Normal mode splitting in a coupled system of nanomechanical oscillator and parametric amplifier cavity

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We study how an optical parametric amplifier inside the cavity can affect the normal mode splitting behavior of the coupled movable mirror and the cavity field. We work in the resolved sideband regime. The spectra exhibit a double-peak structure as the parametric gain is increased. Moreover, for a fixed parametric gain, the double-peak structure of the spectrum is more pronounced with increasing the input laser power. We give results for mode splitting. The widths of the split lines are sensitive to parametric gain.

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I. INTRODUCTION

Recently there has been a major effort in applying many of the well tested ideas from quantum optics such as squeezing, quantum entanglement to optomechanical systems which are macroscopic systems. Thus observation of entanglement [1, 2, 3, 4, 5, 6], squeezing [7, 8] etc in optomechanical systems would enable one to study quantum behavior at macroscopic scale. This of course requires cooling such systems to their ground state and significant advances have been made in cooling the mechanical mirror to far below the temperature of the environment [9, 10, 11, 12, 13, 14, 15]. Further it has been pointed out that using optical back action one can possibly achieve the ground state cooling in the resolved sideband regime where the frequency of the mechanical mirror is much larger than the cavity decay rate, that is $\omega_m \gg \kappa$ [16, 17, 18].

Another key idea from quantum optics is the vacuum Rabi splitting [19, 20] which is due to strong interaction between the atoms and the cavity mode. The experimentalists have worked hard over the years to produce stronger and stronger couplings to produce larger and larger splittings [21, 22, 23]. Application of these ideas to macroscopic systems is challenging as well. In a recent paper Kippenberg *et al.* [24] proposed the possibility of normal mode splitting in the resolved sideband regime using optomechanical oscillators. In this paper, we propose placing a type I optical parametric amplifier inside the cavity to increase the coupling between the movable mirror and the cavity field, and this should make the observation of the normal mode splitting of the movable mirror and the output field more accessible.

The paper is structured as follows. In Sec. II we present the model, derive the quantum Langevin equations, and give the steady-state mean values. In Sec. III we present solution to the linearized Langevin equations and give the spectrum of the movable mirror. In Sec. IV we analyse and estimate the amount of the normal mode splitting of the spectra. In Sec. V we calculate the spectra of the output field. In Sec. VI we discuss the mode splitting of the spectra of the movable mirror and the output field.

II. MODEL

The system under consideration, sketched in Fig. 1, is an optical parametric amplifier (OPA) placed within a Fabry-Perot cavity formed by one fixed partially transmitting mirror and one movable perfectly reflecting mirror in equilibrium with its environment at a low temperature. The movable mirror is treated as a quantum mechanical harmonic oscillator with effective mass m, frequency ω_m, and energy decay rate γ_m. An external laser enters the cavity through the fixed mirror, then the photons in the cavity will exert a radiation pressure force on the movable mirror due to momentum transfer. This force is proportional to the instantaneous photon number in the cavity.

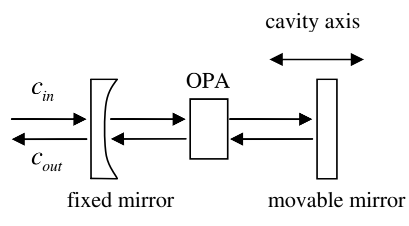

FIG. 1: Sketch of the studied system. The cavity contains a nonlinear crystal which is pumped by a laser (not shown) to produce parametric amplification and to change photon statistics in the cavity.

In the adiabatic limit, the frequency ω_m of the movable mirror is much smaller than the free spectral range of the cavity $\frac{c}{2L}$ (c is the speed of light in vacuum and L is the cavity length), the scattering of photons to other cavity modes can be ignored, thus only one cavity mode ω_c is considered [25, 26]. The Hamiltonian for the system in a frame rotating at the laser frequency ω_L can be written as

$$\begin{aligned} H &= \hbar(\omega_c-\omega_L)n_c - \hbar\omega_m\chi n_c Q + \frac{\hbar\omega_m}{4}(Q^2+P^2) \\ &\quad + i\hbar\varepsilon(c^\dagger - c) + i\hbar G(e^{i\theta}c^{\dagger 2} - e^{-i\theta}c^2). \end{aligned} \tag{1}$$

Here Q and P are the dimensionless position and momentum operators for the movable mirror, defined by $Q=\sqrt{\frac{2m\omega_m}{\hbar}}q$ and $P=\sqrt{\frac{2}{m\hbar\omega_m}}p$ with $[Q,P]=2i$. In Eq. (1), the first term is the energy of the cavity field, $n_c=c^\dagger c$ is the number of the photons inside the cavity, c and $c^\dagger$ are the annihilation and creation operators for the cavity field satisfying the commutation relation $[c,c^\dagger]=1$. The second term comes from the coupling of the movable mirror to the cavity field via radiation pressure, the dimensionless parameter $\chi=\frac{1}{\omega_m}\frac{\omega_c}{L}\sqrt{\frac{\hbar}{2m\omega_m}}$ is the optomechanical coupling constant between the cavity and the movable mirror. The third term corresponds the energy of the movable mirror. The fourth term describes the coupling between the input laser field and the cavity field, ε is related to the input laser power $\wp$ by $\varepsilon=\sqrt{\frac{2\kappa\wp}{\hbar\omega_L}}$, where κ is the cavity decay rate. The last term is the coupling between the OPA and the cavity field, G is the nonlinear gain of the OPA, and θ is the phase of the field driving the OPA. The parameter G is proportional to the pump driving the OPA.

Using the Heisenberg equations of motion and adding the corresponding damping and noise terms, we obtain the quantum Langevin equations as follows,

$$\begin{aligned}
\dot{Q}&=\omega_m P,\\
\dot{P}&=2\omega_m\chi n_c-\omega_m Q-\gamma_m P+\xi,\\
\dot{c}&=-i(\omega_c-\omega_L-\omega_m\chi Q)c+\varepsilon+2Ge^{i\theta}c^\dagger-\kappa c+\sqrt{2\kappa}c_{in},\\
\dot{c}^\dagger&=i(\omega_c-\omega_L-\omega_m\chi Q)c^\dagger+\varepsilon+2Ge^{-i\theta}c-\kappa c^\dagger+\sqrt{2\kappa}c_{in}^\dagger.
\end{aligned}\tag{2}$$

Here we have introduced the input vacuum noise operator c_{in} with zero mean value, which obeys the correlation function in the time domain [27]

$$\begin{aligned}
&\langle\delta c_{in}(t)\delta c_{in}^\dagger(t')\rangle=\delta(t-t'),\\
&\langle\delta c_{in}(t)\delta c_{in}(t')\rangle=\langle\delta c_{in}^\dagger(t)\delta c_{in}(t')\rangle=0.
\end{aligned}\tag{3}$$

The force ξ is the Brownian noise operator resulting from the coupling of the movable mirror to the thermal bath, whose mean value is zero, and it has the following correlation function at temperature T [28]:

$$\langle\xi(t)\xi(t^{'})\rangle=\frac{1}{\pi}\frac{\gamma_m}{\omega_m}\int\omega e^{-i\omega(t-t^{'})}\left[1+\coth\left(\frac{\hbar\omega}{2k_BT}\right)\right]d\omega,\tag{4}$$

where k_B is the Boltzmann constant and T is the thermal bath temperature. Following standard methods from quantum optics [29], we derive the steady-state solution to Eq. (2) by setting all the time derivatives in Eq. (2) to zero. They are

$$P_s=0,\ Q_s=2\chi|c_s|^2,\ c_s=\frac{\kappa-i\Delta+2Ge^{i\theta}}{\kappa^2+\Delta^2-4G^2}\varepsilon,\tag{5}$$

where

$$\Delta=\omega_c-\omega_L-\omega_m\chi Q_s\tag{6}$$

is the effective cavity detuning, depending on Q_s. The Q_s denotes the new equilibrium position of the movable mirror relative to that without the driving field. Further c_s represents the steady-state amplitude of the cavity field. From Eq. (5) and Eq. (6), we can see Δ satisfies a fifth order equation, it can at most have five real solutions. Therefore, the movable mirror displays an optical multistable behavior [30, 31, 32], which is a nonlinear effect induced by the radiation-pressure coupling of the movable mirror to the cavity field.

III. RADIATION PRESSURE AND QUANTUM FLUCTUATIONS

In order to investigate the normal mode splitting of the movable mirror and the output field, we need to calculate the fluctuations of the system. Since the problem is nonlinear, we assume that the nonlinearity is weak. Thus we can focus on the dynamics of small fluctuations around the steady state of the system. Each operator of the system can be written as the sum of its steady-state mean value and a small fluctuation with zero mean value,

$$Q=Q_s+\delta Q,\quad P=P_s+\delta P,\quad c=c_s+\delta c.\tag{7}$$

Inserting Eq. (7) into Eq. (2), then assuming $|c_s|\gg 1$, the linearized quantum Langevin equations for the fluctuation operators take the form

$$\begin{aligned}
\delta\dot{Q}&=\omega_m\delta P,\\
\delta\dot{P}&=2\omega_m\chi(c_s^*\delta c+c_s\delta c^\dagger)-\omega_m\delta Q-\gamma_m\delta P+\xi,\\
\delta\dot{c}&=-(\kappa+i\Delta)\delta c+i\omega_m\chi c_s\delta Q+2Ge^{i\theta}\delta c^\dagger+\sqrt{2\kappa}\delta c_{in},\\
\delta\dot{c}^\dagger&=-(\kappa-i\Delta)\delta c^\dagger-i\omega_m\chi c_s^*\delta Q+2Ge^{-i\theta}\delta c+\sqrt{2\kappa}\delta c_{in}^\dagger.
\end{aligned}\tag{8}$$

Introducing the cavity field quadratures $\delta x=\delta c+\delta c^\dagger$ and $\delta y=i(\delta c^\dagger-\delta c)$, and the input noise quadratures $\delta x_{in}=\delta c_{in}+\delta c_{in}^\dagger$ and $\delta y_{in}=i(\delta c_{in}^\dagger-\delta c_{in})$, Eq. (8) can be rewritten in the matrix form

$$\dot{f}(t)=Af(t)+\eta(t),\tag{9}$$

in which $f(t)$ is the column vector of the fluctuations, $\eta(t)$ is the column vector of the noise sources. Their transposes are

$$\begin{aligned}
f(t)^T&=(\delta Q,\delta P,\delta x,\delta y),\\
\eta(t)^T&=(0,\xi,\sqrt{2\kappa}\delta x_{in},\sqrt{2\kappa}\delta y_{in});
\end{aligned}\tag{10}$$

and the matrix A is given by

$$A=\begin{pmatrix}
0 & \omega_m & 0 & 0\\
-\omega_m & -\gamma_m & \omega_m\chi(c_s+c_s^*) & -i\omega_m\chi(c_s-c_s^*)\\
i\omega_m\chi(c_s-c_s^*) & 0 & 2G\cos\theta-\kappa & 2G\sin\theta+\Delta\\
\omega_m\chi(c_s+c_s^*) & 0 & 2G\sin\theta-\Delta & -(2G\cos\theta+\kappa)
\end{pmatrix}.\tag{11}$$

The system is stable only if all the eigenvalues of the matrix A have negative real parts. The stability conditions for the system can be derived by applying the Routh-Hurwitz criterion [33, 34]. This gives

$$
\begin{aligned}
&2\kappa(\kappa^2-4G^2+\Delta^2+2\kappa\gamma_m)+\gamma_m(2\kappa\gamma_m+\omega_m^2)>0,\\
&2\omega_m^3\chi^2(2\kappa+\gamma_m)^2[|c_s|^2\Delta+iG(c_s^2e^{-i\theta}-c_s^{*2}e^{i\theta})]\\
&\quad+\kappa\gamma_m\{(\kappa^2-4G^2+\Delta^2)^2+(2\kappa\gamma_m+\gamma_m^2)\\
&\quad\times(\kappa^2-4G^2+\Delta^2)+\omega_m^2[2(\kappa^2+4G^2-\Delta^2)\\
&\quad+\omega_m^2+2\kappa\gamma_m]\}>0,\\
&\kappa^2-4G^2+\Delta^2-4\omega_m\chi^2[|c_s|^2\Delta+iG(c_s^2e^{-i\theta}-c_s^{*2}e^{i\theta})]>0.
\end{aligned}
\tag{12}
$$

All the external parameters must be chosen to satisfy the stability conditions (12).

Taking Fourier transform of Eq. (8) by using $f(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}f(\omega)e^{-i\omega t}d\omega$ and $f^\dagger(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}f^\dagger(-\omega)e^{-i\omega t}d\omega$, where $f^\dagger(-\omega)=[f(-\omega)]^\dagger$, then solving it, we obtain the position fluctuations of the movable mirror

$$
\begin{aligned}
\delta Q(\omega)=&-\tfrac{\omega_m}{d(\omega)}[2\sqrt{2\kappa}\omega_m\chi\{[(\kappa-i(\Delta+\omega))c_s^*\\
&+2Ge^{-i\theta}c_s]\delta c_{in}(\omega)+[(\kappa+i(\Delta-\omega))c_s\\
&+2Ge^{i\theta}c_s^*]\delta c_{in}^\dagger(-\omega)\}\\
&+[(\kappa-i\omega)^2+\Delta^2-4G^2]\xi(\omega)],
\end{aligned}
\tag{13}
$$

where

$$
\begin{aligned}
d(\omega)=&4\omega_m^3\chi^2[\Delta|c_s|^2+iG(c_s^2e^{-i\theta}-c_s^{*2}e^{i\theta})]\\
&+(\omega^2-\omega_m^2+i\gamma_m\omega)[(\kappa-i\omega)^2+\Delta^2-4G^2].
\end{aligned}
\tag{14}
$$

In Eq. (13), the first term proportional to χ originates from radiation pressure, while the second term involving $\xi(\omega)$ is from the thermal noise. So the position fluctuations of the movable mirror are now determined by radiation pressure and the thermal noise. In the case of no coupling with the cavity field, the movable mirror will make Brownian motion, $\delta Q(\omega)=\omega_m\xi(\omega)/(\omega_m^2-\omega^2-i\gamma_m\omega)$, whose susceptibility has a Lorentzian shape centered at frequency ω_m with width γ_m.

The spectrum of fluctuations in position of the movable mirror is defined by

$$
\frac{1}{2}(\langle\delta Q(\omega)\delta Q(\Omega)\rangle+\langle\delta Q(\Omega)\delta Q(\omega)\rangle)=2\pi S_Q(\omega)\delta(\omega+\Omega).
\tag{15}
$$

To calculate the spectrum, we require the correlation functions of the noise sources in the frequency domain,

$$
\begin{aligned}
&\langle\delta c_{in}(\omega)\delta c_{in}^\dagger(-\Omega)\rangle=2\pi\delta(\omega+\Omega),\\
&\langle\xi(\omega)\xi(\Omega)\rangle=4\pi\tfrac{\gamma_m}{\omega_m}\omega\left[1+\coth(\tfrac{\hbar\omega}{2k_BT})\right]\delta(\omega+\Omega).
\end{aligned}
\tag{16}
$$

Substituting Eq. (13) and Eq. (16) into Eq. (15), we obtain the spectrum of fluctuations in position of the movable mirror [35]

$$
\begin{aligned}
S_Q(\omega)=&\tfrac{\omega_m^2}{|d(\omega)|^2}\{8\omega_m^2\chi^2\kappa[(\kappa^2+\omega^2+\Delta^2+4G^2)|c_s|^2\\
&+2Ge^{i\theta}c_s^{*2}(\kappa-i\Delta)+2Ge^{-i\theta}c_s^2(\kappa+i\Delta)]\\
&+2\tfrac{\gamma_m}{\omega_m}\omega[(\Delta^2+\kappa^2-\omega^2-4G^2)^2+4\kappa^2\omega^2]\\
&\times\coth(\tfrac{\hbar\omega}{2k_BT})\}.
\end{aligned}
\tag{17}
$$

In Eq. (17), the first term involving χ arises from radiation pressure, while the second term originates from the thermal noise. So the spectrum $S_Q(\omega)$ of the movable mirror depends on radiation pressure and the thermal noise.

IV. NORMAL MODE SPLITTING AND THE EIGENVALUES OF THE MATRIX A

The structure of all the spectra is determined by the eigenvalues of iA (Eq. (11)) or the complex zeroes of the function $d(\omega)$ defined by Eq. (14). Clearly we need the eigenvalues of iA as the solution of (Eq. (9)) in Fourier domain is $f(\omega)=i(\omega-iA)^{-1}\eta(\omega)$. Let us analyse the eigenvalues of Eq. (11). Note that in the absence of the coupling χ=0, the eigenvalues of iA are

$$
\pm\sqrt{\omega_m^2-\tfrac{\gamma_m^2}{4}}-\tfrac{i\gamma_m}{2};\pm\sqrt{\Delta^2-4G^2}-i\kappa.
\tag{18}
$$

Thus the positive frequencies of the normal modes are given by $\sqrt{\Delta^2-4G^2}$, $\sqrt{\omega_m^2-\frac{\gamma_m^2}{4}}$ $(\Delta>2G,\omega_m>\frac{\gamma_m}{2})$. The case that we consider in this paper corresponds to

$$
\omega_m\gg\tfrac{\gamma_m}{2};\Delta>2G;\kappa\gg\gamma_m;\omega_m>\kappa.
\tag{19}
$$

The coupling between the normal modes would be most efficient in the degenerate case i.e. when $\omega_m=\sqrt{\Delta^2-4G^2}$. It is known from cavity QED that the normal mode splitting leads to symmetric (asymmetric) spectra in the degenerate (nondegenerate) case, provided that the dampings of the individual modes are much smaller than the coupling constant. Thus the mechanical oscillator is like the atomic oscillator, the cavity mode in the rotating frame acquires the effective frequency $\sqrt{\Delta^2-4G^2}$ which is dependent on the parametric coupling. An estimate of the splitting can be made by using the approximations given by Eq. (19) and the zeroes of $d(\omega)$. We find that the frequency splitting is given by [37]

$$
\omega_\pm^2\cong\tfrac{\omega_m^2+\Delta^2-4G^2}{2}\pm\sqrt{(\tfrac{\omega_m^2-\Delta^2+4G^2}{2})^2+4\omega_m^2g^2},
\tag{20}
$$

where we have defined

$$
g^2=\omega_m\chi^2|c_s|^2[\Delta+2G\sin(\theta-2\varphi)],\ e^{2i\varphi}=c_s^2/|c_s|^2.
\tag{21}
$$

It should be borne in mind that c_s, Δ etc are dependent on the parametric coupling G. The splitting is determined by the pump power, the couplings χ and G.

The parameters used are the same as those in the recent successful experiment on optomechanical normal mode splitting [36]: the wave length of the laser $\lambda = 2\pi c/\omega_L = 1064$ nm, $L = 25$ mm, $m = 145$ ng, $\kappa = 2\pi \times 215 \times 10^3$ Hz, $\omega_m = 2\pi \times 947 \times 10^3$ Hz, $T = 300$ mK, the mechanical quality factor $Q' = \omega_m/\gamma_m = 6700$, parametric phase $\theta = \pi/4$. And in the high temperature limit $k_BT \gg \hbar\omega_m$, we have $\coth(\hbar\omega/2k_BT) \approx 2k_BT/\hbar\omega$.

Figure 2 shows the roots of $d(\omega)$ in the domain $\mathrm{Re}(\omega) > 0$ for different values of G. Figure 3 shows imaginary parts of the roots of $d(\omega)$ for different values of G. The parametric coupling affects the width of the lines and this for certain range of parameters aids in producing well split lines. One root broadens and the other root narrows. The root that broadens is the one that moves further away from the position for $G = 0$.

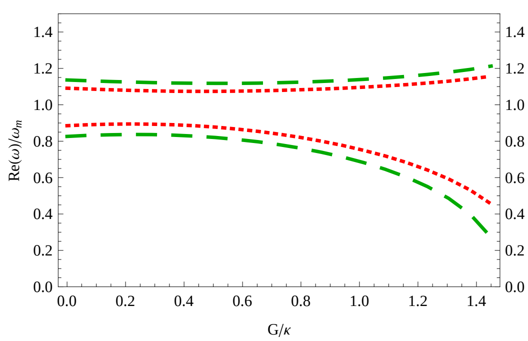

FIG. 2: (Color online) The roots of $d(\omega)$ in the domain $\mathrm{Re}(\omega) > 0$ as a function of parametric gain. $\wp = 6.9$ mW (dotted line), $\wp = 10.7$ mW (dashed line). Parameters: the cavity detuning $\Delta = \omega_m$.

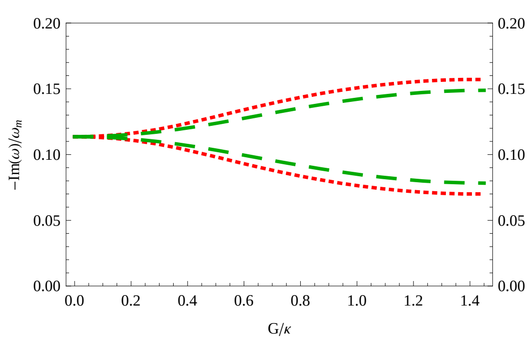

FIG. 3: (Color online) The imaginary parts of the roots of $d(\omega)$ as a function of parametric gain. $\wp = 6.9$ mW (dotted line), $\wp = 10.7$ mW (dashed line). Parameters: the cavity detuning $\Delta = \omega_m$.

V. THE SPECTRA OF THE OUTPUT FIELD

In this section, we would like to calculate the spectra of the output field. The fluctuations $\delta c(\omega)$ of the cavity field can be obtained from Eq. (8). Further using the input-output relation [38] $c_{out}(\omega) = \sqrt{2\kappa}c(\omega) - c_{in}(\omega)$, the fluctuations of the output field are given by

$$\delta c_{out}(\omega) = V(\omega)\xi(\omega) + E(\omega)\delta c_{in}(\omega) + F(\omega)\delta c^{\dagger}_{in}(-\omega), \tag{22}$$

where

$$\begin{aligned}
V(\omega) &= -\tfrac{\sqrt{2\kappa}\omega_m^2\chi}{d(\omega)} i\{[\kappa - i(\omega+\Delta)]c_s - 2Ge^{i\theta}c_s^*\},\\
E(\omega) &= \tfrac{2\kappa}{(\kappa-i\omega)^2+\Delta^2-4G^2}[-\tfrac{2\omega_m^3\chi^2}{d(\omega)} i\{[\kappa - i(\omega+\Delta)]c_s\\
&\quad -2Ge^{i\theta}c_s^*\}\{[\kappa - i(\omega+\Delta)]c_s^* + 2Ge^{-i\theta}c_s\}\\
&\quad +\kappa - i(\omega+\Delta)] - 1,\\
F(\omega) &= \tfrac{2\kappa}{(\kappa-i\omega)^2+\Delta^2-4G^2}[-\tfrac{2\omega_m^3\chi^2}{d(\omega)} i\{[\kappa - i(\omega+\Delta)]c_s\\
&\quad -2Ge^{i\theta}c_s^*\}\{[\kappa - i(\omega-\Delta)]c_s + 2Ge^{i\theta}c_s^*\}\\
&\quad +2Ge^{i\theta}].
\end{aligned} \tag{23}$$

In Eq. (22), the first term associated with $\xi(\omega)$ stems from the thermal noise of the mechanical oscillator, while the other two terms are from the input vacuum noise. So the fluctuations of the output field are influenced by the thermal noise and the input vacuum noise.

The spectra of the output field are defined as

$$\begin{aligned}
\langle\delta c^{\dagger}_{out}(-\Omega)\delta c_{out}(\omega)\rangle &= 2\pi S_{cout}(\omega)\delta(\omega+\Omega),\\
\langle\delta x_{out}(\Omega)\delta x_{out}(\omega)\rangle &= 2\pi S_{xout}(\omega)\delta(\omega+\Omega),\\
\langle\delta y_{out}(\Omega)\delta y_{out}(\omega)\rangle &= 2\pi S_{yout}(\omega)\delta(\omega+\Omega).
\end{aligned} \tag{24}$$

where $\delta x_{out}(\omega)$ and $\delta y_{out}(\omega)$ are the Fourier transform of the fluctuations $\delta x_{out}(t)$ and $\delta y_{out}(t)$ of the output field , which are defined by $\delta x_{out}(t) = \delta c_{out}(t) + \delta c^{\dagger}_{out}(t)$ and $\delta y_{out}(t) = i[\delta c^{\dagger}_{out}(t) - \delta c_{out}(t)]$ [29]. Here $S_{cout}(\omega)$ denotes the spectral density of the output field, $S_{xout}(\omega)$ means the spectrum of fluctuations in the x quadrature of the output field, and $S_{yout}(\omega)$ is the spectrum of fluctuations in the y quadrature of the output field.

Combining Eq. (16), Eq. (22), and Eq. (24), we obtain the spectra of the output field

$$\begin{aligned}
S_{cout}(\omega) &= V^*(\omega)V(\omega) \times 2\tfrac{\gamma_m}{\omega_m}\omega[-1 + \coth(\tfrac{\hbar\omega}{2k_BT})]\\
&\quad +F^*(\omega)F(\omega),\\
S_{xout}(\omega) &= [V(-\omega) + V^*(\omega)][V(\omega) + V^*(-\omega)]\\
&\quad \times 2\tfrac{\gamma_m}{\omega_m}\omega[-1 + \coth(\tfrac{\hbar\omega}{2k_BT})]\\
&\quad +[E(-\omega) + F^*(\omega)][F(\omega) + E^*(-\omega)],\\
S_{yout}(\omega) &= -[V^*(\omega) - V(-\omega)][V^*(-\omega) - V(\omega)]\\
&\quad \times 2\tfrac{\gamma_m}{\omega_m}\omega[-1 + \coth(\tfrac{\hbar\omega}{2k_BT})]\\
&\quad -[F^*(\omega) - E(-\omega)][E^*(-\omega) - F(\omega)].
\end{aligned} \tag{25}$$

From Eq. (25), it is seen that any spectrum of the output field includes two terms, the first term is from the contribution of the thermal noise of the mechanical oscillator, the second term is from the contribution of the input vacuum noise.

We note that the spectra $S_Q(\omega)$, $S_{cout}(\omega)$, $S_{xout}(\omega)$, and $S_{yout}(\omega)$ are determined by the detuning Δ, parametric gain G, parametric phase θ, input laser power $\wp$, and cavity length L. In the following we will concentrate on discussing the dependence of the spectra on parametric gain and input laser power.

VI. NUMERICAL RESULTS

In this section, we numerically evaluate the spectra $S_Q(\omega)$, $S_{cout}(\omega)$, $S_{xout}(\omega)$, and $S_{yout}(\omega)$ given by Eq. (17) and Eq. (25) to show the effect of an OPA in the cavity on the normal mode splitting of the movable mirror and the output field.

We consider the degenerate case $\Delta = \omega_m$ for $G = 0$, and choose $\wp = 6.9$ mW to satisfy the stability conditions (12), parametric gain must satisfy $G \leq 1.62\kappa$. Figures 4 − 7 shows the spectra $S_Q(\omega)$, $S_{cout}(\omega)$, $S_{xout}(\omega)$, and $S_{yout}(\omega)$ as a function of the normalized frequency ω/ω_m for various values of parametric gain. When the OPA is absent ($G = 0$), the spectra barely show the normal mode splitting. As parametric gain is increased, the normal mode splitting becomes observable. This is due to significant changes in the line widths and position. When $G = 1.3\kappa$, two peaks can be found in the spectra. Note that the separation between two peaks becomes larger as parametric gain increases. The reason is that increasing the parametric gain causes a stronger coupling between the movable mirror and the cavity field due to an increase in the photon number in the cavity. The values of intercavity photon number $|c_s|^2$ are 2.68×10^9, 4.30×10^9, 5.65×10^9 for $G = 0$, 1.3κ, and 1.45κ respectively. We have examined the contributions of various terms in Eq. (25) to the output spectrum. The dominant contribution comes from the mechanical oscillator. Note further the similarity [36] of the spectrum of the output quadrature y (Fig. 7) to the spectrum of the mechanical oscillator (Fig. 4). It should be borne in mind that the strong asymmetries in the spectra for $G \neq 0$ arise from the fact that by fixing Δ at ω_m, the frequencies of the cavity mode and the mechanical oscillator do not coincide if $G \neq 0$; $\chi = 0$. Besides the damping term κ, κ being not negligible compared to Δ, also contributes to asymmetries.

Now we fix parametric gain $G = 1.3\kappa$, and choose $\Delta = \sqrt{\omega_m^2 + 4G^2}$, the input laser power must satisfy $\wp \leq 55$ mW. The spectrum $S_Q(\omega)$ as a function of the normalized frequency ω/ω_m for increasing the input laser power is shown in Fig. 8. As we increase the laser power from 0.6 mW to 10.7 mW, the spectrum exhibits a doublet and the peak separation is proportional to the laser power, because the coupling between the movable mirror and

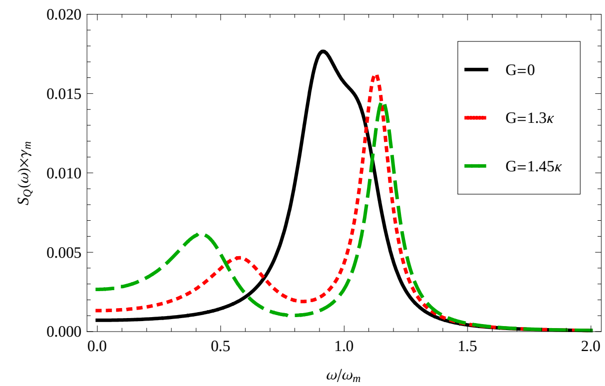

FIG. 4: (Color online) The scaled spectrum $S_Q(\omega)\times\gamma_m$ versus the normalized frequency ω/ω_m for different parametric gain. G= 0 (solid curve), 1.3κ (dotted curve), 1.45κ (dashed curve). Parameters: the cavity detuning $\Delta = \omega_m$, the laser power $\wp = 6.9$ mW.

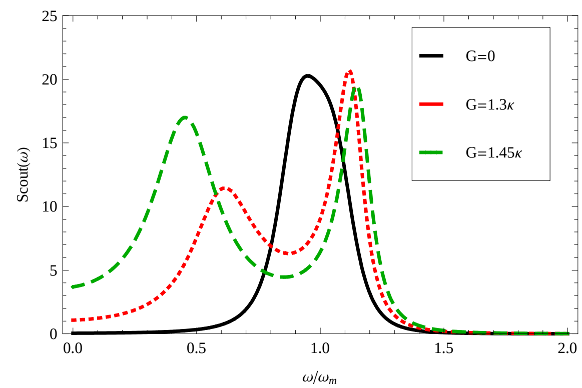

FIG. 5: (Color online) The spectrum $S_{cout}(\omega)$ versus the normalized frequency ω/ω_m for different parametric gain. G= 0 (solid curve), 1.3κ (dotted curve), 1.45κ (dashed curve). Parameters: the cavity detuning $\Delta = \omega_m$, the laser power $\wp = 6.9$ mW.

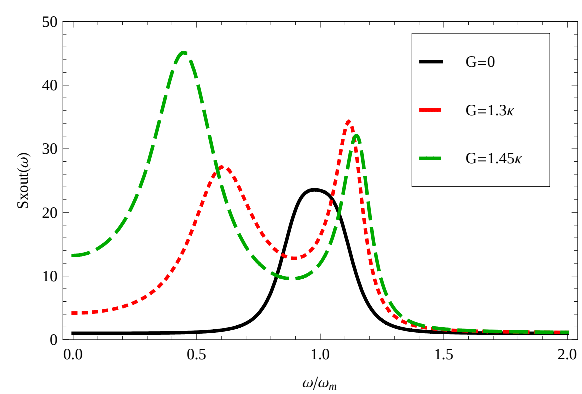

FIG. 6: (Color online) The spectrum $S_{xout}(\omega)$ versus the normalized frequency ω/ω_m for different parametric gain. G= 0 (solid curve), 1.3κ (dotted curve), 1.45κ (dashed curve). Parameters: the cavity detuning $\Delta = \omega_m$, the laser power $\wp = 6.9$ mW.

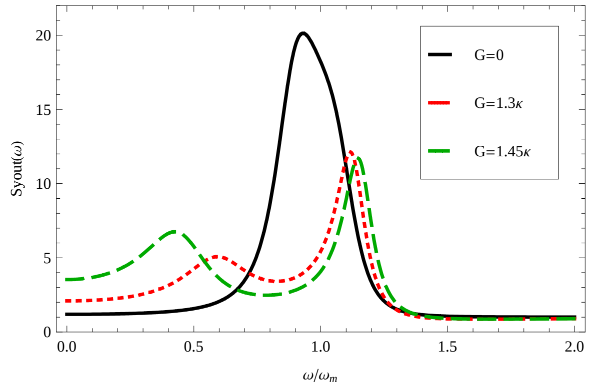

FIG. 7: (Color online) The spectrum $S_{yout}(\omega)$ versus the normalized frequency ω/ω_m for different parametric gain. $G=$ 0 (solid curve), 1.3κ (dotted curve), 1.45κ (dashed curve). Parameters: the cavity detuning $\Delta = \omega_m$, the laser power $\wp = 6.9$ mW.

the cavity field for a given parametric gain G is increased with increasing the input laser power due to an increase in photon number.

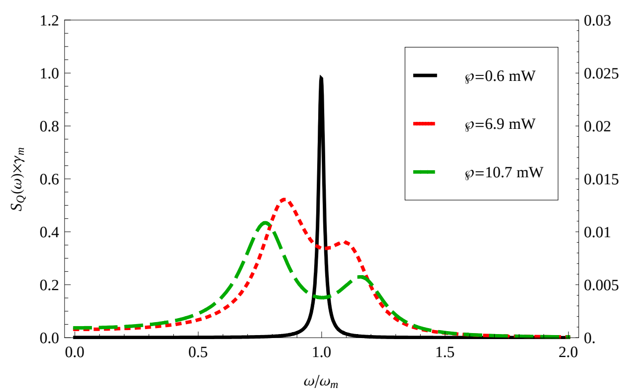

FIG. 8: (Color online) The scaled spectrum $S_Q(\omega) \times \gamma_m$ versus the normalized frequency ω/ω_m, each curve corresponds to a different input laser power. $\wp=$ 0.6 mW (solid curve, leftmost vertical scale), 6.9 mW (dotted curve, rightmost vertical scale), 10.7 mW (dashed curve, rightmost vertical scale). Parameters: the cavity detuning $\Delta = \sqrt{\omega_m^2 + 4G^2}$, parametric gain $G = 1.3\kappa$.

For comparison, we also consider the case of the cavity without OPA ($G = 0$), the spectrum $S_Q(\omega)$ as a function of the normalized frequency ω/ω_m for increasing the input laser power at $\Delta = \omega_m$ is plotted in Fig. 9. We can see if the laser power is increased from 0.6 mW to 10.7 mW, the spectrum also displays normal mode splitting. However the normal mode with OPA (Fig. 8) are more pronounced than that in the absence of OPA (Fig. 9).

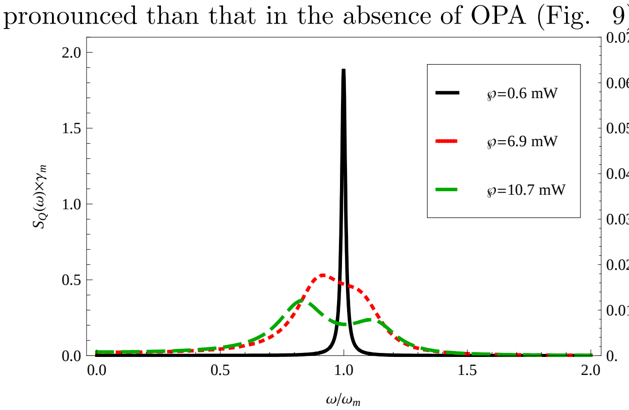

FIG. 9: (Color online) The scaled spectrum $S_Q(\omega) \times \gamma_m$ versus the normalized frequency ω/ω_m, each curve corresponds to a different input laser power. $\wp=$ 0.6 mW (solid curve, leftmost vertical scale), 6.9 mW (dotted curve, rightmost vertical scale), 10.7 mW (dashed curve, rightmost vertical scale). Parameters: the cavity detuning $\Delta = \omega_m$, parametric gain $G = 0$.

VII. CONCLUSIONS

In conclusion, we have shown how the normal mode splitting behavior of the movable mirror and the output field is affected by the OPA in the cavity. We work in the resolved sideband regime and operate under the stability conditions (12). We find that increasing parametric gain can make the spectra $S_Q(\omega)$, $S_{cout}(\omega)$, $S_{xout}(\omega)$, and $S_{yout}(\omega)$ evolve from a single peak to two peaks. Furthermore, for a given parametric gain, increasing input laser power can increase the amount of normal mode splitting of the movable mirror due to the stronger coupling between the movable mirror and the cavity field.

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[37] This is derived by setting κ and γ_m in $d(\omega)$ zero. A better estimate can be obtained by dropping $i\gamma_m\omega$ and κ^2, but keeping the term $-2i\kappa\omega$. This is because $\kappa/\omega_m(\simeq 0.22)$ is not much smaller than 1.
[38] See Ref. [29], p124, Eq. (7.18).