TESIS - SS14 2501

PEMODELAN PARTIAL PROPORTIONAL ODDS PADA MASA STUDI LULUSAN PROGRAM MAGISTER INSTITUT TEKNOLOGI SEPULUH NOPEMBER

MIKHRATUNNISA
NRP. 1313 201 015

DOSEN PEMBIMBING
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PEMODELAN *PARTIAL PROPORTIONAL ODDS*
PADA MASA STUDI LULUSAN PROGRAM MAGISTER
INSTITUT TEKNOLOGI SEPULUH NOPEMBER

Tesis disusun untuk memenuhi salah satu syarat memperoleh gelar
Magister Sains (M.Si)
di
Institut Teknologi Sepuluh Nopember
Oleh :

MIKHRATUNNISA
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KEMENTERIAN RISET, TEKNOLOGI, DAN PENDIDIKAN TINGGI
INSTITUT TEKNOLOGI SEPULUH NOPEMBER
PASCASARJANA

PEMODELAN PARTIAL PROPORTIONAL ODDS
PADA MASA STUDI LULUSAN PROGRAM MAGISTER
INSTITUT TEKNOLOGI SEPULUH NOPEMBER

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ABSTRAK

Regresi logistik ordinal merupakan salah satu metode statistika untuk menganalisis data dengan variabel respon yang mempunyai skala data ordinal dan terdiri dari tiga kategori atau lebih, serta variabel prediktor yang digunakan dapat berupa data kategori atau kuantitatif. Model yang umum digunakan dalam regresi logistik ordinal adalah *Proportional Odds Model* (POM). POM mempunyai asumsi kuat yang dapat menyebabkan kesalahan interpretasi jika asumsi dilanggar, asumsi umum dari model tersebut adalah bahwa *log-odds ratio* tidak bergantung pada kategori variabel respon, atau lebih dikenal dengan asumsi *proportional odds*. Dalam model regresi logistik ordinal yang memenuhi asumsi *proportional odds* bahwa koefisien dari masing-masing fungsi logit yang terbentuk adalah sama dan yang berbeda adalah pada nilai *intercept* masing-masing fungsi logit tersebut. Model alternatif yang perlu dipertimbangkan ketika asumsi *proportional odds* tidak terpenuhi adalah PPOM (*Partial Proportional Odds Model*), yakni model yang melemahkan asumsi proporsionalitas pada beberapa variabel prediktor dalam model. Tujuan penelitian ini adalah mengkaji estimasi parameter PPOM dan mengaplikasikan PPOM pada masa studi lulusan program magister Institut Teknologi Sepuluh Nopember (ITS). Estimasi parameter PPOM dengan menggunakan metode *Maximum Likelihood Estimation* (MLE) diperoleh hasil yang tidak *close form* sehingga diperlukan iterasi *Newton-Raphson.* Hasil pengujian parameter PPOM menunjukkan bahwa variabel prediktor yang berpengaruh terhadap masa studi lulusan program magister Institut Teknologi Sepuluh Nopember (ITS) adalah nilai Toefl, jenis kelamin, kesesuaian bidang studi antara S1 dan S2, nilai IPK S1, sumber pendanaan, jenis pekerjaan, dan asal fakultas.

Kata kunci : Regresi logistik ordinal, *Proportional Odds Model*, *Partial Proportional Odds Model*, masa studi.

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PARTIAL PROPORTIONAL ODDS MODELLING OF STUDY PERIOD FOR MASTER'S DEGREE PROGRAMME IN INSTITUT TEKNOLOGI SEPULUH NOPEMBER

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ABSTRACT

Ordinal logistic regression is one of statistical method that use to data analysis with ordinal response variable and consist of three or more category, and the predictor variables can be category or quantitative. The general model that used in ordinal logistic regression is Proportional Odds Model (POM) which has strong assumptions may cause wrong interpretation if the assumptions was violated, the common assumption is log-odds ratio not depend on response variable category, called proportional odds assumption. In ordinal logistic regression model that satisfy the proportional odds assumption has a same coefficient of each formed logit function and a different intercept of logit function. The alternative model considering when the proportional odds assumption is not satisfy, that is Partial Proportional Odds Model (PPOM). This model relaxing the proportionality assumption only for some predictor variables in the model. The aims of this research are to review parameter estimation of PPOM and applied PPOM in study period of master's degree programme in Institut Teknologi Sepuluh Nopember. The parameter estimation use Maximum Likelihood Estimation (MLE) and because it is not close form, Newton-Raphson iteration necessary. Parameter PPOM's test show that the influence variables are TOEFL test, Sex, Suitability of S1 and S2, S1 IPK, Funding Resource, Job and Native Faculty.

Keywords : Ordinal logistic regression, Proportional Odds Model, Partial Proportional Odds, Study period

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Surabaya,

Penulis

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BAB 1
PENDAHULUAN

1.1 Latar Belakang

Model regresi merupakan bagian penting dalam beberapa analisis data yang menggambarkan hubungan antara variabel respon dengan satu atau beberapa variabel prediktor. Pada umumnya, analisis regresi digunakan untuk menganalisis data dengan variabel respon berupa data kuantitatif. Akan tetapi sering juga ditemui kasus dengan variabel respon yang bersifat kualitatif atau kategori. Untuk mengatasi masalah tersebut digunakan model regresi logistik. Regresi logistik digunakan jika variabel respon bersifat kualitatif atau kategori (Hair, Black & Barbin, 2006).

Model regresi logistik menurut jenis kategori variabel respon yaitu model regresi logistik biner, model regresi logistik multinomial, dan model regresi logistik ordinal. Model regresi logistik biner digunakan untuk memodelkan variabel respon biner, regresi logistik multinomial adalah perluasan dari model regresi logistik biner dimana variabel respon memiliki lebih dari dua kategori tidak berurutan, dan regresi logistik ordinal merupakan salah satu metode statistika untuk menganalisis data dengan variabel respon yang mempunyai skala data ordinal dan terdiri dari tiga kategori atau lebih, sedangkan variabel prediktor yang digunakan dalam model dapat berupa data kategori atau kuantitatif (Hosmer & Lemeshow, 2000).

Model yang umum digunakan dalam regresi logistik ordinal adalah POM (*Proportional Odds Model*). POM mempunyai asumsi kuat yang dapat menyebabkan kesalahan interpretasi jika asumsi dilanggar (Das & Rahman, 2011). Dolgun & Saracbasi (2014) juga menjelaskan bahwa penggunaan POM dilandasi pada asumsi penting yang harus dipenuhi, asumsi umum dari model tersebut adalah bahwa *log-odds ratio* tidak bergantung pada kategori variabel respon atau lebih dikenal dengan asumsi *proportional odds*.

Dalam model regresi logistik ordinal yang memenuhi asumsi *proportional odds* bahwa koefisien regresi dari masing-masing fungsi logit yang terbentuk adalah sama dan yang berbeda adalah pada nilai *intercept* masing-masing fungsi logit tersebut (Kleinbaum & Klein, 2002). Penyalahgunaan model *proportional odds* meningkat terutama karena penerapan metode ini untuk data dengan *non proportional odds*. Sehingga model alternatif yang perlu dipertimbangkan untuk kasus ini adalah PPOM (*Partial Proportional Odds Model*), yakni model yang melemahkan asumsi proporsionalitas hanya untuk beberapa variabel prediktor dalam model (Dolgun & Saracbasi, 2014). PPOM merupakan perluasan dari *proportional odds model* yang memperbolehkan beberapa prediktor dimodelkan dengan asumsi *proportional adds* dan untuk variabel lain dimana asumsi ini tidak terpenuhi maka parameter tertentu dimasukkan dalam model yang berbeda untuk berbagai kategori yang dibandingkan (Siqueira, Cardoso, Caiaffa, Abreu & Natali, 2008).

Beberapa penelitian berkaitan dengan regresi logistik ordinal dan PPOM telah dilakukan oleh Ananth dan Kleinbaum (1997), Bender dan Grouven (1998), Siqueira *et al*. (2008), Das dan Rahman (2011), dan Citko *et al*. (2012) pada bidang kesehatan, kemudian Liu dan Koirala (2012) pada bidang pendidikan. Penelitian-penelitian tersebut hanya mengaplikasikan model regresi logistik ordinal dan PPOM pada bidang kesehatan dan pendidikan, oleh karena itu pada penelitian ini akan dilakukan kajian estimasi parameter PPOM dan mengaplikasikan PPOM pada bidang pendidikan yakni pada masa studi lulusan program magister Institut Teknologi Sepuluh Nopember (ITS).

Pendidikan adalah suatu aktivitas sosial yang memungkinkan masyarakat tetap ada dan berkembang (Brameld, 1999). Di Indonesia terdapat tiga jenjang pendidikan, yaitu pendidikan dasar (SD/MI/Paket A dan SLTP/MTs/Paket B), pendidikan menengah (SMU/SMK), dan pendidikan tinggi (mencakup program pendidikan diploma, sarjana, magister, doktor, dan spesialis). Salah satu jenjang pendidikan yang menjadi persyaratan dasar dalam mencari pekerjaan adalah perguruan tinggi, yang mana perguruan tinggi akan mempersiapkan calon-calon sarjana yang handal dan mempunyai keterampilan dibidangnya (Padmini, Suciptawati & Susilawati, 2012). Salah satu ukuran kesuksesan pembelajaran pada suatu perguruan tinggi adalah tingkat ketepatan masa studi (Guillory, 2008). Seorang mahasiswa membutuhkan waktu normal selama delapan semester untuk dapat menyelesaikan program sarjana dan empat semester untuk menyelesaikan program magisternya, akan tetapi dalam prakteknya mahasiswa tidak selalu dapat menyelesaikan studinya selama waktu normal yang telah di tentukan. Hal ini menunjukkan bahwa mahasiswa dapat menyelesaikan studinya dalam tiga tingkatan ketepatan masa studi, yakni menyelesaikan studi lebih cepat dari waktu normal, sesuai waktu normal, dan lebih dari waktu normal. Sehingga pada penelitian ini masa studi lulusan program magister ITS dikategorikan menjadi tiga kategori, yaitu lulus dengan masa studi tiga semester, lulus dengan masa studi empat semester, dan lulus dengan masa studi lebih dari empat semester.

1.2 Rumusan Masalah

Adapun rumusan masalah dalam penelitian ini adalah sebagai berikut.

1. Bagaimana hasil estimasi parameter pada PPOM?
2. Bagaimana aplikasi PPOM pada masa studi lulusan program magister Institut Teknologi Sepuluh Nopember?

1.3 Tujuan

Adapun tujuan dalam penelitian ini adalah sebagai berikut.

1. Mengkaji estimasi parameter pada PPOM.
2. Mengaplikasikan PPOM pada masa studi lulusan program magister Institut Teknologi Sepuluh Nopember.

1.4 Manfaat Penelitian

Adapun manfaat yang ingin dicapai pada penelitian ini adalah sebagai berikut.

1. Sebagai pengembangan keilmuwan PPOM pada variabel respon ordinal dan implementasinya pada bidang pendidikan.
2. Mengetahui karakteristik dan faktor-faktor yang berhubungan dengan masa studi lulusan program magister Institut Teknologi Sepuluh Nopember.

1.5 Batasan Masalah

Batasan permasalahan dalam penelitian ini adalah sebagai berikut:

1. PPOM yang digunakan adalah *Unconstrained Partial Proportional Odds Model*.
2. Pada penelitian ini menggunakan tiga kategori variabel respon.
3. Orde deret Taylor yang digunakan adalah sampai pada orde ke dua.

BAB 2
TINJAUAN PUSTAKA

2.1 Regresi Logistik Ordinal

Hosmer dan Lemeshow (2000) mengemukakan bahwa model regresi logistik adalah model regresi dengan variabel respon bersifat kualitatif. Model regresi logistik ada yang bersifat *dichotomous* yang mensyaratkan variabel respon terdiri dari dua kategori dan bersifat *polychotomous* dengan variabel respon lebih dari dua kategori. Regresi logistik *polychotomous* dengan variabel respon yang bertingkat sering dikenal dengan regresi logistik ordinal. Contoh variabel respon pada regresi logistik ordinal dengan tiga kategori adalah kategori rendah, sedang, dan tinggi (ada tingkatan dalam kategori variabel respon).

Regresi logistik ordinal adalah suatu analisis regresi yang digunakan untuk menggambarkan hubungan antara variabel respon dengan sekumpulan variabel prediktor, dimana variabel respon bersifat ordinal, yaitu mempunyai lebih dari dua kategori dan setiap kategori dapat diurutkan, variabel prediktor dapat berupa data kategori atau kontinu yang terdiri dari dua variabel atau lebih. Metode yang sering digunakan untuk variabel respon berskala ordinal adalah membentuk fungsi logit dari peluang kumulatif. Oleh karena itu model regresi logistik untuk respon ordinal sering disebut sebagai model logit kumulatif (Agresti, 2007).

2.2 Proportional Odds Model

Model logit kumulatif pertama kali diperkenalkan oleh Walkel dan Duncan pada tahun 1967 dan kemudian disebut POM (*Proportional Odds Model*) oleh McCullagh pada tahun 1980. POM merupakan jenis regresi logistik ordinal yang umum digunakan (Hosmer & Lemeshow, 2000). Menurut Agresti (2007), POM adalah model yang membandingkan peluang kumulatif yaitu peluang kurang dari atau sama dengan kategori respon ke-*j* pada *p* variabel prediktor yang dinyatakan dalam vektor **x**, $(P[Y \leq j|\mathbf{x}])$, dengan peluang lebih besar dari kategori respon ke-*j*, $(P[Y > j|\mathbf{x}])$. Jika variabel prediktor $\mathbf{x} = (x_1 x_2 ... x_p)^T$, maka peluang kumulatif logit didefinisikan sebagai:

$$\text{Logit } P(Y \le j|\mathbf{x}) = \ln\left[\frac{P(Y \le j|\mathbf{x})}{1 - P(Y \le j|\mathbf{x})}\right]$$

$$= \ln\left[\frac{P(Y \le j|\mathbf{x})}{P(Y > j|\mathbf{x})}\right]$$

$$= \ln\left[\frac{\pi_1(\mathbf{x}) + \pi_2(\mathbf{x}) + \ldots + \pi_j(\mathbf{x})}{\pi_{j+1}(\mathbf{x}) + \pi_{j+2}(\mathbf{x}) + \ldots + \pi_J(\mathbf{x})}\right], j = 1, 2, \ldots, J\text{-}1 \qquad (2.1)$$

dimana $j = 1, 2, \ldots, J\text{-}1$ merupakan kategori variabel respon.

Dependensi peluang kumulatif *y* terhadap **x** untuk *Proportional Odds Model* sering dinyatakan dalam bentuk:

$$\ln\left[\frac{(P \le j|\mathbf{x})}{(P > j|\mathbf{x})}\right] = \theta_{0j} + \mathbf{x}^T\boldsymbol{\beta} \quad , j = 1, 2, \ldots, J\text{-}1 \qquad (2.2)$$

Persamaan (2.1) dapat diubah ke dalam bentuk:

$$\ln\left[\frac{\pi_1(\mathbf{x}) + \pi_2(\mathbf{x}) + \ldots + \pi_j(\mathbf{x})}{\pi_{j+1}(\mathbf{x}) + \pi_{j+2}(\mathbf{x}) + \ldots + \pi_J(\mathbf{x})}\right] = \theta_{0j} + \mathbf{x}^T\boldsymbol{\beta} \quad , j = 1, 2, \ldots, J\text{-}1$$

Model yang secara simultan menggunakan semua kumulatif logit adalah:

$$\text{logit } [P(Y \le j|\mathbf{x})] = \theta_{0j} + \mathbf{x}^T\boldsymbol{\beta} \quad , j = 1, 2, \ldots, J\text{-}1 \qquad (2.3)$$

$\gamma_j(\mathbf{x}) = P(Y \le j|\mathbf{x})$ merupakan peluang kumulatif dari kejadian $(Y \le j)$ dan $\{\theta_{0j}\}$ merupakan parameter intersep yang tidak diketahui yang memenuhi kondisi $\theta_{01} \le \theta_{02} \le \ldots \le \theta_{0,J-1}$ dan $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_p)^T$ merupakan vektor koefisien regresi yang tidak diketahui yang bersesuaian dengan **x**.

Jika $\gamma_j(\mathbf{x}) = \pi_1(\mathbf{x}) + \pi_2(\mathbf{x}) + \pi_3(\mathbf{x}) + \ldots + \pi_j(\mathbf{x})$ maka $\gamma_1(\mathbf{x}) = \pi_1(\mathbf{x})$, $\gamma_2(\mathbf{x}) = \pi_1(\mathbf{x}) + \pi_2(\mathbf{x})$, dan $\gamma_J(\mathbf{x}) = \pi_1(\mathbf{x}) + \pi_2(\mathbf{x}) + \pi_3(\mathbf{x}) + \ldots + \pi_J(\mathbf{x}) = 1$. Model regresi logistik ordinal yang terbentuk jika terdapat J kategori respon adalah:

$$\text{logit } \gamma_1(\mathbf{x}) = \ln\left(\frac{\gamma_1(\mathbf{x})}{1 - \gamma_1(\mathbf{x})}\right) = \theta_{01} + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p$$

$$\text{logit } \gamma_2(\mathbf{x}) = \ln\left(\frac{\gamma_2(\mathbf{x})}{1 - \gamma_2(\mathbf{x})}\right) = \theta_{02} + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p$$

$$\vdots$$

$$\text{logit}\ \text{a}_{J-1}(\mathbf{x}) = \ln\left(\frac{\text{a}_{J-1}(\mathbf{x})}{1-\text{a}_{J-1}(\mathbf{x})}\right) = \quad {}_{0,J-1} + \quad {}_{1}x_1 + \quad {}_{2}x_2 + \ldots + \quad {}_{p}x_p$$

dimana $\text{a}_j(\mathbf{x}) = P(Y \le j|\mathbf{x}) = \dfrac{e^{\ {}_{0j}+\mathbf{x}^T\boldsymbol{\beta}}}{1+e^{\ {}_{0j}+\mathbf{x}^T\boldsymbol{\beta}}}$, j *= 1, 2, …, J-1*

model ini disebut model kumulatif karena *odds rasio* dari suatu kejadian $(Y \le j)$ adalah independen pada setiap indikator kategori.

Jika dimisalkan terdapat tiga kategori variabel respon, maka peluang untuk masing-masing kategori variabel respon adalah:

$$®_1(x) = P(Y \le 1) = \frac{e^{\ {}_{01}+\mathbf{x}^T\boldsymbol{\beta}}}{1+e^{\ {}_{01}+\mathbf{x}^T\boldsymbol{\beta}}}$$

$$®_2(x) = \frac{e^{\ {}_{02}+\mathbf{x}^T\boldsymbol{\beta}}}{1+e^{\ {}_{02}+\mathbf{x}^T\boldsymbol{\beta}}} - \frac{e^{\ {}_{01}+\mathbf{x}^T\boldsymbol{\beta}}}{1+e^{\ {}_{01}+\mathbf{x}^T\boldsymbol{\beta}}}$$

$$®_3(x) = \frac{1}{1+e^{\ {}_{02}+\mathbf{x}^T\boldsymbol{\beta}}}$$

2.2.1 Estimasi Parameter Proportional Odds Model

Metode yang digunakan untuk estimasi parameter regresi logistik ordinal adalah *maximum likelihood estimation* (MLE). Metode ini digunakan karena distribusi dari variabel respon Y diketahui. Selain itu, metode MLE dipilih karena mempunyai beberapa kelebihan dibandingkan dengan metode lainnya, diantaranya dapat digunakan untuk model tidak linier seperti regresi logistik, serta hasil estimasinya mendekati parameternya (Hosmer & Lemeshow, 2000).

Dengan metode ini estimasi parameter dilakukan dengan cara memaksimumkan fungsi *ln-likelihood*. Variabel respon Y dengan J kategori dimana J > 2 dapat dipandang sebagai sebuah percobaan yang menghasilkan J hasil yang mungkin, sehingga variabel random $Y_1, Y_2, \ldots, Y_J$ yang menyatakan terjadinya kejadian $y_1, y_2, \ldots, y_J$ berdistribusi multinomial. Jika diambil *m* sampel random $\mathbf{Y}_1, \mathbf{Y}_2, \ldots, \mathbf{Y}_m$ maka

$\mathbf{Y}_i = (Y_{1i}, Y_{2i}, \ldots, Y_{Ji}) \sim M(1; ®_1(\mathbf{x}_i), ®_2(\mathbf{x}_i), \ldots, ®_J(\mathbf{x}_i))$ dengan distribusi probabilitas sebagai berikut:

$$P\left(Y_{1i}=y_{1i},Y_{2i}=y_{2i},Y_{Ji}=y_{Ji}\middle|1;\text{®}_j\left(\mathbf{x}_i\right)\right)=\left[\text{®}_1\left(\mathbf{x}_i\right)\right]^{y_{1i}}\left[\text{®}_2\left(\mathbf{x}_i\right)\right]^{y_{2i}}\ldots\left[\text{®}_J\left(\mathbf{x}_i\right)\right]^{y_{Ji}}$$

dimana i = 1, 2, …, m dan j = 1, 2, …, J

Karena peluang kumulatif yang digunakan dalam estimasi parameter, maka fungsi *likelihood* yang merupakan fungsi padat peluang bersama dari variabel random $Y_1, Y_2, \ldots, Y_J$ dapat ditulis sebagai perkalian J-1 kategori sebagai berikut:

$$L(\boldsymbol{\theta})=\prod_{i=1}^{m}\left\{\left[\text{®}_1\left(\mathbf{x}_i\right)\right]^{y_{1i}}\left[\text{®}_2\left(\mathbf{x}_i\right)\right]^{y_{2i}}\ldots\left[1-\text{®}_1\left(\mathbf{x}_i\right)-\text{®}_2\left(\mathbf{x}_i\right)-\ldots-\text{®}_{J-1}\left(\mathbf{x}_i\right)\right]^{1-y_{1i}-y_{2i}-\ldots y_{(J-1)i}}\right\} \quad (2.4)$$

dimana

$$\text{®}_J\left(\mathbf{x}_i\right)=1-\text{®}_1\left(\mathbf{x}_i\right)-\text{®}_2\left(\mathbf{x}_i\right)-\ldots-\text{®}_{J-1}\left(\mathbf{x}_i\right)$$

$$y_{Ji}=1-y_{1i}-y_{2i}-\ldots-y_{(J-1)i}$$

Berdasarkan fungsi *likelihood* pada persamaan (2.4) diperoleh fungsi *ln-likelihood* sebagai berikut

$$\begin{aligned}\ell(\boldsymbol{\theta})=\sum_{i=1}^{m}\Big\{&y_{1i}\ln\left[\text{®}_1\left(\mathbf{x}_i\right)\right]+y_{2i}\ln\left[\text{®}_2\left(\mathbf{x}_i\right)\right]+\ldots+y_{(J-1)i}\ln\left[\text{®}_{J-1`}\left(\mathbf{x}_i\right)\right]+\\&\left(1-y_{1i}-y_{2i}-\ldots-y_{(J-1)i}\right)\ln\left[1-\text{®}_1\left(\mathbf{x}_i\right)-\text{®}_2\left(\mathbf{x}_i\right)-\ldots-\text{®}_{J-1}\left(\mathbf{x}_i\right)\right]\Big\}\end{aligned}$$

Sebagaimana diketahui bahwa estimasi parameter dengan menggunakan metode MLE adalah memaksimumkan fungsi *ln-likelihood* dengan melakukan turunan parsial pertama fungsi *ln-likelihood* terhadap parameter yang akan diestimasi, kemudian menyamakannya dengan nol. Akan tetapi, turunan parsial pertama fungsi *ln-likelihood* terhadap parameter yang akan diestimasi merupakan fungsi non linier, oleh karena itu diperlukan metode numerik untuk memperoleh estimasi parameternya. Metode numerik yang digunakan adalah metode iterasi *Newton Raphson*.

Persamaan yang digunakan dalam metode iterasi *Newton Raphson* adalah sebagai berikut

$$\boldsymbol{\theta}^{(t+1)}=\boldsymbol{\theta}^{(t)}-\left[\mathbf{H}^{(l)}\right]^{-1}\mathbf{q}^{(t)}$$

dimana, $\mathbf{q}^T = \left(\frac{\partial \ell(\boldsymbol{\theta})}{\partial \ _0}, \frac{\partial \ell(\boldsymbol{\theta})}{\partial \ _1}, \frac{\partial \ell(\boldsymbol{\theta})}{\partial \ _2}, \ldots, \frac{\partial \ell(\boldsymbol{\theta})}{\partial \ _p} \right)$ dan $\mathbf{H}$ merupakan matriks Hessian dengan elemen $h_{ab} = \left(\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \ _a \partial \ _b} \right)$. Iterasi akan berhenti jika terpenuhi kondisi konvergen, yaitu selisih $\left\| \boldsymbol{\theta}^{(t+1)} - \boldsymbol{\theta}^{(t)} \right\| \leq £$, dimana £ adalah bilangan yang sangat kecil (Didimalayan, 2009).

2.2.2 Asumsi Proportional Odds

Mengingat bahwa kategori dari variabel respon adalah ordinal, pendekatan umum yang digunakan adalah model logit ordinal yang standar. Hasil dari model logit ordinal standar tersebut hanya akan berlaku jika asumsi *parallel lines* (atau lebih dikenal dengan asumsi *proportional odds*) terpenuhi (Soon, 2010). Model logistik ordinal tersebut diartikan dengan konstanta yang berbeda, namun koefisien vektor $\boldsymbol{\beta}$ sama. Hal ini merupakan sifat "asumsi *proportional odds*" yang harus diperiksa dalam penerapan model regresi logistik ordinal. Uji Brant diterapkan untuk menguji asumsi tersebut sebagai berikut (Brant, 1990).

Hipotesis:

$H_0 : \boldsymbol{\beta}_j - \boldsymbol{\beta}_1 = 0$, *j = 2, 3, …, J-1*

atau

$H_0 : \mathbf{R}\boldsymbol{\beta}^* = 0$

$H_1 : \mathbf{R}\boldsymbol{\beta}^* \neq 0$

dimana:

$$\mathbf{R} = \begin{bmatrix} \mathbf{I} & -\mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & -\mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{I} \end{bmatrix}, \ \boldsymbol{\beta}^* = \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \boldsymbol{\beta}_3 \\ \vdots \\ \boldsymbol{\beta}_{J-1} \end{bmatrix}$$

Statistik Uji Wald:

$$\text{i}^2_{[p\times(J-1)]} = \left(\mathbf{R}\hat{\boldsymbol{\beta}}^*\right)^T \left[\mathbf{R} \times \textbf{Asy.Var}\left(\hat{\boldsymbol{\beta}}^*\right) \times \mathbf{R}^T\right]^{-1} \left(\mathbf{R}\hat{\boldsymbol{\beta}}^*\right)$$

dimana matriks asimtotik covarian memuat:

$$\textbf{Asy.Var}\left(\hat{\boldsymbol{\beta}}^*\right)(j,l) = Est.Asy.Cov\left[\hat{\boldsymbol{\beta}}_j, \hat{\boldsymbol{\beta}}_l\right]$$

$$= \left[\sum_{i=1}^{n} \hat{\Lambda}_{ij}\left(1-\hat{\Lambda}_{ij}\right)\mathbf{X}_i\mathbf{X}_i^T\right]^{-1} \times \left[\sum_{i=1}^{n} \hat{\Lambda}_{il}\left(1-\hat{\Lambda}_{ij}\right)\mathbf{X}_i\mathbf{X}_i^T\right] \times \left[\sum_{i=1}^{n} \hat{\Lambda}_{il}\left(1-\hat{\Lambda}_{il}\right)\mathbf{X}_i\mathbf{X}_i^T\right]^{-1}$$

dan $\hat{\Lambda}_{ij} = \Lambda\left(\hat{\alpha}_{0j} + \mathbf{X}_i^T\hat{\boldsymbol{\beta}}_j\right)$. Dibawah hipotesis nol statistik Wald mendekati distribusi *Chi-square* dengan derajat bebas *p(J-1)*.

Daerah penolakan: H_0 ditolak jika $\chi^2 > \chi^2_{(df,\alpha)}$.

Asumsi *proportional odds* membatasi koefisien dari masing-masing variabel prediktor identik pada sejumlah *J-1* persamaan kumulatif, artinya bahwa dalam kasus model logit ordinal yang standar, hanya mengestimasi sekumpulan koefisien yang akan digunakan pada *J-1* persamaan kumulatif. Dengan kata lain, asumsi *proportional odds* menunjukkan bahwa $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \ldots = \boldsymbol{\beta}_{J-1}$, dimana 1, 2,…, *J-1* adalah *J* kategori respon. Oleh karena itu, jika asumsi *proportional odds* terpenuhi maka koefisien yang diestimasi $\hat{\boldsymbol{\beta}}_1 = \hat{\boldsymbol{\beta}}_2 = \ldots = \hat{\boldsymbol{\beta}}_{J-1}$ harus identik atau hampir sama (Long & Freese, 2006).

2.3 Partial Proportional Odds Model

Pengembangan *Partial Proportional Odds Model* (PPOM) adalah untuk melemahkan asumsi yang kuat dari *log-odds ratio* identik untuk hubungan *Y* dengan x_i dalam POM, hal ini berarti bahwa koefisien dari masing-masing variabel prediktor adalah identik pada sejumlah *J-1* persamaan kumulatif. McCullagh menyebut asumsi *log-odds ratio* identik tersebut sebagai asumsi *proportional odds*. Pelanggaran asumsi tersebut dapat menyebabkan perumusan model yang tidak sesuai atau *misspecified* (Ananth & Kleinbaum, 1997).

Jarang sekali variabel prediktor yang dimasukkan dalam model regresi logistik ordinal memenuhi asumsi *proportional odds*. Untuk mangatasi masalah ini digunakan PPOM, dimana PPOM membolehkan beberapa variabel prediktor dimodelkan dengan asumsi *proportional adds* dan untuk variabel lain yang tidak memenuhi asumsi *proportional adds* maka parameter tertentu dimasukkan dalam model yang berbeda untuk berbagai kategori yang dibandingkan. PPOM merupakan perluasan dari POM. PPOM dapat diklasifikasikan menjadi *Unconstained Partial Proportional Odds Model* dan *Constrained Partial Proportional Odds Model* (Siqueira *et al*., 2008).

2.3.1 Klasifikasi Partial Proportional Odds Model

a. Unconstrained Partial Proportional Odds Model

Menurut Siqueira *et al*. (2008) dan Liu (2009), *Unconstrained Partial Proportional Odds Model* digunakan ketika asumsi *proporsional odds* tidak terpenuhi. Model tersebut sebagai berikut.

$$ª_j(\mathbf{x}) = \ln\left[\frac{\Pr(Y=1|\mathbf{x})+\ldots+\Pr(Y=j|\mathbf{x})}{\Pr(Y=j+1|\mathbf{x})+\ldots+\Pr(Y=k|\mathbf{x})}\right] = \ln\left[\frac{\sum_{1}^{j}\Pr(Y=j|\mathbf{x})}{\sum_{j+1}^{J}\Pr(Y=j|\mathbf{x})}\right]$$

$$ª_j(\mathbf{x}) = \sim_j + \{-(\ \ _1 + ¥_{j1})x_1 - \ldots - (\ \ _q + ¥_{jq})x_q - (\ \ _{q+1}X_{q+1}) - \ldots - (\ \ _p X_p)\}\ ,\ j=1,\ldots,J-1$$

Dalam model tersebut pada p variabel prediktor $\mathbf{x} = (x_1, x_2, \ldots, x_p)$, hanya beberapa di antaranya yang memenuhi asumsi *proportional odds*. Tanpa mengurangi keumuman, misalkan q variabel prediktor pertama tidak memenuhi asumsi *proporsional odds*.

Untuk variabel dimana asumsi *proportional odds* tidak terpenuhi, misalkan x_1, $\sim_j + \ \ x_1$ ditambah dengan koefisien $¥_{j1}$ yang merupakan pengaruh asosiasi dengan setiap logit kumulatif. Dengan demikian, koefisien dari variabel prediktor adalah $\sim_j + \ \ x_1 + ¥_{j1}$ (Siqueira *et al*., 2008).

Untuk model ini, yang diestimasi adalah (*J-1*) intersep, p koefisien beta $(\ \ )$ yang independen dari kategori yang dibandingkan, dan *q(J-2)* parameter gamma $(¥)$ yang berhubungan dengan setiap variabel prediktor dan kategori dalam variabel respon. Jika parameter gamma $(¥)$ adalah nol, $¥_j = 0$ untuk setiap j maka model direduksi menjadi model *proportional odds* (Siqueira *et al*., 2008).

Dalam model ini, untuk q variabel prediktor pertama, koefisien $(\ \ )$ bergantung pada j, yang berarti bahwa hubungan antara $\mathbf{x}$ dan Y adalah bergantung pada kategori. Akibatnya *odds rasio* diestimasi untuk semua perbandingan antara kategori variabel respon. Untuk variabel prediktor lain, koefisien$(\quad)$ independen dari *j*, dengan demikian hanya satu *odds rasio* yang diestimasi (Siqueira *et al*., 2008).

Menurut Ananth & Kleinbaum (1997), PPOM memperbolehkan *non-proportional odds* untuk *q* subset dari *p* variabel prediktor ($q < p$). Jika Y variabel respon ordinal dengan J kategori dan $\mathbf{x}$ vektor berdimensi p dari variabel prediktor, maka model probabilitas kumulatif sebagai berikut.

$$\Pr\left(Y \le y_j \middle| \mathbf{x}\right) = \frac{e^{-\sim_j - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_j}}{1 + e^{-\sim_j - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_j}} \text{ , } j=1,2,\ldots J\text{-}1 \qquad (2.5)$$

dimana $\mathbf{x}$ adalah vektor berukuran $(p \times 1)$ yang memuat nilai observasi pada p variabel prediktor, $\boldsymbol{\beta}$ merupakan vektor koefisien regresi yang berukuran $(p \times 1)$ yang terkait dengan p variabel dalam $\mathbf{x}$, $\mathbf{t}$ adalah vektor berukuran $(q \times 1)$, $q < p$, yang memuat nilai observasi pada subset dari p variabel prediktor yang tidak memenuhi asumsi *proportional odds*, $\boldsymbol{\gamma}_j$ adalah vektor berukuran $(q \times 1)$ yang merupakan vektor koefisien regresi yang berhubungan dengan q variabel dalam $\mathbf{t}$, sehingga $\mathbf{t}^T\boldsymbol{\gamma}_j$ adalah peningkatan/penambahan yang hanya berhubungan dengan logit kumulatif ke-*j*, $1 \le j \le J$ dan $\boldsymbol{\gamma}_1 = \mathbf{0}$. Ketika $\boldsymbol{\gamma}_j = \mathbf{0}$ untuk semua *j*, model (2.5) direduksi menjadi POM. Pengujian secara simultan asumsi *proportional odds* untuk q variabel dalam ***t*** didasarkan pada hipotesis nol $H_0 : \boldsymbol{\gamma}_j = \mathbf{0}$ untuk semua j $(2 \le j \le J)$. Karena $\boldsymbol{\gamma}_1 = \mathbf{0}$, model hanya menggunakan $\left(\sim + \mathbf{x}^T\boldsymbol{\beta}\right)$ untuk mengestimasi *odds rasio* yang terkait dengan dikotomisasi dari *Y* menjadi $y_j = 1$ dibandingkan $y_j > 1$. Namun, estimasi *odds rasio* yang terkait dengan probabilitas kumulatif yang tersisa melibatkan peningkatan/penambahan $\left(\sim + \mathbf{x}^T\boldsymbol{\beta}\right)$ oleh $\mathbf{t}^T\boldsymbol{\gamma}_j$.

b. Constrained Partial Proportional Odds Model

Constrained Partial Proportional Odds Model digunakan jika *asumsi proportional odds* tidak valid, dan ada hubungan linier untuk *odds rasio* antara prediktor dan variabel respon (Siqueira *et al*., 2008). Menurut Siqueira *et al*. (2008) dan Liu (2009), model tersebut sebagai berikut.

$$a_j(\mathbf{x}) = \ln\left[\frac{\Pr(Y=1|\mathbf{x})+\ldots+\Pr(Y=j|\mathbf{x})}{\Pr(Y=j+1|\mathbf{x})+\ldots+\Pr(Y=k|\mathbf{x})}\right] = \ln\left[\frac{\sum_{i}^{j}\Pr(Y=j|\mathbf{x})}{\sum_{j+1}^{k}\Pr(Y=j|\mathbf{X})}\right]$$

$$a_j(\mathbf{x}) = \alpha_j + \{\tau_j\left[-(\beta_1+\gamma_{j1})x_1 - \ldots - (\beta_q+\gamma_q)x_q\right] - (\beta_{q+1}x_{q+1}) - \ldots - (\beta_p x_p)\} \;,\; j=1,\ldots,J-1$$

Dalam hal ini, pembatasan (yang ditunjukkan oleh parameter gamma dan *fixed scalar*) dapat dimasukkan sebagai parameter dalam model untuk menggabungkan linieritas ini. Untuk variabel prediktor yang diberikan, koefisien gamma (γ) tidak bergantung pada titik potong, tetapi dikalikan dengan koefisien tau (τ) untuk setiap logit (Siqueira *et al*., 2008).

Peterson dan Harrell (1990) mengusulkan model lain yang disebut "*Constrained Partial Proportional Odds Model*". Model tersebut sebagai berikut.

$$\Pr(Y \le y_j|x) = \frac{e^{-\alpha_j - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}\tau_j}}{1+e^{-\alpha_j - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}\tau_j}} \quad , \quad j=1,2,\ldots,J-1$$

dimana τ_j ditentukan sebelumnya, *fixed scalar*, dan $\tau_1 = 0$. Parameter baru yakni $\boldsymbol{\gamma}$ merupakan vektor berukuran $(q \times 1)$, dan tidak terindeks oleh *j*. Meskipun $\boldsymbol{\gamma}$ tidak bergantung pada *j*, $\boldsymbol{\gamma}$ dikalikan dengan konstanta *fixed scalar* τ_j dalam perhitungan logit kumulatif ke-*j*.

2.3.2 Estimasi Parameter Partial Proportional Odds Model

Umumnya langkah estimasi parameter pada PPOM serupa dengan estimasi parameter pada POM. Estimasi parameter PPOM juga menggunakan metode MLE. Menurut Peterson dan Harrell (1990), fungsi *likelihood* untuk PPOM sebagai berikut.

$$L(\mathbf{\Phi})=\prod_{i=1}^{m}\prod_{j=1}^{J}\left[\Pr(Y=j|\mathbf{x})\right]^{y_{ji}}=\prod_{i=1}^{m}\prod_{j=1}^{J}\left[\circledR_{j}(\mathbf{x}_i)\right]^{y_{ji}} \qquad (2.6)$$

dimana y_{ji} merupakan indikator variabel untuk observasi *i* sehingga $y_{ji}=1$ jika Y=*j* dan $y_{ji}=0$ jika $Y\neq j$, $j = 1, \ldots, J.$ Fungsi *ln-likelihood*, dinotasikan dengan ℓ , yakni

$$\ell(\mathbf{\Phi})=\sum_{i=1}^{m}\sum_{j=1}^{J}y_{ji}\ln\Pr(Y=j|\mathbf{x})=\sum_{i=1}^{m}\sum_{j=1}^{J}y_{ji}\ln\left[\circledR_{j}(\mathbf{x}_i)\right] \qquad (2.7)$$

dimana $\boldsymbol{\gamma}_1=\mathbf{0}$, dan

$$\Pr(Y=1|\mathbf{x})=1-\frac{1}{1+e^{-\sim_j-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_j}}$$

$$=\frac{e^{-\sim_j-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_j}}{1+e^{-\sim_j-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_j}} \quad \text{, jika Y=1}$$

$$\Pr(Y=j|\mathbf{x})=\frac{1}{1+e^{-\sim_j-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_j}}-\frac{1}{1+e^{-\sim_{j+1}-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_{j+1}}}$$

$$=\frac{e^{-\sim_{j+1}-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_{j+1}}-e^{-\sim_j-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_j}}{\left(e^{-\sim_{j+1}-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_{j+1}}\right)\left(1+e^{-\sim_j-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_j}\right)} \quad \text{,1<Y<J}$$

$$\Pr(Y=J|\mathbf{x})=\frac{1}{1+e^{-\sim_J-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_J}} \quad \text{, jika Y = J.}$$

Untuk menentukan nilai $\sim_j$,   , dan $¥_j$ yang memaksimumkan *log-likelohood*, ℓ , maka digunakan metode *Newton-Raphson*. Metode ini merupakan prosedur iterative untuk mendapatkan nilai-nilai dimana fungsi yang sesuai dimaksimalkan. Untuk menggunakannya, nilai estimasi awal harus dibuat. Maka ekspansi Taylor dari fungsi ini dibuat disekitar nilai awal tersebut. Dengan menggunakan dua syarat pertama dari ekspansi Taylor dapat ditentukan nilai dimana aproksimasi dimaksimalkan. Nilai ini menjadi dua, yang telah diperbaiki, estimasi, dan proses iterasi kedua dimulai. Pada iterasi kedua, fungsi ini diaproksimasi disekitar estimasi kedua dengan dua tingkat ekspansi Taylor lain, dan nilai yang memaksimalkan pendekatan ini menjadi estimasi ketiga. Prosedur berlanjut secara demikian sampai dua kali *ln-likelihood* kurang dari suatu konstanta yang ditentukan, misalnya 0,005.

2.4 Pengujian Parameter Model Regresi Logistik

Pengujian parameter dalam model regresi logistik terdiri dari uji parsial dan uji serentak. Berikut ini merupakan langkah-langkah pengujiannya.

a. Uji Serentak

Uji serentak ini dilakukan untuk mengetahui signifikansi parameter terhadap variabel respon secara keseluruhan. Pengujian signifikansi parameter ini menggunakan statistik uji *G* atau *likelihood ratio test*, dimana statistik uji *G* mengikuti distribusi *Chi-Square* (Hosmer & Lemeshow, 2000).

Hipotesis yang digunakan:

$H_0 : \boldsymbol{\beta} = \mathbf{0}$

$H_1 : \boldsymbol{\beta} \neq \mathbf{0}$

Statistik Uji:

$$G = -2\ln\left(\frac{L_0}{L_1}\right)$$

dimana, L_0 adalah *likelihood* tanpa variabel prediktor dan L_1 adalah *likelihood* dengan variabel prediktor.

Daerah penolakan:

H_0 ditolak jika $G > \chi^2_{(\alpha,v)}$ dengan derajat bebas *v*. Dimana *v* menunjukkan banyaknya variabel prediktor dalam model.

b. Uji Parsial

Uji parsial dilakukan jika pada uji serentak diperoleh $\boldsymbol{\beta} \neq \mathbf{0}$ (H_0 ditolak). Uji ini digunakan untuk menguji pengaruh setiap β_k secara individual. Hasil pengujian secara individual akan menunjukkan apakah suatu variabel prediktor layak untuk masuk dalam model atau tidak. Hipotesis yang digunakan sebagai berikut.

$H_0 : \beta_k = 0$

$H_1 : \beta_k \neq 0$, dengan k = 1, 2, 3,…, p

Statistik Uji yang digunakan adalah Statistik Uji *Wald*

$$W^2 = \left(\frac{\hat{\beta}_k}{SE\left(\hat{\beta}_k\right)} \right)^2$$

dimana $SE\left(\hat{\beta}_k\right)$ adalah standar error $\hat{\beta}_k$ yang diperoleh dari $SE\left(\hat{\beta}_k\right) = \sqrt{\operatorname{var} \hat{\beta}_k}$.

Daerah penolakan: H_0 ditolak jika $W^2 > \chi^2_{(\alpha,v)}$.

2.5 Uji Kesesuaian Model

Uji kesesuaian model (*goodness-of-fit*) merupakan suatu suatu uji yang digunakan untuk mengetahui apakah model dengan satu atau lebih variabel prediktor merupakan model yang sesuai atau tidak. Statistik uji yang digunakan adalah *deviance* (Hosmer & Lemeshow, 2000). Hipotesis yang digunakan adalah

H_0 : model sesuai (tidak ada perbedaan antara hasil observasi dengan kemungkinan hasil prediksi model)

H_1 : model tidak sesuai (ada perbedaan antara hasil observasi dengan kemungkinan hasil prediksi model)

Statistik uji :

$$D = -2\sum_{i=1}^{n} \left[y_{ij} \ln\left(\frac{\hat{\pi}_{ij}}{y_{ij}} \right) + \left(1 - y_{ij}\right) \ln\left(\frac{1-\hat{\pi}_{ij}}{1-y_{ij}} \right) \right]$$

dimana $\hat{\pi}_{ij} = \hat{\pi}_j\left(x_i\right)$ merupakan peluang observasi ke-*i* pada kategori ke-*j*. Ringkasan statistik berdasarkan *deviance* mengindikasikan kebaikan model terhadap data. Apabila nilai *deviance* semakin tinggi dan *P-value* semakin rendah, maka mengindikasikan bahwa model tidak fit terhadap data. Adapun derajat bebas pada uji ini adalah *J-(p+1)* dimana *J* merupakan jumlah kovariat dan *p* merupakan banyaknya variabel prediktor. Jika model adalah terbaik, maka *deviance* akan mendekati distribusi ¡$^{2}_{\sim, J-(p+1)}$.

2.6 Interpretasi Parameter

Interpretasi parameter dalam regresi logistik menggunakan nilai *odd rasio*. Interpretasi dalam hal ini bertujuan untuk menentukan hubungan fungsional antara variabel prediktor dengan variabel respon dan menentukan unit perubahan dalam variabel prediktor. Menurut Hosmer & Lemeshow (2000), interpretasi model logit tergantung pada jenis variabel prediktornya, yakni:

a. Variabel prediktor kategori

Pada variabel prediktor kategori, nilai *odd rasio* diartikan sebagai nilai yang menunjukkan perbandingan tingkat kecenderungan dari dua kategori dalam satu variabel prediktor dengan salah satu kategorinya dijadikan pembanding atau kategori dasar, yang dimaksud dengan *odd rasio* dari dua kategori X adalah

$$\Psi(x_1, x_2) = \frac{\left[\dfrac{P(Y \le j|x_2)}{P(Y > j|x_2)}\right]}{\left[\dfrac{P(Y \le j|x_1)}{P(Y > j|x_1)}\right]}$$

$$= \frac{\exp(\ \ _{0j} + \ \ _1(x_2))}{\exp(\ \ _{0j} + \ \ _1(x_1))}$$

$$= \exp\{(\ \ _{0j} + \ \ _1(x_2)) - (\ \ _{0j} + \ \ _1(x_1))\}$$

$$= \exp(\ \ _1(x_2 - x_1))$$

Log dari *odd rasio* pada persamaan adalah $(\ \ _1(x_2 - x_1))$. Nilai *odd rasio* menginterpretasikan bahwa peluang variabel respon pada kategori kurang dari atau sama dengan j dibandingkan dengan suatu respon pada kategori (j+1) sampai dengan *r* untuk $X = x_2$ sebesar $\exp(\ \ _1(x_2 - x_1))$kali dari $X = x_1$.

b. Variabel prediktor kontinu

Nilai *odds ratio* pada variabel prediktor bertipe kontinu dapat diinterpretasikan secara matematis sebagai berikut.

$$y(x) = {}_0 + {}_1 x$$

dimana ${}_1 = y(x+1) - y(x)$ dan $s\ {}_1 = y(x+s) - y(x)$.

Berdasarkan rumusan matematis tersebut dapat diketahui bahwa perubahan satu satuan pada variabel prediktor dapat memberikan perubahan pada $y(x)$ sebesar ${}_1$, sedangkan apabila terdapat perubahan sebesar *s* satuan pada variabel prediktor maka $y(x)$ akan berubah sebesar $s\ {}_1$.

2.7 Faktor-faktor yang Mempengaruhi Masa Studi

Perguruan tinggi dan mahasiswa adalah dua elemen yang tak terpisahkan. Keduanya merupakan subjek yang mempunyai peran strategis untuk suatu perubahan dan perbaikan. Kemampuan dan analisis, itulah kata kunci bagi perguruan tinggi dan mahasiswa. Keberhasilan studi mahasiswa dapat dilihat dari penilaian evaluasi akhir Indeks Prestasi Kumulatif (IPK) dan masa studi yang ditempuh. Peraturan Akademik Institut Teknologi Sepuluh Nopember Surabaya Tahun 2009 BAB VIII tentang Kelulusan Pasal 25 menyebutkan bahwa predikat kelulusan ditetapkan berdasarkan IPK dan masa studi. Menurut Warsa (2004), IPK adalah angka yang diperoleh dari hasil membagian jumlah mutu kumulatif dengan satuan kredit semester kumulatif, sedangkan masa studi adalah masa untuk penyelesaian beban studi dalam mengikuti proses pendidikan pada program studinya.

Peraturan akademik ITS Surabaya tahun 2009 BAB VII tentang Evaluasi Masa Studi bagian ketiga Program Magister menyebutkan bahwa masa studi paling lama untuk mahasiswa program magister adalah delapan semester, dimana evaluasi masa studi mahasiswa dilakukan setiap semester dan dimulai pada semester dua. Faktor-faktor yang mempengaruhi masa studi mahasiswa pernah dilakukan dalam penelitian sebelumnya. Berdasarkan penelitian yang dilakukan oleh Padmini, dkk (2012) bahwa kelulusan mahasiswa dipengaruhi oleh nilai IPK, jenis kelamin, dan fakultas. Kemudian hasil penelitian yang dilakukan oleh Thola

(2003) bahwa masa studi S2 dipengaruhi oleh empat faktor yakni jenis kelamin, status perkawinan, nilai IPK S1, dan jenis pekerjaan. Hal ini serupa dengan hasil penelitian yang dilakukan oleh Wulandari (2004), Hadi dan Suhartono (2012). Hasil penelitian Wulandari (2004) menyimpulkan bahwa masa studi mahasiswa sangat dipengaruhi oleh kesesuaian program studi S1 dan S2, status perguruan tinggi asal, dan sumber biaya. Selanjutnya hasil penelitian Hadi dan Suhartono (2012) menyimpulkan bahwa prestasi mahasiswa pascasarjana dipengaruhi oleh kesesuaian bidang studi, IPK saat S1, status perkawinan, dan asal perguruan tinggi saat S1. Menurut Notodiputro, Mariati & Juanda (2005) bahwa keberhasilan mahasiswa dipengaruhi oleh beberapa faktor yaitu jenis pekerjaan, asal perguruan tinggi S1, nilai mutu rata-rata (NMR) yang dicapai untuk semua mata kuliah selama S1.

Kemampuan IPK pada tingkat pendidikan sebelumnya untuk memprediksi prestasi belajar juga pernah diteliti oleh Newton dan Moore (2007), hasil penelitiannya menunjukkan bahwa skor *Undergraduate Grade Point Average* (UGPA) dan tes bakat akademis *Graduate Record Examination* (GRE) adalah prediktor yang baik dalam menilai kesuksesan sekolah tingkat selanjutnya, tetapi UGPA diduga merupakan prediktor yang lebih baik. Hal ini serupa dengan hasil penelitian yang dilakukan oleh Trail (2006), bahwa nilai IPK S1 merupakan *key predictor* dalam menentukan kinerja akademik mahasiswa dalam program S2 maupun doktoral. Sedangkan menurut penelitian yang dilakukan oleh Eskew dan Faley (1988) membuktikan bahwa kecerdasan akademis yang diukur dengan skor *Scholastic Aptitude Test* (SAT) mempunyai kontribusi yang paling besar dalam mempengaruhi prestasi belajar mahasiswa mahasiswa jurusan akuntansi keuangan di *Purdue University*. Kemudian hasil penelitian Adams dan Hancock (2007) menyimpulkan bahwa pada mahasiswa *Master of Business Administration* (MBA), pengalaman kerja lebih berhubungan dengan keberhasilan belajar dibandingkan dengan nilai *Graduate Management Admission Test* (GMAT) maupun UCPA. Pegawai dengan pengalaman kerja akan lebih mampu menghubungkan konsep dengan pengalaman kerjanya.

Usia seseorang diduga mempunyai pengaruh terhadap kemampuan berpikirnya. Mahasiswa yang berusia lebih tua sering dikatakan mengalami penurunan dalam hal *basic skills* yang diperlukan untuk belajar efektif pada tingkat pendidikan tinggi (Richardson, 1994). Karena itu, mahasiswa yang usianya lebih tua diduga mempunyai prestasi akademis yang lebih rendah dibandingkan mahasiswa yang lebih muda. Sehingga memungkinkan mahasiswa tersebut telat dalam masa studinya. Menurut Rampacher dan Peterson (1999) bahwa perbedaan jenis kelamin dan usia tidak memberikan dampak terhadap kinerja siswa. Dalam penelitiannya, usia 25-29 tahun sebagai usia paling bagus dalam hal kinerja siswa, namun usia selain itu hanya memberikan perbedaan yang tipis dalam mempengaruhi kinerja siswa. Kemudian Eskew dan Faley (1988) dalam penelitiannya menunjukkan bahwa latar belakang pendidikan pada tingkat pendidikan sebelumnya berpengaruh positif dan signifikan terhadap prestasi belajar mahasiswa.

Sebagian besar penelitian yang menguji pengaruh gender terhadap prestasi belajar menunjukkan bahwa perempuan cenderung memiliki prestasi akademik yang lebih bagus daripada laki-laki. Penjelasan teoritis mengenai hal ini antara lain karena setalah zaman emansipasi, pendidikan merupakan kunci utama kemajuan, pemberdayaan, dan kebebasan bagi kaum perempuan. Selain itu, perempuan dikenal cenderung lebih tekun dalam belajar, sedangkan laki-laki lebih menyukai kegiatan kampus yang bersifat refresing dan olah raga (Che, Pino & Smith, 2005).

BAB 3
METODOLOGI PENELITIAN

3.1 Sumber Data

Data yang digunakan dalam penelitian ini adalah data sekunder (Al Fattah, 2012). Data tersebut merupakan data lulusan mahasiswa pascasarjana program magister ITS Surabaya periode Maret 2008 sampai dengan Maret 2011. Adapun jumlah data sebanyak 1284 lulusan. Data dapat dilihat pada Lampiran A.

3.2 Variabel Penelitian

Dalam penelitian ini, variabel respon (*Y*) yang digunakan adalah masa studi lulusan program magister ITS Surabaya. Data masa studi lulusan program magister ITS merupakan data kontinu yang kemudian dikategorikan menjadi tiga kategori. Variabel prediktor (X_k) merupakan variabel-variabel yang diduga mempengaruhi masa studi lulusan program magister ITS Surabaya. Secara ringkas variabel pada penelitian ini disajikan dalam Tabel 3.1.

Tabel 3.1 Variabel Penelitian

Simbol	Variabel	Kategori
Y	Masa Studi	Yang dikategorikan menjadi: 1 = Masa studi lebih dari 4 semester 2 = Masa studi 4 semester 3 = Masa studi 3 semester
X_1	Nilai TPA	-
X_2	Nilai Toefl	-
X_3	Jenis Kelamin	1 = laki-laki 2 = Perempuan
X_4	Kesesuaian bidang studi	1 = Sesuai 2 = Tidak sesuai
X_5	Nilai IPK S1	-
X_6	Lama waktu lulus dari S1 ke S2	-

Lanjutan Tabel 3.1

Simbol	Variabel	Kategori
X_7	Sumber pendanaan	1 = sendiri 2 = beasiswa BPPS 3 = beasiswa instansi 4 = beasiswa lainnya
X_8	Status pernikahan	1 = menikah 2 = belum menikah
X_9	Jenis pekerjaan	1 = pegawai negeri/BUMN 2 = pegawai swasta/wiraswasta 3 = belum bekerja
X_{10}	usia	-
X_{11}	Asal Fakultas	1= FMIPA 2 = FTI 3 = FTSP 4 = FTK 5 = FTIF

Definisi operasional variabel-variabel yang digunakan dalam penelitian ini sebagai berikut:

a. Masa studi (Y) : masa studi merupakan masa untuk penyelesaian beban studi selama mengikuti proses pendidikan dalam program magister.
b. Nilai TPA (X_1) : Tes Potensi Akademik (TPA) adalah sebuah tes yang bertujuan untuk mengetahui bakat dan kemampuan seseorang di bidang keilmuan atau akademis. Nilai TPA yang digunakan dalam penelitian ini merupakan nilai hasil TPA yang diperoleh pada saat seleksi masuk Program Magister ITS.
c. Nilai Toefl (X_2) : Toefl merupakan tes yang digunakan untuk mengetahui tingkat kemampuan seseorang dalam berbahasa inggris. Nilai Toefl yang digunakan dalam penelitian ini merupakan nilai Toefl yang diperoleh pada saat masuk Program Magister ITS.
d. Kesesuaian bidang studi (X_4) : kesesuaian bidang studi yang dimaksud dalam penelian ini adalah kesesuaian bidang studi mahasiswa pada saat S1 dan S2.

e. Lama waktu lulus dari S1 ke S2 (X_6) : merupakan rentang waktu setelah mahasiswa lulus S1 kemudian melanjutkan studi S2.

f. Sumber Pendanaan (X_7) : sumber pendanaan yang dimaksud dalam penelian ini merupakan sumber biaya yang diperoleh mahasiswa selama mengikuti proses pendidikan dalam program magister.

g. Status pernikahan (X_8) : status pernikahan dalam penelitian ini dikategorikan menjadi dua yakni menikah dan belum menikah. Menikah yang dimaksud dalam penelitian ini adalah semua mahasiswa yang pernah dan sudah menikah, sedangkan kategori belum menikah yang dimaksud dalam penelitian ini adalah semua mahasiswa yang belum pernah menikah.

h. Usia (X_{10}) : usia yang digunakan dalam penelitian ini adalah usia mahasiswa pada saat masuk program Magister.

3.3 Struktur Data Penelitian

Adapun struktur data dalam penelitian ini ditampilkan pada Tabel 3.2 berikut.

Tabel 3.2 Struktur Data Penelitian

Individu	Y	X_1	X_2	$\cdots$	X_{11}
1	Y_1	$X_{1.1}$	$X_{2.1}$	$\cdots$	$X_{11.1}$
2	Y_2	$X_{1.2}$	$X_{2.2}$	$\cdots$	$X_{11.2}$
3	Y_3	$X_{1.3}$	$X_{2.3}$	$\cdots$	$X_{11.3}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
1284	Y_{1284}	$X_{1.1284}$	$X_{2.1284}$	$\cdots$	$X_{11.1284}$

3.4 Langkah-langkah Analisis Data

Langkah-langkah analisis data yang dilakukan dalam penelitian ini sebagai berikut:

1. Estimasi parameter *Partial Proportional Odds Model* (PPOM) sebagai berikut:
   a. Membentuk fungsi *likelihood* dari model *Partial Proportional Odds*
   b. Menentukan fungsi *ln-likelihood* berdasarkan fungsi *likelihood* pada langkah (a)
   c. Melakukan turunan parsial pertama dari fungsi *ln-likelihood* terhadap parameter yang akan diestimasi, kemudian disama dengankan nol
   d. Melakukan turunan parsial kedua dari fungsi *ln-likelihood* terhadap parameter yang akan diestimasi
   e. Jika pada langkah (c) diperoleh hasil yang tidak *close form* maka digunakan metode *Newton-Raphson* untuk mendapatkan estimasi parameter. Adapun tahapan dalam metode *Newton-Raphson* sebagai berikut:
      - Menentukan nilai awal $\hat{\mathbf{\Phi}}^{(0)}$ dengan menggunakan nilai $\hat{\mathbf{\Phi}}$ yang diperoleh dari model regresi logistik ordinal.
      - Membuat pendekatan Taylor dari fungsi *ln-likelihood* disekitar nilai estimasi awal,

        $$l(\mathbf{\Phi}) = l\left(\mathbf{\Phi}^{(0)}\right) + \mathbf{g}^{(0)^T}\left(\mathbf{\Phi} - \mathbf{\Phi}^{(0)}\right) + \frac{1}{2}\left(\mathbf{\Phi} - \mathbf{\Phi}^{(0)}\right)^T \mathbf{H}^{(0)}\left(\mathbf{\Phi} - \mathbf{\Phi}^{(0)}\right) \qquad (3.1)$$

        dimana **H** adalah matriks nonsingular dan merupakan turunan parsial kedua dari fungsi *ln-likelihood* terhadap parameter yang akan diestimasi, dan **g** merupakan vektor turunan parsial pertama dari fungsi *ln-likelihood* terhadap parameter yang akan diestimasi.
      - Persamaan (3.1) diturunkan terhadap $\mathbf{\Phi}$, sehingga diperoleh $\mathbf{g}^{(0)} + \mathbf{H}^{(0)}\left(\mathbf{\Phi} - \mathbf{\Phi}^{(0)}\right) = \mathbf{0}$

        Kemudian diperoleh $\mathbf{\Phi} = \mathbf{\Phi}^{(0)} - \left[\mathbf{H}^{(0)}\right]^{-1}\mathbf{g}^{(0)}$
      - Nilai $\mathbf{\Phi}$ berikutnya dapat diperoleh dengan persamaan $\mathbf{\Phi}^{(t+1)} = \mathbf{\Phi}^{(t)} - \left[\mathbf{H}^{(t)}\right]^{-1}\mathbf{g}^{(t)}$, dan prosedur berlanjut sampai mencapai konvergen.

2. Analisis data dilakukan dengan bantuan *software* statistika yaitu Stata dan SPSS. Analisis regresi logistik ordinal PPOM dengan tahapan sebagai berikut:
   a. Menguji signifikansi parameter yaitu untuk memeriksa ada atau tidaknya pengaruh variabel prediktor terhadap model. Pada tahap ini diawali dengan melakukan analisis dengan *proportional odds model*. Ada dua pengujian signifikansi parameter yakni:
      - Uji Serentak : uji yang digunakan untuk mengetahui variabel-variabel prediktor yang berpengaruh signifikan secara bersama-sama terhadap model.
      - Uji Parsial : uji yang digunakan untuk mengetahui variabel-variabel prediktor yang berpengaruh signifikan secara individu terhadap model.

   b. Menguji asumsi *proportional odds*
   c. Melakukan *parameterization*, dimana dilakukan penambahan parameter gamma pada variabel yang tidak memenuhi asumsi *proportional odds*
   d. Membuat model regresi logistik berdasarkan variabel-variabel yang signifikan
   e. Menguji kesesuaian model, yakni untuk mengetahui kelayakan model yang diperoleh
   f. Menginterpretasikan parameter model regresi logistik menggunakan nilai *odds ratio*

Halaman ini sengaja dikosongkan

BAB 4
ANALISIS DAN PEMBAHASAN

4.1 Estimasi Parameter Partial Proportional Odds Model

Metode yang digunakan untuk mengestimasi parameter *Partial Proportional Odds Model* (PPOM) adalah metode *Maximum Likelihood Estimation* (MLE). Estimasi parameter PPOM diperoleh dengan melakukan turunan parsial fungsi *ln-likelihood* terhadap parameter yang akan diestimasi (Agresti, 2000).

Variabel respon Y dengan J kategori dimana $J > 2$ dapat dipandang sebagai sebuah percobaan yang menghasilkan J hasil yang mungkin, sehingga variabel random $Y_1, Y_2, ..., Y_J$ yang menyatakan terjadinya kejadian $y_1, y_2, ..., y_J$ berdistribusi multinomial. Jika diambil m sampel random $\mathbf{Y}_1, \mathbf{Y}_2, ..., \mathbf{Y}_m$ maka $\mathbf{Y}_i = (Y_{1i}, Y_{2i}, ..., Y_{Ji}) \sim M(1; ®_1(\mathbf{x}_i), ®_2(\mathbf{x}_i), ..., ®_J(\mathbf{x}_i))$ dengan distribusi probabilitas sebagai berikut:

$$P\left(Y_{1i} = y_{1i}, Y_{2i} = y_{2i}, Y_{Ji} = y_{Ji} \middle| 1; ®_j(\mathbf{x}_i)\right) = [®_1(\mathbf{x}_i)]^{y_{1i}} [®_2(\mathbf{x}_i)]^{y_{2i}} ... [®_J(\mathbf{x}_i)]^{y_{Ji}}$$

dimana $i = 1, 2, \ldots, m$ dan $j = 1, 2, \ldots, J$

Karena peluang kumulatif yang digunakan dalam estimasi parameter, maka fungsi *likelihood* yang merupakan fungsi padat peluang bersama dari variabel random $Y_1, Y_2, ..., Y_J$ dapat ditulis sebagai perkalian J-1 kategori sebagai berikut:

$$L(\mathbf{\Phi}) = \prod_{i=1}^{m} \prod_{j=1}^{J} (\Pr(Y = j|\mathbf{x}))^{y_{ji}} = \prod_{i=1}^{m} \prod_{j=1}^{J} (®_j(\mathbf{x}_i))^{y_{ji}}$$

$$= \prod_{i=1}^{m} \left\{ [®_1(\mathbf{x}_i)]^{y_{1i}} [®_2(\mathbf{x}_i)]^{y_{2i}} [®_3(\mathbf{x}_i)]^{y_{3i}} ... [®_{J-1}(\mathbf{x})]^{y_{(J-1)i}} [®_J(\mathbf{x}_i)]^{\Delta_i} \right\} \tag{4.1}$$

dimana $\Delta_i = 1 - y_{1i} - y_{2i} - ... - y_{(J-1)i}$

Prinsip dari metode MLE adalah mengestimasi vektor parameter $\mathbf{\Phi} = [\sim_1 \quad \sim_2 \quad \cdots \quad \sim_{J-1} \quad {}_1 \quad {}_2 \quad \cdots \quad {}_p \quad ¥_1 \quad ¥_2 \quad \cdots \quad ¥_q]^T$ dengan cara memaksimumkan fungsi *likelihood*. Untuk mempermudah perhitungan, maka dilakukan transformasi *ln* pada fungsi *likelihood* sehingga terbentuk fungsi *ln-*

likelihood. Berdasarkan fungsi *likelihood* pada persamaan (4.1) diperoleh fungsi *ln-likelihood* berikut.

$$
\begin{aligned}
\ell(\mathbf{\Phi}) &= \ln[L(\mathbf{\Phi})] \\
&= \sum_{i=1}^{m}\sum_{j=i}^{J} y_{ji}\ln \Pr(Y = j|\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=i}^{J} y_{ji}\ln[\circledR_j(\mathbf{x}_i)] \\
&= \sum_{i=1}^{m}\{y_{1i}\ln[\circledR_1(\mathbf{x}_i)] + y_{2i}\ln[\circledR_2(\mathbf{x}_i)] + y_{3i}\ln[\circledR_3(\mathbf{x}_i)] + \ldots + y_{(J-1)i}\ln[\circledR_{J-1}(\mathbf{x}_i)] + \\
&\qquad (1 - y_1 - y_2 - \ldots - y_{J-1})\ln[\circledR_J(\mathbf{x}_i)]\}
\end{aligned}
\tag{4.2}
$$

dimana

$$\circledR_1(\mathbf{x}) = 1 - \frac{1}{1+e^{-\sim_j - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_j}} = \frac{e^{-\sim_j - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_j}}{1+e^{-\sim_j - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_j}} \quad \text{, jika Y = 1}$$

$$\circledR_j(\mathbf{x}) = \frac{1}{1+e^{-\sim_j - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_j}} - \frac{1}{1+e^{-\sim_{j+1} - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_{j+1}}} = \frac{e^{-\sim_{j+1} - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_{j+1}} - e^{-\sim_j - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_j}}{\left(e^{-\sim_{j+1} - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_{j+1}}\right)\left(1+e^{-\sim_j - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_j}\right)} \text{, jika 1<Y< J}$$

$$\circledR_J(\mathbf{x}) = \frac{1}{1+e^{-\sim_J - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_J}} \quad \text{, jika Y = J}$$

Jika pada variabel respon terdapat J kategori, maka fungsi *ln-likelihood* yang terbentuk sebagai berikut.

$$
\begin{aligned}
\ell(\mathbf{\Phi}) = \sum_{i=1}^{m}\Bigg\{ & y_{1i}\ln\left(\frac{e^{-\sim_1 - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1 - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_1}}\right) + y_{2i}\ln\left(\frac{1}{1+e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}} - \frac{1}{1+e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right) + \\
& y_{3i}\ln\left(\frac{1}{1+e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}} - \frac{1}{1+e^{-\sim_3 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right) + y_{4i}\ln\left(\frac{1}{1+e^{-\sim_3 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}} - \frac{1}{1+e^{-\sim_4 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right) + \\
& \ldots + y_{(J-1)i}\ln\left(\frac{1}{1+e^{-\sim_{J-2} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}} - \frac{1}{1+e^{-\sim_{J-1} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right) + y_{Ji}\ln\left(\frac{1}{1+e^{-\sim_{J-1} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right)\Bigg\}
\end{aligned}
$$

$$= \sum_{i=1}^{m}\Bigg\{ y_{1i}\ln\left(\frac{e^{-\sim_1 - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1 - \mathbf{x}^T\boldsymbol{\beta} - \mathbf{t}^T\boldsymbol{\gamma}_1}}\right) + y_{2i}\ln\left(\frac{e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)\left(1+e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)}\right) +$$

$$y_{3i}\ln\left(\frac{e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)\left(1+e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)}\right)+y_{4i}\ln\left(\frac{e^{-\theta_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{\left(1+e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)\left(1+e^{-\theta_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}\right)}\right)+$$

$$\ldots+y_{(J-1)i}\ln\left(\frac{e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\theta_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{\left(1+e^{-\theta_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)\left(1+e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)}\right)+$$

$$\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-\ldots-y_{(J-1)i}\right)\ln\left(\frac{1}{1+e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right)\Bigg\}$$

$$=\sum_{i=1}^{m}\Big\{y_{1i}\ln\left(e^{-\theta_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}\right)-y_{1i}\ln\left(1+e^{-\theta_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}\right)+y_{2i}\ln\left(e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)-$$
$$y_{2i}\ln\left(1+e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)-y_{2i}\ln\left(1+e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)+y_{3i}\ln\left(e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)-$$
$$y_{3i}\ln\left(1+e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)-y_{3i}\ln\left(1+e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)+y_{4i}\ln\left(e^{-\theta_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)-$$
$$y_{4i}\ln\left(1+e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)-y_{4i}\ln\left(1+e^{-\theta_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}\right)+\ldots+$$
$$y_{(J-1)i}\ln\left(e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\theta_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)-y_{(J-1)i}\ln\left(1+e^{-\theta_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)-$$
$$y_{(J-1)i}\ln\left(1+e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)+\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-\ldots-y_{(J-1)i}\right)\ln(1)-$$
$$\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-\ldots-y_{(J-1)i}\right)\ln\left(1+e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)\Big\}$$

$$\ell(\boldsymbol{\Phi})=\sum_{i=1}^{m}\Big\{y_{1i}\left(-\theta_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1\right)-\left(y_{1i}+y_{2i}\right)\ln\left(1+e^{-\theta_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}\right)-$$
$$\left(y_{2i}+y_{3i}\right)\ln\left(1+e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)-\left(y_{3i}+y_{4i}\right)\ln\left(1+e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)-\ldots-$$
$$\left(y_{(J-2)i}+y_{(J-1)i}\right)\ln\left(1+e^{-\theta_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)+y_{2i}\ln\left(e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)+$$
$$y_{3i}\ln\left(e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)+y_{4i}\ln\left(e^{-\theta_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)+\ldots+$$
$$y_{(J-1)i}\ln\left(e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\theta_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)-$$
$$\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-\ldots-y_{(J-2)i}\right)\ln\left(1+e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)\Big\} \qquad (4.3)$$

selanjutnya dari fungsi *ln-likelihood* pada persamaan (4.3) dilakukan turunan parsial pertama terhadap parameter yang akan diestimasi dan kemudian disama dengankan nol, sehingga diperoleh hasil sebagai berikut.

$$\frac{\partial\ell(\boldsymbol{\Phi})}{\partial\theta_1}=\sum_{i=1}^{m}\left\{-y_{1i}+\left(y_{1i}+y_{2i}\right)\left(\frac{e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\left(\frac{e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)\right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \sim_2}=\sum_{i=1}^{m}\left\{\left(y_{2i}+y_{3i}\right)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)-y_{2i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+\right.$$

$$\left. y_{3i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \sim_3}=\sum_{i=1}^{m}\left\{\left(y_{3i}+y_{4i}\right)\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)-y_{3i}\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)+\right.$$

$$\left. y_{4i}\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)\right\}$$

$$\vdots$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \sim_{J-1}}=\sum_{i=1}^{m}\left\{-y_{(J-1)i}\left(\frac{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{\left(e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)}\right)+\right.$$

$$\left.\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-....-y_{(J-2)i}\right)\left(\frac{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right)\right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\beta}}=\sum_{i=1}^{m}\mathbf{x}_i^T\left\{-y_{1i}-y_{2i}-y_{3i}-y_{4i}+\ldots-y_{(J-1)i}+\left(y_{1i}+y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+\right.$$

$$\left(y_{2i}+y_{3i}\right)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)+\left(y_{3i}+y_{4i}\right)\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)+\ldots+$$

$$\left(y_{(J-2)i}+y_{(J-1)i}\right)\left(\frac{e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{1+e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}\right)+$$

$$\left.\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-....-y_{(J-2)i}\right)\left(\frac{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right)\right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_1}=\sum_{i=1}^{m}\mathbf{t}_i^T\left\{-y_{1i}+\left(y_{1i}+y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)\right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_2}=\sum_{i=1}^{m}\mathbf{t}_i^T\left\{\left(y_{2i}+y_{3i}\right)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)-y_{2i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+\right.$$

$$\left. y_{3i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_3}=\sum_{i=1}^{m}\mathbf{t}_i^T\left\{(y_{3i}+y_{4i})\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)-y_{3i}\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)+\right.$$

$$\left.y_{4i}\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)\right\}$$

$$\vdots$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_{J-2}}=\sum_{i=1}^{m}\mathbf{t}_i^T\left\{(y_{(J-2)i}+y_{(J-1)i})\left(\frac{e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{1+e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}\right)-y_{(J-2)i}\left(\frac{e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}-e^{-\sim_{J-3}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-3}}}\right)+\right.$$

$$\left.y_{(J-1)i}\left(\frac{e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}\right)\right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_{J-1}}=\sum_{i=1}^{m}\mathbf{t}_i^T\left\{-y_{(J-1)i}\left(\frac{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{\left(e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)}\right)+\right.$$

$$\left.(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-....-y_{(J-2)i})\left(\frac{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right)\right\}$$

Hasil lengkap turunan parsial pertama dari persamaan (4.3) terhadap parameter yang akan diestimasi dapat dilihat pada Lampiran B. Hasil turunan parsial pertama yang diperoleh merupakan fungsi yang nonlinier terhadap parameter yang akan diestimasi, sehingga diperlukan suatu metode numerik untuk memperoleh estimasi parameternya. Metode numerik yang dapat digunakan adalah metode *Newton-Raphson*, dalam metode ini diperlukan turunan parsial kedua dari fungsi *ln-likelihood* terhadap parameter yang akan diestimasi. Hasil turunan parsial kedua dari fungsi *ln-likelihood* sebagai berikut.

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_1^2}=\sum_{i=1}^{m}\left\{-(y_{1i}+y_{2i})\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)-y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)\right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_1 \partial \sim_2}=\sum_{i=1}^{m}y_{2i}\left(\frac{e^{-\sim_2-\sim_1-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

dan $\dfrac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_1 \partial \sim_j}=0$, j = 3, 4, …, *J-1*

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_2^2} = \sum_{i=1}^{m} \left\{ -\left(y_{2i} + y_{3i}\right) \left(\frac{e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2}}{\left(1 + e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2}\right)^2} \right) - y_{2i} \left(\frac{e^{-\sim_1 - \sim_2 - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1 - \mathbf{t}_i^T \boldsymbol{\gamma}_2}}{\left(e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1}\right)^2} \right) - \right.$$

$$\left. y_{3i} \frac{e^{-\sim_2 - \sim_3 - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2 - \mathbf{t}_i^T \boldsymbol{\gamma}_3}}{\left(e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1}\right)^2} \right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_2 \partial \sim_3} = \sum_{i=1}^{m} y_{3i} \left(\frac{e^{-\sim_3 - \sim_2 - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3 - \mathbf{t}_i^T \boldsymbol{\gamma}_2}}{\left(e^{-\sim_3 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3} - e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2}\right)^2} \right)$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_3^2} = \sum_{i=1}^{m} \left\{ -\left(y_{3i} + y_{4i}\right) \left(\frac{e^{-\sim_3 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3}}{\left(1 + e^{-\sim_3 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3}\right)^2} \right) - y_{3i} \left(\frac{e^{-\sim_3 - \sim_2 - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3 - \mathbf{t}_i^T \boldsymbol{\gamma}_2}}{\left(e^{-\sim_3 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3} - e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2}\right)^2} \right) - \right.$$

$$\left. y_{4i} \left(\frac{e^{-\sim_3 - \sim_4 - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3 - \mathbf{t}_i^T \boldsymbol{\gamma}_4}}{\left(e^{-\sim_4 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_4} - e^{-\sim_3 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3}\right)^2} \right) \right\}$$

$$\vdots$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_j^2} = \sum_{i=1}^{m} \left\{ -\left(y_{ji} + y_{(j+1)i}\right) \left(\frac{e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j}}{\left(1 + e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j}\right)^2} \right) - y_{ji} \left(\frac{e^{-\sim_j - \sim_{j-1} - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}}{\left(e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j} - e^{-\sim_{j-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}\right)^2} \right) - \right.$$

$$\left. y_{(j+1)i} \left(\frac{e^{-\sim_j - \sim_{j+1} - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j - \mathbf{t}_i^T \boldsymbol{\gamma}_{j+1}}}{\left(e^{-\sim_{j+1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j+1}} - e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j}\right)^2} \right) \right\} \quad , \ j = 2, 3, \ldots, J\text{-}2$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_{J-1}^2} = \sum_{i=1}^{m} \left\{ - y_{(J-1)i} \left(\frac{e^{-\sim_{J-1} - \sim_{J-2} - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{J-1} - \mathbf{t}_i^T \boldsymbol{\gamma}_{J-2}}}{\left(e^{-\sim_{J-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{J-1}} - e^{-\sim_{J-2} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{J-2}}\right)^2} \right) - \right.$$

$$\left. \left(1 - y_{1i} - y_{2i} - y_{3i} - y_{4i} - \ldots . - y_{(J-2)i}\right) \left(\frac{e^{-\sim_{J-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{J-1}}}{\left(1 + e^{-\sim_{J-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{J-1}}\right)^2} \right) \right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_j \partial \sim_{j-1}} = \sum_{i=1}^{m} y_{ji} \left(\frac{e^{-\sim_j - \sim_{j-1} - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}}{\left(e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j} - e^{-\sim_{j-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}\right)^2} \right) \quad , \ j = 2, 3, \ldots, J\text{-}1$$

dan $\dfrac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_n \partial \sim_j} = 0\,, \quad |n - j| \geq 2$

kemudian

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_1 \partial \boldsymbol{\beta}} = \sum_{i=1}^{m} - \mathbf{x}_i^T \left(y_{1i} + y_{2i}\right) \left(\frac{e^{-\sim_1 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1}}{\left(1 + e^{-\sim_1 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1}\right)^2} \right)$$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_2 \partial \boldsymbol{\beta}} = \sum_{i=1}^{m} -\mathbf{x}_i^T \left(y_{2i} + y_{3i}\right)\left(\frac{e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)$$

$$\vdots$$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_j \partial \boldsymbol{\beta}} = \sum_{i=1}^{m} -\mathbf{x}_i^T \left(y_{ji} + y_{(j+1)i}\right)\left(\frac{e^{-\sim_j - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_j}}{\left(1+e^{-\sim_j - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_j}\right)^2}\right) \quad , j = 1, 2, \ldots, J\text{-}2$$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_{J-1} \partial \boldsymbol{\beta}} = \sum_{i=1}^{m} -\mathbf{x}_i^T \left(1 - y_{1i} - y_{2i} - y_{3i} - y_{4i} - \ldots. - y_{(J-2)i}\right)\left(\frac{e^{-\sim_{J-1} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{\left(1+e^{-\sim_{J-1} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)^2}\right)$$

$$\begin{aligned}\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^T} = \sum_{i=1}^{m} \mathbf{x}_i \mathbf{x}_i^T \Bigg\{ &\left(-y_{1i} - y_{2i}\right)\left(\frac{e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right) - \left(y_{2i} + y_{3i}\right)\left(\frac{e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right) - \\ &\left(y_{3i} + y_{4i}\right)\left(\frac{e^{-\sim_3 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_3}}{\left(1+e^{-\sim_3 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)^2}\right) - \ldots - \left(y_{(J-2)i} + y_{(J-1)i}\right)\left(\frac{e^{-\sim_{J-2} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{\left(1+e^{-\sim_{J-2} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)^2}\right) - \\ &\left(1 - y_{1i} - y_{2i} - y_{3i} - y_{4i} - \ldots. - y_{(J-2)i}\right)\left(\frac{e^{-\sim_{J-1} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{\left(1+e^{-\sim_{J-1} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)^2}\right)\Bigg\}\end{aligned}$$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_1 \partial \boldsymbol{\gamma}_1} = \sum_{i=1}^{m} \mathbf{t}_i^T \left\{ -\left(y_{1i} + y_{2i}\right)\left(\frac{e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right) - y_{2i}\left(\frac{e^{-\sim_1 - \sim_2 - 2\mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1 - \mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)\right\}$$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_1 \partial \boldsymbol{\gamma}_2} = \sum_{i=1}^{m} \mathbf{t}_i^T y_{2i}\left(\frac{e^{-\sim_2 - \sim_1 - 2\mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2 - \mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

dan $\dfrac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_1 \partial \boldsymbol{\gamma}_j} = 0$, $j = 3, 4, \ldots, J\text{-}1$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_2 \partial \boldsymbol{\gamma}_1} = \sum_{i=1}^{m} \mathbf{t}_i^T y_{2i}\left(\frac{e^{-\sim_1 - \sim_2 - 2\mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1 - \mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

$$\begin{aligned}\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_2 \partial \boldsymbol{\gamma}_2} = \sum_{i=1}^{m} \mathbf{t}_i^T \Bigg\{ &-\left(y_{2i} + y_{3i}\right)\left(\frac{e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right) - y_{2i}\left(\frac{e^{-\sim_2 - \sim_1 - 2\mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2 - \mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right) - \\ &y_{3i}\left(\frac{e^{-\sim_2 - \sim_3 - 2\mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2 - \mathbf{t}_i^T\boldsymbol{\gamma}_3}}{\left(e^{-\sim_3 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_3} - e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)\Bigg\}\end{aligned}$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_2\partial\boldsymbol{\gamma}_3}=\sum_{i=1}^{m}\mathbf{t}_i^T y_{3i}\left(\frac{e^{-\sim_3-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)$$

$$\vdots$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_{j-1}\partial\boldsymbol{\gamma}_j}=\sum_{i=1}^{m}\mathbf{t}_i^T y_{ji}\left(\frac{e^{-\sim_j-\sim_{j-1}-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_j-\mathbf{t}_i^T\boldsymbol{\gamma}_{j-1}}}{\left(e^{-\sim_j-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_j}-e^{-\sim_{j-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{j-1}}\right)^2}\right) \quad , j = 2, 3, \ldots, J\text{-}1$$

dan $\dfrac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_n\partial\boldsymbol{\gamma}_j}=0\,,\quad |n-j|\geq 2$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_1\partial\boldsymbol{\gamma}_1^T}=\sum_{i=1}^{m}\mathbf{t}_i\mathbf{t}_i^T\left\{-(y_{1i}+y_{2i})\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)-y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)\right\}$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_1\partial\boldsymbol{\gamma}_2^T}=\sum_{i=1}^{m}\mathbf{t}_i\mathbf{t}_i^T y_{2i}\left(\frac{e^{-\sim_2-\sim_1-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

dan $\dfrac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_1\partial\boldsymbol{\gamma}_j^T}=0 \quad , j = 3, 4, \ldots, J\text{-}1$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_2\partial\boldsymbol{\gamma}_2^T}=\sum_{i=1}^{m}\mathbf{t}_i\mathbf{t}_i^T\left\{-(y_{2i}+y_{3i})\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)-y_{2i}\left(\frac{e^{-\sim_2-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)-\right.$$
$$\left. y_{3i}\left(\frac{e^{-\sim_2-\sim_3-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{\left(e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)}\right)\right\}$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_2\partial\boldsymbol{\gamma}_3^T}=\sum_{i=1}^{m}\mathbf{t}_i\mathbf{t}_i^T y_{3i}\left(\frac{e^{-\sim_3-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_3\partial\boldsymbol{\gamma}_3^T}=\sum_{i=1}^{m}\mathbf{t}_i\mathbf{t}_i^T\left\{-(y_{3i}+y_{4i})\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{\left(1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)^2}\right)-y_{3i}\left(\frac{e^{-\sim_3-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)-\right.$$
$$\left. y_{4i}\left(\frac{e^{-\sim_3-\sim_4-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3-\mathbf{t}_i^T\boldsymbol{\gamma}_4}}{\left(e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)^2}\right)\right\}$$

$$\vdots$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_{j-1}\partial\boldsymbol{\gamma}_j^T}=\sum_{i=1}^{m}\mathbf{t}_i\mathbf{t}_i^T y_{ji}\left(\frac{e^{-\sim_j-\sim_{j-1}-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_j-\mathbf{t}_i^T\boldsymbol{\gamma}_{j-1}}}{\left(e^{-\sim_j-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_j}-e^{-\sim_{j-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{j-1}}\right)^2}\right) \quad , j = 2, 3, \ldots, J\text{-}1$$

dan $\dfrac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_n \partial \boldsymbol{\gamma}_j^T} = 0 \quad , \; |n - j| \geq 2$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_j \partial \boldsymbol{\gamma}_j^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T \left\{ -\left(y_{ji} + y_{(j+1)i}\right) \left(\frac{e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j}}{\left(1 + e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j}\right)^2} \right) - y_{ji} \left(\frac{e^{-\sim_j - \sim_{j-1} - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}}{\left(e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j} - e^{-\sim_{j-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}\right)^2} \right) - \right.$$

$$\left. y_{(j+1)i} \left(\frac{e^{-\sim_j - \sim_{j+1} - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j - \mathbf{t}_i^T \boldsymbol{\gamma}_{j+1}}}{\left(e^{-\sim_{j+1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j+1}} - e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j}\right)^2} \right) \right\} \quad , \; j = 2, 3, \ldots, J\text{-}2$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_{J-1} \partial \boldsymbol{\gamma}_{J-1}^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T \left\{ - y_{(j-1)i} \left(\frac{e^{-\sim_{j-1} - \sim_{j-2} - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-2}}}{\left(e^{-\sim_{j-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}} - e^{-\sim_{j-2} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-2}}\right)^2} \right) - \right.$$

$$\left. \left(1 - y_{1i} - y_{2i} - \ldots - y_{(j-1)i}\right) \left(\frac{e^{-\sim_{j-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}}{1 + e^{-\sim_{j-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}} \right) \right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\gamma}_1^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{x}_i^T \left\{ -\left(y_{1i} + y_{2i}\right) \left(\frac{e^{-\sim_1 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1}}{\left(1 + e^{-\sim_1 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1}\right)^2} \right) \right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\gamma}_2^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{x}_i^T \left\{ -\left(y_{2i} + y_{3i}\right) \left(\frac{e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2}}{\left(1 + e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2}\right)^2} \right) \right\}$$

$$\vdots$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\gamma}_j^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{x}_i^T \left\{ -\left(y_{ji} + y_{(j+)i}\right) \left(\frac{e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j}}{\left(1 + e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j}\right)^2} \right) \right\} \quad , \; j = 2, 3, \ldots, J\text{-}2$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\gamma}_{J-1}^T} = \sum_{i=1}^{m} -\mathbf{t}_i \mathbf{x}_i^T \left(1 - y_{1i} - y_{2i} - \ldots - y_{(J-2)i}\right) \left(\frac{e^{-\sim_{J-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{J-1}}}{\left(1 + e^{-\sim_{J-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{J-1}}\right)^2} \right)$$

Hasil lengkap turunan parsial kedua dapat dilihat pada Lampiran B.

Misalkan pada variabel respon terdapat tiga kategori, maka fungsi *ln-likelihood* yang terbentuk sebagai berikut.

$$\ell(\boldsymbol{\Phi}) = \sum_{i=1}^{m} \left\{ y_{1i} \ln\left[\circledR_i(\mathbf{x}_1)\right] + y_{2i} \ln\left[\circledR_i(\mathbf{x}_2)\right] + \left(1 - y_{1i} - y_{2i}\right) \ln\left[\circledR_i(\mathbf{x}_3)\right] \right\}$$

$$
\ell(\mathbf{\Phi})=\sum_{i=1}^{m}\left\{y_{1i}\ln\left(\frac{e^{-\sim_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\ln\left(\frac{1}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}-\frac{1}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)+\right.
$$

$$
\left.y_{3i}\ln\left(\frac{1}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}
$$

$$
=\sum_{i=1}^{m}\left\{y_{1i}\ln\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\ln\left(\frac{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)}\right)+\right.
$$

$$
\left.y_{3i}\ln\left(\frac{1}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\} \tag{4.4}
$$

Persamaan (4.4) dapat disederhanakan menjadi

$$
\ell(\mathbf{\Phi})=\sum_{i=1}^{m}\left\{y_{1i}\left(-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1\right)-\left(y_{1i}+y_{2i}\right)\ln\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)+\right.
$$

$$
\left.y_{2i}\ln\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)+\left(y_{1i}-1\right)\ln\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)\right\} \tag{4.5}
$$

selanjutnya dari fungsi *ln-likelihood* pada persamaan (4.5) dilakukan turunan parsial pertama terhadap parameter yang akan diestimasi dan kemudian disama dengankan nol, sehingga diperoleh hasil sebagai berikut.

$$
\frac{\partial\ell(\mathbf{\Phi})}{\partial\sim_1}=\sum_{i=1}^{m}\left\{-y_{1i}+\left(y_{1i}+y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)\right\}
$$

$$
\frac{\partial\ell(\mathbf{\Phi})}{\partial\sim_2}=\sum_{i=1}^{m}\left\{-y_{2i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)-\left(y_{1i}-1\right)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}
$$

$$
\frac{\partial\ell(\mathbf{\Phi})}{\partial\boldsymbol{\beta}}=\sum_{i=1}^{m}\mathbf{x}_i^T\left\{y_{1i}-\left(y_{1i}+y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)-y_{2i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)-\right.
$$

$$
\left.\left(y_{1i}-1\right)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}
$$

$$
=\sum_{i=1}^{m}\mathbf{x}_i^T\left\{-y_{1i}-y_{2i}+\left(y_{1i}+y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)-\left(y_{1i}-1\right)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}
$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_1}=\sum_{i=1}^{m}\left\{-y_{1i}\mathbf{t}_i^T-\left(y_{1i}+y_{2i}\right)\left(\frac{-\mathbf{t}_i^T\left(e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\left(\frac{-\mathbf{t}_i^T\left(-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)\right\}$$

$$=\sum_{i=1}^{m}\mathbf{t}_i^T\left(-y_{1i}+\left(y_{1i}+y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)\right)$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_2}=\sum_{i=1}^{m}\left\{y_{2i}\left(\frac{-\mathbf{t}_i^T e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+\left(y_{1i}-1\right)\left(\frac{-\mathbf{t}_i^T e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

$$=\sum_{i=1}^{m}\mathbf{t}_i^T\left\{-y_{2i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)-\left(y_{1i}-1\right)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

Hasil lengkap turunan parsial pertama dari persamaan (4.5) terhadap parameter yang akan diestimasi dapat dilihat pada Lampiran C. Hasil turunan parsial pertama yang diperoleh merupakan fungsi yang nonlinier terhadap parameter yang akan diestimasi, sehingga diperlukan metode *Newton-Raphson* untuk memperoleh estimasi parameternya. Dalam metode *Newton-Raphson* diperlukan turunan parsial kedua dari fungsi *ln-likelihood* terhadap parameter yang akan diestimasi. Hasil turunan parsial kedua dari fungsi *ln-likelihood* sebagai berikut.

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_1^2}=\sum_{i=1}^{m}\left\{-\left(y_{1i}+y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)-y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)\right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_1 \partial \sim_2}=\sum_{i=1}^{m}\left\{y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_2}-e^{-\sim_1-x^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}\right)^2}\right)\right\}$$

$$\frac{\partial^2 l(\boldsymbol{\Phi})}{\partial \sim_1 \partial \boldsymbol{\beta}}=\sum_{i=1}^{m}\mathbf{x}_i^T\left(-y_{1i}-y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

$$\frac{\partial^2 l(\boldsymbol{\Phi})}{\partial \sim_1 \partial \boldsymbol{\gamma}_1}=\sum_{i=1}^{m}\mathbf{t}_i^T\left(\left(-y_{1i}-y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)-y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)\right)$$

$$\frac{\partial^2 l(\boldsymbol{\Phi})}{\partial \sim_1 \partial \boldsymbol{\gamma}_2}=\sum_{i=1}^{m}\mathbf{t}_i^T y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_2^2} = \sum_{i=1}^{m} \left\{ -y_{2i} \left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) + \left(y_{1i}-1\right) \left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right) \right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_2 \partial \boldsymbol{\beta}} = \sum_{i=1}^{m} \mathbf{x}_i^T \left(y_{1i}-1\right) \left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right)$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_2 \partial \boldsymbol{\gamma}_1} = \sum_{i=1}^{m} \mathbf{t}_i^T y_{2i} \left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right)$$

$$\frac{\partial^2 l(\boldsymbol{\Phi})}{\partial \sim_2 \partial \boldsymbol{\gamma}_2} = \sum_{i=1}^{m} \mathbf{t}_i^T \left\{ -y_{2i} \left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) + \left(y_{1i}-1\right) \left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right) \right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^T} = \sum_{i=1}^{m} \mathbf{x}_i \mathbf{x}_i^T \left\{ \left(-y_{1i}-y_{2i}\right) \left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) + \left(y_{1i}-1\right) \left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right) \right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\gamma}_1^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{x}_i^T \left(\left(-y_{1i}-y_{2i}\right) \left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) \right)$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\gamma}_2^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{x}_i^T \left(y_{1i}-1\right) \left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right)$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_1 \partial \boldsymbol{\gamma}_1^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T \left(\left(-y_{1i}-y_{2i}\right) \left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) - y_{2i} \left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) \right)$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_1 \partial \boldsymbol{\gamma}_2^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T y_{2i} \left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right)$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_2 \partial \boldsymbol{\gamma}_2^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T \left\{ -y_{2i} \left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) + \left(y_{1i}-1\right) \left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right) \right\}$$

Hasil lengkap turunan parsial kedua dapat dilihat pada Lampiran C. Berikut adalah langkah-langkah bagaimana metode *Newton-Raphson* mendapatkan nilai $\hat{\boldsymbol{\Phi}}$ dari sebuah fungsi $\ell(\boldsymbol{\Phi})$ yang telah dimaksimumkan (dalam hal ini untuk tiga kategori variabel repson).

Langkah pertama	:	yakni menentukan nilai awal untuk $\hat{\mathbf{\Phi}}^{(0)}$ dengan menggunakan nilai $\hat{\mathbf{\Phi}}$ yang diperoleh dari model regresi logistik ordinal
Langkah kedua	:	Membentuk vektor gradien $\mathbf{g}$

$$\mathbf{g}(\mathbf{\Phi}) = \left[\frac{\partial \ell(\mathbf{\Phi})}{\partial \sim_1} \quad \frac{\partial \ell(\mathbf{\Phi})}{\partial \sim_2} \quad \frac{\partial \ell(\mathbf{\Phi})}{\partial \boldsymbol{\beta}} \quad \frac{\partial \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_1} \quad \frac{\partial \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_2} \right]^T$$

Langkah ketiga	:	Membentuk matriks Hessian $\mathbf{H}$

$$\mathbf{H}(\boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_1^2} & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_1 \sim_2} & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_1 \boldsymbol{\beta}} & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_1 \boldsymbol{\gamma}_1} & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_1 \boldsymbol{\gamma}_2} \\ & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_2^2} & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_2 \boldsymbol{\beta}} & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_2 \boldsymbol{\gamma}_1} & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_2 \boldsymbol{\gamma}_2} \\ & & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^T} & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\gamma}_1^T} & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\gamma}_2^T} \\ & & & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_1 \partial \boldsymbol{\gamma}_1^T} & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_1 \partial \boldsymbol{\gamma}_2^T} \\ \textit{simetris} & & & & \frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_2 \partial \boldsymbol{\gamma}_2^T} \end{bmatrix}$$

Jika $\mathbf{g}^{(t)}$ dan $\mathbf{H}^{(t)}$ adalah $\mathbf{g}$ dan $\mathbf{H}$ yang dievaluasi pada $\mathbf{\Phi}^{(t)}$ yang merupakan estimasi t untuk $\hat{\mathbf{\Phi}}$. Langkah t dalam proses iteratif (0, 1, 2, …) mengestimasi $\ell(\mathbf{\Phi})$ mendekati $\mathbf{\Phi}^{(t)}$ yang dihubungkan dengan orde kedua ekspansi deret Taylor

$$\ell(\mathbf{\Phi}) = \ell\left(\mathbf{\Phi}^{(t)}\right) + \mathbf{g}^{(t)T}\left(\mathbf{\Phi} - \mathbf{\Phi}^{(t)}\right) + \frac{1}{2}\left(\mathbf{\Phi} - \mathbf{\Phi}^{(t)}\right)^T \mathbf{H}^{(t)}\left(\mathbf{\Phi} - \mathbf{\Phi}^{(t)}\right) \tag{4.6}$$

Jika persamaan (4.6) diturunkan terhadap $\mathbf{\Phi}$, maka diperoleh

$$\frac{\partial \ell(\mathbf{\Phi})}{\partial \mathbf{\Phi}} \approx \mathbf{g}^{(t)} + \mathbf{H}^{(t)}\left(\mathbf{\Phi} - \mathbf{\Phi}^{(t)}\right) = \mathbf{0}$$

$$\mathbf{H}^{(t)}\left(\mathbf{\Phi} - \mathbf{\Phi}^{(t)}\right) = -\mathbf{g}^{(t)}$$

$$\left(\mathbf{\Phi} - \mathbf{\Phi}^{(t)}\right) = -\left[\mathbf{H}^{(t)}\right]^{-1} \mathbf{g}^{(t)}$$

$$\mathbf{\Phi} = \mathbf{\Phi}^{(t)} - \left[\mathbf{H}^{(t)}\right]^{-1} \mathbf{g}^{(t)}$$

Sehingga nilai $\mathbf{\Phi}$ berikutnya dapat diperoleh dengan persamaan (4.7).

$$\mathbf{\Phi}^{(t+1)} = \mathbf{\Phi}^{(t)} - \left[\mathbf{H}^{(t)}\right]^{-1} \mathbf{g}^{(t)} \qquad (4.7)$$

dimana $\mathbf{H}^{(t)}$ adalah matriks nonsingular dan merupakan matriks turunan parsial kedua dari fungsi *ln-likelihood* terhadap parameter yang akan diestimasi, sedangkan $\mathbf{g}^{(t)}$ adalah vektor turunan parsial pertama dari fungsi *ln-likelihood* terhadap parameter yang akan diestimasi (Agresti, 2000). Persamaan (4.7) digunakan sebagai panduan dalam proses iterasi *Newton-Raphson*.

Langkah keempat : Mulai dari t = 0 dilakukan iterasi pada persamaan (4.7)

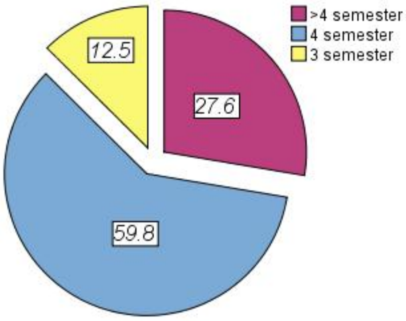
Langkah kelima : Proses iterasi akan berhenti jika terpenuhi kondisi konvergen, yaitu $\left\|\mathbf{\Phi}^{(t+1)} - \mathbf{\Phi}^{[t]}\right\| \leq £$, dimana £ adalah bilangan yang sangat kecil.

4.2 Aplikasi Partial Proportional Odds Model

Pada penelitian ini *Partial Proportional Odds Model* diaplikasikan pada kasus masa studi lulusan program magister Insitut Teknologi Sepuluh Nopember (ITS), dengan terlebih dahulu melakukan analisis deskriptif terhadap variabel-variabel yang digunakan. Analisis deskriptif pada penelitian ini digunakan untuk mengetahui seluruh informasi mengenai variabel-variabel yang digunakan dalam penelitian sebagai gambaran awal agar dapat diketahui karakteristik dari lulusan program magister ITS.

4.2.1 Deskripsi Variabel Penelitian

Variabel respon pada penelitian ini adalah masa studi lulusan program magister ITS pada periode Maret 2008 sampai dengan Maret 2011. Masa studi dikategorikan menjadi tiga kategori, yaitu masa studi lebih dari 4 semester, masa studi 4 semester, dan masa studi 3 semester.



Gambar 4.1 Persentase Masa Studi Lulusan Program Magister ITS

Berdasarkan Gambar 4.1 dapat diketahui bahwa dari 1284 lulusan pada periode Maret 2008 sampai dengan Maret 2011 paling banyak mahasiswa lulus dengan masa studi 4 semester yakni sebesar 59,8% atau sebanyak 768 mahasiswa dari total keseluruhan lulusan, 27,6% atau sebanyak 355 mahasiswa lulus dengan masa studi lebih dari 4 semester, dan 12,5% atau sebanyak 161 mahasiswa lulus dengan masa studi 3 semester.

a. Cross Tabulasi Variabel Kategori

Pada bagian ini akan dilakukan analisis cross tabulasi antara variabel kategori, yakni antara variabel respon dengan variabel prediktor yang bersifat kategori. Adapun variabel prediktor yang bersifat kategori pada penelitian ini yaitu jenis kelamin, kesesuaian bidang studi, sumber pendanaan, status pernikahan, jenis pekerjaan, dan asal fakultas.

Tabel 4.1 Cross Tabulasi Masa Studi dan Jenis Kelamin

Masa Studi	Jenis Kelamin		Total
	Perempuan	Laki-laki	
>4 Semester	127	228	355
4 Semester	311	457	768
3 Semester	45	116	161
Total	483	801	1284

Berdasarkan Tabel 4.1 diketahui bahwa lulusan program magister ITS yang berjenis kelamin laki-laki lebih banyak dari pada perempuan yakni sebesar 62,38% lulusan berjenis kelamin laki-laki dan 37,62% berjenis kelamin perempuan dari total keseluruhan lulusan. Jika ditinjau dari masa studi dapat diketahui bahwa jumlah lulusan program magister ITS lebih banyak yang berjenis kelamin laki-laki dari pada perempuan.

Tabel 4.2 Cross Tabulasi Masa Studi dan Kesesuaian Bidang Studi

Masa Studi	Kesesuaian Bidang Studi		Total
	Tidak Sesuai	Sesuai	
>4 Semester	134	221	355
4 Semester	229	539	768
3 Semester	46	115	161
Total	409	875	1284

Berdasarkan Tabel 4.2 diketahui bahwa sebagian besar lulusan program magister ITS memiliki kesesuaian bidang studi pada saat S1 dengan bidang studi pada saat S2, yakni sebesar 68,15% dari total keseluruhan lulusan memiliki kesesuaian bidang studi pada saat S1 dengan bidang studi S2 dan 31,85% dari total keseluruhan lulusan tidak memiliki kesesuaian bidang studi pada saat S1 dengan bidang studi S2. Jika ditinjau dari masa studi dapat diketahui bahwa sebagian besar lulusan yang memiliki kesesuaian bidang studi pada saat S1 dengan bidang studi S2 lulus dengan masa studi empat semester dan yang paling sedikit adalah lulusan yang tidak memiliki kesesuaian bidang studi pada saat S1 dengan bidang studi S2 lulus dengan masa studi tiga semester.

Tabel 4.3 Cross Tabulasi Masa Studi dan Sumber Pendanaan

Masa Studi	Sumber Pendanaan				Total
	Sendiri	BPPS	Instansi	Lainnya	
>4 Semester	112	120	83	40	355
4 Semester	192	249	159	168	768
3 Semester	43	26	61	31	161
Total	347	395	303	239	1284

Tabel 4.3 menunjukkan bahwa paling banyak lulusan program magister ITS pernah mendapat pendanaan yang bersumber dari beasiswa BPPS yakni sebesar 30,76% dari total keseluruhan lulusan dan paling sedikit adalah lulusan yang

pernah mendapat pendanaan yang bersumber dari beasiswa lainnnya yakni sebesar 18,61% dari total keseluruhan lulusan. Jika dilihat dari masa studinya dapat diketahui bahwa lulusan yang mendapat beasiswa BPPS lebih banyak lulus dengan masa studi empat semester dan lebih dari empat semester dibandingkan dengan lulusan dengan sumber pendanaan lainnya, yakni sebesar 19,39% dan 9,35% dari total keseluruhan lulusan, sedangkan lulusan yang mendapat beasiswa dari instansi lebih banyak lulus dengan masa studi tiga semester yakni sebesar 4,75% dari total keseluruhan lulusan.

Tabel 4.4 Cross Tabulasi Masa Studi dan Status Pernikahan

Masa Studi	Status Pernikahan		Total
	Menikah	Belum Menikah	
>4 Semester	196	159	355
4 Semester	405	363	768
3 Semester	92	69	161
Total	693	591	1284

Berdasarkan Tabel 4.4 dapat diketahui bahwa lulusan program magister ITS lebih banyak yang sudah menikah daripada yang belum menikah, yakni sebesar 53,97% yang sudah menikah dari total keseluruhan lulusan dan sebesar 46,03% yang belum menikah dari total keseluruhan lulusan. Jika dilihat dari masa studi dapat diketahui bahwa paling banyak lulusan yang sudah menikah lulus dengan masa studi empat semester dan yang paling sedikit adalah lulusan yang belum menikah lulus dengan masa studi tiga semester.

Tabel 4.5 Cross Tabulasi Masa Studi dan Jenis Pekerjaan

Masa Studi	Jenis Pekerjaan			Total
	PN/BUMN	Swasta/ wiraswasta	Belum Bekerja	
>4 Semester	152	121	82	355
4 Semester	437	191	140	768
3 Semester	94	39	28	161
Total	683	351	250	1284

Berdasarkan Tabel 4.5 diketahui bahwa lulusan program magister ITS lebih banyak yang sudah bekerja daripada yang belum bekerja, yakni sebesar 80,53% yang sudah bekerja dari total keseluruhan lulusan dan sebesar 19,47% yang belum bekerja dari total keseluruhan lulusan. Jika dilihat dari masa studi dapat diketahui bahwa sebagian besar lulusan yang sudah bekerja lulus dengan masa studi empat semester yakni sebesar 45,91% dari total keseluruhan lulusan, sedangkan yang paling sedikit adalah lulusan yang belum bekerja lulus dengan masa studi tiga semester yakni sebesar 2,18% dari total keseluruhan lulusan.

Tabel 4.6 Cross Tabulasi Masa Studi dan Asal Fakultas

Masa Studi	Asal Fakultas					Total
	FMIPA	FTI	FTSP	FTK	FTIF	
>4 Semester	52	137	105	26	35	355
4 Semester	308	248	95	64	53	768
3 Semester	63	55	15	20	8	161
Total	423	440	215	110	96	1284

Berdasarkan Tabel 4.6 dapat diketahui bahwa paling banyak lulusan program magister ITS berasal dari FTI yakni 34,27% dari total keseluruhan lulusan dan yang paling sedikit berasal dari FTIF yakni 7,48% dari total keseluruhan lulusan. Jika dilihat dari masa studi dapat diketahui bahwa paling banyak lulusan yang berasal dari FMIPA lulus dengan masa studi empat semester yakni 23,99% dari total keseluruhan lulusan, sedangkan yang paling sedikit adalah lulusan yang berasal dari FTIF lulus dengan masa studi tiga semester yakni 0,62% dari total keseluruhan lulusan.

b. Deskripsi Variabel Prediktor Kontinu

Pada penelitian ini terdapat lima variabel prediktor yang bersifat kontinu yaitu nilai TPA, nilai Toefl, Nilai IPK S1, lama waktu lulus S1 ke S2, dan usia.

Tabel 4.7 Deskripsi Variabel Prediktor Kontinu

Variabel	Minimum	Maksimum	Rata-rata	Standar Deviasi
Nilai TPA	43,21	212,11	131,46	29,16
Nilai Toefl	183	656	435,16	65,43
Nilai IPK S1	2,00	4,00	3,09	0,33
Lama waktu lulus S1 ke S2	0	32	5,34	4,77
Usia	18	60	30,10	6,32

Berdasarkan Tabel 4.7 dapat diketahui bahwa pada saat masuk program magister ITS, lulusan program magister ITS rata-rata berusia 30,10 tahun dengan usia minimum 18 tahun dan usia maksimum adalah 60 tahun. Lulusan program magister ITS tersebut rata-rata memiliki selang waktu dari lulus S1 ke S2 selama 5 tahun, dengan selang waktu minimum adalah 0 tahun artinya setelah lulus S1 langsung melanjutkan studi S2. Dari segi kemampuan akademik yang dilihat dari nilai IPK S1, lulusan tersebut memiliki nilai rata-rata IPK S1 sebesar 3,09 dengan nilai IPK minimum adalah 2,00 dan nilai IPK maksimum adalah 4,00. Selain itu, dilihat dari nilai TPA dan Toefl pada saat masuk program magister ITS dapat diketahui bahwa mahasiswa lulusan program magister ITS memiliki nilai rata-rata TPA sebesar 131,46 dengan nilai TPA minimum adalah 43,21 dan nilai TPA maksimum adalah 212,11 sedangan nilai rata-rata Toefl sebesar 435,16 dengan nilai Toefl minimum adalah 183 dan nilai Toefl maksimum adalah 656.

4.2.2 Proportional Odds Model

Sebelum dilakukan analisis dengan *partial proportional odds model* (PPOM), terlebih dahulu dilakukan identifikasi awal variabel-variabel yang berpengaruh signifikan terhadap variabel respon dengan melakukan pengujian parameter secara serentak dan parsial untuk *proportional odds model* (POM) dan kemudian dilanjutkan dengan pengujian asumsi *proportional odds* pada variabel prediktor untuk mengetahui ada atau tidaknya variabel yang tidak memenuhi asumsi tersebut.

a. Pengujian Secara Serentak

Pengujian parameter secara serentak dilakukan untuk mengetahui signifikansi parameter terhadap variabel respon secara keseluruhan. Pengujian ini dilakukan dengan menggunakan statistik uji *G* atau *likelihood ratio*. Hasil pengujian secara serentak diperoleh bahwa nilai statistik uji *G* sebesar 166,45 yang lebih besar dari $\chi^2_{(17;0,05)} = 27,587$ dan *P-value* yang lebih kecil dari $\alpha = 0,05$. Hal ini menunjukkan bahwa H_0 ditolak, sehingga dapat disimpulkan bahwa minimal ada satu variabel prediktor yang berpengaruh terhadap variabel respon. Untuk mengetahui variabel-variabel yang berpengaruh tersebut dilakukan pengujian parameter secara parsial. Hasil pengujian serentak dapat dilihat pada Lampiran D.

b. Pengujian Secara Parsial

Pengujian ini dilakukan untuk mengetahui pengaruh setiap variabel prediktor secara individual. Hasil pengujian secara parsial akan menunjukkan apakah suatu variabel prediktor layak untuk masuk dalam model atau tidak. Hasil pengujian secara parsial ditunjukkan dalam Tabel 4.8.

Tabel 4.8 Hasil Pengujian Secara Parsial

Variabel	Koefisien	*P-value*	Keputusan
X_1	0,0032	0,139	Gagal Tolak H_0
X_2	0,0023	0,017	Tolak H_0
$X_{3(1)}$	0,2709	0,032	Tolak H_0
$X_{4(1)}$	0,3154	0,013	Tolak H_0
X_5	1,1619	0,000	Tolak H_0
X_6	0,0237	0,374	Gagal Tolak H_0
$X_{7(1)}$	0,3706	0,108	Gagal Tolak H_0
$X_{7(2)}$	-0,0653	0,703	Tolak H_0
$X_{7(4)}$	0,6100	0,002	Tolak H_0
$X_{8(1)}$	0,0747	0,633	Gagal Tolak H_0
$X_{9(1)}$	0,4232	0,073	Gagal Tolak H_0
$X_{9(2)}$	0,0734	0,741	Gagal Tolak H_0
X_{10}	0,0001	0,995	Gagal Tolak H_0

Lanjutan Tabel 4.8

Variabel	Koefisien	*P-value*	Keputusan
$X_{11(1)}$	1,0151	0,000	Tolak H_0
$X_{11(2)}$	0,4264	0,066	Gagal Tolak H_0
$X_{11(3)}$	-0,2679	0,294	Gagal Tolak H_0
$X_{11(4)}$	0,8755	0,003	Tolak H_0
Alpha (1)	5,4697		
Alpha (2)	8,6912		

Tabel 4.8 merupakan hasil pengujian parameter secara parsial yang menunjukkan bahwa pada tingkat kepercayaan 95% terdapat enam variabel prediktor yang signifikan berpengaruh terhadap variabel respon, yakni nilai Toefl pada saat masuk program magister ITS (X_2), jenis kelamin (X_3), kesesuaian bidang studi (X_4), nilai IPK S1 (X_5), sumber pendanaan (X_7), dan asal fakultas (X_{11}). Hal tersebut dapat dilihat dari *P-value* yang lebih kecil dari $\sim = 0{,}05$ yang menunjukkan bahwa H_0 ditolak. Selanjutnya variabel-variabel yang tidak signifikan tersebut dikeluarkan, sehingga diperoleh variabel yang signifikan seperti yang ditunjukkan dalam Tabel 4.9.

Tabel 4.9 Variabel-variabel yang Signifikan

Variabel	Koefisien	*P-value*	Keputusan
X_2	0,0025	0,008	Tolak H_0
$X_{3(1)}$	0,2768	0,026	Tolak H_0
$X_{4(1)}$	0,3187	0,012	Tolak H_0
X_5	1,0556	0,000	Tolak H_0
$X_{7(1)}$	0,2784	0,214	Gagal Tolak H_0
$X_{7(2)}$	-0,0317	0,852	Gagal Tolak H_0
$X_{7(4)}$	0,4854	0,007	Tolak H_0
$X_{9(1)}$	0,5139	0,023	Tolak H_0
$X_{9(2)}$	0,1430	0,507	Gagal Tolak H_0
$X_{11(1)}$	1,0497	0,000	Tolak H_0
$X_{11(2)}$	0,4265	0,063	Gagal Tolak H_0
$X_{11(3)}$	-0,2541	0,315	Gagal Tolak H_0
$X_{11(4)}$	0,8753	0,003	Tolak H_0
Alpha (1)	4,6583		
Alpha (2)	7,8678		

Berdasarkan Tabel 4.9 dapat diketahui bahwa variabel-variabel yang signifikan adalah nilai Toefl pada saat masuk program magister ITS (X_2), jenis kelamin (X_3), kesesuaian bidang studi antara S1 dan S2 (X_4), nilai IPK S1 (X_5), sumber pendanaan (X_7), jenis pekerjaan (X_9), dan asal fakultas (X_{11}).

4.2.3 Pengujian Asumsi Proportional Odds

Pada tahap ini dilakukan pengujian asumsi *proportional odds* pada variabel-variabel yang digunakan dalam penelitian untuk mengetahui ada atau tidaknya variabel yang tidak memenuhi asumsi *proportional odds*. Pada tahap awal akan dilakukan pengujian asumsi *proportional odds* pada semua variabel prediktor yang digunakan dalam penelitian. Hasil tersebut ditunjukkan dalam Tabel 4.10.

Tabel 4.10. Pengujian Asumsi *Proportional Odds* untuk Semua Variabel

Variabel	*Chi-Square*	*P-value*	Df
All	83,77	0,000	17
X_1	10,94	0,001	1
X_2	0,17	0,676	1
$X_{3(1)}$	2,29	0,130	1
$X_{4(1)}$	0,41	0,521	1
X_5	1,83	0,176	1
X_6	0,00	0,990	1
$X_{7(1)}$	5,59	0,018	1
$X_{7(2)}$	34,51	0,000	1
$X_{7(4)}$	17,45	0,000	1
$X_{8(1)}$	0,06	0,805	1
$X_{9(1)}$	1,13	0,287	1
$X_{9(2)}$	0,01	0,909	1
X_{10}	0,49	0,484	1
$X_{11(1)}$	6,09	0,014	1
$X_{11(2)}$	0,36	0,550	1
$X_{11(3)}$	0,28	0,594	1
$X_{11(4)}$	0,04	0,842	1

Berdasarkan hasil pengujian asumsi *proportional odds* pada semua variabel prediktor yang ditunjukkan pada Tabel 4.10 diketahui bahwa nilai *Chi-Square* =

83,77 lebih besar dari $\chi^2_{(df,\alpha)} = 27,587$ dan *P-value* yang lebih kecil dari $\alpha = 0,05$. Hal ini menunjukkan bahwa H_0 ditolak, sehingga dapat disimpulkan bahwa minimal ada satu variabel prediktor yang tidak memenuhi asumsi *proportional odds*. Variabel-variabel yang tidak memenuhi asumsi *proportional odds* tersebut adalah nilai TPA (X_1), variabel sumber pendanaan (X_7) dan asal fakultas (X_{11}).

Pada tahap sebelumnya telah dilakukan pengujian signifikansi parameter variabel prediktor sehingga diperoleh variabel prediktor yang berpengaruh terhadap variabel respon seperti yang ditunjukkan pada Tabel 4.9, kemudian dilakukan kembali pengujian asumsi *proportional odds* pada variabel-variabel tersebut untuk mengetahui ada atau tidaknya variabel yang tidak memenuhi asumsi *proportional odds*. Hasil pengujian asumsi *proportional odds* untuk semua variabel prediktor yang signifikan ditunjukkan dalam Tabel 4.11.

Tabel 4.11 Pengujian Asumsi *Proportional Odds* untuk Variabel yang Signifikan

Variabel	*Chi-Square*	*P-value*	Df
All	76,00	0,000	13
X_2	0,06	0,801	1
$X_{3(1)}$	2,77	0,096	1
$X_{4(1)}$	0,14	0,709	1
X_5	1,59	0,207	1
$X_{7(1)}$	7,24	0,007	1
$X_{7(2)}$	32,72	0,000	1
$X_{7(4)}$	23,92	0,000	1
$X_{9(1)}$	0,84	0,360	1
$X_{9(2)}$	0,05	0,815	1
$X_{11(1)}$	5,38	0,020	1
$X_{11(2)}$	0,31	0,579	1
$X_{11(3)}$	0,29	0,588	1
$X_{11(4)}$	0,01	0,917	1

Berdasarkan hasil pengujian asumsi *proportional odds* yang ditunjukkan pada Tabel 4.11 diketahui bahwa minimal ada satu variabel yang tidak memenuhi asumsi *proportional odds*. Hal ini dapat dilihat dari nilai *Chi-Square* = 76,00 yang lebih besar dari $\chi^2_{(df,\alpha)} = 22{,}362$ dan *P-value* yang lebih kecil dari $\alpha = 0{,}05$. Variabel-variabel yang tidak memenuhi asumsi *proportional odds* tersebut adalah variabel sumber pendanaan (X_7) dan asal fakultas (X_{11}). Karena dari hasil pengujian tersebut terdapat variabel yang tidak memenuhi asumsi *proportional odds* maka analisis selanjutnya dilakukan dengan PPOM.

Berdasarkan Tabel 4.8 dan Tabel 4.10 diketahui bahwa nilai TPA (X_1) merupakan salah satu variabel yang tidak signifikan berpengaruh terhadap variabel respon dan tidak memenuhi asumsi *proportional odds*. Maka pada penelitian ini analisis dilakukan dengan tetap menggunakan variabel nilai TPA/tidak mengeluarkan variabel nilai TPA (X_1) karena mempertimbangkan tidak terpenuhinya asumsi *proportional odds*, dan mengeluarkan variabel nilai TPA (X_1) karena tidak signifikan berpengaruh terhadap variabel respon. Hasil pengujian asumsi *proportional odds* dengan tidak mengeluarkan variabel nilai TPA (X_1) ditunjukkan dalam Tabel 4.12.

Tabel 4.12 Pengujian Asumsi *Proportional Odds* dengan Tidak Mengeluarkan X_1

Variabel	*Chi-Square*	*P-value*	Df
All	83,49	0,000	14
X_1	10,96	0,001	1
X_2	0,34	0,561	1
$X_{3(1)}$	2,77	0,096	1
$X_{4(1)}$	0,39	0,543	1
X_5	3,56	0,059	1
$X_{7(1)}$	6,54	0,011	1
$X_{7(2)}$	35,17	0,000	1
$X_{7(4)}$	20,50	0,000	1
$X_{9(1)}$	0,70	0,404	1
$X_{9(2)}$	0,11	0,742	1
$X_{11(1)}$	6,04	0,014	1
$X_{11(2)}$	0,50	0,479	1
$X_{11(3)}$	0,37	0,542	1
$X_{11(4)}$	0,07	0,790	1

Berdasarkan Tabel 4.12 dapat diketahui bahwa variabel-variabel yang tidak memenuhi asumsi *proportional odds* adalah variabel nilai TPA (X_1), sumber pendanaan (X_7) dan asal fakultas (X_{11}).

4.2.4 Partial Proportional Odds Model

Pada bagian ini akan dilakukan pemodelan faktor-faktor yang mempengaruhi masa studi lulusan program magister ITS menggunakan *Partial Proportional Odds Model* (PPOM). PPOM merupakan model alternatif yang digunakan ketika asumsi *proportional odds* tidak dipenuhi oleh satu atau beberapa variabel prediktor.

Pada penelitian ini akan dilakukan pemodelan dengan tidak mengeluarkan variabel nilai TPA (X_1) dan mengeluarkan variabel nilai TPA (X_1). Hasil PPOM dengan mengeluarkan variabel nilai TPA (X_1) ditunjukkan pada Tabel 4.13.

Tabel 4.13 Hasil PPOM dengan MengeluarkanVariabel X_1

Variabel Prediktor	Kategori: Lebih dari 4 semester		Variabel Prediktor	Kategori: Kurang dari 4 semester	
	Koefisian	*P-value*		Koefisian	*P-value*
X_2	0,0026	0,006	X_2	0,0026	0,006
$X_{3(1)}$	0,2675	0,036	$X_{3(1)}$	0,2675	0,036
$X_{4(1)}$	0,3391	0,009	$X_{4(1)}$	0,3391	0,009
X_5	1,1097	0,000	X_5	1,1097	0,000
$X_{7(1)}$	**0,5149**	0,033	$X_{7(1)}$	**-0,1769**	0,521
$X_{7(2)}$	**0,4024**	0,036	$X_{7(2)}$	**-1,2036**	0,000
$X_{7(4)}$	**1,0332**	0,000	$X_{7(4)}$	**-0,3726**	0,138
$X_{9(1)}$	0,5470	0,017	$X_{9(1)}$	0,5470	0,017
$X_{9(2)}$	0,1812	0,402	$X_{9(2)}$	0,1812	0,402
$X_{11(1)}$	**1,5737**	0,000	$X_{11(1)}$	**0,1255**	0,660
$X_{11(2)}$	0,4226	0,061	$X_{11(2)}$	0,4226	0,061
$X_{11(3)}$	-0,2255	0,366	$X_{11(3)}$	-0,2255	0,366
$X_{11(4)}$	0,8179	0,005	$X_{11(4)}$	0,8179	0,005
cons	-5,3161	0,000	cons	-7,1752	0,000

Tabel 4.13 menunjukkan bahwa variabel prediktor bergantung pada kategori variabel respon, dimana variabel yang tidak memenuhi asumsi *proportional odds*

yakni $X_{7(1)}$, $X_{7(2)}$, $X_{7(4)}$, dan $X_{11(1)}$ memiliki nilai koefisien yang berbeda pada tiap kategori variabel respon, William (2006) menyebut model tersebut sebagai *non proportional odds model*. Peterson & Harrel (1990) dan Lall *et al*. (2002) dalam William (2006) menyajikan *parameterization* yang setara dengan model yang diperkenalkan oleh William (2006), yang disebut *unconstrained partial proportional odds model*. Pada *unconstrained partial proportional odds model* terdapat menambahakan suatu parameter gamma, dimana parameter gamma menunjukkan penyimpangan proporsionalitas. Hasil *parameterization* dengan mengeluarkan variabel nilai TPA (X_1) ditunjukkan pada Tabel 4.14.

Selanjutnya dilakukan analisis PPOM dengan tidak mengeluarkan variabel nilai TPA (X_1). Hasil PPOM dengan tidak mengeluarkan variabel nilai TPA (X_1) ditunjukkan pada Tabel 4.15.

Tabel 4.14 Hasil *Parameterization* dengan Mengeluarkan Variabel X_1

Parameter	Koefisien	*P-value*
Beta		
X_2	0,0026	0,006
$X_{3(1)}$	0,2675	0,036
$X_{4(1)}$	0,3391	0,009
X_5	1,1097	0,000
$X_{7(1)}$	0,5149	0,033
$X_{7(2)}$	0,4024	0,036
$X_{7(4)}$	1,0332	0,000
$X_{9(1)}$	0,5470	0,017
$X_{9(2)}$	0,4226	0,061
$X_{11(1)}$	1,5637	0,000
$X_{11(2)}$	0,4226	0,061
$X_{11(3)}$	-0,2255	0,366
$X_{11(4)}$	0,8179	0,005
Gamma		
$X_{7(1)}$	-0,6919	0,007
$X_{7(2)}$	-1,6060	0,000
$X_{7(4)}$	-1,4058	0,000
$X_{11(1)}$	-1,4482	0,000
Alpha (1)	-5,3161	0,000
Alpha (2)	-7,1752	0,000

Berdasarkan Tabel 4.14 dapat dibentuk *unconstrained partial proportional odds model* seperti yang ditunjukkan pada persamaan (4.8).

$$\left.\begin{aligned}
a_1(\mathbf{x}) = {} & -5{,}316 - 0{,}003x_2 - 0{,}268x_{3(1)} - 0{,}339x_{4(1)} - 1{,}109x_5 + 0{,}177x_{7(1)} + \\
& 1{,}204x_{7(2)} + 0{,}373x_{7(4)} - 0{,}547x_{9(1)} - 0{,}423x_{9(2)} - 0{,}116x_{11(1)} - \\
& 0{,}423x_{11(2)} + 0{,}226x_{11(3)} - 0{,}818x_{11(4)} \\
a_2(\mathbf{x}) = {} & -7{,}175 - 0{,}003x_2 - 0{,}268x_{3(2)} - 0{,}339x_{4(1)} - 1{,}109x_5 + 0{,}177x_{7(1)} + \\
& 1{,}204x_{7(2)} + 0{,}373x_{7(4)} - 0{,}547x_{9(1)} - 0{,}423x_{9(2)} - 0{,}116x_{11(1)} - \\
& 0{,}423x_{11(2)} + 0{,}226x_{11(3)} - 0{,}818x_{11(4)}
\end{aligned}\right\} \quad (4.8)$$

Berdasarkan model yang yang ditunjukkan pada persamaan (4.8) dapat diketahui bahwa masa studi lulusan program magister ITS dipengaruhi secara signifikan oleh tujuh variabel prediktor, yakni nilai Toefl pada saat masuk program magister ITS, jenis kelamin, kesesuaian bidang studi antara S1 dan S2, nilai IPK S1, sumber pendanaan, jenis pekerjaan (pegawai negeri/BUMN), dan asal fakultas (FMIPA dan FTK).

Tabel 4.15 Hasil PPOM dengan Tidak Mengeluarkan Variabel X_1

Variabel Prediktor	Kategori: Lebih dari 4 semester		Variabel Prediktor	Kategori: Kurang dari 4 semester	
	Koefisian	*P-value*		Koefisian	*P-value*
X_1	**-0,0007**	0,793	X_1	**0,0092**	0,003
X_2	0,0024	0,014	X_2	0,0024	0,014
$X_{3(1)}$	0,2578	0,044	$X_{3(1)}$	0,2578	0,044
$X_{4(1)}$	0,3266	0,012	$X_{4(1)}$	0,3266	0,012
X_5	1,0585	0,000	X_5	1,0585	0,000
$X_{7(1)}$	**0,5447**	0,024	$X_{7(1)}$	**-0,1858**	0,506
$X_{7(2)}$	**0,4024**	0,027	$X_{7(2)}$	**-1,2992**	0,000
$X_{7(4)}$	**1,0501**	0,000	$X_{7(4)}$	**-0,2711**	0,292
$X_{9(1)}$	0,5461	0,017	$X_{9(1)}$	0,5461	0,17
$X_{9(2)}$	0,1818	0,400	$X_{9(2)}$	0,1818	0,400
$X_{11(1)}$	**1,5794**	0,000	$X_{11(1)}$	**0,0049**	0,986
$X_{11(2)}$	0,4224	0,060	$X_{11(2)}$	0,4224	0,060
$X_{11(3)}$	-0,2391	0,335	$X_{11(3)}$	-0,2391	0,335
$X_{11(4)}$	0,8309	0,004	$X_{11(4)}$	0,8309	0,004
cons	-4,9979	0,000	cons	-8,1255	0,000

Tabel 4.15 serupa dengan Tabel 4.13 yang menunjukkan bahwa variabel prediktor bergantung pada kategori variabel respon, dimana pada Tabel 4.15 variabel yang tidak memenuhi asumsi *proportional odds* adalah X_1, $X_{7(1)}$, $X_{7(2)}$, $X_{7(4)}$, dan $X_{11(1)}$ memiliki nilai koefisien yang berbeda pada tiap kategori variabel respon. Hasil *parameterization* dari variabel pada Tabel 4.15 ditunjukkan dalam Tabel 4.16.

Tabel 4.16 Hasil *Parameterization* dengan Tidak Mengeluarkan Variabel X_1

Parameter	Koefisien	*P-value*
Beta		
X_1	-0,0007	0,793
X_2	0,0024	0,014
$X_{3(1)}$	0,2578	0,044
$X_{4(1)}$	0,3266	0,012
X_5	1,0585	0,000
$X_{7(1)}$	0,5447	0,024
$X_{7(2)}$	0,4286	0,027
$X_{7(4)}$	1,0501	0,000
$X_{9(1)}$	0,5461	0,017
$X_{9(2)}$	0,1818	0,400
$X_{11(1)}$	1,5794	0,000
$X_{11(2)}$	0,4224	0,060
$X_{11(3)}$	-0,2391	0,335
$X_{11(4)}$	0,8309	0,004
Gamma		
X_1	0,0099	0,004
$X_{7(1)}$	-0,7305	0,005
$X_{7(2)}$	-1,7278	0,000
$X_{7(4)}$	-1,3212	0,000
$X_{11(1)}$	-1,5744	0,000
Alpha (1)	-4,9979	0,000
Alpha (2)	-8,1255	0,000

Berdasarkan Tabel 4.16 dapat dibentuk *unconstrained partial proportional odds model* seperti yang ditunjukkan pada persamaan (4.9).

$$
\left.
\begin{aligned}
\mathrm{a}_1(\mathbf{x}) = {} & -4{,}998 - 0{,}009x_1 - 0{,}002x_2 - 0{,}258x_{3(1)} - 0{,}327x_{4(1)} - 1{,}059x_5 + \\
& 0{,}186x_{7(1)} + 1{,}299x_{7(2)} + 0{,}271x_{7(4)} - 0{,}546x_{9(1)} - 0{,}182x_{9(2)} - \\
& 0{,}005x_{11(1)} - 0{,}422x_{11(2)} + 0{,}239x_{11(3)} - 0{,}831x_{11(4)} \\
\mathrm{a}_2(\mathbf{x}) = {} & -8{,}126 - 0{,}009x_1 - 0{,}002x_2 - 0{,}258x_{3(1)} - 0{,}327x_{4(1)} - 1{,}059x_5 + \\
& 0{,}186x_{7(1)} + 1{,}299x_{7(2)} + 0{,}271x_{7(4)} - 0{,}546x_{9(1)} - 0{,}182x_{9(2)} - \\
& 0{,}005x_{11(1)} - 0{,}422x_{11(2)} + 0{,}239x_{11(3)} - 0{,}831x_{11(4)}
\end{aligned}
\right\} \quad (4.9)
$$

Berdasarkan model yang yang ditunjukkan pada persamaan (4.9) dapat diketahui bahwa masa studi lulusan program magister ITS dipengaruhi secara signifikan oleh tujuh variabel prediktor, yakni nilai Toefl pada saat masuk program magister ITS, jenis kelamin, kesesuaian bidang studi antara S1 dan S2, nilai IPK S1, sumber pendanaan, jenis pekerjaan (pegawai negeri/BUMN), dan asal fakultas (FMIPA dan FTK), serta tidak dipengaruhi secara signifikan oleh variabel nilai TPA.

4.2.5 Uji Kesesuaian Model

Uji kesesuaian model digunakan untuk mengetahui apakah model dengan satu atau lebih variabel prediktor merupakan model yang sesuai atau tidak. Dengan menggunakan statistik uji *deviance* diperoleh bahwa model dengan mengeluarkan variabel X_1 dan model dengan tidak mengeluarkan variabel X_1 mempunyai *P-value* sebesar 1,000 dan nilai *deviance* berturut-turut sebesar 2209,97 dan 2207,311. Hal ini menunjukkan bahwa model sesuai atau fit dengan data.

4.2.6 Interprestasi Parameter

Interpretasi parameter dari model yang terbentuk menggunakan nilai *odds ratio*. Nilai *odds ratio* dihitung berdasarkan nilai koefisien dari masing-masing variabel prediktor. Nilai *odds ratio* berdasarkan model yang terbentuk ditunjukkan dalam Tabel 4.17 dan 4.18.

Tabel 4.17 Nilai *Odds Ratio* untuk Model dengan Mengeluarkan Variabel X_1

Parameter	*Odds Ratio*
Nilai Toefl (X_2)	0,997
Jenis kelamin (laki-laki) ($X_{3(1)}$)	0,765
Kesesuaian bidang studi (sesuai) ($X_{4(1)}$)	0,712
Nilai IPK S1 (X_5)	0,329
Sumber Pendanaan dari biaya sendiri ($X_{7(1)}$)	1,194
beasiswa BPPS ($X_{7(2)}$)	3,333
beasiswa lainnya ($X_{7(4)}$)	1,452
Jenis pekerjaan : Pengawai Negeri/BUMN ($X_{9(1)}$)	0,579
Pegawai swasta/wiraswasta ($X_{9(2)}$)	0,655
Asal Fakultas dari FMIPA ($X_{11(1)}$)	0,890
FTI ($X_{11(2)}$)	0,655
FTSP ($X_{11(3)}$)	1,254
FTK ($X_{11(4)}$)	0,441

Berdasarkan nilai *odds ratio* pada Tabel 4.17 dapat diketahui bahwa setiap peningkatan nilai Toefl sebesar 50 pada saat masuk program magister ITS maka kecenderungan mahasiswa lulus dengan masa studi lebih dari empat semester dan empat semester sebesar $e^{50\times(-0,003)} = 0,861$ kali dibandingkan masa studi tiga semester, dengan kata lain peningkatan nilai Toefl sebesar 50 pada saat masuk program magister ITS maka kecenderungan mahasiswa lulus dengan masa studi tiga semester sebesar $\frac{1}{0,861} = 1,162$ kali dibandingkan masa studi lebih dari empat semester dan empat semester. Jika dilihat dari IPK S1 maka setiap peningkatan nilai IPK S1 maka kecenderungan mahasiswa lulus dengan masa studi tiga semester sebesar $\frac{1}{0,329} = 3,031$ kali dibandingkan masa studi lebih dari empat semester dan empat semester.

Apabila ditinjau dari jenis kelamin dan kesesuaian bidang studi antara S1 dan S2 maka kecenderungan mahasiswa lulus dengan masa studi tiga semester dibandingkan dengan masa studi lebih dari empat semester dan empat semester, untuk mahasiswa berjenis kelamin laki-laki sebesar $\frac{1}{0,765} = 1,307$ kali daripada perempuan, sedangkan untuk mahasiswa yang memiliki kesesuaian bidang studi antara S1 dan S2 sebesar $\frac{1}{0{,}712} = 1{,}404$ kali daripada mahasiswa yang tidak memiliki kesesuaian bidang studi.

Berdasarkan sumber pendanaan maka kecenderungan mahasiswa lulus dengan masa studi lebih dari empat semester dan empat semester dibandingkan dengan masa studi 3 semester, untuk mahasiswa dengan biaya sendiri sebesar 1,194 kali daripada mahasiswa yang mendapat beasiswa dari instansi, untuk mahasiswa yang mendapat beasiswa BPPS sebesar 3,333 kali daripada mahasiswa yang mendapat beasiswa dari instansi, dan untuk mahasiswa yang mendapat beasiswa lainnya sebesar 1,452 kali daripada mahasiswa yang mendapat beasiswa dari instansi.

Apabila ditinjau dari jenis pekerjaan dan asal fakultas maka kecenderungan mahasiswa lulus dengan masa studi tiga semester dibandingkan dengan masa studi lebih dari empat semester dan empat semester, untuk mahasiswa yang bekerja sebagai pengawai negeri/BUMN sebesar $\frac{1}{0{,}579} = 1{,}727$ kali daripada mahasiswa yang belum bekerja, untuk mahasiswa yang berasal dari FMIPA sebesar $\frac{1}{0{,}890} = 1{,}124$ kali daripada mahasiswa yang berasal dari FTIF, dan untuk mahasiswa yang berasal dari FTK sebesar $\frac{1}{0{,}441} = 2{,}268$ kali daripada mahasiswa yang berasal dari FTIF.

Tabel 4.18 Nilai *Odds Ratio* untuk Model dengan Tidak Mengeluarkan Variabel X_1

Parameter	*Odds Ratio*
Nilai TPA (X_1)	0,991
Nilai Toefl (X_2)	0,998
Jenis kelamin (laki-laki) ($X_{3(1)}$)	0,773
Kesesuaian bidang studi (sesuai) ($X_{4(1)}$)	0,721
Nilai IPK S1 (X_5)	0,347
Sumber Pendanaan dari biaya sendiri ($X_{7(1)}$)	1,204
beasiswa BPPS ($X_{7(2)}$)	3,666
beasiswa lainnya ($X_{7(4)}$)	1,311

Lanjutan Tabel 4.18

Parameter	*Odds Ratio*
Jenis pekerjaan : Pengawai Negeri/BUMN ($X_{9(1)}$)	0,579
Pegawai swasta/wiraswasta ($X_{9(2)}$)	0,834
Asal Fakultas dari FMIPA ($X_{11(1)}$)	0,995
FTI ($X_{11(2)}$)	0,656
FTSP ($X_{11(3)}$)	1,269
FTK ($X_{11(4)}$)	0,436

Berdasarkan nilai *odds ratio* pada Tabel 4.18 dapat diketahui bahwa setiap peningkatan nilai Toefl sebesar 50 pada saat masuk program magister ITS maka kecenderungan mahasiswa lulus dengan masa studi lebih dari empat semester dan empat semester sebesar $e^{50\times(-0,002)} = 0,905$ kali dibandingkan masa studi tiga semester, dengan kata lain peningkatan nilai Toefl sebesar 50 pada saat masuk program magister ITS maka kecenderungan mahasiswa lulus dengan masa studi tiga semester sebesar $\frac{1}{0,905} = 1,105$ kali dibandingkan masa studi lebih dari empat semester dan empat semester. Jika dilihat dari IPK S1 maka setiap peningkatan nilai IPK S1 maka kecenderungan mahasiswa lulus dengan masa studi tiga semester sebesar $\frac{1}{0,347} = 2,882$ kali dibandingkan masa studi lebih dari empat semester dan empat semester.

Apabila ditinjau dari jenis kelamin dan kesesuaian bidang studi antara S1 dan S2 maka kecenderungan mahasiswa lulus dengan masa studi tiga semester dibandingkan dengan masa studi lebih dari empat semester dan empat semester, untuk mahasiswa berjenis kelamin laki-laki sebesar $\frac{1}{0,765} = 1,294$ kali daripada perempuan, sedangkan untuk mahasiswa yang memiliki kesesuaian bidang studi antara S1 dan S2 sebesar $\frac{1}{0,721} = 1,387$ kali daripada mahasiswa yang tidak memiliki kesesuaian bidang studi.

Berdasarkan sumber pendanaan maka kecenderungan mahasiswa lulus dengan masa studi lebih dari empat semester dan empat semester dibandingkan dengan masa studi 3 semester, untuk mahasiswa dengan biaya sendiri sebesar 1,204 kali daripada mahasiswa yang mendapat beasiswa dari instansi, untuk mahasiswa yang mendapat beasiswa BPPS sebesar 3,666 kali daripada mahasiswa yang mendapat beasiswa dari instansi, dan untuk mahasiswa yang mendapat beasiswa lainnya sebesar 1,311 kali daripada mahasiswa yang mendapat beasiswa dari instansi.

Apabila ditinjau dari jenis pekerjaan dan asal fakultas maka kecenderungan mahasiswa lulus dengan masa studi tiga semester dibandingkan dengan masa studi lebih dari empat semester dan empat semester, untuk mahasiswa yang bekerja sebagai pengawai negeri/BUMN sebesar $\frac{1}{0,579} = 1,727$ kali daripada mahasiswa yang belum bekerja, untuk mahasiswa yang berasal dari FMIPA sebesar $\frac{1}{0,995} = 1,005$ kali daripada mahasiswa yang berasal dari FTIF, dan untuk mahasiswa yang berasal dari FTK sebesar $\frac{1}{0,436} = 2,294$ kali daripada mahasiswa yang berasal dari FTIF.

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BAB 5
KESIMPULAN DAN SARAN

5.1 Kesimpulan

Berdasarkan hasil dan pembahasan yang telah dilakukan sebelumnya, maka dapat diperoleh beberapa kesimpulan sebagai berikut.

1. PPOM (*Partial Proportional Odds Model*) merupakan generalisasi dari POM (*Proportional Odds Model*), dimana PPOM melemahkan asumsi proporsionalitas pada beberapa variabel prediktor. Sehingga dalam estimasi parameter PPOM (*unconstrained partial proportional odds model*) terdapat parameter tambahan yakni parameter gamma yang menunjukkan penyimpangan proporsionalitas atau tidak terpenuhinya asumsi *proportional odds* pada beberapa variabel prediktor.
2. Hasil analisis regresi logistik ordinal dengan PPOM (*unconstrained partial proportional odds model*) diperoleh faktor-faktor yang berpengaruh terhadap masa studi lulusan program magister ITS yaitu nilai Toefl, jenis kelamin, kesesuaian bidang studi antara S1 dan S2, nilai IPK S1, sumber pendanaan, jenis pekerjaan, dan asal fakultas. Berdasarkan nilai *odds ratio* dapat diketahui bahwa peningkatan nilai Toefl dan IPK S1 maka kecenderungan mahasiswa lulus dengan masa studi tiga semester lebih besar dibandingkan masa studi lebih dari empat semester dan empat semester. Selain itu mahasiswa yang berjenis kelamin laki-laki dan mahasiswa yang memiliki kesesuaian bidang studi antara S1 dan S2 memiliki kecenderungan lulus dengan masa studi tiga semester daripada mahasiswa yang berjenis kelamin perempuan dan mahasiswa yang tidak memiliki kesesuaian bidang studi antara S1 dan S2. Berdasarkan sumber pendanaan yakni mahasiswa dengan biaya sendiri, beasiswa BPPS, dan beasiswa lainnya memiliki kecenderungan lulus dengan masa studi empat semester dan lebih dari empat semester daripada mahasiswa yang mendapat beasiswa dari instansi, Apabila ditinjau dari jenis pekerjaan dan asal fakultas maka mahasiswa yang bekerja sebagai pengawai negeri/BUMN serta mahasiswa yang berasal dari FMIPA

dan FTK memiliki kecenderungan lulus dengan masa studi tiga semester daripada mahasiswa yang belum bekerja dan mahasiswa yang berasal dari FTIF.

5.2 Saran

Pada penelitian ini membahas analisis regresi logistik ordinal dengan PPOM yang dibatasi hanya pada *unconstrained partial proportional odds model*, sehingga diharapkan ada penelitian selanjutnya yang mengkaji dan membahas PPOM selain *unconstrained partial proportional odds model*, misalnya *constrained partial proportional odds model* dan mengaplikasikannya pada kasus lain selain pada penelitian ini, serta mengkaji dan membahas PPOM pada model probit ordinal.

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DAFTAR LAMPIRAN

LAMPIRAN A. DATA PENELITIAN

No	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	Y
1	149,26	444	1	1	3,53	2	2	1	0	24	0	1
2	160,07	407	1	1	3,29	7	2	0	0	29	0	1
3	142,39	393	1	1	3,05	6	2	0	0	32	0	1
4	133,55	526	1	1	2,72	10	3	0	0	39	0	1
5	115,52	453	1	0	2,75	15	3	0	0	39	0	1
6	112,77	262	1	0	2,78	14	3	0	0	38	0	1
7	129,28	214	1	0	2,82	17	3	0	0	41	0	1
8	118,27	397	0	0	3,24	9	3	0	0	34	0	1
9	123,78	294	0	1	3,19	1	3	1	1	24	0	1
10	90,77	294	0	1	2,89	8	3	0	0	32	0	1
11	123,78	262	1	1	2,86	16	3	0	0	39	0	1
12	112,77	453	1	0	2,75	11	3	0	0	37	0	1
13	99,02	262	1	0	2,81	13	3	0	0	37	0	1
14	132,03	373	0	0	2,84	13	3	0	0	31	0	1
15	126,53	294	0	0	2,82	11	3	0	0	35	0	1
16	104,52	294	0	1	3,05	1	3	1	0	26	0	1
17	132,03	262	1	1	2,97	9	3	0	0	34	0	1
18	137,53	262	1	1	2,78	2	3	0	0	27	0	1
19	93,52	342	0	0	2,38	9	3	0	0	38	0	1
20	132,03	238	0	0	3,23	8	3	0	0	38	0	1
21	129,28	238	0	1	3,08	2	3	0	0	25	0	1
22	121,18	262	0	0	3,36	2	3	1	0	23	0	1
23	118,27	262	1	1	2,75	10	3	0	0	33	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1284	145,34	360	1	0	2,87	2	1	1	1	27	2	0

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Lampiran B. Fungsi *Likelihood*, Fungsi *ln-Likelihood*, Turunan Parsial Pertama, Turunan Parsial Kedua untuk *J* Kategori Variabel Respon

Fungsi *likelihood*:

$$L(\boldsymbol{\Phi}) = \prod_{i=1}^{m}\prod_{j=1}^{J}\left(\Pr(Y=j|\mathbf{x})\right)^{y_{ji}} = \prod_{i=1}^{m}\prod_{j=1}^{J}\left(\circledR_j(\mathbf{x}_i)\right)^{y_{ji}}$$

$$= \prod_{i=1}^{m}\left\{[\circledR_1(\mathbf{x}_i)]^{y_{1i}}[\circledR_2(\mathbf{x}_i)]^{y_{2i}}[\circledR_3(\mathbf{x}_i)]^{y_{3i}}\ldots[\circledR_{J-1}(\mathbf{x})]^{y_{(J-1)i}}[\circledR_J(\mathbf{x}_i)]^{\Delta_i}\right\}$$

dimana $\Delta_i = 1 - y_{1i} - y_{2i} - \ldots - y_{(J-1)i}$

Fungsi *ln-likelihood* sebagai berikut:

$$\ell(\boldsymbol{\Phi}) = \ln[L(\boldsymbol{\Phi})]$$

$$= \sum_{i=1}^{m}\sum_{j=i}^{J} y_{ji}\ln\Pr(Y=j|\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=i}^{J} y_{ji}\ln[\circledR_j(\mathbf{x}_i)]$$

$$= \sum_{i=1}^{m}\{y_{1i}\ln[\circledR_1(\mathbf{x}_i)] + y_{2i}\ln[\circledR_2(\mathbf{x}_i)] + y_{3i}\ln[\circledR_3(\mathbf{x}_i)] + y_{4i}\ln[\circledR_4(\mathbf{x}_i)] + \ldots +$$

$$y_{(J-1)i}\ln[\circledR_{J-1}(\mathbf{x}_i)] + (1 - y_1 - y_2 - \ldots - y_{J-1})\ln[\circledR_J(\mathbf{x}_i)]\}$$

$$\ell(\boldsymbol{\Phi}) = \sum_{i=1}^{m}\left\{y_{1i}\ln\left(\frac{e^{-\sim_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}}\right) + y_{2i}\ln\left(\frac{1}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}} - \frac{1}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right) +\right.$$

$$y_{3i}\ln\left(\frac{1}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}} - \frac{1}{1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right) +$$

$$y_{4i}\ln\left(\frac{1}{1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}} - \frac{1}{1+e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right) + \ldots +$$

$$\left.y_{(J-1)i}\ln\left(\frac{1}{1+e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}} - \frac{1}{1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right) + y_{Ji}\ln\left(\frac{1}{1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right)\right\}$$

$$= \sum_{i=1}^{m}\left\{y_{1i}\ln\left(\frac{e^{-\sim_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}}\right) + y_{2i}\ln\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)}\right) +\right.$$

$$y_{3i}\ln\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3} - e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)\left(1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)}\right) +$$

$$y_{4i}\ln\left(\frac{e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{\left(1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)\left(1+e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}\right)}\right)+\ldots+$$

$$y_{(J-1)i}\ln\left(\frac{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{\left(1+e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)\left(1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)}\right)+$$

$$\left.\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-\ldots-y_{(J-1)i}\right)\ln\left(\frac{1}{1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right)\right\}$$

$$=\sum_{i=1}^{m}\left\{y_{1i}\ln\left(e^{-\sim_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}\right)-y_{1i}\ln\left(1+e^{-\sim_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}\right)+y_{2i}\ln\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)-\right.$$

$$y_{2i}\ln\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)-y_{2i}\ln\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)+y_{3i}\ln\left(e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)-$$

$$y_{3i}\ln\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)-y_{3i}\ln\left(1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)+y_{4i}\ln\left(e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)-$$

$$y_{4i}\ln\left(1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)-y_{4i}\ln\left(1+e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}\right)+\ldots+$$

$$y_{(J-1)i}\ln\left(e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)-y_{(J-1)i}\ln\left(1+e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)-$$

$$y_{(J-1)i}\ln\left(1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)+\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-\ldots-y_{(J-1)i}\right)\ln(1)-$$

$$\left.\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-\ldots-y_{(J-1)i}\right)\ln\left(1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)\right\}$$

$$\ell(\boldsymbol{\Phi})=\sum_{i=1}^{m}\left\{y_{1i}\left(-\sim_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1\right)-\left(y_{1i}+y_{2i}\right)\ln\left(1+e^{-\sim_1-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}\right)-\right.$$

$$\left(y_{2i}+y_{3i}\right)\ln\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)-\left(y_{3i}+y_{4i}\right)\ln\left(1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)-\ldots-$$

$$\left(y_{(J-2)i}+y_{(J-1)i}\right)\ln\left(1+e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)+y_{2i}\ln\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)+$$

$$y_{3i}\ln\left(e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)+y_{4i}\ln\left(e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)+\ldots+$$

$$y_{(J-1)i}\ln\left(e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)-$$

$$\left.\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-\ldots-y_{(J-2)i}\right)\ln\left(1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)\right\}$$

Turunan Parsial Pertama *ln-likelihood*

$$\frac{\partial\ell(\boldsymbol{\Phi})}{\partial\sim_1}=\sum_{i=1}^{m}\left\{-y_{1i}-\left(y_{1i}+y_{2i}\right)\left(\frac{-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)\right\}$$

$$=\sum_{i=1}^{m}\left\{-y_{1i}+\left(y_{1i}+y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)\right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \sim_2} = \sum_{i=1}^{m}\left\{-\left(y_{2i}+y_{3i}\right)\left(\frac{-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)+y_{2i}\left(\frac{-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+\right.$$

$$\left.y_{3i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

$$= \sum_{i=1}^{m}\left\{\left(y_{2i}+y_{3i}\right)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)-y_{2i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+\right.$$

$$\left.y_{3i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \sim_3} = \sum_{i=1}^{m}\left\{-\left(y_{3i}+y_{4i}\right)\left(\frac{-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)+y_{3i}\left(\frac{-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)+\right.$$

$$\left.y_{4i}\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)\right\}$$

$$= \sum_{i=1}^{m}\left\{\left(y_{3i}+y_{4i}\right)\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)-y_{3i}\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)+\right.$$

$$\left.y_{4i}\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)\right\}$$

$$\vdots$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \sim_{J-1}} = \sum_{i=1}^{m}\left\{y_{(J-1)i}\left(\frac{-e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{\left(e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)}\right)-\right.$$

$$\left.\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-....-y_{(J-2)i}\right)\left(\frac{-e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{1+-e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right)\right\}$$

$$= \sum_{i=1}^{m}\left\{-y_{(J-1)i}\left(\frac{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{\left(e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)}\right)+\right.$$

$$\left.\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-....-y_{(J-2)i}\right)\left(\frac{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right)\right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\beta}} = \sum_{i=1}^{m}\left\{-y_{1i}\mathbf{x}_i^T-\left(y_{1i}+y_{2i}\right)\left(\frac{-\mathbf{x}_i^T e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)-\left(y_{2i}+y_{3i}\right)\left(\frac{-\mathbf{x}_i^T e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right.$$

$$-\left(y_{3i}+y_{4i}\right)\left(\frac{-\mathbf{x}_i^T e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)-\ldots-\left(y_{(J-2)i}+y_{(J-1)i}\right)\left(\frac{-\mathbf{x}_i^T e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{1+e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}\right)+$$

$$y_{2i}\left(\frac{-\mathbf{x}_i^T e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}+\mathbf{x}_i^T e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{3i}\left(\frac{-\mathbf{x}_i^T e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}+\mathbf{x}_i^T e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)+$$

$$y_{4i}\left(\frac{-\mathbf{x}_i^T e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}+\mathbf{x}_i^T e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)+\ldots+$$

$$y_{(J-1)i}\left(\frac{-\mathbf{x}_i^T e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}+\mathbf{x}_i^T e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}\right)-$$

$$\left.\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-\ldots.-y_{(J-2)i}\right)\left(\frac{-\mathbf{x}_i^T e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right)\right\}$$

$$=\sum_{i=1}^{m}\mathbf{x}_i^T\left\{-y_{1i}+\left(y_{1i}+y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+\left(y_{2i}+y_{3i}\right)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)+\right.$$

$$\left(y_{3i}+y_{4i}\right)\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)-\ldots+\left(y_{(J-2)i}+y_{(J-1)i}\right)\left(\frac{e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{1+e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}\right)-$$

$$y_{2i}-y_{3i}-y_{4i}-\ldots-y_{(J-1)i}+$$

$$\left.\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-\ldots.-y_{(J-2)i}\right)\left(\frac{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right)\right\}$$

$$=\sum_{i=1}^{m}\mathbf{x}_i^T\left\{-y_{1i}-y_{2i}-y_{3i}-y_{4i}+\ldots-y_{(J-1)i}+\left(y_{1i}+y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+\right.$$

$$\left(y_{2i}+y_{3i}\right)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)+\left(y_{3i}+y_{4i}\right)\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}\right)+\ldots+$$

$$\left(y_{(J-2)i}+y_{(J-1)i}\right)\left(\frac{e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{1+e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}\right)+$$

$$\left.\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-\ldots.-y_{(J-2)i}\right)\left(\frac{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}\right)\right\}$$

$$\frac{\partial\ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_1}=\sum_{i=1}^{m}\left\{-y_{1i}\mathbf{t}_i^T-\left(y_{1i}+y_{2i}\right)\left(\frac{-\mathbf{t}_i^T e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\left(\frac{\mathbf{t}_i^T e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)\right\}$$

$$= \sum_{i=1}^{m} \mathbf{t}_i^T \left\{ -y_{1i} + \left(y_{1i} + y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}} \right) + y_{2i}\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}} \right) \right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_2} = \sum_{i=1}^{m} \left\{ -\left(y_{2i} + y_{3i}\right)\left(\frac{-\mathbf{t}_i^T e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}} \right) + y_{2i}\left(\frac{-\mathbf{t}_i^T e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}} \right) \right.$$

$$\left. + y_{3i}\left(\frac{\mathbf{t}_i^T e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3} - e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}} \right) \right\}$$

$$= \sum_{i=1}^{m} \mathbf{t}_i^T \left\{ \left(y_{2i} + y_{3i}\right)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}} \right) - y_{2i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}} \right) + \right.$$

$$\left. y_{3i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3} - e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}} \right) \right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_3} = \sum_{i=1}^{m} \left\{ -\left(y_{3i} + y_{4i}\right)\left(\frac{-\mathbf{t}_i^T e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}} \right) + y_{3i}\left(\frac{-\mathbf{t}_i^T e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3} - e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}} \right) + \right.$$

$$\left. y_{4i}\left(\frac{\mathbf{t}_i^T e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4} - e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}} \right) \right\}$$

$$= \sum_{i=1}^{m} \mathbf{t}_i^T \left\{ \left(y_{3i} + y_{4i}\right)\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}} \right) - y_{3i}\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3} - e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}} \right) + \right.$$

$$\left. y_{4i}\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4} - e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}} \right) \right\}$$

$$\vdots$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_{J-2}} = \sum_{i=1}^{m} \left\{ -\left(y_{(J-2)i} + y_{(J-1)i}\right)\left(\frac{-\mathbf{t}_i^T e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{1+e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}} \right) + y_{(J-2)i}\left(\frac{-\mathbf{t}_i^T e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}} - e^{-\sim_{J-3}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-3}}} \right) + \right.$$

$$\left. y_{(J-1)i}\left(\frac{\mathbf{t}_i^T e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}} - e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}} \right) \right\}$$

$$= \sum_{i=1}^{m} \mathbf{t}_i^T \left\{ \left(y_{(J-2)i} + y_{(J-1)i}\right)\left(\frac{e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{1+e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}} \right) - y_{(J-2)i}\left(\frac{e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}} - e^{-\sim_{J-3}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-3}}} \right) + \right.$$

$$\left. y_{(J-1)i}\left(\frac{e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}} - e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}} \right) \right\}$$

$$\frac{\partial \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\gamma}_{J-1}} = \sum_{i=1}^{m}\left\{ y_{(J-1)i}\left(\frac{-\mathbf{t}_i^T e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{\left(e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}} - e^{-\theta_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)} \right) - \right.$$

$$\left. \left(1 - y_{1i} - y_{2i} - y_{3i} - y_{4i} - \ldots . - y_{(J-2)i}\right)\left(\frac{-\mathbf{t}_i^T e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{1 + e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}} \right) \right\}$$

$$= \sum_{i=1}^{m}\mathbf{t}_i^T\left\{ -y_{(J-1)i}\left(\frac{e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{\left(e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}} - e^{-\theta_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)} \right) + \right.$$

$$\left. \left(1 - y_{1i} - y_{2i} - y_{3i} - y_{4i} - \ldots . - y_{(J-2)i}\right)\left(\frac{e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{1 + e^{-\theta_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}} \right) \right\}$$

Turunan Parsial Kedua *ln-likelihood*

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \theta_1^2} = \sum_{i=1}^{m}\left\{ \left(y_{1i} + y_{2i}\right)\left(-\frac{e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1 + e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) + y_{2i}\left(-\frac{e^{-\theta_1-\theta_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) \right\}$$

$$= \sum_{i=1}^{m}\left\{ -\left(y_{1i} + y_{2i}\right)\left(\frac{e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1 + e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) - y_{2i}\left(\frac{e^{-\theta_1-\theta_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) \right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \theta_1 \partial \theta_2} = \sum_{i=1}^{m} y_{2i}\left(\frac{e^{-\theta_2-\theta_1-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right)$$

dan $\dfrac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \theta_1 \partial \theta_j} = 0$, j = 3, 4, …, *J-1*

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \theta_2^2} = \sum_{i=1}^{m}\left\{ -\left(y_{2i} + y_{3i}\right)\left(\frac{e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1 + e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right) - y_{2i}\left(\frac{e^{-\theta_1-\theta_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) - \right.$$

$$\left. y_{3i}\left(\frac{e^{-\theta_2-\theta_3-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{\left(e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\theta_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) \right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \theta_2 \partial \theta_3} = \sum_{i=1}^{m} y_{3i}\left(\frac{e^{-\theta_3-\theta_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3} - e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right)$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \theta_3^2} = \sum_{i=1}^{m}\left\{ \left(y_{3i} + y_{4i}\right)\left(-\frac{e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{\left(1 + e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)^2} \right) - y_{3i}\left(\frac{e^{-\theta_3-\theta_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\theta_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3} - e^{-\theta_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right) + \right.$$

$$y_{4i}\left(-\frac{e^{-\sim_3-\sim_4-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3-\mathbf{t}_i^T\boldsymbol{\gamma}_4}}{\left(e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)^2}\right)\Bigg\}$$

$$=\sum_{i=1}^{m}\Bigg\{-\left(y_{3i}+y_{4i}\right)\left(\frac{e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}}{\left(1+e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)^2}\right)-y_{3i}\left(\frac{e^{-\sim_3-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)-$$

$$y_{4i}\left(\frac{e^{-\sim_3-\sim_4-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3-\mathbf{t}_i^T\boldsymbol{\gamma}_4}}{\left(e^{-\sim_4-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_4}-e^{-\sim_3-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)^2}\right)\Bigg\}$$

$$\vdots$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_j^2}=\sum_{i=1}^{m}\Bigg\{-\left(y_{ji}+y_{(j+1)i}\right)\frac{e^{-\sim_j-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_j}}{\left(1+e^{-\sim_j-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_j}\right)^2}-y_{ji}\frac{e^{-\sim_j-\sim_{j-1}-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_j-\mathbf{t}_i^T\boldsymbol{\gamma}_{j-1}}}{\left(e^{-\sim_j-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_j}-e^{-\sim_{j-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{j-1}}\right)^2}-$$

$$y_{(j+1)i}\left(\frac{e^{-\sim_j-\sim_{j+1}-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_j-\mathbf{t}_i^T\boldsymbol{\gamma}_{j+1}}}{\left(e^{-\sim_{j+1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{j+1}}-e^{-\sim_j-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_j}\right)^2}\right)\Bigg\}\quad,\quad j=2,3,\ldots,J\text{-}2$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_{J-1}^2}=\sum_{i=1}^{m}\Bigg\{-y_{(J-1)i}\left(\frac{e^{-\sim_{J-1}-\sim_{J-2}-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{\left(e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}-e^{-\sim_{J-2}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)^2}\right)-$$

$$\left(1-y_{1i}-y_{2i}-y_{3i}-y_{4i}-\ldots-y_{(J-2)i}\right)\left(\frac{e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{\left(1+e^{-\sim_{J-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)^2}\right)\Bigg\}$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_j\partial\sim_{j-1}}=\sum_{i=1}^{m}y_{ji}\left(\frac{e^{-\sim_j-\sim_{j-1}-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_j-\mathbf{t}_i^T\boldsymbol{\gamma}_{j-1}}}{\left(e^{-\sim_j-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_j}-e^{-\sim_{j-1}-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_{j-1}}\right)^2}\right)\quad,\ j=2,3,\ldots,J\text{-}1$$

$$\text{dan }\ \frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_n\partial\sim_j}=0\,,\quad |n-j|\geq 2$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_1\partial\boldsymbol{\beta}}=\sum_{i=1}^{m}-\mathbf{x}_i^T\left(y_{1i}+y_{2i}\right)\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_2\partial\boldsymbol{\beta}}=\sum_{i=1}^{m}-\mathbf{x}_i^T\left(y_{2i}+y_{3i}\right)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)$$

$$\vdots$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_j \partial \boldsymbol{\beta}} = \sum_{i=1}^{m} -\mathbf{x}_i^T \left(y_{ji} + y_{(j+1)i}\right)\left(\frac{e^{-\sim_j - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_j}}{\left(1+e^{-\sim_j - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_j}\right)^2}\right) \quad , j = 1, 2, \ldots, J\text{-}2$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_{J-1} \partial \boldsymbol{\beta}} = \sum_{i=1}^{m} -\mathbf{x}_i^T \left(1 - y_{1i} - y_{2i} - y_{3i} - y_{4i} - \ldots - y_{(J-2)i}\right)\left(\frac{e^{-\sim_{J-1} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{\left(1+e^{-\sim_{J-1} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)^2}\right)$$

$$\begin{aligned}\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^T} = \sum_{i=1}^{m} \mathbf{x}_i \mathbf{x}_i^T \Bigg\{ & \left(-y_{1i} - y_{2i}\right)\left(\frac{e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right) - \left(y_{2i} + y_{3i}\right)\left(\frac{e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right) - \\ & \left(y_{3i} + y_{4i}\right)\left(\frac{e^{-\sim_3 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_3}}{\left(1+e^{-\sim_3 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_3}\right)^2}\right) - \ldots - \left(y_{(J-2)i} + y_{(J-1)i}\right)\left(\frac{e^{-\sim_{J-2} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}}{\left(1+e^{-\sim_{J-2} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-2}}\right)^2}\right) - \\ & \left(1 - y_{1i} - y_{2i} - y_{3i} - y_{4i} - \ldots - y_{(J-2)i}\right)\left(\frac{e^{-\sim_{J-1} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}}{\left(1+e^{-\sim_{J-1} - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_{J-1}}\right)^2}\right)\Bigg\}\end{aligned}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_1 \partial \boldsymbol{\gamma}_1} = \sum_{i=1}^{m} \mathbf{t}_i^T \left\{ -\left(y_{1i} + y_{2i}\right)\left(\frac{e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right) - y_{2i}\left(\frac{e^{-\sim_1 - \sim_2 - 2\mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1 - \mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)\right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_1 \partial \boldsymbol{\gamma}_2} = \sum_{i=1}^{m} \mathbf{t}_i^T y_{2i}\left(\frac{e^{-\sim_2 - \sim_1 - 2\mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2 - \mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

dan $\dfrac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_1 \partial \boldsymbol{\gamma}_j} = 0 \quad , j = 3, 4, \ldots, J\text{-}1$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_2 \partial \boldsymbol{\gamma}_1} = \sum_{i=1}^{m} \mathbf{t}_i^T y_{2i}\left(\frac{e^{-\sim_1 - \sim_2 - 2\mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1 - \mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

$$\begin{aligned}\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_2 \partial \boldsymbol{\gamma}_2} = \sum_{i=1}^{m} \mathbf{t}_i^T \Bigg\{ & -\left(y_{2i} + y_{3i}\right)\left(\frac{e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right) - y_{2i}\left(\frac{e^{-\sim_2 - \sim_1 - 2\mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2 - \mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right) - \\ & y_{3i}\left(\frac{e^{-\sim_2 - \sim_3 - 2\mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2 - \mathbf{t}_i^T\boldsymbol{\gamma}_3}}{\left(e^{-\sim_3 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_3} - e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)\Bigg\}\end{aligned}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial \sim_2 \partial \boldsymbol{\gamma}_3} = \sum_{i=1}^{m} \mathbf{t}_i^T y_{3i}\left(\frac{e^{-\sim_3 - \sim_2 - 2\mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_3 - \mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_3 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_3} - e^{-\sim_2 - \mathbf{x}_i^T\boldsymbol{\beta} - \mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)$$

$\vdots$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_{j-1} \partial \boldsymbol{\gamma}_j} = \sum_{i=1}^{m} \mathbf{t}_i^T y_{ji} \left(\frac{e^{-\sim_j - \sim_{j-1} - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}}{\left(e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j} - e^{-\sim_{j-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}} \right)^2} \right) \quad , j = 2, 3, \ldots, J\text{-}1$$

dan $\dfrac{\partial^2 \ell(\mathbf{\Phi})}{\partial \sim_n \partial \boldsymbol{\gamma}_j} = 0, \quad |n - j| \geq 2$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_1 \partial \boldsymbol{\gamma}_1^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T \left\{ -(y_{1i} + y_{2i}) \left(\frac{e^{-\sim_1 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1}}{\left(1 + e^{-\sim_1 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1}\right)^2} \right) - y_{2i} \left(\frac{e^{-\sim_1 - \sim_2 - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1 - \mathbf{t}_i^T \boldsymbol{\gamma}_2}}{\left(e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1} \right)^2} \right) \right\}$$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_1 \partial \boldsymbol{\gamma}_2^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T y_{2i} \left(\frac{e^{-\sim_2 - \sim_1 - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2 - \mathbf{t}_i^T \boldsymbol{\gamma}_1}}{\left(e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1} \right)^2} \right)$$

dan $\dfrac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_1 \partial \boldsymbol{\gamma}_j^T} = 0 \quad , j = 3, 4, \ldots, J\text{-}1$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_2 \partial \boldsymbol{\gamma}_2^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T \left\{ -(y_{2i} + y_{3i}) \left(\frac{e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2}}{\left(1 + e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2}\right)^2} \right) - y_{2i} \left(\frac{e^{-\sim_2 - \sim_1 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2 - \mathbf{t}_i^T \boldsymbol{\gamma}_1}}{\left(e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2} - e^{-\sim_1 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_1} \right)^2} \right) - \right.$$
$$\left. y_{3i} \left(\frac{e^{-\sim_2 - \sim_3 - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2 - \mathbf{t}_i^T \boldsymbol{\gamma}_3}}{\left(e^{-\sim_3 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3} - e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2} \right)^2} \right) \right\}$$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_2 \partial \boldsymbol{\gamma}_3^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T y_{3i} \left(\frac{e^{-\sim_3 - \sim_2 - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3 - \mathbf{t}_i^T \boldsymbol{\gamma}_2}}{\left(e^{-\sim_3 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3} - e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2} \right)^2} \right)$$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_3 \partial \boldsymbol{\gamma}_3^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T \left\{ -(y_{3i} + y_{4i}) \left(\frac{e^{-\sim_3 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3}}{\left(1 + e^{-\sim_3 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3}\right)^2} \right) - y_{3i} \left(\frac{e^{-\sim_3 - \sim_2 - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3 - \mathbf{t}_i^T \boldsymbol{\gamma}_2}}{\left(e^{-\sim_3 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3} - e^{-\sim_2 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_2} \right)^2} \right) - \right.$$
$$\left. y_{4i} \left(\frac{e^{-\sim_3 - \sim_4 - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3 - \mathbf{t}_i^T \boldsymbol{\gamma}_4}}{\left(e^{-\sim_4 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_4} - e^{-\sim_3 - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_3} \right)^2} \right) \right\}$$

$\vdots$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_{j-1} \partial \boldsymbol{\gamma}_j^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T y_{ji} \left(\frac{e^{-\sim_j - \sim_{j-1} - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}}{\left(e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j} - e^{-\sim_{j-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}} \right)^2} \right) \quad , j = 2, 3, \ldots, J\text{-}1$$

dan $\dfrac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_n \partial \boldsymbol{\gamma}_j^T} = 0 \quad , |n - j| \geq 2$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_j \partial \boldsymbol{\gamma}_j^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T \left\{ -\left(y_{ji} + y_{(j+1)i}\right) \left(\frac{e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j}}{\left(1 + e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j}\right)^2} \right) - y_{ji} \left(\frac{e^{-\sim_j - \sim_{j-1} - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}}{\left(e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j} - e^{-\sim_{j-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}\right)^2} \right) - \right.$$

$$\left. y_{(j+1)i} \left(\frac{e^{-\sim_j - \sim_{j+1} - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j - \mathbf{t}_i^T \boldsymbol{\gamma}_{j+1}}}{\left(e^{-\sim_{j+1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j+1}} - e^{-\sim_j - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_j}\right)^2} \right) \right\} \quad , \; j = 2, 3, \ldots, J\text{-}2$$

$$\frac{\partial^2 \ell(\mathbf{\Phi})}{\partial \boldsymbol{\gamma}_{J-1} \partial \boldsymbol{\gamma}_{J-1}^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T \left\{ - y_{(j-1)i} \left(\frac{e^{-\sim_{j-1} - \sim_{j-2} - 2\mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-2}}}{\left(e^{-\sim_{j-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}} - e^{-\sim_{j-2} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-2}}\right)^2} \right) - \right.$$

$$\left. \left(1 - y_{1i} - y_{2i} - \ldots - y_{(j-1)i}\right) \left(\frac{e^{-\sim_{j-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}}{1 + e^{-\sim_{j-1} - \mathbf{x}_i^T \boldsymbol{\beta} - \mathbf{t}_i^T \boldsymbol{\gamma}_{j-1}}} \right) \right\}$$

Lampiran C. Fungsi *Likelihood*, Fungsi *ln-Likelihood*, Turunan Parsial Pertama, Turunan Parsial Kedua untuk 3 Kategori Variabel Respon

Fungsi *likelihood*:

$$L(\mathbf{\Phi}) = \prod_{i=1}^{m}\prod_{j=1}^{J}\left(\Pr\left(Y = j|\mathbf{x}\right)\right)^{y_{ji}} = \prod_{i=1}^{m}\prod_{j=1}^{J}\left(\boldsymbol{\pi}_j\left(\mathbf{x}_i\right)\right)^{y_{ji}}$$

$$= \prod_{i=1}^{m}\left\{\left[\circledR_1\left(\mathbf{x}_i\right)\right]^{y_{1i}}\left[\circledR_2\left(\mathbf{x}_i\right)\right]^{y_{2i}}\left[\circledR_3\left(\mathbf{x}_i\right)\right]^{y_{3i}}\ldots\left[\circledR_{J-1}\left(\mathbf{x}\right)\right]^{y_{(J-1)i}}\left[\circledR_J\left(\mathbf{x}_i\right)\right]^{1-y_{1i}-y_{2i}-\ldots-y_{(J-1)i}}\right\}$$

Fungsi *ln-likelihood*:

$$\ell(\mathbf{\Phi}) = \sum_{i=1}^{m}\left\{y_{1i}\ln\left(\circledR_1\right) + y_{2i}\ln\left(\circledR_2\right) + y_{3i}\ln\left(\circledR_3\right)\right\}$$

$$= \sum_{i=1}^{m}\left\{y_{1i}\ln\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right) + y_{2i}\ln\left(\frac{1}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}} - \frac{1}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right) + \right.$$

$$\left. y_{3i}\ln\left(\frac{1}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

$$= \sum_{i=1}^{m}\left\{y_{1i}\ln\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right) + y_{2i}\ln\left(\frac{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - \left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)}\right) + \right.$$

$$\left. y_{3i}\ln\left(\frac{1}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

$$= \sum_{i=1}^{m}\left\{y_{1i}\left(-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1\right) - y_{1i}\ln\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right) + y_{2i}\ln\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right) - \right.$$

$$\left. y_{2i}\ln\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right) - y_{2i}\ln\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right) + y_{3i}\ln(1) - y_{3i}\ln\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)\right\}$$

$$= \sum_{i=1}^{m}\left\{y_{1i}\left(\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1\right) - y_{1i}\ln\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right) + y_{2i}\ln\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right) - \right.$$

$$\left. y_{2i}\ln\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right) - y_{2i}\ln\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right) + -\left(1-y_{1i}-y_{2i}\right)\ln\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)\right\}$$

$$\ell(\mathbf{\Phi}) = \sum_{i=1}^{m}\left\{y_{1i}\left(-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1\right) - \left(y_{1i}+y_{2i}\right)\ln\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right) + \right.$$

$$\left. y_{2i}\ln\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right) + \left(y_{1i}-1\right)\ln\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)\right\}$$

Turunan Parsial Pertama *ln-likelihood*

$$\frac{\partial\ell(\boldsymbol{\Phi})}{\partial\sim_1}=\sum_{i=1}^{m}\left\{-y_{1i}-(y_{1i}+y_{2i})\left(\frac{-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)\right\}$$

$$=\sum_{i=1}^{m}\left\{-y_{1i}+(y_{1i}+y_{2i})\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)\right\}$$

$$\frac{\partial\ell(\boldsymbol{\Phi})}{\partial\sim_2}=\sum_{i=1}^{m}\left\{y_{2i}\left(\frac{-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+(y_{1i}-1)\left(\frac{-e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

$$=\sum_{i=1}^{m}\left\{-y_{2i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)-(y_{1i}-1)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

$$\frac{\partial\ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\beta}}=\sum_{i=1}^{m}\left\{-y_{1i}\mathbf{x}_i^T-(y_{1i}+y_{2i})\left(\frac{-\mathbf{x}_i^Te^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+\right.$$

$$\left.y_{2i}\left(\frac{-\mathbf{x}_i^Te^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}+\mathbf{x}_i^Te^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+(y_{1i}-1)\left(\frac{-\mathbf{x}_i^Te^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

$$=\sum_{i=1}^{m}\mathbf{x}_i^T\left\{-y_{1i}-y_{2i}+(y_{1i}+y_{2i})\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)-(y_{1i}-1)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

$$\frac{\partial\ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_1}=\sum_{i=1}^{m}\left\{-y_{1i}\mathbf{t}_i^T-(y_{1i}+y_{2i})\left(\frac{-\mathbf{t}_i^T\left(e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\left(\frac{-\mathbf{t}_i^T\left(-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)\right\}$$

$$=\sum_{i=1}^{m}\mathbf{t}_i^T\left(-y_{1i}+(y_{1i}+y_{2i})\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+y_{2i}\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)\right)$$

$$\frac{\partial\ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_2}=\sum_{i=1}^{m}\left\{y_{2i}\left(\frac{-\mathbf{t}_i^Te^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)+(y_{1i}-1)\left(\frac{-\mathbf{t}_i^Te^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

$$=\sum_{i=1}^{m}\mathbf{t}_i^T\left\{-y_{2i}\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}\right)-(y_{1i}-1)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}\right)\right\}$$

Turunan Parsial Kedua Fungsi *ln-likelihood*

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_1^2}=\sum_{i=1}^{m}\left\{(y_{1i}+y_{2i})\left(-\frac{\left(e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)+y_{2i}\left(-\frac{\left(e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)\right\}$$

$$=\sum_{i=1}^{m}\left\{-(y_{1i}+y_{2i})\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)-y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)\right\}$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_1\partial\sim_2}=\sum_{i=1}^{m}y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_2}-e^{-\sim_1-x^T\boldsymbol{\beta}-\mathbf{t}^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_1\partial\boldsymbol{\beta}}=\sum_{i=1}^{m}\mathbf{x}_i^T(-y_{1i}-y_{2i})\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

$$\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\sim_1\partial\boldsymbol{\gamma}_1}=\sum_{i=1}^{m}\mathbf{t}_i^T\left((y_{1i}+y_{2i})\left(-\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)+y_{2i}\left(-\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)\right)$$

$$=\sum_{i=1}^{m}\mathbf{t}_i^T\left((-y_{1i}-y_{2i})\left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)-y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)\right)$$

$$\frac{\partial^2 l(\boldsymbol{\Phi})}{\partial\sim_1\partial\boldsymbol{\gamma}_2}=\sum_{i=1}^{m}\mathbf{t}_i^T y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_2^2}=\sum_{i=1}^{m}\left\{-y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)-(y_{1i}-1)\left(-\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)\right\}$$

$$=\sum_{i=1}^{m}\left\{-y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)+(y_{1i}-1)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)\right\}$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_2\partial\boldsymbol{\beta}}=\sum_{i=1}^{m}\mathbf{x}_i^T(y_{1i}-1)\left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)$$

$$\frac{\partial^2\ell(\boldsymbol{\Phi})}{\partial\sim_2\partial\boldsymbol{\gamma}_1}=\sum_{i=1}^{m}\mathbf{t}_i^T y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)$$

$$\frac{\partial^2 l(\boldsymbol{\Phi})}{\partial\sim_2\partial\boldsymbol{\gamma}_2}=\sum_{i=1}^{m}\mathbf{t}_i^T\left\{-y_{2i}\left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}-e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2}\right)-(y_{1i}-1)\left(-\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2}\right)\right\}$$

$$= \sum_{i=1}^{m} \mathbf{t}_i^T \left\{ -y_{2i} \left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) + \left(y_{1i}-1\right) \left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right) \right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\beta}\partial\boldsymbol{\beta}^T} = \sum_{i=1}^{m} \mathbf{x}_i \left\{ \left(y_{1i}+y_{2i}\right) \left(-\frac{\mathbf{x}_i^T e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) - \left(y_{1i}-1\right) \left(-\frac{\mathbf{x}_i^T e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right) \right\}$$

$$= \sum_{i=1}^{m} \mathbf{x}_i \mathbf{x}_i^T \left\{ \left(-y_{1i}-y_{2i}\right) \left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) + \left(y_{1i}-1\right) \left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right) \right\}$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\beta}\partial\boldsymbol{\gamma}_1^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{x}_i^T \left(-y_{1i}-y_{2i}\right) \left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right)$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\beta}\partial\boldsymbol{\gamma}_2^T} = \sum_{i=1}^{m} \mathbf{x}_i^T \mathbf{t}_i \left(y_{1i}-1\right) \left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right)$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_1\partial\boldsymbol{\gamma}_1^T} = \sum_{i=1}^{m} \mathbf{t}_i \left(\left(y_{1i}+y_{2i}\right) \left(-\frac{\mathbf{t}_i^T e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) + y_{2i} \left(-\frac{\mathbf{t}_i^T e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) \right)$$

$$= \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T \left(\left(-y_{1i}-y_{2i}\right) \left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) - y_{2i} \left(\frac{e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) \right)$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_1\partial\boldsymbol{\gamma}_2^T} = \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T y_{2i} \left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right)$$

$$\frac{\partial^2 \ell(\boldsymbol{\Phi})}{\partial\boldsymbol{\gamma}_2\partial\boldsymbol{\gamma}_2^T} = \sum_{i=1}^{m} \mathbf{t}_i \left\{ -y_{2i} \left(\frac{\mathbf{t}_i^T e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) - \left(y_{1i}-1\right) \left(-\frac{\mathbf{t}_i^T e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right) \right\}$$

$$= \sum_{i=1}^{m} \mathbf{t}_i \mathbf{t}_i^T \left\{ -y_{2i} \left(\frac{e^{-\sim_1-\sim_2-2\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2} - e^{-\sim_1-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_1}\right)^2} \right) + \left(y_{1i}-1\right) \left(\frac{e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{-\sim_2-\mathbf{x}_i^T\boldsymbol{\beta}-\mathbf{t}_i^T\boldsymbol{\gamma}_2}\right)^2} \right) \right\}$$

LAMPIRAN D : OUTPUT ANALISIS DATA (mengeluarkan X_1)

```
. ologit Y X1 X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X8_1 X9_1 X9_2 X10 X11_1 X11_2
  X11_3 X11_4

Iteration 0:   log likelihood = -1185.3939
Iteration 1:   log likelihood = -1104.0195
Iteration 2:   log likelihood = -1102.1739
Iteration 3:   log likelihood = -1102.1677
Iteration 4:   log likelihood = -1102.1677

Ordered logistic regression                       Number of obs   =       1284
                                                  LR chi2(17)     =     166.45
                                                  Prob > chi2     =     0.0000
Log likelihood = -1102.1677                       Pseudo R2       =     0.0702

------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          X1 |   .0032487    .002196     1.48   0.139    -.0010554    .0075528
          X2 |   .0023246   .0009751     2.38   0.017     .0004135    .0042357
        X3_1 |   .2708682   .1260774     2.15   0.032     .0237611    .5179753
        X4_1 |    .315383   .1273709     2.48   0.013     .0657406    .5650255
          X5 |   1.161923   .2151769     5.40   0.000     .7401845    1.583662
          X6 |   .0236657   .0265958     0.89   0.374    -.0284611    .0757924
        X7_1 |   .3705851   .2305801     1.61   0.108    -.0813436    .8225138
        X7_2 |   -.065294   .1712124    -0.38   0.703    -.4008641    .2702761
        X7_4 |   .6100024   .1931305     3.16   0.002     .2314735    .9885312
        X8_1 |   .0747022   .1562464     0.48   0.633     -.231535    .3809394
        X9_1 |   .4231663    .236101     1.79   0.073    -.0395832    .8859158
        X9_2 |   .0733912   .2217645     0.33   0.741    -.3612592    .5080416
         X10 |    .000146   .0214556     0.01   0.995    -.0419061    .0421982
       X11_1 |   1.015143   .2467069     4.11   0.000     .5316069     1.49868
       X11_2 |   .4263795   .2319125     1.84   0.066    -.0281605    .8809196
       X11_3 |  -.2679512   .2550994    -1.05   0.294    -.7679368    .2320343
       X11_4 |   .8754772   .2992577     2.93   0.003      .288943    1.462011
-------------+----------------------------------------------------------------
       /cut1 |   5.469699   1.103109                      3.307646    7.631752
       /cut2 |   8.691161   1.124621                      6.486945    10.89538
------------------------------------------------------------------------------

. brant, detail

Brant Test of Parallel Regression Assumption

    Variable |      chi2   p>chi2    df
-------------+--------------------------
         All |     83.77    0.000    17
-------------+--------------------------
          X1 |     10.94    0.001     1
          X2 |      0.17    0.676     1
        X3_1 |      2.29    0.130     1
        X4_1 |      0.41    0.521     1
          X5 |      1.83    0.176     1
          X6 |      0.00    0.990     1
        X7_1 |      5.59    0.018     1
        X7_2 |     34.51    0.000     1
        X7_4 |     17.45    0.000     1
        X8_1 |      0.06    0.805     1
        X9_1 |      1.13    0.287     1
        X9_2 |      0.01    0.909     1
         X10 |      0.49    0.484     1
       X11_1 |      6.09    0.014     1
       X11_2 |      0.36    0.550     1
       X11_3 |      0.28    0.594     1
       X11_4 |      0.04    0.842     1
----------------------------------------
```

```
. fitstat

Measures of Fit for ologit of Y

Log-Lik Intercept Only:      -1185.394   Log-Lik Full Model:          -1102.168
D(1265):                      2204.335   LR(17):                        166.452
                                         Prob > LR:                       0.000
McFadden's R2:                   0.070   McFadden's Adj R2:               0.054
ML (Cox-Snell) R2:               0.122   Cragg-Uhler(Nagelkerke) R2:      0.144
McKelvey & Zavoina's R2:         0.145
Variance of y*:                  3.849   Variance of error:               3.290
Count R2:                        0.609   Adj Count R2:                    0.027
AIC:                             1.746   AIC*n:                        2242.335
BIC:                         -6850.200   BIC':                          -44.771
BIC used by Stata:            2340.332   AIC used by Stata:            2242.335


. gologit2  Y X1 X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X8_1 X9_1 X9_2 X10 X11_1 X11_2
  X11_3 X11_4, autofit gamma lrf

------------------------------------------------------------------------------
Testing parallel lines assumption using the .05 level of significance...

Step  1:  Constraints for parallel lines imposed for X2 (P Value = 0.9632)
Step  2:  Constraints for parallel lines imposed for X6 (P Value = 0.8475)
Step  3:  Constraints for parallel lines imposed for X11_4 (P Value = 0.8363)
Step  4:  Constraints for parallel lines imposed for X9_2 (P Value = 0.8453)
Step  5:  Constraints for parallel lines imposed for X8_1 (P Value = 0.6482)
Step  6:  Constraints for parallel lines imposed for X11_3 (P Value = 0.4432)
Step  7:  Constraints for parallel lines imposed for X4_1 (P Value = 0.4386)
Step  8:  Constraints for parallel lines imposed for X11_2 (P Value = 0.4903)
Step  9:  Constraints for parallel lines imposed for X5 (P Value = 0.2601)
Step  10: Constraints for parallel lines imposed for X3_1 (P Value = 0.2177)
Step  11: Constraints for parallel lines imposed for X9_1 (P Value = 0.0560)
Step  12: Constraints for parallel lines imposed for X7_1 (P Value = 0.0587)
Step  13: Constraints for parallel lines are not imposed for
          X1 (P Value = 0.00151)
          X7_2 (P Value = 0.00000)
          X7_4 (P Value = 0.00083)
          X10 (P Value = 0.00425)
          X11_1 (P Value = 0.00000)

Wald test of parallel lines assumption for the final model:

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X6 - [4_semester]X6 = 0
 ( 3)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 4)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 5)  [>4_semester]X8_1 - [4_semester]X8_1 = 0
 ( 6)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 7)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 8)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 9)  [>4_semester]X5 - [4_semester]X5 = 0
 (10)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 (11)  [>4_semester]X9_1 - [4_semester]X9_1 = 0
 (12)  [>4_semester]X7_1 - [4_semester]X7_1 = 0

           chi2( 12) =   12.34
         Prob > chi2 =    0.4184


pl(X2 X6 X11_4 X9_2 X8_1 X11_3 X4_1 X11_2 X5 X3_1 X9_1 X7_1)
------------------------------------------------------------------------------

Generalized Ordered Logit Estimates              Number of obs   =       1284
                                                 LR chi2(22)     =     235.42
                                                 Prob > chi2     =     0.0000
Log likelihood = -1067.6857                      Pseudo R2       =     0.0993

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X6 - [4_semester]X6 = 0
 ( 3)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
```

(4) [>4_semester]X9_2 - [4_semester]X9_2 = 0
(5) [>4_semester]X8_1 - [4_semester]X8_1 = 0
(6) [>4_semester]X11_3 - [4_semester]X11_3 = 0
(7) [>4_semester]X4_1 - [4_semester]X4_1 = 0
(8) [>4_semester]X11_2 - [4_semester]X11_2 = 0
(9) [>4_semester]X5 - [4_semester]X5 = 0
(10) [>4_semester]X3_1 - [4_semester]X3_1 = 0
(11) [>4_semester]X9_1 - [4_semester]X9_1 = 0
(12) [>4_semester]X7_1 - [4_semester]X7_1 = 0

Y	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
>4_semester						
X1	-.0013955	.0025458	-0.55	0.584	-.0063853	.0035942
X2	.0026436	.0010021	2.64	0.008	.0006795	.0046076
X3_1	.256097	.1295624	1.98	0.048	.0021593	.5100347
X4_1	.3328007	.1301939	2.56	0.011	.0776253	.587976
X5	1.16893	.2186903	5.35	0.000	.7403053	1.597555
X6	.0228157	.0264376	0.86	0.388	-.029001	.0746323
X7_1	.3368117	.2247812	1.50	0.134	-.1037514	.7773747
X7_2	.3315182	.1899346	1.75	0.081	-.0407468	.7037832
X7_4	.9863335	.2380848	4.14	0.000	.5196959	1.452971
X8_1	.0540334	.1582932	0.34	0.733	-.2562156	.3642824
X9_1	.4577791	.2362133	1.94	0.053	-.0051905	.9207487
X9_2	.0909087	.2196146	0.41	0.679	-.3395281	.5213455
X10	-.0165933	.0219647	-0.76	0.450	-.0596434	.0264568
X11_1	1.529488	.2707881	5.65	0.000	.9987534	2.060223
X11_2	.4163849	.2275166	1.83	0.067	-.0295394	.8623093
X11_3	-.2300493	.2507329	-0.92	0.359	-.7214768	.2613782
X11_4	.8071347	.2923746	2.76	0.006	.234091	1.380178
_cons	-4.802937	1.131667	-4.24	0.000	-7.020963	-2.584911
4_semester						
X1	.0097919	.0032309	3.03	0.002	.0034594	.0161244
X2	.0026436	.0010021	2.64	0.008	.0006795	.0046076
X3_1	.256097	.1295624	1.98	0.048	.0021593	.5100347
X4_1	.3328007	.1301939	2.56	0.011	.0776253	.587976
X5	1.16893	.2186903	5.35	0.000	.7403053	1.597555
X6	.0228157	.0264376	0.86	0.388	-.029001	.0746323
X7_1	.3368117	.2247812	1.50	0.134	-.1037514	.7773747
X7_2	-1.108413	.2658594	-4.17	0.000	-1.629488	-.5873385
X7_4	.0500376	.2542627	0.20	0.844	-.4483082	.5483833
X8_1	.0540334	.1582932	0.34	0.733	-.2562156	.3642824
X9_1	.4577791	.2362133	1.94	0.053	-.0051905	.9207487
X9_2	.0909087	.2196146	0.41	0.679	-.3395281	.5213455
X10	.0267049	.0237322	1.13	0.260	-.0198093	.0732192
X11_1	.1442592	.2855822	0.51	0.613	-.4154717	.70399
X11_2	.4163849	.2275166	1.83	0.067	-.0295394	.8623093
X11_3	-.2300493	.2507329	-0.92	0.359	-.7214768	.2613782
X11_4	.8071347	.2923746	2.76	0.006	.234091	1.380178
_cons	-9.810482	1.220197	-8.04	0.000	-12.20203	-7.418939

Alternative parameterization: Gammas are deviations from proportionality

Y	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
Beta						
X1	-.0013955	.0025458	-0.55	0.584	-.0063853	.0035942
X2	.0026436	.0010021	2.64	0.008	.0006795	.0046076
X3_1	.256097	.1295624	1.98	0.048	.0021593	.5100347
X4_1	.3328007	.1301939	2.56	0.011	.0776253	.587976
X5	1.16893	.2186903	5.35	0.000	.7403053	1.597555
X6	.0228157	.0264376	0.86	0.388	-.029001	.0746323
X7_1	.3368117	.2247812	1.50	0.134	-.1037514	.7773747
X7_2	.3315182	.1899346	1.75	0.081	-.0407468	.7037832
X7_4	.9863335	.2380848	4.14	0.000	.5196959	1.452971
X8_1	.0540334	.1582932	0.34	0.733	-.2562156	.3642824
X9_1	.4577791	.2362133	1.94	0.053	-.0051905	.9207487
X9_2	.0909087	.2196146	0.41	0.679	-.3395281	.5213455
X10	-.0165933	.0219647	-0.76	0.450	-.0596434	.0264568
X11_1	1.529488	.2707881	5.65	0.000	.9987534	2.060223

```
       X11_2 |   .4163849   .2275166     1.83   0.067    -.0295394    .8623093
       X11_3 |  -.2300493   .2507329    -0.92   0.359    -.7214768    .2613782
       X11_4 |   .8071347   .2923746     2.76   0.006      .234091    1.380178
-------------+----------------------------------------------------------------
Gamma_2      |
          X1 |   .0111874   .0035257     3.17   0.002     .0042772    .0180976
        X7_2 |  -1.439931   .2626463    -5.48   0.000    -1.954709   -.9251541
        X7_4 |   -.936296   .2801834    -3.34   0.001    -1.485445   -.3871466
         X10 |   .0432982   .0151428     2.86   0.004     .0136187    .0729776
       X11_1 |  -1.385229   .2339696    -5.92   0.000    -1.843801   -.9266571
-------------+----------------------------------------------------------------
Alpha        |
     _cons_1 |  -4.802937   1.131667    -4.24   0.000    -7.020963   -2.584911
     _cons_2 |  -9.810482   1.220197    -8.04   0.000    -12.20203   -7.418939
------------------------------------------------------------------------------

==============================================================================

. ologit Y X1 X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X8_1 X9_1 X9_2 X11_1 X11_2 X11_3
  X11_4

Iteration 0:   log likelihood = -1185.3939
Iteration 1:   log likelihood = -1104.0189
Iteration 2:   log likelihood = -1102.1739
Iteration 3:   log likelihood = -1102.1677
Iteration 4:   log likelihood = -1102.1677

Ordered logistic regression                       Number of obs   =       1284
                                                  LR chi2(16)     =     166.45
                                                  Prob > chi2     =     0.0000
Log likelihood = -1102.1677                       Pseudo R2       =     0.0702
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          X1 |   .0032466   .0021744     1.49   0.135    -.0010151    .0075083
          X2 |   .0023236   .0009643     2.41   0.016     .0004335    .0042137
        X3_1 |    .270989    .124822     2.17   0.030     .0263424    .5156357
        X4_1 |   .3153977   .1273527     2.48   0.013      .065791    .5650043
          X5 |    1.16172   .2130962     5.45   0.000     .7440593    1.579381
          X6 |   .0238039   .0171796     1.39   0.166    -.0098676    .0574753
        X7_1 |   .3705269   .2304213     1.61   0.108    -.0810906    .8221443
        X7_2 |  -.0652964    .171212    -0.38   0.703    -.4008658    .2702731
        X7_4 |   .6100244   .1931031     3.16   0.002     .2315494    .9884995
        X8_1 |   .0749266   .1527275     0.49   0.624    -.2244137     .374267
        X9_1 |    .423311   .2351416     1.80   0.072     -.037558    .8841801
        X9_2 |   .0734328     .22168     0.33   0.740    -.3610521    .5079177
       X11_1 |   1.015086    .246562     4.12   0.000     .5318334    1.498339
       X11_2 |   .4264994   .2312421     1.84   0.065    -.0267268    .8797257
       X11_3 |  -.2678462   .2546314    -1.05   0.293    -.7669145    .2312222
       X11_4 |   .8755791    .298883     2.93   0.003     .2897792    1.461379
-------------+----------------------------------------------------------------
       /cut1 |   5.465044   .8655352                      3.768626    7.161462
       /cut2 |   8.686503   .8924652                      6.937303     10.4357
------------------------------------------------------------------------------

Brant Test of Parallel Regression Assumption

    Variable |      chi2   p>chi2    df
-------------+--------------------------
         All |     82.99    0.000    16
-------------+--------------------------
          X1 |     10.52    0.001     1
          X2 |      0.27    0.600     1
        X3_1 |      2.59    0.107     1
        X4_1 |      0.39    0.531     1
          X5 |      2.11    0.146     1
          X6 |      0.66    0.415     1
        X7_1 |      5.79    0.016     1
        X7_2 |     34.64    0.000     1
        X7_4 |     17.45    0.000     1
        X8_1 |      0.01    0.908     1
        X9_1 |      1.01    0.316     1
```

```
        X9_2 |      0.02    0.891     1
       X11_1 |      6.25    0.012     1
       X11_2 |      0.41    0.524     1
       X11_3 |      0.31    0.578     1
       X11_4 |      0.05    0.824     1
----------------------------------------

. fitstat

Measures of Fit for ologit of Y

Log-Lik Intercept Only:      -1185.394   Log-Lik Full Model:          -1102.168
D(1266):                      2204.335   LR(16):                        166.452
                                         Prob > LR:                       0.000
McFadden's R2:                   0.070   McFadden's Adj R2:               0.055
ML (Cox-Snell) R2:               0.122   Cragg-Uhler(Nagelkerke) R2:      0.144
McKelvey & Zavoina's R2:         0.145
Variance of y*:                  3.849   Variance of error:               3.290
Count R2:                        0.609   Adj Count R2:                    0.027
AIC:                             1.745   AIC*n:                        2240.335
BIC:                         -6857.358   BIC':                          -51.929
BIC used by Stata:            2333.175   AIC used by Stata:            2240.335


. gologit2 Y X1 X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X8_1 X9_1 X9_2 X11_1 X11_2 X11_3
  X11_4, autofit gamma lrf

------------------------------------------------------------------------------
Testing parallel lines assumption using the .05 level of significance...

Step  1:  Constraints for parallel lines imposed for X2 (P Value = 0.8999)
Step  2:  Constraints for parallel lines imposed for X11_4 (P Value = 0.8085)
Step  3:  Constraints for parallel lines imposed for X9_2 (P Value = 0.7938)
Step  4:  Constraints for parallel lines imposed for X6 (P Value = 0.5572)
Step  5:  Constraints for parallel lines imposed for X11_3 (P Value = 0.3974)
Step  6:  Constraints for parallel lines imposed for X4_1 (P Value = 0.4446)
Step  7:  Constraints for parallel lines imposed for X11_2 (P Value = 0.4342)
Step  8:  Constraints for parallel lines imposed for X8_1 (P Value = 0.4298)
Step  9:  Constraints for parallel lines imposed for X3_1 (P Value = 0.2035)
Step  10: Constraints for parallel lines imposed for X9_1 (P Value = 0.1030)
Step  11: Constraints for parallel lines are not imposed for
          X1 (P Value = 0.00108)
          X5 (P Value = 0.04286)
          X7_1 (P Value = 0.00814)
          X7_2 (P Value = 0.00000)
          X7_4 (P Value = 0.00002)
          X11_1 (P Value = 0.00000)

Wald test of parallel lines assumption for the final model:

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 3)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 4)  [>4_semester]X6 - [4_semester]X6 = 0
 ( 5)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 6)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 7)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 8)  [>4_semester]X8_1 - [4_semester]X8_1 = 0
 ( 9)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 (10)  [>4_semester]X9_1 - [4_semester]X9_1 = 0

           chi2( 10) =    7.34
         Prob > chi2 =    0.6933


pl(X2 X11_4 X9_2 X6 X11_3 X4_1 X11_2 X8_1 X3_1 X9_1)
------------------------------------------------------------------------------
```

```
Generalized Ordered Logit Estimates                Number of obs   =       1284
                                                   Wald chi2(22)   =     199.53
                                                   Prob > chi2     =     0.0000
Log likelihood = -1065.5569                        Pseudo R2       =     0.1011

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 3)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 4)  [>4_semester]X6 - [4_semester]X6 = 0
 ( 5)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 6)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 7)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 8)  [>4_semester]X8_1 - [4_semester]X8_1 = 0
 ( 9)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 (10)  [>4_semester]X9_1 - [4_semester]X9_1 = 0
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
>4_semester  |
          X1 |  -.0013925   .0025302    -0.55   0.582    -.0063516    .0035667
          X2 |   .0025672   .0009916     2.59   0.010     .0006237    .0045107
        X3_1 |   .2602472     .12894     2.02   0.044     .0075294     .512965
        X4_1 |   .3368713   .1309634     2.57   0.010     .0801877    .5935548
          X5 |   1.397938   .2473644     5.65   0.000      .913113    1.882764
          X6 |   .0201208   .0175244     1.15   0.251    -.0142265    .0544681
        X7_1 |   .5741729   .2456849     2.34   0.019     .0926394    1.055706
        X7_2 |    .424019    .195099     2.17   0.030      .041632    .8064059
        X7_4 |    1.11012    .243181     4.56   0.000     .6334944    1.586746
        X8_1 |   .0522916   .1547275     0.34   0.735    -.2509686    .3555519
        X9_1 |   .4598244   .2374548     1.94   0.053    -.0055785    .9252274
        X9_2 |   .1048788   .2229224     0.47   0.638    -.3320411    .5417986
       X11_1 |   1.556177   .2715557     5.73   0.000     1.023937    2.088416
       X11_2 |   .4073846   .2270879     1.79   0.073    -.0376996    .8524687
       X11_3 |  -.2396498   .2511834    -0.95   0.340    -.7319601    .2526606
       X11_4 |   .8082851   .2914676     2.77   0.006     .2370191    1.379551
       _cons |  -6.082514   .9482689    -6.41   0.000    -7.941087   -4.223942
-------------+----------------------------------------------------------------
4_semester   |
          X1 |   .0101964   .0032359     3.15   0.002      .003854    .0165387
          X2 |   .0025672   .0009916     2.59   0.010     .0006237    .0045107
        X3_1 |   .2602472     .12894     2.02   0.044     .0075294     .512965
        X4_1 |   .3368713   .1309634     2.57   0.010     .0801877    .5935548
          X5 |   .7667769   .2956827     2.59   0.010     .1872495    1.346304
          X6 |   .0201208   .0175244     1.15   0.251    -.0142265    .0544681
        X7_1 |  -.1130625   .2819702    -0.40   0.688    -.6657138    .4395889
        X7_2 |  -1.303789   .2781391    -4.69   0.000    -1.848932   -.7586465
        X7_4 |  -.1920127   .2629888    -0.73   0.465    -.7074614     .323436
        X8_1 |   .0522916   .1547275     0.34   0.735    -.2509686    .3555519
        X9_1 |   .4598244   .2374548     1.94   0.053    -.0055785    .9252274
        X9_2 |   .1048788   .2229224     0.47   0.638    -.3320411    .5417986
       X11_1 |   .0389872   .2921101     0.13   0.894    -.5335381    .6115126
       X11_2 |   .4073846   .2270879     1.79   0.073    -.0376996    .8524687
       X11_3 |  -.2396498   .2511834    -0.95   0.340    -.7319601    .2526606
       X11_4 |   .8082851   .2914676     2.77   0.006     .2370191    1.379551
       _cons |  -7.498187    1.09127    -6.87   0.000    -9.637037   -5.359336
------------------------------------------------------------------------------

Alternative parameterization: Gammas are deviations from proportionality
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Beta         |
          X1 |  -.0013925   .0025302    -0.55   0.582    -.0063516    .0035667
          X2 |   .0025672   .0009916     2.59   0.010     .0006237    .0045107
        X3_1 |   .2602472     .12894     2.02   0.044     .0075294     .512965
        X4_1 |   .3368713   .1309634     2.57   0.010     .0801877    .5935548
          X5 |   1.397938   .2473644     5.65   0.000      .913113    1.882764
          X6 |   .0201208   .0175244     1.15   0.251    -.0142265    .0544681
        X7_1 |   .5741729   .2456849     2.34   0.019     .0926394    1.055706
        X7_2 |    .424019    .195099     2.17   0.030      .041632    .8064059
        X7_4 |    1.11012    .243181     4.56   0.000     .6334944    1.586746
        X8_1 |   .0522916   .1547275     0.34   0.735    -.2509686    .3555519
```

```
        X9_1 |   .4598244   .2374548     1.94   0.053    -.0055785    .9252274
        X9_2 |   .1048788   .2229224     0.47   0.638    -.3320411    .5417986
       X11_1 |   1.556177   .2715557     5.73   0.000     1.023937    2.088416
       X11_2 |   .4073846   .2270879     1.79   0.073    -.0376996    .8524687
       X11_3 |  -.2396498   .2511834    -0.95   0.340    -.7319601    .2526606
       X11_4 |   .8082851   .2914676     2.77   0.006     .2370191    1.379551
-------------+----------------------------------------------------------------
Gamma_2      |
          X1 |   .0115888   .0035466     3.27   0.001     .0046376    .0185401
          X5 |  -.6311615   .3116697    -2.03   0.043    -1.242023   -.0203001
        X7_1 |  -.6872353   .2597113    -2.65   0.008     -1.19626   -.1782105
        X7_2 |  -1.727808   .2933089    -5.89   0.000    -2.302683   -1.152933
        X7_4 |  -1.302133   .3064027    -4.25   0.000    -1.902671   -.7015947
       X11_1 |   -1.51719   .2451831    -6.19   0.000     -1.99774   -1.036639
-------------+----------------------------------------------------------------
Alpha        |
     _cons_1 |  -6.082514   .9482689    -6.41   0.000    -7.941087   -4.223942
     _cons_2 |  -7.498187    1.09127    -6.87   0.000    -9.637037   -5.359336
------------------------------------------------------------------------------

=================================================================================

. ologit Y X1 X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X9_1 X9_2 X11_1 X11_2 X11_3 X11_4

Iteration 0:   log likelihood = -1185.3939
Iteration 1:   log likelihood =  -1104.133
Iteration 2:   log likelihood = -1102.2942
Iteration 3:   log likelihood = -1102.2881
Iteration 4:   log likelihood = -1102.2881

Ordered logistic regression                       Number of obs   =       1284
                                                  LR chi2(15)     =     166.21
                                                  Prob > chi2     =     0.0000
Log likelihood = -1102.2881                       Pseudo R2       =     0.0701
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          X1 |   .0033351   .0021667     1.54   0.124    -.0009115    .0075817
          X2 |   .0022816   .0009605     2.38   0.018     .0003991    .0041642
        X3_1 |   .2760998   .1243812     2.22   0.026     .0323171    .5198825
        X4_1 |   .3157508    .127363     2.48   0.013     .0661239    .5653777
          X5 |   1.162501   .2130738     5.46   0.000     .7448838    1.580118
          X6 |   .0266864   .0161525     1.65   0.099    -.0049719    .0583447
        X7_1 |   .3504995   .2266898     1.55   0.122    -.0938043    .7948034
        X7_2 |  -.0666011   .1711883    -0.39   0.697     -.402124    .2689218
        X7_4 |   .5880401   .1878072     3.13   0.002     .2199447    .9561354
        X9_1 |   .4381553   .2331225     1.88   0.060    -.0187564     .895067
        X9_2 |   .0810892   .2210664     0.37   0.714     -.352193    .5143714
       X11_1 |   1.005093   .2456998     4.09   0.000     .5235302    1.486656
       X11_2 |   .4197486   .2308102     1.82   0.069    -.0326311    .8721282
       X11_3 |  -.2755719   .2541617    -1.08   0.278    -.7737197    .2225759
       X11_4 |   .8756514   .2989348     2.93   0.003       .28975    1.461553
-------------+----------------------------------------------------------------
       /cut1 |   5.432771   .8629196                       3.74148    7.124063
       /cut2 |   8.653755   .8897925                      6.909794    10.39772
------------------------------------------------------------------------------

Brant Test of Parallel Regression Assumption

    Variable |      chi2   p>chi2    df
-------------+--------------------------
         All |     83.13    0.000    15
-------------+--------------------------
          X1 |     10.60    0.001     1
          X2 |      0.26    0.608     1
        X3_1 |      2.59    0.108     1
        X4_1 |      0.40    0.529     1
          X5 |      2.11    0.147     1
          X6 |      0.69    0.407     1
        X7_1 |      5.86    0.016     1
        X7_2 |     34.71    0.000     1
        X7_4 |     18.09    0.000     1
```

```
        X9_1 |      1.06    0.303     1
        X9_2 |      0.02    0.900     1
       X11_1 |      6.22    0.013     1
       X11_2 |      0.42    0.518     1
       X11_3 |      0.32    0.573     1
       X11_4 |      0.05    0.821     1
----------------------------------------

. fitstat

Measures of Fit for ologit of Y

Log-Lik Intercept Only:      -1185.394   Log-Lik Full Model:          -1102.288
D(1267):                      2204.576   LR(15):                        166.212
                                         Prob > LR:                       0.000
McFadden's R2:                   0.070   McFadden's Adj R2:               0.056
ML (Cox-Snell) R2:               0.121   Cragg-Uhler(Nagelkerke) R2:      0.144
McKelvey & Zavoina's R2:         0.145
Variance of y*:                  3.848   Variance of error:               3.290
Count R2:                        0.607   Adj Count R2:                    0.023
AIC:                             1.743   AIC*n:                        2238.576
BIC:                         -6864.275   BIC':                          -58.846
BIC used by Stata:            2326.258   AIC used by Stata:            2238.576


. gologit2 Y X1 X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X9_1 X9_2 X11_1 X11_2 X11_3
  X11_4,autofit gamma lrf

------------------------------------------------------------------------------
Testing parallel lines assumption using the .05 level of significance...

Step  1:  Constraints for parallel lines imposed for X2 (P Value = 0.8677)
Step  2:  Constraints for parallel lines imposed for X11_4 (P Value = 0.7909)
Step  3:  Constraints for parallel lines imposed for X9_2 (P Value = 0.7619)
Step  4:  Constraints for parallel lines imposed for X11_3 (P Value = 0.4488)
Step  5:  Constraints for parallel lines imposed for X4_1 (P Value = 0.4382)
Step  6:  Constraints for parallel lines imposed for X11_2 (P Value = 0.4825)
Step  7:  Constraints for parallel lines imposed for X6 (P Value = 0.3516)
Step  8:  Constraints for parallel lines imposed for X3_1 (P Value = 0.2019)
Step  9:  Constraints for parallel lines imposed for X9_1 (P Value = 0.1065)
Step  10: Constraints for parallel lines are not imposed for
          X1 (P Value = 0.00106)
          X5 (P Value = 0.04170)
          X7_1 (P Value = 0.00808)
          X7_2 (P Value = 0.00000)
          X7_4 (P Value = 0.00002)
          X11_1 (P Value = 0.00000)

Wald test of parallel lines assumption for the final model:

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 3)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 4)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 5)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 6)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 7)  [>4_semester]X6 - [4_semester]X6 = 0
 ( 8)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 ( 9)  [>4_semester]X9_1 - [4_semester]X9_1 = 0

           chi2(  9) =    7.00
         Prob > chi2 =    0.6366


pl(X2 X11_4 X9_2 X11_3 X4_1 X11_2 X6 X3_1 X9_1)
------------------------------------------------------------------------------
```

```
Generalized Ordered Logit Estimates               Number of obs   =       1284
                                                  LR chi2(21)     =     239.56
                                                  Prob > chi2     =     0.0000
Log likelihood =  -1065.614                       Pseudo R2       =     0.1010

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 3)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 4)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 5)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 6)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 7)  [>4_semester]X6 - [4_semester]X6 = 0
 ( 8)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 ( 9)  [>4_semester]X9_1 - [4_semester]X9_1 = 0
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
>4_semester  |
          X1 |  -.0013324   .0025235    -0.53   0.598    -.0062783    .0036136
          X2 |    .002538   .0009878     2.57   0.010     .0006019    .0044741
        X3_1 |   .2638797   .1284936     2.05   0.040     .0120369    .5157225
        X4_1 |   .3368852   .1309744     2.57   0.010     .0801802    .5935903
          X5 |   1.399772   .2472998     5.66   0.000     .9150733    1.884471
          X6 |   .0220815   .0165409     1.33   0.182    -.0103381     .054501
        X7_1 |   .5610483   .2425383     2.31   0.021      .085682    1.036415
        X7_2 |   .4237599   .1950967     2.17   0.030     .0413775    .8061423
        X7_4 |   1.096425   .2397578     4.57   0.000     .6265084    1.566342
        X9_1 |   .4706847   .2352335     2.00   0.045     .0096355    .9317339
        X9_2 |   .1105682    .222254     0.50   0.619    -.3250417     .546178
       X11_1 |   1.550175   .2709426     5.72   0.000     1.019137    2.081213
       X11_2 |   .4029698   .2266844     1.78   0.075    -.0413235     .847263
       X11_3 |  -.2448785   .2507046    -0.98   0.329    -.7362505    .2464934
       X11_4 |   .8089695   .2914829     2.78   0.006     .2376734    1.380266
       _cons |  -6.064751   .9467675    -6.41   0.000    -7.920381    -4.20912
-------------+----------------------------------------------------------------
4_semester   |
          X1 |   .0102762   .0032275     3.18   0.001     .0039504    .0166019
          X2 |    .002538   .0009878     2.57   0.010     .0006019    .0044741
        X3_1 |   .2638797   .1284936     2.05   0.040     .0120369    .5157225
        X4_1 |   .3368852   .1309744     2.57   0.010     .0801802    .5935903
          X5 |   .7655319   .2956202     2.59   0.010     .1861269    1.344937
          X6 |   .0220815   .0165409     1.33   0.182    -.0103381     .054501
        X7_1 |   -.126669   .2789959    -0.45   0.650    -.6734908    .4201528
        X7_2 |  -1.304118   .2780459    -4.69   0.000    -1.849078   -.7591581
        X7_4 |  -.2060582   .2596197    -0.79   0.427    -.7149036    .3027871
        X9_1 |   .4706847   .2352335     2.00   0.045     .0096355    .9317339
        X9_2 |   .1105682    .222254     0.50   0.619    -.3250417     .546178
       X11_1 |   .0331623   .2915269     0.11   0.909    -.5382198    .6045445
       X11_2 |   .4029698   .2266844     1.78   0.075    -.0413235     .847263
       X11_3 |  -.2448785   .2507046    -0.98   0.329    -.7362505    .2464934
       X11_4 |   .8089695   .2914829     2.78   0.006     .2376734    1.380266
       _cons |  -7.473321   1.088419    -6.87   0.000    -9.606583   -5.340059
------------------------------------------------------------------------------

Alternative parameterization: Gammas are deviations from proportionality
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Beta         |
          X1 |  -.0013324   .0025235    -0.53   0.598    -.0062783    .0036136
          X2 |    .002538   .0009878     2.57   0.010     .0006019    .0044741
        X3_1 |   .2638797   .1284936     2.05   0.040     .0120369    .5157225
        X4_1 |   .3368852   .1309744     2.57   0.010     .0801802    .5935903
          X5 |   1.399772   .2472998     5.66   0.000     .9150733    1.884471
          X6 |   .0220815   .0165409     1.33   0.182    -.0103381     .054501
        X7_1 |   .5610483   .2425383     2.31   0.021      .085682    1.036415
        X7_2 |   .4237599   .1950967     2.17   0.030     .0413775    .8061423
        X7_4 |   1.096425   .2397578     4.57   0.000     .6265084    1.566342
        X9_1 |   .4706847   .2352335     2.00   0.045     .0096355    .9317339
        X9_2 |   .1105682    .222254     0.50   0.619    -.3250417     .546178
       X11_1 |   1.550175   .2709426     5.72   0.000     1.019137    2.081213
       X11_2 |   .4029698   .2266844     1.78   0.075    -.0413235     .847263
```

```
       X11_3 |  -.2448785   .2507046    -0.98   0.329    -.7362505    .2464934
       X11_4 |   .8089695   .2914829     2.78   0.006     .2376734    1.380266
-------------+----------------------------------------------------------------
Gamma_2      |
          X1 |   .0116086    .003546     3.27   0.001     .0046586    .0185586
          X5 |  -.6342402   .3114435    -2.04   0.042    -1.244658    -.023822
        X7_1 |  -.6877173   .2596558    -2.65   0.008    -1.196633   -.1788014
        X7_2 |  -1.727878   .2932406    -5.89   0.000    -2.302619   -1.153137
        X7_4 |  -1.302483   .3063666    -4.25   0.000    -1.902951   -.7020158
       X11_1 |  -1.517013   .2451226    -6.19   0.000    -1.997444   -1.036581
-------------+----------------------------------------------------------------
Alpha        |
     _cons_1 |  -6.064751   .9467675    -6.41   0.000    -7.920381    -4.20912
     _cons_2 |  -7.473321   1.088419    -6.87   0.000    -9.606583   -5.340059
------------------------------------------------------------------------------

==================================================================================

. ologit Y X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X9_1 X9_2 X11_1 X11_2 X11_3 X11_4

Iteration 0:   log likelihood = -1185.3939
Iteration 1:   log likelihood = -1105.2682
Iteration 2:   log likelihood = -1103.4799
Iteration 3:   log likelihood = -1103.4737
Iteration 4:   log likelihood = -1103.4737

Ordered logistic regression                       Number of obs   =       1284
                                                  LR chi2(14)     =     163.84
                                                  Prob > chi2     =     0.0000
Log likelihood = -1103.4737                       Pseudo R2       =     0.0691
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          X2 |   .0026395   .0009306     2.84   0.005     .0008154    .0044635
        X3_1 |   .2718967   .1243376     2.19   0.029     .0281994     .515594
        X4_1 |   .3247797   .1271803     2.55   0.011     .0755108    .5740485
          X5 |   1.224135    .209192     5.85   0.000      .814126    1.634144
          X6 |   .0279969   .0161236     1.74   0.082    -.0036048    .0595987
        X7_1 |   .3277809   .2261799     1.45   0.147    -.1155235    .7710854
        X7_2 |  -.0432598   .1704259    -0.25   0.800    -.3772884    .2907688
        X7_4 |   .5607512   .1867267     3.00   0.003     .1947736    .9267288
        X9_1 |   .4180533   .2326477     1.80   0.072    -.0379278    .8740344
        X9_2 |   .0618469   .2207159     0.28   0.779    -.3707483    .4944421
       X11_1 |   1.015345   .2455155     4.14   0.000     .5341438    1.496547
       X11_2 |   .3983426   .2302393     1.73   0.084    -.0529181    .8496032
       X11_3 |  -.2832996   .2539214    -1.12   0.265    -.7809765    .2143772
       X11_4 |   .8525102   .2984123     2.86   0.004     .2676328    1.437388
-------------+----------------------------------------------------------------
       /cut1 |    5.32726   .8587523                      3.644136    7.010383
       /cut2 |   8.543474   .8852795                      6.808358    10.27859
------------------------------------------------------------------------------

Brant Test of Parallel Regression Assumption

    Variable |      chi2   p>chi2    df
-------------+--------------------------
         All |     75.64    0.000    14
-------------+--------------------------
          X2 |      0.10    0.757     1
        X3_1 |      2.52    0.113     1
        X4_1 |      0.14    0.705     1
          X5 |      0.73    0.392     1
          X6 |      0.95    0.331     1
        X7_1 |      6.52    0.011     1
        X7_2 |     32.45    0.000     1
        X7_4 |     20.86    0.000     1
        X9_1 |      1.31    0.253     1
        X9_2 |      0.00    0.997     1
       X11_1 |      5.58    0.018     1
       X11_2 |      0.23    0.628     1
       X11_3 |      0.25    0.620     1
       X11_4 |      0.01    0.938     1
----------------------------------------
```

```
. fitst at

Measures of Fit for ologit of Y

Log-Lik Intercept Only:      -1185.394   Log-Lik Full Model:          -1103.474
D(1268):                      2206.947   LR(14):                        163.840
                                         Prob > LR:                       0.000
McFadden's R2:                   0.069   McFadden's Adj R2:               0.056
ML (Cox-Snell) R2:               0.120   Cragg-Uhler(Nagelkerke) R2:      0.142
McKelvey & Zavoina's R2:         0.143
Variance of y*:                  3.838   Variance of error:               3.290
Count R2:                        0.611   Adj Count R2:                    0.033
AIC:                             1.744   AIC*n:                        2238.947
BIC:                         -6869.061   BIC':                          -63.632
BIC used by Stata:            2321.471   AIC used by Stata:            2238.947


. gologit2 Y X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X9_1 X9_2 X11_1 X11_2 X11_3 X11_4,
  autofit gamma lrf

------------------------------------------------------------------------------
Testing parallel lines assumption using the .05 level of significance...

Step  1:  Constraints for parallel lines imposed for X11_4 (P Value = 0.8060)
Step  2:  Constraints for parallel lines imposed for X9_2 (P Value = 0.7921)
Step  3:  Constraints for parallel lines imposed for X2 (P Value = 0.5932)
Step  4:  Constraints for parallel lines imposed for X5 (P Value = 0.6024)
Step  5:  Constraints for parallel lines imposed for X4_1 (P Value = 0.4938)
Step  6:  Constraints for parallel lines imposed for X11_3 (P Value = 0.4867)
Step  7:  Constraints for parallel lines imposed for X11_2 (P Value = 0.6254)
Step  8:  Constraints for parallel lines imposed for X3_1 (P Value = 0.1936)
Step  9:  Constraints for parallel lines imposed for X6 (P Value = 0.0965)
Step  10: Constraints for parallel lines imposed for X9_1 (P Value = 0.0839)
Step  11: Constraints for parallel lines are not imposed for
          X7_1 (P Value = 0.00766)
          X7_2 (P Value = 0.00000)
          X7_4 (P Value = 0.00001)
          X11_1 (P Value = 0.00000)

Wald test of parallel lines assumption for the final model:

 ( 1)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 2)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 3)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 4)  [>4_semester]X5 - [4_semester]X5 = 0
 ( 5)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 6)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 7)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 8)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 ( 9)  [>4_semester]X6 - [4_semester]X6 = 0
 (10)  [>4_semester]X9_1 - [4_semester]X9_1 = 0

           chi2( 10) =    9.39
         Prob > chi2 =    0.4952

pl(X11_4 X9_2 X2 X5 X4_1 X11_3 X11_2 X3_1 X6 X9_1)
------------------------------------------------------------------------------

Generalized Ordered Logit Estimates               Number of obs   =       1284
                                                  LR chi2(18)     =     225.92
                                                  Prob > chi2     =     0.0000
Log likelihood = -1072.4356                       Pseudo R2       =     0.0953

 ( 1)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 2)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 3)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 4)  [>4_semester]X5 - [4_semester]X5 = 0
 ( 5)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 6)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 7)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 8)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 ( 9)  [>4_semester]X6 - [4_semester]X6 = 0
```

```
 (10)  [>4_semester]X9_1 - [4_semester]X9_1 = 0
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
>4_semester  |
          X2 |   .0027327   .0009514     2.87   0.004      .000868    .0045974
        X3_1 |   .2653901   .1277866     2.08   0.038     .0149329    .5158472
        X4_1 |   .3414758   .1299079     2.63   0.009      .086861    .5960907
          X5 |   1.237014   .2134162     5.80   0.000     .8187259    1.655302
          X6 |   .0219186   .0164516     1.33   0.183    -.0103259    .0541632
        X7_1 |   .5478774   .2424752     2.26   0.024     .0726347     1.02312
        X7_2 |   .3866611   .1923827     2.01   0.044     .0095979    .7637243
        X7_4 |    1.07893   .2393411     4.51   0.000     .6098303     1.54803
        X9_1 |   .4704254   .2354188     2.00   0.046      .009013    .9318379
        X9_2 |     .11354   .2222103     0.51   0.609    -.3219842    .5490643
       X11_1 |   1.541375   .2699172     5.71   0.000     1.012347    2.070403
       X11_2 |   .3999188   .2259342     1.77   0.077    -.0429041    .8427418
       X11_3 |  -.2485503   .2500519    -0.99   0.320     -.738643    .2415425
       X11_4 |   .8019109   .2898365     2.77   0.006     .2338418     1.36998
       _cons |  -5.810147   .8726852    -6.66   0.000    -7.520578   -4.099715
-------------+----------------------------------------------------------------
4_semester   |
          X2 |   .0027327   .0009514     2.87   0.004      .000868    .0045974
        X3_1 |   .2653901   .1277866     2.08   0.038     .0149329    .5158472
        X4_1 |   .3414758   .1299079     2.63   0.009      .086861    .5960907
          X5 |   1.237014   .2134162     5.80   0.000     .8187259    1.655302
          X6 |   .0219186   .0164516     1.33   0.183    -.0103259    .0541632
        X7_1 |   -.140053   .2777463    -0.50   0.614    -.6844258    .4043197
        X7_2 |  -1.205714   .2739732    -4.40   0.000    -1.742692   -.6687368
        X7_4 |  -.3132936   .2555404    -1.23   0.220    -.8141435    .1875564
        X9_1 |   .4704254   .2354188     2.00   0.046      .009013    .9318379
        X9_2 |     .11354   .2222103     0.51   0.609    -.3219842    .5490643
       X11_1 |   .0972441   .2865448     0.34   0.734    -.4643735    .6588616
       X11_2 |   .3999188   .2259342     1.77   0.077    -.0429041    .8427418
       X11_3 |  -.2485503   .2500519    -0.99   0.320     -.738643    .2415425
       X11_4 |   .8019109   .2898365     2.77   0.006     .2338418     1.36998
       _cons |  -7.680949   .8987273    -8.55   0.000    -9.442422   -5.919476
------------------------------------------------------------------------------

Alternative parameterization: Gammas are deviations from proportionality
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Beta         |
          X2 |   .0027327   .0009514     2.87   0.004      .000868    .0045974
        X3_1 |   .2653901   .1277866     2.08   0.038     .0149329    .5158472
        X4_1 |   .3414758   .1299079     2.63   0.009      .086861    .5960907
          X5 |   1.237014   .2134162     5.80   0.000     .8187259    1.655302
          X6 |   .0219186   .0164516     1.33   0.183    -.0103259    .0541632
        X7_1 |   .5478774   .2424752     2.26   0.024     .0726347     1.02312
        X7_2 |   .3866611   .1923827     2.01   0.044     .0095979    .7637243
        X7_4 |    1.07893   .2393411     4.51   0.000     .6098303     1.54803
        X9_1 |   .4704254   .2354188     2.00   0.046      .009013    .9318379
        X9_2 |     .11354   .2222103     0.51   0.609    -.3219842    .5490643
       X11_1 |   1.541375   .2699172     5.71   0.000     1.012347    2.070403
       X11_2 |   .3999188   .2259342     1.77   0.077    -.0429041    .8427418
       X11_3 |  -.2485503   .2500519    -0.99   0.320     -.738643    .2415425
       X11_4 |   .8019109   .2898365     2.77   0.006     .2338418     1.36998
-------------+----------------------------------------------------------------
Gamma_2      |
        X7_1 |  -.6879304   .2579555    -2.67   0.008    -1.193514   -.1823468
        X7_2 |  -1.592375   .2896191    -5.50   0.000    -2.160018   -1.024732
        X7_4 |  -1.392224    .305267    -4.56   0.000    -1.990536   -.7939113
       X11_1 |  -1.444131   .2395107    -6.03   0.000    -1.913563   -.9746986
-------------+----------------------------------------------------------------
Alpha        |
     _cons_1 |  -5.810147   .8726852    -6.66   0.000    -7.520578   -4.099715
     _cons_2 |  -7.680949   .8987273    -8.55   0.000    -9.442422   -5.919476
------------------------------------------------------------------------------
```

```
. ologit Y X2 X3_1 X4_1 X5 X7_1 X7_2 X7_4 X9_1 X9_2 X11_1 X11_2 X11_3 X11_4

Iteration 0:   log likelihood = -1185.3939
Iteration 1:   log likelihood = -1106.7028
Iteration 2:   log likelihood = -1104.9903
Iteration 3:   log likelihood = -1104.9842
Iteration 4:   log likelihood = -1104.9842

Ordered logistic regression                       Number of obs   =       1284
                                                  LR chi2(13)     =     160.82
                                                  Prob > chi2     =     0.0000
Log likelihood = -1104.9842                       Pseudo R2       =     0.0678
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          X2 |   .0024595   .0009236     2.66   0.008     .0006493    .0042696
        X3_1 |   .2767684   .1242904     2.23   0.026     .0331637     .520373
        X4_1 |   .3186667   .1271244     2.51   0.012     .0695074     .567826
          X5 |   1.055568   .1848029     5.71   0.000     .6933607    1.417775
        X7_1 |   .2783956   .2241018     1.24   0.214    -.1608359     .717627
        X7_2 |  -.0316866   .1702135    -0.19   0.852    -.3652989    .3019257
        X7_4 |    .485404   .1814305     2.68   0.007     .1298067    .8410013
        X9_1 |   .5139217   .2259116     2.27   0.023     .0711431    .9567002
        X9_2 |   .1430055   .2155227     0.66   0.507    -.2794112    .5654222
       X11_1 |   1.049677   .2444888     4.29   0.000     .5704875    1.528866
       X11_2 |   .4264604   .2294577     1.86   0.063    -.0232684    .8761893
       X11_3 |  -.2541186   .2531059    -1.00   0.315    -.7501971    .2419599
       X11_4 |   .8752742   .2983751     2.93   0.003     .2904697    1.460079
-------------+----------------------------------------------------------------
       /cut1 |   4.658305   .7650606                      3.158813    6.157796
       /cut2 |    7.86781   .7916308                      6.316242    9.419378
------------------------------------------------------------------------------

Brant Test of Parallel Regression Assumption

    Variable |      chi2   p>chi2    df
-------------+--------------------------
         All |     76.00    0.000    13
-------------+--------------------------
          X2 |      0.06    0.801     1
        X3_1 |      2.77    0.096     1
        X4_1 |      0.14    0.709     1
          X5 |      1.59    0.207     1
        X7_1 |      7.24    0.007     1
        X7_2 |     32.72    0.000     1
        X7_4 |     23.92    0.000     1
        X9_1 |      0.84    0.360     1
        X9_2 |      0.05    0.815     1
       X11_1 |      5.38    0.020     1
       X11_2 |      0.31    0.579     1
       X11_3 |      0.29    0.588     1
       X11_4 |      0.01    0.917     1
----------------------------------------

. fitstat

Measures of Fit for ologit of Y

Log-Lik Intercept Only:      -1185.394   Log-Lik Full Model:          -1104.984
D(1269):                      2209.968   LR(13):                        160.820
                                         Prob > LR:                       0.000
McFadden's R2:                   0.068   McFadden's Adj R2:               0.055
ML (Cox-Snell) R2:               0.118   Cragg-Uhler(Nagelkerke) R2:      0.140
McKelvey & Zavoina's R2:         0.140
Variance of y*:                  3.826   Variance of error:               3.290
Count R2:                        0.609   Adj Count R2:                    0.027
AIC:                             1.745   AIC*n:                        2239.968
BIC:                         -6873.198   BIC':                          -67.769
BIC used by Stata:            2317.334   AIC used by Stata:            2239.968
```

```
. gologit2 Y X2 X3_1 X4_1 X5 X7_1 X7_2 X7_4 X9_1 X9_2 X11_1 X11_2 X11_3
  X11_4,autofit gamma lrf

------------------------------------------------------------------------------
Testing parallel lines assumption using the .05 level of significance...

Step  1:  Constraints for parallel lines imposed for X11_4 (P Value = 0.7642)
Step  2:  Constraints for parallel lines imposed for X2 (P Value = 0.6622)
Step  3:  Constraints for parallel lines imposed for X9_2 (P Value = 0.5980)
Step  4:  Constraints for parallel lines imposed for X4_1 (P Value = 0.4489)
Step  5:  Constraints for parallel lines imposed for X11_3 (P Value = 0.4776)
Step  6:  Constraints for parallel lines imposed for X11_2 (P Value = 0.5978)
Step  7:  Constraints for parallel lines imposed for X5 (P Value = 0.3180)
Step  8:  Constraints for parallel lines imposed for X3_1 (P Value = 0.1476)
Step  9:  Constraints for parallel lines imposed for X9_1 (P Value = 0.0849)
Step  10: Constraints for parallel lines are not imposed for
          X7_1 (P Value = 0.00711)
          X7_2 (P Value = 0.00000)
          X7_4 (P Value = 0.00000)
          X11_1 (P Value = 0.00000)

Wald test of parallel lines assumption for the final model:

 ( 1)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 2)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 3)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 4)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 5)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 6)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 7)  [>4_semester]X5 - [4_semester]X5 = 0
 ( 8)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 ( 9)  [>4_semester]X9_1 - [4_semester]X9_1 = 0

           chi2(  9) =    8.14
         Prob > chi2 =    0.5205

pl(X11_4 X2 X9_2 X4_1 X11_3 X11_2 X5 X3_1 X9_1)
------------------------------------------------------------------------------

Generalized Ordered Logit Estimates               Number of obs   =       1284
                                                  LR chi2(17)     =     224.14
                                                  Prob > chi2     =     0.0000
Log likelihood = -1073.3243                       Pseudo R2       =     0.0945

 ( 1)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 2)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 3)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 4)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 5)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 6)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 7)  [>4_semester]X5 - [4_semester]X5 = 0
 ( 8)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 ( 9)  [>4_semester]X9_1 - [4_semester]X9_1 = 0
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
>4_semester  |
          X2 |   .0026074   .0009459     2.76   0.006     .0007535    .0044614
        X3_1 |   .2675302   .1277606     2.09   0.036     .0171241    .5179363
        X4_1 |    .339067   .1299267     2.61   0.009     .0844154    .5937187
          X5 |   1.109701   .1904263     5.83   0.000     .7364719    1.482929
        X7_1 |   .5149201    .240855     2.14   0.033     .0428529    .9869872
        X7_2 |   .4024453   .1919516     2.10   0.036      .026227    .7786636
        X7_4 |   1.033176   .2366971     4.36   0.000     .5692583    1.497094
        X9_1 |   .5470127    .228237     2.40   0.017     .0996763     .994349
        X9_2 |   .1811608   .2162102     0.84   0.402    -.2426033    .6049249
       X11_1 |   1.573704   .2688153     5.85   0.000     1.046836    2.100572
       X11_2 |   .4226445   .2252012     1.88   0.061    -.0187417    .8640307
       X11_3 |  -.2254573   .2492805    -0.90   0.366    -.7140382    .2631235
       X11_4 |   .8179382    .289655     2.82   0.005     .2502247    1.385652
       _cons |   -5.31612   .7880921    -6.75   0.000    -6.860752   -3.771488
-------------+----------------------------------------------------------------
```

4_semester						
X2	.0026074	.0009459	2.76	0.006	.0007535	.0044614
X3_1	.2675302	.1277606	2.09	0.036	.0171241	.5179363
X4_1	.339067	.1299267	2.61	0.009	.0844154	.5937187
X5	1.109701	.1904263	5.83	0.000	.7364719	1.482929
X7_1	-.1769416	.2757459	-0.64	0.521	-.7173936	.3635104
X7_2	-1.203575	.2734248	-4.40	0.000	-1.739478	-.6676722
X7_4	-.3725938	.2511095	-1.48	0.138	-.8647593	.1195717
X9_1	.5470127	.228237	2.40	0.017	.0996763	.994349
X9_2	.1811608	.2162102	0.84	0.402	-.2426033	.6049249
X11_1	.125524	.2852533	0.44	0.660	-.4335622	.6846102
X11_2	.4226445	.2252012	1.88	0.061	-.0187417	.8640307
X11_3	-.2254573	.2492805	-0.90	0.366	-.7140382	.2631235
X11_4	.8179382	.289655	2.82	0.005	.2502247	1.385652
_cons	-7.175213	.8117141	-8.84	0.000	-8.766143	-5.584282

Alternative parameterization: Gammas are deviations from proportionality

Y	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
Beta						
X2	.0026074	.0009459	2.76	0.006	.0007535	.0044614
X3_1	.2675302	.1277606	2.09	0.036	.0171241	.5179363
X4_1	.339067	.1299267	2.61	0.009	.0844154	.5937187
X5	1.109701	.1904263	5.83	0.000	.7364719	1.482929
X7_1	.5149201	.240855	2.14	0.033	.0428529	.9869872
X7_2	.4024453	.1919516	2.10	0.036	.026227	.7786636
X7_4	1.033176	.2366971	4.36	0.000	.5692583	1.497094
X9_1	.5470127	.228237	2.40	0.017	.0996763	.994349
X9_2	.1811608	.2162102	0.84	0.402	-.2426033	.6049249
X11_1	1.573704	.2688153	5.85	0.000	1.046836	2.100572
X11_2	.4226445	.2252012	1.88	0.061	-.0187417	.8640307
X11_3	-.2254573	.2492805	-0.90	0.366	-.7140382	.2631235
X11_4	.8179382	.289655	2.82	0.005	.2502247	1.385652
Gamma_2						
X7_1	-.6918617	.2570611	-2.69	0.007	-1.195692	-.1880311
X7_2	-1.60602	.2888728	-5.56	0.000	-2.172201	-1.03984
X7_4	-1.40577	.3046873	-4.61	0.000	-2.002946	-.8085937
X11_1	-1.44818	.2389602	-6.06	0.000	-1.916533	-.9798264
Alpha						
_cons_1	-5.31612	.7880921	-6.75	0.000	-6.860752	-3.771488
_cons_2	-7.175213	.8117141	-8.84	0.000	-8.766143	-5.584282

Halaman ini sengaja dikosongkan

LAMPIRAN E : OUTPUT ANALISIS DATA (tidak mengeluarkan X_1)

```
. ologit Y X1 X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X8_1 X9_1 X9_2 X10 X11_1 X11_2
  X11_3 X11_4

Iteration 0:   log likelihood = -1185.3939
Iteration 1:   log likelihood = -1104.0195
Iteration 2:   log likelihood = -1102.1739
Iteration 3:   log likelihood = -1102.1677
Iteration 4:   log likelihood = -1102.1677

Ordered logistic regression                       Number of obs   =       1284
                                                  LR chi2(17)     =     166.45
                                                  Prob > chi2     =     0.0000
Log likelihood = -1102.1677                       Pseudo R2       =     0.0702
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          X1 |   .0032487    .002196     1.48   0.139    -.0010554    .0075528
          X2 |   .0023246   .0009751     2.38   0.017     .0004135    .0042357
        X3_1 |   .2708682   .1260774     2.15   0.032     .0237611    .5179753
        X4_1 |    .315383   .1273709     2.48   0.013     .0657406    .5650255
          X5 |   1.161923   .2151769     5.40   0.000     .7401845    1.583662
          X6 |   .0236657   .0265958     0.89   0.374    -.0284611    .0757924
        X7_1 |   .3705851   .2305801     1.61   0.108    -.0813436    .8225138
        X7_2 |   -.065294   .1712124    -0.38   0.703    -.4008641    .2702761
        X7_4 |   .6100024   .1931305     3.16   0.002     .2314735    .9885312
        X8_1 |   .0747022   .1562464     0.48   0.633     -.231535    .3809394
        X9_1 |   .4231663    .236101     1.79   0.073    -.0395832    .8859158
        X9_2 |   .0733912   .2217645     0.33   0.741    -.3612592    .5080416
         X10 |    .000146   .0214556     0.01   0.995    -.0419061    .0421982
       X11_1 |   1.015143   .2467069     4.11   0.000     .5316069     1.49868
       X11_2 |   .4263795   .2319125     1.84   0.066    -.0281605    .8809196
       X11_3 |  -.2679512   .2550994    -1.05   0.294    -.7679368    .2320343
       X11_4 |   .8754772   .2992577     2.93   0.003      .288943    1.462011
-------------+----------------------------------------------------------------
       /cut1 |   5.469699   1.103109                      3.307646    7.631752
       /cut2 |   8.691161   1.124621                      6.486945    10.89538
------------------------------------------------------------------------------

Brant Test of Parallel Regression Assumption

    Variable |      chi2   p>chi2    df
-------------+--------------------------
         All |     83.77    0.000    17
-------------+--------------------------
          X1 |     10.94    0.001     1
          X2 |      0.17    0.676     1
        X3_1 |      2.29    0.130     1
        X4_1 |      0.41    0.521     1
          X5 |      1.83    0.176     1
          X6 |      0.00    0.990     1
        X7_1 |      5.59    0.018     1
        X7_2 |     34.51    0.000     1
        X7_4 |     17.45    0.000     1
        X8_1 |      0.06    0.805     1
        X9_1 |      1.13    0.287     1
        X9_2 |      0.01    0.909     1
         X10 |      0.49    0.484     1
       X11_1 |      6.09    0.014     1
       X11_2 |      0.36    0.550     1
       X11_3 |      0.28    0.594     1
       X11_4 |      0.04    0.842     1
----------------------------------------

. fitstat

Measures of Fit for ologit of Y

Log-Lik Intercept Only:      -1185.394   Log-Lik Full Model:          -1102.168
D(1265):                      2204.335   LR(17):                        166.452
```

```
                                        Prob > LR:                      0.000
McFadden's R2:                   0.070   McFadden's Adj R2:              0.054
ML (Cox-Snell) R2:               0.122   Cragg-Uhler(Nagelkerke) R2:     0.144
McKelvey & Zavoina's R2:         0.145
Variance of y*:                  3.849   Variance of error:              3.290
Count R2:                        0.609   Adj Count R2:                   0.027
AIC:                             1.746   AIC*n:                       2242.335
BIC:                         -6850.200   BIC':                         -44.771
BIC used by Stata:            2340.332   AIC used by Stata:           2242.335


. gologit2 Y X1 X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X8_1 X9_1 X9_2 X10 X11_1 X11_2
  X11_3 X11_4, autofit gamma lrf

------------------------------------------------------------------------------
Testing parallel lines assumption using the .05 level of significance...

Step  1:  Constraints for parallel lines imposed for X2 (P Value = 0.9632)
Step  2:  Constraints for parallel lines imposed for X6 (P Value = 0.8475)
Step  3:  Constraints for parallel lines imposed for X11_4 (P Value = 0.8363)
Step  4:  Constraints for parallel lines imposed for X9_2 (P Value = 0.8453)
Step  5:  Constraints for parallel lines imposed for X8_1 (P Value = 0.6482)
Step  6:  Constraints for parallel lines imposed for X11_3 (P Value = 0.4432)
Step  7:  Constraints for parallel lines imposed for X4_1 (P Value = 0.4386)
Step  8:  Constraints for parallel lines imposed for X11_2 (P Value = 0.4903)
Step  9:  Constraints for parallel lines imposed for X5 (P Value = 0.2601)
Step  10: Constraints for parallel lines imposed for X3_1 (P Value = 0.2177)
Step  11: Constraints for parallel lines imposed for X9_1 (P Value = 0.0560)
Step  12: Constraints for parallel lines imposed for X7_1 (P Value = 0.0587)
Step  13: Constraints for parallel lines are not imposed for
          X1 (P Value = 0.00151)
          X7_2 (P Value = 0.00000)
          X7_4 (P Value = 0.00083)
          X10 (P Value = 0.00425)
          X11_1 (P Value = 0.00000)

Wald test of parallel lines assumption for the final model:

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X6 - [4_semester]X6 = 0
 ( 3)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 4)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 5)  [>4_semester]X8_1 - [4_semester]X8_1 = 0
 ( 6)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 7)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 8)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 9)  [>4_semester]X5 - [4_semester]X5 = 0
 (10)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 (11)  [>4_semester]X9_1 - [4_semester]X9_1 = 0
 (12)  [>4_semester]X7_1 - [4_semester]X7_1 = 0

           chi2( 12) =   12.34
         Prob > chi2 =    0.4184

pl(X2 X6 X11_4 X9_2 X8_1 X11_3 X4_1 X11_2 X5 X3_1 X9_1 X7_1)
------------------------------------------------------------------------------

Generalized Ordered Logit Estimates               Number of obs   =       1284
                                                  LR chi2(22)     =     235.42
                                                  Prob > chi2     =     0.0000
Log likelihood = -1067.6857                       Pseudo R2       =     0.0993

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X6 - [4_semester]X6 = 0
 ( 3)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 4)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 5)  [>4_semester]X8_1 - [4_semester]X8_1 = 0
 ( 6)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 7)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 8)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 9)  [>4_semester]X5 - [4_semester]X5 = 0
 (10)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
```

(11) [>4_semester]X9_1 - [4_semester]X9_1 = 0
(12) [>4_semester]X7_1 - [4_semester]X7_1 = 0

Y	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
>4_semester						
X1	-.0013955	.0025458	-0.55	0.584	-.0063853	.0035942
X2	.0026436	.0010021	2.64	0.008	.0006795	.0046076
X3_1	.256097	.1295624	1.98	0.048	.0021593	.5100347
X4_1	.3328007	.1301939	2.56	0.011	.0776253	.587976
X5	1.16893	.2186903	5.35	0.000	.7403053	1.597555
X6	.0228157	.0264376	0.86	0.388	-.029001	.0746323
X7_1	.3368117	.2247812	1.50	0.134	-.1037514	.7773747
X7_2	.3315182	.1899346	1.75	0.081	-.0407468	.7037832
X7_4	.9863335	.2380848	4.14	0.000	.5196959	1.452971
X8_1	.0540334	.1582932	0.34	0.733	-.2562156	.3642824
X9_1	.4577791	.2362133	1.94	0.053	-.0051905	.9207487
X9_2	.0909087	.2196146	0.41	0.679	-.3395281	.5213455
X10	-.0165933	.0219647	-0.76	0.450	-.0596434	.0264568
X11_1	1.529488	.2707881	5.65	0.000	.9987534	2.060223
X11_2	.4163849	.2275166	1.83	0.067	-.0295394	.8623093
X11_3	-.2300493	.2507329	-0.92	0.359	-.7214768	.2613782
X11_4	.8071347	.2923746	2.76	0.006	.234091	1.380178
_cons	-4.802937	1.131667	-4.24	0.000	-7.020963	-2.584911
4_semester						
X1	.0097919	.0032309	3.03	0.002	.0034594	.0161244
X2	.0026436	.0010021	2.64	0.008	.0006795	.0046076
X3_1	.256097	.1295624	1.98	0.048	.0021593	.5100347
X4_1	.3328007	.1301939	2.56	0.011	.0776253	.587976
X5	1.16893	.2186903	5.35	0.000	.7403053	1.597555
X6	.0228157	.0264376	0.86	0.388	-.029001	.0746323
X7_1	.3368117	.2247812	1.50	0.134	-.1037514	.7773747
X7_2	-1.108413	.2658594	-4.17	0.000	-1.629488	-.5873385
X7_4	.0500376	.2542627	0.20	0.844	-.4483082	.5483833
X8_1	.0540334	.1582932	0.34	0.733	-.2562156	.3642824
X9_1	.4577791	.2362133	1.94	0.053	-.0051905	.9207487
X9_2	.0909087	.2196146	0.41	0.679	-.3395281	.5213455
X10	.0267049	.0237322	1.13	0.260	-.0198093	.0732192
X11_1	.1442592	.2855822	0.51	0.613	-.4154717	.70399
X11_2	.4163849	.2275166	1.83	0.067	-.0295394	.8623093
X11_3	-.2300493	.2507329	-0.92	0.359	-.7214768	.2613782
X11_4	.8071347	.2923746	2.76	0.006	.234091	1.380178
_cons	-9.810482	1.220197	-8.04	0.000	-12.20203	-7.418939

Alternative parameterization: Gammas are deviations from proportionality

Y	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
Beta						
X1	-.0013955	.0025458	-0.55	0.584	-.0063853	.0035942
X2	.0026436	.0010021	2.64	0.008	.0006795	.0046076
X3_1	.256097	.1295624	1.98	0.048	.0021593	.5100347
X4_1	.3328007	.1301939	2.56	0.011	.0776253	.587976
X5	1.16893	.2186903	5.35	0.000	.7403053	1.597555
X6	.0228157	.0264376	0.86	0.388	-.029001	.0746323
X7_1	.3368117	.2247812	1.50	0.134	-.1037514	.7773747
X7_2	.3315182	.1899346	1.75	0.081	-.0407468	.7037832
X7_4	.9863335	.2380848	4.14	0.000	.5196959	1.452971
X8_1	.0540334	.1582932	0.34	0.733	-.2562156	.3642824
X9_1	.4577791	.2362133	1.94	0.053	-.0051905	.9207487
X9_2	.0909087	.2196146	0.41	0.679	-.3395281	.5213455
X10	-.0165933	.0219647	-0.76	0.450	-.0596434	.0264568
X11_1	1.529488	.2707881	5.65	0.000	.9987534	2.060223
X11_2	.4163849	.2275166	1.83	0.067	-.0295394	.8623093
X11_3	-.2300493	.2507329	-0.92	0.359	-.7214768	.2613782
X11_4	.8071347	.2923746	2.76	0.006	.234091	1.380178
Gamma_2						
X1	.0111874	.0035257	3.17	0.002	.0042772	.0180976

```
        X7_2 |  -1.439931   .2626463    -5.48   0.000    -1.954709   -.9251541
        X7_4 |   -.936296   .2801834    -3.34   0.001    -1.485445   -.3871466
         X10 |   .0432982   .0151428     2.86   0.004     .0136187    .0729776
       X11_1 |  -1.385229   .2339696    -5.92   0.000    -1.843801   -.9266571
-------------+----------------------------------------------------------------
Alpha        |
     _cons_1 |  -4.802937   1.131667    -4.24   0.000    -7.020963   -2.584911
     _cons_2 |  -9.810482   1.220197    -8.04   0.000    -12.20203   -7.418939
------------------------------------------------------------------------------

==================================================================================
. ologit Y X1 X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X8_1 X9_1 X9_2 X11_1 X11_2 X11_3
  X11_4

Iteration 0:   log likelihood = -1185.3939
Iteration 1:   log likelihood = -1104.0189
Iteration 2:   log likelihood = -1102.1739
Iteration 3:   log likelihood = -1102.1677
Iteration 4:   log likelihood = -1102.1677

Ordered logistic regression                       Number of obs   =       1284
                                                  LR chi2(16)     =     166.45
                                                  Prob > chi2     =     0.0000
Log likelihood = -1102.1677                       Pseudo R2       =     0.0702
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          X1 |   .0032466   .0021744     1.49   0.135    -.0010151    .0075083
          X2 |   .0023236   .0009643     2.41   0.016     .0004335    .0042137
        X3_1 |    .270989    .124822     2.17   0.030     .0263424    .5156357
        X4_1 |   .3153977   .1273527     2.48   0.013      .065791    .5650043
          X5 |    1.16172   .2130962     5.45   0.000     .7440593    1.579381
          X6 |   .0238039   .0171796     1.39   0.166    -.0098676    .0574753
        X7_1 |   .3705269   .2304213     1.61   0.108    -.0810906    .8221443
        X7_2 |  -.0652964    .171212    -0.38   0.703    -.4008658    .2702731
        X7_4 |   .6100244   .1931031     3.16   0.002     .2315494    .9884995
        X8_1 |   .0749266   .1527275     0.49   0.624    -.2244137     .374267
        X9_1 |    .423311   .2351416     1.80   0.072     -.037558    .8841801
        X9_2 |   .0734328     .22168     0.33   0.740    -.3610521    .5079177
       X11_1 |   1.015086    .246562     4.12   0.000     .5318334    1.498339
       X11_2 |   .4264994   .2312421     1.84   0.065    -.0267268    .8797257
       X11_3 |  -.2678462   .2546314    -1.05   0.293    -.7669145    .2312222
       X11_4 |   .8755791    .298883     2.93   0.003     .2897792    1.461379
-------------+----------------------------------------------------------------
       /cut1 |   5.465044   .8655352                      3.768626    7.161462
       /cut2 |   8.686503   .8924652                      6.937303     10.4357
------------------------------------------------------------------------------

Brant Test of Parallel Regression Assumption

    Variable |      chi2   p>chi2    df
-------------+--------------------------
         All |     82.99    0.000    16
-------------+--------------------------
          X1 |     10.52    0.001     1
          X2 |      0.27    0.600     1
        X3_1 |      2.59    0.107     1
        X4_1 |      0.39    0.531     1
          X5 |      2.11    0.146     1
          X6 |      0.66    0.415     1
        X7_1 |      5.79    0.016     1
        X7_2 |     34.64    0.000     1
        X7_4 |     17.45    0.000     1
        X8_1 |      0.01    0.908     1
        X9_1 |      1.01    0.316     1
        X9_2 |      0.02    0.891     1
       X11_1 |      6.25    0.012     1
       X11_2 |      0.41    0.524     1
       X11_3 |      0.31    0.578     1
       X11_4 |      0.05    0.824     1
----------------------------------------
```

```
. fitstat

Measures of Fit for ologit of Y

Log-Lik Intercept Only:      -1185.394   Log-Lik Full Model:          -1102.168
D(1266):                      2204.335   LR(16):                        166.452
                                         Prob > LR:                       0.000
McFadden's R2:                   0.070   McFadden's Adj R2:               0.055
ML (Cox-Snell) R2:               0.122   Cragg-Uhler(Nagelkerke) R2:      0.144
McKelvey & Zavoina's R2:         0.145
Variance of y*:                  3.849   Variance of error:               3.290
Count R2:                        0.609   Adj Count R2:                    0.027
AIC:                             1.745   AIC*n:                        2240.335
BIC:                         -6857.358   BIC':                          -51.929
BIC used by Stata:            2333.175   AIC used by Stata:            2240.335


. gologit2 Y X1 X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X8_1 X9_1 X9_2 X11_1 X11_2 X11_3
  X11_4, autofit gamma lrf

------------------------------------------------------------------------------
Testing parallel lines assumption using the .05 level of significance...

Step  1:  Constraints for parallel lines imposed for X2 (P Value = 0.8999)
Step  2:  Constraints for parallel lines imposed for X11_4 (P Value = 0.8085)
Step  3:  Constraints for parallel lines imposed for X9_2 (P Value = 0.7938)
Step  4:  Constraints for parallel lines imposed for X6 (P Value = 0.5572)
Step  5:  Constraints for parallel lines imposed for X11_3 (P Value = 0.3974)
Step  6:  Constraints for parallel lines imposed for X4_1 (P Value = 0.4446)
Step  7:  Constraints for parallel lines imposed for X11_2 (P Value = 0.4342)
Step  8:  Constraints for parallel lines imposed for X8_1 (P Value = 0.4298)
Step  9:  Constraints for parallel lines imposed for X3_1 (P Value = 0.2035)
Step  10: Constraints for parallel lines imposed for X9_1 (P Value = 0.1030)
Step  11: Constraints for parallel lines are not imposed for
          X1 (P Value = 0.00108)
          X5 (P Value = 0.04286)
          X7_1 (P Value = 0.00814)
          X7_2 (P Value = 0.00000)
          X7_4 (P Value = 0.00002)
          X11_1 (P Value = 0.00000)

Wald test of parallel lines assumption for the final model:

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 3)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 4)  [>4_semester]X6 - [4_semester]X6 = 0
 ( 5)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 6)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 7)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 8)  [>4_semester]X8_1 - [4_semester]X8_1 = 0
 ( 9)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 (10)  [>4_semester]X9_1 - [4_semester]X9_1 = 0

           chi2( 10) =    7.34
         Prob > chi2 =    0.6933

pl(X2 X11_4 X9_2 X6 X11_3 X4_1 X11_2 X8_1 X3_1 X9_1)
------------------------------------------------------------------------------

Generalized Ordered Logit Estimates                Number of obs   =       1284
                                                   LR chi2(22)     =     239.67
                                                   Prob > chi2     =     0.0000
Log likelihood = -1065.5569                        Pseudo R2       =     0.1011

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 3)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 4)  [>4_semester]X6 - [4_semester]X6 = 0
 ( 5)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 6)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 7)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
```

```
 ( 8)  [>4_semester]X8_1 - [4_semester]X8_1 = 0
 ( 9)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 (10)  [>4_semester]X9_1 - [4_semester]X9_1 = 0
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
>4_semester  |
          X1 |  -.0013925   .0025302    -0.55   0.582    -.0063516    .0035667
          X2 |   .0025672   .0009916     2.59   0.010     .0006237    .0045107
        X3_1 |   .2602472     .12894     2.02   0.044     .0075294     .512965
        X4_1 |   .3368713   .1309634     2.57   0.010     .0801877    .5935548
          X5 |   1.397938   .2473644     5.65   0.000      .913113    1.882764
          X6 |   .0201208   .0175244     1.15   0.251    -.0142265    .0544681
        X7_1 |   .5741729   .2456849     2.34   0.019     .0926394    1.055706
        X7_2 |    .424019    .195099     2.17   0.030      .041632    .8064059
        X7_4 |    1.11012    .243181     4.56   0.000     .6334944    1.586746
        X8_1 |   .0522916   .1547275     0.34   0.735    -.2509686    .3555519
        X9_1 |   .4598244   .2374548     1.94   0.053    -.0055785    .9252274
        X9_2 |   .1048788   .2229224     0.47   0.638    -.3320411    .5417986
       X11_1 |   1.556177   .2715557     5.73   0.000     1.023937    2.088416
       X11_2 |   .4073846   .2270879     1.79   0.073    -.0376996    .8524687
       X11_3 |  -.2396498   .2511834    -0.95   0.340    -.7319601    .2526606
       X11_4 |   .8082851   .2914676     2.77   0.006     .2370191    1.379551
       _cons |  -6.082514   .9482689    -6.41   0.000    -7.941087   -4.223942
-------------+----------------------------------------------------------------
4_semester   |
          X1 |   .0101964   .0032359     3.15   0.002      .003854    .0165387
          X2 |   .0025672   .0009916     2.59   0.010     .0006237    .0045107
        X3_1 |   .2602472     .12894     2.02   0.044     .0075294     .512965
        X4_1 |   .3368713   .1309634     2.57   0.010     .0801877    .5935548
          X5 |   .7667769   .2956827     2.59   0.010     .1872495    1.346304
          X6 |   .0201208   .0175244     1.15   0.251    -.0142265    .0544681
        X7_1 |  -.1130625   .2819702    -0.40   0.688    -.6657138    .4395889
        X7_2 |  -1.303789   .2781391    -4.69   0.000    -1.848932   -.7586465
        X7_4 |  -.1920127   .2629888    -0.73   0.465    -.7074614     .323436
        X8_1 |   .0522916   .1547275     0.34   0.735    -.2509686    .3555519
        X9_1 |   .4598244   .2374548     1.94   0.053    -.0055785    .9252274
        X9_2 |   .1048788   .2229224     0.47   0.638    -.3320411    .5417986
       X11_1 |   .0389872   .2921101     0.13   0.894    -.5335381    .6115126
       X11_2 |   .4073846   .2270879     1.79   0.073    -.0376996    .8524687
       X11_3 |  -.2396498   .2511834    -0.95   0.340    -.7319601    .2526606
       X11_4 |   .8082851   .2914676     2.77   0.006     .2370191    1.379551
       _cons |  -7.498187    1.09127    -6.87   0.000    -9.637037   -5.359336
------------------------------------------------------------------------------

Alternative parameterization: Gammas are deviations from proportionality
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Beta         |
          X1 |  -.0013925   .0025302    -0.55   0.582    -.0063516    .0035667
          X2 |   .0025672   .0009916     2.59   0.010     .0006237    .0045107
        X3_1 |   .2602472     .12894     2.02   0.044     .0075294     .512965
        X4_1 |   .3368713   .1309634     2.57   0.010     .0801877    .5935548
          X5 |   1.397938   .2473644     5.65   0.000      .913113    1.882764
          X6 |   .0201208   .0175244     1.15   0.251    -.0142265    .0544681
        X7_1 |   .5741729   .2456849     2.34   0.019     .0926394    1.055706
        X7_2 |    .424019    .195099     2.17   0.030      .041632    .8064059
        X7_4 |    1.11012    .243181     4.56   0.000     .6334944    1.586746
        X8_1 |   .0522916   .1547275     0.34   0.735    -.2509686    .3555519
        X9_1 |   .4598244   .2374548     1.94   0.053    -.0055785    .9252274
        X9_2 |   .1048788   .2229224     0.47   0.638    -.3320411    .5417986
       X11_1 |   1.556177   .2715557     5.73   0.000     1.023937    2.088416
       X11_2 |   .4073846   .2270879     1.79   0.073    -.0376996    .8524687
       X11_3 |  -.2396498   .2511834    -0.95   0.340    -.7319601    .2526606
       X11_4 |   .8082851   .2914676     2.77   0.006     .2370191    1.379551
-------------+----------------------------------------------------------------
Gamma_2      |
          X1 |   .0115888   .0035466     3.27   0.001     .0046376    .0185401
          X5 |  -.6311615   .3116697    -2.03   0.043    -1.242023   -.0203001
        X7_1 |  -.6872353   .2597113    -2.65   0.008     -1.19626   -.1782105
        X7_2 |  -1.727808   .2933089    -5.89   0.000    -2.302683   -1.152933
```

```
        X7_4 |  -1.302133   .3064027    -4.25   0.000    -1.902671   -.7015947
       X11_1 |   -1.51719   .2451831    -6.19   0.000     -1.99774   -1.036639
-------------+----------------------------------------------------------------
Alpha        |
     _cons_1 |  -6.082514   .9482689    -6.41   0.000    -7.941087   -4.223942
     _cons_2 |  -7.498187    1.09127    -6.87   0.000    -9.637037   -5.359336
------------------------------------------------------------------------------

==================================================================================

. ologit Y X1 X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X9_1 X9_2 X11_1 X11_2 X11_3 X11_4

Iteration 0:   log likelihood = -1185.3939
Iteration 1:   log likelihood =  -1104.133
Iteration 2:   log likelihood = -1102.2942
Iteration 3:   log likelihood = -1102.2881
Iteration 4:   log likelihood = -1102.2881

Ordered logistic regression                       Number of obs   =       1284
                                                  LR chi2(15)     =     166.21
                                                  Prob > chi2     =     0.0000
Log likelihood = -1102.2881                       Pseudo R2       =     0.0701
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          X1 |   .0033351   .0021667     1.54   0.124    -.0009115    .0075817
          X2 |   .0022816   .0009605     2.38   0.018     .0003991    .0041642
        X3_1 |   .2760998   .1243812     2.22   0.026     .0323171    .5198825
        X4_1 |   .3157508    .127363     2.48   0.013     .0661239    .5653777
          X5 |   1.162501   .2130738     5.46   0.000     .7448838    1.580118
          X6 |   .0266864   .0161525     1.65   0.099    -.0049719    .0583447
        X7_1 |   .3504995   .2266898     1.55   0.122    -.0938043    .7948034
        X7_2 |  -.0666011   .1711883    -0.39   0.697     -.402124    .2689218
        X7_4 |   .5880401   .1878072     3.13   0.002     .2199447    .9561354
        X9_1 |   .4381553   .2331225     1.88   0.060    -.0187564     .895067
        X9_2 |   .0810892   .2210664     0.37   0.714     -.352193    .5143714
       X11_1 |   1.005093   .2456998     4.09   0.000     .5235302    1.486656
       X11_2 |   .4197486   .2308102     1.82   0.069    -.0326311    .8721282
       X11_3 |  -.2755719   .2541617    -1.08   0.278    -.7737197    .2225759
       X11_4 |   .8756514   .2989348     2.93   0.003       .28975    1.461553
-------------+----------------------------------------------------------------
       /cut1 |   5.432771   .8629196                       3.74148    7.124063
       /cut2 |   8.653755   .8897925                      6.909794    10.39772
------------------------------------------------------------------------------

Brant Test of Parallel Regression Assumption

    Variable |      chi2   p>chi2    df
-------------+--------------------------
         All |     83.13    0.000    15
-------------+--------------------------
          X1 |     10.60    0.001     1
          X2 |      0.26    0.608     1
        X3_1 |      2.59    0.108     1
        X4_1 |      0.40    0.529     1
          X5 |      2.11    0.147     1
          X6 |      0.69    0.407     1
        X7_1 |      5.86    0.016     1
        X7_2 |     34.71    0.000     1
        X7_4 |     18.09    0.000     1
        X9_1 |      1.06    0.303     1
        X9_2 |      0.02    0.900     1
       X11_1 |      6.22    0.013     1
       X11_2 |      0.42    0.518     1
       X11_3 |      0.32    0.573     1
       X11_4 |      0.05    0.821     1
----------------------------------------
```

```
. fitstat

Measures of Fit for ologit of Y

Log-Lik Intercept Only:      -1185.394   Log-Lik Full Model:          -1102.288
D(1267):                      2204.576   LR(15):                        166.212
                                         Prob > LR:                       0.000
McFadden's R2:                   0.070   McFadden's Adj R2:               0.056
ML (Cox-Snell) R2:               0.121   Cragg-Uhler(Nagelkerke) R2:      0.144
McKelvey & Zavoina's R2:         0.145
Variance of y*:                  3.848   Variance of error:               3.290
Count R2:                        0.607   Adj Count R2:                    0.023
AIC:                             1.743   AIC*n:                        2238.576
BIC:                         -6864.275   BIC':                          -58.846
BIC used by Stata:            2326.258   AIC used by Stata:            2238.576


. gologit2 Y X1 X2 X3_1 X4_1 X5 X6 X7_1 X7_2 X7_4 X9_1 X9_2 X11_1 X11_2 X11_3
  X11_4, autofit gamma lrf
------------------------------------------------------------------------------
Testing parallel lines assumption using the .05 level of significance...

Step  1:  Constraints for parallel lines imposed for X2 (P Value = 0.8677)
Step  2:  Constraints for parallel lines imposed for X11_4 (P Value = 0.7909)
Step  3:  Constraints for parallel lines imposed for X9_2 (P Value = 0.7619)
Step  4:  Constraints for parallel lines imposed for X11_3 (P Value = 0.4488)
Step  5:  Constraints for parallel lines imposed for X4_1 (P Value = 0.4382)
Step  6:  Constraints for parallel lines imposed for X11_2 (P Value = 0.4825)
Step  7:  Constraints for parallel lines imposed for X6 (P Value = 0.3516)
Step  8:  Constraints for parallel lines imposed for X3_1 (P Value = 0.2019)
Step  9:  Constraints for parallel lines imposed for X9_1 (P Value = 0.1065)
Step  10: Constraints for parallel lines are not imposed for
          X1 (P Value = 0.00106)
          X5 (P Value = 0.04170)
          X7_1 (P Value = 0.00808)
          X7_2 (P Value = 0.00000)
          X7_4 (P Value = 0.00002)
          X11_1 (P Value = 0.00000)

Wald test of parallel lines assumption for the final model:

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 3)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 4)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 5)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 6)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 7)  [>4_semester]X6 - [4_semester]X6 = 0
 ( 8)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 ( 9)  [>4_semester]X9_1 - [4_semester]X9_1 = 0

           chi2(  9) =    7.00
         Prob > chi2 =    0.6366

pl(X2 X11_4 X9_2 X11_3 X4_1 X11_2 X6 X3_1 X9_1)
------------------------------------------------------------------------------

Generalized Ordered Logit Estimates               Number of obs   =       1284
                                                  LR chi2(21)     =     239.56
                                                  Prob > chi2     =     0.0000
Log likelihood =  -1065.614                       Pseudo R2       =     0.1010

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 3)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 4)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 5)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 6)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 7)  [>4_semester]X6 - [4_semester]X6 = 0
 ( 8)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 ( 9)  [>4_semester]X9_1 - [4_semester]X9_1 = 0
```

```
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
>4_semester  |
          X1 |  -.0013324   .0025235    -0.53   0.598    -.0062783    .0036136
          X2 |    .002538   .0009878     2.57   0.010     .0006019    .0044741
        X3_1 |   .2638797   .1284936     2.05   0.040     .0120369    .5157225
        X4_1 |   .3368852   .1309744     2.57   0.010     .0801802    .5935903
          X5 |   1.399772   .2472998     5.66   0.000     .9150733    1.884471
          X6 |   .0220815   .0165409     1.33   0.182    -.0103381     .054501
        X7_1 |   .5610483   .2425383     2.31   0.021      .085682    1.036415
        X7_2 |   .4237599   .1950967     2.17   0.030     .0413775    .8061423
        X7_4 |   1.096425   .2397578     4.57   0.000     .6265084    1.566342
        X9_1 |   .4706847   .2352335     2.00   0.045     .0096355    .9317339
        X9_2 |   .1105682    .222254     0.50   0.619    -.3250417     .546178
       X11_1 |   1.550175   .2709426     5.72   0.000     1.019137    2.081213
       X11_2 |   .4029698   .2266844     1.78   0.075    -.0413235     .847263
       X11_3 |  -.2448785   .2507046    -0.98   0.329    -.7362505    .2464934
       X11_4 |   .8089695   .2914829     2.78   0.006     .2376734    1.380266
       _cons |  -6.064751   .9467675    -6.41   0.000    -7.920381    -4.20912
-------------+----------------------------------------------------------------
4_semester   |
          X1 |   .0102762   .0032275     3.18   0.001     .0039504    .0166019
          X2 |    .002538   .0009878     2.57   0.010     .0006019    .0044741
        X3_1 |   .2638797   .1284936     2.05   0.040     .0120369    .5157225
        X4_1 |   .3368852   .1309744     2.57   0.010     .0801802    .5935903
          X5 |   .7655319   .2956202     2.59   0.010     .1861269    1.344937
          X6 |   .0220815   .0165409     1.33   0.182    -.0103381     .054501
        X7_1 |   -.126669   .2789959    -0.45   0.650    -.6734908    .4201528
        X7_2 |  -1.304118   .2780459    -4.69   0.000    -1.849078   -.7591581
        X7_4 |  -.2060582   .2596197    -0.79   0.427    -.7149036    .3027871
        X9_1 |   .4706847   .2352335     2.00   0.045     .0096355    .9317339
        X9_2 |   .1105682    .222254     0.50   0.619    -.3250417     .546178
       X11_1 |   .0331623   .2915269     0.11   0.909    -.5382198    .6045445
       X11_2 |   .4029698   .2266844     1.78   0.075    -.0413235     .847263
       X11_3 |  -.2448785   .2507046    -0.98   0.329    -.7362505    .2464934
       X11_4 |   .8089695   .2914829     2.78   0.006     .2376734    1.380266
       _cons |  -7.473321   1.088419    -6.87   0.000    -9.606583   -5.340059
------------------------------------------------------------------------------

Alternative parameterization: Gammas are deviations from proportionality
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Beta         |
          X1 |  -.0013324   .0025235    -0.53   0.598    -.0062783    .0036136
          X2 |    .002538   .0009878     2.57   0.010     .0006019    .0044741
        X3_1 |   .2638797   .1284936     2.05   0.040     .0120369    .5157225
        X4_1 |   .3368852   .1309744     2.57   0.010     .0801802    .5935903
          X5 |   1.399772   .2472998     5.66   0.000     .9150733    1.884471
          X6 |   .0220815   .0165409     1.33   0.182    -.0103381     .054501
        X7_1 |   .5610483   .2425383     2.31   0.021      .085682    1.036415
        X7_2 |   .4237599   .1950967     2.17   0.030     .0413775    .8061423
        X7_4 |   1.096425   .2397578     4.57   0.000     .6265084    1.566342
        X9_1 |   .4706847   .2352335     2.00   0.045     .0096355    .9317339
        X9_2 |   .1105682    .222254     0.50   0.619    -.3250417     .546178
       X11_1 |   1.550175   .2709426     5.72   0.000     1.019137    2.081213
       X11_2 |   .4029698   .2266844     1.78   0.075    -.0413235     .847263
       X11_3 |  -.2448785   .2507046    -0.98   0.329    -.7362505    .2464934
       X11_4 |   .8089695   .2914829     2.78   0.006     .2376734    1.380266
-------------+----------------------------------------------------------------
Gamma_2      |
          X1 |   .0116086    .003546     3.27   0.001     .0046586    .0185586
          X5 |  -.6342402   .3114435    -2.04   0.042    -1.244658    -.023822
        X7_1 |  -.6877173   .2596558    -2.65   0.008    -1.196633   -.1788014
        X7_2 |  -1.727878   .2932406    -5.89   0.000    -2.302619   -1.153137
        X7_4 |  -1.302483   .3063666    -4.25   0.000    -1.902951   -.7020158
       X11_1 |  -1.517013   .2451226    -6.19   0.000    -1.997444   -1.036581
-------------+----------------------------------------------------------------
Alpha        |
     _cons_1 |  -6.064751   .9467675    -6.41   0.000    -7.920381    -4.20912
     _cons_2 |  -7.473321   1.088419    -6.87   0.000    -9.606583   -5.340059
```

```
. ologit Y X1 X2 X3_1 X4_1 X5 X7_1 X7_2 X7_4 X9_1 X9_2 X11_1 X11_2 X11_3 X11_4

Iteration 0:   log likelihood = -1185.3939
Iteration 1:   log likelihood =   -1105.43
Iteration 2:   log likelihood = -1103.6619
Iteration 3:   log likelihood = -1103.6557
Iteration 4:   log likelihood = -1103.6557

Ordered logistic regression                       Number of obs   =       1284
                                                  LR chi2(14)     =     163.48
                                                  Prob > chi2     =     0.0000
Log likelihood = -1103.6557                       Pseudo R2       =     0.0690

------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          X1 |   .0035215   .0021614     1.63   0.103    -.0007148    .0077579
          X2 |    .002091   .0009523     2.20   0.028     .0002245    .0039576
        X3_1 |   .2809397   .1243281     2.26   0.024     .0372611    .5246182
        X4_1 |   .3093379   .1273009     2.43   0.015     .0598328     .558843
          X5 |   .9985843   .1881312     5.31   0.000      .629854    1.367315
        X7_1 |   .3047735   .2246873     1.36   0.175    -.1356055    .7451525
        X7_2 |  -.0570784   .1710357    -0.33   0.739    -.3923023    .2781454
        X7_4 |   .5176456    .182728     2.83   0.005     .1595054    .8757859
        X9_1 |   .5306558   .2261923     2.35   0.019     .0873269    .9739846
        X9_2 |   .1593907   .2157232     0.74   0.460     -.263419    .5822004
       X11_1 |   1.037232   .2447093     4.24   0.000     .5576101    1.516853
       X11_2 |   .4478988   .2299945     1.95   0.051    -.0028821    .8986797
       X11_3 |  -.2472489   .2533508    -0.98   0.329    -.7438073    .2493095
       X11_4 |   .8981603   .2988513     3.01   0.003     .3124225    1.483898
-------------+----------------------------------------------------------------
       /cut1 |   4.802276   .7715693                      3.290028    6.314524
       /cut2 |   8.017129   .7986157                      6.451871    9.582387
------------------------------------------------------------------------------

Brant Test of Parallel Regression Assumption

    Variable |      chi2   p>chi2    df
-------------+--------------------------
         All |     83.49    0.000    14
-------------+--------------------------
          X1 |     10.96    0.001     1
          X2 |      0.34    0.561     1
        X3_1 |      2.77    0.096     1
        X4_1 |      0.39    0.534     1
          X5 |      3.56    0.059     1
        X7_1 |      6.54    0.011     1
        X7_2 |     35.17    0.000     1
        X7_4 |     20.50    0.000     1
        X9_1 |      0.70    0.404     1
        X9_2 |      0.11    0.742     1
       X11_1 |      6.04    0.014     1
       X11_2 |      0.50    0.479     1
       X11_3 |      0.37    0.542     1
       X11_4 |      0.07    0.790     1
----------------------------------------

. fitstat

Measures of Fit for ologit of Y

Log-Lik Intercept Only:      -1185.394   Log-Lik Full Model:          -1103.656
D(1268):                      2207.311   LR(14):                        163.476
                                         Prob > LR:                       0.000
McFadden's R2:                   0.069   McFadden's Adj R2:               0.055
ML (Cox-Snell) R2:               0.120   Cragg-Uhler(Nagelkerke) R2:      0.142
McKelvey & Zavoina's R2:         0.143
Variance of y*:                  3.837   Variance of error:               3.290
Count R2:                        0.611   Adj Count R2:                    0.031
AIC:                             1.744   AIC*n:                        2239.311
BIC:                         -6868.697   BIC':                          -63.268
BIC used by Stata:            2321.835   AIC used by Stata:            2239.311
```

```
. gologit2 Y X1 X2 X3_1 X4_1 X5 X7_1 X7_2 X7_4 X9_1 X9_2 X11_1 X11_2 X11_3 X11_4,
  autofit gamma lrf

------------------------------------------------------------------------------
Testing parallel lines assumption using the .05 level of significance...

Step  1:  Constraints for parallel lines imposed for X2 (P Value = 0.8019)
Step  2:  Constraints for parallel lines imposed for X11_4 (P Value = 0.7309)
Step  3:  Constraints for parallel lines imposed for X9_2 (P Value = 0.6179)
Step  4:  Constraints for parallel lines imposed for X11_3 (P Value = 0.4436)
Step  5:  Constraints for parallel lines imposed for X4_1 (P Value = 0.4092)
Step  6:  Constraints for parallel lines imposed for X11_2 (P Value = 0.4809)
Step  7:  Constraints for parallel lines imposed for X3_1 (P Value = 0.1910)
Step  8:  Constraints for parallel lines imposed for X9_1 (P Value = 0.1104)
Step  9:  Constraints for parallel lines imposed for X5 (P Value = 0.0530)
Step  10: Constraints for parallel lines are not imposed for
          X1 (P Value = 0.00388)
          X7_1 (P Value = 0.00505)
          X7_2 (P Value = 0.00000)
          X7_4 (P Value = 0.00002)
          X11_1 (P Value = 0.00000)

Wald test of parallel lines assumption for the final model:

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 3)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 4)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 5)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 6)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 7)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 ( 8)  [>4_semester]X9_1 - [4_semester]X9_1 = 0
 ( 9)  [>4_semester]X5 - [4_semester]X5 = 0

           chi2(  9) =   10.48
         Prob > chi2 =    0.3130

pl(X2 X11_4 X9_2 X11_3 X4_1 X11_2 X3_1 X9_1 X5)
------------------------------------------------------------------------------

Generalized Ordered Logit Estimates               Number of obs   =       1284
                                                  LR chi2(19)     =     234.12
                                                  Prob > chi2     =     0.0000
Log likelihood = -1068.3363                       Pseudo R2       =     0.0988

 ( 1)  [>4_semester]X2 - [4_semester]X2 = 0
 ( 2)  [>4_semester]X11_4 - [4_semester]X11_4 = 0
 ( 3)  [>4_semester]X9_2 - [4_semester]X9_2 = 0
 ( 4)  [>4_semester]X11_3 - [4_semester]X11_3 = 0
 ( 5)  [>4_semester]X4_1 - [4_semester]X4_1 = 0
 ( 6)  [>4_semester]X11_2 - [4_semester]X11_2 = 0
 ( 7)  [>4_semester]X3_1 - [4_semester]X3_1 = 0
 ( 8)  [>4_semester]X9_1 - [4_semester]X9_1 = 0
 ( 9)  [>4_semester]X5 - [4_semester]X5 = 0
------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
>4_semester  |
          X1 |  -.0006544   .0024973    -0.26   0.793    -.0055491    .0042402
          X2 |   .0024116   .0009824     2.45   0.014     .0004861     .004337
        X3_1 |   .2578131   .1278138     2.02   0.044     .0073027    .5083235
        X4_1 |   .3266435   .1304463     2.50   0.012     .0709733    .5823136
          X5 |   1.058505    .194377     5.45   0.000     .6775332    1.439477
        X7_1 |   .5446661   .2410267     2.26   0.024     .0722624     1.01707
        X7_2 |   .4286017   .1939713     2.21   0.027     .0484249    .8087785
        X7_4 |   1.050081   .2365454     4.44   0.000     .5864605    1.513701
        X9_1 |   .5461143   .2279381     2.40   0.017     .0993638    .9928649
        X9_2 |   .1818499   .2158981     0.84   0.400    -.2413027    .6050024
       X11_1 |   1.579369   .2684862     5.88   0.000     1.053145    2.105592
       X11_2 |    .422416   .2243413     1.88   0.060    -.0172848    .8621169
       X11_3 |  -.2391083   .2481534    -0.96   0.335      -.72548    .2472635
       X11_4 |   .8309618   .2901162     2.86   0.004     .2623446    1.399579
```

_cons	-4.99792	.801885	-6.23	0.000	-6.569586	-3.426254
4_semester						
X1	.0092944	.0031795	2.92	0.003	.0030626	.0155262
X2	.0024116	.0009824	2.45	0.014	.0004861	.004337
X3_1	.2578131	.1278138	2.02	0.044	.0073027	.5083235
X4_1	.3266435	.1304463	2.50	0.012	.0709733	.5823136
X5	1.058505	.194377	5.45	0.000	.6775332	1.439477
X7_1	-.1858086	.279096	-0.67	0.506	-.7328267	.3612096
X7_2	-1.299184	.2789528	-4.66	0.000	-1.845921	-.7524463
X7_4	-.2710785	.2571584	-1.05	0.292	-.7750997	.2329428
X9_1	.5461143	.2279381	2.40	0.017	.0993638	.9928649
X9_2	.1818499	.2158981	0.84	0.400	-.2413027	.6050024
X11_1	.0049631	.2891256	0.02	0.986	-.5617126	.5716388
X11_2	.422416	.2243413	1.88	0.060	-.0172848	.8621169
X11_3	-.2391083	.2481534	-0.96	0.335	-.72548	.2472635
X11_4	.8309618	.2901162	2.86	0.004	.2623446	1.399579
_cons	-8.125549	.8800683	-9.23	0.000	-9.850451	-6.400646

Alternative parameterization: Gammas are deviations from proportionality

Y	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
Beta						
X1	-.0006544	.0024973	-0.26	0.793	-.0055491	.0042402
X2	.0024116	.0009824	2.45	0.014	.0004861	.004337
X3_1	.2578131	.1278138	2.02	0.044	.0073027	.5083235
X4_1	.3266435	.1304463	2.50	0.012	.0709733	.5823136
X5	1.058505	.194377	5.45	0.000	.6775332	1.439477
X7_1	.5446661	.2410267	2.26	0.024	.0722624	1.01707
X7_2	.4286017	.1939713	2.21	0.027	.0484249	.8087785
X7_4	1.050081	.2365454	4.44	0.000	.5864605	1.513701
X9_1	.5461143	.2279381	2.40	0.017	.0993638	.9928649
X9_2	.1818499	.2158981	0.84	0.400	-.2413027	.6050024
X11_1	1.579369	.2684862	5.88	0.000	1.053145	2.105592
X11_2	.422416	.2243413	1.88	0.060	-.0172848	.8621169
X11_3	-.2391083	.2481534	-0.96	0.335	-.72548	.2472635
X11_4	.8309618	.2901162	2.86	0.004	.2623446	1.399579
Gamma_2						
X1	.0099488	.0034452	2.89	0.004	.0031964	.0167012
X7_1	-.7304747	.26051	-2.80	0.005	-1.241065	-.2198845
X7_2	-1.727785	.2947907	-5.86	0.000	-2.305565	-1.150006
X7_4	-1.321159	.3074562	-4.30	0.000	-1.923762	-.7185564
X11_1	-1.574406	.2447004	-6.43	0.000	-2.05401	-1.094802
Alpha						
_cons_1	-4.99792	.801885	-6.23	0.000	-6.569586	-3.426254
_cons_2	-8.125549	.8800683	-9.23	0.000	-9.850451	-6.400646

BIODATA PENULIS

MIKHRATUNNISA lahir pada 14 Maret 1989 di Rabangodu Bima, Nusa Tenggara Barat sebagai anak pertama dari empat bersaudara. Jenjang pendidikan yang telah ditempuh penulis yaitu Sekolah Dasar di SDN N0. 1 Kandai II pada tahun 1995-1999 dan ditamatkan di SD INPRES Manggelewa. Tahun pertama dan kedua Pendidikan Menengah Pertama ditempuh di SLTP Negeri 1 Dompu dan ditamatkan di SLTP Negeri 1 Kempo. Pada tahun 2004-2007 melanjutkan Pendidikan Menengah Atas di SMA Negeri 1 Dompu. Pendidikan tinggi dimulai pada tahun 2007 di Program Studi Matematika Fakultas Matematika dan Ilmu Pengetahuan Alam (FMIPA) Universitas Mataram, melalui jalur PMDK dan lulus pada bulan Februari tahun 2012. Pada tahun 2013 melanjutkan pendidikan S2 di Institut Teknologi Sepuluh Nopember (ITS) Surabaya, Jurusan Statistika Fakultas Matematika dan Ilmu Pengetahuan Alam (FMIPA) melalui program Beasiswa Pendidikan Pascasarjana Dalam Negeri tahun 2013-2015 oleh Direktorat Pendidikan Tinggi (Ditjen Dikti).

Surabaya, Juni 2015

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