RELATIVISTIC WAVE EQUATIONS FOR THE PROTON

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I recently gave* a scheme of equations for describing the behaviour of the elementary particles, different from that given by Dirac (1936) and developed by Fierz (1939) and Pauli (1939). The scheme is based on the postulate that the description of any elementary particle shall be provided by a first order wave equation of the form

$$\{\alpha^k p_k + \chi\} \psi = 0 \tag{1}$$

without any further auxiliary conditions, and that all physical and mathematical properties of the particle shall be derivable from (1). The operators p_k stand for $i\partial/\partial x^k$, the α^k are four matrices giving the spin properties of the particle and χ is a constant connected with the various values of the mass of the particle. $\hbar$ and c are put equal to unity in this paper. It has been shown in A that, with the assumption that the α's themselves in their six antisymmetric combinations $(\alpha^k\alpha^l - \alpha^l\alpha^k)$ shall define the way the wave function transforms under any transformation of the Lorentz group, the problem of finding all irreducible equations of the form (1) can be connected with that of finding all irreducible representations of the Lorentz group in five dimensions, the solution of which is already known. All irreducible representations of the α's in (1) satisfying the general postulate have been specified in B, and the general structure of the spin matrices analysed. In fact, the nucleus of every irreducible representation of the Lorentz group in five dimensions provides one irreducible set of α's in (1). Now every irreducible representation $R_5(n, n')$ of the five dimensional Lorentz group is characterized by two numbers $n \geqslant n' \geqslant 0$ both of which are integers, or both half odd integers. The former give the tensor, the latter the spinor representations. It was shown in B that this representation describes a particle of *maximum* spin n, irrespective of the value of n'. It was further shown that for a particle of maximum spin n there are $n+1$ different inequivalent equations if n is an integer, and $n+\frac{1}{2}$ if n is half an odd integer.

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* *Current Science*, 1945, **14**, 89–90, referred to in this paper as A. The full paper on the subject will appear in the number of the *Reviews of Modern Physics* which is being brought out in celebration of Prof. N. Bohr's sixtieth birthday. It will be referred to here as B.