# INCOMPREHENSION OF THE INDONESIAN ELEMENTARY SCHOOL STUDENTS ON FRACTION DIVISION PROBLEM

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WORD COUNT 6661
TIME SUBMITTED 08-MAR-2019 10:37AM
PAPER ID 44996593

# Incomprehension of the Indonesian Elementary School Students on Fraction Division Problem

**Article Info**

***Article history:***

Received Jun 12, 201x
Revised Aug 20, 201x
Accepted Aug 26, 201x

***Keywords:***

Fraction division
Fractional parts
Conceptual knowledge
Procedural knowledge
Elementary school students
Indonesia

**ABSTRACT**

The purpose of the study is to investigate the Indonesian students' performance in solving fraction division case including the difficulties, relations, and implications for classroom instruction. This study employed a descriptive case study to achieve it. The procedures of data collecting were initiated by giving a context-based problem to 40 elementary school students and it then according to the test result was selected three students for semi-structure interviewed. The findings of the study showed that the tendency of students' procedural knowledge dominated to their conceptual knowledge in solving the fraction division problem. Furthermore, it was found several mistakes. First, the students were not accurate when solving the problem and unsuccessful to figure out the problem. Second, students' conceptual knowledge was incomplete. The last was is to apply the laws and strategies of fraction division irrelevant. These findings emphasized other sub-construct of fractions instead of part-to-whole in the teaching and learning process. Teaching and learning of fraction in the mathematics classroom should take both conceptual and procedural knowledge into account as an attempt to prevent faults and misconceptions. In conclusion, it was substantial to present context-based problems at the beginning of the lesson in order for students to be able to learn fraction division meaningfully.



***Corresponding Author:***

XXXXXXXXXXXXXXXXX **(Hiden by Editor)**

***How to Cite:***

XXXXXXXXXXXXXXXXX (2019). Incomprehension of the Indonesian Elementary School Students on Fraction Division Problem. *Infinity*, *X*(X), XX-XX.

## 1. INTRODUCTION

Fractions are one of the topics studied in elementary mathematics which serves as a cornerstone for comprehending further mathematics topics such as arithmetics, algebra, probability, data analysis, geometry, and measurements. They are also capitalized on for communicating and solving daily life problems. Nevertheless, fractions and the operation are the most challenging elementary mathematics topics as they are difficult to understand. In addition, most elementary school students are presented with meaningless instruction (Geller, Son, & Stigler, 2017; Lin, Becker, Ko, & Byun, 2013; Wang, Chen, & Lin, 2014), especially in fractions division (Kribs-Zaleta, 2006; Sidney, Hattikudur, & Alibali, 2015; Tirosh, 2000). Fractions division learning emphasizing heavily on algorithms acquisition inevitably tends to be accepted and dominated teaching and learning process.

The most common procedures used in coming to grips with fractions division problems are the invert and multiply (or keep-flip-change) strategies. These procedures are perceived as the most mysterious rules in elementary mathematics topics (van de Walle, Karp, & Bay-William, 2010) as they are not frequently understood by the teachers and the students. Skemp (1987) name these procedures as rules without reason. The ability to manipulate symbols and implement rules without understanding may create trouble in making sense and students may make mistakes when encountering problems which have to be solved using unfamiliar procedures (Purnomo, Kowiyah, Alyani, & Assiti, 2014).

### 1.1. Procedural and Conceptual Knowledge of Fraction

Procedural knowledge has been described as knowledge about how to do something (Hallett, Nunes, & Bryant, 2010). It refers to students' ability to implement, calculate, and execute symbols representation

system and algorithms to solve problem accurately, efficiently, and appropriately (Lauritzen, 2012; Rittle-Johnson, Siegler, & Alibali, 2001). Skemp has identified this knowledge as instrumental understanding which is described as rules without reason. Procedural knowledge deals with symbols, rules, formulas, and algorithms in a discrete manner while conceptual knowledge refers to a knowledge that is rich in connections (Rittle-Johnson et al., 2001). Skemp (1976) label it as a relational understanding. Connection or relation between mathematical concepts and integration to a contextual situation is the heart of conceptual knowledge. It is able to assist students in making sense of the fraction concept.

Conceptual knowledge and procedural knowledge support each other. Concept-first and procedures-first were acknowledged by kinds of literature (Rittle-Johnson et al., 2001). Development of students' conceptual knowledge at the outset of a lesson may contribute to the ability to address varieties of mathematical tasks successfully. Moreover, students are able to build conceptual knowledge by establishing various types of algorithm initially. Focusing heavily on procedures in fraction learning may impede students to engage with the real-world context used as a bridge for developing conceptual knowledge. Without context, students may encounter puzzlement in coming to grips with the concept of fraction and its application in various situations (Sharp & Adams, 2002; Yim, 2010).

Traditionally, in the case of fraction division, students mostly are presented by procedure-oriented, memo based, and meaningless instructional approaches. The common algorithm used for fraction division is by multiplying the dividend by the reciprocal of the divisor. This algorithm is straightforward. Hence, students are able to use it easily in dealing with routine problems that have been exemplified by their teachers. However, they get trouble in an attempt to solve unfamiliar problems such as word problems and non-routine problems. The difficulties can be caused by students' lack to understand the concept of fractions. Therefore, procedural knowledge and conceptual knowledge should be interwoven and integrated with each other (Kilpatrick, Swafford, & Findell, 2001).

### 1.2. Interpretation of Fraction Division

Traditionally, fraction division is able to be explained by the extension of the division interpretation of whole numbers, namely, partitive and measurement concepts (Alenazi, 2016; Purnomo, 2015a). Furthermore, recent studies have discussed others interpretation of fraction division, namely, the determination of a unit rate, the inverse of multiplication, and the inverse of a Cartesian product (Alenazi, 2016; Sinicrope, Mick, & Kolb, 2002). In our study, we focus on the traditional concepts of fraction division that are partitive and measurement concepts because these concepts are typically introduced to learn the fraction division in first because of related concept whole number division. In addition, these concepts are considered to be relevant and appropriate for developing the understanding of elementary school students on fraction division problem.

***Fractions division as measurement (repeated subtraction)***

This model explains fraction as the number of times we can subtract the denominator from the numerator before we attain 0 (zero). This meaningful interpretation can be applied in case of fraction division. For instance, in the case of division of 1/2 by 1/4, students may reason it as a quarter goes into 1/2 two times. It can be interpreted contextually, for instance, if someone has half of a cake and she/he wants to divide it into quarters, then you have two pieces of 1/4.

***Fractions division as partitive (equal share)***

This model is mostly known as partitioning or equal sharing. It represents to share activity which distributes a collection or quantity equally among some number of people. In the case of fraction division, for example, division 1/2 by 1/4, students may reason it contextually as a process of distributing half of a cake to several numbers of people in order that each person gets 1/4 of cake equally.

### 1.3. The Present Study

In Indonesia, the elementary school consists of classes from 1st grade up to 6th grade. It is commonly classified into a lower elementary (1st – 3rd grade) an upper elementary (4th – 6th grade). Indonesia has two simultaneously applied curricula, namely the School-based curriculum (known as Kurikulum Tingkat Satuan Pendidikan; in abbreviated as KTSP) that has been implemented since 2006 and the curriculum of 2013. One of the fundamental differences of both curricula is a pattern of material organizing. At the KTSP, a relationship among subjects is more mutually exclusive, while for the curriculum of 2013, it is integrative thematic. Nevertheless, the implementation of the 2013 curriculum is still a limited trial phase applied in

selected schools. At the elementary school level, this curriculum is only applied in grade 2 to represent the lower elementary and grade 4 to represent the upper elementary.

In Indonesia and most of the international curriculum, fractions are firstly introduced in the third grade of the elementary school (Wijaya, 2017), while the divisions are introduced in the fifth grade of elementary school (see also Purnomo, Widowati, Aziz, & Pramudiani, 2017). Further, Wijaya (2017) states that the introduction of the fractional concept as parts of the whole becomes the only construct having a space to learn from both mathematical textbooks and the teacher's teaching method, while the fractional operations are dominated by rigid rules to solve problems.

In Indonesia, some previous studies have mentioned that the fractions topic becomes a difficult topic for the students and a serious concern because of the weakness of the students' performance on this topic (Purnomo et al., 2014; Trivena, Ningsih, & Jupri, 2017; Wijaya, 2017). In his study, Wijaya (2017) has analyzed students' difficulties on the fractions topic from TIMSS results in 2015 and attributed it to the students' opportunities to study the fractional at school. Based on the analysis of TIMSS results, his research has found that Indonesian students have a weakness on the understanding of fractions, particularly in story problem cases. Students do not have space to explore their ideas because teachers resist getting out of the content and sequence of material in the book. Similarly, Trivena, Ningsih, and Jupri (2017) also have found that elementary school students are oftentimes misconception the concept of addition and subtraction of fractions. These studies show the fraction is one of the materials requiring attention and handling. However, those studies and literature related to it have not focused on more specific content that is fractions division. Focusing on more specific issues helps to handle the problems more precise at hand. In addition, it is also substantial to know how the students' strategy in dealing with the case of division and what the difficulties are.

Based on the above description, this study aims to explore the Indonesian elementary school students' performance in solving fractions division cases including the difficulties, relations, and implications for the classroom instruction. Research questions may arise i.e. how is Indonesian elementary school students' understanding of fraction division?

## 2. METHOD

### 2.1 Context and Participants

The method of this research employed two phases. The first one was a descriptive study to gain insight into students' performance in coming to grips with fraction division problems. The participant of this phase was 40 fifth grade elementary school students in Jakarta. It was collected fifth-grade students as the participant because they had learned a fraction from the definition up to the operation of fractions division. The second one was a case study to investigate the students' knowledge further about fraction division and its underlying epistemological factors. Several participants were selected based on their achievement in the test and teacher's suggestions such as their ability of verbal expression and confidence. According to these considerations, three students were selected and pseudonyms were used to address ethical issues. The first student was Ummu, an 11-years-old, Javanese girl, she was an outstanding student being a top three in her class every academic year. She comes from a middle-income family. The second one was Nunu, an 11-years-old, Javanese boy, he was an average student. He comes from a low-income family. His salient characteristics are that he is an active student selected as a leader in his class. The last was Cici, a 12-years-old, Sundanese girl, she was a student categorized as a low-achiever. However, she is involved actively in several school activities such as flag hoisting troop. She comes from a low-income family. A similarity among them is that they have settled in Jakarta City since they were born.

### 2.2 Data Colletion

The data collection processes were done by using a written test and an interview. The written test was administered to obtain data from the participants' performance in dealing with fraction division problems. Meanwhile, the semi-structured interview was conducted to explore epistemological factors related to the understanding of fraction and its difficulty.

The written test composed of three question items about fractions division with different indicators. Table 1 below demonstrates the question items.

**Table 1.** Indicators of the written test

| Case | Descriptions |
|---|---|
| 1. $a \div \frac{1}{k}$, with $k \neq 0$ (This case relates to a measurement | You have 2 birthday cakes given to your friends, each of them is 1/2 parts. How do you know the number of your |

| | | |
|---|---|---|
| | concept of the fraction division) | friends who will get the cakes? |
| 2. | $\frac{1}{k} \div a$, with $a \neq 0$ and $k \neq 0$ (This case relates to partitive concept of the fraction division) | Father has 1/2 pizzas given to 2 of his children named Mila and Damar equally. How do you know how big parts will Damar get? |
| 3. | $\frac{1}{a} \div \frac{1}{k}$, with $a \neq 0$ and $k \neq 0$ (This case relates to measurement concept of fraction division) | Mrs. Vivi has 3/4 kgs flavor. To make 1 donut, Mrs. Vivi spent 1/4 kgs flavor. How do you know that how many donuts can be made? |

The interviewing questions based on the items of the question above and responses of the students on each question. First, we re-questioned "how do they solve the problem". Then, we asked, "why do they choose those strategies". The third was "where do they know the knowledge".

### 2.3 Data Analysis

We used a rubric to analyze participants' written answers. The rubric (see Table 2) took indicators into consideration that were, understanding the problems, planning, and the answer's accuracy. Each indicator had 2 for the maximum score and 0 for the minimum score, so the highest score possible for each item is 6. In total, the highest score possible is 18 and the lowest score possible is 0.

**Table 2.** Assessment rubric for written test

| Assessment Criteria | Assessed Indicators | Score |
|---|---|---|
| Problem Understanding | Comprehensive and organized understanding | 2 |
| | There is an effort to organize but some problems could not be figured out | 1 |
| | Do not understand the problems, are not organized and systematic | 0 |
| Strategic Planning | The strategy used is relevant and well explained (if it is implemented, it will be valid) | 2 |
| | Some strategies are relevant but are not well explained | 1 |
| | The strategies are irrelevant, unclear and difficult to get to the point | 0 |
| Accuracy of Calculation | Using the right strategy leading to the right answer | 2 |
| | Some algorithms applied are correct but there are errors found. As the result, the answer is not valid | 1 |
| | There are no answer | 0 |

The percentage of students' correct answers on every item is counted and classified into the whole right answer, partly right answer, and wrong answer. The written test data also were analyzed descriptively including mean and standard deviation. The mean and standard deviation are used to determine selected students for a case study. The written test data is grouped into high, medium, and low category. The high category had a score more than $\bar{X} + \frac{1}{2}SD$. The low category had a score less than $\bar{X} - \frac{1}{2}SD$. The scores between both criteria are categorized as a medium category. One student of each category would be chosen for the case study.

On the other hand, the interview result data were transcribed and coded based on pattern responses to explore further. Triangulation was done by confirmation the written test data and interview result. We also used students' worksheets as additional data. The students' worksheets were taken during the interview when students explained their understanding to the interviewer.

## 3. RESULTS

### 3.1. Profile of the fraction division performance

The students' written answers represented their understanding of each case given. Descriptive statistic of students' answers is shown in Table 3.

**Table 3.** Percentage of student responses for each case of fraction division

| No. Item | Correct | Partially correct | Incorrect |
|---|---|---|---|
| **1** | 10% | 50% | 40% |
| **2** | 8% | 25% | 68% |

| | | | |
|---|---|---|---|
| **3** | 15% | 28% | 58% |

Based on Table 3, most participants encountered difficulties when dealing with the presented items. Item number 2 obtained the lowest response among others. This item was only whole right answered by 8% of students and 68% of them responded wrong. Table 3 also shows that at most only 15% of respondents answered a whole right answer for each given item. Overall, the number of participants who correctly answered the items was much less than that of those who obtained partially correct and incorrect answers. It might indicate that fractions division is a problematic and challenging elementary mathematics topic for students.

The results of the descriptive analysis on the written test have obtained a mean at 4.525 and a standard deviation at 4.391 with the highest score was 14 and the lowest score was 0. Based on the criteria for each category that we had previously set, there were 7 students (18%) as the high category, 21 students (53%) as the medium category, and 13 students (33%) in the low category.

### 3.2. A case on the natural numbers divided by fractions

***Ummu's responses***

In accordance with her written response addressing the first case, Ummu sliced each cake into two pieces equally, then four half-pieces were obtained from two cakes. In the interview, she said, "*This cake is sliced into two similar parts, so does this (another cake) one. After slicing the two, we have four similar pieces. This one (by referring to the shaded part) is for one person. So, there will be four*". Ummu tried to address the problem by considering the number of half piece of cakes. Therefore, a strategy implemented by Ummu is likely to be in line with the concept of division as repeated subtraction.

Nevertheless, we discern that Ummu's explanation in the interview session tends to be different from her written answer in the test. The same approach also she did when answering subsequent questions we gave. Ummu was asked to give explanations of how to share twelve doughnuts with her friends in which each of them would get two doughnuts. She explained, "*It will be two doughnuts for one person, two for the other, so do this, this, this (putting marks to every two objects)*". Ummu's strategy was that she tried to distribute two for each person so that it would end at twelve for the number of doughnuts and six for the number of the person. Based on this, Ummu's knowledge of the concept of division tends to converge on the idea of division as an inverse of repeated addition.

***Nunu's Responses***

When dealing with the first case, Nunu employed invert-and-multiply rule. Figure 1 describes Nunu's efforts to address the question given.

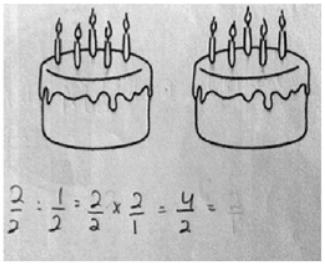


**Figure 1.** Nunu's response in written test

Through the interview, we tried to gain deep information related to Nunu's written response during the test. The following are excerpts of the interview with Nunu conducted after the test.

Interviewer : *Nunu, could you explain your answer?*
Nunu : *This was division, I mean that 2/2 is divided by 1/2.*
Interviewer : *Which 2/2 did you mean?*
Nunu : *As there were two cakes, so 2/2.*
*Because we divided this, the second one was reversed and then we multiplied.*
*The result was 4/2.*
Interviewer : *So, you thought that 2/2 was two cakes, how about 5/5?*

Nunu : *There were five cakes.*

Based on the excerpt above, it is obvious that invert-and-multiply rule is capitalized on by her to solve the question. However, even though she was able to solve it in a correct manner, she did not seem to have an accurate comprehension concerning a concept of fractional parts. This inaccuracy occurs as she tends to assume that the value of a fraction, whose numerator and denominator are equal numbers, is the same as its numerator and denominator. For instance, Nunu presumed that 2/2 could be represented as two units. By this method, she also regarded that 5/5 could be represented as five units. Based on this interesting fact, further queries were posed to find out about Nunu's comprehension of the concept of fractional parts.

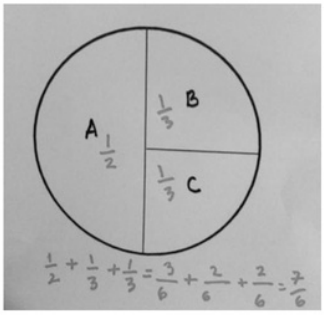


**Figure 2.** Nunu's response in interview session

Fig. 2 shows that Nunu's understanding of the fractional concept parts is weak. Nunu focused on how many parts are divided and then she presumes that each part B and part C are 1/3. She said, "Because we have three parts from one cake". However, this statement is in contradiction with Nunu's answer that is 1/2 for part A. Nunu realized that she added up the three parts and turned out to produce more than 1.

***Cici's Responses***

Cici's responses to the first case were similar to what Ummu undertook. In the interview, she said, "This cake is cut into two parts. Then, I get two 1/2s. The other is also divided by two, so I get 1/2 and 1/2. Thus, 1/2 + 1/2 = 2/4. Since there are two cakes, thus 2/4 × 2/4 = 4/4. So, the result is 4/4 person. 4/4 refers to four persons". When we elaborated on an aforementioned explanation by posing a further question, unfortunately, she was not able to reveal her argumentation concerning the reason why she used addition and multiplication. "I don't know why?" she replied.

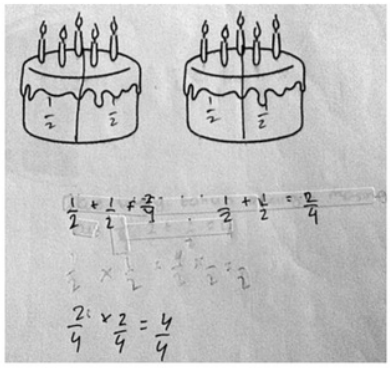


**Figure 3.** Cici's response in written test

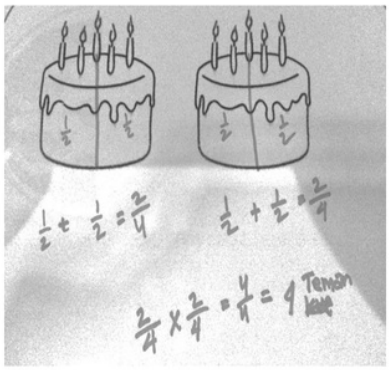


**Figure 4.** Cici's response in interview session

Based on the aforementioned description, we disclosed several mistakes and misconceptions made by Cici's work. Firstly, Cici did mistake when adding two fractions with like denominator, for instance, 1/2 and 1/2. Secondly, Cici was likely to have an inaccurate understanding of addition and multiplication

conception of numbers. Thirdly, similar to what Nunu did, Cici claimed that the value of a fraction with similar numerator and denominator were equal to its numerator and denominator.

### 3.3. A case on the fractions divided by the natural numbers

***Ummu's Responses***

In the second case, Ummu tried to respond by splitting pizza out into two equal parts and wrote 1/4 in each part. Therefore, according to Ummu's responses, the result of a division of 1/2 by 4 is 1/4. The following figure presents Ummu's written response to this case.

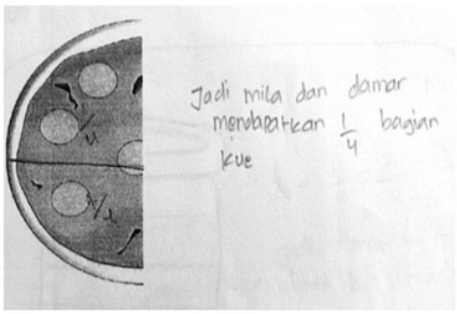


**Figure 5.** Ummu's response in written test

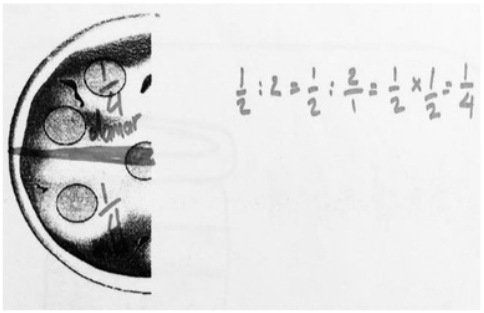


**Figure 6.** Ummu's response in interview session

Ummu's written response in the test was likely to reflect her understanding of division as equal sharing. However, its conception was not demonstrated when she was interviewed. She tended to present her procedural knowledge. It was obvious when she tried to solve the question presented. Even though her procedures and obtained answer were correct, she seemed to express her puzzlement concerning her answer. The following was an excerpt from the interview with Ummu.

| | |
|---|---|
| Interviewer | : *Could you explain the way how you get 1/4 as your final answer?* |
| Ummu | : *By dividing. 1/2 is divided by 2 equals 1/4* |
| | *Wait, it is divided by 2… It is 1/2, isn't?* |
| Interviewer | : *What do you mean by 1/2?* |
| Ummu | : *This pizza is cut into two. It is 1/2, isn't?* |
| | *Damar has one part, and … …* |
| Interviewer | : *Your previous obtained answer was 1/4. Why do you have the different answer?* |
| Ummu | : *Because a half pizza is cut into two.* |
| Interviewer | : *Could you show me, which part of the figure indicates 1/4?* |
| Ummu | : *This one (She refers to Damar's part), but I am a bit confused because it is divided by two. But I am sure that 1/4 is the correct answer (by showing her written response)* |

Based on the above excerption, Ummu was not able to convince herself that 1/4 was the result of division 1/2 by 2. Ummu tended to rely heavily on her procedural knowledge and got confused when there was a contradiction between her work showing that 1/2 divided 2 equals to 1/4 and her mental image presuming that something divided by 2 equals to 1/2. The reason might lay in the fact that Ummu's primary focus was the result of division instead of paying attention to its dividend and divisor. In addition, a similar response was presented by Ummu when she was asked further questions during the interview.

| | |
|---|---|
| Ummu | : *Each person gets one. The rest is cut into four like this. Let me sign these, 1, 2, 3, and 4. Then, I add one cake with this. So, each person will get* $1\frac{1}{4}$. |
| Ummu | : *it's the same. I divide this by three. As each person gets one, then we divide the rest by 3. So the answer is* $1\frac{1}{3}$. |

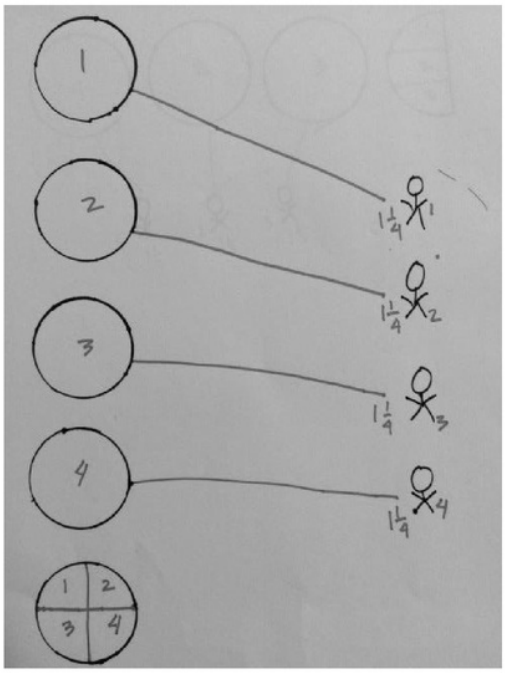


**Figure 7.** Ummu's response in interview session

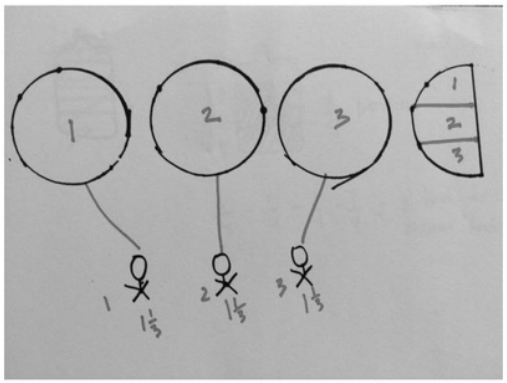


**Figure 8.** Ummu's response in interview session

It is apparent from Fig. 7 that Ummu was able to address a case in which the dividend of the fraction is whole numbers. However, she was likely to have difficulty in coming to grips with the division problem in which its dividend was a rational number. From Fig. 8, Ummu presumed that division of a half circle by three results in 1/3. In addition, Ummu seemed to have a weak understanding of fractional parts concept. It was obviously observed when she was asked about the fraction representing each sector in Fig. 2 and her answers were that sector B and C were 1/3 and sector A was 2/3. Fractional parts concept is a fundamental aspect in comprehending division of fraction and other fraction operation.

***Nunu's Responses***

Nunu's response to the second case indicated that Nunu capitalized on procedural knowledge, yet her works were difficult to interpret. Based on the interview response, it seemed that Nunu's concept of fraction and division of fraction were still weak.

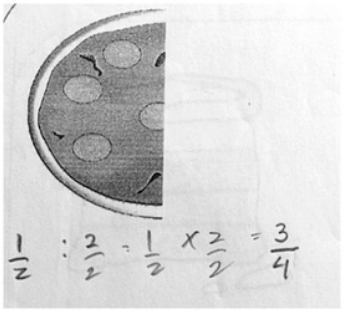


**Figure 9.** Nunu's response in written test

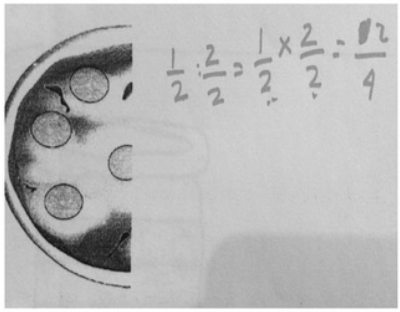


**Figure 10.** Nunu's response in written test

Interviewer : *Nunu, could you explain this answer you have obtained during the test?*
Nunu : *1/2 is divided by 2/2*

Interviewer : *What does 2/2 mean?*
Nunu : *two children*
Interviewer : *Was 3/4 the answer to the question?*
Nunu : *Yes, it was. 3/4 for Damar. But, wait. It is wrong. It should be 2, not 3. So, the correct answer is 2/4.*

The dominance of procedural knowledge over conceptual knowledge in fraction concept might lead Nunu to make mistake as she was not able to catch on what the presented problem was. For instance, she presumed that 2/2 stood for two units.

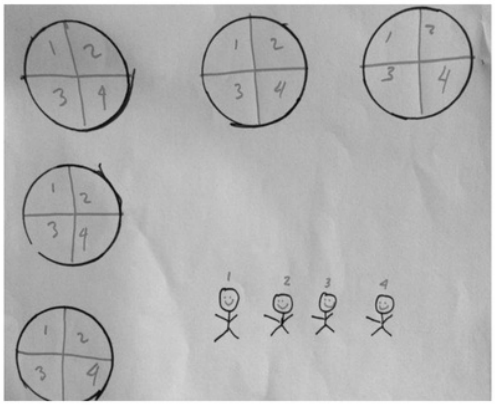

**Figure 11.** Nunu's response in interview session

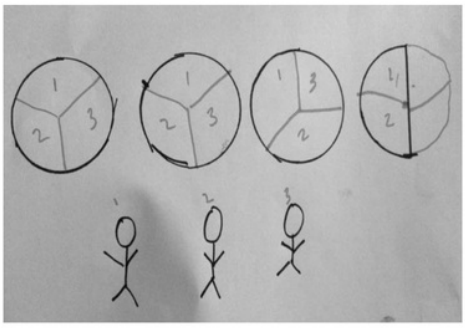

**Figure 12.** Nunu's response in interview session

In the further interview, Nunu said *I got 1/4 as we divide a cake by four. Then, 1/4 + 1/4 + 1/4 + 1/4 + 1/4 = 5/4. Therefore, each child will have 5/4 cake* (see Fig. 11). In this case, Nunu was able to deal with division case whose dividend was whole numbers, yet from Fig. 12 we knew that she encountered difficulty as the dividend was not whole numbers.

***Cici's Responses***

On the third case, Cici tried to solve the question by cutting the cake into two equal parts. According to Cici's statement during the interview, we found that Cici was likely to focus her attention on the result of the fraction instead of noticing the dividend and divisor.

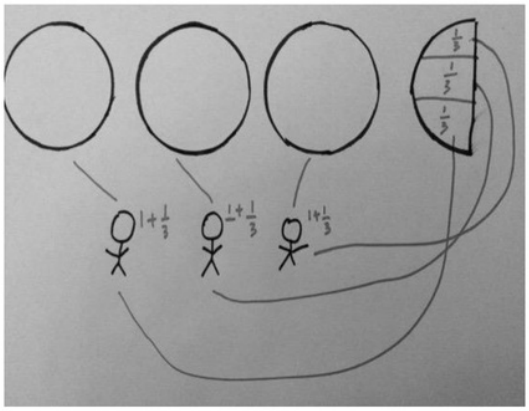

**Figure 13.** Cici's response in interview session

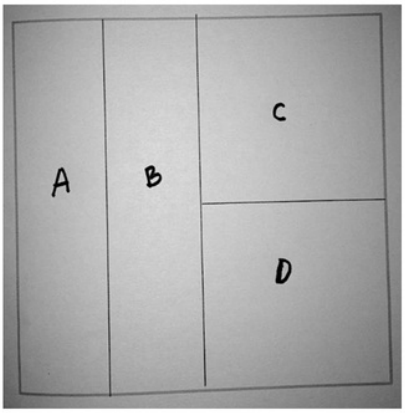


**Figure 14.** Cici's response in interview session

Based on the Cici's work and statements, one issue was paramount when it came to her misconceptions about the fraction, that was, she presumed that if all things were cut into three equal parts, then the result would be 1/3. She passed over the form of the thing being divided. Another Cici's weaknesses were found when she assumed that a region C and D were greater than region A and B as illustrated in Fig. 14. However, surprisingly this response was at odds with her statement when she was asked about the fraction unit that named each part of the divided whole. She claimed that each part represented 1/4. This state of an affair might be attributed to Cici's lack of understanding of equal sharing concept at the fractional parts.

### 3.4. A case on fractions divide by fractions

***Ummu's Responses***

Ummu tried to address the third case using the subtraction method. The following figures illustrate Ummu's written responses during the test and the interview session.

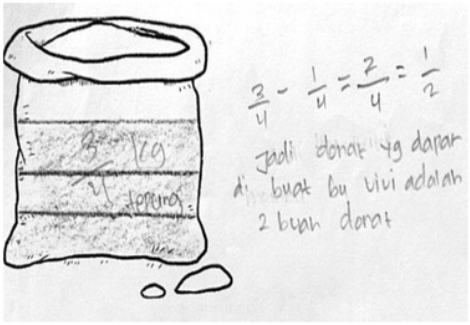


**Figure 15.** Ummu's response in written test

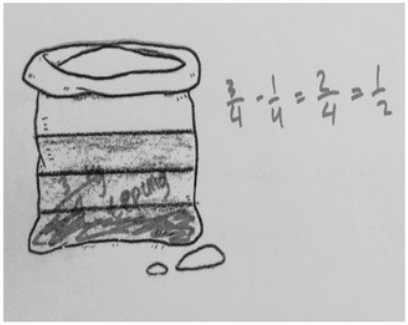


**Figure 16.** Ummu's response in interview session

We were not able to identify Ummu's argumentation, as she viewed this problem as a case of subtracting 1/4 from 3/4. We tried to have Ummu read the question meticulously, yet she was likely to stick with its stance in her opinion.

***Nunu's Responses***

Nunu's responses tended to be similar to that of Ummu in which the third case could be addressed using subtraction. It could be discerned clearly when interviewing Nunu. Her answer was 2/4 as a result of subtraction of 1/4 from 3/4. She accounted for it as 1/4 floor used for making a doughnut. She performed single subtraction in lieu of multiple subtractions.

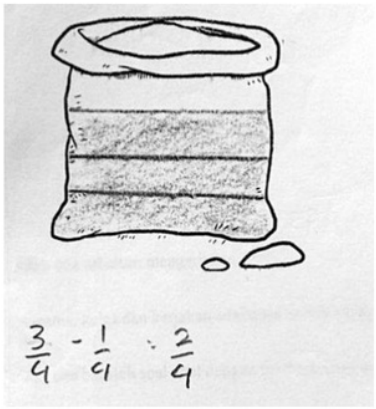


**Figure 17.** Nunu's response in written test

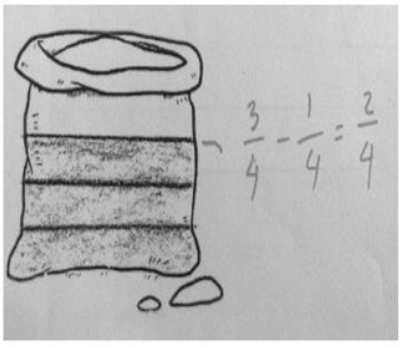


**Figure 18.** Nunu's response in interview test

***Cici's Responses***

Cici's response to the third case indicated that she made the use of her procedural knowledge to address the question. Fig. 19 shows Cici's written response in the test. She multiplied 3/4 and 1/4 and during the interview, she was not able to uncover the reason behind her strategy. Therefore, it seemed that Cici encountered difficulty in grasping the problem presented thoroughly.

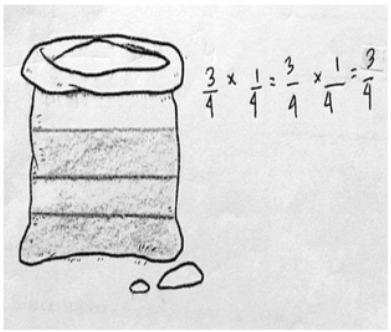


**Figure 19.** Cici's response in written test

## 4. DISCUSSION

Tirosh (2000) has summarized that there are at least three main categories of mistakes made by children when solving fractions division problems, to wit: algorithm-based mistakes, intuition-based mistakes, and mistakes derived from formal knowledge. In our study, we explored how children knowledge solves fraction division regarding Tirosh's work and any possibility beyond his work.

The findings of the study indicated that most of the participants encountered difficulties in solving problems of fraction division. They face difficulties in solving problems related to the case of dividing fractions by whole numbers. In other words, their equal sharing concept is still weak. This finding is not in accordance with our predictions who reveal that this case is the easiest case compared with other cases. Nevertheless, the other cases also need an attention because 58% of the students provided an incorrect answer for the case of dividing fractions by fractions and only 10% answered correctly for the case of dividing of whole numbers by fractions. In addition to being weak in equal sharing's conceptual knowledge, the majority of the students in this sample still depend on procedural knowledge which is not accompanied by a strong conceptual knowledge. This finding was also supported by Purnomo et al. (2014) when examining primary school students number sense. In their study, Purnomo found that the students encountered some difficulties in understanding the meaning and the concept of numbers, especially the fractions and decimals. Students have a misconception about the fractional concepts and make some errors when performing calculations as they pay more attention to its rules and algorithms.

The findings were clarified and reinforced by the response of the three case study samples, namely Ummu, Cici, and Nunu. All three participants are weak in equal sharing concept in the division of fractions. The equal sharing concept is used to interpret divisions which involving whole numbers as divisors. More precisely, this difficulty occurs when they encounter the case of an incomplete part of an object and they asked to determine how much each part divides the incomplete part. They focus on how many parts have been divided regardless of the shared part. This case can be exemplified by Ummu when responding to what part was received by three children when they shared the $3\frac{1}{2}$ cakes equally. Ummu assumed that the 1/2 part divided by 3 is 1/3 (Fig.8). This is also similar to Cici's response to the problem (Fig 13).

We connect this equal sharing conceptual problem to an intuition-based mistake from Tirosh (2000). The intuition-based mistake encounters stem from students' tendency to generalize the concept of equal sharing overly. The students in the sample of this study think that "everything shared by a certain number of a part is one per part of a dividing part". We also recognize that the conceptual problem for equal sharing of these fractions is related to students' misconceptions on the concept of fractions part. The students often focus only on how many parts are shared but they do not notice whether a value of fractions is equal (see Fig. 2; Fig. 14). The concept of the fraction part is a foundation for children to learn a fraction meaning, fractions operations, and advanced concepts of fractions. Therefore, when these fundamentals are not robust, it will affect the understanding of fraction operations including fraction divisions. In an attempt to reduce this problem, it is critical not only to focus on the meaning of the fractions as part-to-whole, but also to emphasize

other sub-fraction constructs in the learning process of fraction concepts (Clarke, Clarke, & Roche, 2011; Purnomo, 2015a; Siebert & Gaskin, 2006), among others fraction as division, fraction as ratio, fraction as operator, fraction as measure. This is also alluded to by Wijaya (2017) in which most mathematics textbooks in Indonesia only introduce the concept of fractions with the concept of fractions as part-to-whole. The intuition-based mistakes and misconceptions about the concept of fractional parts also discourage students from using the correct terminology. We encounter these things when the child considers 2 as 2/2, 5 as 5/5, and so on (see Fig. 1; Fig. 3; Fig. 4).

Concept issues for equal sharing are not stand-alone. There are other obstacles related to student difficulties when students face fraction divisions. Moreover, we have found that most students still depend on procedural knowledge without being aligned with conceptual knowledge. Some researchers agree that focusing only on procedural knowledge may block a development of intuitive sense and the conceptual knowledge itself (Forrester & Chinnappan, 2010; Purnomo et al., 2014, 2017). These problems can be verified by participants' work on the fractions division case which most of the students did it using invert and multiple rules. These rules are not based on a comprehensive explanation, students employ these rules to obey and apply procedures properly. However, applying the rules by ignoring the conceptual knowledge often causes errors in calculations. One example of the errors in employing this strategy can be seen in Nunu (see Fig.1 and Fig. 9) and Cici work (see Fig. 3 and Fig. 19). This has been mentioned by Tirosh's study (2000) that he has categorized it as an algorithm-based mistake. Generally, obedience to the rules and how to perform procedures properly require the students to memorize them. When they forget a few steps then it will certainly lead them to make mistake.

The last problem encountered is an inability of the children to comprehend the fraction division case, particularly the fractions divided by fractions. This makes sense because they are not accustomed to confronting-context related to sources in both teaching and learning process. In Indonesia, context-based teaching is still unique because education systems still focus on performance and result (Purnomo, 2015b, 2016; Purnomo, Suryadi, & Darwis, 2016; Wijaya, 2017). In addition, mathematics textbooks in Indonesia tend to consist of a set of rules and the use of performance-oriented algorithms (Purnomo et al., 2014; Wijaya, 2017). Consequently, students are more likely to cope with regular problems and they encounter hardness in dealing with context-based problems.

## 5. CONCLUSION

The results of this study indicate that most of the participants of this study still tend to grape with difficulties in working on the fractions division. There are at least three crucial problems creating students difficulties in working on the case of fractions division. First, students' struggles are based on a shortage of conceptual understanding about equal sharing and fraction parts. Second, the difficulty is based on an overemphasis on procedural understanding but not guided by a solid conceptual understanding. The last is students' unfamiliarity on the context-based problems leading to difficulties in interpreting the problem.

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