# Copredication, Quantification and Individuation 

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I, Matthew Graham Haigh Gotham, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

## Abstract

This thesis addresses the various problems of copredication: the phenomenon whereby two predicates are applied to a single argument, but they appear to require that their argument denote different things. For instance, in the sentence 'The lunch was delicious but went on for hours', the predicate 'delicious' appears to require that 'the lunch' denote food, while 'went on' appears to require that it denote an event. Copredication raises philosophical issues regarding the place of a reference relation in semantic theory. It also raises issues concerning the ascription of sortal requirements to predicates in framing a theory of semantic anomaly. Finally, many quantified copredication sentences have truth conditions that cannot be accounted for given standard assumptions, because the predicates used impose distinct criteria of individuation on the objects to which they apply. For instance, the sentence 'Three books are heavy and informative' cannot be true in a situation involving only a trilogy (informationally three books, but physically only one), nor in a situation involving only three copies of the same book (physically three books, but informationally only one): the three books involved must be both physically and informationally distinct.

The central claims of this thesis are that nouns supporting copredication denote sets of complex objects, and that lexical entries incorporate information about their criteria of individuation, defined in terms of equivalence relations on subsets of the domain of discourse. Criteria of individuation are combined during semantic composition, then accessed and exploited by quantifiers in order to specify that the objects quantified over are distinct in defined ways. This novel approach is presented formally in Chapters 2 and 3, then compared with others in the literature in Chapter 4. In Chapter 5, the discussion is extended to the question of the implications of this approach for the form that a semantic theory should take.

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## Abbreviations

CCG Combinatory Categorial Grammar

LF Logical Form
MTT Modern Type Theory
NSC Noun Supporting Copredication
RMA Revised Mereological Approach (to copredication)
TCL Type Composition Logic
TTR Type Theory with Records

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## Chapter 1

## Introduction

### 1.1 Copredication

There are sentences that are coherent and possibly true, but in which it appears that incompatible properties are being attributed to a single object. For example, it seems to be the case that in (1) properties of information and of an event are being attributed to the lecture, in (2) properties of a physical object and of an agent are being attributed to the bank, and in (3) all manner of properties are being attributed to London.
(1) Nobody understood the lecture, which lasted an hour.
(2) The bank was vandalised after calling in Bob's debt.
(3) London is so unhappy, ugly and polluted that it should be destroyed and rebuilt 100 miles away. (Chomsky, 2000, p. 37)

This phenomenon, illustrated by (1)-(3), is known as 'copredication' (Pustejovsky, 1995, p. 236). It is the subject matter of this thesis.

There are several different ways in which copredication poses challenges to linguistic theory, and also theorizing about linguistic theory. Copredication raises philosophical questions about the place of a reference relation in semantics. What, if anything, should the referents of 'lecture', 'bank' and 'London' be taken to be in sentences like (1)-(3)
respectively? It presents difficulties concerning the ascription of selectional requirements to predicates in framing a theory of semantic anomaly. How can apparently sortally conflicting predicates, such as 'understood' and 'lasted' for example, be applied to a single syntactic argument? Finally, many quantified copredication sentences have truth conditions that cannot be accounted for given standard assumptions, because the predicates used impose distinct criteria of individuation on the objects to which they apply.

What these challenges have in common is that they demonstrate that there are natural assumptions based on which we would expect copredication sentences to be defective in ways that they are not, or to have properties that they do not have.

The range of reactions in the literature to copredication data includes: the suggestion that the data should be taken as confirmation of the need for a type theory based on many-sorted logic (Luo, 2010), the idea that predicate meanings are very flexible and can adapt to resolve incongruity (Brandtner, 2011), the claim that the understanding of predication itself should be significantly complicated so as to take account of how objects are conceptualised (Asher, 2011), and the contention that the whole field of semantics is in need of a drastic overhaul (Pietroski, 2005). Consequently, existing analyses of copredication are divided over the level in linguistic theory at which an explanation for the phenomenon should be provided: that of syntax, lexical semantics, compositional semantics, pragmatics or some combination of the above.

The challenges posed by copredication will be introduced in turn in the rest of this introduction: philosophical challenges in Section 1.2.1, compositional challenges in Section 1.2.2 and issues to do with individuation and counting in Section 1.2.3. The formal approach to copredication that will be developed in the rest of this thesis will then be outlined in Section 1.3.

### 1.2 Issues raised by copredication

### 1.2.1 Philosophical

The phenomenon of copredication has been cited as evidence in a philosophical debate about exactly what successful semantic theories are supposed to be theories of. Chomsky (2000), Collins (2009), and Pietroski (2005), for example, have argued that copredication makes it difficult to maintain an 'externalist' (Collins, 2009, p. 55, for example) view of semantic theory; that is, one according to which a proper explanation of semantic competence must include relations between either words or their mental encodings and things in the world. Reference as commonly understood would be such a relation. ${ }^{1}$ The difficulty comes from the fact that, according to such a view, semantic theory involves ontological commitments, of the following kind:

Semantic theory [...] can tell us what the costs would be of denying the existence of certain kinds of entities [...] If a straightforward semantic theory for arithmetic is true, then a sentence such as 'There is a prime number between two and five' entails the existence of numbers. As a result, a nominalist who rejects the existence of numbers is committed either to rejecting the simple semantics, or to rejecting the truth of 'There is a prime number between two and five'. (Kennedy and Stanley, 2009, p. 584)

By Kennedy and Stanley's logic, if a straightforward semantic theory is true then (1) entails the existence of things that can both be understood and have a duration, i.e. be both information and an event. Similarly, if a straightforward semantic theory is true then (2) entails the existence of things that can both be vandalised and raise interests rates, i.e. both a building and an agent. Finally, if a straightforward semantic theory is true, then (3) entails the existence of something that can be unhappy, ugly, polluted and capable of being destroyed and rebuilt, i.e. all manner of things (people, buildings and the air?). In short, the argument goes: if we take this externalist view, then (1)-(3) would appear to commit speakers to belief in objects with contradictory properties. If nothing can be both information and an event, then being incomprehensible and lasting

[^0]an hour are properties both of which no single object can have. This would mean that a straightforward semantic theory would predict (1) to be always false, and mutatis mutandis for (2) and (3). However, for each of these sentences there are situations in which competent speakers of English will judge them to be true. Regarding (3), Chomsky (2000, p. 37) contends that 'there neither are nor are believed to be things-in-the-world with the properties of the intricate modes of reference that a city name encapsulates', and concludes that a 'mode of reference' is therefore not the right way to think about meaning. If Chomsky is right, then these examples demonstrate that Kennedy and Stanley's claim should be abandoned, and instead some form or other of 'internalism' with respect to semantics should be adopted; that is, a view according to which a theory of semantic competence need make no mention of anything external to language and the mind of the speaker/hearer. ${ }^{2}$

In order to respond to this argument, one can either (i) deny that the properties involved really are incompatible, or (ii) concede that perhaps the properties are incompatible, but contend that the sentence in question has a structure such that those properties are not really attributed to the same object - either a syntactic structure at some level of representation, or logical structure given a proper understanding of its interpretation. Pietroski (2005, p. 277) describes (ii) as the programme of associating problematic sentences like (1)-(3) with an 'ontologically respectable paraphrase'-i.e. one that does not commit the speaker to the existence of objects with incompatible properties. Taking this idea seriously, a paraphrase of what speakers are assenting to when they judge (1) true might be:

There was some information that nobody understood, and there was an event that lasted an hour. We call both of these 'a lecture' and they are linked inasmuch as the information in question was communicated during the event in question.

As Pietroski points out, the project of of associating every copredication sentence with an ontologically-respectable paraphrase is by no means straightforward, still less

[^1]so to do so in a principled way. ${ }^{3}$ Copredication sentences are in many ways just like ordinary sentences for which no paraphrase is sought, and it is not obvious that the same paraphrasing strategy will work for all of them, given the diverse form in which they appear.

## A look forward

I will defer a proper discussion of these philosophical issues until Section 5.4. What should be stressed at the outset, however, is that it is not the case that copredication raises difficulties only for defenders of externalism about semantics. Any theory of copredication will have to explain precisely what the interpretations of sentences like (1)-(3) are, and how speakers effortlessly and reliably arrive at those interpretations. Furthermore, whatever we make of the argumentation about the philosophical implications of copredication, we need to have some theory to address the compositional issues to do with anomaly introduced in Section 1.2.2 below, and to address the quantificational issues to do with counting and individuation discussed in Section 1.2.3 below. In the discussion of issues of counting and individuation in particular, I introduce data that has not been appreciated in the literature.

The theory of copredication that I will develop in this thesis happens to be compatible with a particular method for making the commitments attributed to speakers 'ontologically respectable': the claim that nouns supporting copredication (henceforth NSCs), such as 'lecture' and 'bank', denote sets of complex objects made up of parts. ${ }^{4}$ For example, a lecture is an information+event composite object, and so a sentence like (1) can be true because the informational part of it was incomprehensible, while the event part of it lasted an hour. The motivation for adopting this theory is not simply that the theory would allow a defender of externalism in semantics to sidestep the philosophical challenge of copredication; rather, it is that it gets the facts right regarding the issues of individuation and counting in a way that no account proposed

[^2]so far does.

### 1.2.2 Compositional

Chomsky (1965) noted that, in between clear cases of ungrammaticality like (4) and 'standard examples of purely semantic (or "pragmatic") incongruity' (ibid., p. 76) like (5) there is a class of strings that are deviant for reasons that seem to fall somewhere in between the two.
(4) * John became Bill to leave (ibid., p. 149)
(5) I knew you would come, but I was wrong (ibid., p. 77)

An example of such an 'in between' sentence is given in (6).
(6) \# The meeting was delicious.

I will describe sentences like (6) as 'anomalous' and from now on will annotate them with a hash \#. In so doing I do not mean to claim (yet) that the source or kind of their deviancy is different to that of ungrammatical stings (4), which I will annotate with an asterisk.

What accounts for the anomalousness of (6)? The natural response is to say that it involves the ascription of a property to an object that is not of the right kind to have it: meetings, being events, are not the kind of thing that can be tasted and hence have the property of deliciousness.

In the system described by Chomsky (ibid.), the unacceptability of (6) would be due to a syntactic violation, albeit one that is different in kind from the violation that explains the unacceptability of (4) - the former would involve failure to observe a 'selectional rule', the latter a 'subcategorization rule'. However, the subsequent trend has very much been to follow McCawley (1968) and Grimshaw (1979), who proposed taking selectional rules in this sense out of the syntax and assuming that if there is a principled explanation for the anomaly of (6), then it comes from somewhere else. The obvious place to look, then, is the semantic component of the grammar.

If the requirement that the argument of 'delicious' have some property that 'the meeting' lacks is a semantic requirement, then the explanation for the anomaly of (6) is that it is unsemantical. On this view the ideal semantic theory should be able to give as systematic an account of when and why a sentence is unsemantical as the ideal syntactic theory would of when and why a sentence is ungrammatical.

Now, nothing in the familiar use of the simply-typed lambda calculus for semantic composition accounts for the unacceptability of (6), as (7) shows.


However, the compositional framework can be refined in such a way as to formalise the idea of certain objects being 'of the right kind' (or not) to have certain properties. An approach that has been widely adopted in implementing this kind of refinement, especially by linguists with an interest in computational modelling, such as Asher (2011), Chatzikyriakidis and Luo (2012), Cooper (2007), Luo (2012b), and Pustejovsky (1995), is that of elaborating the system of types. For example, ${ }^{5}$ in a typed lambda calculus with subtyping, there can be subtypes of $e$, which means that predicates can place more restrictive type requirements on their potential arguments than is possible under a simply-typed system. For instance, on the assumptions that (i) the type of physical objects (type $p$, say) and that of events (type $v$ ) are disjoint subtypes of the type of entities, (ii) the predicate 'delicious' semantically subcategorises for physical objects and (iii) meetings are events, (6) would fail to be interpreted, as shown in (8), since ' $x x$ (meeting $(x)$ )' is no longer of the right type to be an argument of the predicate ' $\lambda y$.delicious $(y)$ '. This formalises the notion that meetings are not objects 'of the right kind' to bear the property of deliciousness. We thus would have an analysis from

[^3]within the semantic theory for the anomalousness of certain sentences like (6): on this approach, they are anomalous because they are actually uninterpretable.


This view of the type system underlying composition also correctly predicts that (9) and (11) are interpretable, assuming the type assignments indicated in (10) and (12) respectively.
(9) The cake was delicious.

(11) The meeting went on for hours.


But this raises the question of what we should make of a sentence like (13).
(13) The lunch was delicious but went on for hours.

This sentence has the predicates 'delicious' and 'went on for hours' being applied to a single argument. (13) is not anomalous, which, given the type assignments adopted above for (8), (10) and (12), would lead us to expect that the types $p$ and $v$ are compatible after all-in which case we cannot use their supposed incompatibility as an explanation for the anomaly of (6). This kind of observation underlies the definition
of copredication offered by Asher and Pustejovsky (2006, p. 2): 'where apparently incompatible types of predicates are applied to a single type of object' (my emphasis). The predicates involved in a copredication sentence are apparently incompatible.

Note that the explanation for the acceptability of (13) cannot have anything essential to do with coordination, since (14) is just as acceptable.
(14) The delicious lunch went on for hours.

Of course, one can understand the copredication data as indicating that the attempt to explain the anomaly of sentences like (6) by means of type restrictions imposed by predicates is on the wrong track. But we would surely like to have some explanation for the anomaly of sentences like (6), and (15).
(15) \# A table talks.

Luo (2010, p. 45) say of this example:
the term [interpreting (15) within in a conventional theory] is well-typed (and false), while the term [interpreting (15) within Luo's theory] is simply not well-typed, i.e., meaningless. We contend that, in this respect, the type-theoretical semantics [Luo's theory] captures the meanings in a better way: the sentence $[(15)]$ is usually regarded as meaningless (unless in some fictional world), as in the type-theoretical semantics.

One might disagree (as I do) with the claim that (15) is actually meaningless, but it is certainly defective (like (6)) in a way that goes beyond being merely false, even necessarily false, and a theory that accounts for this-without over-predicting anomaly in the case of copredication-is desirable.

Defenders of the kind of type system under discussion have proposed various ways of addressing the problem posed by copredication for that kind of system. Some of those ways of addressing the problem will be discussed in Sections 4.1, 4.2 and 4.3. Moreover, the type-theoretical account of anomaly is, of course, by no means the only one available. But what the discussion in this section is designed to show is that there is something special about NSCs: they allow apparently incompatible predicates to be
applied to a single grammatical argument. In the type-theoretical accounts referred to above, this specialness tends to be implemented by the introduction into the system of a dedicated type constructor specifically for NSCs. ${ }^{6}$ The question then naturally arises: does this innovation explain the other unusual properties of copredication sentences? Can it be used to gain any insight into the philosophical issues raised by copredication? The issues of counting and individuation outlined in Section 1.2.3 below will also have to be addressed.

The account proposed in this thesis runs in the other direction. It begins by setting up (in Section 2.3) an architecture that resolves the counting and individuation issues of copredication described below, and then (in Section 5.1) applies that architecture to issues of anomaly. The theory described in Section 5.1 predicts that sentences like (15) and (6) are both false and anomalous (not meaningless), while predicting that (13) and (14) and other copredication sentences are not anomalous. According to this theory, nouns supporting copredication are special in a way that predicts their unusual ability to appear in apparently conflicting predicational environments. However, this specialness comes from what it is that the noun denotes, rather than its being an expression of a special complex type or possessing particular internal grammatical features.

### 1.2.3 Individuation and counting

Because the predicates applied in a copredication sentence are apparently incompatible, they can impose distinct criteria of individuation, and hence counting, on the objects to which they apply. This can be illustrated by comparing the truth conditions of (16)-(19), which contain different combinations of predicates.
(16) Fred picked up three books.
(17) Fred mastered three books.
(18) Fred picked up and mastered three books.

[^4](19) Fred mastered three heavy books.

In each case we have 'three books'; however, what counts as three books differs. For example, (16) can be true if Fred picked up three copies of the same book: in that case, individuated physically there are three books. However, it would not be true if Fred (merely) picked up a single physical volume that was a trilogy: in that case, individuated physically there is only one book (although, individuated informationally, there are three books). Conversely, (17) can be true if Fred mastered the contents of a trilogy: in that case, individuated informationally there are three books. However, it would not be true if Fred mastered the contents of three copies of the same book (even if three times over): in that case, individuated informationally there is only one book (although, individuated physically, there are three books).

These truth-value judgements must in some sense follow from the fact that the verb 'pick up' requires or expects its grammatical object to denote something physical, while the verb 'master' requires or expects its grammatical object to denote something informational. ${ }^{7}$ If it were not for copredication then it would make sense to say that 'book' is simply ambiguous and that in (16) a different sense of the word (or a different word) is used than in (17). That is to say, it would make sense to say that in (16) 'book' denotes a set of physical books, while in (17) it denotes a set of informational books. However, this approach cannot be maintained for (18) and (19), because in those sentences neither of the aforementioned senses is adequate to account for the truth conditions of the sentence. (18) is true neither if Fred picked up three copies of the same book and mastered it, nor if Fred picked up a trilogy and mastered the contents. For (18) to be true there must be three books individuated both physically and informationally. Likewise, for (19) to be true, the three books that Fred picked up must be both physically and informationally distinct from one another.

The account of copredication to be presented in this thesis begins by asking what kind of theory is needed in order to predict the truth conditions and entailment pat-

[^5]terns displayed by numerically quantified sentences such as (16)-(19), in particular the copredication sentences (18) and (19).

## A false step

Many of the examples of copredication given so far have involved coordination. For example, in (18) the predicates 'picked up' and 'mastered' are joined by the conjunction 'and'. This might invite the supposition that copredication in this case is to be explained by coordination reduction (Haspelmath, 2007, pp. 38-39). The argument would be that the sentence has an underlying structure like that shown in $(18$ '), and that 'books' is resolved differently in each coordinand, i.e. that 'books,' denotes physical objects and 'books ${ }_{2}$ ' denotes informational objects.
$\left(18^{\prime}\right)$ Fred picked up three books ${ }_{1}$ and Fred mastered three books $_{2}$.
$\left(18^{\prime}\right)$ on its own is not sufficient as an analysis of (18), as there is nothing in (18 )
 writing printed on it 'instantiates' the information communicated by that writing). And even if the analysis underling $\left(18^{\prime}\right)$ can be extended so as to account for this, it would not suffice as an explanation of copredication, since copredication is not limited to a particular syntactic structure like coordination, as examples like (19) show. From this point on I am going to assume that the right explanation for copredication in the cases in which conjunction reduction cannot plausibly be postulated extends to those in which it might.

### 1.3 Outline of the rest of the thesis

In Chapter 2 a system will be described that derives the correct truth conditions for numerically quantified copredication sentences. A starting assumption is that NSCs denote sets of complex objects made up of parts corresponding to the objects that
those nouns are conventionally thought to denote. A further assumption is that predicates encode, as part of their meanings, a specification of how their arguments are to be individuated. For example, 'heavy' includes a specification that its argument be individuated physically. Numerical quantifiers access these specifications and combine them in such a way that the truth conditions for sentences like (16)-(19) state that the books in question are distinct from each other in the required ways. For instance, they specify in (16) that there are three books that are distinct from each other in terms of their physical parts, and in (19) they specify that there are three books that are distinct from each other in terms of their physical and informational parts.

This is primarily a lexical-semantic theory of copredication. The desired truth conditions are obtained by the interaction of the lexical entries for NSCs and those of numerical determiners according to conventional compositional rules, namely function application and, depending on the implementation (Section 2.4), some combination of lambda abstraction, type raising and/or function composition. There is no positing of unpronounced syntactic structure or accommodation processes particular to copredication. The choice of syntactic theory is immaterial, provided that its semantics can be specified in the lambda calculus. While it is part of this account that nouns supporting copredication are special, this specialness resides entirely in what the noun denotes, not its possession of particular grammatical features.

Making this theory work does require increasing the complexity, not only of the lexical entries for NSCs and determiners, but of those of nearly all lexical entries. It also crucially relies on product types being included in the calculus of composition. What is added is a second dimension of lexical meaning, called a construction, that in effect acts as a store of criteria of individuation. Because of the focus on issues of counting and individuation, the presentation in Chapter 2 exclusively addresses sentences with bare numerical determiners; in Chapter 3 this treatment is extended to other sentence types.

In Chapter 4 the theory described in Chapters 2 and 3 is compared with others
that have been proposed, both on their own terms and in terms of how they deal with the counting and individuation data described above and more fully in Chapter 2. I conclude that the theory proposed in this thesis has better empirical coverage and also that it compares favourably with existing accounts in conceptual terms.

In Chapter 5 various formal and conceptual issues raised by the treatment in Chapter 2 are addressed. It is proposed in Section 5.1 that construction is independently motivated, in that it can also be used as the basis for a predictive theory of anomaly that is not susceptible to the difficulties in previous accounts mentioned in Section 1.2.2. Those difficulties are avoided by a combination of two factors: first, the mereological view of NSCs hinted at above (and elaborated at the beginning of Chapter 2), and second, the fact that construction at no point prevents composition from happening (unlike in the type-theoretical approach taken by Luo, for example). Rather, constructions can serve as a post-compositional check for anomaly mediated by an association between entities and equivalence relations on subsets of the domain of discourse, making it possible for a sentence to be both false (or even true) and anomalous. I also examine the interaction of anomaly with quantifier domain restriction, and discuss how that kind of domain restriction might be implemented in this system. The discussion in Section 5.2 concerns situations in which copredication is not acceptable. Section 5.3 addresses some criticisms that have been made of the mereological approach to NSCs taken in this thesis. Finally, Section 5.4 deals with the question of the implications of this approach for the form that a semantic theory should take, confronting the critique of externalism in semantics based on copredication.

## Chapter 2

## A compositional theory of criteria of individuation for copredication

### 2.1 The scope of this chapter

In this chapter I will describe a system the predicts the correct truth conditions for numerically quantified copredication sentences. As discussed in Section 1.2.3, the truth conditions of sentences like (1)-(4) (repeated from chapter 1) are problematic for semantic theories because the different predicates impose different criteria of individuation on their arguments. In particular, the criteria of individuation for 'book' in the copredication sentences (3) and (4) do not depend uniquely on the criteria of individuation associated with either verb, but rather emerge from both.
(1) Fred picked up three books.
(2) Fred mastered three books.
(3) Fred picked up and mastered three books.
(4) Fred mastered three heavy books.

The account to be presented assumes that there is no clash of properties in copredication sentences; that is to say, there is no inherent syntactic or semantic incompatibility involved in the predications made in sentences like (3) or (4) requiring repair or
adjustment at the level of categories, types or meanings. To this extent, I am in agreement with those (see Section 1.2.1) who view copredication as involving no resources in addition to those employed in ordinary predication. But unlike them, I do not draw the conclusion that copredication necessitates a move away from externalist views of what semantic theories are theories of. Rather, I take a position that is consistent with externalism about semantics: a noun supporting copredication (NSC), such as 'book', has complex objects in its extension. So for example, the reason that a sentence like (4) (for example) can be acceptable and true is that there can be objects in the extension of 'book' that are be both heavy and mastered by someone.

Saying that NSCs have complex objects in their extensions still leaves many questions to be answered. What is the structure of those objects? And what are their properties? Here I will lay out the answers to those questions that I am assuming, which will enable us to deal with sentences like those shown in (1)-(4). In Section 3.6.2 I will revisit these assumptions and look at some ways in which they might need to be revised in future work.

I will assume that an NSC has in its extension a set of objects, each member of which is made up of two parts. For example, 'book' denotes the set of composite objects $p+i$, where $p$ is a physical book and $i$ is an informational book instantiated by $p .{ }^{1}$ I also assume that any property that holds of $p$ holds of $p+i$, and likewise any property that holds of $i$ also holds of $p+i$. So for example, if $v_{1}$ is a physical volume instantiating War and Peace (conceived of as a purely informational (or abstract) object), and $v_{1}$ is heavy, then $v_{1}+W a r$ and Peace is heavy. Likewise, $v_{1}+W a r$ and Peace is by Tolstoy, in virtue of War and Peace being by Tolstoy.

I will have to defer philosophical discussion of this mereological approach to Chapter 5, where the argument against externalism in semantics, based on copredication, will

[^6]be addressed directly. In this chapter I will show that the mereological approach that I describe does empirical work in deriving the correct truth conditions for numerically quantified sentences like (1)-(4). Viewing the extension of NSCs as sets of complex objects allows us to compare those objects across different dimensions determined by their parts.

The chapter is structured as follows. First I will outline the theory being proposed in Section 2.2 in general terms, and then in Section 2.3 in more detail. In Section 2.4 it will be implemented in different syntactic frameworks.

### 2.2 A revised mereological approach

The idea that NSCs might denote sets of complex objects is not a new one. In an analysis of (13) from chapter 1, Cooper (2007, p. 4) suggests that
the lunch is delicious in virtue of the food which is part of the lunch being delicious. It is common in natural language for us to make predications of objects in terms of predications that hold of some of their parts, though not all of them.

However, there is an obvious problem with this kind of approach, as Asher (2011, pp. 146-7) points out. According to one of the most widely-accepted axioms of mereology, two objects are distinct if any of their parts are distinct (Varzi, 2012). On this basis, there are three complex objects listed in (5) (in fact, there are four), since each of them is distinct from the others in at least one of its parts.
(5) $\left\{v_{1}+N f U, v_{2}+N f U, v_{1}+T G, v_{2}+T G\right\}$
(5) represents a small Dostoyevsky library consisting of two volumes, each of which contains both the novellas Notes from Underground and The Gambler (illustrated in figure 2.1). ${ }^{2}$ Suppose that Fred picked up and mastered the books in that library (and

[^7]volume 1

| Notes from Underground |
| :---: |
| The Gambler |

Notes from Underground The Gambler
volume 2 Figure 2.1: A small Dostoyevsky library
no others). None of (1)-(3) would be true: he did not pick up three books (only two), he did not master three books (only two), and a fortiori he did not pick up and master three books. However, if the simple mererological picture painted above is right, then all three sentences should be true, since (5) indicates that, conceived of as complex objects made up of physical and informational parts, there are four books in this situation that Fred both picked up and mastered. ${ }^{3}$ Partly for this reason, Cooper (2011) has subsequently adopted a different approach to copredication, which I will discuss in Section 4.2.

However, this mereological approach to NSCs can be improved, so that truth conditions are derived for (1)-(4) that express what is shown in (TC 1)-(TC 4) respectively. In (TC 1)-(TC 4), 'book' is to be taken to mean a complex object made up of one part that is a physical book and one part that is an informational book instantiated by it.
(TC 1) There is a plurality $s$ of three books such that

- Every member of $s$ is physically distinct from every other member.
- Fred picked up every member of $s$.
(TC 2) There is a plurality $s$ of three books such that
- Every member of $s$ is informationally distinct from every other member.
- Fred mastered every member of $s$.
(TC 3) There is a plurality $s$ of three books such that

[^8]- Every member of $s$ is physically and informationally distinct from every other member.
- Fred picked up and mastered every member of $s$.
(TC 4) There is a plurality $s$ of three books such that
- Every member of $s$ is physically and informationally distinct from every other member.
- Each member of $s$ is heavy.
- Fred mastered every member of $s$.

As can be seen from (TC 1)-(TC 4), an aim of this approach is to avoid the pitfalls of a naïve mereological account by introducing the requirement that the individual books being counted must all be distinct from each other in defined ways. The approach that I will adopt in confronting this issue is to formalize and refine this notion of distinctness, and use it for the purposes of counting when computing the truth conditions of sentences like (1)-(4). The basic idea is that, while (5) may well accurately represent the set of books in a given situation, this set is never used in determining truth conditions without being somehow modified. I call this approach to copredication a 'revised mereological approach', or RMA.

The question then becomes, how are the right distinctness requirements, as outlined in (TC 1)-(TC 4), to be introduced compositionally in each case? Clearly, in order to derive (TC 1)-(TC 4) respectively as the truth conditions for (1)-(4) in a compositional manner, the notions of physical and informational distinctness must play a role in the theory. The approach to be taken in order to achieve this relies on the following 2 elements:

1. Lexical entries are more complex than is conventionally thought. In addition to determining extension ${ }^{4}$, there is another 'part' to them that plays a role in determining the distinctness requirement.

[^9]2. Numerical quantifiers ${ }^{5}$ are sensitive to this second 'part' of their arguments and use it to restrict the extension of their first argument in the way indicated in (TC 1)-(TC 4).

These 2 points will be explained in Section 2.3 below.

### 2.3 Composing criteria of individuation

### 2.3.1 Basic concepts

In this section I will introduce the formal elements of the approach to be taken and then use them to derive the truth conditions of some copredication sentences and some non-copredication sentences.

In what follows I will use $a+b$ to indicate the single complex object made up of parts $a$ and $b$, and $a \oplus b$ to indicate the plurality made up of single objects $a$ and $b$. So e.g. $a+b \oplus c+d$ indicates the two-membered plurality made up of the complex objects $a+b$ and $c+d$.

Above, I claimed that (TC 4) describes the truth conditions of (4). (6) shows the metalanguage translation of the these truth conditions within the system to be presented. ${ }^{6,7}$

$$
\begin{align*}
& \exists x\left(|x| \geq 3 \wedge \text { * } \operatorname{book}^{\prime}(x) \wedge \text { * }^{\prime} \operatorname{heavy}^{\prime}(x) \wedge{ }_{2}^{*} \text { master }^{\prime}\left(f^{\prime}, x\right)\right.  \tag{6}\\
& \wedge \neg \exists y \exists z\left(y \neq z \wedge \mathrm{i}-\operatorname{part}^{\prime}(y, x) \wedge \mathrm{i}-\operatorname{part}^{\prime}(z, x) \wedge \mathrm{i}-\operatorname{atom}^{\prime}(y) \wedge \mathrm{i}-\operatorname{atom}^{\prime}(z)\right. \\
& \left.\left.\wedge\left(\operatorname{phys}^{-e q u i v}{ }^{\prime}(y, z) \vee \operatorname{info-equiv}^{\prime}(y, z)\right)\right)\right)
\end{align*}
$$

In English: there is a plurality of three heavy books each of which Fred mastered, and no two distinct singular objects in that plurality are physically or informationally equivalent to each other. I am assuming the ontology of plurals described by Link

[^10](1983), such that the domain of type $e$ contains both singular objects and proper pluralities. ${ }^{*} P$ is the (characteristic function of) the set of (possibly singular) pluralities formed from entities in the extension of $P$ :
\[

$$
\begin{aligned}
& \forall x\left(P(x) \rightarrow{ }^{*} P(x)\right) \\
& \forall x \forall y\left(\left({ }^{*} P(x) \wedge^{*} P(y)\right) \leftrightarrow{ }^{*} P(x \oplus y)\right)
\end{aligned}
$$
\]

Slightly non-standardly, I will also define a star operator that applies to any twoplace predicate, in which case all its argument positions are pluralised in the following way: ${ }^{8}$

$$
{ }_{2}^{*} R(x, y) \stackrel{\text { def }}{=} *\left(\lambda v_{e} \cdot R(x, v)\right)(y) \vee *\left(\lambda z_{e} \cdot R(z, y)\right)(x)
$$

In practice, I will omit the subscript ' 2 ' where there is no risk of confusion.
'i-part' $(x, y)$ ' is to be read as saying that $x$ is an individual part of $y$ :

$$
\text { i- } \operatorname{part}^{\prime}(x, y) \stackrel{\text { def }}{=} x \oplus y=y
$$

'i-part' is a predicate corresponding to the inclusion relation $\leq_{i}$ in the join-semilattice constituted by the (singular and properly plural) entities of type $e .{ }^{9}$ ' $\mathrm{i}-\mathrm{atom}^{\prime}(x)$ ' is to be read as saying that $x$ is an individual atom, i.e. an atom in that semilattice, i.e. a singular object:

$$
\mathrm{i}-\operatorname{atom}^{\prime}(x) \stackrel{\text { def }}{=} \forall y\left(\mathrm{i}-\operatorname{part}^{\prime}(y, x) \rightarrow x=y\right)
$$

N.B., by this I mean that it is an atom in terms of plurality, so the books qua complex objects made up of physical and informational parts, as well as those parts themselves, can be atoms in this algebra. This is illustrated in Figure 2.2. Here, we have i-atom ${ }^{\prime}(a)$ and i-atom ${ }^{\prime}(a+b)$, but $\neg \mathrm{i}-\operatorname{atom}^{\prime}(a \oplus b)$. Above (e.g. in (TC 1$)-($ TC 4$)$ ), I glossed ' $x$ is an atomic individual part of $y$ ' by saying that $x$ is a 'member' of $y$.

[^11]

Figure 2.2: Complex objects are atoms in terms of plurality

Below, I will often abbreviate the name of the relation of physical equivalence as 'PHYS'. This relation holds between (singular) objects $a$ and $b$ if and only if they both have a physical part and the physical part of $a$ is identical to the physical part of $b$. Similarly, INFO is a relation that holds between objects $a$ and $b$ if and only if they both have an informational part and the informational part of $a$ is identical to the informational part of $b$. Relations like PHYS and INFO I will call 'individuation relations' or 'ind-relations'. ${ }^{10}$

$$
\begin{aligned}
& \text { PHYS }=\lambda x \cdot \lambda y \cdot \text { phys-equiv }^{\prime}(x, y) \\
& \mathrm{INFO}=\lambda x \cdot \lambda y \cdot \text { info-equiv }^{\prime}(x, y)
\end{aligned}
$$

Although I have called this a relation of 'physical equivalence' it is not strictly speaking an equivalence relation, as objects that are not even partly physical (do not have at least one physical part) are physically equivalent to nothing at all, not even themselves; therefore PHYS is not reflexive. ${ }^{11}$

For instance, consider the small Dostoyevsky library described at the beginning of Section 2.2 and depicted in (5) and Figure 2.1. Here, phys-equiv ${ }^{\prime}\left(v_{1}+N f U, v_{1}+T G\right)-$

[^12]they are physically equivalent as they have the same physical part: $v_{1}$. However, $\neg$ phys-equiv ${ }^{\prime}\left(v_{1}+N f U, v_{2}+N f U\right)$.

In some of what follows I will express the fact that no two members (atomic individual parts) of a plurality $x$ bear relation $R$ to each other by saying that $x$ is 'not $R$ compressible', or in metalanguage formulae as ' $\neg(R) \operatorname{comp}(x)$ '—which I will sometimes refer to as a 'compressibility statement'. A definition of the notion of compressibility is given in Definition 1 below.

## Definition 1 (Compressibility).

A plurality $x$ is $R$-compressible if and only if there are two distinct atomic individual parts of $x$ that bear relation $R$ to each other.
$(R) \operatorname{comp}(x) \stackrel{\text { def }}{=} \exists y \exists z\left(y \neq z \wedge \mathrm{i}-\operatorname{part}^{\prime}(y, x) \wedge \mathrm{i}-\operatorname{part}^{\prime}(z, x) \wedge \mathrm{i}-\operatorname{atom}^{\prime}(y) \wedge \mathrm{i}-\operatorname{atom}^{\prime}(z) \wedge R(y, z)\right)$

By way of illustration, note that (7) is PHYS-compressible, because $v_{1}+N f U$ and $v_{1}+T G$ are both atomic parts of it, and $v_{1}+N f U$ is physically equivalent to $v_{1}+T G$. In contrast, (8) is not PHYS-compressible, because no two atomic parts of it are physically equivalent to each other.
(7) $v_{1}+N f U \oplus v_{1}+T G$
(8) $v_{1}+N f U \oplus v_{2}+T G$

Therefore, (6) can be abridged as shown in (9), where INFO is the relation of informational equivalence.
(9) $\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{master}^{\prime}\left(f^{\prime}, x\right) \wedge \neg(\right.$ PHYS $\left.\sqcup \operatorname{INFO}) \operatorname{comp}(x)\right)$
$\sqcup$ is the join operation that can apply in any boolean algebra, such as is formed for example by the set of inhabitants of type $e \rightarrow(e \rightarrow t)$ (Keenan and Faltz, 1985). ${ }^{12}$ This can be implemented in the 'generalized conjunction' approach put forward by Partee

[^13]and Rooth (1983), because in that approach $e \rightarrow(e \rightarrow t)$ is a 'conjoinable type' as defined in (10).
(10) $t$ is a conjoinable type.

For all $a$ and $b$, if $b$ is a conjoinable type then $a \rightarrow b$ is a conjoinable type.

The recursive definition of generalized disjunction is given in (11). The definitions give in (10) and (11) are adapted from equivalent ones given by Partee and Rooth (1983).
(11) If $X: t$ and $Y: t$, then $X \sqcup Y \stackrel{\text { def }}{=} X \vee Y$.

$$
\text { If } X: a \rightarrow b \text { and } Y: a \rightarrow b, \text { then } X \sqcup Y \stackrel{\text { def }}{=} \lambda v_{a}(X(v) \sqcup Y(v)) .
$$

So for example, PHYS $\sqcup$ INFO has the interpretation shown in (12)
(12) PHYS $\sqcup$ INFO $=\lambda x_{e}\left(\lambda y_{e}\left(\operatorname{phys}-e q u i v^{\prime}(x, y)\right)\right) \sqcup \lambda x_{e}\left(\lambda y_{e}\left(\operatorname{info-equiv}^{\prime}(z, v)\right)\right)$

$$
\begin{aligned}
& =\lambda v_{e}\left(\lambda y_{e}\left(\operatorname{phys}^{-e q u i v^{\prime}}(v, y)\right) \sqcup \lambda y_{e}\left(\operatorname{info-equiv}^{\prime}(v, y)\right)\right) \\
& =\lambda v_{e}\left(\lambda u_{e}\left(\operatorname{phys-equiv}^{\prime}(v, u) \sqcup \operatorname{info-equiv}^{\prime}(v, u)\right)\right) \\
& =\lambda v_{e}\left(\lambda u_{e}\left(\text { phys-equiv}^{\prime}(v, u) \vee \operatorname{info-equiv}^{\prime}(v, u)\right)\right)
\end{aligned}
$$

In order to build the interpretation shown in (9) I propose, firstly, that propertydenoting expressions also carry one of these individuation relations as part of their meaning, as shown in the provisional lexical entries (13)-(16).
(13) $\llbracket t a b l e \rrbracket=\lambda x_{e}\left\langle\operatorname{table}^{\prime}(x)\right.$, PHYS $\rangle$
(14) $\llbracket b o o k \rrbracket=\lambda x_{e}\left\langle\operatorname{book}^{\prime}(x)\right.$, PHYS $\left.\sqcap \mathrm{INFO}\right\rangle$
(15) $\llbracket b o o k s \rrbracket=\lambda y_{e}\left\langle{ }^{*} \operatorname{book}^{\prime}(y)\right.$, PHYS $\left.\sqcap \mathrm{INFO}\right\rangle$

$\square$ is the generalized conjunction operator corresponding to the generalized disjunction operator defined above. Its inclusion in the lexical entries for 'book(s)' allows
books, unlike (say) tables, to be individuated in more than one way; describing how this variable individuation is achieved compositionally is the aim of the present section. This is the extent to which the ability to support copredication is marked in the lexical entries for nouns. PHYS $\sqcap$ INFO is not a type specification in the sense described in Section 1.2.2; it simply means that the lexical entry for 'book' includes the relation that is the generalized conjunction of the relations of physical and informational equivalence. Importantly, no matching between function and argument with respect to these relations is required to take place during composition and, while it does make a (crucial) contribution to the theory of semantic anomaly to be presented in Section 5.1, this contribution is only indirect.

The expressions shown in (13)-(16) are of type $e \rightarrow(t \times \mathcal{R})$, where $\mathcal{R}$ is an abbreviation of $e \rightarrow(e \rightarrow t) .(t \times \mathcal{R})$ is a product type, and so inhabitants of this type will be ordered pairs $\langle a, b\rangle$, where $a$ is a truth value - standard sentence extensional meaning-and $b$ is an individuation relation. As per conventional usage, $\pi_{1}(\langle a, b\rangle)=a$ and $\pi_{2}(\langle a, b\rangle)=b$ : the 'first projection' of $\langle a, b\rangle$ and the 'second projection' of $\langle a, b\rangle$, respectively. In each case, there is a simple method for going back from the interpretation shown above to a meaning of the type conventionally assumed for the lexical item. For instance, $\lambda y \cdot \pi_{1}(P(y))$, where $P=\llbracket b o o k \rrbracket$, is $\lambda y \cdot \operatorname{book}^{\prime}(y)$, as shown in (17).

$$
\begin{align*}
\lambda y \cdot \pi_{1}(\llbracket b o o k \rrbracket(y)) & =\lambda y \cdot \pi_{1}\left(\lambda x_{e} \cdot\left\langle\operatorname{book}^{\prime}(x), \text { PHYS } \sqcap \operatorname{INFO}\right\rangle(y)\right)  \tag{17}\\
& =\lambda y \cdot \pi_{1}\left(\left\langle\operatorname{book}^{\prime}(y), \text { PHYS } \sqcap \mathrm{INFO}\right\rangle\right) \\
& =\lambda y \cdot \operatorname{book}^{\prime}(y)
\end{align*}
$$

The second projection function $\pi_{2}$ will be used to access the ind-relations associated with interpretations. For example, $\pi_{2}(\llbracket b o o k \rrbracket(y))$ is as shown in (18).

$$
\begin{align*}
\pi_{2}(\llbracket b o o k \rrbracket(y)) & =\pi_{2}\left(\lambda x_{e} \cdot\left\langle\operatorname{book}^{\prime}(x), \text { PHYS } \sqcap \mathrm{INFO}\right\rangle(y)\right)  \tag{18}\\
& =\pi_{2}\left(\left\langle\operatorname{book}^{\prime}(y), \text { PHYS } \sqcap \mathrm{INFO}\right\rangle\right) \\
& =\operatorname{PHYS} \sqcap \mathrm{INFO}
\end{align*}
$$

Secondly, I propose that determiners can exploit those ind-relations, as shown for example in (19), the provisional lexical entry for $\llbracket t h r e e \rrbracket .{ }^{13}$

$$
\begin{align*}
& \lambda P_{e \rightarrow(t \times \mathcal{R})} \cdot \lambda Q_{e \rightarrow(t \times \mathcal{R})}  \tag{19}\\
& \quad\left\langle\exists x\left(|x| \geq 3 \wedge \pi_{1}(P(x)) \wedge \pi_{1}(Q(x)) \wedge \neg\left(\pi_{2}(P(x)) \sqcup \pi_{2}(Q(x))\right) \operatorname{comp}(x)\right),\right. \\
& \left.\quad \pi_{2}(P(x)) \sqcap \pi_{2}(Q(x))\right\rangle
\end{align*}
$$

## Some examples

Let us see how the lexical entries given above conspire to generate interpretations for (20) and (21). We want (20) to require only that the books involved be informationally distinct, but we want (21) to require that the books involved be both informationally and physically distinct.
(20) Three books are informative.
(21) Three heavy books are informative.

In order to obtain the interpretation of 'three books' we apply (19) to (15). With $P=\llbracket b o o k s \rrbracket$, we have the following ...
(22) $\pi_{1}(P(x))=\pi_{1}(\llbracket b o o k s \rrbracket(x))$

$$
\begin{aligned}
& =\pi_{1}\left(\lambda y_{e}\left\langle * \operatorname{book}^{\prime}(y), \text { PHYS } \sqcap \mathrm{INFO}\right\rangle(x)\right) \\
& =\pi_{1}\left(\left\langle * \operatorname{book}^{\prime}(x), \text { PHYS } \sqcap \mathrm{INFO}\right\rangle\right) \\
& ={ }^{*} \operatorname{book}^{\prime}(x)
\end{aligned}
$$

$$
\begin{align*}
\pi_{2}(P(x)) & =\pi_{2}(\llbracket b o o k s \rrbracket(x))  \tag{23}\\
& =\pi_{2}\left(\lambda y_{e}\left\langle * \operatorname{book}^{\prime}(y), \text { PHYS } \sqcap \operatorname{INFO}\right\rangle(x)\right) \\
& =\pi_{2}\left(\left\langle * \operatorname{book}^{\prime}(x), \text { PHYS } \sqcap \mathrm{INFO}\right\rangle\right) \\
& =\text { PHYS } \sqcap \mathrm{INFO}
\end{align*}
$$

[^14]\[

$$
\begin{align*}
& \therefore \llbracket \text { three books } \rrbracket \lambda Q_{e \rightarrow(t \times \mathcal{R})}  \tag{24}\\
& \left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge \pi_{1}(Q(x)) \wedge \neg\left((\operatorname{PHYS} \sqcap \mathrm{INFO}) \sqcup \pi_{2}(Q(x))\right) \operatorname{comp}(x)\right),\right. \\
& \left.\quad(\mathrm{PHYS} \sqcap \mathrm{INFO}) \sqcap \pi_{2}(Q(x))\right\rangle
\end{align*}
$$
\]

Note that the compressibility statement of the whole sentence depends on both arguments to the determiner, and hence is not finalized at this stage.

In order to obtain the interpretation of (20) we apply (24) to (16). With $Q=$【be informative ${ }_{p l} \rrbracket$, we have the following:

$$
\begin{align*}
\pi_{1}(Q(x)) & =\pi_{1}\left(\llbracket \text { be informative }_{p l} \rrbracket(x)\right)  \tag{25}\\
& =\pi_{1}\left(\lambda y_{e}\left\langle{ }^{*} \operatorname{inform}^{\prime}(y), \text { INFO }\right\rangle(x)\right) \\
& =\pi_{1}\left(\left\langle{ }^{*} \operatorname{inform}^{\prime}(x), \text { INFO }\right\rangle\right) \\
& =*_{\operatorname{inform}^{\prime}(x)}
\end{align*}
$$

$$
\begin{align*}
\pi_{2}(Q(x)) & =\pi_{2}\left(\llbracket \text { be informative }_{p l} \rrbracket(x)\right)  \tag{26}\\
& =\pi_{2}\left(\lambda y_{e}\left\langle *^{*} \operatorname{inform}^{\prime}(y), \text { INFO }\right\rangle(x)\right) \\
& =\pi_{2}\left(\left\langle *^{*} \operatorname{inform}^{\prime}(x), \text { INFO }\right\rangle\right) \\
& =\text { INFO }
\end{align*}
$$

(27) $\therefore \llbracket$ three books are informative $\rrbracket=$ $\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{inform}^{\prime}(x) \wedge \neg((\operatorname{PHYS} \sqcap \operatorname{INFO}) \sqcup \operatorname{INFO}) \operatorname{comp}(x)\right)\right.$, (PHYS $\sqcap$ INFO) $\sqcap$ INFO $\rangle$

Because of the boolean equalities shown in (28)-(29), (27) can be simplified to (30).
(28) $(A \sqcap B) \sqcup B=B$
(29) $(A \sqcap B) \sqcap B=A \sqcap B$
(30) $\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{inform}^{\prime}(x) \wedge \neg(\right.\right.$ INFO $\left.) \operatorname{comp}(x)\right)$, PHYS $\sqcap$ INFO $\rangle$

What the first projection of (30) says is that there is a plurality of three informative books, and this plurality is not informationally compressible. These are the right truth
conditions for (20).
It should now be clear why the lexical entry for 'books' in (15) took the form that it did, with PHYS $\Pi$ INFO. By requiring the determiner to combine the ind-relations from its two arguments by taking their boolean join ( $\sqcup$ ), the 'PHYS' part of $\llbracket b o o k s \rrbracket$ is effectively cancelled when it is combined (only) with an informational predicate like $\llbracket b e$ informative $e_{p l} \rrbracket$ in (20).

In the case of copredication, things are different. To see this in the case of (21), we need to add the following entry to the toy lexicon:

$$
\begin{equation*}
\llbracket h e a v y_{p l} \rrbracket=\lambda P_{e \rightarrow(t \times \mathcal{R})} \cdot \lambda z_{e}\left\langle\left(\text { heavy }^{\prime}(z) \wedge \pi_{1}(P(z))\right), \pi_{2}(P(z)) \sqcup \mathrm{PHYS}\right\rangle \tag{31}
\end{equation*}
$$

Therefore we get:
(32) $\llbracket h e a v y b o o k s \rrbracket=\llbracket h e a v y_{p l} \rrbracket(\llbracket b o o k s \rrbracket)$

$$
\begin{aligned}
& =\lambda y_{e}\left\langle * \operatorname{heavy}^{\prime}(y) \wedge * \operatorname{book}^{\prime}(y),(\text { PHYS } \sqcap \mathrm{INFO}) \sqcup \mathrm{PHYS}\right\rangle \\
& =\lambda y_{e}\left\langle{ }^{*} \operatorname{heavy}^{\prime}(y) \wedge * \operatorname{book}^{\prime}(y), \text { PHYS }\right\rangle
\end{aligned}
$$

This time the 'INFO' part of $\llbracket b o o k s \rrbracket$ has been cancelled after being combined with the physical predicate $\llbracket h e a v y_{p l} \rrbracket$.

Applying (19) to (32), we obain the interpretation of 'three heavy books' shown in (33).

$$
\begin{align*}
& \lambda Q_{e \rightarrow(t \times \mathcal{R})}\left\langle\exists x \left(|x| \geq 3 \wedge \text { *heavy }^{\prime}(x) \wedge{ }^{*} \operatorname{book}^{\prime}(x) \wedge \pi_{1}(Q(x))\right.\right.  \tag{33}\\
&\left.\left.\wedge \neg\left(\text { PHYS } \sqcup \pi_{2}(Q(x))\right) \operatorname{comp}(x)\right), \text { PHYS } \sqcap \pi_{2}(Q(x))\right\rangle
\end{align*}
$$

Note how the compressibility statement at this stage is different from that of (24). Therefore, if we apply (33) to (16) then we obtain (34) as the interpretation of (21).

$$
\begin{align*}
& \left\langle\exists x\left(|x| \geq 3 \wedge{ }^{*} \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{inform}^{\prime}(x) \wedge \neg(\operatorname{PHYS} \sqcup \operatorname{INFO}) \operatorname{comp}(x)\right)\right.  \tag{34}\\
& \text { PHYS } \sqcap \text { INFO }\rangle
\end{align*}
$$

What the first projection of (34) says is that there is a plurality of three heavy, informative books, and this plurality is neither physically nor informationally compressible.

These are the right truth conditions for (21).
So from these examples we can see how lexical entries can combine to generate the appropriate compression statements for a non-copredication sentence (20) and a copredication sentence (21).

### 2.3.2 Keeping track of individuation relations

However, things are not quite so simple, because of course we also want to be able to assign ind-relations to each of the argument positions of predicates that have more than one. For instance, with the lexical entries already given and the constituent interpretation shown in (35), we can derive interpretations for (2) and (4) as shown in (36) and (37) respectively, but it is not obvious how to get the constituent interpretation shown in (35).

$$
\begin{align*}
& \llbracket \lambda_{1}\left[\text { Fred mastered } t_{1}\right] \rrbracket=\lambda x_{e}\left\langle{ }^{*} \operatorname{master}^{\prime}\left(f^{\prime}, x\right), \text { INFO }\right\rangle  \tag{35}\\
& \left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge{ }^{*} \operatorname{master}^{\prime}\left(f^{\prime}, x\right) \wedge \neg((\text { PHYS } \sqcap \mathrm{INFO}) \sqcup \mathrm{INFO}) \operatorname{comp}(x)\right),\right.  \tag{36}\\
& \quad(\text { PHYS } \sqcap \mathrm{INFO}) \sqcap \mathrm{INFO}\rangle \\
& =\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{master}^{\prime}\left(f^{\prime}, x\right) \wedge \neg(\mathrm{INFO}) \operatorname{comp}(x)\right), \text { PHYS } \sqcap \mathrm{INFO}\right\rangle \\
& \left\langle\exists x \left(|x| \geq 3 \wedge * \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{master}^{\prime}\left(f^{\prime}, x\right)\right.\right.  \tag{37}\\
& \quad \wedge \neg(\text { PHYS } \sqcup \mathrm{INFO}) \operatorname{comp}(x)), \text { PHYS } \sqcap \operatorname{INFO}\rangle
\end{align*}
$$

What the first projection of (36) says is that there is a plurality of three books that Fred mastered, which is not informationally compressible. The truth conditions are equivalent to (TC 2). And the first projection of (37) is identical to (9), and as such accurately represents the truth conditions of (4) by being equivalent to (TC 4).

All this is as desired, but how are we supposed to get (35)? There has to be some way of associating the first argument of $\llbracket$ mastered $\rrbracket$ with INFO and the second with ANI, the relation of animate equivalence, which holds between objects $a$ and $b$ if and only if they both have an animate part and the animate part of $a$ is identical to the animate
part of $b .{ }^{14}$. There also must be a way of recovering these individuation relations under appropriate circumstances. What we want is something like the schematic entry shown in (38).

$$
\llbracket \text { mastered } \rrbracket=\lambda x_{e} \cdot \lambda y_{e}\left\langle *^{\text {master }^{\prime}(y, x),}, \begin{array}{l}
x \leadsto \mathrm{INFO}  \tag{38}\\
y \leadsto \mathrm{ANI}
\end{array}\right\rangle
$$

The $\leadsto$ symbol here is simply meant to indicate that there is some connection between the variable shown and the individuation relation shown. Specifying just what that connection is is the purpose of the current subsection.

To do this we are going to need to make further use of the algebraic properties of the domain of type $e \rightarrow(e \rightarrow t)$. Specifically, in addition to the meet $\square$ and join $\sqcup$ operations in this algebra that we are already using, we need to make use of the partial order $\sqsubseteq$.

Like $\Pi$ and $\sqcup$, for our purposes $\sqsubseteq$ can be given a recursive definition in terms of functions, as shown in (39) below. It is, in effect, a form of generalized implication.
(39) If $X: t$ and $Y: t$, then $X \sqsubseteq Y \stackrel{\text { def }}{=} X \rightarrow Y$.

$$
\text { If } X: a \rightarrow b \text { and } Y: a \rightarrow b \text {, then } X \sqsubseteq Y \stackrel{\text { def }}{=} \forall v_{a}(X(v) \sqsubseteq Y(v)) \text {. }
$$

It follows that in the domain for the type that we are interested in, $e \rightarrow(e \rightarrow t)$, we have $A \sqsubseteq B=\forall x \forall y(A(x, y) \rightarrow B(x, y))$.

As this is a boolean algebra, the following proofs hold:
(40) (a) $\vdash(A \sqcap B) \sqsubseteq A$
(b) $\vdash(A \sqcap B) \sqsubseteq B$
(c) $A \sqsubseteq B, A \sqsubseteq C \vdash A \sqsubseteq(B \sqcap C)$
(41) (a) $\vdash A \sqsubseteq(A \sqcup B)$
$(\mathrm{b}) \vdash B \sqsubseteq(A \sqcup B)$
(c) $A \sqsubseteq(B \sqcup C) \vdash A \sqsubseteq B, A \sqsubseteq C$

[^15]I am now in a position to state the lexical entry for 'master':

$$
\begin{equation*}
\llbracket \text { master } \rrbracket=\lambda x_{e} \cdot \lambda y_{e}\left\langle\operatorname{master}^{\prime}(y, x), \lambda f_{e \rightarrow \mathcal{R}}(f(y) \sqsubseteq \mathrm{ANI} \wedge f(x) \sqsubseteq \mathrm{INFO})\right\rangle \tag{42}
\end{equation*}
$$

The second member of the ordered pair shown is no longer simply an individuation relation, but rather (the characteristic function of) a set of functions of type $e \rightarrow \mathcal{R}$, each of which maps $x$ to some relation $R_{1}$ such that $R_{1} \sqsubseteq$ INFO and maps $y$ to some relation $R_{2}$ such that $R_{2} \sqsubseteq$ Ans. In what follows I will sometimes refer to this set of functions as a 'construction'.

## Definition 2 (Construction).

If $e$ is an expression such that $\llbracket e \rrbracket=\lambda \ldots\langle a, b\rangle$, the construction of $e$ is $b$.

The reason that the partial order $\sqsubseteq$ has been chosen to do the role of $\leadsto$ in (38) is that this formulation allows constructions to be combined monotonically. So for example, we want it to be the case that if there are requirements that $x$ be mapped to PHYS and also that $x$ be mapped to INFO, then $x$ is mapped to PHYS $\sqcap$ INFO. (40c) guarantees this, as instantiated in (43) below.

$$
\begin{equation*}
f(x) \sqsubseteq \mathrm{PHYS}, f(x) \sqsubseteq \mathrm{INFO} \vdash f(x) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO}) \tag{43}
\end{equation*}
$$

The lexical entry shown in (42) is of type $e \rightarrow(e \rightarrow(t \times((e \rightarrow \mathcal{R}) \rightarrow t)))$. I will henceforth abbreviate $t \times((e \rightarrow \mathcal{R}) \rightarrow t)$ as $T$, as this is the (extensional) type of sentence meaning. (42) is therefore of type $e \rightarrow(e \rightarrow T)$. Correspondingly, an example of a lexical entry of type $e \rightarrow T$ is our revised entry for 'books', as shown in (44).

$$
\begin{equation*}
\llbracket b o o k s \rrbracket=\lambda y_{e}\left\langle * \operatorname{book}^{\prime}(y), \lambda f_{e \rightarrow \mathcal{R}} \cdot f(y) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO})\right\rangle \tag{44}
\end{equation*}
$$

But now how do we access ind-relations, as we need to do for quantification for example? In Section 2.3.1 this was achieved simply with the use of $\pi_{2}$ (e.g. in (19)), but given the complication of lexical entries that has just been made we now need something else.

In order to do this, let us first look at a version of (44) from which all information not relevant to construction has been removed. ${ }^{15}$ This is shown in (45).

$$
\begin{equation*}
\lambda x_{e} \cdot \lambda f_{e \rightarrow \mathcal{R}}(f(x) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO})) \tag{45}
\end{equation*}
$$

What we want to do is to get at the the 'PHYS $\square$ INFO' in (45), i.e. the individuation relation associated with the abstracted variable. In order to do this:

- We note that for an arbitrary object $o,(45)[o]$ is (the characteristic function of) a set of functions $\{f: f(o) \sqsubseteq($ PHYS $\sqcap$ INFO $)\}$.
- We map this set of functions to the set of its values with respect to $o$, and take the least upper bound of that set, which will be PHYS $\Pi$ INFO.

Formally, we use the function $\Omega$, of type $(e \rightarrow((e \rightarrow \mathcal{R}) \rightarrow t)) \rightarrow \mathcal{R}$. Its definition is given in (46). ${ }^{16}$

$$
\begin{equation*}
\Omega(A) \stackrel{\text { def }}{=} \bigsqcup\left\{R: \exists x_{e} \exists f_{e \rightarrow \mathcal{R}}(A(x)(f) \wedge f(x)=R)\right\} \tag{46}
\end{equation*}
$$

The definition of least upper bound is as follows (Partee, Meulen, and Wall, 1990, p. 278):
(47) If $A$ is a set and $\leq$ is a partial order on $A$ and $B$ is a subset of $A$, then the least upper bound of $B$ (if there is one) is the element $x$ of $A$ such that

- $C$ is the set of elements $z$ such that for every element $y$ of $B, y \leq z$ (the set of upper bounds of $B$ ), and
- for every element $z$ of $C, x \leq z$.

So for example, if $A$ is the set of $e \rightarrow t$-type predicates $\left\{\right.$ tall $^{\prime}$, fat ${ }^{\prime}$, strong $^{\prime}$, (tall ${ }^{\prime} \sqcup$ fat $\left.^{\prime}\right),\left(\right.$ tall $^{\prime} \sqcup$ strong $\left.{ }^{\prime}\right),\left(\right.$ fat $^{\prime} \sqcup$ strong $\left.^{\prime}\right),\left(\right.$ tall $^{\prime} \sqcup$ fat $^{\prime} \sqcup$ strong $\left.\left.{ }^{\prime}\right)\right\}$, then $\sqsubseteq$ is a partial order on $A$. If $B$ is $\left\{\operatorname{tall}^{\prime}\right.$, fat $\left.^{\prime}\right\}$, then the least upper bound of $B$ is $\operatorname{tall}^{\prime} \sqcup \mathrm{fat}^{\prime}$. In this

[^16]case, $C$ is $\left\{\left(\right.\right.$ tall $^{\prime} \sqcup$ fat $\left.^{\prime}\right),\left(\right.$ tall $^{\prime} \sqcup$ fat $^{\prime} \sqcup$ strong $\left.\left.^{\prime}\right)\right\}$, and $\left(\right.$ tall $^{\prime} \sqcup$ fat $\left.^{\prime}\right) \sqsubseteq\left(\right.$ tall $^{\prime} \sqcup$ fat $\left.^{\prime}\right)$ and $\left(\right.$ tall $^{\prime} \sqcup$ fat $\left.^{\prime}\right) \sqsubseteq\left(\right.$ tall $^{\prime} \sqcup$ fat $^{\prime} \sqcup$ strong $\left.{ }^{\prime}\right)$.

Now that we have the $\Omega$ function, we can think about how to apply it within lexical entries like (42). For lexical entries of type $e \rightarrow T$ I will use the function $\Omega_{1}$, of type $(e \rightarrow T) \rightarrow \mathcal{R}$, as defined in (48).

$$
\begin{equation*}
\Omega_{1}(A) \stackrel{\text { def }}{=} \Omega\left(\lambda v_{e} \cdot \pi_{2}(A(v))\right) \tag{48}
\end{equation*}
$$

$\Omega_{1}(\llbracket b o o k s \rrbracket)$ is therefore PHYS $\Pi$ INFO, as can be seen from the working below.

$$
\begin{align*}
\Omega_{1}(\llbracket b o o k s \rrbracket) & =\Omega\left(\lambda v_{e} \cdot \pi_{2}(\llbracket b o o k s \rrbracket(v))\right)  \tag{49}\\
& =\Omega\left(\lambda v_{e} \cdot \pi_{2}\left(\lambda y_{e}\left\langle{ }^{*} \operatorname{book}^{\prime}(y), \lambda f_{e \rightarrow \mathcal{R}} \cdot f(y) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO})\right\rangle(v)\right)\right) \\
& =\Omega\left(\lambda v_{e} \cdot \pi_{2}\left(\left\langle * \operatorname{book}^{\prime}(v), \lambda f_{e \rightarrow \mathcal{R}} \cdot f(v) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO})\right\rangle\right)\right) \\
& =\Omega\left(\lambda v_{e} \cdot \lambda f_{e \rightarrow \mathcal{R}} \cdot f(v) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO})\right) \\
& =\bigsqcup\left\{R: \exists x_{e} \exists f_{e \rightarrow \mathcal{R}}((f(x) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO})) \wedge f(x)=R)\right\} \\
& =\bigsqcup\{R: R \sqsubseteq(\operatorname{PHYS} \sqcap \mathrm{INFO})\} \\
& =\operatorname{PHYS} \sqcap \mathrm{INFO}
\end{align*}
$$

The last line of working is justified because:
(i) The set in the penultimate line of (49) is the set of relations $R$ such that $R \sqsubseteq$ (PHYS $\sqcap$ INFO), corresponding to $B$ in (47).
(ii) Therefore, the set corresponding to $C$ in (47) contains PHYS $\sqcap$ INFO as a member.
(iii) From (i), the set corresponding to $B$ in (47) also contains PHYS $\Pi$ INFO as a member, because (by definition) $\sqsubseteq$ is reflexive.
(iv) Therefore, PHYS $\sqcap$ INFO is an upper bound of the set in the penultimate line of (49) (from (ii)).
(v) And for every upper bound $U$ of the set in the penultimate line of (49), (PHYS $\sqcap$ INFO) $\sqsubseteq U($ from $(i i i))$.
(vi) Therefore, PHYS $\sqcap$ INFO is the least upper bound of the set in the penultimate line
of (49).

For lexical entries of type $e \rightarrow(e \rightarrow T)$, such as (42), we can define the function $\Omega_{2}$ as shown in (50).

$$
\begin{equation*}
\Omega_{2}(R) \stackrel{\text { def }}{=} \Omega\left(\lambda v_{e} \cdot \lambda f_{e \rightarrow \mathcal{R}} \cdot \exists z\left(\pi_{2}(R(v)(z))(f)\right)\right) \tag{50}
\end{equation*}
$$

The idea is to get the criterion of individuation associated with the most oblique argument of the verb. $\Omega_{2}(\llbracket$ master $\rrbracket)$ is therefore INFO, as can be seen from the working in (51).

$$
\begin{align*}
& \Omega_{2}(\llbracket \text { master } \rrbracket)=\Omega\left(\lambda v_{e} \cdot \lambda f_{e \rightarrow \mathcal{R}} \cdot \exists z\left(\pi_{2}(\llbracket \text { master } \rrbracket(v)(z))(f)\right)\right)  \tag{51}\\
& \quad=\Omega\left(\lambda v _ { e } \cdot \lambda f _ { e \rightarrow \mathcal { R } } \cdot \exists z \left(\pi _ { 2 } \left(\lambda x _ { e } \cdot \lambda y _ { e } \left\langle\operatorname{master}^{\prime}(y, x),\right.\right.\right.\right. \\
& \\
& \left.\left.\left.\left.\quad \lambda g_{e \rightarrow \mathcal{R}}(g(y) \sqsubseteq \mathrm{ANI} \wedge g(x) \sqsubseteq \mathrm{INFO})\right\rangle(v)(z)\right)(f)\right)\right) \\
& =\Omega\left(\lambda v_{e} \cdot \lambda f_{e \rightarrow \mathcal{R}} \cdot \exists z\left(\pi_{2}\left(\left\langle\text { master }^{\prime}(z, v),(f(z) \sqsubseteq \mathrm{ANI} \wedge f(v) \sqsubseteq \mathrm{INFO})\right\rangle\right)\right)\right) \\
& =\Omega\left(\lambda v_{e} \cdot \lambda f_{e \rightarrow \mathcal{R}} \cdot \exists z(f(z) \sqsubseteq \mathrm{ANI} \wedge f(v) \sqsubseteq \mathrm{INFO})\right) \\
& \quad=\bigsqcup\left\{R: \exists x_{e} \exists f_{e \rightarrow \mathcal{R}}(\exists z(f(z) \sqsubseteq \mathrm{ANI} \wedge f(x) \sqsubseteq \mathrm{INFO}) \wedge f(x)=R)\right\} \\
& \quad=\bigsqcup\{R: R \sqsubseteq \mathrm{INFO}\} \\
& \quad=\mathrm{INFO}
\end{align*}
$$

These functions $\Omega_{1}$ and $\Omega_{2}$ come in useful when giving lexical entries for determiners, which need to be adjusted accordingly. For instance, the final version of the lexical entry for the determiner 'three' is shown in (52).

$$
\begin{align*}
& \llbracket \text { three }=  \tag{52}\\
& \begin{aligned}
& \lambda A_{e \rightarrow T} \cdot \lambda B_{e \rightarrow T}\left\langle\exists x\left(|x| \geq 3 \wedge \pi_{1}(A(x)) \wedge \pi_{1}(B(x)) \wedge \neg\left(\Omega_{1}(A) \sqcup \Omega_{1}(B)\right) \operatorname{comp}(x)\right)\right. \\
&\left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(A(v)) \wedge \pi_{2}(A(v))(h) \wedge \pi_{2}(B(v))(h)\right)\right\rangle
\end{aligned}
\end{align*}
$$

Since it has already been established (in (49)) that $\Omega_{1}(\llbracket b o o k s \rrbracket)=$ PHYS $\sqcap$ INFO, we can see that $\llbracket t h r e e ~ b o o k s \rrbracket$ is as shown in (53).
(53) $\llbracket$ three books】 $=(52)[(44)]$

$$
\begin{gathered}
=\lambda B_{e \rightarrow T}\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge \pi_{1}(B(x)) \wedge \neg\left((\mathrm{PHYS} \sqcap \mathrm{INFO}) \sqcup \Omega_{1}(B)\right) \operatorname{comp}(x)\right),\right. \\
\left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO}) \wedge \pi_{2}(B(v))(h)\right)\right\rangle
\end{gathered}
$$

The lexical entry of the adjective 'heavy' can also be adapted accordingly, as shown in (54).
(54) $\llbracket h e a v y_{p l} \rrbracket=$

$$
\begin{aligned}
\lambda Q_{e \rightarrow T} \cdot \lambda x_{e}\langle & \left(* \text { heavy }^{\prime}(x) \wedge \pi_{1}(Q(x))\right), \\
& \left.\lambda g_{e \rightarrow R}\left(\exists h\left(\pi_{2}(Q(x))(h) \wedge g \sim_{x} h\right) \wedge g(x) \sqsubseteq\left(\text { PHYS } \sqcup \Omega_{1}(Q)\right)\right)\right\rangle
\end{aligned}
$$

$g \sim_{x} h$ indicates that $g$ and $h$ differ at most with respect to $x$. We can now see how these lexical entries combine to give us an interpretation for 'three heavy books'.

$$
\begin{aligned}
& \llbracket h e a v y \text { books } \rrbracket=(54)[(44)] \\
& =\lambda x_{e}\left\langle\left(* \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x)\right),\right. \\
& \left.\lambda g_{e \rightarrow R}\left(\exists h\left(h(x) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO}) \wedge g \sim_{x} h\right) \wedge g(x) \sqsubseteq(\mathrm{PHYS} \sqcup(\mathrm{PHYS} \sqcap \mathrm{INFO}))\right)\right\rangle \\
& =\lambda x_{e}\left\langle\left(\text { *heavy }^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x)\right),\right. \\
& \left.\quad \lambda g_{e \rightarrow R}\left(\exists h\left(h(x) \sqsubseteq(\text { PHYS } \sqcap \mathrm{INFO}) \wedge g \sim_{x} h\right) \wedge g(x) \sqsubseteq \mathrm{PHYS}\right)\right\rangle
\end{aligned}
$$

The expression ' $\exists h\left(h(x) \sqsubseteq(\right.$ PHYS $\left.\sqcap \mathrm{INFO}) \wedge g \sim_{x} h\right)$ ' is redundant in the interpretation of 【heavy books】: to say that $g$ is an $x$-variant of some function $h$ such that $h(x) \sqsubseteq($ PHYS $\sqcap \mathrm{INFO})$ is to say that $g$ could in fact be anything. But if the expression were to contain more information about the function $h$ in addition to the value of $h(x)$, then the expression would not be redundant in that case. This would happen for example in the interpretation of 'heavy books that Steve likes', because in that case $h$ would contain information regarding the value of $h\left(s^{\prime}\right)$ (Steve). We will see something similar in Section 3.1.1.

In any case, given the redundancy noted above, the interpretation of 'heavy books' is as shown in (55). Therefore, the interpretation of 'three heavy books' is as shown in (56).

$$
\begin{equation*}
\llbracket h e a v y \text { books } \rrbracket=\lambda x_{e}\left\langle\left({ }^{*} \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x)\right), \lambda g_{e \rightarrow R}(g(x) \sqsubseteq \mathrm{PHYS})\right\rangle \tag{55}
\end{equation*}
$$

$\llbracket$ three heavy books $\rrbracket=(52)[(55)]$

$$
\begin{gather*}
=\lambda B_{e \rightarrow T}\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge \pi_{1}(B(x)) \wedge \neg\left(\operatorname{PHYS} \sqcup \Omega_{1}(B)\right) \operatorname{comp}(x)\right),\right.  \tag{56}\\
\left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{heavy}^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq \operatorname{PHYS} \wedge \pi_{2}(B(v))(h)\right)\right\rangle
\end{gather*}
$$

It is worth comparing (53) to (56) to see how they make different contributions towards compression statements. The difference is the same as that between (24) and (33) in Section 2.3.1.

With this much in place, we can see how these assumptions come together to derive the correct truth conditions for a copredication sentence.

### 2.4 Implementation

In this section, I will show how the lexical entries given in this approach, together with standard compositional principles, can be applied to derive the correct truth conditions for (4) within two different frameworks: a Logical Form (LF)-based approach and Combinatory Categorial Grammar (CCG). These two presentations are chosen in order to show that the lexical/compositional system described here can be adapted equally well to theories involving movement, type raising, flexibility and compositional mechanisms other than function application.

### 2.4.1 Interpretation via Logical Form

If we were to adopt the approach to the syntax/semantics interface according to which the level of syntactic representation that is the input to interpretation (Logical Form) is the result of moving quantified DPs out of their surface positions, then the semantically relevant structure of (4) would be something like that shown in (57). Here, the DP 'three heavy books' has moved from the object position, leaving a trace with index 1 , adjoined to its containing sentence (TP) and adjoined an index 1 to that sentence.


In this presentation I will ignore tense and treat the T node as semantically null.
The presence of movement in this system means that we need to relativize interpretation to an assignment function $g$-a function with domain the set of natural numbers and with range the domain of discourse $D_{e} \cdot{ }^{17}$ This can be done as shown in (58)-(59), requiring no addition to the system described by Heim and Kratzer (1998).
(58) If $t_{i}$ is a trace, then $\llbracket t_{i} \rrbracket^{g}=g(i)$.
(59) If $\alpha$ is a binary branching node with daughters $\beta$ and (numerical index) $i$, then

$$
\llbracket \alpha \rrbracket^{g}=\lambda v_{e} \cdot \llbracket \beta \rrbracket^{g^{i / v}} .
$$

Where $g^{i / v}$ is the function that is just like $g$, except that $g(i)=v$.
Based on (58) and the lexical entries stated above, we can see that the interpretation of the lower TP is as shown in (60).

$$
\begin{equation*}
\llbracket T P \rrbracket^{g}=\left\langle *^{*} \operatorname{master}^{\prime}\left(f^{\prime}, g(1)\right), \lambda h_{e \rightarrow \mathcal{R}}\left(h\left(f^{\prime}\right) \sqsubseteq \operatorname{ANI} \wedge h(g(1)) \sqsubseteq \mathrm{INFO}\right)\right\rangle \tag{60}
\end{equation*}
$$

Based on (59) and (60), we can see that the interpretation of its mother is as shown in (61).
(61) $\lambda v_{e}\left\langle{ }^{*} \operatorname{master}^{\prime}\left(f^{\prime}, v\right), \lambda h_{e \rightarrow \mathcal{R}}\left(h\left(f^{\prime}\right) \sqsubseteq \operatorname{ANI} \wedge h(v) \sqsubseteq \operatorname{INFO}\right)\right\rangle$

[^17]Given that the semantic value of the moved DP 'three heavy books' is the same as shown in (56), the interpretation of the whole sentence is as shown in (62).
$\llbracket$ Fred mastered three heavy books】=(56)[(61)]

$$
\begin{align*}
= & \left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{master}^{\prime}\left(f^{\prime}, x\right) \wedge \neg(\text { PHYS } \sqcup \mathrm{INFO}) \operatorname{comp}(x)\right),\right.  \tag{62}\\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{heavy}^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq \operatorname{PHYS} \wedge h\left(f^{\prime}\right) \sqsubseteq \operatorname{ANI} \wedge h(v) \sqsubseteq \operatorname{INFO}\right)\right\rangle \\
= & \left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge * \text { master }^{\prime}\left(f^{\prime}, x\right) \wedge \neg(\text { PHYS } \sqcup \mathrm{INFO}) \operatorname{comp}(x)\right),\right. \\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{heavy}^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\text { PHYS } \sqcap \mathrm{INFO}) \wedge h\left(f^{\prime}\right) \sqsubseteq \operatorname{ANI}\right)\right\rangle
\end{align*}
$$

In the last line of working the inference has been made from $h(v) \sqsubseteq \operatorname{PHYS} \wedge h(v) \sqsubseteq$ INFO to $h(v) \sqsubseteq$ (PHYS $\sqcap$ INFO), which is licensed by (40c). This shows the utility of using generalized implication in these lexical entries.

The first projection of (62) - the extensional meaning of the sentence - is identical to (9), as desired. The significance of the second projection will be examined in Section 5.1.

### 2.4.2 Combinatory Categorial Grammar

As an example of a theory of the syntax/semantics interface that is surface-compositional, I choose Combinatory Categorial Grammar (CCG). ${ }^{18}$ In CCG, syntactic categories can be atomic or complex. For the examples addressed in this section, the categories are as defined in (63).

- The set of atomic categories is $\{N, N P, S\}$.
- Every atomic category is a category.
- If $A$ is a category and $B$ is a category, the $A / B$ and $A \backslash B$ are categories.
- Nothing else is a category.

[^18]Informally, an expression of category $A / B$ is looking for an expression of category $B$ to its right to form an expression of category $A$, and $A \backslash B$ is looking for an expression of category $B$ to its left to form an expression of category $A$.

There is a mapping from syntactic categories to semantic types. The mapping Ty that I will use is as shown in (64).

- $\operatorname{Tr}(N P)=e$
- $\operatorname{Tr}(N)=e \rightarrow T$
- $\operatorname{Tr}(S)=T$
- $\operatorname{Ty}(A / B)=\operatorname{Ty}(A \backslash B)=\operatorname{Tr}(A) \rightarrow \operatorname{Ty}(B)$
- If $e x$ is an expression of syntactic category $A$, then $\llbracket e x \rrbracket$ is of type $\operatorname{Ty}(A)$.

The mapping shown in (64) differs from that conventionally used in that $N$ maps to $e \rightarrow T$ (rather than $e \rightarrow t$ as is conventional) and $S$ maps to $T$ (rather than $t$ as is conventional). These changes do not have any deleterious effects on the surfacecompositonality of the system.

The combinatory rules that will be needed for the example at hand are given in (65)-(68). ${ }^{19} x: Y$ indicates that $x$ is the interpretation of an expression and $Y$ is that expression's syntactic category.
(65) Forward application

$$
\frac{f: X / Y \quad a: Y}{f(a): X}>
$$

(66) Backward application

$$
\frac{a: Y \quad f: X \backslash Y}{f(a): X}<
$$

## (67) Forward composition

$$
\frac{f: X / Y \quad g: Y / Z}{\lambda v_{\operatorname{TY}(Z)} f(g(v)): X / Z}>\mathrm{B}
$$

[^19]Forward type raising

$$
\begin{equation*}
\frac{a: X}{\lambda f_{\operatorname{Ty}(Y \backslash X)} f(a): Y /(Y \backslash X)}>\mathrm{T} \tag{68}
\end{equation*}
$$

(69) and (70) show two possible derivations of (4) in CCG, showing only syntactic categories.



The boxed numbers are only there to label points in the derivations for future reference and are not in any way part of the derivation.

In the version of CCG applied here, determiners are of the flexible syntactic category $(C a t \backslash(C a t / N P)) / N,{ }^{20}$ where Cat can be any category ending in $S$. The determiner shown in (52) is the simplest one possible, used in (69) (i.e. where Cat is $S$ ). This needs to be generalised for (70) (i.e. where Cat is $S \backslash N P$ ) as shown in (71).

$$
\begin{align*}
\lambda A_{e \rightarrow T} \cdot & \lambda R_{e \rightarrow(e \rightarrow T)} \cdot \lambda z_{e}  \tag{71}\\
& \left\langle\exists x\left(|x| \geq 3 \wedge \pi_{1}(A(x)) \wedge \pi_{1}(R(x)(z)) \wedge \neg\left(\Omega_{1}(A) \sqcup \Omega_{2}(R)\right) \operatorname{comp}(x)\right)\right. \\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(A(v)) \wedge \pi_{2}(A(v))(h) \wedge \pi_{2}(R(v)(z))(h)\right)\right\rangle
\end{align*}
$$

Notwithstanding the fact that in the current system the semantic counterpart of the syntactic category $S$ is the type $T$ (and not $t$ ), the generalisation to higher types is predictable in same way that it is for determiners in more conventional theories. A definition of generalised determiners is given in Section A. 1 of the Appendix.

[^20]We are now in a position to show the semantic composition for (69) and (70).

## Semantic composition of (69)

6 As (56):

$$
\begin{aligned}
& \lambda B_{e \rightarrow T}\langle\exists x\left(|x| \geq 3 \wedge * \text { heavy }^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge \pi_{1}(B(x))\right. \\
&\left.\wedge \neg\left(\text { PHYS } \sqcup \Omega_{1}(B)\right) \operatorname{comp}(x)\right), \\
&\left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left({ }^{*} \operatorname{heavy}^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq \operatorname{PHYS} \wedge \pi_{2}(B(v))(h)\right)\right\rangle
\end{aligned}
$$

5 As (42):
$\lambda z_{e} \cdot \lambda y_{e}\left\langle{ }^{*}\right.$ master $^{\prime}(y, z), \lambda g_{e \rightarrow \mathcal{R}}(g(y) \sqsubseteq$ ANI $\left.\wedge g(z) \sqsubseteq \mathrm{INFO})\right\rangle$

$$
4=f^{\prime}
$$

$$
3=\lambda P_{e \rightarrow T} \cdot P\left(f^{\prime}\right)
$$

$$
2=\lambda z_{e} \cdot \sqrt{3}(\boxed{5}(z))
$$

$$
=\lambda z_{e}\left\langle{ }^{*} \operatorname{master}^{\prime}\left(f^{\prime}, z\right), \lambda g_{e \rightarrow \mathcal{R}}\left(g\left(f^{\prime}\right) \sqsubseteq \mathrm{ANI} \wedge g(z) \sqsubseteq \mathrm{INFO}\right)\right\rangle
$$

$$
1=6(\sqrt{2})
$$

$$
=\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{master}^{\prime}\left(f^{\prime}, x\right) \wedge \neg(\text { PHYS } \sqcup \operatorname{INFO}) \operatorname{comp}(x)\right)\right.
$$

$$
\left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{heavy}^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq \operatorname{PHYS} \wedge h\left(f^{\prime}\right) \sqsubseteq \mathrm{ANI} \wedge h(v) \sqsubseteq \mathrm{INFO}\right)\right\rangle
$$

$$
=\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{master}^{\prime}\left(f^{\prime}, x\right) \wedge \neg(\mathrm{PHYS} \sqcup \operatorname{INFO}) \operatorname{comp}(x)\right)\right.
$$

$$
\left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left({ }^{*} \operatorname{heavy}^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO}) \wedge h\left(f^{\prime}\right) \sqsubseteq \mathrm{ANI}\right)\right\rangle
$$

## Semantic composition of (70)

7 As (55):

$$
\lambda x_{e}\left\langle\left({ }^{*} \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x)\right), \lambda g_{e \rightarrow R}(g(x) \sqsubseteq \mathrm{PHYS})\right\rangle
$$

6 As (71):

$$
\begin{aligned}
& \lambda A_{e \rightarrow T} \cdot \lambda R_{e \rightarrow(e \rightarrow T)} \cdot \lambda z_{e}\langle\exists x(|x| \geq 3 \wedge \pi_{1}(A(x)) \wedge \pi_{1}(R(x)(z)) \\
&\left.\wedge \neg\left(\Omega_{1}(A) \sqcup \Omega_{2}(R)\right) \operatorname{comp}(x)\right) \\
&\left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(A(v)) \wedge \pi_{2}(A(v))(h) \wedge \pi_{2}(R(v)(z))(h)\right)\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
& 5=6(7) \\
& =\lambda R_{e \rightarrow(e \rightarrow T)} \cdot \lambda z_{e}\left\langle\exists x \left(|x| \geq 3 \wedge{ }^{*} \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge \pi_{1}(R(x)(z))\right.\right. \\
& \left.\wedge \neg\left(\text { PHYS } \sqcup \Omega_{2}(R)\right) \operatorname{comp}(x)\right), \\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left({ }^{*} \operatorname{heavy}^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq \text { PHYS } \wedge \pi_{2}(B(v)(z))(h)\right)\right\rangle \\
& 4 \text { As (42): } \\
& \lambda z_{e} \cdot \lambda y_{e}\left\langle * \text { master }^{\prime}(y, z), \lambda g_{e \rightarrow \mathcal{R}}(g(y) \sqsubseteq \text { ANI } \wedge g(z) \sqsubseteq \mathrm{INFO})\right\rangle \\
& 5=5(\sqrt{4}) \\
& =\lambda z_{e}\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{master}^{\prime}(z, x) \wedge \neg(\operatorname{PHYS} \sqcup \operatorname{INFO}) \operatorname{comp}(x)\right),\right. \\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{heavy}^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq \text { PHYS } \wedge h(z) \sqsubseteq \text { ANI } \wedge h(v) \sqsubseteq \text { INFO }\right)\right\rangle \\
& =\lambda z_{e}\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{master}^{\prime}(z, x) \wedge \neg(\operatorname{PHYS} \sqcup \operatorname{INFO}) \operatorname{comp}(x)\right),\right. \\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\text { 'heavy }^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\text { PHYS } \sqcap \text { INFO }) \wedge h(z) \sqsubseteq \text { ANI }\right)\right\rangle \\
& 2=f^{\prime} \\
& 1=3(\boxed{2}) \\
& =\left\langle\exists x\left(|x| \geq 3 \wedge{ }^{*} \operatorname{heavy}^{\prime}(x) \wedge{ }^{*} \operatorname{book}^{\prime}(x) \wedge * \operatorname{master}^{\prime}\left(f^{\prime}, x\right) \wedge \neg(\text { PHYS } \sqcup \mathrm{INFO}) \operatorname{comp}(x)\right),\right. \\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left({ }^{*} \operatorname{heavy}^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\text { PHYS } \sqcap \mathrm{INFO}) \wedge h\left(f^{\prime}\right) \sqsubseteq \text { ANI }\right)\right\rangle
\end{aligned}
$$

We can see that in both derivations we end up with the same interpretation, as desired. This is the same as the interpretation derived in Section 2.4.1, shown in (62), and so again it accords with (9) (and (6)).

## Chapter 3

## Expanding the system

In this chapter the basic system described in Chapter 2 will be extended to take account of grammatical constructions not considered there. In Section 3.1 I will describe how one major source of copredication, namely coordination structures, should be accommodated. In Chapter 2 the focus was on numerically-quantified copredication sentences with bare numerals, because those bring out most clearly the problems of counting and individuation raised by copredication. Sections 3.2-3.5 extend the empirical coverage of the theory to other quantifiers and sentence types. In Section 3.6 I outline some ways in which the system needs to be improved, and suggest some avenues to pursue in order to do so in future work.

### 3.1 Coordination

### 3.1.1 Conjunction

In order to ensure that the books are individuated in the same way for (1) as they are for (2) (repeated from Chapter 2), i.e. requiring both physical and informational distinctness, we have to provide a lexical entry for the transitive verb conjunction 'and' that combines the constructions of each transitive verb in the right way. This is shown in (3).
(1) Fred picked up and mastered three books.
(2) Fred mastered three heavy books.
(3) $\lambda A_{e \rightarrow(e \rightarrow T)} \cdot \lambda B_{e \rightarrow(e \rightarrow T)} \cdot \lambda x_{e} \cdot \lambda y_{e}$

$$
\begin{aligned}
& \left\langle\pi_{1}(A(x)(y)) \wedge \pi_{1}(B(x)(y)),\right. \\
& \quad \lambda g_{e \rightarrow \mathcal{R}}\left(\exists h\left(\pi_{2}(A(x)(y))(h) \wedge h \sim_{x, y} g\right) \wedge g(x) \sqsubseteq\left(\Omega_{2}(A) \sqcup \Omega_{2}(B)\right)\right. \\
& \left.\left.\quad \wedge \exists f\left(\pi_{2}(B(x)(y))(f) \wedge f \sim_{x, y} g\right) \wedge g(y) \sqsubseteq\left(\Omega_{1}(A(x)) \sqcup \Omega_{1}(B(x))\right)\right)\right\rangle
\end{aligned}
$$

So then we have
(4) $\llbracket p i c k$ up and master $\rrbracket=$

$$
\begin{aligned}
& \lambda x_{e} \cdot \lambda y_{e}\left\langle\left({ }^{*} \text { pick-up }{ }^{\prime}(y, x) \wedge{ }^{*} \operatorname{master}^{\prime}(y, x)\right),\right. \\
& \lambda g_{e \rightarrow \mathcal{R}}\left(\exists h\left(h(x) \sqsubseteq \mathrm{PHYS} \wedge h(y) \sqsubseteq \text { ANI } \wedge h \sim_{x, y} g\right) \wedge g(x) \sqsubseteq(\text { PHYS } \sqcup \mathrm{INFO})\right. \\
& \left.\left.\wedge \exists f\left(f(x) \sqsubseteq \mathrm{INFO} \wedge f(y) \sqsubseteq \mathrm{ANI} \wedge f \sim_{x, y} g\right) \wedge g(y) \sqsubseteq(\text { ANI } \sqcup \text { ANI })\right)\right\rangle \\
& =\lambda x_{e} \cdot \lambda y_{e}\left\langle\left({ }^{*} \text { pick-up }{ }^{\prime}(y, x) \wedge *^{*} \operatorname{master}^{\prime}(y, x)\right),\right. \\
& \left.\lambda g_{e \rightarrow \mathcal{R}}(g(x) \sqsubseteq(\text { PHYS } \sqcup \mathrm{INFO}) \wedge g(y) \sqsubseteq(\text { ANI } \sqcup \text { ANI }))\right\rangle \\
& =\lambda x_{e} \cdot \lambda y_{e}\left\langle\left({ }^{*} \operatorname{pick}^{\prime} \text { up }^{\prime}(y, x) \wedge{ }^{*} \operatorname{master}^{\prime}(y, x)\right), \lambda g_{e \rightarrow \mathcal{R}}(g(x) \sqsubseteq(\mathrm{PHYS} \sqcup \mathrm{INFO}) \wedge g(y) \sqsubseteq \text { ANI })\right\rangle
\end{aligned}
$$

This means that $\llbracket \lambda_{1}\left[\right.$ Fred picked up and mastered $\left.t_{1}\right] \rrbracket=(5)$. I've illustrated this in terms of a transformational theory, but of course 'Fred picked up and mastered' is also a possible constituent in CCG without the use of traces if composition proceeds in the manner shown in (69) in Section 2.4.2.

$$
\begin{align*}
& \lambda x_{e}\left\langle\left({ }^{*} \text { pick-up }{ }^{\prime}\left(f^{\prime}, x\right) \wedge{ }^{*} \text { master }^{\prime}\left(f^{\prime}, x\right)\right),\right.  \tag{5}\\
& \left.\quad \lambda g_{e \rightarrow \mathcal{R}}\left(g(x) \sqsubseteq(\mathrm{PHYS} \sqcup \mathrm{INFO}) \wedge g\left(f^{\prime}\right) \sqsubseteq \text { ANI }\right)\right\rangle
\end{align*}
$$

Given that $\Omega_{1}(5)=$ PHYS $\sqcup \mathrm{INFO}$, and the fact that 'three books' has the interpretation shown in $(6),{ }^{1}(1)$ has the interpretation shown in (7).
(6) $\lambda B_{e \rightarrow T}\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge \pi_{1}(B(x)) \wedge \neg\left((\right.\right.\right.$ PHYS $\left.\left.\sqcap \operatorname{INFO}) \sqcup \Omega_{1}(B)\right) \operatorname{comp}(x)\right)$,

$$
\left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\text { PHYS } \sqcap \mathrm{INFO}) \wedge \pi_{2}(B(v))(h)\right)\right\rangle
$$

[^21](7) $\llbracket$ Fred picked up and mastered three books $\rrbracket=(6)[(5)]$
\[

$$
\begin{aligned}
= & \left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge * \text { pick-up }{ }^{\prime}\left(f^{\prime}, x\right) \wedge * \operatorname{master}^{\prime}\left(f^{\prime}, x\right) \wedge \neg(\text { PHYS } \sqcup \mathrm{INFO}) \operatorname{comp}(x)\right),\right. \\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\text { PHYS } \sqcap \mathrm{INFO}) \wedge h(v) \sqsubseteq(\text { PHYS } \sqcup \mathrm{INFO}) \wedge h\left(f^{\prime}\right) \sqsubseteq \mathrm{ANI}\right)\right\rangle \\
= & \left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge * \text { pick-up }\left(f^{\prime}, x\right) \wedge * \operatorname{master}^{\prime}\left(f^{\prime}, x\right) \wedge \neg(\text { PHYS } \sqcup \mathrm{INFO}) \operatorname{comp}(x)\right),\right. \\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\text { PHYS } \sqcap \mathrm{INFO}) \wedge h\left(f^{\prime}\right) \sqsubseteq \mathrm{ANI}\right)\right\rangle
\end{aligned}
$$
\]

In the last line of working the inference has been made from $h(v) \sqsubseteq($ PHYS $\sqcap \mathrm{INFO}) \wedge$ $h(v) \sqsubseteq($ PHYS $\sqcup \mathrm{INFO})$ to $h(v) \sqsubseteq($ PHYS $\sqcap \mathrm{INFO})$, which is licensed by the algebraic properties of $\sqsubseteq$ and $\sqcap$.
$\pi_{1}(7)$ gives correct truth conditions for (1): it says that Fred picked up and mastered three books that are both physically and informationally distinct. For any conjoinable category a conjunction like (3) can be stated which preserves the individuation relation(s) present in its argument(s) in the right way. In Section A. 2 of the appendix the conjunction entry is stated in a general form from which these entries can be derived.

### 3.1.2 Disjunction

I propose that disjunction differs from conjunction only in extensional meaning and not in construction. This means that the lexical entry for transitive verb disjunction is as shown in (8).

$$
\begin{align*}
& \lambda A_{e \rightarrow(e \rightarrow T)} \cdot \lambda B_{e \rightarrow(e \rightarrow T)} \cdot \lambda x_{e} \cdot \lambda y_{e}  \tag{8}\\
&\langle \\
& \pi_{1}(A(x)(y)) \vee \pi_{1}(B(x)(y)), \\
& \lambda g_{e \rightarrow \mathcal{R}}\left(\exists h\left(\pi_{2}(A(x)(y))(h) \wedge h \sim_{x, y} g\right) \wedge g(x) \sqsubseteq\left(\Omega_{2}(A) \sqcup \Omega_{2}(B)\right)\right. \\
&\left.\left.\wedge \exists f\left(\pi_{2}(B(x)(y))(f) \wedge f \sim_{x, y} g\right) \wedge g(y) \sqsubseteq\left(\Omega_{1}(A(x)) \sqcup \Omega_{1}(B(x))\right)\right)\right\rangle
\end{align*}
$$

The general treatment of conjunction in Section A. 2 is adapted to disjunction in the same way.

On this approach it follows that the interpretation of (9) is as shown in (10).
(9) Fred picked up or mastered three books.

$$
\begin{align*}
&\left\langle\exists x \left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge\left({ }^{*} \text { pick-up }{ }^{\prime}\left(f^{\prime}, x\right) \vee * \operatorname{master}^{\prime}\left(f^{\prime}, x\right)\right)\right.\right.  \tag{10}\\
&\wedge \neg(\text { PHYS } \sqcup \mathrm{INFO}) \operatorname{comp}(x)), \\
&\left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left({ }^{*} \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO}) \wedge h\left(f^{\prime}\right) \sqsubseteq \mathrm{ANI}\right)\right\rangle
\end{align*}
$$

'There is a plurality of at least three books which is neither informationally nor physically compressible, and Fred picked all of them up or he mastered all of them'

It might be contended that the truth conditions given in (10) are too restrictive, i.e. that it should not be required that the three books in question be both physically and informationally distinct. On this view, the requirement expressed by the compressibility statement should just be that if Fred picked them up, then they are physically distinct, while if he mastered them, they are informationally distinct. However, I take it that this impression is due to a reading of (9) on which the disjunction takes scope over the numerical quantifier. If sentential disjunction is as shown in (11), then the interpretation of (9) with wide scope for disjunction is as shown in (12), which accords with the intuitions reported above.
(11) or $\stackrel{\text { def }}{=} \lambda A_{T} \cdot \lambda B_{T}\left\langle\left(\pi_{1}(A) \vee \pi_{1}(B)\right), \lambda f_{e \rightarrow T}\left(\pi_{2}(A)(f) \wedge \pi_{2}(B)(f)\right)\right\rangle$
(12) $\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{pick}^{\prime}\right.\right.$ up $^{\prime}\left(f^{\prime}, x\right) \wedge \neg($ PHYS $\left.) \operatorname{comp}(x)\right)$

$$
\begin{aligned}
& \vee \exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge{ }^{*} \operatorname{master}^{\prime}\left(f^{\prime}, x\right) \wedge \neg(\mathrm{INFO}) \operatorname{comp}(x)\right), \\
& \left.\lambda f_{e \rightarrow T} \cdot \exists v\left(f(v) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO}) \operatorname{comp}(v) \wedge f\left(f^{\prime}\right) \sqsubseteq \mathrm{ANI}\right)\right\rangle
\end{aligned}
$$

### 3.2 Other plural quantifiers

### 3.2.1 Other numerical quantifiers

Numerical quantifiers that are not monotone-increasing will be treated by combining monotone-increasing quantifier meanings with negation in the appropriate way, i.e. by instantiating quite explicitly the equivalences shown in (13)-(14), where not ${ }^{\prime}$ and and $^{\prime}$ are logical constants defined in (15) and (16) respectively.
$\llbracket$ fewer than $\mathrm{n} \rrbracket(A)(B) \equiv \operatorname{not}^{\prime}(\llbracket$ at least $\mathrm{n} \rrbracket(A)(B))$

$$
\begin{equation*}
\llbracket \text { exactly } \mathrm{n} \rrbracket(A)(B) \equiv \operatorname{and}^{\prime}(\llbracket \text { at least } \mathrm{n} \rrbracket(A)(B))\left(\text { not }^{\prime}(\llbracket \text { more than } \mathrm{n} \rrbracket(A)(B))\right) \tag{13}
\end{equation*}
$$

（15） $\operatorname{not}^{\prime} \stackrel{\text { def }}{=} \lambda T_{T}\left\langle\neg\left(\pi_{1}(T)\right), \pi_{2}(T)\right\rangle$
（16）$\quad$ and ${ }^{\prime} \stackrel{\text { def }}{=} \lambda T_{T} \cdot \lambda U_{T}\left\langle\left(\pi_{1}(T) \wedge \pi_{1}(U)\right), \lambda f_{e \rightarrow \mathcal{R}}\left(\pi_{2}(T)(f) \wedge \pi_{2}(U)(f)\right)\right\rangle$

So for example we have the interpretation in（17）for＇fewer than three＇，and that in（18）for＇exactly three＇．
（17）$[$ fewer than three】 $=$

$$
\begin{aligned}
& \lambda A_{e \rightarrow T} \cdot \lambda B_{e \rightarrow T}\left\langle\neg \exists x\left(|x| \geq 3 \wedge \pi_{1}(A(x)) \wedge \pi_{1}(B(x)) \wedge \neg\left(\Omega_{1}(A) \sqcup \Omega_{1}(B)\right) \operatorname{comp}(x)\right),\right. \\
&\left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(A(v)) \wedge \pi_{2}(A(v))(f) \wedge \pi_{2}(B(v))(f)\right)\right\rangle
\end{aligned}
$$

（18）$\llbracket$ exactly three $\rrbracket=$

$$
\begin{gathered}
\lambda A_{e \rightarrow T} \cdot \lambda B_{e \rightarrow T}\left\langle\exists x\left(|x| \geq 3 \wedge \pi_{1}(A(x)) \wedge \pi_{1}(B(x)) \wedge \neg\left(\Omega_{1}(A) \sqcup \Omega_{1}(B)\right) \operatorname{comp}(x)\right)\right. \\
\wedge \neg \exists x\left(|x|>3 \wedge \pi_{1}(A(x)) \wedge \pi_{1}(B(x))\right. \\
\left.\wedge \neg\left(\Omega_{1}(A) \sqcup \Omega_{1}(B)\right) \operatorname{comp}(x)\right), \\
\left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(A(v)) \wedge \pi_{2}(A(v))(f) \wedge \pi_{2}(B(v))(f)\right)\right\rangle
\end{gathered}
$$

Throughout Section 2．3，a simple numeral＇$n$＇was taken to mean＇at least $n$＇，and lexical entries given accordingly．This was a simplifying assumption that can now be refined．This change can be implemented by taking the set of basic types to additionally contain a type $n$ for natural numbers，and implementing the interpretations shown below in（19）－（23）．
（19）$\llbracket$ three $\rrbracket=3$
（20）$\llbracket$ at least $\rrbracket=\lambda n_{n} \cdot \lambda A_{e \rightarrow T} \cdot \lambda B_{e \rightarrow T}$

$$
\begin{aligned}
& \left\langle\exists x\left(|x| \geq n \wedge \pi_{1}(A(x)) \wedge \pi_{1}(B(x)) \wedge \neg\left(\Omega_{1}(A) \sqcup \Omega_{1}(B)\right) \operatorname{comp}(x)\right)\right. \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(A(v)) \wedge \pi_{2}(A(v))(f) \wedge \pi_{2}(B(v))(f)\right)\right\rangle
\end{aligned}
$$

（21）$\llbracket$ more than】 $=\lambda n_{n} \cdot \lambda A_{e \rightarrow T} \cdot \lambda B_{e \rightarrow T}$

$$
\begin{aligned}
& \left\langle\exists x\left(|x|>n \wedge \pi_{1}(A(x)) \wedge \pi_{1}(B(x)) \wedge \neg\left(\Omega_{1}(A) \sqcup \Omega_{1}(B)\right) \operatorname{comp}(x)\right)\right. \\
& \left.\quad \lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(A(v)) \wedge \pi_{2}(A(v))(f) \wedge \pi_{2}(B(v))(f)\right)\right\rangle
\end{aligned}
$$

（22）$\llbracket$ fewer than』＝

$$
\lambda n_{n} \cdot \lambda P_{e \rightarrow T} \cdot \lambda Q_{e \rightarrow T}\left\langle\neg\left(\pi_{1}(\llbracket \text { at least } \rrbracket(n)(P)(Q))\right), \pi_{2}(\llbracket \text { at least } \rrbracket(n)(P)(Q))\right\rangle
$$

$$
\begin{align*}
& \llbracket \text { exactly } \rrbracket=  \tag{23}\\
& \begin{aligned}
\lambda n_{n} \cdot \lambda P_{e \rightarrow T} \cdot \lambda Q_{e \rightarrow T}\langle & \pi_{1}(\llbracket \text { at least } \rrbracket(n)(P)(Q)) \wedge \neg \pi_{1}(\llbracket \text { more than } \rrbracket(n)(P)(Q)), \\
& \left.\pi_{2}(\llbracket \text { at least } \rrbracket(n)(P)(Q))\right\rangle
\end{aligned}
\end{align*}
$$

If we assume that the interpretation shown in (20) can be phonologically null, then the treatment of numerically quantified sentences in Section 2.3 follows, i.e. that of taking ' $n$ ' to mean 'at least $n$ '.

### 3.2.2 Proportional quantifiers

The general form of proportional quantifiers can be shown by giving a lexical entry for 'most', as shown in (24).

$$
\begin{align*}
& \llbracket m o s t \rrbracket=\lambda P_{e \rightarrow T} \cdot \lambda Q_{e \rightarrow T}  \tag{24}\\
& \left\langle\exists x \left(\pi_{1}(P(x)) \wedge \pi_{1}(Q(x)) \wedge \neg\left(\Omega_{1}(P) \sqcup \Omega_{1}(Q)\right) \operatorname{comp}(x)\right.\right. \\
& \left.\wedge \forall y\left(\left(\pi_{1}(P(y)) \wedge \neg\left(\Omega_{1}(P) \sqcup \Omega_{1}(Q)\right) \operatorname{comp}(y)\right) \rightarrow|x|>\frac{|y|}{2}\right)\right), \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(A(v)) \wedge \pi_{2}(A(v))(f) \wedge \pi_{2}(B(v))(f)\right)\right\rangle
\end{align*}
$$

Examples of copredication and non-copredication sentences are given in (25) and (26) respectively.
(25) $\llbracket$ Most heavy books are informative $\rrbracket=$

$$
\begin{aligned}
& \left\langle\exists x \left(\text { heavy }^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{inform}^{\prime}(x) \wedge \neg(\text { PHYS } \sqcup \mathrm{INFO}) \operatorname{comp}(x)\right.\right. \\
& \quad \wedge \forall y\left(\left({ }^{\left.\left.\left.* \operatorname{heavy}^{\prime}(y) \wedge * \operatorname{book}^{\prime}(y) \wedge \neg(\text { PHYS } \sqcup \mathrm{INFO}) \operatorname{comp}(y)\right) \rightarrow|x|>\frac{|y|}{2}\right)\right),}\right.\right. \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\text { heavy }^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq(\text { PHYS } \sqcap \mathrm{INFO})\right)\right\rangle
\end{aligned}
$$

(26) $\llbracket$ Most books are informative $\rrbracket=$

$$
\begin{aligned}
& \left\langle\exists x \left( * \operatorname{book}^{\prime}(x) \wedge *^{*} \operatorname{inform}^{\prime}(x) \wedge \neg(\operatorname{INFO}) \operatorname{comp}(x)\right.\right. \\
& \left.\quad \wedge \forall y\left(\left(* \operatorname{book}^{\prime}(y) \wedge \neg(\mathrm{INFO}) \operatorname{comp}(y)\right) \rightarrow|x|>\frac{|y|}{2}\right)\right), \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left({ }^{*} \operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO})\right)\right\rangle
\end{aligned}
$$

(25) says: 'there is some plurality of heavy books that are informative, which is neither physically nor informationally compressible, and the cardinality of which is greater
than half of that of any plurality of heavy books that is neither physically nor informationally compressible'. (26) says 'there is some plurality of books that are informative, which is not informationally compressible, and the cardinality of which is greater than half of that of any plurality of books that is not informationally compressible'.

Other proportional quantifiers should be similarly definable. ${ }^{2}$

### 3.2.3 'All'

I propose to treat the meaning of 'all' via a lexical entry that predicts the equivalence between 'every A Bs' and 'all As B' where B is a distributive predicate, but which can also apply to collective predicates.

As I will go on to argue in Section 3.4, I do not believe that there is a need for construction-based information to be incorporated into truth conditions for sentences of the form 'every A B', and so the lexical entry shown (27) has no compressibility statement.

$$
\begin{align*}
& \llbracket a l l \rrbracket=\lambda A_{e \rightarrow T} \cdot \lambda B_{e \rightarrow T}  \tag{27}\\
& \qquad \quad \forall x\left(\pi_{1}(A(x)) \rightarrow \exists y\left(\pi_{1}(B(y)) \wedge \text { i-part }{ }^{\prime}(x, y)\right)\right), \\
& \left.\quad \lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(A(v)) \wedge \pi_{2}(A(v))(f) \wedge \pi_{2}(B(v))(f)\right)\right\rangle
\end{align*}
$$

An example of the interpretation that this lexical entry gives for a sentence requiring a collective interpretation is shown in (28), where similar' is true of a plurality $x$ if and only if every individual part of $x$ is similar to every other individual part.
(28) $\llbracket$ all red books are similar $\rrbracket=$

$$
\begin{gathered}
\left\langle\forall x\left(\left({ }^{*} \operatorname{red}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x)\right) \rightarrow \exists y\left(\operatorname{similar}^{\prime}(y) \wedge \mathrm{i}-\operatorname{part}^{\prime}(x, y)\right)\right),\right. \\
\\
\left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left({ }^{*} \operatorname{red}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge f(v \sqsubseteq(\text { PHYS } \sqcap \operatorname{INFO}))\right)\right\rangle
\end{gathered}
$$

(28) says that every plurality of red books is an individual part of some plurality of similar things.

[^22]
### 3.3 Expletives

In all the 'book' sentences that we have seen so far, it has been clear how books are being individuated, and hence what the target truth conditions of any proposed analysis should be. But there are some instances where things are not as clear-cut as this, for example in the basic expletive sentence (29).
(29) There are three books.

It is commonplace in semantic theories to say that expletive 'there' denotes a trivial property or else is semantically inert. For instance, Barwise and Cooper (1981) have this expression denote the domain, while Carpenter (1998) assigns it to the unit type. In order to incorporate this idea into the current system a little more needs to be said, because we have constructions in addition to truth-conditional content. In (30), a construction has been given in such a way that it adds no new information to interpretation, by leaving the construction of its DP argument unchanged.

$$
\begin{equation*}
\llbracket \text { there }_{\text {expletive }} \rrbracket=\lambda D_{(e \rightarrow T) \rightarrow T} \cdot D\left(\lambda x_{e}\left\langle\text { true }, \Omega_{1}(D)\right\rangle\right) \tag{30}
\end{equation*}
$$

The interpretation of 'three books' is as shown in (31), repeated from (6).
(31) $\lambda B_{e \rightarrow T}\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge \pi_{1}(B(x)) \wedge \neg\left((\right.\right.\right.$ PHYS $\left.\left.\sqcap \operatorname{INFO}) \sqcup \Omega_{1}(B)\right) \operatorname{comp}(x)\right)$,

$$
\left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO}) \wedge \pi_{2}(B(v))(h)\right)\right\rangle
$$

If we interpret (29) as (30)[(31)], ${ }^{3}$ then we end up with (32).

$$
\begin{align*}
& \left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge \neg((\mathrm{PHYS} \sqcap \mathrm{INFO}) \sqcup(\mathrm{PHYS} \sqcap \mathrm{INFO})) \operatorname{comp}(x)\right),\right.  \tag{32}\\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq((\mathrm{PHYS} \sqcap \mathrm{INFO}) \sqcap(\mathrm{PHYS} \sqcap \mathrm{INFO}))\right)\right\rangle \\
= & \left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\mathrm{PHYS} \sqcap \mathrm{INFO}) \operatorname{comp}(x)\right),\right. \\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO})\right)\right\rangle
\end{align*}
$$

This is a problematic result. No utterance of (29) is interpreted by hearers as (32). Its compressibility statement is too weak. The set shown in (33), depicting the books

[^23]volume 1

| Notes from Underground |
| :---: |
| The Gambler |

volume 2
Figure 3.1: Is 'there are three books' true?
in the situation shown in Figure 3.1, has three members, no two of which are physically and informationally equivalent. So the prediction is that (29) would be true in that situation. But hearers will judge (29) false in the situation shown in Figure 3.1, because there are neither three physical volumes nor three informational books in that situation.
(33) $\left\{v_{1}+N f U, v_{2}+N f U, v_{2}+T G\right\}$

Note that this problem does not arise when we move on from bare existence claims to expletive sentences that have additional predicational information, as in (34).
(34) There are three books on the table.

Assuming that the interpretation of the prepositional phrase in (34) is as shown in (36), i.e. that it is a nominal modifier, ${ }^{4}$ the interpretation of (34) is as shown in (37).
(36) $\llbracket$ on the table】=

$$
\begin{aligned}
\lambda Q_{e \rightarrow T} \cdot \lambda x_{e} & \left\langle\left(*_{\text {on-table }^{\prime}(x)} \wedge \pi_{1}(Q(x))\right),\right. \\
& \left.\lambda g_{e \rightarrow R}\left(\exists h\left(\pi_{2}(Q(x))(h) \wedge g \sim_{x} h\right) \wedge g(x) \sqsubseteq\left(\operatorname{PHYS} \sqcup \Omega_{1}(Q)\right)\right)\right\rangle
\end{aligned}
$$

[^24]\[

$$
\begin{align*}
& \left\langle\exists x\left(|x| \geq 3 \wedge *{ }^{\text {on- }} \operatorname{table}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\text { PHYS }) \operatorname{comp}(x)\right),\right.  \tag{37}\\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left({ }^{*} \text { on-table }^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq \text { PHYS }\right)\right\rangle
\end{align*}
$$
\]

(37) says that the plurality in question is not physically compressible - i.e., it forces physical individuation. (33) is physically compressible, and so (34) is correctly predicted to be false in this case. Given the interpretation for 'by Dostoyevsky' shown in (39), the theory also makes the correct predicion that (38) requires that there be three books individuated informationally, as shown by its interpretation in (40).
(38) There are three books by Dostoyevsky.

$$
\begin{align*}
& \llbracket b y \text { Dostoyevsky }=  \tag{39}\\
& \qquad \begin{array}{l}
\lambda Q_{e \rightarrow T} \cdot \lambda x_{e}\left\langle\left(\operatorname{by}^{\prime}\left(d^{\prime}, x\right) \wedge \pi_{1}(Q(x))\right),\right. \\
\\
\left.\lambda g_{e \rightarrow R}\left(\exists h\left(\pi_{2}(Q(x))(h) \wedge g \sim_{x} h\right) \wedge g(x) \sqsubseteq\left(\operatorname{INFO} \sqcup \Omega_{1}(Q)\right)\right)\right\rangle \\
\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{by}^{\prime}\left(d^{\prime}, x\right) \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\operatorname{INFO}) \operatorname{comp}(x)\right),\right. \\
\left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{by}^{\prime}\left(d^{\prime}, v\right) \wedge * \operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq \operatorname{INFO}\right)\right\rangle
\end{array}
\end{align*}
$$

And the theory makes the correct predicion that (41) requires that there be three books that are both physically and informationally distinct, as shown in (42).
(41) There are three books by Dostoyevsky on the table.

$$
\begin{align*}
&\langle\exists x\left(|x| \geq 3 \wedge * \text { on- }^{\operatorname{table}}(x) \wedge * \operatorname{by}^{\prime}\left(d^{\prime}, x\right)\right.  \tag{42}\\
&\left.\wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\text { PHYS } \sqcup \mathrm{INFO}) \operatorname{comp}(x)\right), \\
&\left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left({ }^{*} \text { on-table }^{\prime}(v) \wedge * \operatorname{by}^{\prime}\left(d^{\prime}, v\right) \wedge * \operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq(\text { PHYS } \sqcup \mathrm{INFO})\right)\right\rangle
\end{align*}
$$

What causes a problem in the case of (29) is that the lexical entry for 'book' is effectively underspecified with respect to how books are to be individuated, which is precisely what makes copredication possible. In sentences like (34), (38) and (41), further specification comes from elsewhere in the sentence, delivering appropriate truth conditions. However, in sentences like (29) further specification is not forthcoming from anywhere else in the sentence, apparently.

I want to maintain that expletive 'there' has the lexical entry shown in (30), given that this derives the expected interpretations shown in (37), (40) and (42). With respect to (29), I see three possible ways in which the problematic prediction of having (32) as its interpretation can be addressed. The first is to make an adjustment to the lexical semantics of NSCs in their plural form. The second is to appeal to ellipsis, and the third to domain restriction.

## Revising the lexical entry for 'books'

Suppose that we had (43) as a lexical entry.

$$
\begin{gather*}
\llbracket \text { books } \rrbracket \lambda x_{e}\left\langle\left({ }^{*} \operatorname{book}^{\prime}(x) \wedge(\neg(\mathrm{PHYS}) \operatorname{comp}(x) \vee \neg(\mathrm{INFO}) \operatorname{comp}(x))\right),\right.  \tag{43}\\
\left.\lambda f_{e \rightarrow T} \cdot f(x) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO})\right\rangle
\end{gather*}
$$

(29) would now be interpreted as $(30)(\llbracket t h r e e \rrbracket[(43) \rrbracket)=(44)$, and so would be false in the situation shown in Figure 3.1, because (33) is both physically compressible and informationally compressible.

$$
\begin{align*}
& \left\langle\exists x \left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge(\neg(\mathrm{PHYS}) \operatorname{comp}(x) \vee \neg(\mathrm{INFO}) \operatorname{comp}(x))\right.\right.  \tag{44}\\
& \quad \wedge \neg(\mathrm{PHYS} \sqcap \mathrm{INFO}) \operatorname{comp}(x)), \\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO})\right)\right\rangle \\
& =\left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge(\neg(\mathrm{PHYS}) \operatorname{comp}(x) \vee \neg(\mathrm{INFO}) \operatorname{comp}(x))\right),\right. \\
& \left.\quad \lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO})\right)\right\rangle
\end{align*}
$$

The disadvantage of this approach (apart from being rather ad-hoc) is that (43) cannot be generated from the singular form of the noun by a general rule, and so this approach would require NSCs (or at least some of them) in their plural form to be lexically marked as exceptions semantically, rather than having the plural interpretation come from a general rule associated with plural morphology that applies the star. With this in mind it is worth considering other potential solutions.

## Ellipsis

The essence of this response is simply to bite the bullet and accept that the interpretation of (29) is as shown in (32). Of course, it would then have to be explained why speakers would judge (29) false in the situation shown in Figure 3.1. The suggestion would be that, in so doing, speakers are interpreting (29) as if it were an ellided form of a more informative sentence, like (37) or (40).
(29) does sound somewhat odd without some contextual help in determining the purpose for which it was uttered. It certainly requires more help of this kind than (37), (40) or (42) do. A plausible pragmatic explanation for this is that, in uttering (29), a speaker is violating some conversational priniciple relating to informativeness, such as Grice's (1975) maxim of quantity. Bare existence claims tend to have this property, as can also be seen from (45)-(47). In each of these cases the (a) sentence is somewhat odd-sounding in the same way as (29), but the (b) sentence is not.
(45) (a) There are twenty desks.
(b) There are twenty desks in the classroom.
(46) (a) There is a mug.
(b) There is a mug on the desk.
(47) (a) There are twelve houses.
(b) There are twelve houses on this street.

There are bare existence claims that are not odd-sounding in the same way, such as (48). However, this tends to be when making an existence claim of metaphysical import and hence there is not the same need of contextual help in determining the purpose of the utterance.
(48) There is a God.

Back to (29). If there is a (linguistic) context in which an utterance of (29) does not sound odd, it must be because the context supplies enough information to be
relied upon for the speaker not to be violating any conversational priniciples relating to informativeness. With this in mind, it is worth asking what kind of conversation (29) could felicitiously appear in. (49) is a minimal example.
(49) A: What is in that bag?

B: There are three books.

In this case, B's utterance is surely interpreted as expressing 'there are three books in that bag', and that sentence receives an interpretation that would not be true in the situation shown in Figure 3.1, the interpretation shown in (50) (where ' $b$ '' denotes the demonstrated bag).

$$
\begin{align*}
& \left\langle\exists x\left(|x| \geq 3 \wedge *_{\operatorname{in}^{\prime}}\left(x, b^{\prime}\right) \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\mathrm{PHYS}) \operatorname{comp}(x)\right)\right.  \tag{50}\\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left({ }^{*} \operatorname{in}^{\prime}\left(v, b^{\prime}\right) \wedge * \operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq \mathrm{PHYS}\right)\right\rangle
\end{align*}
$$

Any linguistic context providing enough information such that (29) does not sound odd will provide linguistic material for a more specific criterion of individuation than is present in (32). In contrast, (48) requires no such context.

## Domain restriction

One might well argue that the reason the B's utterance in (49) is interpreted in the given context as meaning the same thing 'there are three books in that bag' is because of restriction of the domain of quantification to objects in the bag, rather than actual ellipsis of the PP 'in that bag'.

Given this, it is worth exploring the possibility that domain restriction can also account for the fact that, in context, (29) is not interpreted as having the interpretation shown in (32).

According to a well-known theory of quantifier domain restriction due to Stanley and Szabó (2000), nominals co-habit a terminal node with a set-denoting contextual variable, ${ }^{5}$ which restricts the domain of quantification by intersection with the interpre-

[^25]tation of its co-habiting nominal expression. For example, according to this account, the reason that 'every bottle is open' can mean in context 'every bottle in the house is open' (and not 'every bottle in the world is open') is that it is interpreted as shown in (51), where $X$ is the variable, and $C(X)$ denotes the value that the context supplies to $X$.


If $C(X)$ denotes the set of objects in the house, then the sentence receives the intended interpretation.

This idea can be applied to the case of (29). In the present system, though, context would have to assign to the variable co-habiting a terminal node with 'books' something of type $e \rightarrow T$ rather than a set; and interpretation would have to proceed not by straightforward intersection of it with the meaning of the nominal, but rather by joining them together with the conjunction for expressions of type $e \rightarrow T$ in this system, ${ }^{6}$ as shown in (52).

$$
\begin{align*}
\lambda A_{e \rightarrow T} \cdot \lambda B_{e \rightarrow T} \cdot \lambda x_{e}\left\langle\left(\pi_{1}(A(x))\right.\right. & \left.\wedge \pi_{1}(B(x))\right)  \tag{52}\\
\lambda f_{e \rightarrow \mathcal{R}}(\exists g & \left(\pi_{2}(A(x))(g) \wedge f \sim_{x} g\right) \\
& \wedge \exists h\left(\pi_{2}(B(x))(h) \wedge f \sim_{x} h\right) \\
& \left.\left.\wedge f(x) \sqsubseteq\left(\Omega_{1}(A) \sqcup \Omega_{1}(B)\right)\right)\right\rangle
\end{align*}
$$

The expression of type $e \rightarrow T$ that provides the intuitive truth conditions in (49) is shown in (53); again, where $b^{\prime}$ denotes the demonstrated bag.

$$
\begin{equation*}
\lambda x_{e}\left\langle *^{*} \mathrm{in}^{\prime}\left(x, b^{\prime}\right), \lambda f_{e \rightarrow \mathcal{R}} f . f(x) \sqsubseteq \mathrm{PHYS}\right\rangle \tag{53}
\end{equation*}
$$

$(52)[(53)](\llbracket b o o k s \rrbracket)=(54)$, and therefore in the given context the utterance of (29) is

[^26]interpreted as $\llbracket$ there $e_{\text {expletive }} \rrbracket(\llbracket$ three $\rrbracket[(54)])=(55)$.
\[

$$
\begin{align*}
& \lambda x_{e}\left\langle\left(\operatorname{in}^{\prime}\left(x, b^{\prime}\right) \wedge * \operatorname{book}^{\prime}(x)\right), \lambda f_{e \rightarrow \mathcal{R}} f \cdot f(x) \sqsubseteq \text { PHYS }\right\rangle  \tag{54}\\
& \left\langle\exists x\left(|x| \geq 3 \wedge *^{*} \operatorname{in}^{\prime}\left(x, b^{\prime}\right) \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\text { PHYS }) \operatorname{comp}(x)\right),\right.  \tag{55}\\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(*_{\mathrm{in}^{\prime}}\left(v, b^{\prime}\right) \wedge * \operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq \mathrm{PHYS}\right)\right\rangle
\end{align*}
$$
\]

A possible objection to this idea is that it makes a leap of plausibility that Stanley and Szabó's (2000) theory does not. The objection would be that it is much easier to imagine how a set of entities could be made contextually salient than to imagine how a function of the form shown in (53) could be made contextually salient.

With respect to this objection, it's important to remember just what the essential ingredients of an expression like (53) are: (the characteristic function of) a set, and an individuation relation. These are the things that ultimately need to be made contextually salient. Furthermore, it is not the case that any arbitrary combination of the two could be made contextually salient. (53) has the property that every object in the set it determines (the things in the bag) bears the individuation relation it determines to itself-they are all physical objects. This can be a restriction on possible contextually-determined type $e \rightarrow T$ functions.

Can context make the criterion of individuation of an expletive sentence more restrictive when the sentence is not a bare existential claim like (29)? The answer seems to be yes. The interpretation of (38) shown in (40) is true is a situation in which three books by Dostoyevsky are bound in a single volume: it requires informational individuation. But now imagine that (38) is uttered in the context shown in (56).
(56) A: What is in that bag?

B: There are three books by Dostoyevsky.

It is my judgement that (38) would actually be false as uttered by B in the context shown in (56) if in fact what is in the bag is a single volume instantiating three books by Dostoyevsky. This would be explained if in the interpretation of $(56)$ the domain of quantification had been restricted by applying (52) to (53) to $\llbracket b o o k s \rrbracket$ in the interpretive
process. We therefore would end up with the interpretation shown in (57), which requires the three books in question to be both physically and informationally distinct.

$$
\begin{align*}
& \left\langle\exists x \left(|x| \geq 3 \wedge * \operatorname{in}^{\prime}\left(x, b^{\prime}\right) \wedge * \operatorname{by}^{\prime}\left(d^{\prime}, x\right) \wedge * \operatorname{book}^{\prime}(x)\right.\right.  \tag{57}\\
& \quad \wedge \neg(\text { PHYS } \sqcup \mathrm{INFO}) \operatorname{comp}(x)), \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(*_{\operatorname{in}^{\prime}}\left(v, b^{\prime}\right) \wedge * \operatorname{by}^{\prime}\left(d^{\prime}, v\right) \wedge * \operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq(\text { PHYS } \sqcup \mathrm{INFO})\right)\right\rangle
\end{align*}
$$

On either the ellipsis account or the domain restriction account there are many details to be worked out. One particular empirical question that remains is that of whether or not the problematic interpretation of (29), shown in (32), is in principle available. Only the lexical entry account rules this out entirely: it should be available on the ellipsis account given a conversational context in which it conforms to all conversational principles relating to informativeness (although such a conversational context is very difficult to imagine), and it should be available on the domain restriction account in which the restriction that context supplies does not change the relevant criterion of individuation. One disadvantage of the ellipsis account is that it is difficult to imagine how it could be extended to other cases in which further specification of how books are to be individuated does not appear to be forthcoming from elsewhere in the sentence, such as e.g. 'Fred requested three books'.

### 3.4 Singular nouns

So far we have only been considering plural quantifiers. But it has been contended that issues of individuation arise with singular quantifiers as well, in particular with 'every'. Consider (58) and (59).
(58) Bob defaced every book on the table.
(59) Bob memorised every book on the table.

Suppose that on the table there are two copies of The Language Instinct, and no other books, in a situation $s_{1}$. That is to say, the extension of 'book on the table' is as
shown in (60).
(60) $\llbracket b o o k$ on the table $\rrbracket^{s_{1}}=\left\{v_{1}+T L I, v_{2}+T L I\right\}$

If both (58) and (59) are true, then Bob defaced more books than he memorised. For (59) to be true, it must only(!) be the case that Bob memorised The Language Instinct, whereas for (58) to be true, it must be the case that Bob defaced both copies. But, given the assumptions made so far, this does not actually necessitate a special treatment of the universal quantifier. For instance, suppose we use the lexical entry for 'every' shown in (61), such that no construction-based information is incorporated into truth conditions.
(61) $\llbracket$ every $\rrbracket=\lambda P_{e \rightarrow T} \cdot \lambda Q_{e \rightarrow T} \cdot\left\langle\forall x\left(\pi_{1}(P(x)) \rightarrow \pi_{1}(Q(x))\right)\right.$,

$$
\left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(A(v)) \wedge \pi_{2}(A(v))(f) \wedge \pi_{2}(B(v))(f)\right)\right\rangle
$$

(61) has no compressibility statement, but it does not matter. If Fred memorised The Language Instinct, then, given the assumptions we have been making he memorised $v_{1}+T L I$ and $v_{2}+T L I$.

Of course, it does not follow from this that 'Bob memorised two books' is true. $v_{1}+T L I$ and $v_{2}+T L I$ may be distinct books, but they are informationally equivalent, which is the relevant consideration for the truth conditions of 'Bob memorised two books'. It is a welcome feature of this treatment that it captures the invalidity of the argument shown in (62).

Bob memorised every book on the table.
There are (at least) two books on the table.
Bob memorised (at least) two books.
The reason for this is that, although the argument form shown in (63) is valid, (62) actually follows the argument form shown in (64), which is invalid. The interpretation assumed for 'on the table' is as shown in (36).

$$
\begin{align*}
& \forall x\left(\left(\operatorname{on-table}^{\prime}(x) \wedge \operatorname{book}^{\prime}(x)\right) \rightarrow \operatorname{memorise}^{\prime}\left(b^{\prime}, x\right)\right)  \tag{63}\\
& \exists x\left(|x| \geq 2 \wedge *{ }^{*} \operatorname{on-table}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x)\right) \\
& \exists x\left(|x| \geq 2 \wedge * \operatorname{memorise}^{\prime}\left(b^{\prime}, x\right) \wedge * \operatorname{book}^{\prime}(x)\right) \\
& \forall x\left(\left(\operatorname{on-table}^{\prime}(x) \wedge \operatorname{book}^{\prime}(x)\right) \rightarrow \operatorname{memorise}^{\prime}\left(b^{\prime}, x\right)\right)  \tag{64}\\
& \exists x\left(|x| \geq 2 \wedge * \text { on-table }^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\text { PHYs }) \operatorname{comp}(x)\right) \\
& \exists x\left(|x| \geq 2 \wedge * \operatorname{memorise}^{\prime}\left(b^{\prime}, x\right) \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\text { INFO }) \operatorname{comp}(x)\right)
\end{align*}
$$

By the same logic, the argument shown in (65) is valid, as it has the form shown in (66). This is another welcome result of the current system.

Bob defaced every book on the table.
There are (at least) two books on the table.
Bob defaced (at least) two books.

$$
\begin{align*}
& \forall x\left(\left(\text { on-table }^{\prime}(x) \wedge \operatorname{book}^{\prime}(x)\right) \rightarrow \operatorname{deface}^{\prime}\left(b^{\prime}, x\right)\right)  \tag{66}\\
& \exists x\left(|x| \geq 2 \wedge *{ }^{*} \text { on-table }^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\text { PHYS }) \operatorname{comp}(x)\right) \\
& \exists x\left(|x| \geq 2 \wedge * \operatorname{deface}^{\prime}\left(b^{\prime}, x\right) \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\text { PHYS }) \operatorname{comp}(x)\right)
\end{align*}
$$

Issues of individuation do not seem to arise except in cases of numerical quantification. No wrong predictions emerge from simply taking the domain of quantification to include complex objects in (58), (59) and cases like these. That is to say, if we simply view $\llbracket b o o k \rrbracket$ as the set of complex objects made up of a part that is a physical book and a part that is an informational book that instantiates it, then no individuation conundrums emerge as they do for sentences involving numerical quantification.

### 3.4.1 Objections

I can anticipate two possible objections. ${ }^{7}$ The first concerns the predictions that this system makes in certain cases of ellipsis, and the second concerns ambiguities that arise with the verb 'read'.

[^27]
## The ellipsis objection

Consider a situation in which there are two copies of some informative book (say, The Language Instinct again), one (hardback) which is heavy, and one (paperback) which is light. Then imagine that someone utters (67).
(67) One informative book is heavy, and one is light.

It is my judgement that (67) is false, or at least somehow deviant, in this situation. However, the account developed so far does not predict this. Assuming that (however ellipsis construal works) what is interpreted in (67) is 'one informative book is heavy, and one informative book is light', then the interpretation predicted would be as shown in (68).

$$
\begin{align*}
& \left\langle\exists x\left(\operatorname{book}^{\prime}(x) \wedge \operatorname{inform}^{\prime}(x) \wedge \operatorname{heavy}^{\prime}(x)\right) \wedge \exists y\left(\operatorname{book}^{\prime}(y) \wedge \operatorname{inform}^{\prime}(y) \wedge \operatorname{light}^{\prime}(y)\right),\right.  \tag{68}\\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{book}^{\prime}(v) \wedge \operatorname{inform}^{\prime}(v) \wedge f(v) \sqsubseteq(\operatorname{PHYS} \sqcap \operatorname{INFO})\right)\right\rangle
\end{align*}
$$

(68) is true in the situation described, given the other assumptions made so farthere is a book that is informative and heavy, and there is a book that is informative and light. The the system does not predict the judgement that I give for (67).

The reason for this, I think is that (67) has the implication that 'there are two informative books'-which the account developed so far does predict to be false in this situation, as a working through of previous examples with suitable adaptations will show. Given the theory of expletives outlined in Section 3.3, we end up with an interpretation of (69) as shown in (70).
(69) There are two informative books.

$$
\begin{gather*}
\left\langle\exists x\left(|x| \geq 2 \wedge * \operatorname{inform}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\text { INFO }) \operatorname{comp}(z)\right),\right.  \tag{70}\\
\left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(*^{*} \operatorname{inform}^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq \operatorname{INFO}\right)\right\rangle
\end{gather*}
$$

If we think about the form of (67), we can see that this kind of conundrum is actually not special to copredication sentences. For instance, consider (71).
(71) One student is tall, and one is freckled.
(71) is likewise interpreted as false, or at least deviant, in a situation in which there is just one student, who is both tall and freckled.

Perhaps, then, there is a common explanation for the two cases. My claim is that (67) is strictly speaking true in the situation involving two copies of The Language Instinct, and that (71) is strictly speaking true in the situation involing just one (tall and freckled) student, but that both sentences are somehow degraded on pragmatic grounds in their respective situations, because of some strong implicature that each of them has, which is false in the situation in question. In the case of (67), the implicature is 'there are two informative books'. In the case of (71), the implicature is 'there are two students'.

## The ambiguity of 'read'

The second objection concerns the ambiguity of sentences like (72).
(72) Susannah read every book in the library.

On one reading, (72) is true if Susannah read at least one copy of every book in the library. On the other reading, (72) can only be true if Susannah read every copy of every book in the library, which normally would require reading some informational books several times. Suppose that the library (only) has the two copies of The Language Instinct mentioned above. If Susannah took out one copy $\left(v_{1}\right)$ and read it, but never touched the other one $\left(v_{2}\right)$, then there is a reading of (72) on which it is true, and also a reading on which it is false.

The obvious way to address this issue is to attribute ambiguity to the word 'read'. We can leave the treatment of quantification as it is, and locates the ambiguity in the extension of the predicate when it comes to complex objects.

The situation is that we have the two copies of The Language Instinct mentioned above. The model in this situation $s_{2}$ is as follows:
(73) Domain $\left(s_{2}\right)=\left\{s^{\prime}, v_{1}, v_{2}, T L I, v_{1}+T L I, v_{2}+T L I\right\}$
(74) $\llbracket$ book in the library $\rrbracket^{s_{2}}=\left\{v_{1}+T L I, v_{2}+T L I\right\}$
(75) $\llbracket$ informative $\rrbracket^{s_{2}}=\left\{T L I, v_{1}+T L I, v_{2}+T L I\right\}$
(76) $\llbracket h e a v y \rrbracket^{s_{2}}=\left\{v_{1}, v_{1}+T L I\right\}$
$\llbracket l i g h t \rrbracket^{s_{2}}=\left\{v_{2}, v_{2}+T L I\right\}$
$v_{1}$ is itself an object in the domain of discourse, and is in the extension of 'heavy', but it is not in the extension of 'book in the library'. By virtue of the fact that $v_{1}$ is in the extension of 'heavy', $v_{1}+T L I$ is in the extension of 'heavy'. TLI is an object in the domain of discourse, and is in the extension of 'informative', but it is not in the extension of 'book in the library'. By virtue of the fact that TLI is in the extension of 'informative', $v_{1}+T L I$ and $v_{2}+T L I$ are in the extension of 'informative'.

The approach to the ambiguity of (72) proceeds as follows. There is a meaning of 'read' that is like this: modulo construction, it is a relation between individuals and informational objects. Let's call this 'read ${ }_{1}$ '. On this view, if Susannah has read The Language Instinct, then $T L I$ is in $\llbracket \lambda_{1}\left[\right.$ Susannah $\left.^{\operatorname{read}}{ }_{1} t_{1}\right] \rrbracket$, and therefore so are all those composites of which $T L I$ is a part - so in the domain in question, $v_{1}+T L I$ and $v_{2}+T L I$. We would therefore have (78).

$$
\begin{equation*}
\llbracket \lambda_{1}\left[\text { Susannah } \text { read }_{1} t_{1}\right] \rrbracket^{s_{2}}=\left\{\mathrm{TLI}, v_{1}+T L I, v_{2}+T L I\right\} \tag{78}
\end{equation*}
$$

It follows that (72) is true in the situation being described, because (74) $\subseteq(78)$.
To get the reading of (72) on which it is not true in the situation being described, we must posit the existence of a second verb, ' $\mathrm{read}_{2}$ ', which takes complex physical+informational objects in its extension directly, i.e. not via inheritance from their informational parts in the sense outlined above. On this view, $\llbracket \mathrm{read}_{2} \rrbracket$ is (modulo construction) a relation between individuals and physical+informational composite objects. Therefore, in $s^{2}$ the only object that Susannah $\operatorname{read}_{2}$ is $v_{1}+T L I$. TLI by itself is not in $\llbracket \lambda_{1}$ [Susannah read $\left._{2} t_{1}\right] \rrbracket$ - no simple object is-and $v_{2}+T L I$ is not in $\llbracket \lambda_{1}\left[\right.$ Susannah read $\left.{ }_{2} t_{1}\right] \rrbracket$, because when Susannah read The Language Instinct it did
not involve $v_{2}$. We would therefore have (79).

$$
\begin{equation*}
\llbracket \lambda_{1}\left[\text { Susannah } \text { read }_{2} t_{1}\right] \rrbracket^{s^{2}}=\left\{v_{1}+T L I\right\} \tag{79}
\end{equation*}
$$

Because $(74) \nsubseteq(79)$, on this reading (72) is not true in situation $s_{2}$.

### 3.5 The definite article

In this context, the definite article deserves a special mention because it is, in a sense, both singular and numerical. As a first pass, we might attempt an essentialy Russellian treatment, and suppose that the logical form of 'the $A B$ ' is 'there is exactly one $A$, and every A is B'. That would be the treatment shown in (80).

$$
\begin{equation*}
\llbracket \text { the } A B \rrbracket=\operatorname{and}^{\prime}\left(\llbracket \text { there } e_{\text {expletive }} \rrbracket(\llbracket \text { exactly one } \rrbracket(\llbracket A \rrbracket))\right)(\llbracket \text { every } \rrbracket(\llbracket A \rrbracket)(\llbracket B \rrbracket)) \tag{80}
\end{equation*}
$$

However, as can be gathered from the discussion in Section 3.3, a bit more needs to be said about the 'there is' part of this. We want how the As are individuated to be determined by $\llbracket A \rrbracket$ and $\llbracket B \rrbracket$, not just $\llbracket A \rrbracket$.

With this in mind, the definite article can be given the lexical entry shown in (81). Here, ${ }^{*} P$ is shorthand for ${ }^{*}\left(\lambda x_{e} . \pi_{1}(P(x))\right)$ : a method for taking singular predicates to plural predicates.
(81) $\llbracket$ the $\rrbracket=\lambda P_{e \rightarrow T} \cdot \lambda Q_{e \rightarrow T}$

$$
\begin{aligned}
& \left\langle\exists x\left(\pi_{1}(P(x))\right) \wedge \neg \exists x\left(|x|>1 \wedge \hat{*} P(x) \wedge \neg\left(\Omega_{1}(P) \sqcup \Omega_{1}(Q)\right) \operatorname{comp}(x)\right)\right. \\
& \quad \wedge \forall y\left(\pi_{1}(P(y)) \rightarrow \pi_{1}(Q(y))\right) \\
& \left.\quad \lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(P(v)) \wedge \pi_{2}(P(v))(f) \wedge \pi_{2}(Q(v))(f)\right)\right\rangle
\end{aligned}
$$

On this basis, (82)-(84) would have the interpretations shown in (85)-(87) respectively.
(82) The book is heavy.
(83) The book is informative.
(84) The informative book is heavy.

$$
\begin{align*}
& \left\langle\exists x\left(\operatorname{book}^{\prime}(x)\right) \wedge \neg \exists x\left(|x|>1 \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\mathrm{PHYS}) \operatorname{comp}(x)\right)\right.  \tag{85}\\
& \wedge \forall y\left(\operatorname{book}^{\prime}(y) \rightarrow \operatorname{heavy}^{\prime}(y)\right), \\
& \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq(\operatorname{PHYS} \sqcap \operatorname{INFO})\right)\right\rangle  \tag{86}\\
& \left\langle\exists x\left(\operatorname{book}^{\prime}(x)\right) \wedge \neg \exists x\left(|x|>1 \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\mathrm{INFO}) \operatorname{comp}(x)\right)\right. \\
& \\
& \wedge \forall y\left(\operatorname{book}^{\prime}(y) \rightarrow \operatorname{inform}^{\prime}(y)\right),  \tag{87}\\
& \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq(\text { PHYS } \sqcap \operatorname{INFO})\right)\right\rangle \\
& \left\langle\exists x\left(\operatorname{inform}^{\prime}(x) \wedge \operatorname{book}^{\prime}(x)\right)\right. \\
& \\
& \wedge \neg \exists x\left(|x|>1 \wedge * \operatorname{inform} \wedge^{*} \operatorname{book}^{\prime}(x) \wedge \neg(\text { PHYS } \sqcup \mathrm{INFO}) \operatorname{comp}(x)\right) \\
& \\
& \wedge \forall y\left(\operatorname{book}^{\prime}(y) \rightarrow \operatorname{heavy}^{\prime}(y)\right), \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq(\operatorname{PHYS} \sqcap \mathrm{INFO})\right)\right\rangle
\end{align*}
$$

As desired, (85) requires that there be exactly one book-individuated-physically, and (86) requires that there be exactly one book-individuated-informationally. However, some of the results in the copredication case are problematic. (87) is true in a situation in which we have a single (heavy) physical volume instantiating two informational books, both of which are informative. In that case we have the books shown in (88).

$$
\begin{equation*}
\left\{v_{1}+i_{1}, v_{1}+i_{2}\right\} \tag{88}
\end{equation*}
$$

(87) is true in the situation described because there is no plurality formed from members of (88) of cardinality greater than 1 that is not physically compressible. But intuitively we would probably say that (84) is not true in that situation. The interpretation shown in (87) does make the requirement that either every informative book is physically equivalent to every other (which is the situation in (88)), or every informative book is physically equivalent to every other. ${ }^{8}$ This underspecification in individuation seems to be acceptable in cases like (89) where the physical and information-selecting predicates are introduced in a coordinate structure; but for (84) it does seem to be the case that informational individuation is required. ${ }^{9}$

[^28](89) The book is informative and heavy.

Informational individuation could be enforced in the case of (84) be applying (80) more straightforwardly, i.e. by having how the As are individuated be determined just by $\llbracket A \rrbracket$ and not $\llbracket B \rrbracket$ as well. This would give the lexical entry shown in (90), from which the interpretation of (84) shown in (91) would follow.

$$
\begin{align*}
& \lambda P_{e \rightarrow T} \cdot \lambda Q_{e \rightarrow T}  \tag{90}\\
& \qquad \quad \exists x\left(\pi_{1}(P(x))\right) \wedge \neg \exists x\left(|x|>1 \wedge \hat{*} P(x) \wedge \neg\left(\Omega_{1}(P)\right) \operatorname{comp}(x)\right) \\
& \wedge \forall y\left(\pi_{1}(P(y)) \rightarrow \pi_{1}(Q(y))\right), \\
& \left.\quad \lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(P(v)) \wedge \pi_{2}(P(v))(f) \wedge \pi_{2}(Q(v))(f)\right)\right\rangle \\
& \left\langle\exists x\left(\operatorname{inform}^{\prime}(x) \wedge \operatorname{book}^{\prime}(x)\right)\right.  \tag{91}\\
& \wedge \neg \exists x\left(|x|>1 \wedge *_{\text {inform }} \wedge * \operatorname{book}^{\prime}(x) \wedge \neg(\text { INFO }) \operatorname{comp}(x)\right) \\
& \wedge \forall y\left(\operatorname{book}^{\prime}(y) \rightarrow \operatorname{heavy}^{\prime}(y)\right), \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq(\text { PHYS } \sqcap \mathrm{INFO})\right)\right\rangle
\end{align*}
$$

However, adopting (90) universally would generate bad predictions in the noncopredication cases like (82) and (83), and in fact for any sentence of the form 'the book A'.

For the case of (84), it seems that we want the ind-relation for the sentence to be determined solely by the construction of the complement of the definite article without involving that of the VP. There are other cases where similar judgements arise, which is the topic of Section 3.6.1.

### 3.6 Some unresolved issues

### 3.6.1 Are the requirements too strong?

The system developed in this chapter and the last predicts the interpretation for (92) shown in (93).
(92) Two heavy books are informative.

$$
\begin{align*}
& \left\langle\exists x \left(|x| \geq 2 \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{inform}^{\prime}(x) \wedge \neg(\mathrm{PHYS} \sqcup \mathrm{INFO}) \operatorname{comp}(x)\right.\right.  \tag{93}\\
& \left.\left.\lambda f_{e \rightarrow R} \cdot f(x) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO})\right)\right\rangle
\end{align*}
$$

(93) would be false in a situation in which there are two heavy copies of the same informational book, which is informative (because the plurality consisting of those two copies is informationally compressible). While I am not outraged by this prediction, many people find it to be extremely dubious. It seems that for many speakers, (92) is interpreted in such as way that books are individuated in the way specified by the nominal modifier, without taking the ind-relation contributed by the VP into consideration. There are similarities between this case and that of (84), in that in both cases structural closeness appears to be a factor. This observation is reinforced by the distinction between (92) and (94). (92) and (94) are predicted to have identical truth conditions (although the sentential constructions are slightly different), but in the case of (94) this prediction is much less controversial. That is to say, in the situation described above in which there are two heavy copies of the same informational book, people are generally happier with the prediction that (94) is false than with the prediction that (92) is false.
(94) Two books are heavy and informative.

That is not to say that it must be that the connection between structural closeness and these judgements is necessarily direct. It may be that the contribution of an expression's construction to a sentence's compressibility statement is in some measure affected by pragmatic interference. I leave the source of variability in judgements for (92) and (94) to future research.

### 3.6.2 The nature of complex objects

At the outset of Chapter 2 I introduced the following assumptions about NSCs, and promised to revisit them later on:
(A 1) An NSC has in its extension a set of objects, each member of which is made up of two parts.
(A 2) Any property that holds of one of those parts hold of the object as a whole.

These assumptions have been adequate for the examples examined in this chapter and the last, and crucially are sufficient for criteria of individuation to be determined compositionally. But there are certain cases in which the predictions that they make are questionable. In this section I will review some of those cases and offer some suggestions for refining these assumptions to deal with them. The discussion in the section is necessarily tentative and the discussion in subsequent chapters is not based on suggestions made here. ${ }^{10}$

Consider a situation $s_{3}$ in which one heavy physical volume $p_{1}$ instantiates five informational books $i_{1}-i_{5}$, one (and only one) of which Fred masters: $i_{1}$. The system developed so far predicts (95) to be true, because, based on (A 1)-(A 2), $p_{1}+i_{1}$ is a book that is heavy and that Fred mastered.
(95) Fred mastered a heavy book.

This predicion is problematic. Intuitively, one would say that Fred has to have mastered all of $i_{1}-i_{5}$ in order for (95) to be true in $s_{3}$. Plausibly, this requirement comes from the lexical semantics of 'master'. In contrast, if, in the same situation, only $i_{1}$ is informative, then (96) would be true, as predicted by (A 1)-(A 2).
(96) A heavy book is informative.

This distinction between 'informative' and 'master' invites the following idea for revising (A 2): for someone to master a book (for example) $p+i$, that person has to master not only $i$ but also any other informational object that is also instantiated by $p$. In contrast, for $p+i$ to be informative, it suffices for $i$ to be informative, irrespective of what else $p$ instantiates.

The suggested distinction could be formalised by means of different meaning postulates for 'informative' and 'master', which would be supposed to describe how the

[^29]properties of complex objects are determined by the properties of their parts, and hence replace the informal description given in (A 2). To do this, we would need to make explicit reference to two different kinds of parthood relation:

- As outlined in Section 2.3.1, we have the sense in which the (complex) object $p+i$ is part of the plurality $p+i \oplus p$ : an $\mathrm{i}($ ndividual)-part relation. According to this relation, $p$ is an i-atom and so is $p+i$, which just means that they are not pluralities.
- Now, additionally, we need to be able to talk about the sense in which $p$ is part of $p+i$, i.e. the relation of parts of complex objects to complex objects: a m (aterial)-part relation. According to this relation, $p$ is an m -atom but $p+i$ is not. ${ }^{11}$

The suggested meaning postulates, then, are shown in (97)-(98).

$$
\begin{equation*}
\square \forall y\left(\exists x\left(\mathrm{~m}-\operatorname{part}^{\prime}(x, y) \wedge \operatorname{inform}^{\prime}(x)\right) \leftrightarrow \operatorname{inform}^{\prime}(y)\right) \tag{97}
\end{equation*}
$$

' $y$ is informative if and only if there is some m-part of $y$ that is informative'

$$
\begin{align*}
& \square \forall x \forall y\left(\left(\forall z \left(\text { phys-equiv }{ }^{\prime}(z, y) \rightarrow\right.\right.\right.(\neg \operatorname{info-equiv}(z, z) \vee  \tag{98}\\
&\left.\left.\exists u\left(\text { m-part}(u, z) \wedge \operatorname{master}^{\prime}(x, u)\right)\right)\right) \\
&\left.\left.\wedge \exists v\left(\text { m-part }^{\prime}(v, y) \wedge \operatorname{master}^{\prime}(x, v)\right)\right) \leftrightarrow \operatorname{master}^{\prime}(x, y)\right)
\end{align*}
$$

' $x$ masters $y$ if and only if there is an m-part of $y$ that $x$ masters and everything that is physically equivalent to $y$ either has no informational m-part or has an m-part that $x$ masters'
(97) is simply an instantiation of (A 2), whereas (98) marks a change from this general statement (A 2). If (98) were to be adopted, then (95) would no longer be predicted to be true in the situation $s_{3}$. Under these circumstances, $\operatorname{book}^{\prime}\left(p_{1}+i_{1}\right)$ would still be true, as would $\operatorname{heavy}^{\prime}\left(p_{1}+i_{1}\right)$, but master $\left(f^{\prime}, p_{1}+i_{1}\right)$ would not. This is because

[^30]$p_{1}+i_{1}$ is physically equivalent to (for example) $p_{1}+i_{2}$, which has an informational part that Fred did not master.

There is a problem with this approach, though, in that it seems that the requirements imposed by (98) would be too strict. Given that Fred mastered $i_{1}$, we would want to say that (99) is true in $s_{3}$. But given (98), (99) is now predicted to be false, as any book of which $i_{1}$ is an m-part will be physically equivalent to some book that has an informational m-part that Fred did not master.

## (99) Fred mastered a book

So on the one hand there are reasons for thinking that 'master' is not different from 'informative' in terms of how properties of complex objects are determined by properties of their informational parts, based on intuitions about (99); while on the other hand, there are reasons for thinking that 'master' is different from 'informative', as described in (97)-(98), based on intuitions about (95). The difference between the cases appears to be simply that (95) involves copredication, while (99) does not.

As an attempt at enforcing this difference, we can try revising (A 1) as well as (A 2), as shown in (A 3) and (A 4) respectively.
(A 3) The set of books is the union of the following sets:
(a) The set of complex objects of the form $p+i_{1}+\ldots+i_{n}$, where $p$ is a physical book and $\left\{i_{1}, \ldots, i_{n}\right\}$ is the set of informational books instantiated by $p$, and
(b) The set of complex objects of the form $p_{1}+\ldots+p_{n}+i$, where $i$ is an informational book and $\left\{p_{1}, \ldots, p_{n}\right\}$ is the set of physical books that instantiate $i$.
(A 4) Different properties project differently from parts to the whole. In particular, all informational m-parts of a complex object have to be mastered for the complex object to be mastered. But for a complex object to be heavy, or to be informative, it is sufficient for one of its m-parts to be heavy or informative respectively.

According to (A 3), in the situation $s_{3}$ the books are $p_{1}+i_{1}+i_{2}+i_{3}+i_{4}+i_{5}$ (the physical volume $p_{1}$ plus all the informational books that it instantiates), $p_{1}+i_{1}, p_{1}+i_{2}$, $p_{1}+i_{3}, p_{1}+i_{4}$ and $p_{1}+i_{5}$ (in each case, $i_{n}$ plus all the physical books that instantiate it).

The difference between 'master' and 'informative' can now be expressed in the difference between what each contributes to the properties of a complex object with more than one informational part. Intuitively, if a book has several informational parts and only one of them is informative, that is enough for the book as a whole to be informative. On the other hand if a book has several informational parts then Fred has to have mastered all of them to have mastered the book as a whole.

$$
\begin{equation*}
\square \forall y\left(\exists x\left(\mathrm{~m}-\operatorname{part}^{\prime}(x, y) \wedge \operatorname{inform}^{\prime}(x)\right) \leftrightarrow \operatorname{inform}^{\prime}(y)\right) \tag{100}
\end{equation*}
$$

' $y$ is informative if and only if there is some m-part of $y$ that is informative'

$$
\begin{align*}
\square \forall x \forall y( & \forall z\left(\left(\operatorname{info-equiv}^{\prime}(z, z) \wedge \mathrm{m}-\operatorname{part}^{\prime}(z, y) \wedge \mathrm{m}^{-\operatorname{atom}^{\prime}}(z)\right) \rightarrow \operatorname{master}^{\prime}(x, z)\right)  \tag{101}\\
& \left.\leftrightarrow \operatorname{master}^{\prime}(x, y)\right)
\end{align*}
$$

' $x$ masters $y$ if and only if $x$ masters every informational atomic m-part of $y$ '
(100) is the same as (97). (101) guarantees that if Fred mastered only $i_{1}$, then $\operatorname{master}^{\prime}\left(f^{\prime}, p_{1}+i_{1}+i_{2}+i_{3}+i_{4}+i_{5}\right)$ is false, because $i_{2}-i_{5}$ are all informational atomic parts of $p_{1}+i_{1}+i_{2}+i_{3}+i_{4}+i_{5}$ that Fred did not master.

Alone, this modification would not be sufficient to block (95) being true in $s_{3}: p_{1}+i_{1}$ is still a book, heavy and something that Fred mastered. What we would need is some way to exclude $p_{1}+i_{1}$ from consideration in the case of (95) but not in the case of (99). The fact that (95) involves copredication but (99) does not is marked in the difference between the constructions of the interpretations of the two sentences. In order to use this information to exclude $p_{1}+i_{1}$ from consideration in the case of (95) but not in the case of (99) some additional requirement would have to be encoded in the lexical entry
for the determiner. An attempt to do this is shown in (102).

$$
\begin{align*}
& \llbracket a \rrbracket=\lambda P_{e \rightarrow T} \cdot \lambda Q_{e \rightarrow T}\left\langle\exists x \left(\pi_{1}(P(x)) \wedge \pi_{1}(Q(x))\right.\right.  \tag{102}\\
& \wedge \neg \exists y\left(x \neq y \wedge \pi_{1}(P(x)) \wedge m-\operatorname{part}^{\prime}(x, y)\right. \\
& \left.\left.\wedge\left(\Omega_{1}(P) \sqcup \Omega_{1}(Q)\right)(x)(y) \wedge \neg \pi_{1}(Q(x))\right)\right), \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(P(v)) \wedge \pi_{2}(P(v))(f) \wedge \pi_{2}(P(v))(f)\right)\right\rangle
\end{align*}
$$

If (102) were to be used, then the interpretations for (95) and (99) would be as shown in (103) and (104) respectively.

$$
\begin{align*}
& \left\langle\exists x \left(\operatorname{book}^{\prime}(x) \wedge \operatorname{heavy}^{\prime}(x) \wedge \operatorname{master}^{\prime}\left(f^{\prime}, x\right)\right.\right.  \tag{103}\\
& \wedge \neg \exists y\left(x \neq y \wedge \operatorname{book}^{\prime}(x) \wedge \operatorname{heavy}^{\prime}(x) \wedge \operatorname{m-part}^{\prime}(x, y)\right. \\
& \left.\left.\wedge\left(\operatorname{phys-equiv}^{\prime}(x, y) \vee \operatorname{info-equiv}^{\prime}(x, y)\right) \wedge \neg \operatorname{master}^{\prime}\left(f^{\prime}, x\right)\right)\right), \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{book}^{\prime}(v) \wedge \operatorname{heavy}^{\prime}(v) \wedge f(v) \sqsubseteq(\text { PHYS } \sqcap \operatorname{INFO})\right)\right\rangle
\end{align*}
$$

'There is some heavy book that Fred mastered, which is not an m-part of any other heavy book to which it is physically or informationally equivalent and which Fred did not master'.

$$
\begin{align*}
& \left\langle\exists x \left(\operatorname{book}^{\prime}(x) \wedge \operatorname{master}^{\prime}\left(f^{\prime}, x\right)\right.\right.  \tag{104}\\
& \quad \wedge \neg \exists y\left(x \neq y \wedge \operatorname{book}^{\prime}(x) \wedge \mathrm{m}-\operatorname{part}^{\prime}(x, y)\right. \\
& \left.\left.\wedge \operatorname{info-equiv}^{\prime}(x, y) \wedge \neg \operatorname{master}^{\prime}\left(f^{\prime}, x\right)\right)\right), \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{book}^{\prime}(v) \wedge f(v) \sqsubseteq(\text { PHYS } \sqcap \operatorname{INFO})\right)\right\rangle
\end{align*}
$$

'There is some book that Fred mastered, which is not an m-part of any other book to which it is informationally equivalent and which Fred did not master'.

So in $s_{3}, p_{1}+i_{1}$ would be excluded from consideration in the case of (95) because it is an m-part of a heavy book to which it is physically equivalent and which Fred did not master: $p_{1}+i_{1}+i_{2}+i_{3}+i_{4}+i_{5}$. Therefore, (95) would be predicted to be false in $s_{3}$. However, $p_{1}+i_{1}$ would not be excluded from consideration in the case of (99), because it is not an m-part of any book to which it is informationally equivalent and which Fred did not master (since it is not informationally equivalent to $p_{1}+i_{1}+i_{2}+i_{3}+i_{4}+i_{5}$ ). ${ }^{12}$

[^31]Therefore, (73) would be predicted to be true in $s_{3}$.
More investigation is needed in order to see how this kind of system would interact with the treatment of plurality given in the rest of this chapter. The definition of compressibility would have to be revised, probably along the lines shown in (105).
(105) $(R) \operatorname{comp}(x) \stackrel{\text { def }}{=} \exists y \exists z\left(y \neq z \wedge \mathrm{i}-\operatorname{part}^{\prime}(y, x) \wedge \mathrm{i}-\operatorname{part}^{\prime}(z, x) \wedge \mathrm{i}-\operatorname{atom}^{\prime}(y) \wedge \mathrm{i}-\operatorname{atom}^{\prime}(z)\right.$

$$
\left.\wedge \exists v\left(\mathrm{~m}-\operatorname{part}^{\prime}(v, y) \wedge R(v, z)\right)\right)
$$

This definition would guarantee for example that the plurality $p_{1}+p_{2}+i_{1} \oplus p_{2}+i_{2}$ is physically compressible, because $p_{2}+i_{1}$ is an m-part of $p_{1}+p_{2}+i_{1}$ and $p_{2}+i_{2}$ is physically equivalent to $p_{2}+i_{1}$.

However, it is an open question as to whether or not the definition shown in (105) coupled with (A 3)-(A 4) will work as well as the definition given in Chapter 2 coupled with (A 1)-(A 2) does when it comes to predicting the truth conditions of numerically quantified copredication sentences. There is also the question of whether or not, given (A 3)-(A 4), plural determiners would have to incorporate something like the 'not an m-part of any other $x$ ' requirement from (103) into their meanings in addition to compressibility statements. I leave all these questions to future research.
and $b$ if and only if they both have a physical part and the physical part of $a$ is identical to the physical part of $b$, we would now have to say that it is the relation that holds between (singular) objects $a$ and $b$ if and only if they both have at least one physical part each and there is a one-to-one correspondence between the physical parts of $a$ and the physical parts of $b$.

## Chapter 4

## Comparison with other theories

In this chapter I compare the formal approach to copredication presented in Chapter 2 with other prominent approaches in the literature, focusing primarily on issues of quantification and individuation.

### 4.1 Asher's Type Composition Logic

According to the theory described by Asher (2011), nouns supporting copredication do not denote composite objects, but rather 'dot objects' (Pustejovsky, 1995)—that is, objects that can be conceptualised in different ways or, in his terminology, viewed under different 'aspects'. Asher describes his view of 'aspects' as follows:

Tropes [aspects] are thus not part of objects; rather it's the other way around - the object is a constituent of a trope or aspect. The sum of the tropes of an object is not identical to that object, since each trope contains the object together with some property that it has. (Asher, 2008, p. 165)

Given the way I have defined aspects, the sum of an object's aspects cannot be identical to the object itself (since each aspect contains the object together with some property that it has). A lunch object is wholly an event (under one aspect) and wholly food (under another aspect). (Asher, 2011, pp. 149-150)

To get a handle on this way of thinking, let's imagine a situation in which we have three (informational) books printed in a single (physical) volume. Concretely,


Figure 4.1: A trilogy, under physical and informational criteria of individuation respectively
let's imagine that we have a volume containing the Dostoyevsky novellas Notes from Underground, The Gambler and The Double. In Asher's view, this situation can be conceptualised in two different ways, as indicated in Figure 4.1. The informational objects are listed as ' $N f U^{\prime}$ ' ' $T G$ ' and ' $T D$ ' and the single physical volume is 'vol 1 '.

Conceptualised in one way, shown on the left hand side of the figure, there is one book. Conceptualised in another way, shown on the right hand side of the figure, there are three books. In both cases $a \longrightarrow b$ is supposed to indicate that $b$ is an aspect of $a$. In Asher's metalanguage formulae, this is represented as ' $\mathrm{o}-\mathrm{elab}^{\prime}(b, a)$ ' $-b$ is an 'object elaboration' of $a$.

Although the number of aspects of each type is the same under the two conceptualisations, then, the number of books is different. That means that changing conceptualisation can change the domain of quantification:

The bare objects of $\bullet$ type [e.g. books] are counted and individuated relative to one of their constituent types [...] We should relativize the domain of quantification in a world to a criterion of individuation (Asher, 2011, pp. 157-159)

For the situation described, then, there are two possible criteria of individuation ${ }^{1}$ to which the domain of quantification can be relativised: physical and informational. The model for these cases is partially described in (1) and (2) respectively.
(1) Domain(Fig. 4.1) Physical $=\{\operatorname{vol} 1, N f U, T G, T D$, book 1$\}$

[^32]\[

$$
\begin{aligned}
& \llbracket \text { o-elab' } \rrbracket^{\text {Physical }=}\{\langle\text { vol } 1, \text { book } 1\rangle,\langle N f U, \text { book } 1\rangle,\langle T G, \text { book } 1\rangle, \\
&\langle T D, \text { book } 1\rangle\} \\
& \llbracket \text { book }^{\prime} \rrbracket^{\text {Physical }}=\{\text { book } 1\}
\end{aligned}
$$
\]

(2) Domain(Fig. 4.1) ${ }^{\text {Informational }}=\{\operatorname{vol} 1, N f U, T G, T D$, book 1, book 2, book 3\}

$$
\begin{aligned}
\llbracket \text { o-elab }^{\prime} \rrbracket \text { Informational }=\{ & \langle\text { vol } 1, \text { book } 1\rangle,\langle\text { vol } 1, \text { book } 2\rangle,\langle\text { vol } 1, \text { book } 3\rangle, \\
& \langle N f U, \text { book } 1\rangle,\langle T G, \text { book } 2\rangle,\langle T D, \text { book } 3\rangle\}
\end{aligned}
$$

$\llbracket$ book $^{\prime} \rrbracket^{\text {Informational }}=\{$ book 1, book 2, book 3 $\}$

One might wonder what the difference is between book 1 and vol 1 in (1), or between e.g. book 2 and The Gambler in (2). Relative to a physical criterion of individuation (as in (1) or on the left hand side of Figure 4.1) there is a one-to-one correspondence between books and physical aspects of books, and relative to an informational criterion of individuation (as in (2) or on the right hand side of Figure 4.1) there is a one-to-one correspondence between books and informational aspects of books. However, these are distinct objects: one is a 'bare particular' while the other is a 'thick individual' (ibid., p. 149).

In most cases the truth conditions predicted in Asher's system will involve quantification over aspects rather than over books as such, so that the way a particular situation is conceptualised will not make a truth-conditional difference. However, there are exceptions to this, which will be discussed below.

The compositional system is built on a framework that involves subtypes of $e$ (the type of entities), for instance $P$ the type of physical objects and I the type of informational objects. Dot objects are of a special dot type, for example $\llbracket b o o k \rrbracket$ is of type $(\mathrm{P} \bullet \mathrm{I}) \rightarrow t$. So for example, in (1), book 1 is an inhabitant of type $\mathrm{P} \bullet \mathrm{I}$, vol 1 is an inhabitant of type P, and Notes from Underground, The Gambler and The Double are inhabitants of type I.

Dot types $\alpha \bullet \beta$ do not, in general, stand in a subtyping relationship to their constituent types $\alpha$ and $\beta$ and therefore cannot be used directly in a context requiring either
of those constituent types; however, there are particular compositional accommodation rules that apply so that they can be used in those contexts. In non-copredication sentences, quantification is over aspects of the appropriate type, e.g. over physical objects or informational objects as appropriate. For instance, the truth conditions that his theory predicts for (1) from Chapter 2, repeated as (3) below, are as shown in (4).
(3) Fred picked up three books.

$$
\begin{align*}
& \lambda \pi \cdot \exists v\left(v=f^{\prime}(\pi) \wedge \exists_{3} x\left(\operatorname{pick}-\operatorname{up}^{\prime}(v, x, \pi) \wedge \exists z\left(\operatorname{book}^{\prime}(z, \pi) \wedge \text { o-elab }(x, z, \pi)\right)\right)\right)  \tag{4}\\
& \pi: \quad v: \mathrm{A}, \quad x: \mathrm{P}, z: \mathrm{P} \bullet \mathrm{I}
\end{align*}
$$

In Asher's theory, $\pi$ is a presupposition parameter that assigns types to argument positions of predicates, to be discussed further in Section 4.1.1. ${ }^{2}$ What (4) says is that there are three objects ${ }^{3}$ of type P (physical), each of which is an aspect of a book and each of which Fred picked up. (4) is therefore not true in the situation shown in Figure 4.1: under neither conceptualisation are there three physical aspects-indeed, changing conceptualisation never changes the number of aspects. The only inhabitant of type $P$ in (1) or (2) is vol 1 .

However, (4) is true in the situation shown in Figure 4.2, where we have three (physical) copies of the same (informational) book (in this case, Crime and Punishment), provided that Fred picked up those volumes. Here, under either conceptualisation there are three physical aspects, as can be seen from the partial model descriptions in (5) and (6).
(5) Domain(Fig. 4.2) Physical $=\{\operatorname{vol} 1, \operatorname{vol} 2, \operatorname{vol} 3, C \mathcal{G} P$, book 1, book 2, book 3$\}$
$\llbracket$ o-elab $\rrbracket^{\text {Physical }}=\{\langle\operatorname{vol} 1$, book 1$\rangle,\langle C \mathcal{B} P$, book 1$\rangle,\langle\operatorname{vol} 2$, book 2$\rangle$, $\langle C \mathcal{G} P$, book 2$\rangle,\langle\operatorname{vol} 3$, book 3$\rangle,\langle C \mathcal{G} P$, book 3$\rangle\}$
$\llbracket$ book $^{\prime} \rrbracket^{\text {Physical }}=\{$ book 1, book 2, book 3\}
(6) Domain(Fig. 4.2) $)^{\text {Informational }}=\{\operatorname{vol} 1$, vol 2, vol 3, book 1, $C \mathcal{E} P\}$

[^33]

Figure 4.2: Three copies of one informational book, under physical and informational criteria of individuation respectively

$$
\begin{aligned}
& \llbracket \mathrm{o}-\mathrm{elab}^{\prime} \rrbracket^{\text {Informational }=}\{\langle\operatorname{vol} 1, \text { book } 1\rangle,\langle\text { vol } 2, \text { book } 1\rangle,\langle\text { vol } 3, \text { book } 1\rangle, \\
&\langle C \mathcal{E} P, \text { book } 1\rangle\} \\
& \llbracket \text { book }^{\prime} \rrbracket^{\text {Informational }}=\{\text { book } 1\}
\end{aligned}
$$

In both (5) and (6), vol 1, vol 2 and vol 3 are inhabitants of type $P$ and are aspects of some book. So (4) is true in this situation, irrespective of conceptualisation.

These are welcome results, as we would want to say that (3) is unequivocally true in the situation shown in Figure 4.2 and unequivocally false in the situation shown in Figure 4.1. However, in some cases the results are not so welcome, as we will see in Section 4.1.1.

### 4.1.1 The system of accommodation

The type assignments for (4) stored in the $\pi$ parameter, which above I summarised as shown (repeated) in (7), are actually of the form shown in (8).
(7) $v: \mathrm{A}, x: \mathrm{P}, z: \mathrm{P} \bullet \mathrm{I}$

$$
\begin{equation*}
* \arg _{1}^{\text {pick-up }}: \mathrm{A} * \arg _{2}^{\text {pick-up }}: \mathrm{P} * \arg _{1}^{\text {book }^{\prime}}: \mathrm{P} \bullet \mathrm{I} * \arg _{1}^{\text {o-elab }}: \mathrm{P} * \arg _{1}^{\text {o-lab }}: \mathrm{P} \bullet \mathrm{I} \tag{8}
\end{equation*}
$$

(8) states that:

- The first argument position of pick-up' is of type A (for 'animate')
- The second argument position of pick-up' is of type $P$
- The first argument position of book ${ }^{\prime}$ is of type P $\bullet$ I
- The first argument position of o-elab' is of type $P$
- The second argument position of o-elab' is of type P $\bullet$ I

So, for example, $\operatorname{book}^{\prime}(x, \pi)$ means that $x$ is a book, and the sequence of type assignments stored in $\pi$ is coherent. In this case, the sequence of type assignments listed in (8) is coherent when applied to (4).

In some circumstances where the sequence of type assignments stored in $\pi$ is not coherent, special accommodation rules apply. This is what happens when $\alpha \bullet \beta$ is expected and $\alpha$ is provided (or vice versa). The way these rules work is by introducing extra 'o-elab' arguments into metalanguage interpretations-so (4) shows an interpretation derived in part by the application of these accommodation rules.

In these cases, there remains the question of which typing expectation $-\alpha \bullet \beta$ or $\alpha$-to accommodate to which, as both kinds of shift are possible within Asher's system. The system is set up so that by default, if $\alpha \bullet \beta$ was introduced by an expression of syntactic category X and $\alpha$ was introduced by an expression of category Y , then $\alpha \bullet \beta$ is accommodated to $\alpha$ if Y projects and $\alpha$ is accommodated to $\alpha \bullet \beta$ if X projects. ${ }^{4}$ For instance, what has happened in the derivation of (4) is that the accommodation functor shown in (9) has applied to convert (10) (which is derived by abstraction from a part of the metalanguage formula introduced by 'three books') to (11).
(9) $\quad \lambda P \cdot \lambda u \cdot \lambda \pi \cdot \exists z(P(\pi)(z) \wedge$ o-elab $(u, z, \pi))$
(10) $\lambda y \cdot \lambda \pi_{2} \cdot \operatorname{book}^{\prime}\left(y, \pi_{2}\right)$
(11) $\lambda u \cdot \lambda \pi \cdot \exists z\left(\operatorname{book}^{\prime}(z, \pi) \wedge o-\operatorname{elab}(u, z, \pi)\right)$

This means that what was the 'book' property, (10), is now the 'book conceptualised as physical object' property, (11). We have moved from the middle line of Figure 4.1 to the bottom line. The result is that in (4) the numerical quantifer (Asher's ' $\exists_{3}$ ') binds a

[^34]variable of type P, preserving the typing introduced by the verbal projection of 'picked up'.

Now let us look at some copredication sentences: (3) and (4) from Chapter 2, repeated as (12) and (13) respectively below.
(12) Fred picked up and mastered three books.
(13) Fred mastered three heavy books.

By the Head Typing Princple, what has to happen compositionally in (13) is that first the physical typing of the adjective 'heavy' has to be accommodated to the dottyping of the noun 'books', and then the dot typing of the DP 'three heavy books' has to be accommodated to the informational typing of the verbal projection. This leaves us with the interpretation shown in (14).

$$
\begin{align*}
\lambda \pi . \exists v(v & =f^{\prime}(\pi) \wedge \exists_{3} x\left(\operatorname { m a s t e r } ^ { \prime } ( v , x , \pi ) \wedge \exists z \left(\operatorname{book}^{\prime}(z, \pi) \wedge \text { o-elab }(x, z, \pi)\right.\right.  \tag{14}\\
& \left.\left.\left.\wedge \exists y\left(\operatorname{heavy}^{\prime}(y, \pi) \wedge \text { o-elab }(y, z, \pi)\right)\right)\right)\right) \\
\pi: & x: \mathrm{I}, y: \mathrm{P}, z: \mathrm{P} \bullet \mathrm{I}
\end{align*}
$$

What (14) says is that there are three objects of type I (information), each of which was mastered by Fred, and each of which is an aspect of a book that (also) has an aspect of type $P$ (physical) that is heavy. By looking at (1) and (2), we can see that the prediction here is that (13) should be true in a situation in which Fred mastered the informational contents of three books printed in a single physical volume, such as the situation shown in Figure 4.1. There, on either conceptualisation, we have three objects of type I (Notes from Underground, The Gambler, and The Double), each of which is an aspect of some book that itself has a(nother) aspect of type P (vol 1, vol 2 or vol 3). But clearly (13) would not be true in such a situation, as there are not three heavy books.

One might think that there is a way to remedy this situation by reformulation of the conditions determining what type is accommodated to what. But analysis of (12) shows that this is not the case.

In (12) the type conflict occurs within the verbal coordination 'picked up and mastered' and is not one involving a clash between $\alpha \bullet \beta$ and $\alpha$, but rather between $\alpha$ and $\beta$ (in this case, P and I). Concretely, the coordination of the two transitive verbs initially delivers the interpretation shown in (15), where $\Phi$ and $\Psi$ are variables ranging over DP denotations.

$$
\begin{align*}
& \lambda \Phi \cdot \lambda \Psi \cdot \lambda \pi_{1} \cdot \Psi\left(\pi_{1} * \arg _{1}^{\text {pick-up' }}: \mathrm{H} * \arg _{1}^{\text {master }^{\prime}}: \mathrm{H}\right)  \tag{15}\\
& \left(\lambda x \cdot \lambda \pi_{2} \cdot \Phi\left(\pi_{2} * \arg _{2}^{\text {pick-up }}: \mathrm{P} * \arg _{2}^{\text {master }}: \mathrm{I}\right)\right. \\
& \left.\left(\lambda y \cdot \lambda \pi_{3}\left(\operatorname{pick}^{\prime} \text { up }^{\prime}\left(x, y, \pi_{3}\right) \wedge \operatorname{master}^{\prime}\left(x, y, \pi_{3}\right)\right)\right)\right)
\end{align*}
$$

In (15), the sequence of type presuppositions shown in $\pi_{2}$ cannot be jointly satisfied. To deal with this situation, Asher (2011, p. 176) proposes an additional rule for coordination structures that effectively splits up $\pi_{2}$ and allows those type requirements to remain unresolved until the formation of the whole VP. So (15) is transformed into (16).

$$
\begin{align*}
& \lambda \Phi \cdot \lambda \Psi \cdot \lambda \pi_{1} \cdot \Psi\left(\pi_{1} * \arg _{1}^{\text {pick-up }}:\right.\left.\mathrm{H} * \arg _{1}^{\text {master }^{\prime}}: \mathrm{H}\right)  \tag{16}\\
&\left(\lambda x \cdot \lambda \pi _ { 2 } \cdot \Phi ( \pi _ { 2 } ) \left(\lambda y \cdot \lambda \pi _ { 3 } \left(\text { pick-up }^{\prime}\left(x, y, \pi_{3} * \arg _{2}^{\text {pick-up' }}: \mathrm{P}\right)\right.\right.\right. \\
&\left.\left.\left.\wedge \operatorname{master}^{\prime}\left(x, y, \pi_{3} * \arg _{2}^{\text {master }}: \mathrm{I}\right)\right)\right)\right)
\end{align*}
$$

When (16) is combined with the interpretation of 'three books', the type presuppositions $\arg _{2}^{\text {pick-up }}$ : P and $\arg _{2}^{\text {master }}$ : I can each individually be accommodated by shifting to P • I. The effect is that that the Head Typing Principle is overridden, as now the whole sentence inherits the typing of the object DP and not that of the verbal projection.

We therefore end up with the interpretation shown in (17) (adapted from ibid., p. 178).

$$
\begin{align*}
\lambda \pi . \exists v(v & =\operatorname{Fred}^{\prime}(\pi) \wedge \exists_{3} x\left(\operatorname{book}^{\prime}(x, \pi) \wedge \exists z(\operatorname{pick}-\mathrm{up}\right.  \tag{17}\\
& (v, z, \pi) \wedge \operatorname{o-elab}(z, x, \pi)) \\
& \left.\left.\wedge \exists y\left(\operatorname{master}^{\prime}(v, y, \pi) \wedge \operatorname{o-elab}(y, x, \pi)\right)\right)\right) \\
\pi: & x: \mathrm{P} \bullet \mathrm{I}, y: \mathrm{I}, z: \mathrm{P}
\end{align*}
$$

In this case, the variable bound by the $\exists_{3}$ quantifier is the dot-typed one itself. Importantly, then, (17) is ambiguous: on one reading, it requires the existence of three
books-individuated-physically, while on the other reading it requires the existence of three books-individuated-informationally. On neither reading does it require the existence of both.

This is problematic. Consider a situation in which Fred picks up three copies of the same (informational) book, and masters the contents. Suppose that he picked up all the physical volumes and masters all the informational objects shown in Figure 4.2. (12) is not true in this situation. But now look at (5). Given a physical criterion of individuation that situation would be one in which (17) is true: there are three objects in the extension of book' ${ }^{\prime}$, each of which has an aspect of physical type that Fred picked up and each of which has an aspect of informational type that Fred mastered.

Likewise, (12) is not true in a situation in which Fred (only) picked up a trilogy and mastered the contents. However, if Fred picked up all the physical volumes and mastered all the informational objects shown in Figure 4.1 then, as indicated in (2), given an informational criterion of individuation that situation would then be one in which (17) is true: there are three objects in the extension of book', each of which has an aspect of physical type that Fred picked up and each of which has an aspect of informational type that Fred mastered.

So (17) does not accurately represent the truth conditions of (12), nor does (14) accurately represent the truth conditions of (13). Some other approach is needed to derive the truth conditions of numerical quantified copredication sentences.

Another way of putting this is to note that the approach to copredication described in Chapter 2 predicts the entailments shown in (18)-(19), but TCL does not.
(18) Fred picked up and mastered three books. $\Rightarrow$ Fred picked up three books.
(19) Fred picked up and mastered three books. $\Rightarrow$ Fred mastered three books.

The cost of denying that (18) and (19) really are entailments is that of denying that (20) and (21) are contradictions.
(20) Fred picked up and mastered three books, but he didn't pick up three books.
(21) Fred picked up and mastered three books, but he didn't master three books.

One might think that the appropriate response here is to appeal to some sort of implicit modalisation such that, for (17) to be true, it has to be true on both (or all) possible criteria of individuation. However, on Asher's own terms this is problematic, because it removes some amibiguity in an unwelcome way. For example, take (22), the metalanguage interpretation of which ${ }^{5}$ is shown in (23) (Asher, 2011, p. 174). ${ }^{6,7}$
(22) A student read every book in the library.

$$
\begin{align*}
& \lambda \pi \cdot \exists y(\text { student }(y) \wedge \forall v(\exists u \exists x \exists z(\operatorname{library}(x, \pi) \wedge \operatorname{in}(u, z, \pi) \wedge \mathrm{o}-\mathrm{elab}(z, x, \pi)  \tag{23}\\
& \quad \wedge \operatorname{book}(v, \pi) \wedge \mathrm{o}-\mathrm{elab}(u, v, \pi) \rightarrow \operatorname{read}(y, v, \pi))) \\
& \pi: \quad y: \mathrm{A}, v: \mathrm{P} \bullet \mathrm{I}, x: \mathrm{P} \bullet \mathrm{~L}, u: \mathrm{P}, z: \mathrm{L}
\end{align*}
$$

On the informational criterion of individuation, (23) is true if a student read every informational book of which there is a physical copy in the library. On the physical criterion of individuation, however, (23) can only be true if the student read every physical book in the library, which normally would require reading some informational books several times. It seems that this tracks a genuine ambiguity in the English sentence. However, if we require (23) to be true on both criteria of individuation for the variable $v$, then (23) can only be true if some student read every physical copy of every book in the library, which is actually the less favourable of the two readings.

Additionally, even if we go down the route of requiring truth relative to all criteria of individuation, the truth conditions predicted for this amendment to the TCL account will still sometimes differ from those predicted by the account given in Chapter 2. To see this, we have to consider a slightly more complex situation.

Figure 4.3 shows a situation that is a partial combination of those shown in Figures 4.1 and 4.2: we have two copies of Crime and Punishment, and also Notes from Underground, The Gambler and The Double in a single volume.

[^35]volume $1 \quad$ Crime and Punishment
volume $2 \quad$ Crime and Punishment
volume 3

| Notes from Underground |
| :---: |
| The Gambler |
| The Double |

Figure 4.3: Two copies of one book, and a trilogy


Figure 4.4: Figure 4.3 according to physical and informational criteria of individuation

Now suppose that Fred picked up volumes 1-3, and that he mastered Crime and Punishment, Notes from Underground, The Gambler and The Double. As Figure 4.4 shows, on both criteria of individuation there are at least three books that meet the following criteria: they have an aspect of type P that Fred picked up, and they have an aspect of type I that Fred mastered. Therefore, according to the proposed revision to the TCL system, (12) would be true in this situation. ${ }^{8}$

In contrast, according to the account described in Chapter 2, (12) is not true in that situation, because every three-or-more-membered plurality that can be formed from the set shown in (24) is physically or informationally compressible, and so the

[^36]truth conditions for (2) as predicted in the account from Chapter 2, repeated below as (25), are not satisfied.
\[

$$
\begin{align*}
& \left\{v_{1}+C B P, v_{2}+C B P, v_{3}+N f U, v_{3}+T G, v_{3}+T D\right\}  \tag{24}\\
& \exists x\left(|x| \geq 3 \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{pick}-\operatorname{up}^{\prime}\left(f^{\prime}, x\right) \wedge * \operatorname{master}^{\prime}\left(f^{\prime}, x\right) \wedge \neg(\text { PHYS } \sqcup \operatorname{INFO}) \operatorname{comp}(x)\right)
\end{align*}
$$
\]

It is my judgement that the revised mereological approach to copredication makes better predictions here than the proposed revisions to the TCL system: (12) is false in the situation shown in Figure 4.3.

### 4.1.2 Accommodation functors and syntax

An additional cause of concern with the TCL system lies the nature of the accommodation functors, which I skirted over while discussing how (4) is derived on the basis of (9)-(11). In Asher's actual presentation, what happens is as follows. First, you derive (26) as the interpretation of the VP 'picked up three books'.

$$
\begin{equation*}
\lambda \Phi \cdot \lambda \pi \cdot \Phi(\pi)\left(\lambda u \cdot \lambda \pi_{1} \cdot \exists_{3} x\left(\operatorname{book}^{\prime}\left(x, \pi_{1} * \arg _{1}^{\text {book }^{\prime}}: \mathrm{P} \bullet \mathrm{I} * \arg _{2}^{\text {pick-up }}: \mathrm{P}\right) \wedge \text { pick-up }\left(u, x, \pi_{1}\right)\right)\right) \tag{26}
\end{equation*}
$$

$\Phi$ is a variable ranging over DP denotations. $\pi_{1} * \arg _{1}^{\text {book' }}: \mathrm{P} \bullet \mathrm{I} * \arg _{2}^{\text {pick-up }}: \mathrm{P}$ says that you update $\pi_{1}$ with the additional requirements that the first argument of book ${ }^{\prime}$ be of type P $\bullet$ I and that the second argument of pick-up' be of type P. This sequence of type presuppositions is inconsistent: the same variable occupies both of those argument positions, and $\mathrm{P} \bullet \mathrm{I} \sqcap \mathrm{P}=\perp$. So some sort of accommodation is required.

Next, you note that the part of (26) that needs to be changed is (27), so you abstract this to (28).
(27) $\operatorname{book}^{\prime}\left(x, \pi_{1} \ldots\right)$
(28) $\lambda y \cdot \lambda \pi_{2} \cdot \operatorname{book}^{\prime}\left(y, \pi_{2}\right)(x)\left(\pi_{1} \ldots\right)$

Then, you apply the accommodation functor (9) as shown below:

$$
\begin{aligned}
& \lambda P \cdot \lambda u \cdot \lambda \pi \cdot \exists z(P(\pi)(z) \wedge \mathrm{o}-\mathrm{elab}(u, z, \pi))\left[\lambda y \cdot \lambda \pi_{2} \cdot \operatorname{book}^{\prime}\left(y, \pi_{2}\right)\right] \\
& =\lambda u \cdot \lambda \pi \cdot \exists z\left(\operatorname{book}^{\prime}(z, \pi) \wedge \operatorname{o-elab}^{\prime}(u, z, \pi)\right)
\end{aligned}
$$

and then you re-integrate the abstracted variables from (28)

$$
\lambda u \cdot \lambda \pi \cdot \exists z\left(\operatorname{book}^{\prime}(z, \pi) \wedge \operatorname{o}^{-\operatorname{elab}^{\prime}}(u, z, \pi)\right)[x]\left[\pi_{1} \ldots\right]=\exists z\left(\operatorname{book}^{\prime}\left(z, \pi_{1} \ldots\right) \wedge \operatorname{o-elab}{ }^{\prime}\left(x, z, \pi_{1}\right)\right)
$$

The expression you are left with, (29), you then substitute back into (26) in place of (27), leaving you with (30) as the shifted meaning of the VP.

$$
\begin{align*}
& \exists z\left(\operatorname{book}^{\prime}\left(z, \pi_{1} * \arg _{1}^{\text {book' }}: \mathrm{P} \bullet \mathrm{I} * \arg _{2}^{\text {pick-up }}: \mathrm{P}\right) \wedge \mathrm{o}^{\left.-\mathrm{elab}^{\prime}\left(x, z, \pi_{1}\right)\right)}\right.  \tag{29}\\
& \lambda \Phi \cdot \lambda \pi \cdot \Phi(\pi)\left(\lambda u \cdot \lambda \pi _ { 1 } \cdot \exists _ { 3 } x \left(\exists z \left(\operatorname{book}^{\prime}\left(z, \pi_{1} * \arg _{1}^{\text {book }}: \mathrm{P} \bullet \mathrm{I} * \arg _{2}^{\text {pick-up' }}: \mathrm{P}\right)\right.\right.\right.  \tag{30}\\
& \left.\left.\left.\wedge \operatorname{o-elab}^{\prime}\left(x, z, \pi_{1}\right)\right) \wedge \operatorname{pick}^{\prime} \operatorname{up}^{\prime}\left(u, x, \pi_{1}\right)\right)\right)
\end{align*}
$$

Now, it is no longer the case that the same variable occupies both the first argument position of book' and the second argument position of pick-up', so the conflict has been resolved.

This way of putting things is rather involved and appears to make indispensable use of the metalanguage as a level of representation: the system as stated involves extracting pieces of metalanguage formulae, applying functors to them and then re-inserting them into the original formulae.

Is this use of the syntax of the metalanguage really indispensable for Asher's account? In many of the instances where accommodation functors are needed, the presentation in terms of substitutions into metalanguage formulae could be replaced with one in terms of type-changing operations on interpretations at specific points of derivation. This would come at the expense of needing to state accommodation functors in addition to the two Asher defines (one for moving from $\alpha$ to $\alpha \bullet \beta$ and one for moving in the opposite direction), in order to account for accomodation of expression meanings of various types. But of course, it is not a fatal problem for Asher's account if it ends up needing multiple accommodation functors of various types.

However, there are other structures for which the accommodation process is less straightforward than this. These are illustrated schematically in (31)-(33) below.




In each case, I have circled the label(s) of the constituent(s) to which an accommodation functor has to be applied, and boxed the label of the constituent that triggers this process. In none of (31)-(33) do these nodes stand in a sisterhood relation, and so if the process of accommodation is to be driven by a type clash then this action will have to be non-local.

One possible solution that suggests itself at this stage is to change the system so that accommodation functors can apply freely rather than being coerced by an inconsistent sequence of type assignments. But this change would seriously undermine the ability of the type system to do one of the major jobs that it is designed to do: namely, to provide a predictive account of anomaly.

If accommodation functors can apply freely, then what is to stop some expression shifting from type $\alpha$ (or a higher-typed analogue) to type $\alpha \bullet \beta$ to type $\beta$ whenever the type $\alpha \bullet \beta$ exists? In other words, what is to stop an expression of any type shifting to an expression of any other type if the intermediate dot type(s) exist(s)?

To give a concrete example from the type of syntactic structure shown in (33): the shifting of both the P-type-selecting predicate 'picked up' and the I-type-selecting predicate 'mastered' to predicates that will accept an argument of type P •I ${ }^{9}$ in the analysis of (12) above crucially depends on this process being justified by their common argument ('book') itself being of P • I type. If instead the accommodation processes were allowed to apply freely without that kind of justification, then it is difficult to see what would prevent the types in (34) matching up in the right way.
\# Fred picked up and mastered a stone.

If accommodation functors could apply freely, then one could apply to $\llbracket p i c k e d u p \rrbracket$ and another to $\llbracket$ mastered $\rrbracket$ so as to coerce both into predicates that select an argument of type P - I. They would then be conjoinable without the need for a special coordination rule. A (freely applying) accommodation functor could then apply to $\llbracket a$ stone』 so as to shift it from type P to type P $\bullet$ I, and all the type requirements in (34) would be satisfied. But clearly this is unacceptable: (34) is anomalous and the type system is supposed to account for semantic anomaly such as that shown by (34). Asher's system as stated correctly predicts the anomaly of (34) because it does not allow for free application of accommodation functors, and no accommodation functor is provided for a situation in which type I is expected and type $P$ is provided.

In summary: the truth conditions that the TCL account predicts for numerically quantified copredication sentences are too permissive - it predicts some sentences to be true in situations where they are not. There are ways in which the system could be improved so as to make the truth conditions more restrictive, but these will not work in every case. Furthermore, the system of accommodation that enables predication and copredication to work relies crucially on the syntax of the metalanguage in which truth conditions are stated, and must do so in order for the type hierarchy to fulfil its function of ensuring semantic well-formedness.

[^37]
### 4.2 Type Theory with Records

### 4.2.1 Dot types and record types

Type Theory with Records (TTR) is an application of intuitionistic type theory to formal semantic composition motivated by the desire to unify it with other approaches in linguistics and artificial intelligence (Cooper, 2005).

In this formalism, records are sets of ordered pairs (called fields) of labels and values, and record types are records where all the values are types. Record types are sets of type judgements, and they correspond to propositions in more standard theories. The counterpart in TTR of a proposition being true is a record type being witnessed. A record type $r_{t}$ is witnessed if and only if there is a record $r$ in which all of the type judgements in $r_{t}$ are satisfied. In this case, $r$ is a record of the type $r_{t}$. In a record the values (paired with labels) can be objects, proofs or types.

To give a concrete example: if $a$ is an individual (an object of type Ind) and $p_{1}$ is a proof that $a$ is food, then the record type (36) is witnessed by the record (37). ${ }^{10}$

$$
\begin{align*}
& \left\{\langle\mathrm{x}, \text { Ind }\rangle,\left\langle\mathrm{c}_{1}, \text { food }(\mathrm{x})\right\rangle\right\} \text {, usually represented as }\left[\begin{array}{lll}
\mathrm{x} & : & \operatorname{Ind} \\
\mathrm{c}_{1} & : & \text { food }(\mathrm{x})
\end{array}\right]  \tag{36}\\
& \left\{\langle\mathrm{x}, a\rangle,\left\langle\mathrm{c}_{1}, p_{1}\right\rangle\right\} \text {, usually represented as }\left[\begin{array}{lll}
\mathrm{x} & =a \\
\mathrm{c}_{1} & =p_{1}
\end{array}\right] \tag{37}
\end{align*}
$$

Cooper (2011, p. 76) suggests that the dot types of Generative Lexicon theory (Pustejovsky, 1995) and Asher's extension of it 'can be usefully construed as record types'. For example, the lexical entry for 'lunch' that he suggests is as follows:

$$
\llbracket \text { lunch } \rrbracket=\lambda r:[\mathrm{x}: I n d]\left(\left[\begin{array}{lll}
\text { event } & : & \text { Event }  \tag{38}\\
\text { food } & : & \text { Food } \\
c_{\text {lunch }} & : & \text { lunch_ev_fd }(r \cdot \mathrm{x}, \text { event, food })
\end{array}\right]\right)
$$

[^38]What (38) represents is a function from records $r$ in which ' $x$ ' labels an individual to record types containing proofs ( $\mathrm{c}_{\text {lunch }}$ ) that that individual is a lunch and that it has an aspect (labelled 'event') of type Event and an aspect (labelled 'food') of type Food. We are to imagine the predicate 'lunch_ev_fd' holding true of ordered triples of objects $\langle x, y, z\rangle$ such that $x$ is a lunch individual with event aspect $y$ and food aspect $z$. ' $r$. x ' indicates the label ' $x$ ' in the record ' $r$ ' that is the argument to the function. ${ }^{11}$ The idea is that you could apply (38) to the record shown in (39) (where $a$ is an individual) and obtain the record type shown in (40).

$$
\begin{align*}
& {[\mathrm{x}=a]}  \tag{39}\\
& {\left[\begin{array}{ll}
\text { event } & : \\
\text { food } & : \\
c_{\text {lunch }} & \text { : Foord } \\
\text { lunch_ev_fd }(a, \text { event, food })
\end{array}\right]} \tag{40}
\end{align*}
$$

More generally, (38) is a function from records introducing an individual to record types; that is, it is of the type shown in (41) (Cooper, 2011, p. 68).

$$
\begin{equation*}
[\mathrm{x}: I n d] \rightarrow \text { RecType } \tag{41}
\end{equation*}
$$

The type shown in (41) can be referred to as Ppty, for 'property'. Cooper sees it as an advantage of his strategy for copredication that this type is not a special kind of unusual type that only nouns supporting copredication have - which is the status that dot types have in Asher's (2011) theory. Rather, it is the normal type of nouns in TTR: essentially, (38) can be seen as representing the property of being a lunch, for example. It is, in fact, a function from something like individuals to something like propositions.

The exact form of the record types involved in this formalisation has changed as a result of the attempt to incorporate copredication. For instance, in previous versions (Cooper, 2007, for example), 'lunch' was represented as shown in (42).
(42) $\llbracket l$ unch $\rrbracket=\lambda r:\left[\begin{array}{lll}\mathrm{x} & : & \operatorname{Ind} \\ \mathrm{c}_{1} & : & \operatorname{food}(\mathrm{x}) \\ \mathrm{c}_{2} & : & \operatorname{event}(\mathrm{x})\end{array}\right]\left(\left[\mathrm{c}_{3}: \operatorname{lunch}(r . \mathrm{x})\right]\right)$

The move from (42) to (38) is motivated in part by the fact that (42), and not

[^39](38), makes the (apparently) dubious requirement that something (labelled ' $x$ ') can be both food and an event. As indicated in Section 2.2, Cooper (2007) explained this requirement by saying that the food and the event would both be parts of a lunch, but this mereological account was not further developed and has been abandoned.

Adapting the lexical entry for 'lunch' shown in (38), the lexical entry for 'book' is as shown in (43).

$$
\llbracket b o o k \rrbracket=\lambda r:[\mathrm{x}: I n d]\left(\left[\begin{array}{lll}
\text { pobj }: & \text { PhysObj }  \tag{43}\\
\text { iobj } & : & \operatorname{InfObj} \\
c_{\mathrm{book}} & : & \text { book_phys_inf }(r \cdot \mathrm{x}, \operatorname{pobj}, \operatorname{iobj})
\end{array}\right]\right)
$$

What (43) shows is a function from records $r$ containing the declaration that the object labelled ' $x$ ' is of type Ind (individual) to records, dependent on $r$, declaring that the object labelled ' $x$ ' is a book that has physical aspect labelled 'pobj' and an informational aspect labelled 'iobj'.

### 4.2.2 Determiners

In order to express quantification, determiner meanings are treated by giving formulae that will translate the type-theoretic expressions of TTR into set-theoretic expressions, such that relations between them can be defined as in generalised quantifier theory (Barwise and Cooper, 1981, for example). This is done in two stages (Cooper, 2011, pp. 69-70). First, for any type $T$, we can talk about the the extension of $T$, written $\left[{ }^{\vee} T\right]$, which is the set of things that are of type $T$. A definition is given in (44).

$$
\begin{equation*}
\left[{ }^{\vee} T\right] \stackrel{\text { def }}{=}\{a \mid a: T\} \tag{44}
\end{equation*}
$$

If $T$ is of type Ppty, then (this is the second stage) we can talk about the set of things that have the property expressed by $T$, the property extension of $T$ or $[\downarrow T]$. The definition of this is shown in (45).

$$
\begin{equation*}
[\downarrow T] \stackrel{\text { def }}{=}\left\{a \mid \exists r\left(r:[\mathrm{x}: I n d] \wedge r \cdot \mathrm{x}=a \wedge\left[{ }^{\vee} T(r)\right] \neq \emptyset\right)\right\} \tag{45}
\end{equation*}
$$

This is the set of things $a$ such that $a$ is of type Ind and $[x=a]$ is a record that
bears the property expressed by $T$. For any determiner meaning $q *$ given as a relation between sets, we can define an equivalent determiner meaning $q$ in TTR in terms of a relation between functional record types, as in (46) (where $A$ and $B$ are of type Ppty).

$$
\begin{equation*}
q(A, B) \Leftrightarrow q^{*}([\downarrow A],[\downarrow B]) \tag{46}
\end{equation*}
$$

This means that the truth conditions predicted for (47) are as shown in (48).
(47) Three books are heavy.
(48) $|[\downarrow \llbracket b o o k \rrbracket] \cap[\downarrow \llbracket h e a v y \rrbracket]| \geq 3$

Using the definition of property extension shown in (45) and the lexical entry for 'book' shown in (43), [ $\downarrow \llbracket b o o k \rrbracket]$ is as shown in (49).

$$
\begin{align*}
&\{a \mid \exists r([\mathrm{x}: \text { Ind }] \wedge r . \mathrm{x}=a  \tag{49}\\
&\left.\left.\wedge\left\{b \mid b:\left[\begin{array}{lll}
\text { pobj } & : \text { PhysObj } \\
\text { iobj } & : & \text { InfObj } \\
\mathrm{c}_{\text {book }} & : & \text { book_phs_inf }(r . \mathrm{x}, \text { pobj, iobj })
\end{array}\right]\right\} \neq \emptyset\right)\right\}
\end{align*}
$$

This is the set of things $a$ such that $a$ is of type Ind and there is at least one record proving that $a$ stands in the book_phs_inf relation to something of type PhysObj and something of type InfObj. So it is the set of books.

Likewise, assuming the lexical entry for 'heavy' shown in (50) (based on the entry for 'delicious' (ibid., p. 72)), it follows that $[\downarrow \llbracket h e a v y \rrbracket]$ is as shown in (51).
(50) $\llbracket h e a v y \rrbracket=\lambda r_{1}:\left[\begin{array}{lll}\mathrm{x} & : & \text { Ind } \\ \text { pobj } & : & \text { PhysObj }\end{array}\right]\left(\left[c_{\text {heavy }}: \operatorname{be\_ heavy\_ phs}\left(r_{1} \cdot \mathrm{x}, r_{1} \cdot \operatorname{pobj}\right)\right]\right)$

$$
\begin{align*}
& \left\{a \left\lvert\, \exists r\left(r:\left[\begin{array}{lll}
\mathrm{x} & : & \text { Ind } \\
\text { pobj } & : & \text { PhysObj }
\end{array}\right] \wedge r . \mathrm{x}=a\right.\right.\right.  \tag{51}\\
& \\
& \left.\left.\qquad\left\{b \mid b:\left[c_{\text {heavy }}: \text { be_heavy_phs }(r . \mathrm{x}, r . \text { pobj })\right]\right\} \neq \emptyset\right)\right\}
\end{align*}
$$

But in what sense is (49) the set of books? As I have been at pains to show in Chapter 2, books can be individuated and counted in different ways depending on the predicational context. It cannot be that the truth conditions of (47) are as shown in (48) and that the truth conditions of (52) are as shown in (53), because the relevant set of books in (47) and (52) could have different cardinalities.
(52) Three books are complex.

$$
\begin{equation*}
\mid[\downarrow \llbracket b o o k \rrbracket] \cap[\downarrow \llbracket \text { complex } \rrbracket] \mid \geq 3 \tag{53}
\end{equation*}
$$

In response to these issues of counting and individuation, Cooper (2011, p. 76) suggests that 'given that we now have aspects in separate fields of our frames we could relativize our notion of property extension to labels in the frame,. ${ }^{12}$ A definition of the property extension of property type $T$ relative to label $1,\left[\downarrow_{l} T\right]$, is given in (54).

$$
\begin{equation*}
\left[\downarrow_{l} T\right] \stackrel{\text { def }}{=}\left\{a \mid \exists r\left(r:[\mathrm{x}: \text { Ind }] \wedge r . \mathrm{l}=a \wedge\left[{ }^{\vee} T(r)\right]\right) \neq \emptyset\right\} \tag{54}
\end{equation*}
$$

This now allows us to count books relative to different criteria of indivduation. For instance, to get the set of books individuated physically we can instantiate $l$ as 'pobj', as in (55).

$$
\begin{equation*}
\left[\downarrow_{\mathrm{pobj}} T\right] \stackrel{\text { def }}{=}\left\{a \mid \exists r\left(r:[\mathrm{x}: I n d] \wedge r \cdot \operatorname{pobj}=a \wedge\left[{ }^{\vee} T(r)\right]\right) \neq \emptyset\right\} \tag{55}
\end{equation*}
$$

Then, if we take $\left[\downarrow_{\text {pobj }}(43)\right]$, we have the set shown in (56), which is the set of books individuated physically.

$$
\begin{align*}
&\{a \mid \exists r([\mathrm{x}: \text { Ind }] \wedge r . \text { pobj }=a  \tag{56}\\
&\left.\left.\wedge\left\{b \mid b:\left[\begin{array}{lll}
\text { pobj } & : \text { PhysObj } \\
\text { iobj } & : \text { InfObj } \\
\text { cbook } & : & \text { book_phs_inf }(r . x, \text { pobj, iobj })
\end{array}\right]\right\} \neq \emptyset\right)\right\}
\end{align*}
$$

This is the set of things $a$ such that $a$ is of type PhysObj and there is at least one record proving that something of type Ind stands in the book_phs_inf relation to $a$ and something of type InfObj. So it is the set of physical aspects of books. We therefore have a way of counting relative to a criterion of individuation.

For this strategy of taking a property extension relative to a label to be compositionally implementable, I take it, we would have to integrate it into determiner meanings somehow. Moreover, whatever form this integration took, it would mean that we could not treat (46) as the meaning of a determiner in TTR.

[^40]As a first attempt, let us suppose that there are four options in TTR $\left(q-q^{\prime \prime \prime}\right)$ for the semantic value of some lexical determiner that has the set-theoretic semantic value $q^{*}$, as shown in (46) (repeated), (57), (58) and (59) below. In these examples, $l_{1}$ is some label in $A(r)$ and $l_{2}$ is some label in $B(r)$.

$$
\begin{align*}
& \text { (46) } q(A, B) \Leftrightarrow q^{*}([\downarrow A],[\downarrow B]) \\
& \text { (57) } q^{\prime}(A, B) \Leftrightarrow q^{*}\left(\left[\downarrow_{l_{1}} A\right],[\downarrow B]\right)  \tag{46}\\
& \text { (58) } q^{\prime \prime}(A, B) \Leftrightarrow q^{*}\left(\left[\downarrow_{l_{1}} A\right],\left[\downarrow_{l_{2}} B\right]\right)  \tag{57}\\
& \text { (59) } \\
& q^{\prime \prime \prime}(A, B) \Leftrightarrow q^{*}\left([\downarrow A],\left[\downarrow_{l_{2}} B\right]\right) \tag{59}
\end{align*}
$$

That is to say, we can relativise to a label either the first argument to the determiner, or the second, or both, or neither. Of course, there is nothing in the above definitions that indicates what $l_{1}$ or $l_{2}$ should be in any particular case. I will make some suggestions as to how tackle this issue in Section 4.2.3.

### 4.2.3 Relativising predicates

Suppose that we tried to get the truth conditions for (47) by relativising the property extension of $\llbracket b o o k \rrbracket$ in such a way as to count physical books (as in (56)), but leaving the propery extension of 【heavy】unrelativised. In other words, suppose that we thought that the truth conditions were as shown in (60).

$$
\begin{equation*}
\mid\left[\downarrow_{\text {physobj }} \llbracket b o o k \rrbracket\right] \cap[\downarrow \llbracket \text { heavy } \rrbracket] \mid \geq 3 \tag{60}
\end{equation*}
$$

This means that in (60) we would be looking at the cardinality of the intersection of the set of physical aspects of books with the set of things that have some aspect that is heavy. Is this what is going on in (47)? Without an account of what aspects are, it is not clear how this question should be answered. If everything is an aspect of itself, then it is acceptable - in that case, everything that is a heavy physical aspect of some book has an aspect (itself) that is heavy. However, this is not how Cooper (ibid.,
p. 67) understands talk of aspects (nor how Asher (2008, p. 165) understands it), so (60) must somehow be ruled out from consideration as the interpretation of (47).

It seems that, in Cooper's terms, what we need in order to get correct truth conditions for (47) is to relativise the property extension of both $\llbracket b o o k \rrbracket$ and $\llbracket h e a v y \rrbracket$ to a label that names something of type PhysObj. Likewise, in order to get accurate truth conditions for a sentence like (52), we would have to relativise the property extension of both $\llbracket b o o k \rrbracket$ and $\llbracket c o m p l e x \rrbracket$ to a label that names something of type $\operatorname{InfObj}$ (in (43), this would be the label 'iobj').

There must be something about the predicate 'heavy' that forces individuation by physical objects; in our current terminology, that means forcing the property extension of $\llbracket b o o k \rrbracket$ in (47) to be taken relative to a label that names something of type PhysObj. Likewise, there must be something about the predicate 'complex' that means that when you take the property extension of 'book' in (52), it is relative to a label that names something of type InfObj.

How can this be enforced? We can begin to approach this question by thinking in terms of what constraints there are on what kind of relativised property extension is used in any quantificational sentence.

At the most basic level, we want to guarantee that the sets of things we end up comparing are sets of things of like type. In order to achieve this, we can propose the following constraints on the use of property extension relative to a label (as outlined in (57)-(59)), remembering that in the normal case (46) we have $q(A, B) \Leftrightarrow q^{*}([\downarrow A],[\downarrow$ $B]$ ). I will use the convention that $\operatorname{Tr}(\alpha)$ is the type of $\alpha$, i.e. $\operatorname{Tr}(\alpha)=x \Leftrightarrow \alpha: x$
(C 1) Using $q^{\prime}(A, B)$, i.e. $q^{*}\left(\left[\downarrow_{l_{1}} A\right],[\downarrow B]\right)$, is licit if and only if $(A(r)) \cdot l_{1}:$ Ind.
(C 2) Using $q^{\prime \prime}(A, B)$, i.e. $q^{*}\left(\left[\downarrow_{l_{1}} A\right],\left[\downarrow_{l_{2}} B\right]\right)$, is licit if and only if $\operatorname{Tr}\left((A(r)) . l_{1}\right)=$ $\operatorname{Ty}\left((B(r)) \cdot l_{2}\right)$
(C 3) Using $q^{\prime \prime \prime}(A, B)$, i.e. $q^{*}\left([\downarrow A],\left[\downarrow_{l_{2}} B\right]\right)$, is licit if and only if $(B(r)) \cdot l_{2}:$ Ind.
As an example of how the constraints (C 1)-(C 3) achieve the object laid out above, we can see that (C 1) rules out applying $q^{\prime}$ as shown in (60). (【book』(r)).physobj :

PhysObj, not Ind, and so $q^{\prime}(A, B)$ is not licit in this case. In the case of $q$, no such constraint is needed because the definition of property extension in (45) guarantees that the objects being quantified over are of like type, namely type Ind.

What these constraints do not do is rule out is the use of $q$ in cases like (47). It would be licit according to the constraints (C 1)-(C 3) but, as discussed above in relation to (49), it's just not clear in that case what it is that we are supposed to be counting.

In this case, the difficulty can be avoided by enforcing an additional constraint.
(C 4) $q$ is a last resort. That is to say, if one of $q^{\prime}-q^{\prime \prime \prime}$ can be used without violating any of the constraints (C 1)-(C 3), then it should be.

It follows from (C 4) that the only way to calculate the truth conditions of (47) is as shown in (61). (48) is inappropriate because (C 4) states that it can only be used as a last resort, and (61) is available in this case. (60) is illicit according to (C 1) because $\operatorname{TY}((\llbracket b o o k \rrbracket(r))$.physobj $) \neq I n d$. Any version of (C 3) that involved taking the property extension of $\llbracket h e a v y \rrbracket$ relative to the label 'physobj' is illicit according to (C 3) because $\operatorname{TY}((\llbracket h e a v y \rrbracket(r)) \cdot$.physobj $) \neq$ Ind.

$$
\begin{equation*}
\mid\left[\downarrow_{\text {physobj }} \llbracket b o o k \rrbracket\right] \cap\left[\downarrow_{\text {physobj }} \llbracket \text { heavy } \rrbracket\right] \mid \geq 3 \tag{61}
\end{equation*}
$$

The same considerations show that the only way to calculate the truth conditions of (52) is as shown in (62).
(62) $\mid\left[\downarrow_{\text {infobj }} \llbracket b o o k \rrbracket\right] \cap\left[\downarrow_{\text {infobj }} \llbracket\right.$ complex $\left.\rrbracket\right] \mid \geq 3$

When it comes to a copredication sentence like (63), though, we have little choice but to use $q$, as in (64).
(63) Three books are heavy and complex.

$$
\begin{equation*}
\mid[\downarrow \llbracket \text { book } \rrbracket] \cap([\downarrow \llbracket \text { heavy } \rrbracket] \cap[\downarrow \llbracket \text { complex } \rrbracket]) \mid \geq 3 \tag{64}
\end{equation*}
$$

If the property extension of $\llbracket b o o k \rrbracket$ were relativised to either the label 'physobj' or the label 'infobj', then there would be no way to satisfy (C 1) (or (C 2)). In fact, this
is just another way of stating the problem of copredication: to the extent that they can place a more fine-grained requirement on their arguments than Ind, 'heavy' and 'complex' impose type restrictions that are apparently incompatible. Thus in this case, we really do want to quantify using standard property extensions as outlined in (46). Indeed, this is one reason for introducing this kind of record type in the first place, as opposed to (42).

### 4.2.4 Organising the domain

Because we have a clear, intuitive idea what physical aspects of books are (just the physical objects that are books, physical volumes), and what informational aspects of books are (just the informational objects that are books, informational books like War and Peace), it is clear what the truth conditions for (61) and (62) are, and that they are as desired.

But it is not clear what books as such are, which we need to know in order to understand the truth conditions of (64). One possibility would be to go down the route of saying that this is itself a context-dependent matter, and that books as such are in a one-to-one correspondence with either physical or informational aspects of books, depending on context. That would essentially be the approach taken by Asher (2011), and hence subject to the criticisms of that approach given in Section 4.1.1.

An alternative would be to say that a book is a physical object+informational object composite or something similar. This is the approach defended in this thesis; however, without making use of quantification over pluralities and compressibility statements about those pluralities, as defined in Chapter 2, this approach would face the problems described in Section 2.2. If a book as such is a physical object+informational object composite then there are four books in the situation shown in Figure 2.1 on page 36: the books shown in (65); and so it should be possible for (64) to be true in that situation.

But clearly (63) cannot be true in that situation.

$$
\begin{align*}
& \left\{v_{1}+\text { Notes from Underground, } v_{1}+\text { The Gambler },\right.  \tag{65}\\
& \left.v_{2}+\text { Notes from Underground, } v_{2}+\text { The Gambler }\right\}
\end{align*}
$$

One might try to rescue this analysis by saying that a book is a physical book+ informational book composite subject to certain requirements, where those requirements are intended to produce the result that the set of books meets the following conditions:

1. No two of its members have the same physical part
2. No two of its members have the same informational part

The problem is that there is, in general, no satisfactory way to meet conditions 1 and 2. That is to say, there is no way to translate the requirements in 1 and 2 into requirements that individual physical+informational composites have to meet in order to qualify as books-as-such. (65) is a case in point: the sets (66) and (67) both meet the conditions 1 and 2, but there's no way to choose between them and so no way to which of the members of (65) should qualify as a book as such.
(66) $\left\{v_{1}+\right.$ Notes from Underground, $v_{2}+$ The Gambler $\}$
(67) $\left\{v_{1}+\right.$ The Gambler, $v_{2}+$ Notes from Underground $\}$

The distinctness requirements we need in order to get the truth conditions of numerically quantified copredication sentences right are requirements of whole pluralities, not their members.

In summary, this TTR approach has a mechanism for delivering the appropriate different counting principles for different criteria of individuation: the mechanism of taking a property extension relative to a label. However, there are copredication sentences for which we need something else, because neither taking the property extension relative to 'physobj' nor taking it relative to 'infobj' would give us truth conditions restricted enough for those sentences. There is no coherent way to talk about the set of books as such that will give the correct truth conditions for numerically quantified copredi-
cation sentences. For that, we need to augment talk of pluralities with compressibility statements about those pluralities, as described in Chapter 2.

### 4.3 Modern Type Theories

Like Asher (see Section 4.1), Luo (2010, 2011, 2012b) and Chatzikyriakidis and Luo (2012, 2013) adopt a compositional framework according to which predicates impose sortal requirements on their arguments, requirements that are encoded in the type system. That system, which they call a 'modern type theory' (MTT), ${ }^{13}$ is based on a many-sorted logic and involves a subtyping relationship $<_{c}$, where $A<_{c} B$ indicates that there is a unique implicit coercion from type $A$ to type $B$. This means that an object of type $A$ can be used in any context requiring an object of type $B .{ }^{14}$ Also like Asher's TCL, the type hierarchy is very fine-grained and it includes dot types.

Unlike the dot types in Asher's TCL, the dot types in this system generally are in a subtyping relationship with their constituent types. For instance, we have the relationships shown in (68).

## (68) Phys • Info $<_{c}$ Phys

Phys • Info $<_{c}$ Info

Taken alone, these subtyping relationships would not go very far at all towards addressing the compositional issues raised by copredication outlined in Section 1.2.2. This is because they do not generalize to higher types in the way we would want, as indicated in (69). ${ }^{15}$

$$
\begin{align*}
& (\text { PHYS } \bullet \text { INFO }) \rightarrow \text { Prop } \nless c_{c} \text { PHYS } \rightarrow \text { Prop }  \tag{69}\\
& (\text { PHYS } \bullet \text { INFO }) \rightarrow \text { Prop } \nless c_{c} \text { InFO } \rightarrow \text { Prop }
\end{align*}
$$

[^41]That is to say, even though we would be able to combine $\llbracket t h e ~ b o o k \rrbracket(t y p e ~ P H Y S ~ \bullet ~$ InFo) with $\llbracket b e$ heavy (type Phys $\rightarrow$ Prop) to gain an interpretation for 'the book is heavy', we would not be able to combine $\llbracket b o o k \rrbracket($ type (PHYS • INFO) $\rightarrow$ Prop) with any expression requiring an argument of type PHYS $\rightarrow$ Prop. Instead, by contravariance, we have the subtyping relationships shown in (70).
(70) Phys $\rightarrow$ Prop $<_{c}($ Phys $\bullet$ Info $) \rightarrow$ Prop

$$
\text { InFO } \rightarrow \text { Prop }<_{c}(\text { PHYS } \bullet \text { Info }) \rightarrow \text { Prop }
$$

The MTT solution to this is to allow abstraction over types and to say that common nouns denote types (Luo, 2012a). So for example the word 'book' denotes the type Book, which is part of the subtype hierarchies shown in (71).
(71) Book $<_{c}$ Phys $\bullet$ Info $<_{c}$ Phys

Book $<_{c}$ Phys •Info $<_{c}$ Info
And so by contravariance we also have the subtyping relationships shown in (72).
(72) Phys $\rightarrow$ Prop $<_{c}$ (Phys $\bullet$ Info) $\rightarrow$ Prop $<_{c}$ Book $\rightarrow$ Prop

$$
\text { InFo } \rightarrow \text { Prop }<_{c}(\text { PHyS } \bullet \text { InFo }) \rightarrow \text { Prop }<_{c} \text { Book } \rightarrow \text { Prop }
$$

Of course this involves taking a novel look at various other lexical entries as well, which I will now illustrate on the basis of some examples. Let us first consider the non-copredication sentences shown in (73)-(74).
(73) John mastered a book.
(74) John picked up a book.
(73) is interpreted as shown in (75). NB this representation has been chosen for ease of comprehension and is not meant to indicate that Chatzikyriakidis and Luo are committed to quantifier raising, or indeed any particular syntactic claims other than that 'a book' is a constituent.


As can be seen from (75), because the determiner takes a type as its first argument, which is then used to determine the type of its second argument, $\llbracket a b o o k \rrbracket$ requires an argument of type Book $\rightarrow$ Prop. The argument provided in fact is of type Info $\rightarrow$ Prop, meaning that composition can proceed because, as can be seen from the subtype hierarchy in (72), InFo $\rightarrow$ Prop is a subtype of Book $\rightarrow$ Prop-so $\lambda_{1}$ [John mastered $t_{1}$ ] can be coerced to be of type Book $\rightarrow$ Prop.

Similarly, (74) is interpreted as shown in (76).


Phys $\rightarrow$ Prop is a subtype of Book $\rightarrow$ Prop, so $\lambda_{1}$ [John picked up $t_{1}$ ] can be coerced to be of type Book $\rightarrow$ Prop.

What should be noted is that, in both these cases, composition can proceed ultimately because some other type can be coerced to the type Book $\rightarrow$ Prop, and so in the end we have quantifiction over objects of type Book. Exactly the same thing is the case for cases of copredication. First, let us consider (77).
(77) John picked up and mastered a book.

According to the theory of coordination outlined by Chatzikyriakidis and Luo (2012), $\llbracket p i c k e d u p \rrbracket$ and $\llbracket$ mastered $\rrbracket$ are of conjoinable types, because both can be coerced to a common type: (PhYs • Info) $\rightarrow$ (HUMAN $\rightarrow$ Prop). This is shown in (78).

$$
\begin{equation*}
\lambda z_{: \operatorname{PHYS} \bullet I N F O} \cdot \lambda x_{: \mathrm{HUMAN}}(\text { pick-up }(x, z) \wedge \text { master }(x, z)) \tag{78}
\end{equation*}
$$



And then, since (PhYs • Info) $\rightarrow$ Prop is a subtype of Book $\rightarrow$ Prop (as shown in (72)), interpretation proceeds as shown in (79).


Once again, we have quantification over objects of type Book.
Although Chatzikyriakidis and Luo (2012, 2013) and Luo (2010, 2011, 2012b) do not discuss plurality, from these examples of existential quantification it should be clear that the system is not set up to make a distinction between physical predication (74), informational predication (73) and copredication (77) with respect to either the domain or the conditions of quantification, and therefore is not prepared to deal with the issues of counting and individuation identifed in Section 1.2.3 and addressed in Chapter 2. In each case, we have quantification over objects of type Book, with no additional specifications being made as to what will count as a book in any case.

In the case of copredication that does not involve coordination, things are different but not importanly so. So suppose we have (80), with the interpretation as shown in (81).

John mastered a heavy book.


According to Chatzikyriakidis and Luo (2013), the constituent 'heavy book' is interpreted as the Sigma type $\Sigma$ (Book, heavy). This is the type of dependent pairs $\langle a, b\rangle$ where $a$ is a book and $b$ is a proof that $a$ is heavy. Dependent pair types are subtypes of their constituent types, so we have the subtyping relationships shown in (82)-(83).
(82) $\Sigma($ Book, heavy $)<_{c}$ Book $^{16}$
(83) Book $\rightarrow$ Prop $<_{c} \Sigma$ (Book, heavy) $\rightarrow$ Prop

Taking (72) and (83) together, we have (84).

$$
\begin{equation*}
\text { InFo } \rightarrow \text { Prop }<_{c}(\text { PhYs } \bullet \text { Info }) \rightarrow \text { Prop }<_{c} \text { Book } \rightarrow \text { Prop }<_{c} \Sigma(\text { Book, heavy }) \rightarrow \text { Prop } \tag{84}
\end{equation*}
$$

(84) shows that the type InFo $\rightarrow$ Prop can be coerced to the type $\Sigma$ (Book, heavy) $\rightarrow$ Prop, and so $\llbracket a$ heavy book $\rrbracket$ will accept $\llbracket \lambda_{1} J o h n$ mastered $t_{1} \rrbracket$ as an argument. Again, though, we end up with quantification over books as such.

The MTT approach has a neat account of the way in which the different type requirements are resolved in both copredication and non-copredication sentences. But one result of that approach is that quantification always ends up being over objects of dot type when the noun in the quantified noun phrase is of dot type. The system is therefore not set up to make the necessary distinctions between e.g. books individuated physically, books individuated informationally, and books in copredication sentences for the purposes of numerical quantification.

[^42]
### 4.4 Pragmatic approaches

One way to understand the TCL approach described in Section 4.1 is that it seeks to derive ontologically respectable truth conditions for sentences like (12) and (13) by means of an increase in complexity of the semantic composition rules, such that in those sentences both predicates do not apply to the object denoted by their grammatical argument, but rather to something that stands in some kind of defined relation to that object. With this in mind, it is worth noting that it has been argued on the basis of apparently quite different cases that there are very many instances in which a predicate can shift meaning in something like this way, and that the triggers for such shifts are pragmatic in nature rather than determined by compositional processes. ${ }^{17}$ Particularly relevant in this respect is the theory of Geoffrey Nunberg relating to examples like (85) (Nunberg, 2004, p. 346).
(85) I'm parked out back.
(85) can be true if uttered by someone who is in fact inside, provided that he or she is the driver of a car or other parkable vehicle which is parked out back. These and similar examples have prompted Nunberg to develop a theory according to which a predicate can 'transfer' its application in context.

Nunberg points out that, although it is initially appealing, the view that it is the denotation of ' $I$ ' that has shifted here (from the speaker to the speaker's car) is actually false, since (86) is acceptable while (87) is not (ibid., p. 347).
(86) I am parked out back and have been waiting for 15 minutes.
(87) \# I am parked out back and may not start.

If it were 'I' that had undergone meaning transfer, then (87) would be acceptable, since the speaker's car could bear both of the properties attributed to it. Therefore it must be 'parked out back' that has undergone meaning transfer, explaining the acceptability

[^43]of (86). It no longer means parked out back, but rather the driver of a vehicle that is parked out back. Formally, the adjusted meaning of 'parked out back' is calculated as follows (Nunberg, 2004, p. 348, where $H$ is the function from cars to their drivers):
\[

$$
\begin{align*}
& \lambda P \cdot \lambda y \cdot \exists x: x \text { is in the domain of } H((H(x)=y) \wedge P(x))\left[\lambda z \cdot \text { parked-out-}-\operatorname{back}^{\prime}(z)\right]  \tag{88}\\
= & \lambda y \cdot \exists x: x \text { is a car }\left((\text { driver-of }(x)=y) \wedge \operatorname{parked}^{\prime}-\operatorname{out}^{\prime}-\operatorname{back}^{\prime}(x)\right)
\end{align*}
$$
\]

Nunberg does not address the issue of copredication as such, but it is striking that (86) is in some ways similar to the copredication sentences that we have been looking at. For instance, there is no way to change the meaning of 'lunch' in (89) (repeated from Chapter 1) that would deal with the problems of copredication.
(89) The lunch was delicious but went on for hours.

However, if it were instead the meaning of one or other of the predicates that changed, then we could imagine that a transfer function had either applied to 'took forever' to transfer its meaning from took forever to participant in an event that took forever if 'lunch' denotes food, or applied to 'was delicious' to transfer its meaning from was delicious to included food that was delicious if 'lunch' denotes an event.

If this kind of analysis is on the right track, what constrains the process by which certain predicates can undergo meaning transfer? Nunberg's story appeals to the pragmatic notions of salience and noteworthiness. For example, $H$ in (88) can be instantiated as the function from cars to their drivers because (i) the relationship between cars and their drivers is contextually salient, and (ii) being the driver of a car that is parked out back makes a person noteworthy in the context.

Some of the pragmatic effects on the acceptability of copredication have been investigated by Regine Brandtner in her dissertation (Brandtner, 2011) on deverbal -ung nominals in German, and an analysis developed on Nunbergian lines. Take the examples (90)-(92) (ibid., p. 169).
(90) \# Die schlecht gemachte Fälschung dauerte lange. The $[\mathrm{bad} \text { done }]^{\text {phys }}$ imitation [lasted long]. event
'The badly-done imitation took a long time.'
(91) Die täuschend echte Fälschung dauerte lange.

The [deceptive true] ${ }^{\text {phys }}$ imitation [lasted long]. .event
'The deceptively real-looking imitation took a long time.'
(92) Die schlecht gemachte Fälschung dauerte trotzdem lange.

The [bad done] ${ }^{\text {phys }}$ imitation [lasted anyway long]. ${ }^{\text {event }}$
'The badly-done imitation still took a long time.'

The copredication of the event (of forging) and resulting object (forgery) senses of 'Fälschung' is not unrestricted, but depends for its acceptability on pragmatic factors; hence (90) is unacceptable. The analysis that Brandtner offers is that in a sentence containing two conflicting predicates applying to a single argument, the second predicate can undergo meaning transfer in such a way as to be compatible with the first, provided that the unshifted predicate and the shifted predicate stand in a predicate coherence relation (ibid., §8.3). The difference between (90) and (91) above is that a forgery's being well-done explains why it would take a long time to complete (in (91)), but that it's being badly-done does not (in (90)); however, (92) is acceptable because a forgery's being badly-done explains why it is surprising that it took a long time to complete (expressed by 'trotzdem'). ${ }^{18}$

The subject of Brandtner's dissertation is exclusively deverbal nominals in German, but it is nevertheless worth examining how well this approach would transfer to other similar cases, since it is worth asking whether or not all copredication could be accounted for on a sense-transfer approach like this.

On Brandtner's account it is always the first predicate that fixes the meaning of the nominal and any subsequent ones that have to shift meaning. So, unlike the accounts considered so far, this kind of pragmatic approach would need to postulate lexical ambiguities for nouns supporting copredication. For instance, we would have one word

[^44]'book' denoting the set of physical books, and a homophone of it denoting the set of informational (or abstract) books. Likewise, we would have one word 'lunch' denoting the set of lunch foods, and a homophone of it denoting the set of lunch events, meaning that (89) would be interpreted as shown in (93), and (94) would be interpreted as shown in (95). ${ }^{19}$
(93) Lunch was delicious but \{eaten during an event that [went on for hours]\}.
(94) Lunch went on for hours but was delicious.
(95) Lunch went on for hours but \{included the serving of food that was delicious $\}$.

The first question to be asked is whether or not copredication in general is subject to the same kind of pragmatic conditions as those of uncontroversially pragmatic process such as the reference transfer in the case of (85). The answer seems to be no. For instance, there is no obvious explanatory relationship between the date on which a book was printed and the coherence or otherwise of its informational content, and yet (96) is perfectly acceptable.
(96) This incoherent book was printed in 1859.

In fact, it does not seem possible to construct an example with the canonical noun supporting copredication 'book' where the combination of physical and informationselecting predicates generates unaccepatability. ${ }^{20}$

Another question worth asking is whether or not we get the same counting effects in cases of reference transfer that we get for copredication, as addressed at length in Chapter 2. That is to say, does (97) require that there are at least two vehicles parked outside, in the same way that (98) requires that there are at least two distinct physical books on the table?
(97) Two angry people are parked outside.

[^45](98) Two books by Dickens are on the table.

Again, the answer seems to be no. (97) can be used to describe a situation in which there is a single car parked outside, and this car has been given a parking ticket, causing its two occupants to become angry. In contrast, (98) cannot be used to describe a situation in which one physical volume, instantiating two informational books by Charles Dickens, is on the table.

It might be objected in this instance that (97) is a collective reading. On this view, it is not the case that each of the two people has been ascribed the property of being the occupant of a car that is parked outside (the same car), but rather that they are considered as one group, which is ascribed the property of being the collective driver of a car that is parked outside.

Two things can be said in response to this point. Firstly, such a reading is not possible in the case of (98). That is to say, (98) cannot be interpreted as saying that two informational books are considered as one group, which is ascribed the property of collectively being instantiated by a single physical volume that is on the table. So there is some difference between the interpretive processes involved in (97) and (98). Secondly, if (97) is a collective reading, then it should not entail (99).
(99) An angry person is parked outside.

It is my judgement that (97) does entail (99); in any situation in which (97) is true (including the one described above), (99) is true. This seems to confirm the analysis of (97) as involving two people each being attributed with the property of standing in some (contextually salient and noteworthy) relation to a car parked outside.

None of this is supposed to cast doubt on Brandtner's analysis of certain sentences containing deverbal -ung nominals in German as involving a pragmatically-driven process of predicate meaning shift that is constrained is various ways. But it intended to be taken as evidence that not all copredication can be analysed in this way.

## Chapter 5

## Further issues

This chapter addresses questions naturally raised by the treatment of copredication described in Chapters 2 and 3. In Section 5.1 I will pursue the idea that the theory developed so far can be used as the basis of a predictive theory of semantic anomaly that does not fall foul of the compositional problem of copredication (cf. Section 1.2.2). Section 5.2 discusses variability in the acceptability of copredication sentences.

The final two sections of this chapter concern the mereological treatment of nouns supporting copredication (NSCs) on which the theory described in Chapter 2 relied. Section 5.3 addresses criticisms made of this mereological treatment, and Section 5.4 assesses its implications for the philosophical issues raised in Section 1.2.1.

### 5.1 Construction and sortal requirements

Given the place that it had in the theory as described in Chapters 2 and 3, it might appear as though construction is acting simply as a store of ind-relations used for determining truth conditions. But in fact constructions themselves have suggestive properties. Look at $\pi_{2} \llbracket$ Fred picked up and mastered three books】, repeated below as (1). ${ }^{1}$
(1) $\quad \lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO}) \wedge h\left(f^{\prime}\right) \sqsubseteq\right.$ ANI $)$

[^46](1) denotes the set of functions that map Fred to a relation $R$ such that $R \sqsubseteq$ Ani, and map some (group of) book(s) to a relation $R$ such that $R \sqsubseteq$ (PHYS $\sqcap$ INFO).

Recall from Section 2.3.1 that for all and only the objects $o$ that are at least partly animate we have $\operatorname{ANI}(o, o)$, and for all and only the objects $o$ that are at least partly physical and at least partly informational we have PHYS $\sqcap \operatorname{INFO}(o, o)$. It therefore seems that (1) can be seen as a record of the sortal requirements introduced by the different predicates in (1), on which a predictive definition of anomaly ${ }^{2}$ can be based. The following definition suggests itself:

Anomaly (first attempt):
(2) A sentence is anomalous if and only if there is no function $f$ (type $e \rightarrow \mathcal{R}$ ) satisfying its construction such that every object $o$ bears the relation $f(o)$ to itself.

$$
S \text { is anomalous } \stackrel{\text { def }}{=} \neg \exists f_{e \rightarrow \mathcal{R}}\left(\pi_{2}(\llbracket S \rrbracket)(f) \wedge \forall x(f(x)(x)(x))\right)
$$

We can see that on the basis of (2), 'Fred picked up and mastered three books' is not anomalous, because there is some function $f$ satisfying (1) such that every object $o$ bears the relation $f(o)$ to itself. For instance, take the function shown schematically in (3), where $b$ is some book and '...' indicates that (3) maps every other entity in the domain to IDENT, the relation of identity.

$$
\left[\begin{array}{lll}
f^{\prime} & \rightarrow & \text { ANI }  \tag{3}\\
b & \rightarrow & \text { PHYS } \sqcap \text { INFO } \\
\ldots & \rightarrow & \text { IDENT }
\end{array}\right]
$$

(3) satisfies (1), Fred is animately equivalent to himself, there is some book that is physically and informationally equivalent to itself, and every object is self-identical. Therefore the existence of (3) shows that (62) is not anomalous.

However, consider (4) below.

[^47](4) $\#$ A number is green.

Given the lexical entries shown in $(5)^{3}-(7)$, the interpretation of (4) would be as shown in (8).

$$
\begin{align*}
& \llbracket a \rrbracket=\lambda P_{e \rightarrow T} \cdot \lambda Q_{e \rightarrow T}\left\langle\exists x\left(\pi_{1}(P(x)) \wedge \pi_{1}(Q(x))\right),\right.  \tag{5}\\
&\left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists x\left(\pi_{1}(P(x)) \wedge f(x) \sqsubseteq\left(\pi_{2}(P(x)(f)) \sqcap \pi_{2}(Q(x)(f))\right)\right)\right\rangle
\end{align*}
$$

(6) $\quad \llbracket n u m b e r \rrbracket=\lambda y_{e}\left\langle\right.$ number $^{\prime}(y), \lambda g_{e \rightarrow \mathcal{R}} \cdot f(y) \sqsubseteq$ MATH $\rangle$
(7) $\llbracket$ be green $\rrbracket=\lambda z_{e}\left\langle\operatorname{green}^{\prime}(z), \lambda h_{e \rightarrow \mathcal{R}} \cdot h(z) \sqsubseteq\right.$ PHYS $\rangle$
(8) $\left\langle\exists x\left(\operatorname{number}^{\prime}(x) \wedge \operatorname{green}^{\prime}(x)\right), \lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{number}^{\prime}(v) \wedge f(v) \sqsubseteq(\right.\right.$ MATH $\sqcap$ PHYS $\left.\left.)\right)\right\rangle$

According to the definition given in (2), (8) is anomalous: there is no function satisfying $\pi_{2}[(8)]$ such that its value with respect to a number is a relation that that number bears to itself (given that no number is partly physical). This is the desired result for (4).

### 5.1.1 Refinement

However, there is a problem with the idea of treating (2) as predictive of semantic anomaly, due to the way in which the restrictor of a quantifier contributes to the construction of the sentence containing it.

Suppose that, intead of 'Fred picked up and masterd three books', we consider 'Fred mastered three heavy books'. $\pi_{2} \llbracket$ Fred mastered three heavy books』 is shown below in (9). ${ }^{4}$

$$
\begin{equation*}
\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{hheavy}^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\operatorname{PHYS} \sqcap \mathrm{INFO}) \wedge h\left(f^{\prime}\right) \sqsubseteq \text { ANI }\right) \tag{9}
\end{equation*}
$$

Now suppose that there are no heavy books in the domain-say, because all the books are light. 'Fred mastered three heavy books' would be false, of course. But according to (2) it would also be anomalous, because there would be no function satisfying its construction, shown in (9)—let alone one that maps every object to a relation

[^48]that that object bears to itself. That is to say, if there are no heavy books, then there is no function with some heavy book in its domain.

The obvious way to go here is to drop the requirement for there to be some function satisfying the construction of a sentence $S$ in order for $S$ to be non-anomalous. This requirement can be weakened to a conditional one, as embodied in (10).

Anomaly (second attempt):
(10) A sentence is anomalous if and only if there is some function satisfying its construction, but no function $g$ satisfying its construction such that every object $o$ bears the relation $g(o)$ to itself.
$S$ is anomalous $\stackrel{\text { def }}{=} \exists f_{e \rightarrow \mathcal{R}}\left(\pi_{2}(\llbracket S \rrbracket)(f)\right) \wedge \neg \exists g\left(\pi_{2}(\llbracket S \rrbracket)(g) \wedge \forall x(g(x)(x)(x))\right)$
The problem now is that the definition of anomaly is too weak. According to the definition given in (10), no existentially quantified sentence with an empty restrictor is anomalous, because no existentially quantified sentence with an empty restrictor would have some function satisfying its construction. For instance, consider (11).
(11) \# Fred attended three heavy books.

The interpretation of (11) would be as shown in (12).

$$
\begin{align*}
& \left\langle\exists x\left(|x| \geq 3 \wedge * \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x) \wedge * \operatorname{attend}^{\prime}\left(f^{\prime}, x\right) \wedge \neg(\operatorname{PHYS} \sqcup \operatorname{EVNT}) \operatorname{comp}(x)\right),\right.  \tag{12}\\
& \left.\lambda h_{e \rightarrow \mathcal{R}} \cdot \exists v\left(* \operatorname{heavy}^{\prime}(v) \wedge * \operatorname{book}^{\prime}(v) \wedge h(v) \sqsubseteq(\operatorname{PHYS} \sqcap \operatorname{EVNT}) \wedge h\left(f^{\prime}\right) \sqsubseteq \operatorname{ANI}\right)\right\rangle
\end{align*}
$$

Now again suppose that there are no heavy books in the domain. The definition given in (10) would fail to predict that (11) is anomalous (as well as false) because there would be no function satisfying $\pi_{2}[(12)]$. The conditions given in (10), then, do not constitute a definition but at best sufficient conditions for anomaly.

The common problem with (2) and (10) as definitions of anomaly is that they ascribe too much importance to the extensions of predicates, which after all are contingent. So according to (2), if the restrictor of a quantifier is empty then the sentence is automatically anomalous, while according to (10), if the restrictor of a quantifier is empty then the sentence is automatically not anomalous. Neither conclusion is warranted.

It seems that this situation should be addressed by an appeal to intensionality. One way to do this is to modalize (2) in such a way as to get (13).

Anomaly (third attempt):
(13) A sentence is anomalous if and only if necessarily there is no function $f$ satisfying its construction such that every object $o$ bears the relation $f(o)$ to itself.
$S$ is anomalous $\stackrel{\text { def }}{=} \square \neg \exists f_{e \rightarrow \mathcal{R}}\left(\pi_{2}(\llbracket S \rrbracket)(f) \wedge \forall x(f(x)(x)(x))\right)$

Unlike (2), the definition given in (13) correctly predicts that (4) is not anomalous, and would not be even in a world with no heavy books. ${ }^{5}$ In order for it to predict that (11) is anomalous, it has to be necessarily the case that there is no (heavy) book that bears the relation EVNT to itself, i.e. no book that is partly an event. This seems right, in that it seems constitutive of the meaning of 'book' that nothing in its extension is an event, and I will proceed on this basis. Spelling this out formally will require the use of meaning postulates for individuation relations such as EVNT, for which see Section A. 3 in the appendix.
(13) inherits a weaker form of one property of (2); it predicts that if the restrictor of a quantifier is necessarily empty then the sentence is automatically anomalous. This means that it does not make a distinction between (14) and (15).
(14) ? Fred has an interesting, uninteresting idea.

$$
\begin{aligned}
& \left\langle\exists x\left(\operatorname{idea}^{\prime}(x) \wedge \operatorname{interest}^{\prime}(x) \wedge \neg \operatorname{interest}^{\prime}(x) \wedge \operatorname{have}^{\prime}\left(f^{\prime}, x\right)\right),\right. \\
& \left.\lambda g_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\text { idea }^{\prime}(v) \wedge \operatorname{interest}^{\prime}(v) \wedge \neg \operatorname{interest}^{\prime}(v) \wedge g(v) \sqsubseteq \operatorname{INFO} \wedge g\left(f^{\prime}\right) \sqsubseteq \mathrm{ANI}\right)\right\rangle
\end{aligned}
$$

(15) \# Fred has a purple idea.

$$
\begin{aligned}
& \left\langle\exists x\left(\operatorname{idea}^{\prime}(x) \wedge \operatorname{purple}^{\prime}(x) \wedge \operatorname{have}^{\prime}\left(f^{\prime}, x\right)\right)\right. \\
& \left.\lambda g_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{idea}^{\prime}(v) \wedge \operatorname{purple}^{\prime}(v) \wedge g(v) \sqsubseteq \operatorname{INFO} \wedge g\left(f^{\prime}\right) \sqsubseteq \mathrm{ANI}\right)\right\rangle
\end{aligned}
$$

[^49]Necessarily, for neither (14) nor (15) is there a function that satisfies its construction, since necessarily there are no ideas that are interesting and uninteresting, nor are there any purple ideas. ${ }^{6}$ But only in (15) is this due to a categorial mismatch of the kind that we have been looking at.

That said, (14) and (15) do both seem odd. One might well argue (as annotated) that they are odd in different ways. However, I will choose at this point to categorise the definition of anomaly to be presented as one that catches both cases, leaving it to future work to distinguish between the two. ${ }^{7}$ That is to say, if necessarily there is no function satisfying the construction of sentence $s$, then $s$ is anomalous. This means that the current theory predicts that both (14) and (15) are anomalous. So the final definition of anomaly is as described in (13), repeated in Definition 3 below along with that of congruity.

Definition 3 (Anomaly and Congruity).
A sentence is anomalous if and only if necessarily there is no function $f$ satisfying its construction such that every object $o$ bears the relation $f(o)$ to itself.

$$
S \text { is anomalous } \stackrel{\text { def }}{=} \square \neg \exists f_{e \rightarrow \mathcal{R}}\left(\pi_{2}(\llbracket S \rrbracket)(f) \wedge \forall x(f(x)(x)(x))\right)
$$

A sentence is congruous if and only if it is not anomalous.

This treatment does, therefore, predict a difference between sentences in which there is a contradiction within the restrictor of a quantifier, like (14), and those in which there is a contradiction within the nuclear scope, like (16).
(16) An idea is interesting and uninteresting.

[^50]The interpretation of (16) would be as shown in (17).

$$
\begin{align*}
& \left\langle\exists x\left(\operatorname{idea}^{\prime}(x) \wedge \text { interesting }^{\prime}(x) \wedge \neg \text { interesting }^{\prime}(x)\right),\right.  \tag{17}\\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\text { idea }^{\prime}(v) \wedge f(v) \sqsubseteq \operatorname{INFO}\right)\right\rangle
\end{align*}
$$

(16) is congruous according to Definition 3: there is some function satisfying $\pi_{2}[(17)]$ that maps every object to a relation that object bears to itself, and hence a fortiori there possibly is. It is my judgement that this distinction between (14) and (16) is a welcome one - (14) is odd in a way that (16) is not, although both are necessarily false.

### 5.1.2 Characteristics of this treatment of anomaly

It is important to see what I am not proposing. This is not a 'naïve pragmatic approach' to anomaly, such as is dispensed with by Magidor (2013, Ch. $5 \S 2$ ). The claim is not that trivial falsity (or trivial truth) itself explains anomaly, as should be clear from the distinction between (14) and (16). If there are pragmatic principles that predict that (16) is infelicitous, then (presumably) (14) is infelicitous according to those principles and also anomalous according to Definition 3. I agree with Magidor that if there are pragmatic principles that predict that (16) is infelicitous because it is trivially false, then they do not account for the anomaly of sentences like (11). Rather, sometimes what it is that makes a sentence trivially false also makes it anomalous according to this definition.

Moreover, although the calculation of construction proceeds alongside that of truthconditional meaning and the two have an effect on each other, it is not the case that anomalousness according to Definition 3 prevents the calculation of truth-conditional meaning. According to this account, the fact that a sentence is anomalous does not preclude it from being meaningful (contra Luo (2010)), having a truth value (contra Thomason (1972) and Shaw (2013)), or even being true. We have already seen examples of sentences that are possibly true and congruous, sentences that are necessarily false and congruous, and sentences that are necessarily false and anomalous. But there are also sentences that are true but anomalous, such as (18), which has the interpretation
shown in (19).
(18) \# No number is green.

$$
\begin{equation*}
\left\langle\neg \exists x\left(\operatorname{number}^{\prime}(x) \wedge \operatorname{green}^{\prime}(x)\right), \lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\text { number }^{\prime}(v) \wedge f(v) \sqsubseteq \mathrm{PHYS}\right)\right\rangle \tag{19}
\end{equation*}
$$

This theory of anomaly does have something in common with Chomsky's (1965) syntactic treatment, to the extent that we can talk about predicates imposing selectional requirements on their arguments, with constructions and individuation relations here fulfilling the role played by selectional features in Aspects. The major difference is that the congruity requirement of this account is entirely model-theoretic and not part of the combinatorics.

### 5.1.3 Anomaly and domain restriction

Shaw (2013) argues that anomaly can cause disruption to expected patterns of inference. For example, there are situations in which speakers will judge (20) true and (21) false, even though we would have thought that their logical forms were as shown in (22) and (23) respectively, and (22) does entail (23).
(20) Bob uprooted everything in his yard and burned it.
(21) Bob burned everything in his yard.
(23) $\forall x\left(\operatorname{in}-\operatorname{yard}^{\prime}(x) \rightarrow \operatorname{burn}^{\prime}\left(b^{\prime}, x\right)\right)$

As an example of such a situation, Shaw (ibid., p. 2) asks us to consider the following:

Bob owns a house with a large yard. In the yard there are six trees and six beautiful hand-carved Scandinavian planks, but nothing else - no bushes, brush, grass or anything of the sort: just dirt. Bob wants to build a fire to keep warm in the winter but is loathe to use those wooden planks. Consequently Bob uproots the six trees and uses them as firewood.

The intuitive explanation for the failure of entailment is that in judging (20), speakers only take into account those things in Bob's yard of which it makes sense to talk about uprooting and burning (so only the trees), whereas in judging (21) they take into account only those things in Bob's yard of which it makes sense to talk about burning (so the trees and the planks). That it does not make sense to talk about uprooting planks is evidenced by the anomalousness of (24).
(24) \# Bob uprooted a plank.

Shaw formalises this intuition by proposing a system according to which predicates are associated with a 'domain of significance' $\langle\rangle\rangle$ in addition to an extension $\llbracket \rrbracket$. The domain of significance of a predicate is the set of things of which it makes sense to attribute that predicate, and consequently for any predicate is a superset of its extension.

The truth conditions for a universally quantified sentence in the system that Shaw develops are then as shown in (25).

$$
\begin{equation*}
\llbracket \text { every } A \quad B \rrbracket=U \text { (ndefined) if }\langle\langle A\rangle\rangle \cap\langle\langle B\rangle\rangle=\emptyset \tag{25}
\end{equation*}
$$

otherwise

$$
\begin{aligned}
& \llbracket \text { every } A B \rrbracket=T \text { if }(\llbracket A \rrbracket \cap(\langle\langle A\rangle\rangle \cap\langle\langle B\rangle\rangle)) \subseteq(\llbracket B \rrbracket \cap(\langle\langle A\rangle\rangle \cap\langle\langle B\rangle\rangle)) \\
& \llbracket \text { every } A B \rrbracket=F \text { if }(\llbracket A \rrbracket \cap(\langle\langle A\rangle\rangle \cap\langle\langle B\rangle\rangle)) \nsubseteq(\llbracket B \rrbracket \cap(\langle\langle A\rangle\rangle \cap\langle\langle B\rangle\rangle))
\end{aligned}
$$

It follows that (20) is true in the situation described, because $\llbracket i n$ Bob's yard $\rrbracket \cap(\langle<$ in Bob's yard $\rangle\rangle \cap\left\langle\left\langle\lambda_{1}\right.\right.$ Bob uprooted $t_{1}$ and Bob burned $\left.\left.\left.t_{1}\right\rangle\right\rangle\right)=$ the set of six trees, which is a subset of $\llbracket \lambda_{1}$ Bob uprooted $t_{1}$ and Bob burned $t_{1} \rrbracket \cap\left(\langle\langle\right.$ in Bob's yard $\rangle\rangle \cap\left\langle\left\langle\lambda_{1}\right.\right.$ Bob uprooted $t_{1}$ and Bob burned $\left.\left.\left.t_{1}\right\rangle\right\rangle\right)=$ the set of six trees. However, (21) is false in the situation described, because 【in Bob's yard $\rrbracket \cap\left(\langle\langle\right.$ in Bob's yard $\rangle\rangle \cap\left\langle\left\langle\lambda_{1}\right.\right.$ Bob burned $\left.\left.\left.t_{1}\right\rangle\right\rangle\right)=$ the set of six trees and six planks, which is not a subset of $\llbracket \lambda_{1}$ Bob burned $t_{1} \rrbracket \cap(\langle\langle$ in Bob's yard $\rangle\rangle \cap\left\langle\left\langle\lambda_{1}\right.\right.$ Bob burned $\left.\left.\left.t_{1}\right\rangle\right\rangle\right)=$ the set of six trees.

It is possible to implement anomaly-driven restricted quantification of this kind in the system proposed in this thesis, by (i) making some amendments to either of the
lexical entries offered for 'every' in Section 3.4, and (ii) making constructions do the work done by Shaw's (2013) domains of significance.

Regarding (ii): for every one-place predicate $P, \Omega_{1}(P)$ is a relation $R$ such that $\{x: R(x, x)\}$ is the set of objects to which $P$ can coherently be attributed. For example, $\Omega_{1}(\llbracket h e a v y \rrbracket)=$ PHYS $=\lambda x \cdot \lambda y$.phys-equiv ${ }^{\prime}(x, y)$. Recall that phys-equiv ${ }^{\prime}(x, x)=T$ for every object $x$ such that $x$ is at least partly physical, and $F$ for every other object. So here we have something like Shaw's domain of significance.

Regarding (i): if we take the lexical entry offered in Section 3.4, then we can adapt it as shown in (26).

$$
\begin{align*}
\llbracket \text { every } \rrbracket=\lambda P_{e \rightarrow T} \cdot \lambda Q_{e \rightarrow T} \cdot\langle\forall x & \left(\left(\pi_{1}(P(x)) \wedge\left(\Omega_{1}(P) \sqcap \Omega_{1}(Q)\right)(x)(x)\right)\right.  \tag{26}\\
& \left.\rightarrow\left(\pi_{1}(Q(x)) \wedge\left(\Omega_{1}(P) \sqcap \Omega_{1}(Q)\right)(x)(x)\right)\right) \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\pi_{1}(A(v)) \wedge \pi_{2}(A(v))(f) \wedge \pi_{2}(B(v))(f)\right)\right\rangle
\end{align*}
$$

In (26), both the restrictor and the nuclear scope of the quantifer are restricted so as to exclude substitution instances that would be anomalous for either of them. For example, on this basis, the interpretation of (20) would be as shown in (27).

$$
\begin{align*}
& \left\langle\forall x \left(\left(\operatorname{in}-\operatorname{yard}^{\prime}(x) \wedge(\operatorname{PLANT} \sqcap \operatorname{PHYS})(x)(x)\right)\right.\right.  \tag{27}\\
& \left.\quad \rightarrow\left(\left(\operatorname{uproot}^{\prime}\left(b^{\prime}, x\right) \wedge \operatorname{burn}^{\prime}\left(b^{\prime}, x\right)\right) \wedge(\text { PLANT } \sqcap \operatorname{PHYS})(x)(x)\right)\right), \\
& \lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{in}^{\left.\left.-\operatorname{yard}^{\prime}(v) \wedge f(v) \sqsubseteq(\text { PLANT } \sqcap \text { PHYS })\right)\right\rangle}\right.
\end{align*}
$$

Or equivalently:

$$
\begin{align*}
& \rightarrow\left(\left(\operatorname{uproot}^{\prime}\left(b^{\prime}, x\right) \wedge \operatorname{burn}^{\prime}\left(b^{\prime}, x\right)\right)\right.  \tag{28}\\
& \wedge\left(\operatorname{plant}^{\text {equiv }}{ }^{\prime}(x, x) \wedge{\left.\left.\left.\operatorname{phys}-\operatorname{equiv}^{\prime}(x, x)\right)\right)\right), ~}_{\text {, }}\right. \text { ) } \\
& \left.\lambda f_{e \rightarrow \mathcal{R}} \cdot \exists v\left(\operatorname{in}-\operatorname{yard}^{\prime}(v) \wedge f(v) \sqsubseteq(\operatorname{PLANT} \sqcap \mathrm{PHYS})\right)\right\rangle
\end{align*}
$$

So on this basis, (20) is true if and only if the set of things in Bob's yard that are plant- and physically equivalent to themselves is a subset of the things that Bob uprooted and burned and that are plant- and physically equivalent to themselves.

The relation of plant-equivalence is evidently much less intuitive than the relation of physical equivalence. I cannot think of any evidence that it plays a role in individuation in the way that physical equivalence does as discussed in Chapter 2. However, it is easy enough to define and understand. $x$ and $y$ are plant-equivalent if and only if both $x$ and $y$ have a plant part, and the plant part of $x$ is the same as the plant part of $y$. Since every plant is physical, we have PLANT $\sqsubseteq$ PHYs. Given that this relation (derivatively) plays a role in the imposition of sortal requirements (as in (24)), it can be included in the lexical entry for 'uproot' as shown in (29).

$$
\begin{equation*}
\llbracket u p r o o t \rrbracket=\lambda x_{e} \cdot \lambda y_{e}\left\langle\operatorname{uproot}^{\prime}(y, x), \lambda f_{e \rightarrow \mathcal{R}}(f(x) \sqsubseteq \operatorname{PLANT} \wedge f(y) \sqsubseteq \text { PHYS })\right\rangle \tag{29}
\end{equation*}
$$

So, we can have quantifier domain restriction determined by anomaly if we want it. But do we want it? I am undecided as to whether the domain restriction that happens in (20) really is different in kind from more general cases of quantifier domain restriction in context, such as is exhibited by (30).
(30) Every mug is chipped.
(30) is never understood as meaning that every mug that there is is chipped. What it means is that every one of some contextually salient group of mugs is chipped-for example, those in a salient box that has just been in transit (for discussion see Stanley and Szabó (2000)). But this domain restriction is not determined by considerations of anomaly.

Shaw (2013, §1) argues that the kind of domain restriction displayed by (20) is different from the kind displayed by (30) on the basis that (i) the domain of quantification is different in (20) compared to (21) irrespective of the context, and (ii) no matter how salient you make the planks prior to uttering (20), hearers do not include them in the domain of quantification for (20). These points are well-taken, but they do not establish that anomaly contributes to a particular kind of domain restriction, as opposed to contributing to contextual salience in a particularly strong way. I am also not entirely convinced that (31), which Shaw's theory would predict has the same truth conditions
as (20), would be true in the situation described by Shaw.
(31) Bob uprooted and burned everything in his yard.

We can, however, have anomaly-driven domain restriction in the system described in this thesis if that proves necessary, in the way described in (26) above.

### 5.2 Varying acceptability in copredication

We saw in Section 4.4 that there are strong reasons for making a distinction between genuine copredication and pragmatically-driven processes that likewise give the impression of involving the coordination of incompatible properties, such as predicate sense transfer and coercion. However, in some cases the acceptability of copredication depends not only on the selectional properties of the predicates applied to an NSC. For example, the 'building' and 'institution' senses of 'bank' can both be used in (32) (repeated from Chapter 1).
(32) The bank was vandalised after calling in Bob's debt.

There is every evidence that 'bank' licenses different counting properties, as can be seen from (33).
(33) Two banks were vandalised after calling in Bob's debt.
(33) would not be true in a situation in which two branches of the same bankinstitution were vandalised after that single bank-institution called in Bob's debt. Therefore the counting data point to a copredicational treatment of 'bank'.

All the same, (34) shows that it is not simply the case that the word 'bank' licenses the coordination of any predicate appropriate of financial institutions with any predicate appropriate of buildings that those institutions operate.
(34) \# A bank is FTSE-100 listed and used to be a police station.

The challenge then becomes to explain the infelicity of (34) while maintaining the claim that copredication is lexically licensed by having complex objects in its extension. If the acceptability of (32), and the truth conditions of (33), are explained by 'bank' (in this sense) denoting a set of institution+building complex objects, then why is (34) unacceptable?

As noted in Section 4.4, some NSCs are not susceptible to this kind of interferencethat is to say, there are NSCs that support any combination of predicates appropriate to any of the constitutive parts of objects in their extension. 'Book' is an example of one of these; as shown by the contrast between (32) and (34), 'bank' is not.

To put my cards on the table: if there is a branch of Barclays (for example) that used to be a police station, then (34) should have a reading on which it is true. ${ }^{8}$ Nevertheless, it is deviant. I think that the source of the deviancy is that the predicate that applies to buildings, 'used to be a police station', is too easily interpreted in this context as if it were being applied to an institution.

I make the following tentative proposal: some NSCs, such as 'book', only have complex objects in their extension. Others, such as 'bank', are ambiguous. 'bank ${ }_{1}$ ' has building+institution complex objects in its extension, 'bank ${ }_{2}$ ' has institutions in its extension, and ' $\mathrm{bank}_{3}$ ' has buildings in its extension. (34) would be acceptable if 'bank' were disambiguated as 'bank ${ }_{1}$ ', but for some reason the interpretive process fixes on 'bank ${ }_{2}$ ' and hence the sentence is anomalous.

The proposal is admittedly ad-hoc. However, some predictions can be teased out from it. If this idea is on the right track, then it is not so much the precise combination of predicates in (34) that causes anomaly, as it is that the first predicate does not accept copredication with any building-selecting predicate. To test this prediction we

[^51]can compare (34) with (36). Recall that the theory described by Brandtner (2011) accounts for the acceptability or otherwise of predicate sense transfer dependent on whether or not the first predicate and the second shifted predicate stand in a discourse coherence relation (see Section 4.4). ${ }^{9}$ The predicates used in (36) do stand in such a relation: the size of the business explains the size of the building. And yet (36) is anomalous. This may be because 'is FTSE-100 listed' fixes the interpretation of 'bank' to be 'bank ${ }_{2}$ '.
(36) \# A bank is FTSE-100 listed and tall.

This is evidence against the idea that what is wrong with (34) is that it has an unacceptable discourse structure. More investigation is needed into just what it is that makes sentences like (34) and (36) unacceptable.

### 5.3 Addressing criticisms of mereological approaches to copredication

Asher (2011, §5.2) makes three arguments against the view that nouns supporting copredication denote composite objects. Firstly, he raises the worry that taking this kind of mereological approach leads to incorrect predictions regarding the truth conditions of numerically quantified sentences. This is the problem raised in Section 2.2. The problem is solved in the rest of Chapter 2 by the introduction of compressibility statements into our talk of pluralities. The point stands that simply taking a mereological approach to copredication is problematic, but that is not the approach taken in this thesis.

The second argument is that speakers do not ordinarily talk as if books or lunches (for example) really have parts of the kind imagined:

[^52][ N$]$ ormal parts of objects have names and can be referred to. This isn't true of the inhabitants of $\bullet$ types like lunches. (ibid., p. 147)

As support for this claim, Asher (ibid., p. 148) offers the oddness of (37).
(37) Part of the lunch is an event, and part of the lunch is a meal.

However, it is actually not at all clear that Asher's (second) claim is true in general. Reference to the food and event parts of a lunch as such by B in (38) seems perfectly acceptable. ${ }^{10}$
(38) A: How was the lunch?

B: The food part was good, but not the event.
In any case, it is not necessary to assume that always and everywhere the parthood relation between objects should be transparent to speakers. Semantic theories often posit a mereological structure to domains without claiming that that mereological structure is transparent to speakers, for example the idea that a situation is a part of a world (Kratzer, 2014, §7).

Asher's third argument is that there is something philosophically objectionable about the claim that the domain of quantification contains objects made up of parts that are of distinct ontological categories:

We readily make sense of a parthood relation among objects of the same type. [...] A much vaguer notion of parthood must be invoked to explain the inhabitants of - objects on the mereological view. Unrestricted mereological composition aside, we normally do not think of objects as having parts of different types.

I readily grant that it is easier, in the abstract, to think of the parthood relation in terms of the composition of physical objects out of physical parts than it is to think of it in terms of composition of objects out of both physical and eventive parts (for example). But as soon as we consider a concrete example, such as the idea that a lunch

[^53]is an object made up of a part that is food and a part that is an event, there is no mystery as to what parthood means in this case.

Asher's final objection takes the form of a challenge. Even if we can say how the parts of a book or lunch relate to the whole, the question remains: how do they relate to each other? If we suppose, for example, that a book is a complex object with a part that is physical and a part that is informational, we have to explain how a change in one part could lead to a change in another, by admission disjoint, part:

In each case we would have to elaborate some sort of special causal or other relation telling us how changes in one part might affect another. But this seems crazy for inhabitants of $\bullet$ types. When I tear pages out of a book, an alteration in the physical part doesn't cause a change in the informational part; it's not that there are two parts - there is just one object, the book, with two aspects. (Asher, 2011, p. 148)

This is a difficult issue to tackle, but a version of it must be faced by any account of copredication. Take Asher's theory as an example (Section 4.1). In this theory, the book is a particular that stands in the o-elab relation to its physical and informational aspects, without being constituted by them. Yes, Asher says that the physical aspect of a book is just a way of conceptualising that book; however, in the model theory those two things are distinct objects (ibid., pp. 156-157). On either theory one would have to add axioms in order to describe how changes to one part of an object, or aspect of an object, leads to changes in the object. I do not see how Asher's theory has an advantage here.

### 5.4 Semantics and ontological commitment

This discussion of the philosophical scruples that one might have with the mereological view of NSCs leads naturally into consideration of the wider philosophical implications of this account of copredication.

The research project that became this thesis was, in large measure, motivated by the following question and observations by Chomsky (2000, p. 16):

Suppose the library has two copies of Tolstoy's War and Peace, Peter takes out one, and John the other. Did Peter and John take out the same book, or different books? If we attend to the material factor of the lexical item, they took out different books; if we focus on its abstract component, they took out the same book. We can attend to both material and abstract factors simultaneously...

When we 'attend to both material and abstract factors simultaneously', we have copredication. The moral that Chomsky (ibid., p. 17) draws from this is

It makes little sense to ask to what thing the expression "Tolstoy's War and Peace" refers, when Peter and John take identical copies out of the library. The answer depends on how the semantic features are used when we think and talk, one way or another. In general, a word, even of the simplest kind, does not pick out an entity of the world, or of our "belief space".

I reject this conclusion. Indeed, to anticipate slightly where this section is heading, it is by asking what it is that an NSC like 'book' refers to that we can get some insight into what the relevant semantic features are, and how they are used when we think and talk.

It is time to consider the argument that copredication makes semantic externalism implausible. We can take Collins's (2011) definition of externalism as a starting point:

Linguistic externalism: The explanations offered by successful linguistic theory (broadly conceived) entail or presuppose externalia (objects or properties individuated independent of speaker-hearers' cognitive states). The externalia include the quotidian objects we take ourselves to talk about each day.

So, I take it, the claim that the meaning of the word 'book' is that it denotes a set of real world objects, would be an externalist claim. ${ }^{11}$ Segal (2012, p. 289) summarises the argument from copredication against externalism in semantics as follows:

An utterance of [(39)] could easily be true.
(39) John gave a book to Mary, but she already had it, so he read it himself then shredded it.

[^54]But then 'book' extends over objects that are both abstract and concrete and 'it' refers to something that is both abstract and concrete. But nothing is both abstract and concrete. So the ideas of extension and reference are kaput.

The response offered by Segal (2012, p. 299) is that
[(39)] means something like [(39')]:
(39') John bought [(a copy of) [a book $]_{\mathrm{i}} \mathrm{i}_{\mathrm{j}}$ for Mary. But Mary already had (a copy of) $i t_{i}$. So he read $i t_{j}$ then shredded $i t_{j}$.

But then nothing in the logical form of [(39)] needs to extend over anything that is both abstract and concrete.

The response offered in this thesis is in some ways similar to Segal's, ${ }^{12}$ but in some ways more simple-minded: according to the theory described in Chapter 2, some things are both (partly) abstract and (partly) concrete, namely books (and other informationbearing objects such as magazines, albums etc.).

As such, the approach taken here is not quite the same as that taken in rebuttal of other arguments against externalism that have been made. For example, the argument has been made (e.g. by Hornstein (1984) and Pietroski (2005)) that externalism about semantics would commit those who accept (40) to the existence of someone who (i) is the average American, and (ii) has 2.3 children-which is obviously an absurd conclusion.
(40) The average American has 2.3 children.

Higginbotham $(1985,1993)$ suggests that this difficulty will go away once we have the appropriate analysis of the logical form of (40), which will reveal that 'the average American' should not be analysed as a referring expression on a par with e.g. 'the tall American'. ${ }^{13}$ This suggestion has been implemented in some detail by Kennedy and Stanley (2009), on whose analysis 'the average American' denotes a function of type

[^55]$(d \rightarrow(e \rightarrow t)) \rightarrow(d \rightarrow t)$, where $d$ is the type of degrees, and degree terms such as this and ' 2.3 ' can undergo quantifier raising.

The revised mereological account of copredication (RMA) is not of the same kind. There is nothing special about the interpreted syntactic form of copredication sentences in comparison with non-copredication sentences, and nouns that support copredication are of the same semantic type as those that do not (although their compositional potential is subtly different). The response to the philosophical problem of copredication is that the supposedly problematic objects involved do in fact exist, as complex objects.

Chomsky (2003, p. 290) is not impressed by the claim that such things exist, or even that speakers believe that they exist:

I doubt that people think that among the constituents of the world are entities that are simultaneously abstract and concrete (like books and banks)

Are we, then, simply left trading intuitions? After all, one might well take the acceptability of sentences like (39) in itself as evidence that speakers are tacitly committed to the existence of physical+informational composite objects. This I take to be in the spirit of what Ludlow (2003, pp. 149,153) suggests:
[an] I-substance is what it appears we are talking about based upon our use of language [...] we may well find that I-substances are entirely plausible candidates for the referents of a semantic theory

As they say, one man's modus ponens is another's modus tollens. What is there to choose between the argument shown in (41) and that shown in (42)?
(41) If the revised mereological account of copredication is correct, then (speakers believe that) there are physical+informational complex objects.

It is not the case that (speakers believe that) there are physical+informational complex objects.

The revised mereological account of copredication is incorrect.
(42) If the revised mereological account of copredication is correct, then (speakers believe that) there are physical+informational complex objects.

The revised mereological account of copredication is correct.
(Speakers believe that) there are physical+informational complex objects.
Importantly, the motivation for the second premise of (42) is not that it enables us to get out of philosophical worries about copredication; the motivation is that it enables us to get the facts right about the truth conditions of numerically quantified copredication sentences.

However, the genuine internalist response is probably not to reject that premise, but rather to reject the conditional premise of both arguments. On this view, although the RMA is couched in terms of reference and truth, it could be re-cast in an internalistically-acceptable way and still have the same explanatory force. Suggestions are often made to this effect; in fact, according to Collins (2009, p. 66), it would not even have to be re-cast in order to be internalistically acceptable:

We are assuming a so-called 'truth-conditional semantics'. The use of 'truth' (or 'satisfaction', 'reference', etc.), however, does not establish externalism. We cannot simply read externalism off of the theory because its central theoretical terms are colloquially read as externalist. One has to see what a semantic theory actually explains.

So the relevant question becomes: are speaker truth-value judgements in given situations one of the things that a semantic theory 'actually explains'? If so, then we have to read the notions of 'truth' (or 'satisfaction', 'reference', etc.) in the straightforward 'colloquial' way. When we ask a respondent for a truth-value judgement in a given situation, what are we asking for if not a judgement regarding the connection between a sentence and the external world? The claim that such a judgement is the result of a 'massive interaction effect' (Pietroski, 2005, p. 254) between (internalist) meaning and myriad other cognitive systems that will escape our understanding for some time yet cannot be conjoined with the claim that our semantic theories nevertheless explain
speaker truth-value judgements, or are answerable to them. And if we say that our theories are not answerable to speaker truth-value judgements, then we have cut ourselves off from our main source of data for constructing semantic theories in the first place. ${ }^{14}$

Suppose, though, that accounting for speaker truth-value judgements is not a burden that a semantic theory has to (or should be made to) bear. In that case, the claim made above, that the RMA 'enables us to get the facts right about the truth conditions of numerically quantified copredication sentences', needs to be re-evaluated. Getting the facts right cannot be a matter of making the correct predictions regarding speaker truthvalue judgements in given situations, but it can involve making the correct predictions 'in order to explain what semantic theories actually explain-for example, facts about entailment relations' (ibid., p. 255).

Let us examine the case of entailment relations. In Section 4.1.1 I criticised Asher's (2011) account of copredication on the grounds that it failed to predict the entailments shown in (43) and (44).
(43) Fred picked up and mastered three books. $\Rightarrow$ Fred picked up three books.
(44) Fred picked up and mastered three books. $\Rightarrow$ Fred mastered three books.

An internalist could well argue that Asher got into the position of failing to predict these entailments because of worrying too much about ontological quandaries. After all, if we did not worry about the interpretation of our metalanguage and simply used it in order to predict relations between sentences such as entailment, then we could treat 'book' like any other noun and represent 'Fred picked up and mastered three books' as shown in (45), from which (46) and (47) follow, thus predicting the correct entailments.

$$
\begin{align*}
& \text { (45) } \exists x\left(|x| \geq 3 \wedge \operatorname{pick-up}^{\prime}\left(f^{\prime}, x\right) \wedge \operatorname{master}^{\prime}\left(f^{\prime}, x\right)\right)  \tag{45}\\
& \text { (46) } \exists x\left(|x| \geq 3 \wedge \operatorname{pick}^{\prime} \operatorname{up}^{\prime}\left(f^{\prime}, x\right)\right)  \tag{46}\\
& \text { (47) } \exists x\left(|x| \geq 3 \wedge \operatorname{master}^{\prime}\left(f^{\prime}, x\right)\right)
\end{align*}
$$

[^56]But it would a mistake to treat 'book' just like every other noun, even on the basis of entailments alone. The account of copredication proposed in this thesis does predict the entailments shown in (43) and (44). Importantly, it also predicts the non-entailment shown in (48), as demonstrated in Section 3.4.
(48) Bob memorised every book on the table. There are at least two books on the table. $\nRightarrow$ Bob memorised at least two books.

Treating 'book' like any other noun would deliver the logical form shown in (49) (repeated from Section3.4) and hence would erroneously predict the argument form shown in (48) to be valid.

$$
\begin{align*}
& \forall x\left(\left(\text { on-table }^{\prime}(x) \wedge \operatorname{book}^{\prime}(x)\right) \rightarrow \operatorname{memorise}^{\prime}\left(b^{\prime}, x\right)\right)  \tag{49}\\
& \exists x\left(|x| \geq 2 \wedge * \text { on-table }^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x)\right) \\
& \exists x\left(|x| \geq 2 \wedge * \text { memorise }^{\prime}\left(b^{\prime}, x\right) \wedge * \operatorname{book}^{\prime}(x)\right)
\end{align*}
$$

I am not claiming that no internalistically-acceptable theory could make the right predictions about these (non-)entailments! One could simply take the theory proposed in this thesis and interpret it internalistically. But I do question how likely one would be to get to that kind of theory without a motivation for keeping (or making) semantic theory ontologically respectable, at least given a suitably generous conception of what is ontologically respectable. Just as a matter of methodology, the path from getting the truth conditions right to getting the entailment relations right is much clearer than that of getting the entailment relations right without concern for truth conditions (or with concern for 'truth conditions' that cannot be tested by speaker truth-value judgements). This applies as much for copredication as it does for the semantics of 'average'. The theory described by Kennedy and Stanley (2009) correctly predicts that (40) and (50) are mutually entailing, because it predicts that both sentences have the (ontologically respectable) truth conditions shown in (51).
(50) Americans have 2.3 children on average.
(51) $\frac{\sum_{\operatorname{american}^{\prime}(x)} \max \left\{d: \exists v\left(\left({ }^{*} \operatorname{child}^{\prime}(v) \wedge|v|=d\right) \wedge \operatorname{have}^{\prime}(x, v)\right)\right\}}{\left|\operatorname{american}^{\prime}\right|}=2.3$

No progress is likely to be made towards predicting the mutual entailment of (40) and (50) by supposing that 'the average American' functions just like 'the tall American' which is a premise of the argument against externalism based on (40). Likewise, no progress is likely to be made towards predicting the entailment relations of numericallyquantified copredication sentences without seeing that something compositionally unusual is going on in those sentences. Even if thoroughgoing externalism is unsustainable in the long run, the attempt to keep semantic theory externalistically viable is methodologically healthy because it forces us to consider analyses that postulate hidden complexity, giving results that internalists and externalists alike can appreciate.

## Chapter 6

## Conclusion

In Chapter 1, the introduction to this thesis, I laid out three issues raised by copredication for linguistic theories and our understanding of them. There is the philosophical issue (Section 1.2.1): prima facie, copredication casts doubt on the place of a reference relation in semantic theory. There is the compositional issue (Section 1.2.2): copredication indicates that some combinations of predicates are acceptable that other data would lead us to expect to be anomalous. And finally there is the issue of counting and individuation (Section 1.2.3): many quantified copredication sentences have truth conditions that cannot be accounted for given standard assumptions, because the predicates used impose distinct criteria of individuation on the objects to which they apply.

I then addressed those three issues in reverse order, dedicating the bulk of the thesis to tackling the issue of counting and individuation before using the resulting system as a perspective from which to engage with the other two issues. The key assumptions involved in developing that system were that (i) nouns supporting copredication have sets of complex objects in their extension, (ii) predicates include in their lexical entries a specification of how their arguments are to be individuated, (iii) quantifiers can access and exploit those specifications. The truth conditions predicted for e.g. 'three N VP' are that there is a plurality $P$ consisting of three N that are VP, and that all the members of $P$ are distinct from each other in a way determined by the semantics of N and VP.

In Chapter 4 I compared this theory with other accounts of copredication in the literature. I showed that none of the existing accounts predicts the correct truth conditions for numerically quantified copredication sentences, unlike the revised mereological account of copredication proposed in Chapter 2. In Chapter 5 I responded to various formal and conceptual issues raised by this treatment of copredication.

The compositional issue of copredication was addressed in Section 5.1. In common with many other accounts in the literature, according to the approach presented in this thesis, nouns supporting copredication are special in a way that predicts their unusual ability to appear in apparently conflicting predicational environments. However, unlike in other accounts, this specialness belongs to the noun's model-theoretic semantic properties, not its location in a type hierarchy or its possession of internal grammatical features. The approach taken to anomaly makes the predictions that sentences in which there is a contradiction within the restrictor of a quantifier pattern together with sentences more usually regarded as containing 'category errors' in being anomalous; it also predicts that sentences in which there is a contradiction within the nuclear scope of a quantifier are not anomalous (unless for some other reason), even though they are necessarily false. I leave it to future work to test these predictions more thoroughly. According to this approach to anomaly, anomalousness and truth value are only tangentially related; it is possible for a sentence to be false and anomalous and also possible for a sentence to be true and anomalous.

The account proposed in this thesis can be regarded as a rebuttal of the philosophical argument from copredication against externalism in semantics, provided that one is willing to countenance the existence of objects that are the mereological fusions of apparently different kinds of things: for example, the fusion of a physical object with an informational object, or of an informational object with an event, or of a territory with its inhabitants with its government with the buildings constructed on it, etc. Of course, many people baulk at this idea for considered philosophical reasons, but it is not obviously self-contradictory. The reasons for either accepting or rejecting this idea take
us outside of linguistics or the philosophy of language and into metaphysics. The one important consideration from linguistics is not that people talk as if there are such things (which I think both internalists and externalists could in principle agree on), but that supposing that there are such things allows us to make the correct predictions regarding speaker truth-value judgements of numerically-quantified copredication sentences. On this basis, in Section 5.4 I advocated methodological externalism: the idea that the attempt to keep semantic theory externalistically viable - given a suitably generous conception of what is externalistically viable - is good methodology for semantic theory.

Geach (1962, pp. 38-40) famously advocated the view that nouns ('substantival terms') distinguish themselves from other predicates ('predicables') in that the former, but not the latter, supply a 'criterion of identity' that makes claims of sameness coherent, and is a necessary condition for making counting possible. This is in addition to a 'criterion of application'. On this view, we need to know not only what things are Fs, but also what things are the same $F$; there might be cases in which ' $a$ is the same $F$ as $b^{\prime}$ is true, but $a$ is the same $G$ as $b^{\prime}$ is false. And of course, whether or not $a$ and $b$ are the same $F$ matters for the truth conditions of numerically quantified sentences involving $F$ s.

Criticism of this view has tended to take the position that in cases where it seems that ' $a$ is the same $F$ as $b$ ' is true, but $a$ is the same $G$ as $b$ ' is false, what is actually happening is that completely different things are being compared in the two cases, for example individuals in one case, but stages or events in the other case (Barker, 2010).

Into this debate, what we have seen can be used to make a subtle point. When investigating the truth conditions of numerically quantified sentences, nominal meaning is not the only place to look in order to see how things are individuated (and neither is nominal meaning plus pragmatics). The phenomenon of copredication indicates that the way in which we as speakers conceptualise the link between words and objects is not uniform. For the most part, (disambiguated) nouns make the same contribution to sentential truth conditions whatever is predicated of them-but that is not the
case for nouns supporting copredication, as can be seen from the varying criteria of individuation and counting for 'book', for example. However, this is not to say that the contribution is unsystematic. Criteria of individuation can be emergent, and determined compositionally.

## Appendix A

## Proofs

This appendix contains definitions of generalised determiners and generalised conjunction for the system described in Chapter 2. It also contains some remarks on how to systematise the constructions.

Here, I will adopt the following additional abbreviatory conventions for type assignments:

1. $a b$ abbreviates $a \rightarrow b$.
2. Brackets associate to the right, so $a b c$ abbreviates $a \rightarrow(b \rightarrow c)$.
3. $a^{n}$ abbreviates $n$ repetitions of $a$.

So for example, $e^{2} t$ abbreviates eet abbreviates $e \rightarrow(e \rightarrow t)$. As before, $\mathcal{R}$ abbreviates $e \rightarrow(e \rightarrow t)$ and $T$ abbreviates $t \times((e \rightarrow \mathcal{R}) \rightarrow t)$.

## A. 1 Determiners

First we define a family of functions of type $\left(e^{n} T\right) \mathcal{R}$ for $n \geq 1$, based on the $\Omega$ function introduced in Chapter 2 and repeated as (1) below.

$$
\begin{align*}
& \Omega\left(A_{\left(e^{2} \mathcal{R}\right) t}\right) \stackrel{\text { def }}{=} \bigsqcup\left\{R: \exists x_{e} \exists f_{e \rightarrow \mathcal{R}}(A(x)(f) \wedge f(x)=R)\right\}  \tag{1}\\
& \Omega_{n}\left(P_{e^{n} T}\right) \stackrel{\text { def }}{=} \Omega\left(\lambda x_{1 e} \cdot \lambda f_{e \rightarrow \mathcal{R}} \cdot \exists x_{2} \ldots \exists x_{n}\left(\pi_{2}\left(P\left(x_{1}\right) \ldots\left(x_{n}\right)\right)(f)\right)\right) \tag{2}
\end{align*}
$$

Now we are in a position to allow determiners to be polymorphic. The example that I will use is the example most used in Chapter 2.

$$
\begin{align*}
& \llbracket(\text { at least } \text { three』 }=  \tag{3}\\
& \begin{aligned}
& \lambda A_{e T} \cdot \lambda B_{e^{n} T} \cdot \lambda x_{1_{e}} \ldots \lambda x_{n-1}\langle\langle\exists y(|y| \geq 3 \wedge \pi_{1}(A(y)) \wedge \pi_{1}\left(B(y)\left(x_{1}\right) \ldots\left(x_{n-1}\right)\right) \\
&\left.\wedge \neg\left(\Omega_{1}(A) \sqcup \Omega_{1}(B)\right) \operatorname{comp}(y)\right) \\
& \lambda f_{e \mathcal{R}} \cdot \exists v\left(\pi_{1}(A(v)) \wedge \pi_{2}(A(v))(f)\right. \\
&\left.\left.\wedge \pi_{2}\left(B(v)\left(x_{1}\right) \ldots\left(x_{n}\right)\right)(f)\right)\right\rangle
\end{aligned}
\end{align*}
$$

(52) and (71) in Chapter 2 follow from the definition given in (3).

## A. 2 Conjunction

There is no single schema from which every form of conjunction can be derived. The basic reason for this is that, due to the form that compressibilty statements have to take, when we conjoin two predicates, e.g. 'heavy' and 'informative', we do not want the individuation function of the resulting complex predicate to be the boolean meet of the individuation functions of each of the conjuncts, but rather the boolean join. Put differently, we want it to be the case that when we have $\Omega_{1}(\llbracket b e ~ h e a v y \rrbracket)=$ PHYS and $\Omega_{1}(\llbracket$ be informative $\rrbracket)=\mathrm{INFO}$, that $\Omega_{1}(\llbracket$ be heavy and informative $\rrbracket)=\mathrm{PHYS} \sqcup \mathrm{INFO}$, not PHYS $\sqcap$ INFO.

What this means is that the the logical constant 'and', introduced in Chapter 5 and repeated as (4) below, cannot be generalized to define conjunctions for $n$-place predicates of individuals or for modifiers of those predicates.

$$
\begin{equation*}
\mathrm{and}^{\prime} \stackrel{\text { def }}{=} \lambda T_{T} \cdot \lambda U_{T}\left\langle\left(\pi_{1}(T) \wedge \pi_{1}(U)\right), \lambda f_{e \rightarrow \mathcal{R}}\left(\pi_{2}(T)(f) \wedge \pi_{2}(U)(f)\right)\right\rangle \tag{4}
\end{equation*}
$$

It can, however, be generalized to define conjunctions for predicates of predicates (e.g. DPs) in the straightforward way shown in (5) below.

$$
\begin{equation*}
\lambda G_{(e T)^{n} T} \cdot \lambda H_{(e T)^{n} T} \cdot \lambda P_{1 e T} \ldots \lambda P_{n e T} \cdot \text { and }^{\prime}\left(G\left(P_{1}\right) \ldots\left(P_{n}\right)\right)\left(H\left(P_{1}\right) \ldots\left(P_{n}\right)\right) \tag{5}
\end{equation*}
$$

So for example, the form for conjoining DPs is as shown in (6).

$$
\begin{equation*}
\llbracket a n d_{D P} \rrbracket=\lambda G_{(e \rightarrow T) \rightarrow T} \cdot \lambda H_{(e \rightarrow T) \rightarrow T} \cdot \lambda D_{e \rightarrow T} \cdot \operatorname{and}^{\prime}(G(P))(H(P)) \tag{6}
\end{equation*}
$$

When it comes to $n$-place predicates of individuals, what we want is a schema that will act like a generalization of (4) except that it achieves the effect described in the first paragraph of this section. A schema that does this is shown in (7).

$$
\begin{align*}
& \lambda A_{e^{n} T \cdot} \cdot \lambda B_{e^{n} T \cdot} \lambda x_{1 e} \ldots \lambda x_{n e}  \tag{7}\\
& \qquad \begin{aligned}
&\left\langle\pi_{1}\left(A\left(x_{1}\right) \ldots\left(x_{n}\right)\right) \wedge \pi_{1}\left(B\left(x_{1}\right) \ldots\left(x_{n}\right)\right),\right. \\
& \lambda f_{e \mathcal{R}}\left(\exists g\left(\pi_{2}\left(A\left(x_{1}\right) \ldots\left(x_{n}\right)\right)(g) \wedge f \sim_{x_{1}, \ldots, x_{n}} g\right)\right. \\
& \wedge \exists h\left(\pi_{2}\left(B\left(x_{1}\right) \ldots\left(x_{n}\right)\right)(h) \wedge f \sim_{x_{1}, \ldots, x_{n}} h\right) \\
& \wedge f\left(x_{1}\right) \sqsubseteq\left(\Omega_{n}(A) \sqcup \Omega_{n}(B)\right) \\
& \wedge \ldots \\
&\left.\left.\wedge f\left(x_{n}\right) \sqsubseteq\left(\Omega_{1}\left(A\left(x_{1}\right) \ldots\left(x_{n-1}\right)\right) \sqcup \Omega_{1}\left(B\left(x_{1}\right) \ldots\left(x_{n-1}\right)\right)\right)\right)\right\rangle
\end{aligned}
\end{align*}
$$

(3) in Chapter 3 follows from the definition given in (7). So does the VP conjunction shown as (8) below.
(8) $\quad \lambda A_{e T} \cdot \lambda B_{e T} \cdot \lambda x_{e}\left\langle\left(\pi_{1}(A(x)) \wedge \pi_{1}(B(x))\right)\right.$,

$$
\begin{aligned}
& \lambda f_{e R}\left(\exists g\left(\pi_{2}(A(x))(g) \wedge f \sim_{x} g\right)\right. \\
& \left.\left.\quad \wedge \exists h\left(\pi_{2}(B(x))(h) \wedge f \sim_{x} h\right) \wedge f(x) \sqsubseteq\left(\Omega_{1}(A) \sqcup \Omega_{1}(B)\right)\right)\right\rangle
\end{aligned}
$$

It follows that $\llbracket b e$ heavy and informative is as shown in (9).
(9) $\quad \lambda x_{e}\left\langle\left(\operatorname{heavy}^{\prime}(x) \wedge \operatorname{inform}^{\prime}(x)\right)\right.$,

$$
\begin{gathered}
\lambda f_{e R}\left(\exists g\left(g(x) \sqsubseteq \operatorname{PHYS} \wedge f \sim_{x} g\right)\right. \\
\left.\left.\wedge \exists h\left(h(x) \sqsubseteq \operatorname{INFO} \wedge f \sim_{x} h\right) \wedge f(x) \sqsubseteq(\text { PHYS } \sqcup \mathrm{INFO})\right)\right\rangle \\
=\lambda x_{e}\left\langle\left(\operatorname{heavy}^{\prime}(x) \wedge \operatorname{inform}^{\prime}(x)\right), \lambda f_{e R} \cdot f(x) \sqsubseteq(\text { PHYS } \sqcup \mathrm{INFO})\right\rangle
\end{gathered}
$$

Call the conjunction that conjoins two expression of type $\tau$ ' $\& \tau$ '. So for example (8) is $\&^{e T}$. The schema for defining conjunctions for modifiers can then be defined as
shown in（10）．
（10）For every type $\tau, \& \tau \tau \stackrel{\text { def }}{=} \lambda M_{\tau \tau} \cdot \lambda N_{\tau \tau} \cdot \lambda P_{\tau} \cdot \&^{\tau}(M(P))(N(P))$

So suppose we wanted to know 【heavy and informative】，where 【heavy】 and 【informative】 are nominal－modifying adjectives（type $(e T)(e T))$ rather than predicatives（type $e T$ ）． We would need to instantiate（10）as shown in（11）．

$$
\begin{align*}
& \&(e T)(e T) \stackrel{\text { def }}{=} \lambda M_{(e T)(e T)} \cdot \lambda N_{(e T)(e T)} \cdot \lambda P_{e T} \cdot \&^{e T}(M(P))(N(P))  \tag{11}\\
& \begin{aligned}
&=\lambda M_{(e T)(e T)} \cdot \lambda N_{(e T)(e T)} \cdot \lambda P_{e T} \cdot \lambda x_{e}\left\langle\left(\pi_{1}(M(P)(x)) \wedge \pi_{1}(N(P)(x))\right),\right. \\
& \lambda f_{e R}\left(\exists g\left(\pi_{2}(M(P)(x))(g) \wedge f \sim_{x} g\right)\right. \\
& \wedge \exists h\left(\pi_{2}(N(P)(x))(h) \wedge f \sim_{x} h\right) \\
&\left.\left.\wedge f(x) \sqsubseteq\left(\Omega_{1}(M(P)) \sqcup \Omega_{1}(N(P))\right)\right)\right\rangle
\end{aligned}
\end{align*}
$$

Given the lexical entries for the adjectival forms of＇heavy＇（（54）in Chapter 2）and ＇informative＇，shown as（12）below，the interpretation of＇heavy and informative＇is as shown in（13）below．

$$
\begin{align*}
& \lambda Q_{e \rightarrow T} \cdot \lambda x_{e}\left\langle\left(\text { inform }^{\prime}(x) \wedge \pi_{1}(Q(x))\right),\right.  \tag{12}\\
& \left.\lambda g_{e \rightarrow R}\left(\exists h\left(\pi_{2}(Q(x))(h) \wedge g \sim_{x} h\right) \wedge g(x) \sqsubseteq\left(\operatorname{INFO} \sqcup \Omega_{1}(Q)\right)\right)\right\rangle \\
& \lambda P_{e T} \cdot \lambda x_{e}\left\langle\left(\text { heavy }^{\prime}(x) \wedge *_{\left.\operatorname{inform}^{\prime}(x)\right)},\right.\right.  \tag{13}\\
& \lambda f_{e \mathcal{R}}\left(\exists g \left(\exists i\left(\pi_{2}(P(x))(i) \wedge g \sim_{x} i\right)\right.\right. \\
& \left.\wedge g(x) \sqsubseteq\left(\operatorname{PHYS} \sqcup \Omega_{1}(P)\right) \wedge f \sim_{x} g\right) \\
& \wedge \exists h\left(\exists j\left(\pi_{2}(P(x))(j) \wedge h \sim_{x} j\right)\right. \\
& \left.\wedge h(x) \sqsubseteq\left(\operatorname{INFO} \sqcup \Omega_{1}(P)\right) \wedge f \sim_{x} h\right) \\
& \left.\left.\wedge f(x) \sqsubseteq\left(\left(\operatorname{PHYS} \sqcup \Omega_{1}(P)\right) \sqcup\left(\operatorname{INFO} \sqcup \Omega_{1}(P)\right)\right)\right)\right\rangle
\end{align*}
$$

If you apply（13）to 【books】 then you get（14）as the interpretation of $\llbracket h e a v y ~ a n d ~$
informative books】.

$$
\begin{align*}
& \lambda x_{e}\left\langle\left({ }^{*} \operatorname{heavy}^{\prime}(x) \wedge{ }^{*} \operatorname{inform}^{\prime}(x) \wedge{ }^{*} \operatorname{book}^{\prime}(x)\right),\right.  \tag{14}\\
& \lambda f_{e \mathcal{R}}\left(\exists g \left(\exists i\left(i(x) \sqsubseteq(\mathrm{PHYS} \sqcap \mathrm{INFO}) \wedge g \sim_{x} i\right)\right.\right. \\
& \left.\wedge g(x) \sqsubseteq(\mathrm{PHYS} \sqcup(\mathrm{PHYS} \sqcap \mathrm{INFO})) \wedge f \sim_{x} g\right) \\
& \wedge \exists h\left(\exists j\left(j(x) \sqsubseteq(\text { PHYS } \sqcap \text { INFO }) \wedge h \sim_{x} j\right)\right. \\
& \left.\wedge h(x) \sqsubseteq(\operatorname{INFO} \sqcup(\mathrm{PHYS} \sqcap \mathrm{INFO})) \wedge f \sim_{x} h\right) \\
& \wedge f(x) \sqsubseteq((\text { PHYS } \sqcup(\mathrm{PHYS} \sqcap \mathrm{INFO})) \sqcup(\mathrm{INFO} \sqcup(\mathrm{PHYS} \sqcap \mathrm{INFO}))))\rangle \\
& =\lambda x_{e}\left\langle\left({ }^{*} \operatorname{heavy}^{\prime}(x) \wedge * \operatorname{inform}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x)\right)\right. \text {, } \\
& \lambda f_{e \mathcal{R}}\left(\exists g\left(\exists i\left(i(x) \sqsubseteq(\text { PHYS } \sqcap \mathrm{INFO}) \wedge g \sim_{x} i\right) \wedge g(x) \sqsubseteq \text { PHYS } \wedge f \sim_{x} g\right)\right. \\
& \wedge \exists h\left(\exists j\left(j(x) \sqsubseteq(\text { PHYS } \sqcap \operatorname{INFO}) \wedge h \sim_{x} j\right) \wedge h(x) \sqsubseteq \operatorname{INFO} \wedge f \sim_{x} h\right) \\
& \wedge f(x) \sqsubseteq(\text { PHYS } \sqcup \mathrm{INFO}))\rangle \\
& =\lambda x_{e}\left\langle\left(\text { heavy }^{\prime}(x) \wedge * \operatorname{inform}^{\prime}(x) \wedge * \operatorname{book}^{\prime}(x)\right), \lambda f_{e \mathcal{R}} \cdot f(x) \sqsubseteq(\text { PHYS } \sqcup \mathrm{INFO})\right\rangle
\end{align*}
$$

## A. 3 Meaning postulates for individuation relations

The relations involved in sortal specifications and hence in the formal characterisation of anomaly in this theory, individuation relations, are model-theoretic objects; they are not filters on the calculus of composition or constraints to be satisfied at some representational level. Nevertheless, in order to fulfil the role required of them in this respect, they have to be in some sense meaning-constitutive. For example, in Section 5.1.1 I claimed that it is a necessarily the case that there are no purple ideas (see (15) of Chapter 2), and this fact is crucial in the definition of anomaly (and congruity) given there.

In order to address this issue, I will follow the well-worn path of introducing meaning postulates to restrict the class of admissible models (Dowty, Wall, and Peters, 1981, $\S 7 . \mathrm{V})$ and hence ensure that bearers of certain properties must fall within certain sorts.

First, we need to place some restrictions on the possible construction of any lexical item (recall that construction is defined in Definition 2 of Chapter 2).

As stated in Section 2.3.1, every relation in the range of a construction of a lexical item must be an equivalence relation on some subset of the domain of discourse, outside of which it is empty. What that means is that it is symmetric (15), it is transitive (16), and if an object bears that relation to anything, then it bears it to itself (17).

For any construction $C$ of a lexical item,

$$
\begin{align*}
& \text { (15) } \forall z(C(z) \rightarrow \forall x \forall y(C(z)(x)(y) \rightarrow C(z)(y)(x))) \\
& \text { (16) } \forall v(C(v) \rightarrow \forall x \forall y \forall z((C(v)(x)(y) \wedge C(v)(y)(z)) \rightarrow C(v)(x)(z)))  \tag{15}\\
& \text { (17) } \forall x(C(x) \rightarrow(\exists y(C(x)(x)(y)) \rightarrow C(x)(x)(x)))
\end{align*}
$$

The qualification 'of a lexical item' is important. For example, PHYS $\sqcup$ INFO is included in the range of the construction of 'picked up and mastered'. This relation is not transitive. That is acceptable. However, it should not be in the range of the construction of any lexical item. The relations that we have seen in the range of constructions of lexical items in this thesis, namely PHYS, INFO, (PHYS $\square$ INFO), ANI, EVNT, IDENT and PLANT, all meet conditions (15)-(17).

Next, in order to be able to state the connection between properties and these relations, we introduce the notions of inclusion and exclusion:
(18) $\operatorname{included}^{\prime}\left(P_{e t}, R_{e(e t)}\right) \stackrel{\text { def }}{=} \square \forall x_{e}(P(x) \rightarrow R(x, x))$
$\operatorname{excluded}^{\prime}(P, R) \stackrel{\text { def }}{=} \square \forall x_{e}(P(x) \rightarrow \neg R(x, x))$
We are now in a position to put inclusion and exclusion relations to use in defining our meaning postulates for relating properties to individuation relations:
(19) included' ${ }^{\prime}\left(\right.$ book $^{\prime}$, PHYS $\sqcap$ INFO)
excluded' ${ }^{\prime}$ (book', ANI)
included $^{\prime}\left(\right.$ man $^{\prime}$, PHYS $\sqcap$ ANI $)$
included ${ }^{\prime}$ (table ${ }^{\prime}$, PHYs)
excluded'(idea', PHYs)
included' ${ }^{\prime}$ (purple ${ }^{\prime}$, PHYS)
(N.B. the list in (19) is not supposed to be exhaustive.)

The combination of (18) with (19) gives us (20) and (21).

$$
\begin{equation*}
\square \forall x\left(\operatorname{idea}^{\prime}(x) \rightarrow \neg \operatorname{PHYS}(x, x)\right) \tag{20}
\end{equation*}
$$

$$
\begin{equation*}
\square \forall x\left(\operatorname{purple}^{\prime}(x) \rightarrow \operatorname{PHYS}(x, x)\right) \tag{21}
\end{equation*}
$$

(20) and (21) together entail (22). A sequent calculus proof of this is given in (23), where $I x \stackrel{\text { def }}{=} \operatorname{idea}^{\prime}(x), P x \stackrel{\text { def }}{=} \operatorname{purple}^{\prime}(x)$ and $R x y \xlongequal{=} \operatorname{def} \operatorname{PHYS}(x, y)$.

$$
\begin{equation*}
\square \neg \exists x\left(\operatorname{idea}^{\prime}(x) \wedge \operatorname{purple}^{\prime}(x)\right) \tag{22}
\end{equation*}
$$

$$
\left.\begin{array}{c}
\frac{\overline{P y \vdash P y} A x \quad \overline{R y y \vdash R y y}}{} A x \\
\frac{P y, P y \rightarrow R y y \vdash R y y}{I y \vdash I y} A x \quad \frac{\overline{I y \wedge P y, P y \rightarrow R y y \vdash R y y}}{} \wedge L \\
\frac{I y \wedge P y \vdash I y}{I} \wedge L \quad \frac{I y \wedge P y, P y \rightarrow R y y, \neg R y y \vdash}{I y \rightarrow \neg R y y, P y \rightarrow R y y, I y \wedge P y, I y \wedge P y \vdash} \rightarrow L \\
\frac{I y \rightarrow \neg R y y, P y \rightarrow R y y, I y \wedge P y \vdash}{I} \rightarrow L
\end{array}\right] L
$$

The meaning postulates therefore guarantee that (22) is true in all admissible models. It in the current system, then, it is provable that it is necessarily the case that there are no purple ideas.

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[^0]:    ${ }^{1}$ For the relevance of the qualification 'as commonly understood', see p. 148 ff .

[^1]:    ${ }^{2}$ Internalist theories are also defended, on various grounds, by Hinzen (2008), Hornstein (1984), Jackendoff (2002), McGilvray (1998), and Stainton (2007).

[^2]:    ${ }^{3}$ A point also made by Ludlow (2003) in his discussion of Higginbotham's (1993) programmatic reliance on logical form to avoid ontological difficulties.
    ${ }^{4}$ That this claim is ontologically respectable is not without controversy. See Section 5.3.

[^3]:    ${ }^{5}$ This simplified formulation should not be attributed to any of the authors just cited, but is simply meant to illustrate the general idea.

[^4]:    ${ }^{6}$ Not always: it is the case for the approaches discussed in Sections 4.1 and 4.3, but not that discussed in 4.2.

[^5]:    ${ }^{7}$ Or perhaps, this is what hearers expect, given the use of these predicates.

[^6]:    ${ }^{1}$ Likewise, (13) from chapter 1 can be true because 'lunch' denotes a set of (lunch event + lunch food) composite objects, and similarly for other NSCs. However, I am going to focus on 'book', because of its particular properties when it comes to counting and individuation, as illustrated in (1)-(4). But to clarify: 'lunch' and 'book' are not semantically different in this respect. Rather, the difference is that it is much easier to think of situations in which one physical book instantiates multiple informational book or one informational book is instantiated by multiple physical books that it is to think of situations in which one lunch meal is spread out of over more than one lunch event.

[^7]:    ${ }^{2}$ The ' + ' symbols here indicates the parts making up a single complex object, not individuals making up a plurality. So for example $a+b$ indicates a singular object, while $a \oplus b$ indicates a plurality. Cf. Figure 2.2.

[^8]:    ${ }^{3}$ Here and throughout this chapter, a simple numeral ' $n$ ' is taken to mean 'at least $n$ ', and lexical entries are given accordingly. Other uses of numerals are discussed in Section 3.2.1.

[^9]:    ${ }^{4}$ Or intension. For the sake of simplicity I will consider only extensional contexts in this paper, but everything should be adaptable to an intensional system in due course.

[^10]:    ${ }^{5}$ And maybe some others. As indicated in Section 3.4, I do not treat singular (first-order) quantifiers 'some' and 'every' this way, but that leaves our options open with respect to 'most' and other proportional quantifiers, for example.
    ${ }^{6}$ Here and throughout this thesis, I will use the lambda calculus as a metalanguage. Expressions in the lambda calculus should be understood as standing in for their interpretations in a model.
    ${ }^{7}$ To say that $|x| \geq n$ is just to say that there are at least $n$ atomic parts of $x$, i.e. $\mid\left\{y\right.$ : a-part $\left.{ }^{\prime}(y, x)\right\} \mid \geq$ $n$.

[^11]:    ${ }^{8}$ This is slightly different to the double star 'cumulation' operator sometimes seen, e.g. in (Beck and Sauerland, 2000), in that it is more restrictive. The difference is probably not crucial.
    ${ }^{9}$ Link (1983) uses the infix predicate ' $\Pi$ ' for this purpose. I have adopted the notation due to Cann, Kempson, and Gregoromichelaki (2009, p. 128) in order to avoid confusion with the projection function ${ }^{\prime} \pi$ '.

[^12]:    ${ }^{10}$ In contrast, the expression 'criterion of individuation' is supposed just to express the pre-theoretical idea 'how things are individuated'. In Section 4.1 the expression 'criterion of individuation' inherits a semi-technical meaning within Asher's (2011) theory of copredication, and in that section I will use the expression in (what I take to be) Asher's intended sense.
    ${ }^{11} \mathrm{Or}$ in other words, PHYs is an equivalence relation on the subset of the domain (of singular entities) consisting of things that have at least one physical part, but not on the whole domain, since it is both transitive and symmetric.

[^13]:    ${ }^{12}$ And $\sqcap$ is the corresponding meet operation.

[^14]:    ${ }^{13}$ This lexical entry is provisional. In the light of the issues discussed in Section 2.3.2, the updated version is given in (52), p. 52.

[^15]:    ${ }^{14}$ The relation of animate equivalence does no work in this section but is relevant in Section 5.1.

[^16]:    ${ }^{15}(45)=\lambda x_{e} \cdot \pi_{2}((44)(x))$.
    ${ }^{16}$ I use $\bigsqcup A$ rather than $\bigvee A$ to indicate the least upper bound of $A$, because I have been using $\sqcup$ rather than $\vee$ to indicate the join operation.

[^17]:    ${ }^{17}$ For ease of presentation I am assuming that traces can only be of type $e$.

[^18]:    ${ }^{18}$ I am basing this presentation on the theory developed by $\operatorname{Steedman}(2000,2011)$, but without adopting the approach to relative quantifer scope and plurality described in those books.

[^19]:    ${ }^{19}$ Function composition and type raising are actually more restricted in their application than I have indicated here. However, they are applicable as shown in (69)

[^20]:    ${ }^{20} \operatorname{Or}(C a t /(C a t \backslash N P)) / N$, but not in these cases because they are in object position.

[^21]:    ${ }^{1}$ See (53) in Chapter 2.

[^22]:    ${ }^{2}$ But see the note of caution in Section 3.6.1.

[^23]:    ${ }^{3}$ I'm assuming that the copula is semantically vacuous or, equivalently in this case, that it denotes the predication relation.

[^24]:    ${ }^{4}$ That is to say, I am broadly assuming the structure shown in (i), rather than that shown in (ii) or any variation on it in which the PP is adjoined higher up.
    
    (ii)
    

    This assumption is made for expository purposes only and is completely dispensible in favour of a different theory of the structure of expletive sentences if that is desired. For example, if the theory of expletives to be employed required the PP to be a secondary predicate rather than a modifier, its interpretation would be as shown in (35).

    $$
    \begin{equation*}
    \lambda x_{e}\left\langle *^{*} \text { on-table }^{\prime}(x), \lambda g_{e \rightarrow \mathcal{R}} \cdot g(x) \sqsubseteq \text { PHYS }\right\rangle \tag{35}
    \end{equation*}
    $$

    (34) would then be interpreted as $(31)[(35)]=(37)$, the expletive being assumed to play no semantic role in this case.

[^25]:    ${ }^{5}$ In fact, it is a context-dependent relation applied to a context-dependent individual variable, giving a contextually-determined set. This difference is not important in the present context.

[^26]:    ${ }^{6}$ See Section A. 2 for discussion.

[^27]:    ${ }^{7}$ Actually, Nathan Klinedinst raised these objections.

[^28]:    ${ }^{8}$ Given that the relations of physical equivalence and informational equivalence are reflexive and symmetric.
    ${ }^{9}$ If not absolute uniqueness, i.e. that there be exactly one informative book tout court.

[^29]:    ${ }^{10}$ The problems reviewed in this subsection are just as much problems for the theories of copredication discussed in Chapter 4.

[^30]:    ${ }^{11}$ This terminology likewise comes from Link (1983), but is being used here slightly differently. Link does not countenance objects that are complexes of physical and non-physical parts, and in any case $p$ would not be an m-atom for him because it can be physically subdivided.

[^31]:    ${ }^{12}$ The definition of informational equivalence would have to be tightened up in the circumstances under consideration now that we have NSC denoting objects made up of more than two parts. Instead of the definition given in Section 2.3.1, where it is the relation that 'holds between (singular) objects $a$

[^32]:    ${ }^{1}$ See footnote 10 on p. 40.

[^33]:    ${ }^{2}$ It is not the projection function that I made use of in Chapter 2.
    ${ }^{3}$ That's the significance of Asher's quantifier ' $\exists_{3}$ '.

[^34]:    ${ }^{4}$ This 'Head Typing Principle', according to which syntactic projection preserves typing, actually follows from more basic assumptions in (Asher, 2011). However, I adopt this simpler presentation (from (Asher and Pustejovsky, 2006) and (Asher, 2008)) for expository purposes and also because deriving the Head Typing Principle seems to be the aim of some of those assumptions.

[^35]:    ${ }^{5}$ Or rather, the interpretation that gives 'a student' wide scope.
    ${ }^{6}$ For a discussion of cases like (22) in the system proposed in this thesis, see Section 3.4.
    ${ }^{7} \mathrm{~L}$ is the type of locations in Asher's system.

[^36]:    ${ }^{8}$ Again, taking 'three' to mean 'at least three'.

[^37]:    ${ }^{9}$ Here and in the rest of the section the actual types are higher than I've indicated, but this is irrelevant for present concerns.

[^38]:    ${ }^{10}$ The field ' $\mathrm{c}_{1}$ : food( x )' in (36) is actually shorthand for the field shown in (35).
    (35) $\mathrm{c}_{1}:\langle\lambda v: \operatorname{Ind}(\operatorname{food}(v)),\langle\mathrm{x}\rangle\rangle$

    That is to say, the arguments to a predicate are not actually labels, but objects. The type of ' $c_{1}$ ' in (35) is a pair, the first member of which is a function (from individuals to proofs) and the second member of which gives us the label indicating where the object that is the argument to that function is to be found (Cooper, 2012, §2.5). In the case of a more-than-one-place predicate, the second member of the pair will of course be a correctly-ordered list of fields of appropriate length.

[^39]:    ${ }^{11}$ This is known as a path to the label x in r .

[^40]:    ${ }^{12}$ In fact, Cooper (2011, pp. 75-76) gives suggestions of various additional ways in which we might individuate and count books over and above the physical and informational ways that we have been considering.

[^41]:    ${ }^{13}$ Modern, because it is an advance on the simple type theory, based on a single-sorted logic, adopted by Montague (1973) and the work in natural language semantics that he ushered in.
    ${ }^{14}$ This is called 'coercive subtyping'.
    ${ }^{15}$ 'Prop' is the type of propositions, which would be type $t$ (or perhaps $s \rightarrow t$ ) in a simply-typed system.

[^42]:    ${ }^{16}$ In this case, the coercion is actually the projection $\pi_{1}$ that we have encountered before.

[^43]:    ${ }^{17}$ Just a few examples are Carston (2002, chapter 5), Nunberg (2004), Recanati (2004, chapter 5), Sperber and Wilson (1998), Wilson and Carston (2007).

[^44]:    ${ }^{18}$ This notion of predicate coherence is inspired by Kehler's (2004) analysis of discourse coherence. There are ways of achieving predicate coherence other than explanation (as in (91) and (92)) -which corresponds to Kehler's (2004) category of 'cause-effect relations'. Kehler (2004) also has categories of 'resemblance relations' and 'contiguity relations', aspects of which are covered by the other predicate coherence relation that Brandtner mentions: 'narration'.

[^45]:    ${ }^{19}$ I have used Brandtner's (2011) notation for shifting, which is not supposed to indicate that what is happening is ellipsis.
    ${ }^{20}$ With some other nouns supporting copredication that have been considered in this thesis, it does seem possible to construct such an example. I will discuss this in Section 5.2.

[^46]:    ${ }^{1}$ See (7) in Chapter 3.

[^47]:    ${ }^{2}$ Throughout this section I mean 'anomaly' in a reasonably restricted sense. The object of study is what was touched up on Section 1.2.2: where we seem to have a property being ascribed to an object that is not of the right kind to have it (sometimes called 'category mistakes' or 'categorial mismatch'). Sentences can be odd-sounding or 'anomalous' for a variety of reasons, including purely pragmatic ones.

[^48]:    ${ }^{3}$ I am not adopting the tentative suggestions made in Section 3.6.2.
    ${ }^{4}$ See (62) in Chapter 2.

[^49]:    ${ }^{5}$ Or even any books, such as the world described in Fahrenheit 451.

[^50]:    ${ }^{6}$ Again, see Section A.3.
    ${ }^{7}$ One point of distinction between (14) and (15) is that (14), but not (15), can be proven to have no function satisfying its construction without needing to look at meaning postulates, in other words without knowing anything about the lexical semantics of the language. This might serve as a basis for isolating anomaly due to categorial mismatch.

[^51]:    ${ }^{8}$ But see the discussion in Section 3.6.2. Robin Cooper (p.c.) has suggested that what is wrong with (34) is that it requires or implies that every branch of some bank used to be a police station. It can be maintained that (35) is acceptable and true if what is being demonstrated is a former police station that is the only branch of some bank.
    (35) That bank is FTSE-100 listed and used to be a police station.

    Even if this is the case, the fact that (34) is judged as not only likely false but also weird needs to be explained. But the contrast between (34) and (35) suggests a possible line of investigation.

[^52]:    ${ }^{9}$ Once again, Brandtner's theory is not designed for cases like (36). It is merely being used as an example of how selectionally distinct predicates can nevertheless be said to cohere with each other.

[^53]:    ${ }^{10}$ Thanks to Tian Ye for this example.

[^54]:    ${ }^{11}$ Collins allows the semanticists often talk like this, but contends that the explanations they offer do not depend on a realist construal of their claims. More on this below.

[^55]:    ${ }^{12}$ It could be seen as implementing Segal's suggestion at the level of the model theory, rather than in the syntax of either English or the chosen metalanguage.
    ${ }^{13} \mathrm{Or}$ as a generalised quantifier on a par with 'every American' either.

[^56]:    ${ }^{14}$ A point made by Stanley (2007, Introduction). See also Partee (2005, p. 10).

