Substituting the value of 7 from (18) in (17), we get

$$\begin{aligned} (4^8+8^8)(x^8+16y^8)^8+(16^8+15^8+2^8)\{(4x^5y^3)^8+(8x^3y^5)^8\} \\ &= (4^8+8^8)\{(x^8-16y^8)^8+(2x^7y)^8+(16xy^7)^8\}+17^8\{(4x^5y^3)^8+(8x^3y^5)^8\}.\end{aligned}$$

Division by 4<sup>8</sup> now gives

$$(1^{8}+2^{8})(x^{8}+16y^{8})^{8}+(16^{8}+15^{8}+2^{8})\{(x^{5}y^{3})^{8}+(2x^{3}y^{5})^{8}\}$$
  
=  $(1^{8}+2^{8})\{(x^{8}-16y^{8})^{8}+(2x^{7}y)^{8}+(16xy^{7})^{8}\}+17^{8}\{(x^{5}y^{3})^{8}+(2x^{3}y^{5})^{8}\},$ 

and the proof of (IX) is complete.

## LEUDESDORF'S GENERALIZATION OF WOLSTENHOLME'S THEOREM

## S. CHOWLA\*.

**THEOREM**<sup>†</sup>. If (n, 6) = 1, then <sup>‡</sup>

$$\sum_{\substack{(m, n)=1\\m < n}} m^{-1} \equiv 0(n^2).$$

Proof§.

$$\sum_{\substack{(m, n) = 1 \\ m < n}} \frac{1}{m} \equiv 0(n^2) \rightleftharpoons \sum_{\substack{(m, n) = 1 \\ m < n}} \frac{1}{m(n-m)} \equiv 0(n),$$

$$\sum_{\substack{(m, n)=1\\m< n}} \frac{1}{m(n-m)} + \sum_{\substack{(m, n)=1\\m< n}} \frac{1}{m^2} \equiv 0(n).$$

It remains to prove that  $\sum' m^{-2} \equiv 0(n)$ , and this follows from the congruence

$$\Sigma' m^{-2} \equiv \Sigma' (am)^{-2} \pmod{n},$$

if a is such that  $a^2 \neq 1(p)$  for all prime factors p of n; it is obviously possible to choose a in this way since n is prime to 6.

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<sup>\*</sup> Received 27 March, 1934; read 26 April, 1934.

<sup>&</sup>lt;sup>†</sup> This is the simplest and most striking case of Leudesdorf's theorem. See Hardy and Wright, *Journal London Math. Soc.*, 9 (1934), 38-41. The theorem was rediscovered by S. S. Pillai.

 $<sup>\</sup>ddagger a \equiv b(c) \text{ means } a \equiv b \pmod{c}$ .

<sup>§</sup> We write  $A \rightleftharpoons B$  when proposition B follows from proposition A and conversely.

 $<sup>\</sup>parallel$  A dash attached to a  $\Sigma$  indicates that *m* takes any incongruent set of values prime to *n*.