Hartman effect in presence of Aharanov Bohm flux

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Abstract: The Hartman effect for the tunneling particle implies the independence of group delay time on the opaque barrier width, with superluminal velocities as a consequence. This effect is further examined on a quantum ring geometry in the presence of Aharonov-Bohm flux. We show that while tunneling through an opaque barrier the group delay time for given incident energy becomes independent of the barrier thickness as well as the magnitude of the flux. The Hartman effect is thereby extended beyond one dimension and in the presence of Aharonov-Bohm flux.

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I. INTRODUCTION

The concept of tunneling in the realm of quantum mechanics has gained much attention over many years. The immense potentiality of this concept has led to the study of various time scales to understand a time the particle takes to tunnel through a barrier [1, 2, 3]. The group delay time associated with potential scattering is one of the important quantities related efficiently to the dynamical aspect of scattering in quantum mechanics. The time delay for scattering processes can be calculated by following the peak of a wavepacket [4]. The phase delay time (τ) is expressed in terms of the derivative of the phase shift of the scattering matrix with respect to energy. Since its inception, Wigner phase delay time has been a quantity of interest from fundamental as well as technological point of view. The delay time statistics is intimately connected with the dynamic admittance of microstructures [5]. This delay time is also directly related to the density of states [6]. The universality of the delay time distributions in random and chaotic systems has been established earlier [7].

The explicit calculation of group delay time in the problem of a particle tunneling through a rectangular barrier becomes independent of the barrier width in the case of an opaque barrier [8, 9]. This phenomenon, often referred as 'Hartman effect', implies that for sufficiently large barriers the effective group velocity of the particle inside the barrier can become arbitrarily large [10]. In other words, the evanescent waves can travel with superluminal speeds.

Though experiments with electrons for verifying this prediction is yet to be done, the formal identity between the Schrödinger equation and the Helmholtz equation for electromagnetic wave correlates the results for electromagnetic and microwaves to that of electrons. Photonic experiments show that electromagnetic pulses travel with group velocities in excess of the speed of light in vacuum as they tunnel through a constriction in a waveguide [11]. Other experiments with photonic band-gap structures also verified that 'tunneling photons' travel with superluminal group velocities [12]. Thus all these experiments directly or indirectly confirmed the occurence of the Hartman effect without violating so called 'Einstein causality'. Operationally, superluminal velocities have been measured in terms of delay time between the appearance of a pulse peak at the input and a pulse peak at the output of a barrier. It should also be noted that the method based on following the peak of the wavepacket looses its significance under strong distortion of wavepacket [1]. Moreover, there is no causal relationship between the peak of transmitted wavepacket and that of the incident wavepacket. This is due to the fact that peak of the transmitted wavepacket can leave the scattering region long before the peak of the incident wavepacket has arrived. However, Hartman effect and its origin is still considered as a poorly resolved problem. Very recently Winful [13, 14] argued that 'the tunneling particle or wave packet is not really traveling with superluminal velocity but actually a standing wave, that just stand and waves!'. The incident wave simply modulates this standing wave. The output adiabatically follows the input with delay proportional to the stored energy. It is shown that the short time delay observed is due to energy storage and release and has nothing to do with real propagation and hence should not be linked with velocity. Thus the origin of the Hartman effect is traced to stored energy. Since the stored energy in the evanescent field decreases exponentially within the barrier after a certain decay distance it becomes independent of the width of the barrier.

All the studies on Hartman effect till date are restricted to one dimension only. Mostly single [10] or successive rectangular barriers [15] have been considered. A very simple semiclassical derivation of Hartman effect valid for potential barrier of general shape in one dimension has been presented in ref. [16]. In our present work we verify Hartman effect beyond one dimension and in the presence of Aharonov-Bohm flux.

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II. DESCRIPTION OF THE SYSTEM

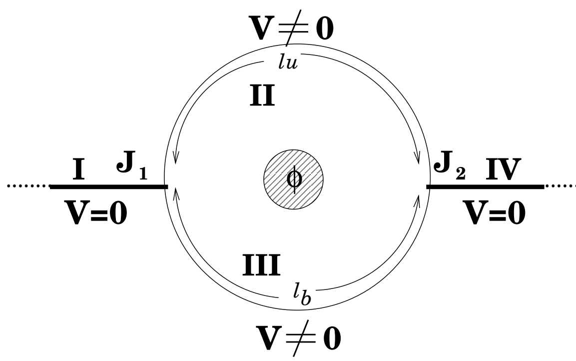

FIG. 1: Schematic diagram of a ring connected to two leads in the presence of an Aharanov-Bohm flux.

We study the scattering problem across a quantum ring geometry as schematized in Fig. 1. Such ring geometry systems have been extensively investigated in mesoscopic physics in analysing normal state Aharanov-Bohm effect which has been observed experimentally [17, 18]. Our system of interest constitute a loop connected to two semi-infinite ideal wires in the presence of a magnetic flux as in Fig. 1. There is a finite quantum mechanical potential V inside the loop while that in the connecting leads are set to be zero. We focus on a situation wherein the incident electrons have an energy E less than V. The impinging electrons in this subbarrier regime travels as an evanescent mode throughout the circumference of the loop and the transmission or the conductance involves contributions from both the Aharanov-Bohm effect as well as quantum tunneling. We are interested in a single channel case where the Fermi energy lies in the lowest subband. To excite the evanescent modes in the ring we have to make the width of the ring much less than that of the connecting wires. The electrons occupying the lowest subband in the connecting wire on entering the ring experience a higher barrier (due to higher quantum zero point energy) and propagate in the loop as evanescent mode. The transmission or conductance across such systems has been studied in detail [19, 20]. In this work an analysis of the phase delay time or the group delay is carried out.

III. THEORETICAL TREATMENT

We approach this scattering problem using the quantum wave guide theory [21, 22]. The wave function in different region in absence of magnetic flux are given below

$$\begin{aligned} \psi_I(x_1) &= e^{ikx_1} + re^{-ikx_1}\,, & (1)\\ \psi_{II}(x_2) &= A\,e^{i\,q\,x_2} + B\,e^{-i\,q\,x_2}\,, & (2)\\ \psi_{III}(x_3) &= C\,e^{i\,q\,x_3} + D\,e^{-i\,q\,x_3}\,, & (3)\\ \psi_{IV}(x_4) &= t\,e^{i\,k\,x_4}\,, & (4) \end{aligned}$$

with $k = \sqrt{2mE/\hbar^2}$ being the wavevector of the incident propagating electrons in the leads and $q = \sqrt{2m(E-V)/\hbar^2}$ the wavevector in the ring. We have assumed the origin of the co-ordinates of x_1 and x_2 to be at J_1 and that for x_3 and x_4 to be at J_2. At J_1, $x_3 = l_b$ and at J_2, $x_2 = l_u$ where l_u and l_b are the lengths of the upper and lower arms of the ring. Total circumference of the ring is $L = l_u + l_b$.

We use the Griffith boundary conditions [23]

$$\psi_I(0) = \psi_{II}(0) = \psi_{III}(l_b)\,, \quad (5)$$

and

$$\Sigma_i \frac{\partial \psi_i}{\partial x_i} = 0\,, \quad (6)$$

at the junction J_1. All the derivatives are either outward or inwards from the junction [21]. Similar boundary conditions hold for J_2 as well. We choose a gauge for the vector potential in which the magnetic field appears only in the boundary conditions rather than explicitly in the Hamiltonian [18, 21]. Thus the electrons propagating clockwise and anticlockwise will pick up opposite phases. The electrons propagating in the clockwise direction in the upper arm from J_1 will pick up a phase $i\,\alpha$ at J_2 and electrons propagating anticlockwise from J_2 to J_1 in the upper arm pick up a phase $-i\,\alpha$ at J_1. Similarly, an electron picks up a phase $i\,\beta$ at J_1 moving in the clockwise direction from J_2 in the lower arm and $-i\,\beta$ at J_2 moving anticlockwise from J_1 in the lower arm of the loop. The total phase around the loop is $\alpha + \beta = 2\,\pi\,\phi/\phi_0$, where ϕ and ϕ_0 are the magnetic flux and flux quantum, respectively. Hence from above mentioned boundary conditions we get for propagating waves [19, 20, 22, 24]

$$\begin{aligned} 1 + r &= A + B\,e^{-i\,\alpha} \\ &= C\,e^{i\,q\,l_b}\,e^{i\,\beta} + D\,e^{-i\,q\,l_b}\,, & (7)\\ t &= A\,e^{i\,q\,l_u}\,e^{i\,\alpha} + B\,e^{-i\,q\,l_u} \\ &= C\, +\, D\,e^{-i\,\beta}\,, & (8) \end{aligned}$$

$$\begin{aligned} i\,k\,(1-r) &= i\,q\,A\, -\, i\,q\,B\,e^{-i\,\alpha} \\ &-\, i\,q\,C\,e^{i\,q\,l_b}\,e^{i\,\beta}\, +\, i\,q\,D\,e^{-i\,q\,l_b}\,, & (9)\\ i\,k\,t &= i\,q\,A\,e^{i\,q\,l_u}\,e^{i\,\alpha} - i\,q\,B\,e^{-i\,q\,l_u} \\ &-\, i\,q\,C\, +\, i\,q\,D\,e^{-i\,\beta}\,. & (10) \end{aligned}$$

IV. RESULTS AND DISCUSSIONS

For the evanescent regime in our problem we replace the wavevector q in the loop by $i\,\kappa$ ($\kappa =$

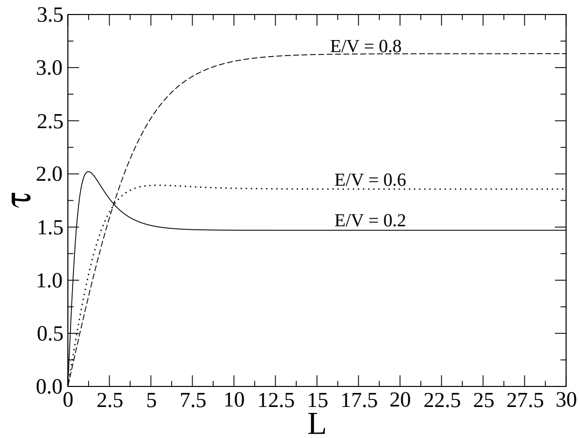

FIG. 2: Plot of τ versus L for three different values of E/V with $\phi = 0$ and $l_u = l_d$.

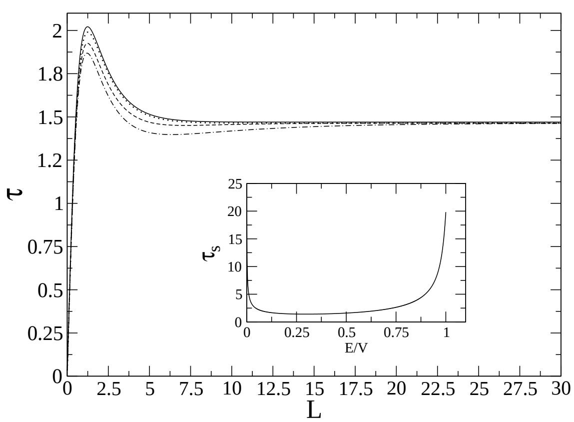

FIG. 3: Plot of τ versus L for different arm length ratios. The ratio $l_u : l_b$ for the solid, dotted, dashed and dot-dashed curves are $1:1$, $3:2$, $4:1$, $9:1$ respectively. Inset shows τ_s versus E/V.

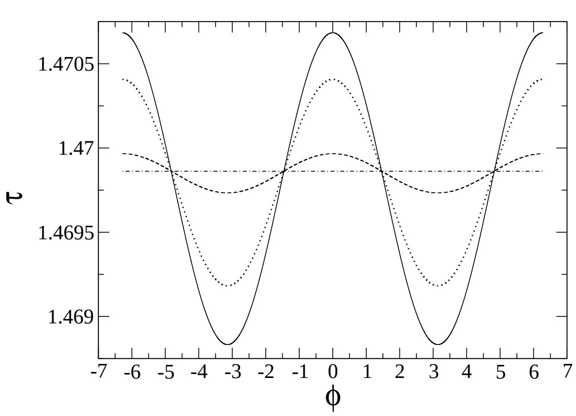

FIG. 4: Plot of τ versus ϕ for different L. The solid, dotted, dashed and dot-dashed curves are for $L = 10$, 10.5, 12.5, 30 respectively.

$\sqrt{2m(V-E)/\hbar^2}$). Solving Eqns. (7) - (10) we obtain an analytical expression for the transmission coefficient t as

$$t = \frac{4ik\kappa e^{i\alpha}\left[Pe^{\kappa l_u} + Qe^{\kappa l_b}\right]}{PQk^2 + 2ik\kappa S_- + 4\kappa^2\left[e^{\kappa(l_u+l_b)}\left(e^{2i(\alpha+\beta)}+1\right) - S_+\right]} \quad (11)$$

where

$$\begin{aligned} P &= e^{i(\alpha+\beta)}\left(e^{2\kappa l_b} - 1\right), \\ Q &= \left(e^{2\kappa l_u} - 1\right), \\ S_\pm &= e^{i(\alpha+\beta)}\left(e^{2\kappa(l_u+l_b)} \pm 1\right). \end{aligned}$$

Next we evaluate the group delay time which is given by the energy derivative of the phase of the transmission coefficient

$$\tau = \hbar\frac{\partial arg[t]}{\partial E}. \quad (12)$$

We have set units of $\hbar$ and $2m$ to be unity. All the physical quantities are taken in dimensionless units ($E \equiv E/V, \tau \equiv V\tau$ and $L \equiv L\sqrt{V}$). In Fig. (2) we plot phase time τ as a function of length L of the ring for different values of incident energies in the absence of magnetic flux ϕ for the case where two armlengths l_u and l_b are equal. From the figure we clearly see that τ evolves as a function of length L and asymptotically saturates to a value (τ_s) which is independent of L thus confirming the Hartman effect. The saturation value increases with increasing incident energy and the corresponding values for $E = 0.2V$, $0.6V$ and $0.8V$ are 1.47, 1.86 and 3.13 respectively. In one dimensional single barrier case in the tunneling region τ saturates to a constant value as a monotonic function of length. In our present case we observe depending on energy it is a monotonic or nonmonotonic function as seen from Fig. (2). Our system differs from the one dimensional case in such a way that electron entering the ring can traverse along different alternative paths before transmitting. The interference between these alternative paths is responsible for this behaviour in the small L regime where contributions from both evanescent and anti-evanescent modes dominate. In Fig. (3) we plot the phase time versus L for a particular energy, $E = 0.2$, in the absence of magnetic flux ϕ but for different length ratios of the upper and lower arms. We observe that the saturation value of the phase time is independent of the arm length ratios for a given energy as one can anticipate. In the inset of Fig. (3) we plot τ_s versus E/V for $\phi = 0$, $L = 30$ with equal upper and lower arm lengths. Plots with different armlength ratios ($l_u : l_b$) with different ϕ in the asymptotic limit were found to overlap with the above curve in the entire energy regime. Moreover, it is given by an analytical expression, $\tau_s = (4\kappa^3 + 5k^2\kappa + (k^4/\kappa))/(2k((2\kappa^2 - (k^2/2))^2 + 4k^2\kappa^2))$ which agrees perfectly well with the numerical results.

In Fig. (4) we have plotted group delay time as a function of flux ϕ for various values of circumference of the ring. In this case $l_u = l_b$ and $E = 0.2V$. We observe that

τ is flux periodic with periodicity ϕ_0. This is consistent with the fact that all the physical properties in presence of Aharonov-Bohm flux across the ring must be periodic function of flux with a period ϕ_0 [6, 17, 25]. However, we observe that as we increase the length of the ring the visibility or the magnitude of Aharonov-Bohm oscillations decreases. Consequently in the large length limit the visibility of Aharonov-Bohm oscillations vanishes as can be seen from Fig. (4). The constant value of τ thus obtained in the presence of flux is identical to τ_s in the absence of the flux (see Fig. (2)) in the large length regime. This result clearly indicates that the delay time in the presence of opaque barrier becomes not only independent of length of the circumference but also it is independent of the Aharonov-Bohm flux thereby observing the Hartman effect in the presence of Aharonov-Bohm flux. We also find that the behaviour of reflection delay time is same as transmission delay time as anticipated from general symmetry laws from the simple geometric structure considered in the present case.

V. CONCLUSIONS

We have verified the Hartman effect in a quantum ring geometry in the presence of Aharanov-Bohm flux. Our studies show that the group delay time for a given incident energy becomes independent of the barrier thickness as well as the magnitude of the flux for the case of opaque barrier. We have also obtained similar results on different geometric structures by including a potential well between two adjacent barriers. These results which will be reported elsewhere [26] were found to agree with the interesting observations made in Ref. [15].

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