THE ASTROPHYSICAL JOURNAL, 590:567–572, 2003 June 10

DOES THE SUN SHRINK WITH INCREASING MAGNETIC ACTIVITY?

H. M. ANTIA
Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India; antia@tifr.res.in
Received 2002 March 21; accepted 2003 February 21

ABSTRACT

It has been demonstrated that frequencies of *f*-modes can be used to estimate the solar radius to a good accuracy. These frequencies have been used to study temporal variations in the solar radius with conflicting results. The variation in *f*-mode frequencies is more complicated than what is assumed in these studies. If a careful analysis is performed, then it turns out that there is no evidence for any variation in the solar radius.

Subject headings: Sun: activity — Sun: oscillations

1. INTRODUCTION

Temporal variation in the solar radius has been a controversial topic, as direct measurements of the solar radius have given conflicting results (Delache, Laclare, & Sadsaoud 1985; Wittmann, Alge, & Bianda 1993; Fiala, Dunham, & Sofia 1994; Laclare et al. 1996; Noël 1997; Emilio et al. 2000). The reported temporal variation in the solar radius ranges from 0 to 700 km. It is important to estimate the radius variations with solar cycle, as these can provide a useful constraint for models to explain the luminosity variation with solar cycle (Gough 2001). In particular, the ratio of the radius variation to the luminosity variation, $W = (\Delta R/R)/(\Delta L/L)$, depends on the theoretical model of luminosity variations. The luminosity variation is known to be about 0.001 (Mecherikunnel 1994) between the maximum and minimum of solar activity. Thus, it is important to obtain a reliable estimate of radius variation over the solar cycle so that we can distinguish between these models.

Recently, Schou et al. (1997) and Antia (1998) have demonstrated that the frequencies of *f*-modes can be used to estimate the solar radius. Since these frequencies have been measured with a relative accuracy of 10^{-5}, we may expect to determine the solar radius to similar accuracy. However, there are systematic errors on the order of 100 km in calibration of the photospheric radius from the measured frequencies (Tripathy & Antia 1999). If these systematic errors are independent of time, then it would be possible to determine the temporal variation in the solar radius using *f*-mode frequencies. The attempts so far (Dziembowski et al. 1998, 2000, 2001; Antia et al. 2000, 2001) give conflicting results. Using the first few data sets from the Michelson Doppler Imager (MDI), Dziembowski et al. (1998) found that the solar radius is increasing with solar activity. They found an increase by about 4 km in 6 months just after the solar minimum. If this variation was indeed correlated to solar activity, we would expect a much larger variation in radius during the solar cycle. Subsequently, using more data, Dziembowski et al. (2000) found no systematic variation in the solar radius. This work used all data sets from MDI that were obtained before the contact with the *Solar and Heliospheric Observatory* (*SOHO*) satellite was lost. Using a few data sets from the Global Oscillation Network Group (GONG), Antia et al. (2000) found the solar radius to be decreasing with activity but, subsequently, using more extensive data sets from GONG and MDI, Antia et al. (2001) found no evidence for any variation in the solar radius. However, using essentially same data sets from MDI, Dziembowski et al. (2001, hereafter DGS) have found a decrease in the solar radius. Unfortunately, the claimed variation, if any, in all these works is of the order of a few kilometers and even a small change in systematic errors can give rise to spurious variations of this order. Clearly, a more careful analysis of *f*-mode frequencies is required before drawing any conclusions about variation of the solar radius.

Antia et al. (2001) have shown that the variation in *f*-mode frequencies is more complex than what is assumed in other studies. These variations can be decomposed into at least two components. One of these components is oscillatory with a period of 1 yr, while the second component is correlated with solar activity. The amplitudes of both these components increase with frequency and hence are not likely to arise from radius variations. Variation in the solar radius will cause frequency shifts that are proportional to frequency, but the observed variations have much steeper dependence on frequency. The oscillatory component is most likely to be an artifact introduced by orbital period of the Earth. Antia et al. (2001) have also shown that most of the discrepancy between different results about radius variation using *f*-mode frequencies can be explained if these two components are invoked in the temporal variations. In particular, Antia et al. (2000) failed to detect the oscillatory component as they used only five data sets covering a period of 3 yr. Further, after accounting for these two components in temporal variations, there is no evidence for any variation in the solar radius. DGS have claimed that the solar radius decreased at a rate of 1.5 km yr^{-1} during 1996–2000. However, they have not removed the oscillatory component in *f*-mode frequency variation, and hence, their claim needs to be examined carefully.

2. RADIUS VARIATION FROM *f*-MODE FREQUENCIES

It can be easily shown that if the solar radius varies by even 1 km over the solar cycle, the rate of resulting release or absorption of gravitational energy would be larger than the solar luminosity. Hence, we can rule out such radius variations. Thus, any possible variation in the solar radius must be confined to the outermost layers of the Sun. DGS have argued that since observed *f*-modes are trapped in a layer beneath the visible surface, they would measure the radius variation at this depth. In particular, the fractional variation in radius could be a function of radial distance. Such a variation is, of course, realistic, but the problem is that it may not be possible to analyze such variations easily. For example, DGS have split the *f*-mode frequency variations into two parts, one arising from radius variation and the other from some variations in the outermost layers, which scales inversely as the mode inertia. Thus, they express the change in *f*-mode frequencies as

$$\Delta\nu_\ell = -\frac{3}{2}\frac{\Delta R}{R}\nu_\ell + \frac{\Delta\gamma}{I_\ell}\ , \tag{1}$$

where ν_ℓ is the frequency of the *f*-mode of degree ℓ, ΔR is the change in radius, $\Delta\gamma$ measures the contribution from surface term, and I_ℓ is the mode inertia. While fitting the expression to the observed data, DGS assume $\Delta R/R$ to be constant, which implies that the radius variations are homologous, at least, in the region where the observed modes are trapped. Thus, as far as *f*-modes are concerned, they have assumed that $\Delta R/R$ is constant, presumably because, otherwise, it is difficult to proceed with the analysis. Subsequently, they claim that these radius variations arise from magnetic field variation in a layer below the outermost surface layers. This is certainly conceivable, but if that is the case, then there should be an additional term in equation (1) that arises from the direct effects of the magnetic field. The effect of the magnetic field on *f*-mode frequencies cannot be assumed to be solely due to those arising from radius variation. Frequency shifts due to magnetic fields (Campbell & Roberts 1989) are not, in general, proportional to frequency as is implied by equation (1): the surface term cannot arise from such fields in the interior. Thus, a more complex model will be required to fit the frequency differences arising from magnetic field. The same applies to frequency variations due to density perturbations (Chitre et al. 1998). Basically, if *f*-mode frequency variations are due to magnetic field or density perturbation, then we need to calculate these shifts explicitly rather than modeling them via radius variation, which cannot account for the entire effect.

To estimate the depth at which *f*-modes are trapped, we can consider the kinetic energy density from the eigenfunctions of *f*-modes in the relevant range of $\ell = 140$–300 that are mainly used in this study. If we assume that the trapping region of each mode covers the layers where the kinetic energy density is greater than $1/e$ of its peak value, then the upper limit of $\ell = 300$ *f*-mode is at a depth of about 1 Mm, while the lower limit of $\ell = 140$ *f*-mode extends to a depth of about 12 Mm. Hence, a depth range of 1–12 Mm is expected to be covered by this study. The *f*-modes are not trapped between a pair of rigid boundaries, and the extent of region covered by them will depend on the definition of boundary as well as on the precise mechanism responsible for frequency variations.

3. RESULTS USING MDI DATA

The inconsistencies pointed out in the previous section arise in attempting to find a physical model that gives rise to the radius variation inferred from *f*-mode frequencies. This model is relevant only if the data actually show any evidence for change in radius from *f*-mode frequencies. Thus, in this section, I ignore the cause of radius variation and just address the question of whether the changes in *f*-mode frequencies imply any change in the solar radius (as defined by DGS). The *f*-mode frequency variation is expressed using two terms, one arising from variation in the solar radius and another from unspecified variations in the outermost surface layers. There are good reasons to look for such a term since it is known that *p*-mode frequency variations largely arise from variations in outer layers (Basu & Antia 2000). Thus, following DGS, I assume the frequency variation to be given by equation (1), where $\Delta R/R$ and $\Delta\gamma$ are constants. In order to test whether the observed data fit this form, I have used the same *f*-mode frequency data (Schou 1999) that DGS have used except for some additional data sets that are now available. These data consist of 30 sets, each covering a period of 72 days starting from 1996 May 1 and ending on 2002 August 21. Note that there is a gap in data sets between 1998.5 and 1999.2, when the contact with the *SOHO* satellite was lost. The only data set during this period has a significantly worse fit and may be ignored. For each data set, I take the difference in frequency with respect to a standard solar model (in the sense of observed−model) with radius $R_\odot = 695.78$ Mm. The model radius is chosen to ensure that the frequency differences are small. These frequency differences are then fitted to equation (1). All modes with $\ell > 140$ are used in these fits. Even if I ignore the data set taken immediately after recovery of the *SOHO* satellite ($\chi^2 = 3.54$), the χ^2 per degree of freedom in these fits varies from 1.2 to 2.5. One such fit for data obtained around 1997.0 is shown in Figure 1. It is clear that the fit is not good and that the variation in frequency differences is more complicated than what is modeled by equation (1). This should be expected from the results of Antia et al. (2001) since, depending on the phase of the oscillatory component in the frequency variations, it can have a different sign as compared to the other component of the variation. Furthermore, the oscillatory component has roughly the same frequency dependence as the second term in equation (1), while the nonoscillatory component has a significantly less steep frequency dependence and hence, may not be adequately represented by equation (1). Thus, at times when both components are in phase, the net frequency variation may be approximated by equation (1), but 6 months later, when the oscillatory component changes sign, the two will be opposite, as is the case around 1997.0, data for which are shown in Figure 1. Since the oscillatory component has a steeper frequency dependence as compared to the

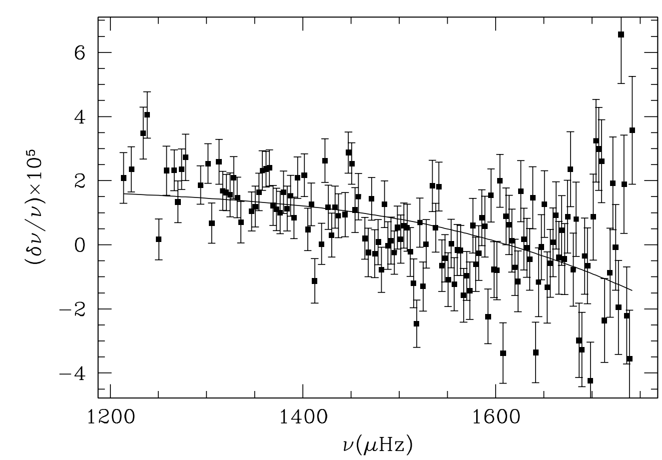

FIG. 1.—Fit to MDI data taken around 1997.0 using eq. (1). *Points with error bars*: Relative frequency difference between observed and model frequencies. *Continuous line*: Best fit using eq. (1).

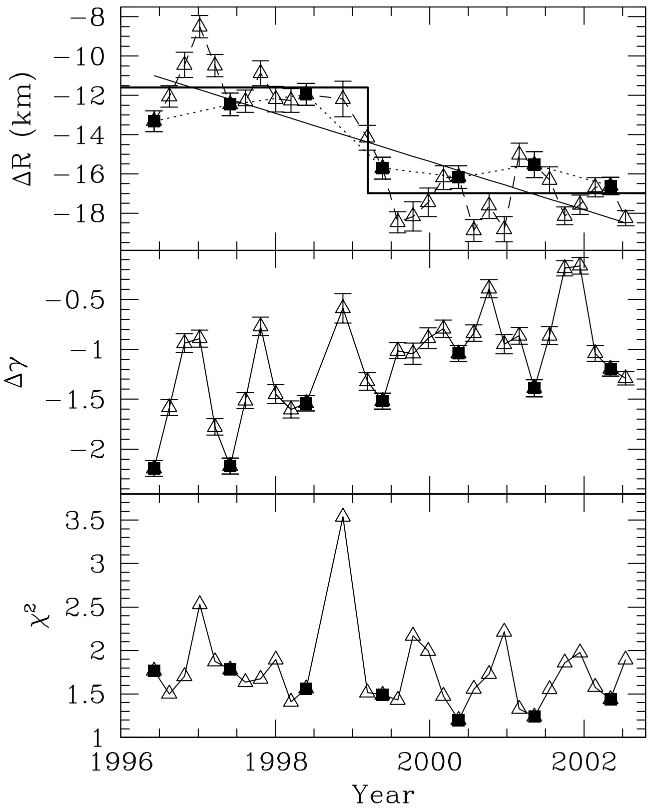

FIG. 2.—Estimated variation in the solar radius, ΔR, and the surface term, $\Delta\gamma$ from *f*-mode frequencies, obtained by fitting eq. (1) to frequency difference between a given MDI set and a solar model. The χ^2 per degree of freedom for each set is shown in the lowest panel. *Filled squares*: Results for data sets at an interval of 360 days for which the fit is relatively good. *Solid line in top panel*: Straight-line fit to all points, similar to that obtained by DGS. The dashed line connects all points in upper panel, while the dotted line connects the filled squares. The bold line shows a step function fit to all points with discontinuity at 1999.2.

nonoscillatory component, at high frequencies, the trend appears to reverse. Such a behavior cannot be modeled by equation (1).

Figure 2 shows the results from fits to all data sets from MDI. The upper panel shows the inferred radius variation, which is similar to Figure 2 of DGS. The middle panel shows the fitted variation in surface term $\Delta\gamma$. This figure also looks similar to Figure 2 of DGS, though the *y*-axis is different. The cause of this difference is not clear, as the surface term cannot be positive, as shown in Figure 2 of DGS. The bottom panel shows the χ^2 per degree of freedom for each of the fits. The oscillatory trend is quite clear in all these panels. Further, comparing different panels, it is clear that the best fits are obtained when the oscillatory and nonoscillatory components are of the same sign and the magnitude of the oscillatory component is close to maximum, which is the case when $\Delta\gamma$ is the lowest. The nonoscillatory component appears to be reducing with increasing solar activity and, as a result, fits improve during high activity periods. For example, the best fits during prerecovery periods have $\chi^2 \approx 1.5$ per degree of freedom, while during 2000–2001 this reduces to 1.2. In their Figure 1, DGS have shown fits to some data sets at intervals of 1 yr. It can be seen from Figure 2 in this paper that these correspond to times when χ^2 is close to a local minima. The fits shown by DGS correspond to filled squares in Figure 2. Some of the intermediate data sets give bad fits, as can be seen from Figure 1. About 20% of the fits have $\chi^2 \gtrsim 2$ and most of these show clear deviation from the assumed form.

These oscillations in ΔR arise because the expression is inadequate to fit the data and do not represent real variations in ΔR. Basically, there is additional contribution to $\delta\nu_\ell$ that cannot be represented by either of the terms in equation (1), and this gets projected onto the two terms, giving spurious results. In particular, the oscillatory trend is also projected onto both terms in equation (1) and further, the division between the two terms is also a function of time. As a result, the oscillations get modulated, and it is not straightforward to isolate the oscillatory part in the fitted results for ΔR. If all these oscillations are ignored and a linear function in time is fitted to the inferred radius variation as DGS have done, then the χ^2 per degree of freedom for this fit is 8.7. This fit is shown by the continuous line in the upper panel of Figure 2 and corresponds to a radius variation of -1.2 km yr^{-1}, slightly less than that inferred by DGS. This difference is because of the additional data that have become available. A slightly smaller χ^2 of 5.5 per degree of freedom is obtained if, instead, these points are fitted by a step function with a discontinuity around 1999.2. This fit is shown by the bold line in upper panel of Figure 2. Looking at the top panel of Figure 2, it appears that inferred radius has suddenly changed around 1999, and the step function fit appears to support this hypothesis. The large χ^2 is to be expected, as equation (1), which is used to calculate ΔR, does not really fit the observed data at all times.

Ideally, one should remove the oscillatory component in frequency variation before considering longer period variations, but for simplicity, I consider fits at interval of 1 yr, which will be at the same phase of oscillatory component and, further, select the phase such that the fits are the best in some sense. These points are marked by filled squares in Figure 2. Table 1 gives the results obtained for these sets, which includes the χ^2 per degree of freedom as well as the average 10.7 cm radio flux during the time interval covered by the data set, which is a measure of solar activity. Looking at this table, it is clear that the radius is not changing continuously. In fact, most of the radius variation has occurred between 1998.4 and 1999.4. Possible radius variation during 1996.4–1998.4 and 1999.4–2002.4 is less than 1 km. The solar activity did increase significantly during the period 1998.4–1999.4, but there has been a comparable change in activity during other periods, too. Hence, that cannot explain the variation seen in Table 1. This happens to be the period during which contact with the *SOHO* satellite was lost, and it is most likely that this variation reflects systematic errors arising from changes in the MDI instrument that

TABLE 1
RADIUS VARIATION AS INFERRED FROM MDI DATA

Time	10.7 cm Radio Flux (sfu)	ΔR (km)	χ^2
1996.4	72.4	-13.3 ± 0.5	1.76
1997.4	74.7	-12.5 ± 0.6	1.78
1998.4	108.5	-12.0 ± 0.6	1.56
1999.4	148.0	-15.7 ± 0.6	1.49
2000.4	186.3	-16.2 ± 0.6	1.20
2001.4	162.4	-15.5 ± 0.6	1.24
2002.4	184.3	-16.6 ± 0.5	1.44

TABLE 2

THE RATE OF RADIUS VARIATION AS INFERRED FROM MDI DATA RESTRICTED TO DIFFERENT RANGE OF DEGREE, ℓ, AND TIME INTERVAL

	Inferred Rate of Radius Variation (km yr^{-1})			
Time Interval	$\ell < 160$	$140 < \ell < 200$	$140 < \ell < 250$	$140 < \ell$
Using Both Terms in Equation (1)				
1996.4–2002.6	...	-0.47 ± 0.17	-0.57 ± 0.08	-1.22 ± 0.06
1996.4–1998.6	...	-1.16 ± 1.01	-0.01 ± 0.41	-0.16 ± 0.28
1999.2–2002.6	...	-0.52 ± 0.40	-0.00 ± 0.17	-0.30 ± 0.12
1996.4–2002.6 (revised)	$+0.11 \pm 0.16$	-0.50 ± 0.11	-0.64 ± 0.06	-1.13 ± 0.04
Using Only the First Term in Equation (1)				
1996.4–2002.6	-0.74 ± 0.09	-1.35 ± 0.05	-1.94 ± 0.04	-2.17 ± 0.03
1996.4–1998.6	-0.46 ± 0.61	-0.50 ± 0.25	-0.57 ± 0.17	-0.62 ± 0.15
1999.2–2002.6	-0.03 ± 0.22	-0.54 ± 0.11	-0.91 ± 0.08	-1.11 ± 0.07
1996.4–2002.6 (revised)	-0.70 ± 0.05	-1.29 ± 0.04	-1.68 ± 0.03	-1.83 ± 0.03

may have occurred during recovery of the satellite. Even if we assume that this variation is real, the rate of shrinking is not 1.5 km yr^{-1}, as claimed by DGS, but something like 3 km yr^{-1} during 1998.4–1999.4 and essentially no variation at other times. Thus, any model to explain this frequency change by a radius variation must explain why there is little radius variation during most of the time and why all variation is confined to less than 1 yr at some intermediate phase of solar cycle.

In order to study the robustness of the inferred radius variation, I attempt the fits by restricting the mode set or the data sets and the results are summarized in Table 2. If high degree modes are neglected, then the fits to data using equation (1) improve to some extent, which is mainly because the total variation in frequencies reduces with degree. Nevertheless, the fit to linear variation in ΔR is still bad and its slope keeps reducing as the upper limit on ℓ is reduced. Thus if only modes with $140 < \ell < 250$ are used the radius variation comes out to be -0.57 ± 0.08 km yr^{-1} (with a $\chi^2 = 2.6$), while if the upper limit on ℓ is reduced to 200, it becomes -0.47 ± 0.17 km yr^{-1} ($\chi^2 = 2$). In these cases if a step function is fitted the χ^2 comes out to be 2.1. Figure 3 shows the fits in these cases. It can be seen that the magnitude of possible discontinuity around 1999 reduces as the upper limit on ℓ is reduced and is hardly visible when the upper limit is reduced to $\ell = 200$. In this case, the errors in inferred radius are rather large, and the 1 year oscillations are essentially wiped out by statistical fluctuations. The reduction in χ^2 is mainly due to the increase in estimated errors in ΔR. Antia et al. (2001) have shown that the amplitude of oscillatory term reduces with decreasing ℓ and that also contributes to improvement in fits. If the ℓ range is reduced still further, the errors in fitted quantities are too large to make any meaningful deductions, and the results

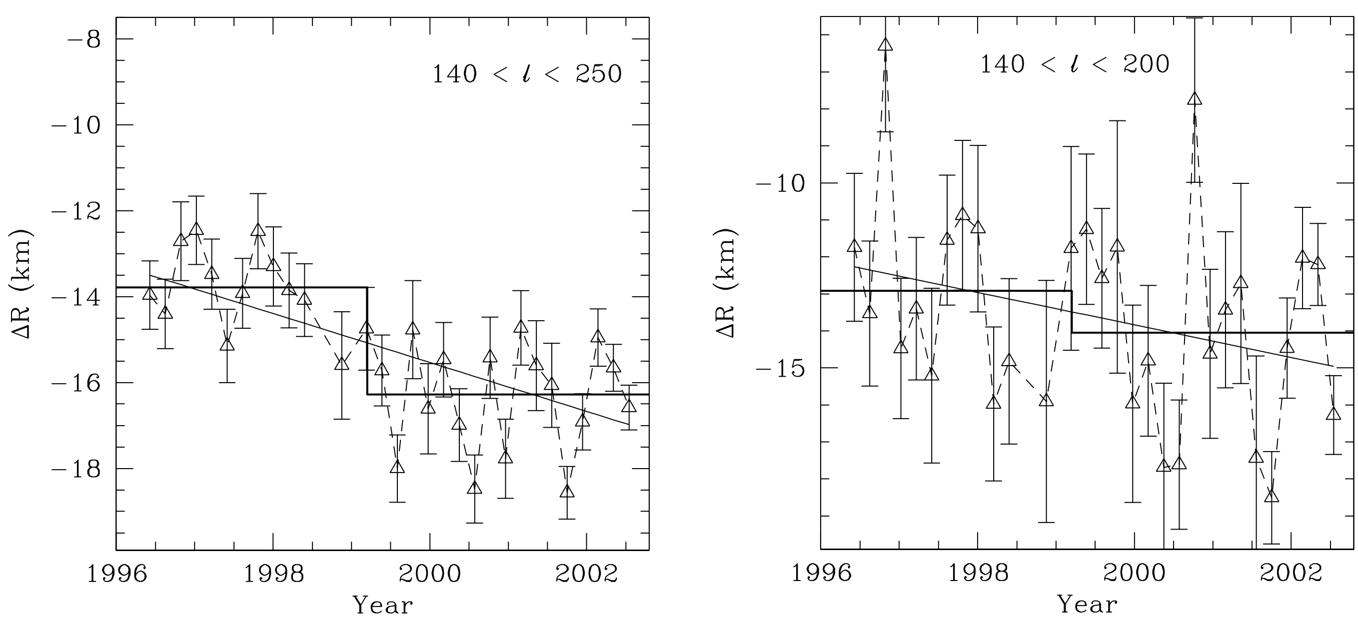

FIG. 3.—Estimated variation in the solar radius, ΔR, from *f*-mode frequencies, obtained by fitting eq. (1) to frequency difference between a given MDI set and a solar model. *Left and right panels, respectively*: Results when modes with $\ell < 250$ and $\ell < 200$ are used. *Solid line*: Linear fit to all points. *Bold line*: Fit to step function.

are not shown in Table 2. This arises mainly because the frequency variation is very small, and it is not easy to distinguish between contributions of the two terms in equation (1) over a small range of ℓ.

If only the first term in equation (1) is used for fitting, then it is equivalent to taking an average of relative frequency variations over all modes. In that case, any variation in frequency will imply a variation in radius. This may not be realistic and the resulting fits (to eq. [1]) are always bad. Since both components in the frequency variations increase steeply with ℓ as the upper limit on ℓ is reduced, the inferred radius variation should reduce, and the limiting value at the lowest frequency range would give an upper limit to any possible radius variation. In this case, since only one parameter is fitted, it is possible to get some fits with only a few low-degree modes, and hence, it is possible to reduce the upper limit on ℓ. The estimated rate of reduction in radius decreases from 2.2 km yr^{-1} when all modes are used to 0.74 km yr^{-1}, when the upper limit on ℓ is reduced to 160. If the upper limit on ℓ is reduced still further, there are very few modes in some data sets, and it is not possible to obtain any meaningful fits. However, recently the MDI data sets have been updated, and the new data has more *f*-modes. With these revised data sets, it is possible to reduce the upper limit to $\ell = 120$, and the inferred radius variation comes out to be $+0.08 \pm 0.11$ km yr^{-1} (Fig. 4). As demonstrated by Antia et al. (2001), at these low frequencies, the oscillatory component in frequency variation also is not observed. This fit appears to be consistent with the results of Basu & Antia (2002), who found that systematic error in MDI data is restricted to modes with $\ell > 120$. When these modes are eliminated, no radius variation is found. In order to enable a direct comparison with DGS, these revised data sets are not generally used in this work, but Table 2 lists the results obtained using these sets also over the full time interval. It is clear that the results are not significantly different from earlier results. If the fits are restricted to include only the postgap data sets (i.e., after 1999) and still use only the first term in equation (1), then the resulting rate of radius decrease is 1.1 km yr^{-1} with all modes and comes down to 0.03 ± 0.22 km yr^{-1} when modes with $\ell < 160$ are considered. This is to be expected, as the frequency variation increases rapidly with degree. Thus, the inferred radius variation is maximum when high degree modes are used and is negligible when only relatively low degree modes are used. If there is any component in frequency differences that corresponds to radius variation, the limiting radius variation will tend to this value when the modes in low ℓ range are used. Since this limiting value happens to be consistent with zero, we can conclude that there is no radius variation during 1999–2002 (or during the entire period when the revised MDI data are used). On the other hand, if the second term in equation (1) is also included and only data sets after 1999.0 are used, then the resulting fit does not show any significant variation in radius irrespective of the upper limit on ℓ. Similar results are obtained when only data sets before the gap (i.e., before 1998.6) are used. Thus, it is clear that most of the inferred variation in solar radius has taken place during the gap in MDI data.

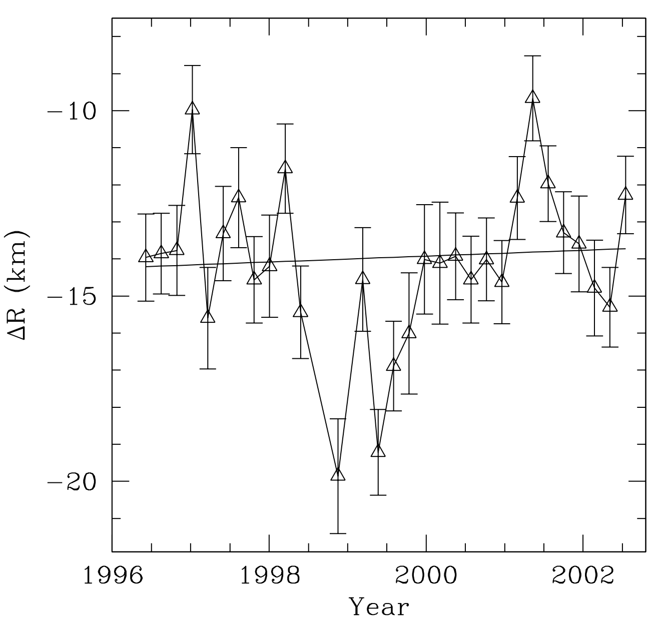

FIG. 4.—Estimated variation in the solar radius, ΔR, from *f*-mode frequencies, obtained by fitting only the first term in eq. (1) to frequency difference between a given MDI set and a solar model. Only modes with $\ell < 120$ from the revised MDI data sets are used. *Solid line*: Linear fit to all points.

From the top panel in Figure 2, it can be seen that in the pregap data (before 1999.0), the filled squares are close to the minimum in ΔR, while after the gap, the filled squares are close to the maximum in ΔR. The reason for this flip is not clear. It could be due to instrumental variations during recovery or, alternatively, it may be because before the gap, the nonoscillatory component had larger amplitude as compared to the oscillatory component; while after the gap, the amplitude of nonoscillatory component is less. There is also some variation in the number of modes and the set of modes between different data sets. If the expression fits the data well, this variation will not matter, but unfortunately, that is not true. It is difficult to assign much significance to these results as, during the time of these data sets, the oscillatory component had maximum magnitude, and this can give rise to spurious results in the fits. Comparing Figures 2 and 4, it can be seen that the inferred radius using $\ell < 120$ modes is in between the pregap and postgap values using all modes. Thus, it is clear that averaging over the oscillations in Figure 2 does not give the correct estimate of radius. Ideally, one should subtract out the oscillatory component in the frequency variation itself before analyzing the data, as has been done by Antia et al. (2001).

4. DISCUSSION AND CONCLUSIONS

There have been a few claims in the recent times about variation in the solar radius from *f*-mode frequencies (Dziembowski et al. 1998, 2001; Antia et al. 2000). Unfortunately, the variation in *f*-mode frequencies is more complex than what is modeled in these studies. From the results shown in Table 2, it is clear that there is no evidence to support a decrease in radius since the result depends on the set of modes included in the study, and the linear fit is generally bad. From Table 2, it can be seen that the estimated rate of radius change varies between -2.2 and $+0.1$ km yr^{-1}, depending on the range of ℓ and the number of terms in equation (1) used for fitting. This large variation is simply because equation (1) does not fit the observed data properly. More terms will be required to obtain a proper estimate of radius variation. In particular, when only $\ell < 120$ *f*-modes are used, the inferred radius variation always comes out to be consistent with zero. Similarly, when only data sets before (or after) the gap are used, the inferred radius variation comes out to be negligible. Basically, the observed data sets do not appear to have any component of relative frequency variation that is independent of degree, as would be required for radius variation (see eq. [1]).

A large part of the inferred variations in solar radius is most probably due to instrumental effects. For example, oscillatory trend with a period of 1 yr is probably due to orbital motion of the Earth and the *SOHO* satellite around the Sun. Similarly, the sharp variation seen between 1998.4 and 1999.4 (depending on the degree of ℓ) is most likely a result of changes in instrumental characteristics during the recovery of the *SOHO* satellite. These two effects can account for all claims of radius variation made earlier. All these instrumental errors need to be eliminated before any claim can be made about the cause of frequency variation. From the results presented above, it is clear that after eliminating these instrumental effects, there is no significant variation in the solar radius as determined by *f*-mode frequencies. A similar conclusion was obtained by Antia et al. (2001) using a more detailed analysis of both GONG and MDI data.

The systematic error between MDI data sets before and after recovery also manifests in other studies (Antia 2002; Basu & Antia 2002; Antia, Chitre, & Thompson 2003). In particular, it is found that this systematic error is mostly confined to modes with $\ell > 120$, which is consistent with the results in this work. When these modes are neglected, no radius variation is found, while if these are included, then we find varying amount of radius variation around 1999. If the radius variation is real, it cannot depend on ℓ. It is quite likely that systematic errors are present in all MDI data sets, but their magnitude has changed during recovery.

If we assume that the inferred radius variation between 1998.4 and 1999.4 is of instrumental origin, then we can put some limits on radius variation. From Table 1, it can be seen that the maximum variation between the three points before the data gap is 1.3 km, while that in the four points after the gap is 1.1 km. Considering an error of about 0.6 km in each data point, this variation is consistent with no radius variation. This would suggest an upper limit comparable to error bars in each point on radius variation during half of the solar cycle. Similar conclusion can be obtained from Figure 4, which shows the results using only $\ell < 120$ modes, which are not expected to be affected by the systematic error in MDI data. From these results, we can put a conservative upper limit of 2 km on radius variations during the last 6 yr. This would yield $\Delta R/R < 3 \times 10^{-6}$ and $W < 0.003$ as the ratio of radius to luminosity variation. Such a small value should favor models involving changes in the outer layers to explain the observed luminosity variations (Gough 1981, 2001; Däppen 1983; Balmforth, Gough, & Merryfield 1996). Of course, the value of W in these models is determined by radius variation at the photosphere, while *f*-modes are sensitive to variations at depths of about 1–12 Mm. But if the photospheric radius variations are much larger than those inferred by the *f*-modes, then the cause is most likely to be near the surface. Emilio et al. (2000) find a much larger increase in photospheric radius by about 15 ± 2 km during the solar cycle. These direct measurements from MDI are also affected by a number of systematic errors, and as they have pointed out, this value should be regarded as an upper limit to radius variation. Thus, our results from *f*-mode frequencies, which effectively measure the solar radius in the subsurface layers, are probably not inconsistent with these measurements.

This work utilizes data obtained by the Solar Oscillations Investigation/Michelson Doppler Imager on the *Solar and Heliospheric Observatory* (*SOHO*). *SOHO* is a project of international cooperation between ESA and NASA.

REFERENCES

Antia, H. M. 1998, A&A, 330, 336
———. 2002, in IAU Colloq. 188, Magnetic Coupling of the Solar Atmosphere, ed. H. Sawaya-Lacoste (ESA SP-505; Noordwijk: ESA), 71
Antia, H. M., Basu, S., Pintar, J., & Pohl, B. 2000, Sol. Phys., 192, 459
Antia, H. M., Basu, S., Pintar, J., & Schou, J. 2001, in Proc. *SOHO* 10/GONG 2000 Workshop, Helio- and Asteroseismology at the Dawn of the Millennium (ESA SP-464; Noordwijk: ESA), 27
Antia, H. M., Chitre, S. M., & Thompson, M. J. 2003, A&A, 399, 329
Balmforth, N. J., Gough, D. O., & Merryfield, W. J. 1996, MNRAS, 278, 437
Basu, S., & Antia, H. M. 2000, Sol. Phys., 192, 449
———. 2002, in Proc. *SOHO* 12/GONG 2002 Local and Global Helioseismology: The Present and Future (ESA SP-517; Noordwijk: ESA)
Campbell, W. R., & Roberts, B. 1989, ApJ, 338, 533
Chitre, S. M., Christensen-Dalsgaard, J., & Thompson, M. J. 1998, in Structure and Dynamics of the Sun and Sun-like Stars, ed. S. G. Korzennik & A. Wilson (ESA SP-418; Noordwijk: ESA), 141
Däppen, W. 1983, A&A, 124, 11
Delache, P., Laclare, F., & Sadsaoud, H. 1985, Nature, 317, 416
Dziembowski, W. A., Goode, P. R., DiMauro, M. P., Kosovichev, A. G., & Schou, J. 1998, ApJ, 509, 456
Dziembowski, W. A., Goode, P. R., Kosovichev, A. G., & Schou, J. 2000, ApJ, 537, 1026
Dziembowski, W. A., Goode, P. R., & Schou, J. 2001, ApJ, 553, 897 (DGS)
Emilio, M., Kuhn, J. R., Bush, R. I., & Scherrer, P. 2000, ApJ, 543, 1007
Fiala, A. D., Dunham, D. W., & Sofia, S. 1994, Sol. Phys., 152, 97
Gough, D. O. 1981, in Variations of the Solar Constant, ed. S. Sofia (NASA CP-2191; Washington, D.C.), 185
———. 2001, Nature, 410, 313
Laclare, F., Delmas, C., Coin, J. P., & Irbah, A. 1996, Sol. Phys., 166, 211
Mecherikunnel, A. T. 1994, Sol. Phys., 155, 211
Noël, F. 1997, A&A, 325, 825
Schou, J. 1999, ApJ, 523, L181
Schou, J., Kosovichev, A. G., Goode, P. R., & Dziembowski, W. A. 1997, ApJ, 489, L197
Tripathy, S. C., & Antia, H. M. 1999, Sol. Phys., 186, 1
Wittmann, A. D., Alge, E., & Bianda, M. 1993, Sol. Phys., 145, 205