The question that naturally arises is whether this peak is related to underlying resonance (stochastic resonance [20]) due to the synchronization of the position of the particle with the external drive induced by the noise. Our analysis of input energy E_{in}, rules out the presence of any resonance features in the dynamics of the position of the particle in these systems in the adiabatic regime [9, 21]. This follows from the earlier works which show that the existence of stochastic resonance in the dynamics of the particle is revealed by a peak in the input energy [22, 23].

The onset of unidirectional currents in ratchet systems can also be viewed as an example of temporal order coming out of disorder. This can happen only at the expense of an overall increase in the entropy production in the system along with its environment. Thus one expects a correlation between the maxima in current and the maxima in entropy production. However, our results show that the maxima in current and entropy production do not correlate with each other.

II. THE MODEL:

A simple model for our ratchet system is described by the stochastic differential equation (Langevin equation) for a Brownian particle in the overdamped regime. This is given by [24]

$$\dot{q} = -\frac{V'(q) - F(t) + L}{\gamma} + \xi(t), \tag{1}$$

where $\xi(t)$ is a randomly fluctuating Gaussian thermal noise with zero mean and correlation, $< \xi(t)\xi(t') >= (2\, k_B T/\gamma)\delta(t-t')$.

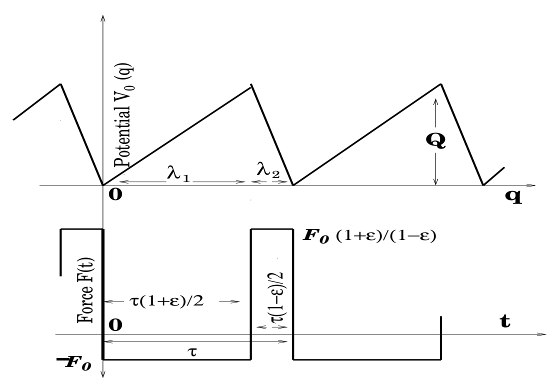

FIG. 1: Schematic representation of the potential and the external force $F(t)$ as a function of space and time respectively.

In the present work we consider the piecewise linear ratchet potential, Fig. 1, as in the case of Magnasco et al. [25] with periodicity λ set equal to unity. This also corresponds to the spacing between the wells. We later on scale all the lengths with respect to λ. $F(t)$ which corresponds to the externally applied time asymmetric force with zero average over the period is also shown in Fig. 1. The force in the gentler and steeper side of the potential are respectively $f^+ = \frac{-Q}{\lambda_1}$ and $f^- = \frac{Q}{\lambda_2}$ and Q is the height of the potential. In the above expression we have also included the presence of an external load L, which is essential for defining thermodynamic efficiency. Following Stratonovich's interpretation [26], the corresponding Fokker-Planck equation [27] is given by

$$\frac{\partial P(q,t)}{\partial t} = \frac{\partial}{\partial q}\Big[k_B T \frac{\partial P(q,t)}{\partial q} + [V'(q) - F(t) + L]P(q,t)\Big]. \quad (2)$$

Since we are interested in the adiabatic limit we first obtain an expression for the probability current density j in the presence of a constant external force F. The expression for current [25] is

$$j(F) = \frac{{P_2}^2 \sinh\{\lambda[F-L]/2k_BT\}}{k_BT(\lambda/Q)^2 P_3 - (\lambda/Q)P_1P_2\sinh\{\lambda[F-L]/2k_BT\}} \quad (3)$$

where

$$P_1 = \Delta + \frac{\lambda^2-\Delta^2}{4}\frac{F-L}{Q} \quad (4)$$

$$P_2 = (1 - \frac{\Delta[F-L]}{2Q})^2 - (\frac{\lambda[F-L]}{2Q})^2 \quad (5)$$

$$P_3 = \cosh(\{Q - 0.5\Delta[F-L]\}/k_BT) - \cosh\{\lambda[F-L]/2k_BT\} \quad (6)$$

where $\lambda = \lambda_1 + \lambda_2$ and $\Delta = \lambda_1 - \lambda_2$, the spatial asymmetry parameter. The current in the stationary regime averaged over the period τ of the driving force $F(t)$ is given by

$$< j >= \frac{1}{\tau}\int_0^\tau j(F(t))\,dt. \quad (7)$$

We assume that $F(t)$ changes slow enough, i.e., its frequency is smaller than any other frequency related to the relaxation rate in the problem such that the system is in a steady state at each instant of time.

In the present work we consider time asymmetric ratchets with a zero mean periodic driving force [14, 16, 18] given by

$$F(t) = \frac{1+\epsilon}{1-\epsilon}F_0,\ \{n\tau \le t < n\tau + \frac{1}{2}\tau(1-\epsilon)\}, \quad (8)$$
$$= -F_0,\ \{n\tau + \frac{1}{2}\tau(1-\epsilon) < t \le (n+1)\tau\}.$$

Here, the parameter ϵ signifies the temporal asymmetry in the periodic forcing, τ the period of the driving force $F(t)$ and $n = 0, 1, 2....$ is an integer. For this forcing in the adiabatic limit the expression for time averaged current is [8, 16]

$$< j >= j^+ + j^-, \quad (9)$$

with

$$j^+ = \frac{1}{2}(1-\epsilon)\,j(\frac{1+\epsilon}{1-\epsilon}F_0), \quad (10)$$
$$j^- = \frac{1}{2}(1+\epsilon)\,j(-F_0)$$

where j^+ is the current fraction in the positive direction over a fraction of time period $(1-\epsilon)/2$ of τ when the external driving force field is $(\frac{1+\epsilon}{1-\epsilon})F_0$ and j^- is the current fraction over the time period $(1+\epsilon)/2$ of τ when the external driving force field is $-F_0$. The input energy E_{in} per unit time is given by [8, 14]

$$E_{in} = F_0[(\frac{1+\epsilon}{1-\epsilon})j^+ - j^-]. \quad (11)$$

In order that the system does useful work a load L is applied in a direction opposite to the direction of current in the ratchet. The overall potential is then $V(q) = [V_0(q) + qL]$. As long as the load is less than the stopping force L_s current flows against the load and the ratchet does work. Beyond the stopping force the current flows in the same direction as the load and hence no useful work is done. Thus in the operating range of the load, $0 < L < L_s$, the Brownian particles move in the direction opposite to the load and the ratchet does useful work (storing energy in the form of potential or say, charging the battery). The average work done over a period is given by [8]

$$E_{out} = L[j^+ + j^-]. \quad (12)$$

The thermodynamic efficiency of energy transduction is [6, 7]

$$\eta = \frac{L[j^+ + j^-]}{F_0[(\frac{1+\epsilon}{1-\epsilon})j^+ - j^-]}. \quad (13)$$

In the limit when the current fraction in the forward direction is much larger than that in the backward direction, $j^+ >> j^-$, and at very low temperature (temperature tending to zero) the efficiency is given by [14] as

$$\eta = \frac{L(1-\epsilon)}{F_0(1+\epsilon)}. \quad (14)$$

The suppression of backward current at low temperature occurs for values of F_0 less than Q/λ_2. However, finite current fraction flows in the positive direction when $\frac{(1+\epsilon)}{(1-\epsilon)}F_0 > -\frac{Q}{\lambda_1}$ or $F_0 > \frac{Q(1-\epsilon)}{\lambda_1(1+\epsilon)}$. Hence, in the operating range of F_0, $\frac{Q}{\lambda_2} > F_0 > \frac{Q(1-\epsilon)}{\lambda_1(1+\epsilon)}$, a high efficiency is expected in the low temperature regime [10].

In the absence of a load the particle moves in a periodic potential without tilt and hence the system does not store any energy. Consequently all the input energy in the steady state is dissipated away. In such a case the energy loss in the medium $E_L = E_{in}$. E_L inturn is equal to the heat Q_h transferred to the bath and thus entropy production $S_p = Q_h/T = E_L/T$ [7]. Thus the total increase in the entropy (or the entropy production) of the bath (universe) integrated over the period of the external drive is given by [7]

$$S_p = \frac{Q_h}{T} = \frac{E_{in}}{T} = \frac{E_L}{T}.$$

As discussed in the introduction, currents in ratchet systems are generated at the expense of entropy and thus we expect a correlation between the magnitude of current and the total entropy production.

In our work all the physical quantities are taken in dimensionless units. Moreover, the energies and lengths are scaled with respect to Q, the barrier height and λ, the spatial period of the potential respectively. In the following section we present our results and the discussion of our calculations.

III. RESULTS AND DISCUSSIONS

We study the motion of an overdamped Brownian particle subjected to a time asymmetric periodic forcing but in presence of a saw tooth potential. We present the noticeable differences between the motion in a smooth potential as in [14] with that in a piecewise linear saw tooth potential. The role of smoothness or regularity in potential on the efficiency of energy transduction is clearly presented here.

To start with, in Fig. 2 we study the behaviour of efficiency with load in a spatially *symmetric* saw tooth potential ($\Delta = 0$) in the presence of time asymmetric driving field for fixed values of $F_0 = 0.1$, $T = 0.01$ and $Q = 1$ for different values of ϵ. Currents in this ratchet model arise solely due to the temporal asymmetry factor. For a given ϵ, the efficiency increases as a function of load and then decreases. The attained value of efficiency is much higher than those attained in other models and it keeps increasing with increasing ϵ. The stopping force L_s too is found to increase with increase in ϵ. Large the ϵ, larger will be the current and efficiency as long as F_0 is less than the critical field, so that the barriers to motion in one direction alone disappears and there will be no current in the opposite direction. We notice that efficiency depends linearly on the load as long as L is much less than L_s where the backward motion is suppressed and the slope is given by $\frac{(1-\epsilon)}{(1+\epsilon)}F_0$ consistent with Eqn. 14. In contrast to the case of smooth sinusoidal potential [14] the locus of the peak in efficiency monotonously increases in the saw tooth case. The value of efficiency is also much higher than that obtained in the smooth potential case. The input energy, E_{in}, output energy, E_{out}, the fraction of currents j^+, j^- and the average current $< j >$ show the same qualitative behaviour as a function of load as is seen in Fig.3 of [14] and the observed behaviour has been discussed in detail in reference [14]. Hence we do not deal with these quantities separately in the present work.

As noted in Fig. 2 one can attain an efficiency of the order of 40% for given physical parameters for the spatially symmetric ($\Delta = 0.0$) rocked ratchet. We now explore the additional role of spatial asymmetry on the above results. For that, in Fig. 3 we plot efficiency as a function of load for various asymmetry in potential (Δ)

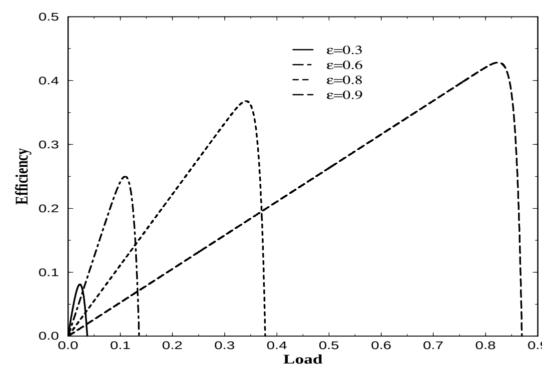

FIG. 2: Efficiency vs load for $\Delta = 0.0$, $F_0 = 0.1$, $T = 0.01$, $Q = 1$ with varying ϵ.

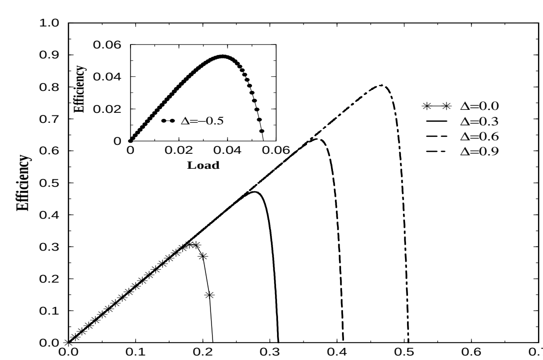

FIG. 3: Efficiency vs load for various $\Delta = 0.9, 0.6, 0.3, 0.0$ with fixed $F_0 = 0.1$ $\epsilon = 0.7$, $T = 0.01$ and $Q = 1$. Inset shows the efficiency for $\Delta = -0.5$ with other parameters remaining the same.

with fixed $F_0 = 0.1$, $T = 0.01$, $Q = 1$ and $\epsilon = 0.7$. We observe that an asymmetry in the potential enhances efficiency and also increases the range of operation of the ratchet. As in the smooth potential case, the higher the ϵ, the larger the current and hence a larger load is necessary for the current to reverse its direction. From this figure it is clear that we can obtain a peak value of efficiency of the order of 30% even in the absence of spatial asymmetry. This peak value of efficiency and the range of operation of the load increases for higher asymmetry. For $\Delta = 0.9$ we obtain a peak value of efficiency of more than 80% which is very high given the fact that ratchet operates in an irreversible mode. It should also be noted that the initial slope of the efficiency versus load curve (for $L < L_s$) is the same, i.e., independent of Δ, again in consistency with Eqn. 14. We can conclude from the above figure that additional spatial asymmetry will further help in enhancing the efficiency of time asymmetric ratchets. This is also due to the fact that currents due to the finiteness in the spatial asymmetry parameter Δ and it being positive enhances the currents in the system as compared to the case when $\Delta = 0.0$. Opposite conclusions will be reached on the effect of Δ on efficiency if Δ is negative, which is obvious. The reduction in currents when Δ is negative and ϵ is positive will be discussed in detail later in connection with current reversals.

In the inset of Fig. 3 we plot efficiency as a function of load for a representative positive and negative value of ϵ and Δ respectively. Here, one can clearly notice that the attained efficiency is in the subpercentage regime. Our further analysis will be restricted to the case wherein Δ and ϵ remains positive as in this parameter space we naturally expect high efficiency.

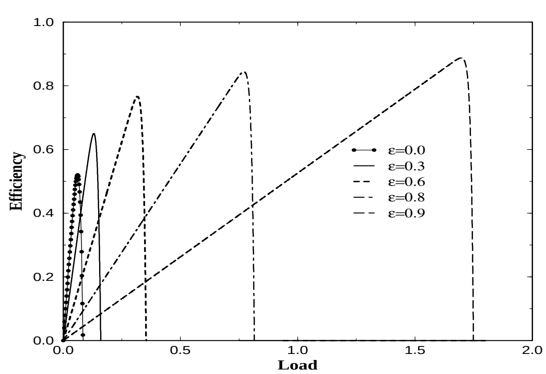

FIG. 4: Efficiency vs load for $\Delta = 0.9$, $F_0 = 0.1$, $T = 0.01$, $Q = 1$ with varying ϵ.

We now study the role of the temporal asymmetry parameter ϵ for the case of spatially asymmetric ($\Delta = 0.9$) ratchets. Fig. 4 shows the behaviour of efficiency as a function of load for varying ϵ for fixed value of $\Delta = 0.9$. It is clear that the inclusion of time asymmetry leads to enhanced value of efficiency and the operational range of load. An efficiency of about $\sim 90\%$ is readily attained as can be seen in Fig. 4. The locus of the peak value in efficiency monotonously increases with increase in ϵ. This is in contrast to the non monotonic behaviour observed in a smooth sinusoidal potential [14]. Moreover, the efficiencies are much higher for these ratchets with discontinuous potential. For the case $\epsilon = 0$ we get an efficiency of $\sim 40\%$. Such a case with $\epsilon = 0$ and finite Δ is discussed in [10]. As has been mentioned earlier, the initial slopes are linear in accordance with Eqn. 14. There are some studies in the deterministic limit where one can attain efficiency to the ideal limit ($\eta = 1$). However, these ratchets work in a reversible quasi-static mode of operation but not in the adiabatic regime [7, 10]. The protocols of the operation rely on synchronizing the dynamics of the particle with external force [7, 10].

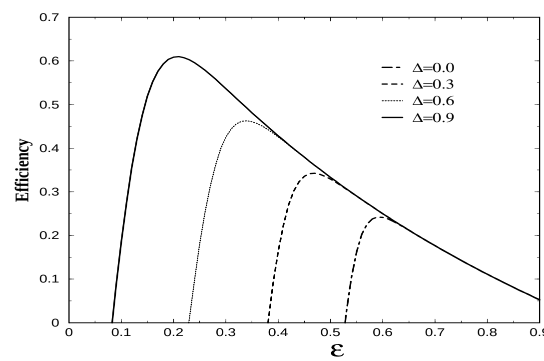

FIG. 5: Efficiency vs ϵ for various values of Δ with fixed $F_0 = 0.1$, $Q = 1$, $L = 0.1$ and $T = 0.01$.

In Fig. 5 we plot the efficiency as a function of ϵ for different strength of potential asymmetry for $F_0 = 0.1$, $L = 0.1$, $Q = 1$ and $T = 0.01$. Similar to the earlier figure we see that the potential asymmetry increases the efficiency value. Larger the asymmetry in potential lower is the value of ϵ for which one gets higher efficiency. This follows from the fact that larger the Δ, the smaller is the critical value of ϵ to get current in the forward direction. The critical value of ϵ, ϵ_c, in the absence of load is given by $\epsilon_c = \frac{Q_0 - F_0\lambda}{Q_0 + F_0\lambda}$. One can notice that this critical value decreases as F_0 increases. In the absence of load the current vanishes for $\epsilon = 0.0$ and moreover the current fraction in the positive direction j^+ vanishes as $\epsilon \to 1$. Hence naturally a peak is expected in efficiency as a function of ϵ. For higher values of ϵ in the regime where the backward current is suppressed the slope in the figure is consistent with Eqn. 14 (which is again independent of Δ as clearly seen in the figure).

In Fig. 6 we plot efficiency as a function of F_0 for the case of symmetric potential ($\Delta = 0.0$) for different values of ϵ with fixed $L = 0.1$, $Q = 1$ and $T = 0.01$. Con-

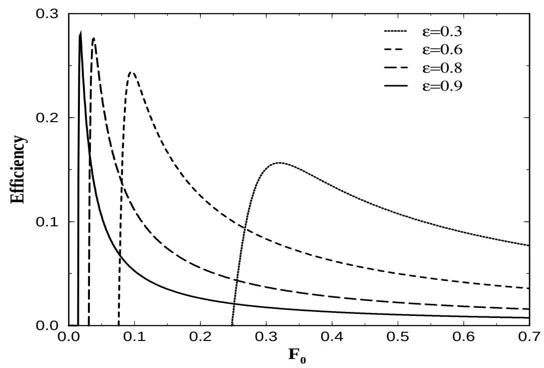

FIG. 6: Efficiency vs F_0 for various values of ϵ for the symmetric case with fixed $L = 0.1$, $Q = 1$ and $T = 0.01$.

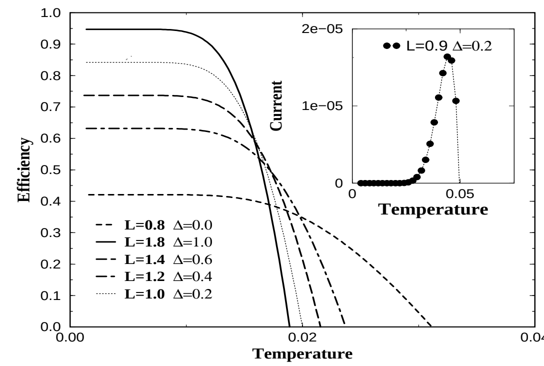

FIG. 8: Efficiency vs temperature for various values of load and Δ with fixed $Q = 1$, $F_0 = 0.1$ and $\epsilon = 0.9$. Inset shows the peaking of current with temperature for $\Delta = 0.2$, $F_0 = 0.1$, $L = 0.9$, $Q = 1.0$ and $\epsilon = 0.9$.

sistent with the general observation of this problem, for lower ϵ values we need larger F_0 to get forward current. Moreover, in the absence of load, the current vanishes in both zero F_0 and large F_0 limit. In the large F_0 limit, the barriers to motion in the forward as well as backward direction disappear and consequently the average current over the period vanishes. Thus a peak in the efficiency as a function of F_0 is obvious. Additional spatial asymmetry enhances the efficiency by a large amount. This can be clearly seen in Fig. 7 where we have plotted efficiency versus F_0 for the case $\Delta = 1.0$. The difference between Figs. 6 and 7 is that the envelope of the peak value of efficiency show opposite behaviour. In the case of smooth potential we had observed earlier [14] that the envelope of the peak of efficiency decreases with increase in ϵ in contrast with that in Fig. 6.

So far we have shown that a large efficiency of the order of unity can be obtained readily in the time asymmetric rocked ratchets in presence of additional spatial asymmetry. Notably, this large efficiency is obtained in the irreversible mode of operation in the adiabatic regime. In presence of both Δ and ϵ we do not have to fine tune the parameters and we get much higher efficiency above the subpercentage limit. In the following we address the question, can thermal fluctuations (noise) facilitate energy transduction?, a subject which has been pursued widely and is of fundamental importance in its own right in the areas in which noise play a constructive role [8].

In Fig. 8 we plot the efficiency as a function of temperature for various load and Δ with fixed $\epsilon = 0.9$. We observe that the efficiency decreases with noise strength (T). We find the value of efficiency at very low temperature to be exactly coinciding with the values obtained from the analytical expression for efficiency in the limit $j^+ >> j^-$, Eqn.14.

In Fig. 9 we plot efficiency as a function of temperature for different spatial asymmetry parameter Δ with fixed $L = 0.77$, $\epsilon = 0.9$, $Q = 1$ and $F_0 = 0.1$. One can notice readily that at low temperatures efficiency is

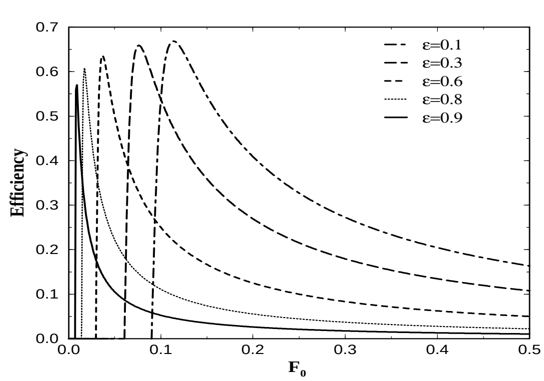

FIG. 7: Efficiency vs F_0 for various values of ϵ with fixed $L = 0.1$, $Q = 1$, $\Delta = 1$ and $T = 0.01$.

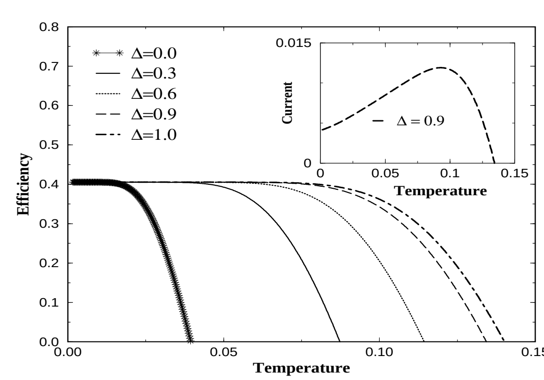

FIG. 9: Efficiency vs temperature for various values of Δ with fixed $\epsilon = 0.9$, $F_0 = 0.1$, $L = 0.77$ and $Q = 1$.

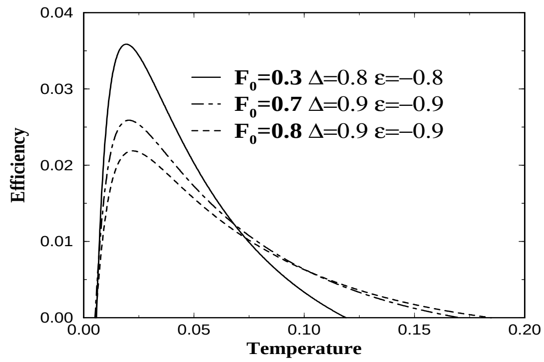

FIG. 10: Efficiency vs temperature for three different cases of physical parameters. (i) $F_0 = 0.3$, $\Delta = 0.8$, $\epsilon = -0.8$ (ii)$F_0 = 0.7$, $\Delta = 0.9$, $\epsilon = -0.9$ (iii) $F_0 = 0.8$, $\Delta = 0.9$, $\epsilon = -0.9$ for fixed $L = 0.01$, and $Q = 1$.

independent of Δ, Eqn.14, and it decreases with temperature. Also, as one increases Δ a larger range of temperature is obtained over which the efficiency value is high. In the parameter range we have considered we generally observe that temperature (noise) cannot facilitate energy transduction i.e., it cannot optimize the efficiency. This is in spite of the fact that in all the cases current as a function of temperature exhibits a peaking behaviour (for example see the inset of Figs. 8 and 9) for a representative parameter value mentioned in the figure captions.

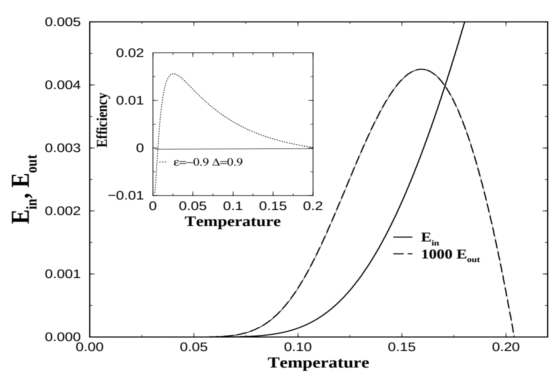

FIG. 11: Efficiency vs temperature for $\Delta = 0.9$, $F_0 = 0.10$, $L = 0.01$, $Q = 1$ and $\epsilon = -0.9$. The inset shows the behaviour of input and output energy for the same set of parameters. The output energy curve is blown up by a factor of 1000 to make it consistent with the scale chosen.

However, with a judicial choice of parameters which require fine tuning we obtain a regime in parameter space where the efficiency exhibits a peak with temperature. In this parameter range, temperature or noise facilitates energy transduction. Fig. 10 shows the peaking behaviour of thermodynamic efficiency with temperature for three representative sets of parameters mentioned in the figure caption. The magnitude of current and efficiency are, however, quite small in this range which we have verified separately.

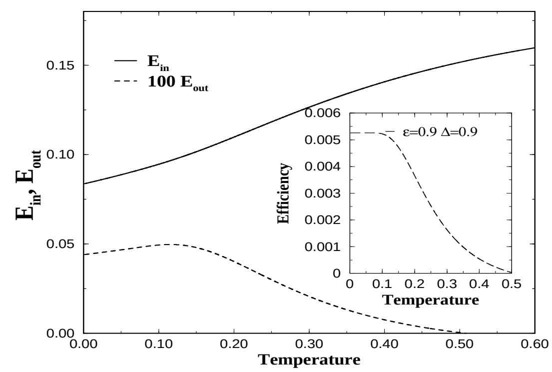

FIG. 12: Efficiency vs temperature for $\Delta = 0.9$, $F_0 = 0.1$, $L = 0.01$, $Q = 1$ and $\epsilon = 0.9$. The inset shows the behaviour of input and output energy for the same set of parameters. The output energy curve is blown up by a factor of 100 to make it consistent with the scale chosen.

To understand this behaviour of efficiency with temperature, in Fig. 11 we plot the input energy, (E_{in}), and the output work, (E_{out}), as a function of temperature. The input energy is found to increase monotonically with temperature. However, the output energy shows a peak with temperature. The output energy curve is blown up by a factor of 1000 to make it comparable with the scale chosen. At very low temperature ($T < 0.006$) the efficiency is negative. The current in the absence of load is very small in this regime. For a given applied load, the current flows in the direction of the load and consequently the output energy is also negative (which could not be seen on the scale we have chosen in the figure). The output energy then increases with temperature and becomes positive for $T > 0.06$. At the crossover points the finite value of the input energy give rise to zero efficiency since the output work is zero. As the temperature is increased the output work increases non monotonically and then becomes zero at a temperature value of about 0.21, beyond which (i.e., beyond the operating range of the load), the current flows in the direction of the load. Thus at $T \sim 0.21$ the output energy and consequently the efficiency is zero. Hence we expect a peaking behaviour in efficiency as a function of temperature as is shown in the inset of the figure. It should be noted that the current in the absence of load shows a peak with temperature.

In Fig. 12 we plot the input and output energy for the case where the efficiency monotonically decreases with temperature. All the physical parameters are mentioned in the figure captions. In contrast to that observed in Fig. 11, we note that both the output energy and the input energy are finite at zero temperature leading in turn to a finite value of efficiency. As we increase the temperature, the input energy increases monotonically whereas the output energy exhibits a small peak. Beyond a temperature of 0.52, E_{out} becomes negative. The rise in input energy is very rapid as compared to that of the output energy and consequently the efficiency decreases monotonically with temperature as shown in the inset of the figure up to $T = 0.52$ beyond which it becomes negative.

So far we have discussed the nature of the efficiency of energy transduction as a function of system variables. We now concentrate on another aspect in ratchet systems, namely, current reversals, which play a central role in designing separation devices. It is known that symmetrically rocked spatially asymmetric ratchets do not exhibit current reversals in the adiabatic regime [2, 28, 29]. However, the presence of system inhomogeneities (frictional or inhomogeneous ratchets) can induce single or multiple current reversals even in the adiabatic regime [14, 30]. In our present case of homogeneous ratchets it is easy to tune current reversals as there are two asymmetric parameters present in the problem. In Fig. 13 we plot current as a function of T for a particular value of ϵ and Δ given in the caption. The parameters are chosen such that the direction of current in presence of either of the parameters alone should be in opposite directions. For example, in Fig. 13 the current is in the positive direction when $\epsilon = 0.8$ and $\Delta = 0$ whereas it is in the reverse direction when $\epsilon = 0.0$ and $\Delta = -0.9$. So by tuning a combination of these two parameter values for $\epsilon = 0.8$ and $\Delta = -0.9$ one gets current reversal as a function of T.

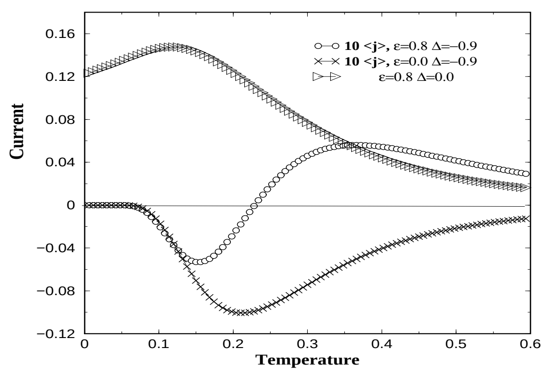

FIG. 13: Current vs temperature for $\Delta = -0.9$, $F_0 = 0.3$, $Q = 1$ and $\epsilon = 0.8$. $< j >$ is multiplied by a factor of 10 for the cases (i) $\epsilon = 0.8$, $\Delta = -0.9$ and (ii) $\epsilon = 0.0$, $\Delta = -0.9$ to make it comparable with the scale chosen.

It should be noted that this is not an additive effect separately arising from ϵ and Δ. The current reversal arises due to complex interplay of these two asymmetry parameters. It should be emphasized that once current inversion upon the variation of one parameter is established, an inversion upon variation of any other parameter can be readily inferred. For details we refer to [2]. In accordance with the above reasoning for current reversals, in Fig. 14 we have plotted current versus temperature with fixed value of $\Delta = -0.9$ and varying ϵ. As we vary ϵ from large value to a small value, in the intermediate range of ϵ we get current reversal.

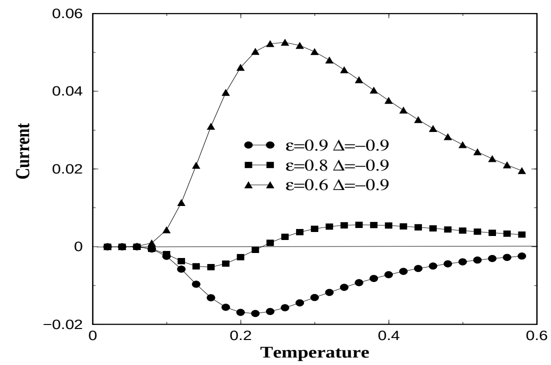

FIG. 14: Current vs temperature for $\Delta = -0.9$, $F_0 = 0.3$, $Q = 1$ and varying ϵ.

Having discussed efficiency and nature of currents and their reversals we now study other thermodynamic quantities namely, input energy and entropy production. We would like to find whether any relation exists among them with the nature of currents as discussed in the introduction. Some recent studies have also tried to reveal the relations between two completely unrelated phenomena, namely, stochastic resonance and Brownian ratchets in a formal way through the consideration of Fokker-Planck equations [31]. Stochastic resonance is a phenomenon where we can obtain optimal output from a system by adding noise to the system [20]. It has been argued that the rate of flow of particles in a Brownian ratchet is analogous to the rate of flow of information in the case of stochastic resonance [32].

In Fig. 15 we plot the entropy production, current and input energy for a representative case, $\Delta = 0.4$, $F_0 = 0.1$ and $\epsilon = 0.8$, as a function of temperature or noise strength. We observe that current exhibits a peak as a function of temperature while the input energy is a monotonously increasing function of temperature [21]. It has been argued earlier that the peak in the input energy is a good measure for the occurrence of stochastic resonance in the dynamics of the particle [22, 23]. It is natural to expect at resonance that the system will extract more input energy from the environment which in turn is consequently dissipated away in the steady state (for details see [22, 23]). However, the observed monotonic behaviour of the input energy, as opposed to the nature of current, rules out the possibility of any resonance in the dynamics of the particle as a function of noise strength.

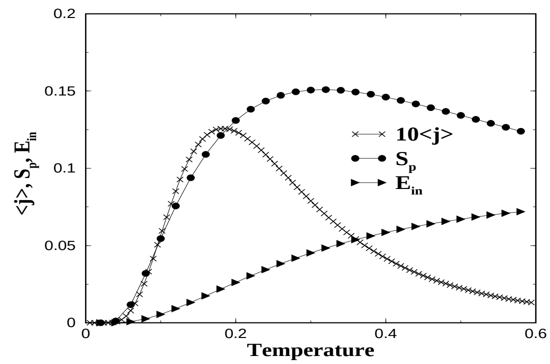

FIG. 15: $< j >$, S_p and E_{in} vs temperature for $\Delta = 0.4$ with fixed $F_0 = 0.1$, $Q = 1$ and $\epsilon = 0.8$. The current $< j >$ is blown up by a factor of ten to make it more clearer.

The presence of net currents in the ratchet increases the amount of known information about the system than otherwise. This extra bit of information comes from the negentropy or the physical information supplied by the external nonequilibrium bath. Since the currents are generated at the expense of entropy one normally expects the maxima in current and the maxima in the overall entropy production to coincide at the same value of noise strength. In fact, in a related development it has been pointed out that the amount of information transferred by the nonequilibrium bath is quantified in terms of algorithmic complexity. Moreover, the algorithmic complexity or Kolmogorov information entropy exhibits a maxima at the same value of physical parameter where the current is maximum [33]. From Fig. 15, we see that the entropy production, S_p, also exhibits a peak as a function of noise strength. The peaks in the average current, $< j >$ and total entropy production S_p do not occur at the same T. This clearly indicates that maxima in the entropy production does not take place at the same value where the current is maximum thereby ruling out the correlation between the entropy production peak and the peak in current maximum [21].

IV. CONCLUSIONS

We have studied in detail the nature of energetic efficiency driven by zero average time asymmetric forcing in the adiabatic limit. The potential is taken to be of the saw tooth type characterized by an asymmetry parameter Δ. In the presence of temporal and spatial asymmetry we have shown that a much higher efficiency ,above the subpercentage regime (known for other ratchets), can be readily obtained. Spatial asymmetry together with temporal asymmetry give larger efficiency as compared to the presence of spatial or temporal asymmetry alone. At low temperatures an efficiency value closer to the ideal limit can be obtained by judicious tuning of physical parameters even though the operation of the ratchet is in the irreversible mode. In the bigger range of parameter space temperature does not facilitate energy transduction. By fine tuning the parameters one can obtain a regime in which temperature facilitates energy transduction. However, in this parameter space the attained value of efficiency is found to be in the subpercentage level.

We also observe current reversals in the adiabatic limit by proper tuning of different parameters. These reversals are attributed to the complex dynamics of the system. From our study of the nature of input energy and currents we conclude that there is no resonance phenomenon occurring in the system. The analysis of current and entropy production results show that the peak in current and entropy production do not coincide.

It is worthwhile exploring whether the transport in these efficient ratchet is coherent or not. Noise induced currents are always accompanied by a diffusive spread. If the diffusive spread of the particle is less than the average distance (say, length of the period of the potential) travelled by the particle in a given time then the transport is said to be coherent. This is quantified in terms of the so-called dimensionless Peclet number [34]. The present work is concentrated mainly on optimizing the thermodynamic efficiency. One can as well optimize maximum work or have the best compromise between maximum work and efficiency. This can be done using known optimizing criterion [11]. Studies in this regard is currently in progress.

[1] F. Jülicher, A. Ajdari and J. Prost, Rev. Mod. Phys. **69**, 1269 (1997).

[2] P. Reimann, Phys. Rep. **361**, 57 (2002) and references therein.

[3] A. M. Jayannavar, cond-mat 0107079; in Frontiers in Condensed Matter Physics, (A commemorative volume to mark the 75^{th} year of Indian Journal of Physics), ed. J. K. Bhattacharjee and B. K. Chakrabarti (in press).

[4] Special issue on "Ratchets and Brownian motors: basics, experiments and applications" ed. H. Linke, Appl. Phys. **A75(2)** 2002.

[5] For details see http://seneca.fis.ucm.es/parr/GAMES/.

[6] K. Sekimoto, J. Phys. Soc. Jpn. **66**, 6335 (1997).
[7] J. M. R. Parrondo and B. J. De Cisneros, Appl. Phys. **A75**, 179 (2002).
[8] H. Kamegawa, T. Hondou and F. Takagi, Phys. Rev. Lett. **80**, 5251 (1998); F. Takagi and T. Hondou, Phys. Rev. **E60**, 4954 (1999); K. Sumithra and T. Sintes, Physica A, **297**, 1 (2001); D. Dan and A. M. Jayannavar, Phys. Rev. **E66**, 41106 (2002).
[9] Raishma Krishnan and A. M. Jayannavar, Physica A, **345**, 61 (2005).
[10] I. M. Sokolov, Phys. Rev. **E63**, 021107, (2001); I. M. Sokolov, cond-mat 0207685v1.
[11] N. Sánchez Salas and A. C. Hernández, Phys. Rev. **E68**, 046125 (2003).
[12] Yu.A. Makhnovskii, V. M. Rozenbaum, D.-Y. Yang, S. H. Lin and T. Y. Tsong, Phys. Rev. **E69**, 021102 (2004).
[13] R. D. Astumian, J. Phys. Chem. **100**, 19075 (1999).
[14] Raishma Krishnan, Mangal C. Mahato and A. M. Jayannavar, Phys. Rev. **E70**, 21102 (2004).
[15] A. Ajdari, D. Mukamel, L. Peliti and J. Prost, J. Phys. I France **4**, 1551 (1994).
[16] D. R. Chialvo, M. M. Millonas, Phys. Lett. A **209**, 26 (1995).
[17] M. C. Mahato and A. M. Jayannavar, Phys. Lett. A **209**, 21 (1995).
[18] Bao-Quan Ai, X. J. Wang, G. T. Liu, D. H. Wen, H. Z. Xie, W. Chen and L. G. Liu, Phys. Rev. **E68**, 061105 (2003).
[19] C. R. Doering and L. A. Dontcheva, Chaos, **8**, 643 (1998).
[20] L. Gammaitoni, P. Hänggi, P. Jung and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).
[21] Raishma Krishnan, Mangal C. Mahato and A. M. Jayannavar, Ind. J. Phys. **78(8)**, 747 (2004).
[22] T. Iwai, Physica A **300**, 350 (2001); T. Iwai, J. Phys. Soc. Jpn. **70**, 353 (2001).
[23] Debasis Dan and A. M. Jayannavar, Physica A, **345**, 404 (2005).
[24] D. Dan, M. C. Mahato and A. M. Jayannavar, Phys. Lett. A **258**, 217 (1999); Int. J. Mod. Phys. B **14** 1585 (2000); Phys. Rev. **E60**, 6421 (1999); Phys. Rev. **E63**, 56307 (2001).
[25] M. O. Magnasco, Phys. Rev. Lett. **71**, 1477 (1993).
[26] R. L. Stratonovich, Radiotekh. Elektron. (Moscow) **3**, 497 (1958). English translation in *Non-Linear Transformations of Stochastic Processes*, edited by P. I. Kuznetsov, R. L. Stratonovich and V. I. Tikhonov (Pergamon, Oxford, 1965).
[27] H. Risken, The Fokker-Planck Equation (Springer Verlag, Berlin, 1984).
[28] D. Dan, M. C. Mahato and A. M. Jayannavar, Phys. Rev. **E63**, 056307 (2001).
[29] P. Reimann, R. Bartussek, R. Häussler and P. Hänggi, Phys. Lett. A 215, 26 (1996).
[30] D. Dan and A. M. Jayannavar, Int. J. Mod. Phys. B **14**, 1585 (2000); D. Dan, M. C. Mahato and A. M. Jayannavar, Phys. Rev. **E60**, 6421 (1999).
[31] M. Bier, Stochastic Dynamics **386**, 81-87, Springer, Berlin (1997); V. Berdichevsky and M. Gitterman, Physica A **249** 88 (1998); R. D. Astumian and F. Moss, Chaos, **8** (3), 533 (1998); Qian Min, Wang Yan and Ahang Xue-Juan, Chin. Phys. Lett. **20**, 810 (2003).
[32] A. Allison and D. Abbott, Fluct. Noise Lett. **1**, L239 (2001).
[33] J. R. Sanchez, F. Family and C. M. Arizmendi, Phys. Lett. **A249**, 281 (1998).
[34] B. Lindner and L Schimansky-Geier, Phys. Rev. Lett. **89**, 230602 (2002); D. Dan and A. M. Jayannavar, Phys. Rev.**E66**, 41106 (2002); Raishma Krishnan, D. Dan and A. M. Jayannavar, cond-mat 0501106, Physica A (in press).