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GENERATION OF SPECTRUM COMPATIBLE ACCELEROGRAMS

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SUMMARY

In this paper a new method is presented for generating earthquake accelerograms which have pre-established response spectra. The non-stationary random nature and other salient features of the accelerograms can be taken care of by the procedure developed. The method leads to a sample spectrum which lies above the given spectrum. The generation of records to suit several spectra simultaneously can also be handled by this approach. The method is detailed first. This is followed by several numerical examples.

INTRODUCTION

Random process models for ground acceleration are widely used in earthquake engineering problems. A variety of stationary and non-stationary Gaussian random processes have been suggested by various investigators[1-4] for this purpose. Generally, these processes incorporate some trends and typical characteristics of the earthquake motion to various levels of accuracy. As a justification of the proposed model, invariably, the response spectra for several samples are constructed to demonstrate that the spectral characteristics are very similar to those of real earthquakes. In recent years, interest is growing in the problem of generating an accelerogram to fit, as precisely as possible, a pre-established response spectrum. This is particularly important in the seismic analysis of nuclear power stations.[5,6] Several research workers have looked into this problem from various angles. Tsai[7] selects an existing real accelerogram whose spectrum matches closely with the smooth design response spectrum (SDRS). The record is then passed successively through suitable filters to reduce the spectrum ordinates wherever necessary. Similarly, to increase the spectrum as required, sinusoidal motions are superposed over the selected record. Rizzo *et al*[8] use a very similar technique. They find it convenient to work in the frequency domain rather than with the accelerogram in the time domain. Scanlan and Sachs[9] use a Fourier series representation with a random phase distribution for the time history to be generated. The Fourier coefficients are found by successive iteration and adjustment such that the time history generated response spectrum (THRS) and the SDRS compare well. To start the iteration, the well-known fact that the response spectrum for zero damping closely resembles the Fourier spectrum is used. Levy and Wilkinson[10] also use a Fourier representation, but without phase shifts. They consider the generation of dual time histories having the same THRS which could represent the two horizontal components to a structure. They have suggested methods to make the two time histories uncorrelated. Shaw *et al*[11] have proposed several guidelines which will be useful in generating spectrum compatible earthquakes. In particular they have brought out the importance of including the basic characteristics and site properties such as peak ground acceleration, duration and non-stationarity. King and Chen[12] in a recent publication have used the power spectral density function for generating spectrum compatible motions. They claim that this technique can help in generating a single sample which is compatible with the spectra for several damping values.

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In the present paper a new method is developed which, retaining the Fourier series representation, ensures that the THRS is always above the SDRS at any stage of the computation process. Further, the procedure developed seems to converge numerically to the target spectrum. Also the technique can be used for the simultaneous satisfaction of several spectra.

THEORY

It is well known that earthquake accelerograms can be represented as non-stationary random processes in the form[1,2]

$$\ddot{x}(t) = v(t)\,S(t) \tag{1}$$

Here, $v(t)$ is a deterministic modulating function and $S(t)$ is a stationary random process which can be represented in a finite interval $(0, T)$ as[13]

$$S(t) = \sum_{i=1}^{N} c_i \sin(\Omega_i t - \phi_i) \tag{2}$$

In this expression the c_i's are deterministic constants which control the power spectral density function of $S(t)$; the ϕ_i's are mutually independent random phase angles uniformly distributed in $(-\pi, \pi)$. The lowest frequency present in $S(t)$ is $2\pi/T$, and hence

$$\left.\begin{aligned} \Omega_i &= 2\pi i/T \\ N &= f_c T \end{aligned}\right\} \tag{3}$$

where f_c is the highest frequency present in $S(t)$. For the modulating function $v(t)$, of the several functional forms in vogue,[1,2] herein the one used by Shinozuka and Sato[3] has been selected. This is

$$v(t) = (\mathrm{e}^{-\alpha t} - \mathrm{e}^{-\beta t}) \tag{4}$$

The simple exponential form of this expression makes response integrations easier. The parameters α and β control the build up, decay and duration of the accelerogram. In terms of the time to the peak and total duration, α and β can be estimated reasonably well beforehand.

It remains to find c_i and ϕ_i such that the supremum of the absolute value of the response of a single degree-of-freedom system, with frequency ω_i and damping η, is equal to the given target spectrum $G(\omega_i)$ for $0 \leqslant \omega_i \leqslant 2\pi f_c$. The relative displacement and velocity of a linear oscillator to the input of equation (1) is

$$Z_i(t) = \int_0^t v(\tau)\,S(\tau)\,h_i(t-\tau)\,\mathrm{d}\tau \tag{5}$$

$$\dot{Z}_i(t) = \int_0^t v(\tau)\,S(\tau)\,\dot{h}_i(t-\tau)\,\mathrm{d}\tau \tag{6}$$

where

$$h_i(t) = (1/\bar{\omega}_i)\,\mathrm{e}^{-\eta\omega_i t} \sin \bar{\omega}_i t \tag{7}$$

$$\bar{\omega}_i = \omega_i(1-\eta^2)^{\frac{1}{2}} \tag{8}$$

$$\dot{h}_i = (1/\bar{\omega}_i)\,\mathrm{e}^{-\eta\omega_i t}(\bar{\omega}_i \cos \bar{\omega}_i t - \eta\omega_i \sin \bar{\omega}_i t) \tag{9}$$

Substitution of equations (2) and (4) in equation (6) leads to the following expression

$$Z_i(t) = \sum_{j=1}^{N} c_j (I_{ij} \cos \phi_j + J_{ij} \sin \phi_j) \tag{10}$$

where

$$\begin{aligned} I_{ij}(t) = {} & (\mathrm{e}^{-\alpha t} A_{ij} + \mathrm{e}^{-\beta t} B_{ij}) \cos \Omega_j t + (\mathrm{e}^{-\alpha t} C_{ij} + \mathrm{e}^{-\beta t} D_{ij}) \sin \Omega_j t + \mathrm{e}^{-\eta\omega_i t} \\ & \times [(E_{ij} + G_{ij}) \cos \bar{\omega}_i t + (F_{ij} + H_{ij}) \sin \bar{\omega}_i t] \end{aligned} \tag{11}$$

$$J_{ij}(t) = (e^{-\alpha t} A_{ij} + e^{-\beta t} B_{ij}) \sin \Omega_j t - (e^{-\alpha t} C_{ij} + e^{-\beta t} D_{ij}) \cos \Omega_j t + e^{-\eta \omega_i t} \times [(G_{ij} - E_{ij}) \sin \bar{\omega}_i t + (F_{ij} - H_{ij}) \cos \bar{\omega}_i t] \quad (12)$$

$$A_{ij} = [(\alpha N_2 - \Omega_j N_4)/(N_2^2 + N_4^2) + (\Omega_j N_5 - \alpha N_2)/(N_2^2 + N_5^2)]/\bar{\omega}_i \quad (13)$$

$$B_{ij} = [(\Omega_j N_4 - \beta N_3)/(N_3^2 + N_4^2) + (\beta N_3 - \Omega_j N_5)/(N_3^2 + N_5^2)]/\bar{\omega}_i \quad (14)$$

$$C_{ij} = [(\Omega_j N_2 + \alpha N_4)/(N_2^2 + N_4^2) - (\Omega_j N_2 + N_5)/(N_2^2 + N_5^2)]/\bar{\omega}_i \quad (15)$$

$$D_{ij} = [(\Omega_j N_3 + \beta N_5)/(N_3^2 + N_5^2) - (\Omega_j N_3 + \beta N_4)/(N_3^2 + N_4^2)]/\bar{\omega}_i \quad (16)$$

$$E_{ij} = [(N_1 N_3 + \bar{\omega}_i N_4)/(N_3^2 + N_4^2) - (N_1 N_2 + \bar{\omega}_i N_4)/(N_2^2 + N_4^2)]/\bar{\omega}_i \quad (17)$$

$$F_{ij} = [(N_1 N_4 - \bar{\omega}_i N_2)/(N_2^2 + N_4^2) + (\bar{\omega}_i N_3 - N_1 N_4)/(N_3^2 + N_4^2)]/\bar{\omega}_i \quad (18)$$

$$G_{ij} = [(N_1 N_2 - \bar{\omega}_i N_5)/(N_2^2 + N_5^2) + (\bar{\omega}_i N_5 - N_1 N_3)/(N_3^2 + N_5^2)]/\bar{\omega}_i \quad (19)$$

$$H_{ij} = [(N_1 N_5 + \bar{\omega}_i N_2)/(N_2^2 + N_5^2) - (N_1 N_5 + \bar{\omega}_i N_3)/(N_3^2 + N_5^2)]/\bar{\omega}_i \quad (20)$$

$$N_1 = \eta \omega_i, \quad N_2 = N_1 - \alpha, \quad N_3 = N_1 - \beta, \quad N_4 = \Omega_j + \bar{\omega}_i, \quad N_5 = \Omega_j - \bar{\omega}_i$$

Let the time to the highest response be a random variable t_i. Further, let the sign S_i of this response be + or − with equal probability. Now, it can be observed that if the equation

$$\dot{Z}_i(t) = S_i G(\omega_i) \quad (21)$$

is satisfied, for some t in $(0, T)$, then for such a response the condition

$$\sup_{(0,T)} |\dot{Z}_i(t)| \geq G(\omega_i) \quad (22)$$

will hold good. Herein this property is extensively used.

SINGLE VELOCITY SPECTRUM

The accelerogram is defined in terms of the $2N$ unknowns c_i and ϕ_i. Equation (22) with the equality sign leads to N conditions only. To overcome this difficulty we proceed as follows. ϕ_i is selected as a random variable uniformly distributed in $(-\pi, \pi)$ to start with. t_i is also selected as a random variable uniformly distributed in a small interval after the modulating function $v(t)$ has reached its peak value. With these quantities determined, the equation

$$\dot{Z}_i(t_i) = S_i G(\omega_i) \quad (i = 1, 2, \ldots, N) \quad (23)$$

is solved simultaneously to arrive at the c_j's. With the new c_j and previous ϕ_j, the accelerogram and the corresponding THRS are computed along with proper t_i and S_i. The spectrum generated will definitely be above the SDRS. However, the discrepancy may be too large calling for further modification in c_j and ϕ_j. If this is necessary the c_j's are scaled down linearly as

$$\bar{c}_j = c_j(\mathrm{SDRS}/\mathrm{THRS}) \quad (24)$$

With this set of c_j's a new set of ϕ_j's is calculated as follows. Equation (2) can be expressed in the form

$$S(t) = \sum_{j=1}^{N} (a_j \sin \Omega_j t + b_j \cos \Omega_j t) \quad (25)$$

where

$$a_j = c_j \cos \phi_j, \quad b_j = -c_j \sin \phi_j \quad (26)$$

Using the modified c_j and the previous ϕ_j, a_j is found from the above equation. At this stage equation (23) is solved with the new values of a_j, t_i, S_i to determine b_j. The accelerogram generated with this a_j and b_j will again lead to an upper bound for the SDRS. A further improvement is quickly obtained by using the current phase angles

$$\phi_j = \tan^{-1}(-b_j/a_j) \tag{27}$$

as a new set of starting values to find the improved c_j. These steps can be repeated till satisfactory convergence is obtained.

Numerical example

To illustrate the application of the above method, the smooth velocity spectrum used by Scanlan and Sachs[9] at 2 per cent damping is taken as the target spectrum. Two earthquake accelerograms of 10 s duration are needed. The parameters for the modulating function can be taken as $\alpha = 0{\cdot}5$ and $\beta = 1{\cdot}0$. Since the lowest frequency is 0·1 Hz, and the cutoff frequency is 10 Hz, the total number of terms to be included in the series representation will be 100. Consideration of 100 unknowns makes the computation somewhat expensive. With this in view only 25 terms are considered here in the numerical work. However, there will be no difficulty in including a larger number of terms at higher computer costs. The target spectrum and the frequencies used in shaping the spectrum are shown in Figure 1. In every cycle of iteration, as pointed out already, two upper bound spectra (one with ϕ_j and c_j and another with a_j and b_j) can be obtained. Spectra for four iterations and the resulting final version of the sample accelerogram are shown in Figures 1 and 2. Figure 3 shows the spectral convergence for a different starting ϕ_j. The corresponding accelerogram is presented in Figure 4.

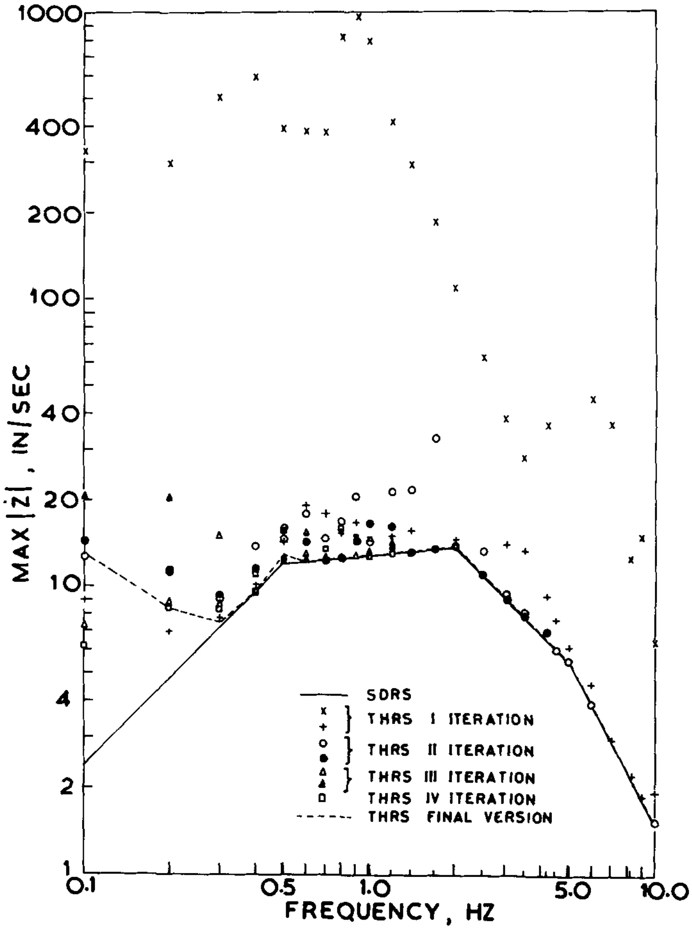

Figure 1. Convergence of THRS to SDRS; sample I

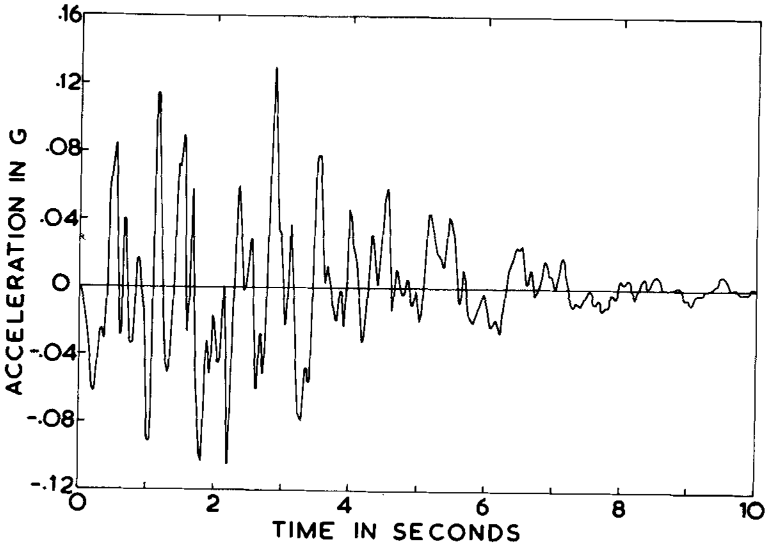

Figure 2. Spectrum compatible accelerogram; sample I

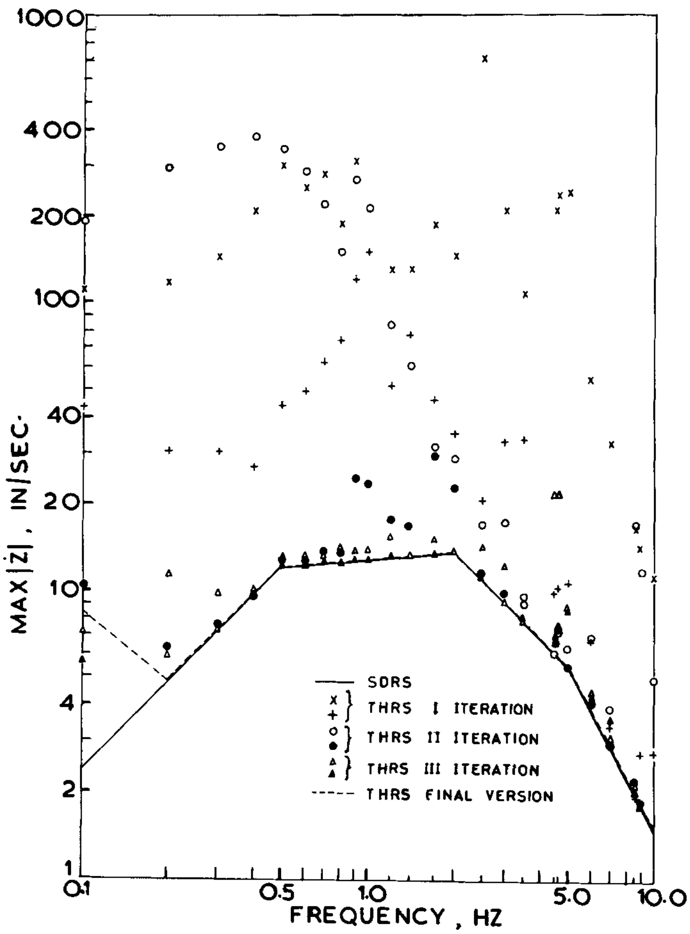

Figure 3. Convergence of THRS to SDRS; sample II

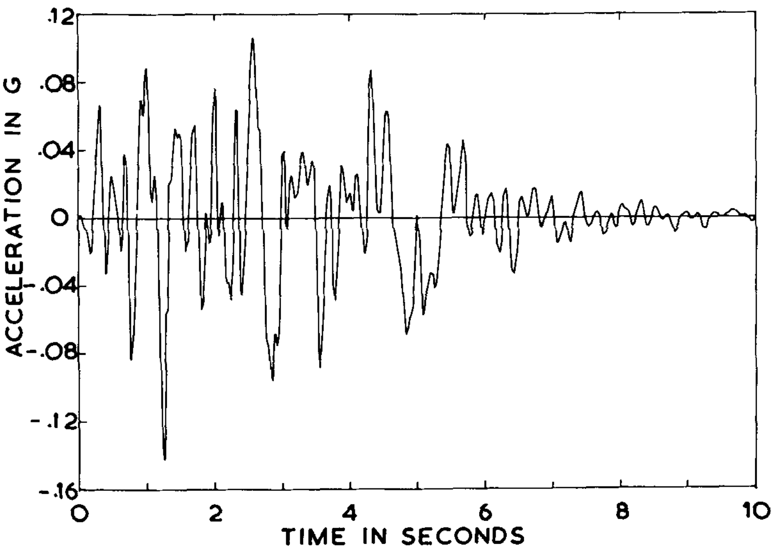

Figure 4. Spectrum compatible accelerogram; sample II

SHAPING OF MULTIPLE SPECTRA

It may be observed from the Fourier representation of equation (25) that $2N$ unknowns are to be determined to prescribe the accelerogram. However, only $N = f_c T$ divisions of the spectrum are sufficient to include all the relevant frequencies. This fact may be exploited to shape two spectra simultaneously. In doing this, iteration with respect to newer t_i and S_i and the constraint that the variance of $S(t)$ should be unity would be necessary. Even though such a procedure looks highly plausible, a large amount of computer time will be needed to obtain meaningful answers. Hence, here an approximate method is used which shapes the SDRS at only a limited number of points. The smooth velocity spectra for two values of damping, namely 2 per cent and 5 per cent shown in Figures 5 and 6, are considered as the targets. In all, 24 frequencies are

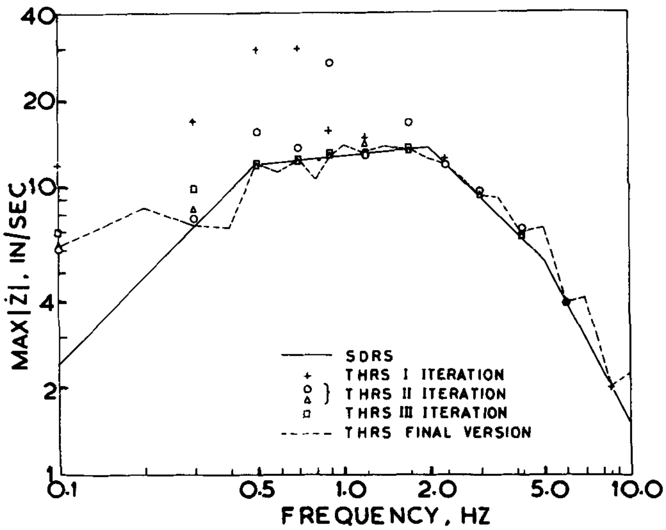

Figure 5. Shaping of multiple velocity spectra, 2 per cent damping

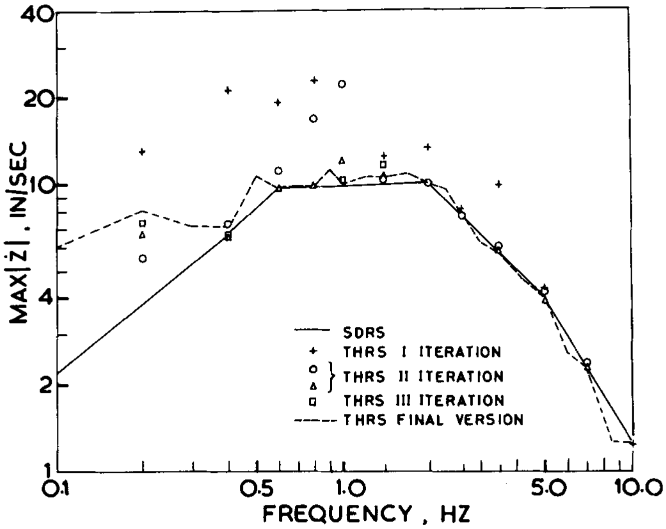

Figure 6. Shaping of multiple velocity spectra, 5 per cent damping

used in the accelerogram representation. The details of the numerical work are the same as with a single spectrum except that now 12 non-overlapping points are selected for each value of damping. The system of equations given by equation (22) is split equally between the two values of damping. The convergence of the THRS to the SDRS and the resulting accelerogram are shown in Figures 5–7.

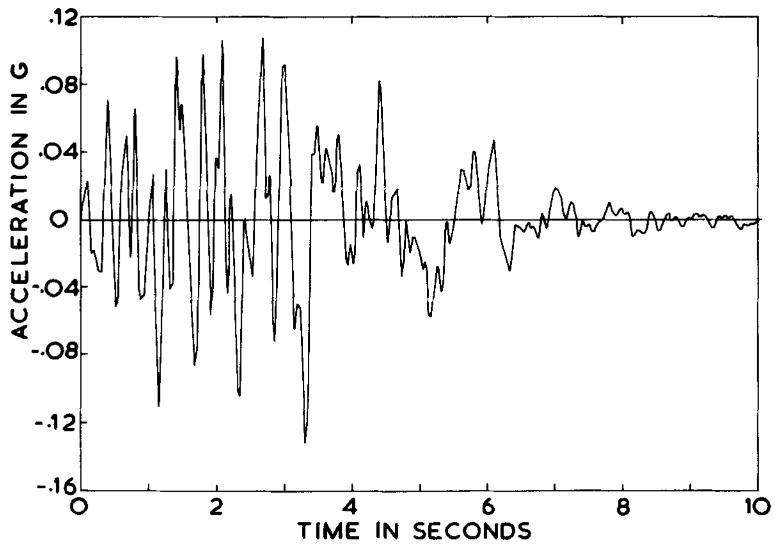

Figure 7. Accelerogram to match simultaneously spectra at two levels of damping

On similar lines, velocity and acceleration spectra can be simultaneously matched. The results of such an exercise are shown in Figures 8–10. Again 24 frequencies have been used in shaping the spectra of Figures 8 and 9.

In Figures 5, 6, 8 and 9, all the frequencies used in the accelerogram are considered in the final version of the spectra. It is observed that at frequencies not included in the shaping process, the THRS sometimes

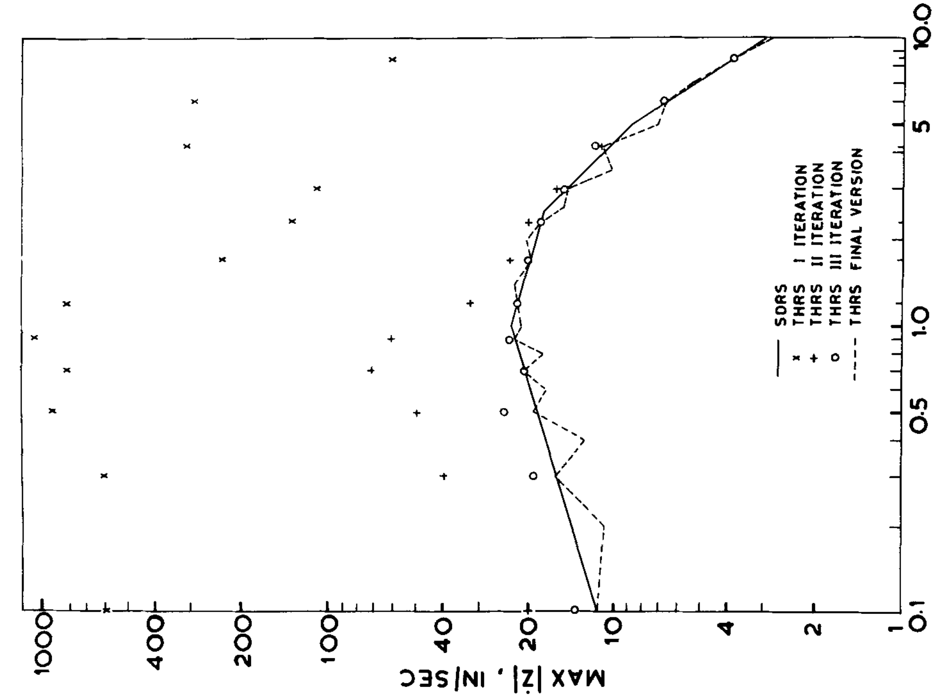

Figure 8. Shaping of multiple spectra simultaneously; velocity spectrum

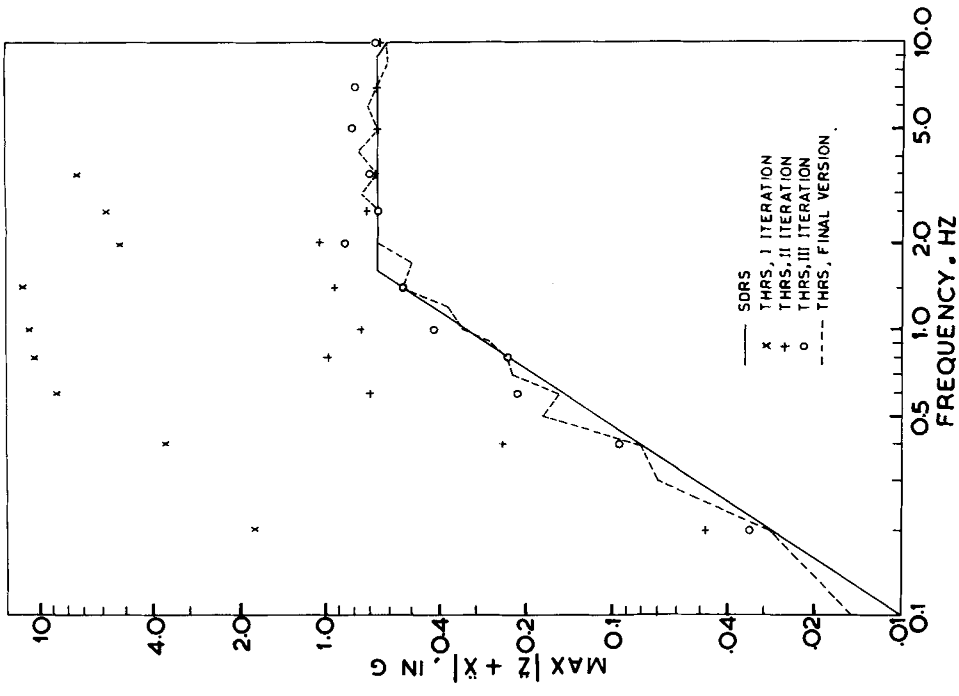

Figure 9. Shaping of multiple spectra simultaneously; acceleration spectrum

goes below the SDRS. However this discrepancy is not very large. Also, this can be overcome by including more frequencies in the iteration procedure. However, there is a possibility that a large number of frequency points may make the convergence erratic. Further investigation on this point is necessary.

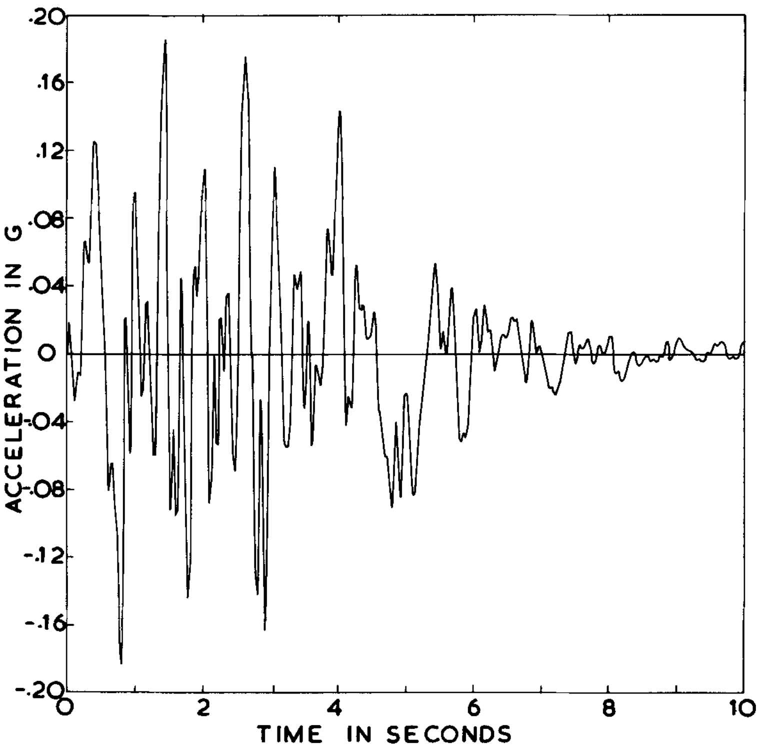

Figure 10. Accelerogram to match simultaneously velocity and acceleration spectra

DISCUSSION AND CONCLUSION

The generation of spectrum compatible earthquake accelerograms has been attempted in the present study from the point of view of a random process without appealing to the power spectral density function. This approach with random phases, amplitudes and signs ensures that the generated spectra will be at least greater than, if not precisely equal to, the target. It is found that inclusion of the random nature of time to the highest response helps the convergence of the results. Generally the results show quicker convergence in the high frequency regions. The random variables t_i are initially selected in a suitable interval after the modulating function $v(t)$ reaches its maximum value. In the examples considered above, the length of this interval has been taken as 1 s. Even though high frequency systems attain their maximum response quickly after the input reaches its peak value, low frequency systems generally need considerable build up time. With this in view it would be better to select t_i in a longer interval for low frequency systems. This may be the reason for the poor convergence of the results presented in the low frequency region. As mentioned previously the S_i's in equation (23) are selected as 1 or -1 with equal probability. On a computer this is easily done by generating a sequence of uniformly distributed random numbers u_i in (0, 1) and taking $S_i = -1$ if $u_i \leqslant 0{\cdot}5$ and $S_i = 1$ if $u_i > 0{\cdot}5$.

In the shaping process, the SDRS has been taken as the controlling quantity and no consideration has been given to the maximum value of the accelerogram. If several records are generated from the same SDRS there is bound to be some variation in the peak acceleration values obtained. Once a sample record is generated, a rough estimate of the mean and variance of the peak acceleration can be obtained easily. As an example, let the record shown in Figure 2 be considered. If this is treated as a sample of a stationary random process, it is found that approximately the mean is zero and the standard deviation $\sigma_s = 0{\cdot}04\,g$. Further, by direct counting the average rate of upward zero crossing is found to be $N_0 = 3$ per second. Now, following Cartwright and Longuet-Higgins,[14] we have for the mean m_e and standard deviation σ_e of the extreme value of a stationary random process the expressions:

$$m_e/\sigma_s = (2\ln N_0 T)^{\frac{1}{2}} + 0{\cdot}5772/(2\ln N_0 T)^{\frac{1}{2}}$$

$$\sigma_e/\sigma_s = \pi/(12\ln N_0 T)^{\frac{1}{2}}$$

This leads to $m_e = 0{\cdot}11\,g$ and $\sigma_e = 0{\cdot}02\,g$. These values seem reasonable as far as the records of Figures 2 and 4 are concerned. This reasoning is no doubt crude and approximate, but it still leads to a useful estimate of the mean and variance of the peak value.

In the simulation of a single spectrum, the duration and the cutoff frequency determine the number of terms in the series representation. In simulating multiple spectra, this procedure generally will not be convenient, since for every spectrum one may like to specify a certain number of matching points. If a number of spectra have to be simulated, each with n_i sample points, one will have to select $N = mn_i = f_c T$ unknown amplitudes in the Fourier series. This leaves either f_c or T an open variable, which cannot be fixed beforehand. Invariably f_c will be known from the spectra and also the frequency step size will be smaller than 0·1 Hz. This would make the duration T, longer than 10 s. However, there is no loss of generality in such a situation since this duration is a mathematical requirement in the series representation for $S(t)$. The duration of an earthquake itself, from the engineering point of view, depends on the rate of decay which is controlled by the modulating function $v(t)$ which is known beforehand.

The limited results presented above lead to the conclusion that the method developed in the present study is a powerful technique for generating a family of accelerograms which is compatible with specified smooth design response spectra.

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APPENDIX

Nomenclature

a_j, b_j, c_j	Fourier coefficients
$A_{ij}, B_{ij}, C_{ij}, \ldots, H_{ij}$	constants
f_c	cutoff frequency
g	acceleration due to gravity
$G(.)$	smoothed design response spectra
$S(t)$	stationary random process
S_i	random variable with value ± 1
t	time
T	duration of earthquake in seconds
$v(t)$	modulating function
$\ddot{x}(t)$	ground acceleration
$Z_i(t)$	relative displacement
SDRS	smooth design response spectra
THRS	time history generated response spectra
α, β	parameters in $v(t)$
η	viscous damping ratio
ω_i	natural frequency
$\bar{\omega}_i$	damped natural frequency
Ω_j	frequency of the jth Fourier component

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