Stochastic resonance and nonlinear response in a dissipative quantum two-state system

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Abstract

We study the dynamics of a dissipative two-level, system driven by a monochromatic ac field, starting from the usual spin-boson Hamiltonian. The quantum Langevin equations for the spin variables are obtained. The amplitude of the coherent oscillations in the average position of the particle is studied in the high temperature limit. The system exhibits quantum stochastic resonance in qualitative agreement with earlier numerical results.

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I. INTRODUCTION

Stochastic resonance is a nonlinear phenomena. It has been predicted and experimentally observed in damped classical systems[1-4]. It is characterized by a maximum in the response of the system to an external input signal as a function of noise strength at the input signal frequency. There have been some recent studies in stochastic resonance in damped quantum double-well systems as well [5-9]. Recently, Makarov and Makri have studied the phenomena in a dissipative two-level system numerically using an iterative path integral scheme and tried to understand the results analytically[8,9]. They demonstrate that it is possible to induce and maintain large amplitude coherent oscillations by exploiting the phenomena of stochastic resonance in quantum systems. In their treatment noise strength is varied, through the coupling to heat bath, to obtain stochastic resonance. The maximum in response is also obtained with respect to the driving field strength indicating breakdown of linear response theory. However, their analytical treatment involves several approximations. They start with the evolution of dynamical variables of a system using Heisenberg equation of motion for a given spin-boson (system-bath) Hamiltonian in the presence of a monochromatic field. In their weak coupling approximation the bath causes stochastic energy fluctuations which are unaffected by the dynamics of the two-level system . To obtain the steady state average position of the particle $\langle \sigma_z \rangle$ they require steady state solution for the population difference between the two-level-system eigenstates $\langle \sigma_x \rangle$. The required expression for $\langle \sigma_x \rangle$ is then taken from the solutions of the phenomenological optical Bloch equations in the rotating wave approximation. The expression for $\langle \sigma_z \rangle$ thus obtained with these approximations seems to be in good agreement with their numerical simulations.

In the present work we study stochastic resonance analytically by systematically deriving the quantum Langevin equation for the system by eliminating bath variables[10-13]. The response of the system to an oscillating field is obtained in the high temperature limit. This limit corresponds to treating the random force operator as a classical c-number variable. Our results are in qualitative agreement with earlier works.

In section II the derivation of quantum Langevin equation is presented. The spin Bloch equations are obtained and the stationary solution of the relevant variables is given in sec. III. The last section IV is devoted to results and discussion.

II. **The Quantum Langevin equation for system variables**

To obtain the Quantum Langevin equations for system variables we consider a symmetric two-level system interacting linearly with a bath of harmonic oscillators in the presence of a time dependent external monochromatic field $V_0 cos(\omega t)$. The total Hamiltonian is given by [8,9]

$$H = -\hbar\Delta_0\sigma_x + \sum \hbar\omega_k a_k^\dagger a_k + \sum s_0 g_k(a_k + a_k^\dagger)\sigma_z + V_0\cos(\omega t)\sigma_z, \tag{1}$$

where σ 's are the Pauli matrices. a_k and $a_k^\dagger$ are annihilation and creation operators for bath variables and g_k is the coupling constant and $2s_0$ is the distance between the two wells. The population difference between the two eigenstates of the two-level system, which are separated by $2\hbar\Delta_0$, is given by $\langle\sigma_x\rangle$. The average value of $\langle\sigma_z\rangle$ represents the average position of the particle, $\langle\sigma_z\rangle$ being +1 or -1 is equivalent to the particle in the right or the left well, respectively.

The quantum equation of motion for any dynamical variable A can be obtained from the following evolution equation

$$\frac{dA}{dt} = \frac{i}{\hbar}[H, A], \tag{2}$$

where [– , –] indicates commutator. The commutaton relations among the spin variables are given by

$$[\sigma_x, \sigma_y] = 2i\sigma_z, \tag{3}$$

and their cyclic permutations. Moreover,

$$[a_{k'}, a_k^\dagger] = \delta_{kk'}. \tag{4}$$

Using equations (1)-(4) one can readily write down the equations of motion for the variables as

$$\frac{d\sigma_x}{dt} = -2\sum \frac{s_0 g_k}{\hbar}(a_k(t) + a_k^\dagger(t))\sigma_y(t) - \frac{2V_0}{\hbar} cos(\omega t)\sigma_y(t), \tag{5}$$

$$\frac{d\sigma_y}{dt} = 2\Delta_0\sigma_z(t) + 2\sum \frac{s_0 g_k}{\hbar}(a_k(t) + a_k^\dagger(t))\sigma_x(t) + \frac{2V_0}{\hbar} cos(\omega t)\sigma_x(t), \tag{6}$$

$$\frac{d\sigma_z}{dt} = -2\Delta_0\sigma_y(t), \tag{7}$$

$$\frac{da_k}{dt} = -i\omega_k a_k(t) - i\frac{s_0 g_k}{\hbar}\sigma_z(t), \tag{8}$$

and,

$$\frac{da_k^\dagger}{dt} = +i\omega_k a_k(t) + i\frac{s_0 g_k}{\hbar}\sigma_z(t). \tag{9}$$

Equations (8) and (9) are linear and therefore can be explicitly integrated, to obtain

$$a_k(t) = a_k(0)e^{-i\omega_k t} - i\frac{s_0 g_k}{\hbar}\int_0^t \sigma_z(t')e^{-i\omega_k(t-t')}dt', \tag{10}$$

$$a_k^\dagger(t) = a_k^\dagger(0)e^{i\omega_k t} + i\frac{s_0 g_k}{\hbar}\int_0^t \sigma_z(t')e^{i\omega_k(t-t')}dt'. \tag{11}$$

Where $a_k(0)$ and $a_k^\dagger(0)$ are the bath operator values at the initial time $t = 0$. Substituting for $a_k(t)$ and $a_k^\dagger(t)$ from equations (10) and (11) in equations (5) and (6), we obtain the following Quantum Langevin equations for the system variables,

$$\begin{aligned}\frac{d\sigma_x}{dt} = &-\frac{2s_0}{\hbar}F(t)\sigma_y(t) - \frac{2V_0}{\hbar}\cos(\omega t)\sigma_y(t)\\ &+\frac{1}{2}\left[\sum_k \frac{4s_0^2 g_k^2}{\hbar^2}\left\{\int_0^t \sin(\omega_k(t-t'))\sigma_z(t')dt'\right\}\sigma_y(t)\right.\\ &\left.+\sum_k \frac{4s_0^2 g_k^2}{\hbar^2}\sigma_y(t)\left\{\int_0^t \sin(\omega_k(t-t'))\sigma_z(t')dt'\right\}\right],\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{d\sigma_y}{dt} = &\,2\Delta_0\sigma_z(t) + \frac{2s_0\sigma_x(t)}{\hbar}F(t) + \frac{2V_0}{\hbar}cos(\omega t)\sigma_x\\ &-\frac{1}{2}\left[\sum_k \frac{4s_0^2 g_k^2}{\hbar^2}\sigma_x(t)\left\{\int_0^t \sin(\omega_k(t-t')\sigma_z(t')dt'\right\}\right.\\ &\left.-\frac{1}{2}\sum_k \frac{4s_0^2 g_k^2}{\hbar^2}\left\{\int_0^t \sin(\omega_k(t-t')\sigma_z(t')dt'(t)\right\}\sigma_x(t)\right],\end{aligned} \tag{13}$$

Here $F(t)$ is given by

$$F(t) = \sum_k g_k(a_k(0)e^{-i\omega_k t} + a_k^\dagger(0)e^{i\omega_k t}). \tag{14}$$

As the dynamical operators ($a_k(0)$, ${a_k}^\dagger(0)$) of the bath are distributed in accordance with the statistical equilibrium distribution for given temperature T, $F(t)$ is referred to as Langevin operator noise term [12,13]. The integrals in equations (12) and (13) can be integrated by parts leading to

$$\frac{d\sigma_x}{dt} = -\frac{2s_0}{\hbar}F(t)\sigma_y(t) - \frac{2V_0}{\hbar}\cos(\omega t)\sigma_y(t)$$
$$+\frac{1}{2}\sum_k \frac{4s_0^2 g_k^2}{\hbar^2\omega_k}\int_0^t \left\{\frac{d\sigma_z(t')}{dt'}\sigma_y(t) + \sigma_y(t)\frac{d\sigma_z(t')}{dt'}\right\} cos(\omega_k(t-t'))dt'$$
$$-\frac{1}{2}\sum \frac{4s_0^2 g_k^2}{\hbar^2\omega_k}\left\{\sigma_z(0)\sigma_y(t) + \sigma_y(t)\sigma_z(0)\right\} cos(\omega_k t), \tag{15}$$

$$\frac{d\sigma_y}{dt} = 2\Delta_0\sigma_z(t) + \frac{2s_0}{\hbar}F(t)\sigma_x(t) + \frac{2V_0}{\hbar}cos(\omega t)\sigma_x(t)$$
$$-\frac{1}{2}\sum_k \frac{4s_0^2 g_k^2}{\hbar^2\omega_k}\int_0^t \left\{\frac{d\sigma_z(t')}{dt'}\sigma_x(t) + \sigma_x(t)\frac{d\sigma_z(t')}{dt'}\right\} cos(\omega_k(t-t'))dt'$$
$$+\frac{1}{2}\sum \frac{4s_0^2 g_k^2}{\hbar^2\omega_k}\left\{\sigma_z(0)\sigma_x(t) + \sigma_x(t)\sigma_z(0)\right\} cos(\omega_k t). \tag{16}$$

In equations (15) and (16) $G(t-t') = \sum(4s_0^2 g_k^2/\hbar^2\omega_k)cos(\omega_k(t-t'))$ represents a damping or memory kernel [10]. The last two terms in equation (15) and (16) are transient terms which can be neglected if one assumes Ohmic spectral density for bath oscillators [10,11],i.e.,

$$\rho(\omega) = \frac{\pi}{2}\sum \frac{4s_0^2 g_k^2}{\hbar^2}\delta(\omega-\omega_k) = \alpha\omega exp(-\frac{\omega}{\omega_c}), \tag{17}$$

where α is a dimension less dissipation coefficient (or Kondo parameter). Notice that the form of the spectral density is not bounded from above and hence for physical reasons one introduces an upper cut-off frequency ω_c, namely $\rho(\omega) = \alpha\omega e^{-(\omega/\omega_c)}$, such that the frequency scale ω_c is assumed to be much larger than the characteristic frequencies of the problem.

With this form of spectral density one can readily show that the transient terms do survive up to a time scale $(1/\omega_c)$, which can be made arbitrarily small and thus can be ignored [11]. Thus for the long time behaviour and for the Ohmic spectral density the equations (15) and (16) get further simplified and we arrive at the equations of motion for spin variables as

$$\frac{d\sigma_x}{dt} = -\frac{2s_0}{\hbar}F(t)\sigma_y(t) - \frac{2V_0}{\hbar}\cos(\omega t)\sigma_y(t)$$

$$+\frac{\alpha}{2}\left\{\frac{d\sigma_z(t)}{dt}\sigma_y(t) + \sigma_y(t)\frac{d\sigma_z(t)}{dt}\right\}, \tag{18}$$

$$\frac{d\sigma_y}{dt} = 2\Delta_0\sigma_z(t) + \frac{2s_0}{\hbar}F(t)\sigma_x(t) + \frac{2V_0}{\hbar}cos(\omega t)\sigma_x$$

$$-\frac{\alpha}{2}\left\{\frac{d\sigma_z(t)}{dt}\sigma_x(t) + \sigma_x(t)\frac{d\sigma_z(t)}{dt}\right\}, \tag{19}$$

$$\frac{d\sigma_z}{dt} = -2\Delta_0\sigma_y(t), \tag{20}$$

with the memory kernel $G(t-t')$=$2\alpha\delta(t-t')$.

Further simplifying equations (18)-(20) and making use of the properties of spin operators, we finally obtain the Langevin equations of motion for spin variables as

$$\frac{d\sigma_x}{dt} = \left\{-\frac{2s_0}{\hbar}F(t) - \frac{2V_0}{\hbar}\cos(\omega t)\right\}\sigma_y(t) - \frac{2\alpha\Delta_0}{\hbar}, \tag{21}$$

$$\frac{d\sigma_y}{dt} = 2\Delta_0\sigma_z(t) + \left\{\frac{2s_0}{\hbar}F(t) + \frac{2V_0}{\hbar}\cos(\omega t)\right\}\sigma_x(t), \tag{22}$$

$$\frac{d\sigma_z}{dt} = -2\Delta_0\sigma_y(t). \tag{23}$$

The above Langevin equations involve the operator random force $F(t)$. The statistical properties of $F(t)$ can be obtained using the equilibrium distribution for bath variables,

$$\langle a_k^\dagger a_k\rangle = \frac{1}{e^{\beta\hbar\omega_k} - 1}, \tag{24}$$

where $\beta = 1/k_B T$. Using this and the Ohmic spectral density for the bath oscillators, the symmetrized autocorrelation for the operator valued random force $F(t)$ is given by[12,13],

$$\frac{1}{2}\langle F(t)F(t') + F(t')F(t)\rangle = \sum g_k^2 \cos\omega_k(t-t')\coth(\hbar\omega_k/2k_BT) \tag{25}$$

and the nonequal time commutator is given by

$$[F(t), F(t')] = -2i\sum g_k^2 \sin\omega_k(t-t'). \tag{26}$$

III. **The Spin Bloch equation and their solutions.**

Owing to the operator nature of the random Langevin force $F(t)$, it is difficult to solve for the expectation values of spin variables using equations (21)-(23). For simplification we make a first approximation in that the operator random force is treated as a classical c-number random variable. One can readily verify that in the classical limit [12,13], taking $\hbar \to 0$, the nonequal time commutator of $F(t)$ vanishes and the autocorrelation of the Gaussian random force $F(t)$ becomes

$$\langle F(t)F(t')\rangle = \eta kT\delta(t-t'), \tag{27}$$

where η is the friction coefficient and is related to Kondo parameter α through the following realation

$$\eta = (\hbar/2s_0^2)\alpha. \tag{28}$$

The classical Markov approximation for $F(t)$ is valid in the high temperature limit, which will become clear later. With the above approximation one can readily write down the equations of motion of spin variables averaged over the ensemble of realizations of random fluctuations $F(t)$, whose autocorrelation is given by equation (27), with the help of Novikov's theorem [14-16]. We get

$$\frac{d\langle\sigma_x(t)\rangle}{dt} = -\delta\langle\sigma_x(t)\rangle - 2\alpha\Delta_0 - A\cos(\omega t)\langle\sigma_y(t)\rangle, \tag{29}$$

$$\frac{d\langle\sigma_y(t)\rangle}{dt} = 2\Delta_0\langle\sigma_z(t)\rangle - \delta\langle\sigma_y(t)\rangle + A\cos(\omega t)\langle\sigma_x(t)\rangle, \tag{30}$$

$$\frac{d\langle\sigma_z(t)\rangle}{dt} = -2\Delta_0\langle\sigma_y(t)\rangle, \tag{31}$$

where $\delta = 2\alpha k_B T/\hbar$, and $A = 2V_0/\hbar$. Equations (29) to (31) represent the Bloch equations for spin variables. Unlike in the standard NMR situation, there is no relaxation term in the evolution of $\langle\sigma_z\rangle_0$. This is because fluctuating environmental fields are exclusively along the z direction [10]. These equations are characterized by a single relaxation time $\tau = \delta^{-1} = \hbar/2\alpha k_B T$. In the absence of an external driving field, the system will relax asymptotically to equilibrium and the equilibrium value of population difference between the two levels separated by an energy value $2\hbar\Delta_0$ is given by $\langle\sigma_x\rangle = (\hbar\Delta_0/k_B T)$ instead of $\tanh(\hbar\Delta_0/k_B T)$. This shows that our approximation of treating the operator random force as classical random c-number variable is valid for high temperature such that $\hbar\Delta_0/k_B T \ll 1$. The stationary solution of Bloch equations in the presence of an external field can be found by using the method of harmonic balance. For this we assume stationary solutions for $\langle\sigma_z\rangle_s$ and $\langle\sigma_x\rangle_s$ to have the form

$$\begin{aligned}\langle\sigma_z(t)\rangle_s &= a\cos(\omega t) + b\sin(\omega t) \\ &\equiv \langle\sigma_z(t)\rangle_0 \cos(\omega t - \phi)\end{aligned} \tag{32}$$

$$\langle\sigma_x(t)\rangle_s = y + c\cos(\omega t) + d\sin(\omega t) \tag{33}$$

where a,b,y, c and d are constants to be determined. Substituting (32) and (33) in the Bloch equations (29) to (31) and using harmonic balance one readily gets an expression for the required steady state amplitude $\langle\sigma_z\rangle_0$ for the average position of the particle $\langle\sigma_z\rangle$ as

$$\begin{aligned}\langle\sigma_z\rangle_0 &= \sqrt{a^2+b^2} \\ &= 4V_1\Delta_1^2 \frac{\sqrt{(\omega_1^2 - 4\Delta_1^2)^2 + 4\alpha^2\omega_1^2}}{(\omega_1^2 - 4\Delta_1^2)^2 + 4\omega_1^2\alpha^2 + 2V_1^2\omega_1^2}.\end{aligned} \tag{34}$$

Here in the above expression we have rescaled all the energy variables in terms of $k_B T$, i.e., $V_1 = V_0/k_B T$, $\Delta_1 = \hbar\Delta_0/k_B T$ and $\omega_1 = \hbar\omega/k_B T$ are dimensionless variables. Note that in the absence of driving field the amplitude $\langle\sigma_z\rangle_0$ vanishes as expected. Away from the resonance ($\omega = 2\Delta_0$), the oscillation amplitude for high frequency field scales as ($1/\omega^2$). In the limit of low frequency (static) driving $\omega \rightarrow 0$ and for small V_0 , i.e., in the adiabatic limit $\langle\sigma_z\rangle_0$ is independent of relaxation time τ ($= \hbar/2\alpha k_B T$). These results are consistent with those obtained in ref. [8,9].

IV. **Results and discussion**

In fig.(1) we have plotted the stationary amplitude of the average position of the particle $\langle\sigma_z\rangle_0$ as a function of the dissipation coefficient (or Kondo parameter) α, for various values of the dimensionless field amplitude V_1 ($\equiv V_0/k_B T$). We have restricted to the case of resonant condition $\omega = 2\Delta_0$. The solid curve is for $V_1 = 0.01$, long dashed curve for $V_1 = 0.04$ and small dashed curve for $V_1 = 0.07$. We see that all these curves exhibit maxima at an optimum value of α, which depends on Δ_1, V_1 and ω_1. The maximum value of the peak (M_p), however, is independent of field strength and is given by $M_p = \sqrt{2}(\Delta_1^2/\omega_1)$, but the position of the peak (α_m=$\sqrt{2V_1^2\omega_1^2 - (\omega_1^2 - 4\Delta_1^2)^2}/2\omega_1$) shifts towards higher values of α as we increase V_1. The independence of the value of peak maxima on the field strength is also noted in reference [8,9], for resonant case. Our expression for M_p is valid for any frequency. The occurrence of the peak or the maximum in the $\langle\sigma_z\rangle_0$ as a function of coupling strength α is attributed to stochastic resonance in quantum two-level systems. This is a result of cooperative phenomena between various competing mechanisms of energy exchange between the two-state system with thermal bath and the external driving field.

In fig.(2) and fig.(3) we have plotted $\langle\sigma_z\rangle_0$ versus ω_1($\equiv \hbar\omega/k_B T$). For this we have taken V_1($\equiv V_0/k_B T$)=0.04 and Δ_1($\equiv \Delta/k_B T$)=0.2. For fig.(2) $\alpha = 0.05$ and for fig.(3) $\alpha = 0.01$. Fig.(2) and (3) should be compared with fig.(7a) and fig.(7b) of respectively ref.[9] for their qualitative behaviour. In fig.(2) $\langle\sigma_z\rangle_0$ exhibits a maximum at the resonant frequency $\omega = 2\Delta_0$, whereas two maxima are seen in fig.(3) at off resonance frequency values. This indicates stochastic resonance can be obtained even for the off resonance conditions. This further indicates that the stochastic resonance is indeed a bonafied resonance [17,18].

In fig(4) we have plotted $\langle\sigma_z\rangle_0$ as a function of external driving field amplitude V_1($\equiv$

V_0/k_BT) for various values of the dissipation parameter, the frequency of the field is $\omega_1 = 2\Delta_1 = 0.4$. The solid, long dashed and small dashed curves are for $\alpha = (0.02), (0.04)$ and (0.06), respectively. We notice that the response of the system, i.e., $\langle\sigma_z\rangle_0$, initially increases with the field amplitude and attains a maximum value at a particular value of V_1 depending on α and on other parameters. After exhibiting the maximum $\langle\sigma_z\rangle_0$ decreases as we increase V_1 further. The maximum value (M_p) of the peak in the $\langle\sigma_z\rangle_0$ for a given value of Δ_1, ω_1 and α occurs at the field amplitude value $V_1 = \sqrt{\frac{\{(\omega_1^2-4\Delta_1^2)^2+4\alpha^2\omega_1^2\}}{2\omega_1^2}}$ and is equal to $M_p = \sqrt{2}(\frac{\Delta_1^2}{\omega_1})$ which is independent of α, the Kondo parameter. The linear response theory is valid for small V_1 to the left side regime of the maximum. In these plots stochastic resonance manifests as a breakdown of linear response theory, thus bringing out the non linear nature of the problem explicitly.

In fig.(5) we have plotted $\langle\sigma_z\rangle_0$ versus $\Delta_1(\equiv \hbar\Delta_0/k_BT)$, for given $\omega_1(\equiv \hbar\omega/k_BT)$=0.4 and a small α=0.04. The maximum appears well within the range of acceptable values of $\Delta_1 \ll 1$, i.e., $\hbar\Delta_0 \ll k_BT$. For very small values of α we do get maximum response at two values of $\Delta_1(\neq 2\omega_1)$ indicating stochastic resonance at off resonance condition (similar to the observation made in fig. (3)).

In conclusion we have derived quantum Langevin equation for a dissipative two-level system, driven by monochromatic ac field, starting from microscopic spin-boson Hamiltonian. The equations of motion for the average values of the spin variables are obtained in the high temperature limit. We have obtained an analytical expression for the amplitude of coherent oscillation in the average position of the particle, which exhibits stochastic resonance with respect to various parameters in the problem. These results are in qualitative agreement with the results obtained by Makarov and Makri using the numerical method of iterative path integral scheme [8,9].

REFERENCES

[1] R. Benzi, G. Parisi, A. Sutera, and A. Vulpiani, Tellus **34**, 10(1982).

[2] B. McNamara and K. Wiesenfeld, Phys. Rev.**A 39**, 4854(1989); A. R. Bulsara and L. Gammaitoni, Physics Today **49**, 39(March 1996).

[3] Proceedings of NATO Workshop on Stochastic Resonance in Physics and Biology, edited by F. Moss, A. Bulsara, and M. F. Shlesinger, J. Stat. Phys. **70**, 1(1993).

[4] S. Fauve and F. Heslot, Phys. Lett. **97A**, 5(1983); B. McNamara, K. Wiesenfeld and R. Roy, Phys. Rev. Lett. **60**, 2626(1988).

[5] R. L$\ddot{o}$fstedt and S. N. Coppersmith, Phys. Rev. **E 49**, 4821(1994); Phys. Rev. Lett. **72**, 1947(1994).

[6] P. H$\ddot{a}$nggi and M. Grifoni, Phys. Rev. Lett. **76**, 1611(1996).

[7] M. Grifoni, M. Sasseti, P. H$\ddot{a}$nggi and U. Weiss, Phys. Rev. **E 52**, 5863(1995).

[8] D. E. Makarov and N. Makri, Phys. Rev. **B 52**, R2257(1995).

[9] D. E. Makarov and N. Makri,Phys. Rev. **E 52**, 5863(1995).

[10] A. J. Legget, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg and M. Zwerger, Rev. Mod. Phys. **59**, 1(1987).

[11] A. M. Jayannavar, Z. Phys. **B 82**, 153(1991).

[12] G. W. Ford, J. T. Lewis and R. F.O′Connell, Phys. Rev. **A 37**, 4419(1988).

[13] K. Lindenberg and B. J. West, Phys. Rev. **A 30**, 568(1984).

[14] E. A. Novikov, Zh. Eksp. Teor. Fiz. **47**, 1991(1947); E. A. Novikon, Sov. Phys. JETP **20**, 1290(1965).

[15] A. M. Jayannavar and N. Kumar, Phys. Rev. Lett. **48**, 553(1982).

[16] A. M. Jayannavar, Phys. Rev. **E 48**, 837(1993).

[17] L. Gammaitoni, F. Marchesoni, and S. Santucci, Phys. Rev. Lett. **74**, 1052(1995).

[18] M. C. Mahato and A. M. Jayannavar, Unpublished.

Figure caption

Fig.1 Plot of the stationary amplitude of average position $\langle\sigma_z\rangle_0$ versus disipation coefficient (Kondo parameter) α for a fixed value of ω=2Δ_0=0.4. For this case $V_1(\equiv V_0/k_BT)$=0.01 (solid curve), 0.04(long dashed curve) and 0.07(small dashed curve).

Fig2. Plot of the stationary amplitude of average position $\langle\sigma_z\rangle_0$ versus ω_1 $(\equiv \hbar\omega_0/k_BT)$ for V_1=0.04, Δ_1=0.2 and α=0.05.

Fig3. Plot of the stationary amplitude of average position $\langle\sigma_z\rangle_0$ versus ω_1 $(\equiv \hbar\omega_0/k_BT)$ for V_1=0.04, Δ_1=0.2 and α=0.01.

Fig.4 Plot of the stationary amplitude of average position $\langle\sigma_z\rangle_0$ versus $V_1(\equiv V_0/k_BT)$ for a fixed value of ω=2Δ_0=0.4. For this case α=0.02 (solid curve), 0.04(long dashed curve) and 0.06(small dashed curve).

Fig.5 Plot of stationary amplitude of average position $\langle\sigma_z\rangle_0$ versus $\Delta_1 \equiv (\hbar\Delta_0/k_BT)$ for a fixed value of V_1=0.04, ω_1 =0.4, and α =0.04.