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IMPERFECT MAINTENANCE MODELS: SIMULATION, ESTIMATION, AND GOODNESS-OF-FIT TEST

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Master Internship Report

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Imperfect Maintenance Models: Simulation, Estimation, and Goodness-of-Fit Test

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Chapter 1

Introduction

The industrial facilities are subject to material maintenance operations in order to keep them in good working condition, with a number of constraints relating to the safety, availability and costs. Maintenance, contributing through its effects on the operational reliability of materials, plays an important role in risk control and is a determinant element performance of an installation. Therefore, to evaluate quantitatively the impact of the maintenance and, if necessary, optimize it, constitutes an essential industrial issue.

Therefore, it is necessary to model the process of the failures and the maintenance of the complex systems using random point process. The first basic model is supposed that the maintenance is minimal, i.e the system after maintenance is in the same state as before. We call this maintenance As Bad As Old (ABAO) and the corresponding random processes are non-homogeneous Poisson processes (NHPP). The second basic model is to assume that maintenance is perfect, i.e the system after maintenance becomes new. We call this maintenance As Good As New (AGAN) and the corresponding random processes are the renewal processes. In reality, the effect of maintenance is between these two extreme cases. This is what we call as imperfect maintenance models.

Électricité de France (EDF) and Laboratoire Jean-Kuntzmann (LJK) work together for several years as part of the strategic project of EDF on extending the life time of the central of the electricity production. This collaboration has developed many imperfect maintenance models, and the methodology of estimation of maintenance efficiency and forecasting of operational reliability [7, 9, 6]. Nowadays, before applying the models proposed, it is important to have the methods to choose the most suitable models for each dataset. Then, we have to do a statistical test, called goodness-of-fit test.

Lindqvist and Rannestad [13] proposed a method for constructing an exact goodness-of-fit test for NHPP (i.e for minimum maintenance). In this project, we are interested in adapting this method to the case of specific imperfect maintenance models. The method is based on the existence of a sufficient statistic S for the model we want to test. The interest of a sufficient statistic lies in the fact that, conditionally on this statistic, the distribution of observations is independent of the unknown parameters of the model. The idea is therefore to be able to simulate dataset conditionally given the observed value of the sufficient statistic s_{obs} on the data we want to test. Then, it is possible to construct a conditional test at $S = s_{obs}$ by using the test statistic Z. If we assume that the large value of Z corresponds to a violation of the null hypothesis, the exact p-value of the test can be estimated by the proportion of simulated datasets for which the value of the test statistic is greater than the value of the observed test statistic on the tested data z_{obs}.

Krit [12] has developed various goodness-of-fit tests for specific imperfect maintenance models using the method proposed by Lindqvist and Rannestad. The aim of the project will, at first, to study these tests and the mathematical framework in which they have been developed. It will then program several goodness-of-fit tests. Before doing the test, we will make a program for the simulation of several imperfect maintenance models first. Then, we will make a program of parameter estimation of imperfect maintenance models by using maximum likelihood estimation (MLE) method. All the programs will be developed under R. Since the tests use Monte Carlo methods and Gibbs sampler algorithms to calculate the p-value for the exact test, it will consume much time. Therefore, we need a high speed computation programming. Then, we will apply the R programming interfaced with C++ by using the Rcpp package. In the next chapter we will also show the comparison between usual R and Rcpp computations. Finally, it will carry different experiments on simulated datasets in order to study the behavior of the tests and especially their power. According to the results, we could develop new mathematical technique in order to improve these tests.

Chapter 2

Point Process in $\mathbb{R}^+$

This chapter presents the definitions and general properties of point process in $\mathbb{R}^+$. It also presents in advance the reliability, the failure rate, the mean time to failure (MTTF), and the distribution in the case of lifetime modeling.

2.1 Reliability, failure rate, MTTF, and distribution

2.1.1 Reliability

Let X be a random variable corresponding to the lifetime, the cumulative distribution function (cdf) is defined by:

$$F(x) = \mathbb{P}(X \leq x). \tag{2.1}$$

The probability density function (pdf) is defined by:

$$f(x) = F'(x). \tag{2.2}$$

The reliability is then the function of time R (R for reliability) defined by:

$$\forall x \geq 0,\ R(x) = \mathbb{P}(X > x) \tag{2.3}$$

We have obviously the fact that $R(x) = 1 - F(x)$ and $R'(x) = -f(x)$.

2.1.2 Failure rate

Recall X be a random variable corresponding to the lifetime, the failure rate or the hazard rate is the function of time h defined by:

$$\forall x \geq 0,\ h(x) = \lim_{\Delta x \to 0} \frac{1}{\Delta x} \mathbb{P}(x < X \leq x + \Delta x | X > x) \tag{2.4}$$

In the expression of (2.4), the desired probability is the probability of the system fails between x and $x + \Delta x$ conditionally given that it is in good condition between 0 and x. We note that the reliability is a probability, but the failure rate is not. It is because $h(x)$ could be greater than 1.

We obviously know that:

$$\begin{aligned} f(x) = F'(x) &= \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \big[\mathbb{P}(X \le x + \Delta x) - \mathbb{P}(X \le x) \big] \\ &= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \big[\mathbb{P}(x < X \le x + \Delta x) \big] \end{aligned}$$

Then, we have for Δx is "very small":

$$f(x)\Delta x \approx \mathbb{P}(x < X \le x + \Delta x)$$

and:

$$h(x)\Delta x \approx \mathbb{P}(x < X \le x + \Delta x | X > x)$$

The quantity of $f(x)\Delta x$ can be considered as the probability of the failure just after the time x as well as $h(x)\Delta x$ can be considered as the probability of the failure just after the time x conditionally given that the system does not fail before time x.

It is also easy to know the relationship between failure rate and the reliability:

$$\begin{aligned} h(x) &= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \mathbb{P}(x < X \le x + \Delta x | X > x) \\ &= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \frac{\mathbb{P}(x < X \le x + \Delta x \cap X > x)}{\mathbb{P}(X > x)} \\ &= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \frac{\mathbb{P}(x < X \le x + \Delta x)}{\mathbb{P}(X > x)} \\ &= \frac{1}{R(x)} \lim_{\Delta x \to 0} \frac{1}{\Delta x} \big[F(x + \Delta x) - F(x) \big] \\ &= \frac{f(x)}{R(x)} = \frac{f(x)}{1 - F(x)} = -\frac{R'(x)}{R(x)} = -\frac{d}{dx} \ln R(x) \end{aligned} \tag{2.5}$$

By integrating each side and taking the initial condition $R(0) = 1$, since we suppose that the system doesn't fail at the initial time, we obtain:

$$R(x) = \exp\left(- \int_0^x h(u) du \right). \tag{2.6}$$

2.1.3 Mean time to failure (MTTF)

The mean time to failure (MTTF) of a non repairable system is the duration of the lifetime in average:

$$\text{MTTF} = \mathbb{E}[X] = \int_0^{+\infty} x\, f(x)\, dx \tag{2.7}$$

By partial integration, then we obtain:

$$\text{MTTF} = \big[- xR(x)\big]_0^{+\infty} + \int_0^{+\infty} R(x)\, dx$$

We suppose that $R(x)$ tends to zero faster than $\frac{1}{x}$, we obtain more usual formula for the MTTF:

$$\text{MTTF} = \int_0^{+\infty} R(x)\, dx \tag{2.8}$$

2.1.4 Probability distribution

In this subsection, we will present the exponential distribution and weibull distribution which are widely used for lifetime modeling.

A random variable X is exponential distributed with parameter $\lambda > 0$, denoted by $\exp(\lambda)$, if and only if the cumulative distribution function is:

$$F(x) = 1 - exp(-\lambda x) \tag{2.9}$$

The reliability is:

$$R(x) = \exp(-\lambda x) \tag{2.10}$$

The probability density function is:

$$f(x) = F'(x) = \lambda \exp(\lambda x) \tag{2.11}$$

The MTTF is:

$$MTTF = \mathbb{E}[X] = \int_0^{+\infty} R(x)\, dx = \int_0^{+\infty} \exp(-\lambda x)\, dx = \frac{1}{\lambda} \tag{2.12}$$

The failure rate is:

$$h(x) = \frac{f(x)}{R(x)} = \lambda \tag{2.13}$$

We also say that the exponential distribution has a "memoryless" property. It means that if the system never fails until time t, it would be as good as it were new at time t. Mathematically, we can express:

$$\forall x \geq 0,\ \mathbb{P}(X > t + x | X > t) = P(X > x) \tag{2.14}$$

A random variable X is Weibull distributed with scale parameter $\eta > 0$ and shape parameter $\beta > 0$, denoted by $\mathcal{W}(\eta, \beta)$, if and only if the cumulative distribution function is:

$$F(x) = 1 - \exp\left(- \left(\frac{x}{\eta}\right)^{\beta} \right) \tag{2.15}$$

The reliability is:

$$R(x) = \exp\left(-\left(\frac{x}{\eta}\right)^{\beta}\right) \tag{2.16}$$

The probability density function is:

$$f(x) = F'(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} \exp\left(-\left(\frac{x}{\eta}\right)^{\beta}\right) \tag{2.17}$$

The MTTF is:

$$\text{MTTF} = \eta\Gamma\left(\frac{1}{\beta} + 1\right) \tag{2.18}$$

where Γ is the gamma function defined by:

$$\Gamma(a) = \int_0^{+\infty} x^{a-1} \exp(-x)\, dx \tag{2.19}$$

The failure rate is:

$$h(x) = \frac{f(x)}{R(x)} = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} \tag{2.20}$$

2.2 Counting process

The point process allow to model the occurrence of events in time. In general, the times between occurrences are neither independent nor identically distributed. The maintenance durations are assumed to be negligible, or not taken into account, then the failures and CM times are the same.

Let the function of a system start from time $T_0 = 0$. The failures occur at time $\{T_i\}_{i\geq 1}$. After a failure occurs, the system is either repaired or not, then is put again into the operation. The duration of maintenance is considered negligible.

Let N_t be the random variable that denotes the number of failures in the interval $[0, t]$. $\{N_t\}_{t\geq 0}$ is called a counting process [1] that satisfies:

- $N_0 = 0$ a.s.
- $\{N_t\}_{t\geq 0}$ is an integer.
- the trajectories of $\{N_t\}_{t\leq 0}$ are increasing, constant piecewise functions, and right continuous with left hand limits.

We consider that the process $\{N_t\}_{t\leq 0}$ is simple, i.e we cannot have more than one failure at once:

$$\forall t \geq 0, \Delta t \geq 0, \mathbb{P}(N_{t+\Delta t} - N_t \leq 2) = o(\Delta t) \tag{2.21}$$

A failure process is defined by one of the following random processes [1, 5]:

- $\{T_i\}_{i\geq 1}$, the sequence of the failure times of the system, with $T_0 = 0$. $\mathbf{T_n}$ denotes the vector of the first n failure times $\mathbf{T_n} = (T_1, ..., T_n)$.
- $\{X_i\}_{i\geq 1}$, the sequence of the successive inter-failure times where $\forall i \geq 1$, $X_i = T_i - T_{i-1}$ is the duration between the $(i-1)^{th}$ and the i^{th} failures.
- $\{N_t\}_{t\geq 0}$, the counting process of the failures, where $\forall t \in \mathbb{R}^+$,

$$N_t = \sum_{i=1}^{+\infty} 1_{\{T_i \leq t\}}$$

is the cumulative number of failures occurred on $[0, t]$. We will denote N_{t^-} as left hand limit of N_t, i.e the number of failures occurred on $[0, t)$. It is supposed that $\forall\ t \in \mathbb{R}^+$, $\mathbb{P}(N_t \leq +\infty) = 1$ which means the number of failures occurred at each time t is always finite.

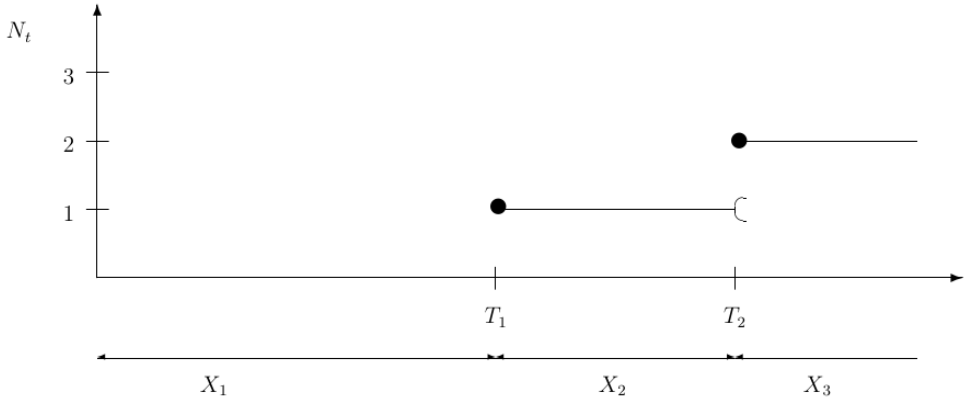

Figure 2.1: Observations of a counting process and corresponding notations

2.3 The failure intensity

In order to be able to predict the future of the process, we need its history. We will introduce the notion of filtration [5]. We consider first that all the random variable N_t, $t > 0$, are defined in the same probability space $(\Omega, \mathcal{A}, \mathcal{P})$. A filtration $\mathcal{H} = \{\mathcal{H}_t\}_{t\leq 0}$ is an increasing sequence of sub-σ-algebras of $\mathcal{A}$:

$$s < t \Rightarrow \mathcal{H}_s \subset \mathcal{H}_t. \tag{2.22}$$

The process $\{N_t\}_{t\geq 0}$ is $\mathcal{H}$-adapted if and only if for all $t \leq 0$, N_t is $\mathcal{H}$-measurable. It means that the filtration $\mathcal{H}_t$ includes of all the information of the history at time t that is likely to influence the random variable N_t. Let $\mathcal{H}_{t^-} = \cap_{s<t}\mathcal{H}_s$. Since the process $\{N_t\}_{t\geq 0}$ is a piecewise constant function that changes its value only at the time $\{T_i\}_{i\geq 1}$, its history

at time t is entirely known by the number and the times of failure occurred between 0 and t. Thus $\mathcal{H}_t$ is the σ-algebra generated by the history of the process at time t:

$$\mathcal{H}_t = \sigma(N_t, T_1, ..., T_{N_t}). \tag{2.23}$$

In this case, the future of the process depends only on its history $\mathcal{H}_t$. It is called self-exciting process [17].

The failure intensity function of the counting process $\{N_t\}_{t\geq 0}$ is:

$$\begin{aligned} \forall t \geq 0, \lambda_t &= \lim_{\Delta t \to 0} \frac{1}{\Delta t} \mathbb{P}(N_{(t+\Delta t)^-} - N_{t^-} = 1 | \mathcal{H}_{t^-}) \\ &= \lim_{\Delta t \to 0} \frac{1}{\Delta t} \mathbb{P}(t \leq T_{N_{t^-}+1} < t + \Delta t | \mathcal{H}_{t^-}) \end{aligned} \tag{2.24}$$

When Δt is "very small", we have:

$$\lambda_t \Delta t \approx \mathbb{P}(N_{(t+\Delta t)^-} - N_{t^-} = 1 | \mathcal{H}_{t^-}) \tag{2.25}$$

The failure intensity function expresses the tendency of the system to have a failure at $[t, t + \Delta t)$, given $\mathcal{H}_t$ which represents all the available information just before t. A self-exciting process is completely characterized by its failure intensity [5].

2.4 The properties of the point process

2.4.1 Reliability

The reliability of the system at time t defines the probability of the system works without failure on any duration start from t, conditionally its history before t. For a self-exciting point process, the reliability function is in the following:

$$\begin{aligned} \forall \tau \in \mathbb{R}^+, R_t(\tau; N_t, T_1, .., T_{N_t}) &= \mathbb{P}(N_{t+\tau} - N_t = 0 | N_t, T_1, .., T_{N_t}) \\ &= \mathbb{P}(T_{N_{t+\tau}} - t > \tau | N_t, T_1, .., T_{N_t}) \end{aligned} \tag{2.26}$$

The reliability at time t is also defined by:

$$R_t(\tau) = \exp\left(- \int_t^{t+\tau} \lambda_s \, ds \right) \tag{2.27}$$

We have also the reliability at the n^{th} failure in the following:

$$R_{T_n}(\tau) = \exp\left(- \int_{T_n}^{T_n+\tau} \lambda_s(n, t_1, ..., t_n) \, ds \right) \tag{2.28}$$

2.4.2 Mean time to failure (MTTF)

The mean time to failure (MTTF) at time t is the conditional expectation of the duration to the next failure start from time t:

$$MTTF_t = \mathbb{E}[T_{N_t+1} - t | N_t, T_1, ..., T_{N_t}] \tag{2.29}$$

We have also the MTTF at time t in the following:

$$MTTF_t = \int_0^{+\infty} R_t(\tau)\, d\tau \tag{2.30}$$

2.5 Likelihood function

In a parametric approach, we assume that the failure intensity is specified using a vector parameters θ. The failure intensity is either denoted λ_t or $\lambda_t(\theta)$. The integral of the failure intensity is called the cumulative intensity function, denoted Λ_t:

$$\Lambda_t = \int_0^t \lambda_s\, ds. \tag{2.31}$$

The estimation of θ can be done by using maximum likelihood estimation (MLE) method. Let $t > 0$ be a deterministic time (time censoring). The likelihood function corresponding to the observation of the failure process on $[0, t]$ is:

$$L_t(\theta) = \left[\prod_{i=1}^{N_t} \lambda_{T_i}(\theta) \right] \exp(-\Lambda_t(\theta)). \tag{2.32}$$

The log-likelihood function is the logarithm of the likelihood:

$$\mathcal{L}_t(\theta) = \sum_{i=1}^{N_t} \ln(\lambda_{T_i}(\theta)) - \int_0^t \lambda_s(\theta)\, ds. \tag{2.33}$$

The maximum likelihood estimator $\hat{\theta}_t$ is defined as the value of $\mathcal{I}_0$ that maximizes the likelihood or equivalently the log-likelihood:

$$\hat{\theta}_t = \underset{\theta \in \mathcal{I}_0}{\operatorname{argmax}}\, \mathcal{L}_t(\theta). \tag{2.34}$$

2.6 Classification of the point processes in $\mathbb{R}^+$

Since the essential characteristic of the point process is its intensity, we can propose a classification of the point process in $\mathbb{R}^+$ in the function of the failure intensity. We will only present 3 usual classes.

2.6.1 Poisson Process

The failure process is a Poisson process if and only if the intensity is a deterministic function of time:

$$\lambda_t = \lambda(t) \tag{2.35}$$

The function in (2.35) expresses that after the maintenance, the system is in the same state as before. The maintenance only put the system into the operation without leaving it more reliable than before. We call this maintenance as minimal or the system is called As Bad As Old (ABAO). The λ function characterizes the wear of the system. When λ is increasing, the system is getting older and degrading. It is usually the case of the material system. When λ is decreasing, the system becomes better and younger. It is usually the case of the software, for which the correction the bugs increase its reliability.

When λ is constant, the Poisson process is called homogeneous (HPP for Homogeneous Poisson Process). Otherwise, we call it as non homogeneous (NHPP for Non Homogeneous Poisson Process).

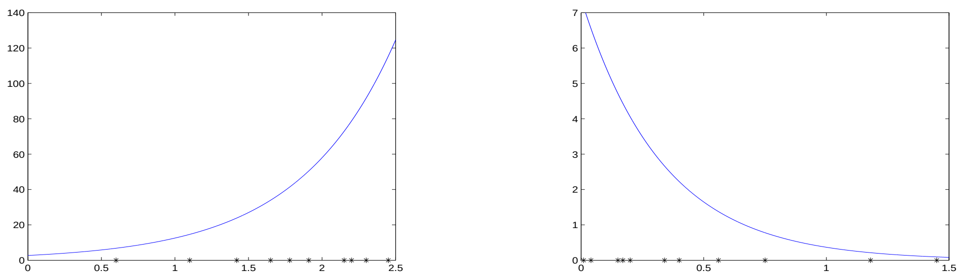

Figure 2.2: NHPP with increasing intensity and decreasing intensity

2.6.2 Renewal process

The failure process is a renewal process if and only if the inter-failure times X_i are independent and identically distributed. The intensity is in the following:

$$\lambda_t = h(t - T_{N_{t^-}}) \tag{2.36}$$

After each failure time T_i, the intensity of the process restarts as the origin. As consequence, it makes the system become like new after each maintenance. The system is also called As Good As New (AGAN), which means the maintenance is perfect.

In the case of the intensity is:

$$\lambda_t = h(N_{t^-}, t - T_{N_{t^-}}), \tag{2.37}$$

the X_i are independent but not identically distributed. The corresponding process is sometimes called quasi-renewal process.

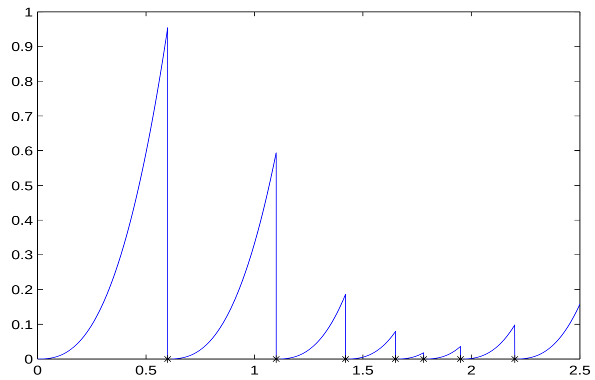

Figure 2.3: Intensity of renewal process

2.6.3 Imperfect maintenance model

In reality, for the materials, we are often situated between both two extreme cases ABAO and AGAN. The maintenance effect is more than minimal, but it does not leave the system as if it were new. It is hence interested to propose the intermediate model between the both cases ABAO and AGAN.

It is the case of arithmetic reduction of age (ARA) model, for which the failure intensity is:

$$\lambda_t = h(t - \rho T_{N_{t^-}}) \tag{2.38}$$

ρ is a parameter characterizing the efficiency of the maintenance:

- $\rho = 0$: the case of ABAO, the maintenance is minimal.
- $\rho = 1$: the case of AGAN, the maintenance is perfect.
- $0 < \rho < 1$: the maintenance is imperfect, but not minimal.

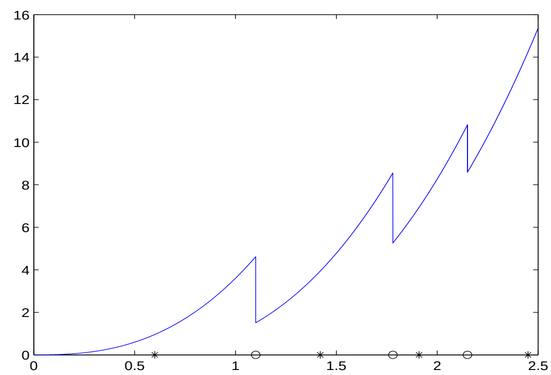

Figure 2.4: Intensity of the imperfect maintenance model

Chapter 3

Repairable Systems

In this chapter, we are interested in system that are repairable and subject to maintenance. There are two kinds of maintenance:

- Corrective maintenance (CM), also called repair, is carried out after a failure and intends to put the system a state in which it can perform its function again.
- Preventive maintenance (PM) is carried out when the system is operating and intends to slow down the wear process and reduce the frequency of occurrence of system failures.

Mathematically, the failure times of a repairable system are random variables and so are the CM. The PM are, in our case, fixed before the system is put into service and they are consequently carried out at deterministic times. These maintenances can have different effects on the system reliability. The basic assumption on maintenance can have different effects in the system reliability. The basic assumption on maintenance efficiency are known as minimal repair or As Bad As Old (ABAO) and perfect repair or As Good As New (AGAN). In the ABAO case, each maintenance leaves the system in the state it was before maintenance. In the AGAN case, each maintenance is perfect and leaves the system as if it were new. It is well known that the reality is between these two extreme cases: standard maintenance reduce failure intensity but does not leave the system AGAN. This is known as imperfect maintenance. The mathematical modeling of the occurence and efficiency of maintenance is done using random point processes. In this framework, the model is completely characterized by its failure intensity. The likelihood function can be written as a function of its intensity.

The most known and widely used models for repairable systems are Nonhomogeneous Poisson Processes (NHPP). They assume that the effect of the CM is ABAO. The two classical intensities are the power-law and the log-linear intensity functions. The objective of our study is to be able to measure the fitness of a given dataset to a give maintenance model.

3.1 Imperfect Maintenance Models

3.1.1 Introduction and Notations

In the context of the imperfect maintenance models, we use several notations as follows.

- PM occur at predetermined deterministic times $\{\tau_i\}_{i\geq 1}$.
- CM occur at random times $\{T_i\}_{i\geq 1}$.
- The number of PM at time t is denoted by m_t.
- The associated failure process is denoted by $\{N_t\}_{t\geq 0}$.
- $\{C_i\}_{i\geq 1}$ denotes the maintenance times (PM and CM).
- $\{W_i\}_{i\geq 1}$ denotes the times between maintenance.
- $\{X_i\}_{i\geq 1}$ denotes the times between two successive CM ($X_i = T_i - T_{i-1}$).
- $\{\chi_i\}_{i\geq 1}$ denotes the times between two successive PM ($\chi_i = \tau_i - \tau_{i-1}$).

Before the first failure, the failure intensity is assumed to be a non null function, non decreasing, deterministic, from $\mathbb{R}^+$ to $\mathbb{R}^+$, and is called the initial intensity and denoted by $\lambda(t)$. The initial intensity represents the intrinsic wear out which means the wear out in the absence of maintenance actions. When the initial intensity is known, an imperfect maintenance model is only characterized by the effect of maintenance actions on the failure intensity. Deterministic PM is a particular case of planned PM, for which Doyen and Gaudoin have proposed a general framework for simultaneous modeling and assessment of aging and maintenance efficiency.

In practice, the effect of maintenance is neither minimal (ABAO) nor maximal (AGAN), it is between these to extreme situations. Indeed, it is more reasonable to think that the maintenance has an effect more than minimal, which means that the system after repair is better than old. It is also less likely that the maintenance leaves the system as good as new. The system in this case after repair is worse than new. This situation is known as better than minimal repair or as imperfect maintenance.

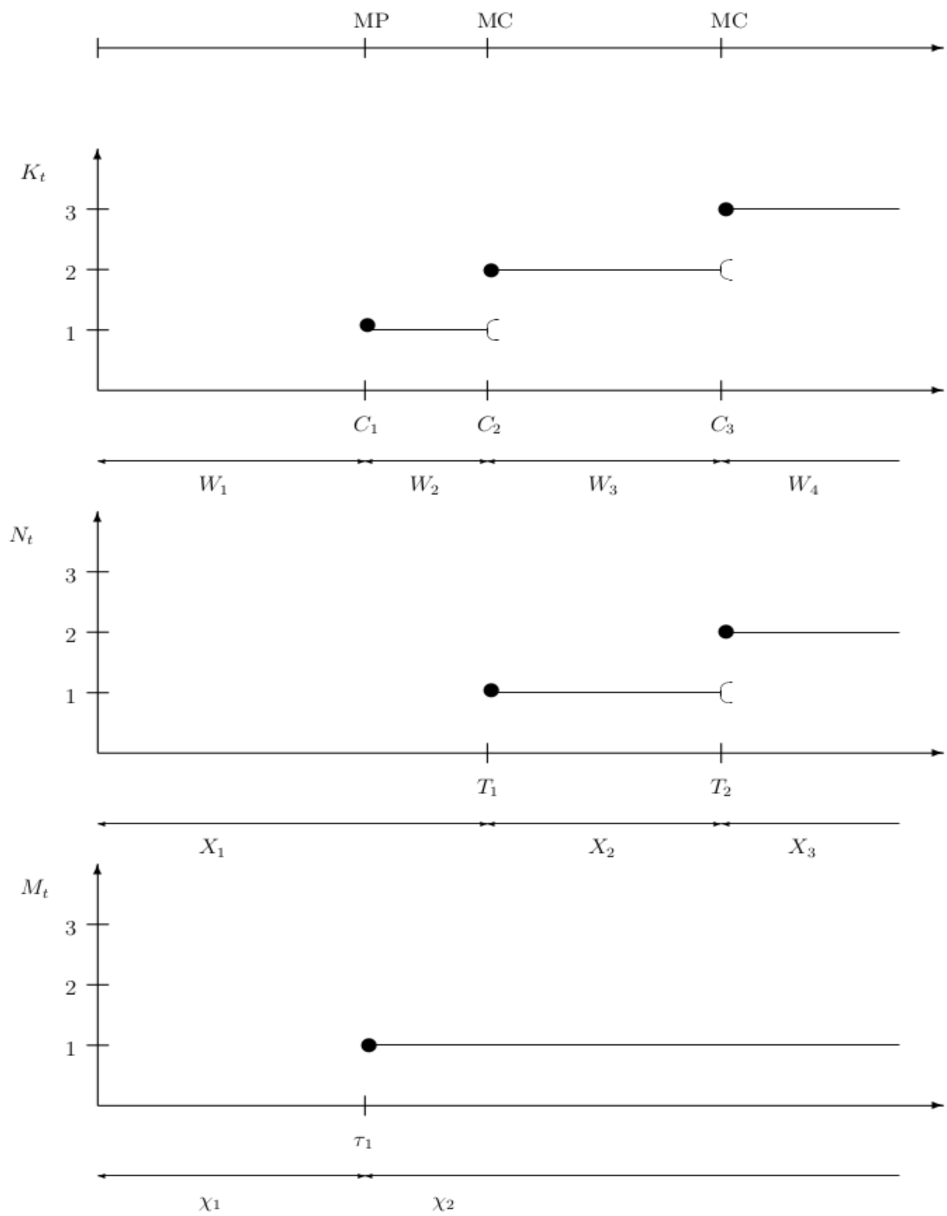

Figure 3.1: Observation of a counting process and the corresponding notations

3.1.2 Generalized Virtual Age Models

Many imperfect maintenance models have been proposed. Virtual age models are among these imperfect maintenance models. They assume that after the i^{th} maintenance the system behaves like a new one that has survived without failure until A_i:

$$\mathbb{P}(W_{i+1} > w | W_1, ..., W_i, A_i) = \mathbb{P}(Y > A_i + w | Y > A_i, A_i) \tag{3.1}$$

where Y is a random variable independent of A_i and with the same distribution as the time to failure of the new unmaintained system. The corresponding failure intensity is:

$$\lambda_t = \lambda(A_{K_{t^-}} + t + C_{K_{t^-}}) \tag{3.2}$$

A_{K_t} is called the effective age at time t and $A_{K_t} + t + C_{K_t}$ is the virtual age at time t. The effective age is the virtual age of the system just after the last maintenance action. The idea that repair actions reduce the age of the system is the basis of Kijima's virtual age models [11]. Several models can be derived. Some of them will be presented in the following and illustrated by a trajectory of the corresponding intensity function.

- AGAN PM-AGAN CM: each maintenance is supposed to be AGAN, i.e each maintenance renews the system. Then, the effective age is $A_i = 0$, $\forall i \geq 1$. The failure intensity is:
$$\lambda_t = \lambda(t - C_{K_{t^-}}) \tag{3.3}$$
One can notice that the failure process is not a renewal process because the failure intensity is not a function of $t - T_{N_{t^-}}$ [17].

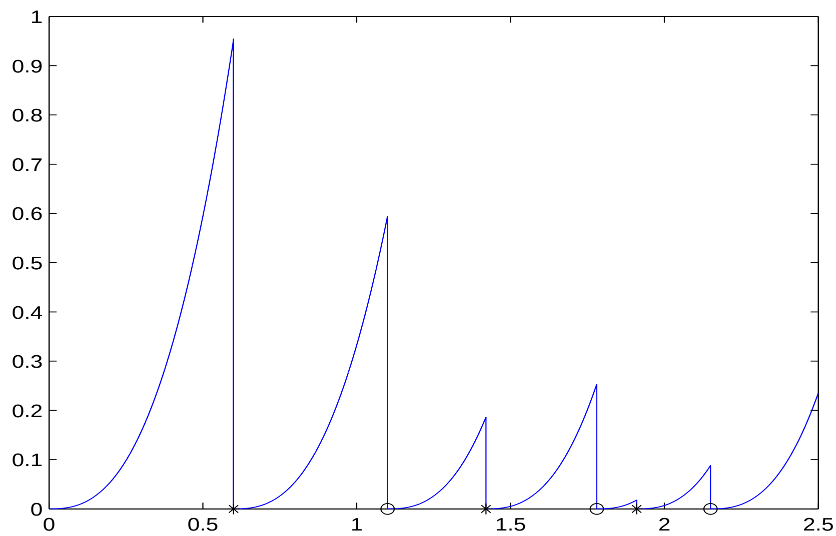

Figure 3.2: The intensity of AGAN PM - AGAN CM

- ABAO PM - ABAO CM: each maintenance is supposed to be minimal. Each maintenance restores the system to the state it was in just before the maintenance action. The effective age is then equal to the last maintenance time $A_i = C_i$, $\forall i \geq 1$. The failure intensity is only a function of time, and the failure process is a NHPP [16].

$$\lambda_t = \lambda(t) \tag{3.4}$$

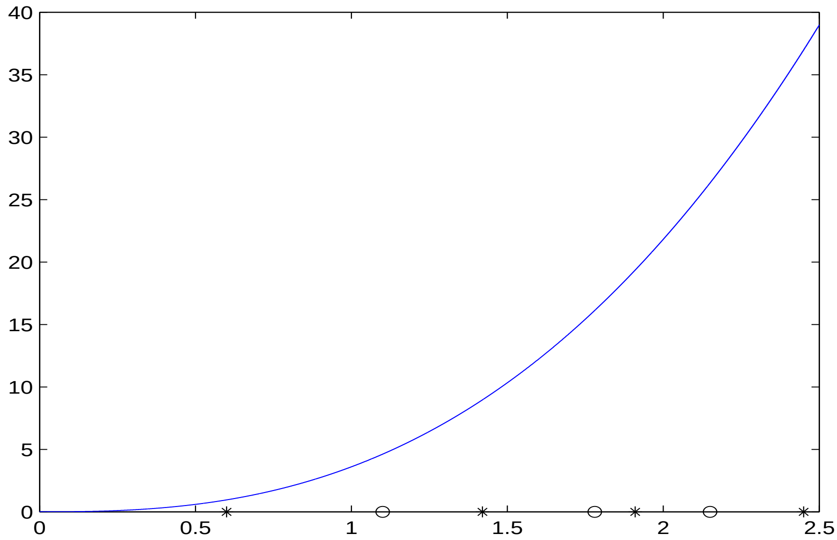

Figure 3.3: The intensity of ABAO PM-ABAO CM

- ABAO PM-AGAN CM: each preventive maintenance is minimal, while each corrective maintenance renews the system. The effective age is then equal to the time elapsed between the last maintenance and the last perfect maintenance, $A_i = C_i - T_{N_{C_i}}$. The failure process is then a renewal process with failure intensity:

$$\lambda_t = \lambda(t - T_{N_{t^-}}) \tag{3.5}$$

The assumption of ABAO PM is unrealistic. Planned PM are carefully prepared in advance, and are expected to have a strong positive effect.Moreover, CM are done after an unplanned failure, and the aim of this type of maintenance is to quickly restore the system into a state in which it can perform its function again. So, the AGAN CM is unlikely. In fact, CM are often assumed to be minimal, as in [18].

- AGAN PM-ABAO CM: the preventive maintenances are perfect and the corrective maintenances are ABAO. Then, the effective ages are $A_i = C_i - \tau_{m_{t^-}}$ and the failure intensity is:

$$\lambda_t = \lambda(t - \tau_{m_{t^-}}) \tag{3.6}$$

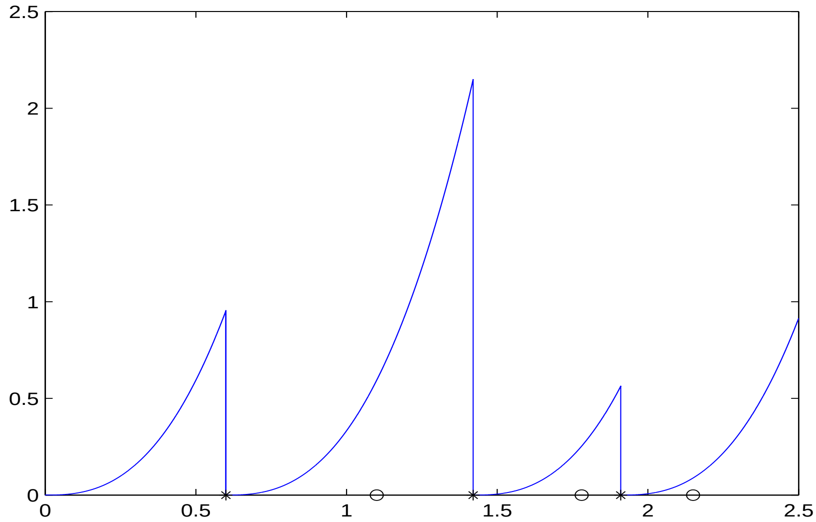

Figure 3.4: The intensity of ABAO PM-AGAN CM

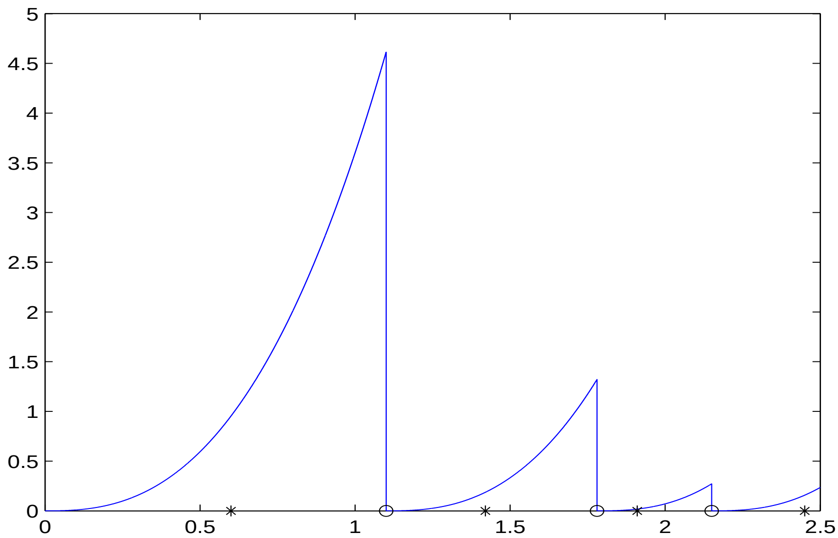

Figure 3.5: The intensity of AGAN PM-ABAO CM

- Virtual age PM effect-ABAO CM: the effective age is equal to the effective age at the time elapsed since the last PM. In this case, the effective age are $A_i = A_{K_{\tau_{m_{C_i}}}} + C_i - \tau_{m_{C_i}}$. Then, the failure intensity is:

$$\lambda_t = \lambda(A_{K_{\tau_{m_{t^-}}}} + t - \tau_{m_{t^-}}) \tag{3.7}$$

- ARA_1 PM-ABAO CM: when preventive maintenance are considered to have the arithmetic redution of age effect with memory one (ARA_1) [7], the effective ages are $A_i = A_{i-1} + (1 - \triangleright)(\tau_i - \tau_{i-1}) = (1 - \rho)\tau_i$. The failure intensity is:

$$\lambda_t = \lambda(t - \rho\tau_{m_{t^-}}) \tag{3.8}$$

  We have the following special cases when the initial failure intensity is increasing:

  - $\rho = 0$: minimal PM (ABAO)
  - $\rho = 1$: perfect PM (AGAN)
  - $0 < \rho < 1$: imperfect PM
  - $\rho < 0$: harmful PM
  - According to the choice of initial intensity, it may be possible to have $\rho > 1$ corresponding to a "better than new" PM. This is possible for a log-linear intensity (because $\exp(a + bt) > 0, \forall t < 0$) but not for the power law intensity (because $\alpha\beta t^{\beta-1}$ is not defined for $t < 0$).

  The ARA_1 PM-ABAO CM with the log-linear intensity is also called ARA_1-LLP.

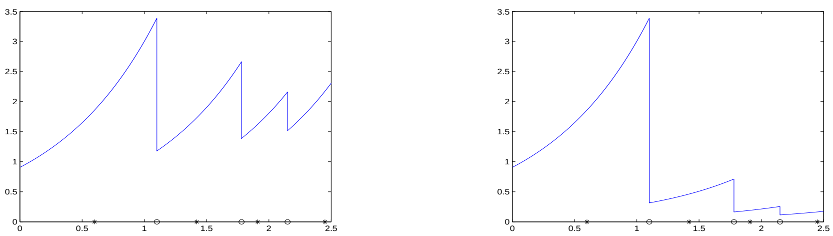

Figure 3.6: The intensity of ARA_1 PM-ABAO CM

- ARA_1 PM-ARA_1 CM: Arithmetic reduction of age with 1 memory. It is assumed that the maintenance effect is to reduce the virtual age of an amount proportional to the supplement of age accumulated since the last maintenance with two different age reduction factors: ρ_p for PM, and ρ_c for CM. Both parameters belong to $(-\infty, 1]$ because a virtual age cannot be negative. The effective age is:

$$A_k = \begin{cases} (A_{k-1} + W_k) - \rho_p W_k & \text{if } U_k = 1 \\ (A_{k-1} + W_k) - \rho_c W_k & \text{if } U_k = 0 \end{cases} \tag{3.9}$$

Then, the failure intensity is [9]:

$$\lambda_t = \lambda\left(t - \sum_{i=1}^{K_{t-}} \rho_p^{U_i} \rho_c^{1-U_i} W_i\right) \tag{3.10}$$

- ARA$_\infty$ PM-ARA$_\infty$ CM: Arithmetic reduction of age with infinite memory. It is assumed that the maintenance effect is to reduce the virtual age of an amount proportional to its value just before maintenance with two different age reduction factors: ρ_p for PM, and ρ_c for CM. Both parameters belong to $(-\infty, 1]$ and the effective age is:

$$A_k = \begin{cases} (1-\rho_p)(A_{k-1} + W_k) & \text{if } U_k = 1 \\ (1-\rho_c)(A_{k-1} + W_k) & \text{if } U_k = 0 \end{cases} \tag{3.11}$$

  Then, the failure intensity is:

$$\lambda_t = \lambda\left(t - C_{K_{t-}} + \sum_{i=1}^{K_{t-}} (1-\rho_p)^{m_{t-} - m_{C_{i-1}}} (1-\rho_c)^{N_{t-} - N_{C_{i-1}}} W_i\right) \tag{3.12}$$

- Brown-Proschan PM - ABAO CM [3]: this model is defined by external random variables $B = \{B_k\}_{i \geq 1}$, independent and bernoulli distributed with parameter p. B_i represents the efficiency of the k^{th} repair:

$$B_k = \begin{cases} 1 & \text{if the } k^{th} \text{ PM is AGAN} \\ 0 & \text{if the } k^{th} \text{ PM is ABAO} \end{cases} \tag{3.13}$$

  The effective ages are:

$$A_k = \sum_{j=1}^{k} \left[\prod_{i=j}^{k} (1 - B_i) \chi_j \right] \tag{3.14}$$

  The failure intensity is:

$$\lambda_t = \lambda(t - \tau_{m_{t-}} + A_{m_{t-}}) \tag{3.15}$$

Chapter 4

Simulation and Parameter Estimation of Imperfect Maintenance Models

This chapter presents the simulation and the parameter estimation of several imperfect maintenance models. We will also study the marginal distributions of the estimated parameters of the models.

4.1 Model Simulations

A simple way to study the behavior of the failure process is by using the Monte Carlo method and calculate the average of the desired behavior from the simulated dataset. In general, we can easily simulate a random variable which follows a specific distribution by using the inverse of the cumulative distribution function, we usually call it as inverse transform sampling. It is a basic method for pseudo-random number sampling, i.e. for generating sample numbers at random from any probability distribution given its cumulative distribution function (cdf).

In order to simulate the desired model, firstly we need to set:

- PM times and censorship time
- The type of the model
- The initial intensity of the model, denoted by $\lambda(.)$
- The cumulative initial intensity of the model, denoted by $\Lambda(.)$
- The inverse of cumulative initial intensity of the model, denoted by $\Lambda^{-1}(.)$
- The function of the effective age model, denoted by A_k

In this section, we use two types of intensity function: Weibull intensity and log-linear intensity. The Weibull intensity function at time t is given as follows.

$$\lambda(t) = \alpha\beta t^{\beta-1} \tag{4.1}$$

The cumulative initial intensity is:

$$\Lambda(t) = \alpha t^{\beta} \tag{4.2}$$

The inverse of cumulative initial intensity is:

$$\Lambda^{-1}(t) = \left(\frac{t}{\alpha}\right)^{\frac{1}{\beta}} \tag{4.3}$$

The log-linear intensity function at time t is given as follows.

$$\lambda(t) = \alpha \exp(\beta t) \tag{4.4}$$

The cumulative initial intensity is:

$$\Lambda(t) = \frac{\exp(\alpha)}{\beta}(\exp(\beta t) - 1) \tag{4.5}$$

The inverse of cumulative initial intensity is:

$$\Lambda^{-1}(t) = \frac{1}{\beta}\ln(1 + \beta t \exp(-a)) \tag{4.6}$$

We have 2 models which will be simulated. Recall the effective age functions of each model are as follows.

- ARA_1 PM - ABAO CM [7]

$$A_k = A_{k-1} + (1-\rho)(\tau_k - \tau_{k-1}) = (1-\rho)\tau_k \tag{4.7}$$

- ARA_∞ PM - ARA_∞ CM

$$A_k = \begin{cases} (1-\rho_p)(A_{k-1} + W_k) & \text{if } U_k = 1 \\ (1-\rho_c)(A_{k-1} + W_k) & \text{if } U_k = 0 \end{cases} \tag{4.8}$$

The simulation algorithm is given as follows.

Algorithm 1: Model simulation

Start with $k = 0$, $C_k = 0$, $A_k = 0$
while $C_k \leq t_{obs}$ *(censorship time)* **do**
  Generate an independent random variable, X_{k+1}, uniformly distributed on $[0, 1]$
  Calculate $Z_{k+1} \leftarrow \Lambda^{-1}(\Lambda(A_k) - \ln(X_{k+1}))$
  if $C_k + Z_{k+1} < \tau_{m_{C_k}+1}$ **then**
    $U_{k+1} \leftarrow 0$, $C_{k+1} \leftarrow C_k + Z_{k+1}$
  else
    $U_{k+1} \leftarrow 1$, $C_{k+1} \leftarrow \tau_{m_{C_k}+1}$
    end if
  Calculate the effective ages A_{k+1}
  $k \leftarrow k + 1$
  end for
Output: All the couples (C_k, U_k) for which C_k are less than or equal the censorship time

4.2 Parameter Estimation

In this section, we will present the algorithm in order to estimate the parameters of the imperfect maintenance models by using maximum likelihood estimation (MLE) method. This method only applies to ARA_1 and ARA_∞ models. In order to apply MLE method, we will first derive the log-likelihood function and its derivative function. Then, we will present an algorithm in order to apply the MLE method.

4.2.1 Log-likelihood function

The log-likelihood function of imperfect maintenance models is given as follows [8].

$$\ln L_t(\theta) = N_t \ln \alpha + \sum_{i=1}^{K_t} (1 - U_i) \ln \psi(A_{i-1} + W_i; \beta) - \alpha \sum_{j=1}^{K_{t^-}+1} \int_{C_{j-1}}^{C_j} \psi(A_{j-1} + s - C_{j-1}; \beta)\, ds \tag{4.9}$$

where $\lambda(t) = \alpha\psi(t; \beta)$. Then, by differentiating (4.9) with respect to α, we obtain that MLE estimator of α can be explicitly computed as a function of the other MLE estimators. Let us denote:

$$\hat{\alpha}_t(\beta, \rho) = \frac{N_t}{\sum_{j=1}^{K_{t^-}+1} [\Psi(A_{j-1} + W_j; \beta) - \Psi(A_{j-1}; \beta)]} \tag{4.10}$$

where $\Psi(t; \beta) = \int_0^t \psi(s; \beta)\, ds$.

Then, the maximization can be done only with respect to β and ρ:

$$\begin{aligned}\ln L_t(\hat{\alpha}_t(\beta,\rho),\beta,\rho) =& N_t \ln N_t - N_t - N_t \ln \sum_{j=1}^{K_{t^-}+1} [\Psi(A_{j-1}+W_j;\beta) - \Psi(A_{j-1};\beta) \\ &+ \sum_{i=1}^{K_t} (1-U_i)\ln\psi(A_{i-1}+C_i-C_{i-1};\beta)\end{aligned} \tag{4.11}$$

More precisely, the ML estimators $\hat{\alpha}, \hat{\beta}, \hat{\rho}$ are obtained by:

$$(\hat{\beta},\hat{\rho}) = \underset{\beta,\rho}{\operatorname{argmax}} \ln L_t(\hat{\alpha}_t(\beta,\rho),\beta,\rho) \tag{4.12}$$

$$\hat{\alpha} = \hat{\alpha}_t(\hat{\beta},\hat{\rho}) \tag{4.13}$$

The algorithm of log-likelihood function is given as follows.

Algorithm 2: Log-likelihood function

Input: Maintenance time C_k, maintenance type U_k, $k = 1, ..., n$
n = number of maintenance times
N = number of failures
Start with $A_1 = 0$, $W_1 = C_1$, $S_1 = 0$, $S_2 = 0$
for $i \in \{1, ..., n\}$ **do**
  Compute $S_1 \leftarrow S_1 + \Psi(A_{i-1} + W_i) - \Psi(A_{i-1})$
  Compute $S_2 \leftarrow S_2 + (1 - U_i)\ln(\psi(A_{i-1} + W_i))$
  Compute A_i
  Compute $W_{i+1} \leftarrow C_{i+1} - C_i$
  end for
Compute $L \leftarrow N\ln(N) - N - N\ln(S_1) + S_2$
Output: The log-likelihood function L

4.2.2 Derivative of log-likelihood function

Recall the log-likelihood function:

$$\ln L_t(\hat{\alpha}_t(\beta,\rho),\beta,\rho) = N_t \ln N_t - N_t - N_t \ln S_1 + S_2 \tag{4.14}$$

where:

$$S_1 = \sum_{j=1}^{K_{t^-}+1} [\Psi(A_{j-1}+W_j;\beta) - \Psi(A_{j-1};\beta)$$

$$S_2 = \sum_{i=1}^{K_t} (1-U_i)\ln\psi(A_{i-1}+C_i-C_{i-1};\beta)$$

Then, the partial derivatives of log-likelihood function with respect to β and ρ are, respectively:

$$\frac{\partial L_t}{\partial \beta} = -\frac{N_t}{S_1}\frac{\partial S_1}{\partial \beta} + \frac{\partial S2}{\partial \beta} \tag{4.15}$$

$$\frac{\partial L_t}{\partial \rho} = -\frac{N_t}{S_1}\frac{\partial S_1}{\partial \rho} + \frac{\partial S2}{\partial \rho} \tag{4.16}$$

where:

$$\frac{\partial S_1}{\partial \beta} = \sum_{i=1}^{n}\left[\frac{\partial \Psi(A_{i-1}+W_i)}{\partial \beta} - \frac{\partial \Psi(A_{i-1})}{\partial \beta}\right]$$

$$\frac{\partial S_1}{\partial \rho} = \sum_{i=1}^{n}\frac{\partial A_{j-1}}{\partial \rho}\left[\frac{\partial \Psi(A_{i-1}+W_i)}{\partial A_{j-1}} - \frac{\partial \Psi(A_{i-1})}{\partial A_{j-1}}\right]$$

$$\frac{\partial S_2}{\partial \beta} = \sum_{i=1}^{n}\left[\frac{1-U_i}{\psi(A_{i-1}+W_i)}\frac{\partial \psi(A_{i-1}+W_i)}{\partial \beta}\right]$$

$$\frac{\partial S_2}{\partial \rho} = \sum_{i=1}^{n}\left[\frac{1-U_i}{\psi(A_{i-1}+W_i)}\frac{\partial \psi(A_{i-1}+W_i)}{\partial A_{i-1}}\frac{\partial A_{i-1}}{\partial \rho}\right]$$

The algorithm of derivative of log-likelihood function is given as follows.

Algorithm 3: Derivative of log-likelihood function

Input: Maintenance time C_k, maintenance type U_k, $k = 1, ..., n$
n = number of maintenance times
N = number of failures
Start with $A_1 = 0$, $\frac{\partial A_1}{\partial \rho}$, $W_1 = C_1$, $S_1 = 0$, $S_2 = 0$, $\frac{\partial S_1}{\partial \rho} = 0$, $\frac{\partial S_1}{\partial \rho} = 0$, $\frac{\partial S_2}{\partial \rho} = 0$, $\frac{\partial S_2}{\partial \rho} = 0$

for $i \in \{1, ..., n\}$ **do**

- Compute $S_1 \leftarrow S_1 + \Psi(A_{i-1} + W_i) - \Psi(A_{i-1})$
- Compute $S_2 \leftarrow S_2 + (1 - U_i)\ln(\psi(A_{i-1} + W_i))$
- Compute $\frac{\partial S_1}{\partial \beta} \leftarrow \frac{\partial S_1}{\partial \beta} + \Psi(A_{i-1} + W_i) - \Psi(A_{i-1})$
- Compute $\frac{\partial S_1}{\partial \rho} \leftarrow \frac{\partial S_1}{\partial \rho} + \frac{\partial A_{i-1}}{\partial \rho}(\psi(A_{i-1} + W_i) - \psi(A_{i-1}))$
- Compute $\frac{\partial S_2}{\partial \beta} \leftarrow \frac{\partial S_2}{\partial \beta} + (1 - U_i)\frac{\partial \psi(A_{i-1}+W_i)}{\partial \beta}/\psi(A_{i-1} + W_i)$
- Compute $\frac{\partial S_2}{\partial \rho} \leftarrow \frac{\partial S_2}{\partial \rho} + (1 - U_i)\frac{\partial A_{i-1}}{\partial \rho}\frac{\partial \psi(A_{i-1}+W_i)}{\partial \rho}/\psi(A_{i-1} + W_i)$
- Compute A_i
- Compute $\frac{\partial A_i}{\partial \rho}$
- Compute $W_{i+1} \leftarrow C_{i+1} - C_i$

end for

Compute $\frac{\partial L}{\partial \beta} \leftarrow -N\frac{\partial S_1}{\partial \beta}S_1 + \frac{\partial S_2}{\partial \beta}$
Compute $\frac{\partial L}{\partial \rho} \leftarrow -N\frac{\partial S_1}{\partial \rho}S_1 + \frac{\partial S_2}{\partial \rho}$
Output: The derivatives of log-likelihood function $\frac{\partial L}{\partial \beta}$ and $\frac{\partial L}{\partial \rho}$

Furthermore, by adapting the algorithm 1 and 2, we construct a parameter estimation algorithm. This algorithm will estimate the parameters α, β, and ρ by optimizing the log-likelihood function, given the partial derivatives of log-likelihood function. We use the BFGS method [10] for the optimization. The parameter estimation algorithm is given as follows.

Algorithm 4: Parameter estimation

Input: Maintenance time C_k, maintenance type U_k, $k = 1, ..., n$
n = number of maintenance times
N = number of failures
initial value of $\beta = \hat{\beta}^0$ and $\rho = \hat{\rho}^0$
Start with $A_1 = 0$, $W_1 = C_1$, $S_1 = 0$
Estimate $\hat{\beta}$ and $\hat{\rho}$ by using BFGS method with initial values $\hat{\beta}^0$ and $\hat{\rho}^0$
for $i \in \{1, ..., n\}$ **do**
  (given the $\hat{\beta}$ and $\hat{\rho}$)
  Compute $S \leftarrow S + \Psi(A_{i-1} + W_i) - \Psi(A_{i-1})$
  Compute A_i
  Compute $W_{i+1} \leftarrow C_{i+1} - C_i$
end for
Compute $\hat{\alpha} \leftarrow N/S$
Output: The estimated parameters $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\rho}$

4.2.3 Application to real dataset

In this subsection, we will apply our algorithms to a real dataset. It is taken from the electricity production plants EDF. The dataset gives the PM and CM times of stubs of the inlet header of the heat exchanger that warms up the feeding water of the boiler of a fossil-fired thermal plant [15]. It is presented on the table below.

Table 4.1: Real dataset

25	50	93	109	114	141	163	164	195	225	264
PM	CM	CM	CM	PM	CM	CM	CM	CM	PM	cens

We assume that the data follows the ARA$_\infty$ PM-ARA$_\infty$ CM model with Weibull intensity. We present the estimation result on the table below.

Table 4.2: Estimation results of ARA_∞ PM-ARA_∞ CM model

Parameter	**Estimated value**
Scale parameter α	1.159e-05
Shape parameter β	3.046
CM efficiency parameter ρ_c	0.564
PM efficiency parameter ρ_p	1
Maximum value of the log-likelihood function	-29.477

According to the result on the table 4.2, compared to the result in [15], the estimation gives the same result. It means that the estimation program is working well. Hence, we can use the program in order to do the monte carlo simulation.

4.3 Monte Carlo simulation

In this section, we will present the simulation of the imperfect maintenance model. We choose the ARA_1-LLP model (ARA_1 PM-ABAO CM with log-linear intensity). Then, we will study the distribution of the number of failures. We will also estimate the parameters of the models and study the marginal distributions of the estimated parameters.

We set firstly $\alpha = 1$, $\beta = 2$, $\rho = 0.9$, with log-linear intensity. We simulate the model 500 times, then we compute the number of failures of each dataset and we present the distribution of the number of failures in a histogram as well as the mean and the variance. Then, we will compare this model with different values of parameters.

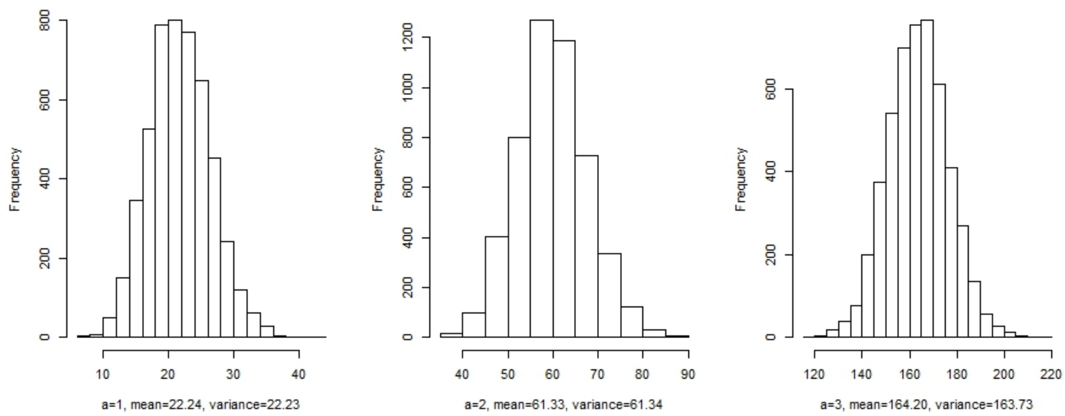

Figure 4.1: Distribution of number of failures with different scale parameter α

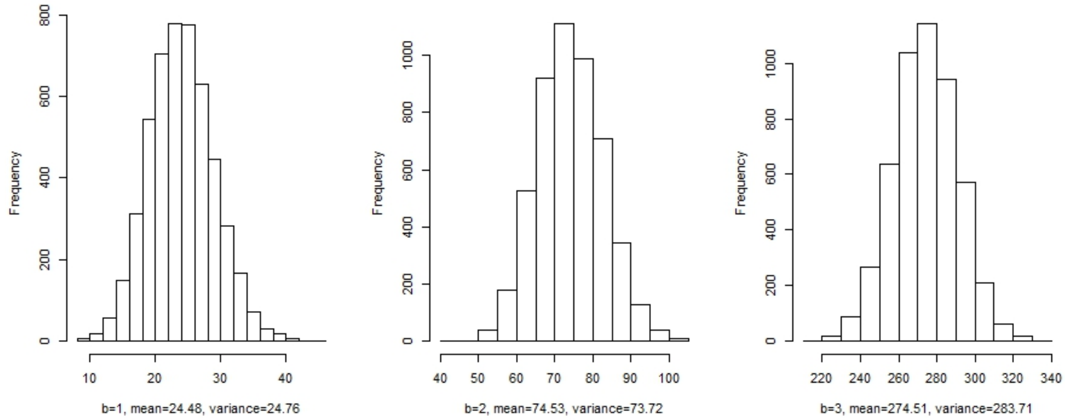

Figure 4.2: Distribution of number of failures with different scale parameter β

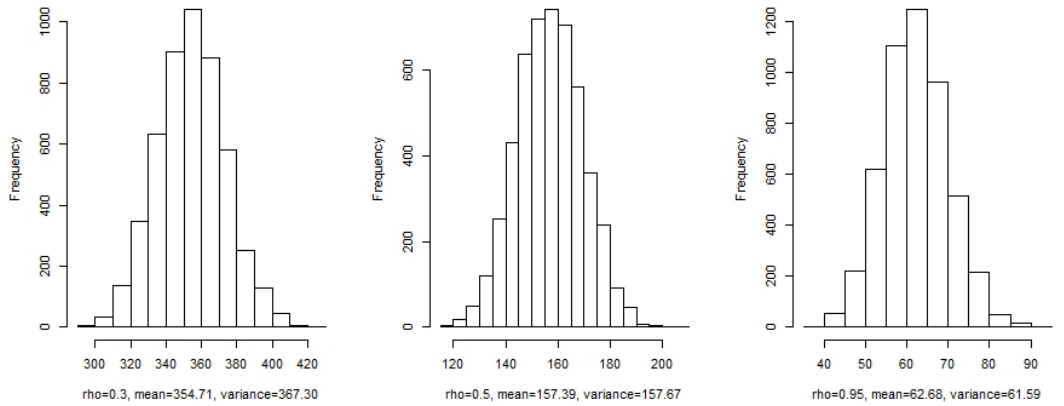

Figure 4.3: Distribution of number of failures with different scale parameter ρ

From figures 4.1 and 4.2, we could see that the large value of parameters α and β increases the number of failures. On the other side, the large value of parameters ρ decreases the number of failures. From all the simulated datasets, we obtain the mean value is statistically the same with the variance.

By using the same parameters as above, we will estimate the parameters from each dataset and present the distribution in histograms as follows.

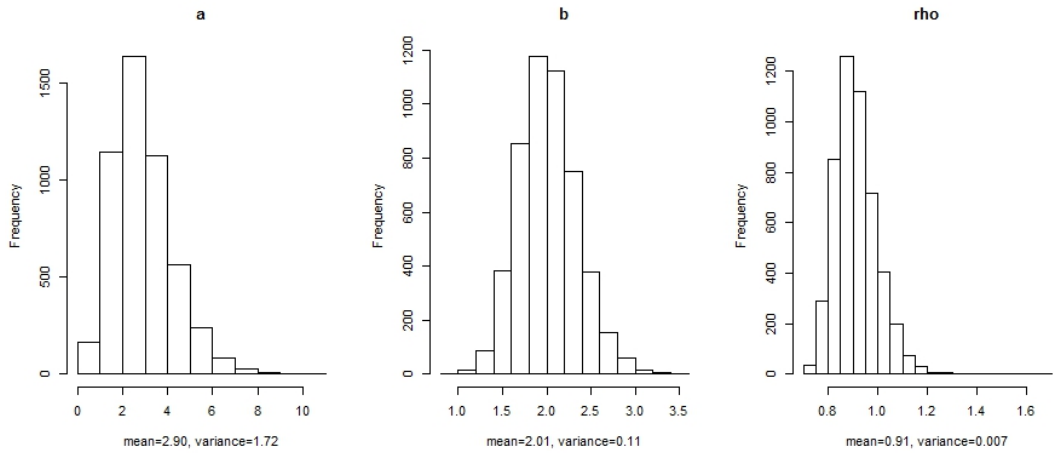

Figure 4.4: Distribution of number of failures with different scale parameter ρ

4.4 Comparison between R and Rcpp

Since our program needs a high speed computation, we use the Rcpp package in order to apply R programming interfaced with C++. In figure 4.5, we compare the simulation program between R and Rcpp. We can see that elapsed time of Rcpp programming is much faster than R. From 3 benchmarks, we obtain the Rcpp elapsed time is respectively, 18, 20, 21 times faster than the R programming.

```
> benchmark(C <- ARAsim (PM,264,2,3.05,1,0.565,1),C <- ARAinf.R (PM,264,2,3.05,1,0.565,1),order=NULL)[,1:4]
                                        test replications elapsed relative
1   C <- ARAsim(PM, 264, 2, 3.05, 1, 0.565, 1)         100    0.22    1.000
2 C <- ARAinf.R(PM, 264, 2, 3.05, 1, 0.565, 1)         100    4.12   18.727
> benchmark(C <- ARAsim (PM,264,3,3.05,1,0.565,1),C <- ARAinf.R (PM,264,3,3.05,1,0.565,1),order=NULL)[,1:4]
                                        test replications elapsed relative
1   C <- ARAsim(PM, 264, 3, 3.05, 1, 0.565, 1)         100    0.23    1.000
2 C <- ARAinf.R(PM, 264, 3, 3.05, 1, 0.565, 1)         100    5.05   21.957
> benchmark(C <- ARAsim (PM,264,4,3.05,1,0.565,1),C <- ARAinf.R (PM,264,4,3.05,1,0.565,1),order=NULL)[,1:4]
                                        test replications elapsed relative
1   C <- ARAsim(PM, 264, 4, 3.05, 1, 0.565, 1)         100    0.28    1.000
2 C <- ARAinf.R(PM, 264, 4, 3.05, 1, 0.565, 1)         100    5.66   20.214
```

Figure 4.5: Comparison between R and Rcpp in simulation programming

Chapter 5

Exact conditional goodness-of-fit (GoF) test for ARA_1-LLP model

In this chapter, we will present a generalization of Lindqvist-Rannestad goodness-of-fit (GoF) tests for a particular imperfect maintenance model with both CM and PM. The CM are assumed to be minimal (ABAO) with log-linear initial intensity. We also assume that PM are carried out at deterministic times with ARA_1 type effect.

5.1 Principle of the test

The construction of an exact conditional GoF test for ARA_1-LLP model is possible. In this model, PM has ARA_1 type effect and the failure intensity is considered to be log-linear. The failure intensity of ARA_1-LLP:

$$\lambda_t(a, b, \rho) = \exp(a + b(t - \rho\tau_{m_{t^-}})). \tag{5.1}$$

The CM effects are assumed to be ABAO. This assumption is meaningful because CM aims to quickly restore the system in working order. It is also common [14] and absolutely necessary in order to be able to apply Lindqvist-Rannestad [13] GoF test method since a NHPP is needed.

The GoF test in this case has the following hypotheses:

$$H_0 : \lambda_t(\theta) \in \mathcal{I} \,\text{versus}\, H_1 : \lambda_t(\theta) \notin \mathcal{I}$$

where $\mathcal{I}$ is the family of failure intensities defined in (5.1) for all $(a, b, \rho) \in \mathbb{R}^3$.

The considered models also needs to have a sufficient statistic. The ARA_1-LLP model has this property. In order to apply the same approach as Lindqvist and Rannestad to ARA_1-LLP model, we need:

- existence of a sufficient statistic,

- conditional simulation of $D|S = s_{obs}$,
- computation of a GoF test statistic Z.

By the definition (5.1), the cumulative intensity function of ARA_1-LLP is:

$$\begin{aligned}\Lambda_t(a,b,\rho) &= \frac{\exp(a)}{b}\sum_{m=1}^{m_t}\exp(-b\rho\tau_{m-1})\big[exp(b\tau_m)-\exp(b\tau_{m-1})\big]\\ &+\frac{\exp(a)}{b}\exp(-b\tau_{m_t})\big[\exp(bt)-\exp(b\tau_{m_t})\big] \qquad \text{if } b\neq 0\\ \Lambda_t(a,0,\rho) &= \exp(a)t\end{aligned} \tag{5.2}$$

We consider that the failure times are observed on the time interval $[0,T]$. For simplification, we denote $\tau_{m_T+1}=T$ and we will use this notation in all the following.

The log-likelihood function of ARA_1-LLP model is:

$$\mathcal{L}_T(a,b,\rho) = aN_T + b\sum_{T_i\leq T}T_i - b\rho\sum_{m=2}^{m_T+1}\tau_{m-1}(N_{\tau_m}-N_{\tau_{m-1}}) - \Lambda_T(a,b,\rho) \tag{5.3}$$

Since Λ_T is a deterministic function, we apply the factorization theorem and deduce the three components of the sufficient statistics $S=(S_1,S_2,S_3)$. The sufficient statistic of the ARA_1-LLP model exists and is:

$$S = \left(N_T, \sum_{T_i\leq T}T_i, \sum_{m=2}^{m_T+1}\tau_{m-1}(N_{\tau_m}-N_{\tau_{m-1}})\right) \tag{5.4}$$

5.2 Extension of the sufficient statistic

The conditional sampling given the sufficient statistic S is too difficult especially given the third component. That is why we use a larger sufficient statistic in order to make the conditional sampling possible. The new sufficient statistic has the following expression:

$$\tilde{S} = \left(N_{\tau_1},...,N_{\tau_{m_T}}, N_T, \sum_{T_i\leq T}T_i\right) \tag{5.5}$$

It is obvious that there is no loss of information when conditioning by the statistic $\tilde{S}$ defined in (5.5) instead of S defined in (5.4). Apparently there is no need to know explicitly $(N_{\tau_1},...,N_{\tau_{m_T}},N_T)$ in order to know $S_3=\sum_{m=2}^{m_T+1}\tau_{m-1}(N_{\tau_m}-N_{\tau_{m-1}})$, but it will be true if the PM times are not periodic.

5.3 Conditional sampling given the sufficient statistic

5.3.1 First step

The conditional sampling is done using the statistic $\tilde{S}$. We will use a classical trick for computational distributions given the sufficient statistic which consists in choosing the parameters values that give rise to particular simple models. This can be done since the conditional distribution, given the sufficient statistic, is the same whatever the parameter values are. For simplification, we will use parameter values ($a = b = 0$) for which the model ARA_1-LLP is an HPP(1).

The objective is to be able to simulate HPP(1) conditionally to the sufficient statistic. Since the statistic in (5.5) includes the number of observed failures at each PM time, our first objective is to condition by $\mathbf{N_T} = (N_{\tau_1}, ..., N_{\tau_{m_T}}, N_T)$. Conditionally on $\mathbf{N_T}$, the event times of HPP(1) are distributed like $(m_T + 1)$ independent samples, the i^{th} sample having the distribution of independent order statistics of $(N_{\tau_i} - N_{\tau_{i-1}})$ variables uniformly distributed on $[\tau_{i-1}, \tau_i]$, for $i \in \{1, ..., m_T + 1\}$ where $\tau_0 = 0$, $\tau_{m_T+1} = T$.

The simulation of $\mathbf{T_n}$ conditionally to $\mathbf{N_T}$ is reduced to simulate independent order statistics of uniforms $(U_1, ..., U_n)$. Our next objective is to simulate these uniforms $(U_1, ..., U_n)$ conditionally to the remaining components of the sufficient statistic $\tilde{S}$ which is $\sum_{i=1}^{n} U_i = \sum_{i=1}^{n} T_i = s_2$. This simulation problem is then transformed into a problem of conditional sampling of uniforms variables. The purpose of the next subsection is to show how this conditional sampling can be carried out.

5.3.2 Second step

We consider the desired sample $U_1, ..., U_n$ composed of $(m_T + 1)$ independent samples of iid random variables. Each sample i is, respectively, of size $(n_i - n_{i-1})$ and follows $\mathcal{U}[\tau_i, \tau_{i-1}]$, $i \in \{1, ..., m_T + 1\}$, where $n_0 = 0$. There is no simple direct way of sampling from the conditional distribution of the uniforms $U_1, ..., U_n$ given $\sum_{i=1}^{n} U_i = s_2$. We will use Gibbs sampler algorithm to simulate the desired samples. There is no simple expression for the pdf of $\sum_{i=1}^{n} U_i$. Since the conditional distribution of $U_1, ..., U_n$ given $\sum_{i=1}^{n} U_i = s_2$ is singular, in order to have a proper conditional pdf we have to leave out one variable, for example U_j. We consider then the conditional distribution of $U_1, ..., U_{j-1}, U_j, ..., U_n$ given $\sum_{i=1}^{n} U_i = s_2$ and deduce $U_j = s_2 - \sum_{k \neq j} U_k$.

We use a modified Gibbs algorithm where in each iteration two of the vector components $(U_i, U_j), i \neq j$, are updated. The algorithm consists of simulating at iteration m the conditional pdf of $U_i^m | U_k^{m-1} = u_k^{m-1}$, $k \neq i$, $k \neq j$, $\sum_{k=1}^{n} U_k^{m-1} = s_2$. This last simulation is equivalent to the simulation of $U_i^m | U_i^{m-1} + U_j^{m-1} = s_2 - \sum_{k \neq i,j} u_k^{m-1}$. Then, we will compute the conditional cdf of $U_i | U_i + U_j$.

Let $0 \leq c_i^1 < c_i^2$, $0 \leq c_j^1 < c_j^2$, U_i and U_j two independent random variables from respectively $\mathcal{U}[c_i^1, c_i^2]$ and $\mathcal{U}[c_j^1, c_j^2]$. The conditional distribution of U_i given $U_i + U_j = s$ is uniform on I where:

- $I = [c_i^1, s - c_j^1]$ if $c_i^1 + c_j^1 \leq s \leq \min(c_i^2 + c_j^1, c_i^1 + c_j^2)$
- $I = [c_i^1, c_i^2]$ if $c_i^2 + c_j^1 \leq s \leq c_i^1 + c_j^2$
- $I = [s - c_j^1, s - c_j^2]$ if $c_i^1 + c_j^2 \leq s \leq c_i^2 + c_j^1$
- $I = [s - c_j^2, c_i^2]$ if $\max(c_i^2 + c_j^1, c_i^1 + c_j^2) \leq s \leq c_i^2 + c_j^2$.

Finally, the Gibbs sampler algorithm is given in the next chapter for the ARA_1-LLP model. It makes conditional sampling of $\mathbf{T_n}|N_{\tau_1} = n_1, ..., N_{\tau_{m_T}} = n_{m_T}, N_T = n, \sum_{i=1}^n T_i = s_2$.

Let $n = n_{m_T+1}$ and $n_0 = 0$.

Algorithm 5: Initialization of Gibbs sampler algorithm

```
for all j ∈ {1, ..., m_T + 1} do
    for all i ∈ {n_{j-1}, ..., n_j} do
        draw u_i^0 ~ U[τ_{j-1}, τ_j]
        d_i^1 ← u_i^0 − τ_{j-1}
        d_i^2 ← τ_j − u_i^0
    end for
end for
if Σ_{i=1}^n u_i^0 > s_2 then
    for all i ∈ 1, ..., n do
        t_i^0 ← u_i^0 − d_i^1 (Σ_{i=1}^n u_i^0 − s_2) / (Σ_{i=1}^n d_i^1)
    end for
else
    for all i ∈ {1, ..., n} do
        t_i^0 ← u_i^0 + d_i^2 (s_2 − Σ_{i=1}^n u_i^0) / (Σ_{i=1}^n d_i^2)
    end for
end if
sort t_1^0, ..., t_n^0
return t_1^0, ..., t_n^0
```

For the initialization of the algorithm, Lindqvist and Rannestad used the same value s_2/n of all the components: $(t_1^0, ..., t_n^0) = (s_2/n, ..., s_2/n)$. We will propose an algorithm in the next chapter for a random initialization which guarantees $\sum_{i=1}^n t_i^0 = s_2$ and $N_{\tau_1} = n_1, ..., N_{\tau_{m_T}} = n_{m_T}, N_T = n$. This initialization is independent of the first configuration of the tested data, which makes the convergence of the Gibbs algorithm faster. Furthermore, our procedure guarantees the independence of the successive simulated values of $\mathbf{T_n}|\sum_{i=1}^{n} T_i = s_2$.

Algorithm 6: Gibbs sampler algorithm for conditional sampling of $\mathbf{T_n}|N_{\tau_1} = n_1, ..., N_{tau_{m_T}}, N_T = n, \sum_{i=1}^{n} T_i = s_2$

Start with initializing t_i^0, $i = 1, ..., n$ (algorithm 1)
for *all* $k \in \{1, ..., nb\}$ **do**
  $t_i^k \leftarrow t_i^{k-1}$, $i = 1, ...n$
  draw integers $1 \leq i < j \leq n$ randomly
  let n_i and n_j of $\{n_1, ..., n_{m_t}, n\}^2$ be such that $n_{i-1} < i \leq n_i$ and $n_{j-1} < j \leq n_j$
  let $s \leftarrow t_i^{k-1} + t_j^{k-1}$, $c_i^1 \leftarrow \tau_{n_{i-1}}$, $c_i^2 \leftarrow \tau_{n_i}$, $c_j^1 \leftarrow \tau_{n_{j-1}}$, $c_j^2 \leftarrow \tau_{n_j}$
  if $c_i^1 + c_j^1 \leq s \leq \min(c_i^2 + c_j^1, c_i^1 + c_j^2)$ **then**
    draw $t_i^k \sim \mathcal{U}[c_i^1, s - c_j^1]$
  else
    if $c_i^2 + c_j^1 \leq s \leq c_i^1 + c_j^2$ **then**
      draw $t_i^k \sim \mathcal{U}[c_i^1, c_i^2]$
    else
      if *if* $c_i^1 + c_j^2 \leq s \leq c_i^2 + c_j^1$ **then**
        draw $t_i^k \sim \mathcal{U}[s - c_j^1, s - c_j^2]$
      else
        if *if* $\max(c_i^2 + c_j^1, c_i^1 + c_j^2) \leq s \leq c_i^2 + c_j^2$ **then**
          draw $t_i^k \sim \mathcal{U}[s - c_j^2, c_i^2]$
        end if
      end if
    end if
  end if
  $t_j^k \leftarrow s - t_i^k$
end for
return $t_1^{nb}, ..., t_n^{nb}$

It has been shown in [4] that the distribution of the sample $(t_1^k, ..., t_n^k)$ converges to the target distribution, whatever the initial vector is. The successive simulated samples are from a Markov chain, and the target distribution is the stationary distribution of this Markov chain. 'Burn in' samples are needed before the samples can be taken to be from the correct distribution.

5.4 Transformation to uniforms

When the conditional sampling is done, a GoF test is chosen to detect the departure from the tested model. Since the parameter (a, b, ρ) of the ARA$_1$-LLP model are unknown, we use the estimated parameter $(\hat{a}, \hat{b}, \hat{\rho})$ from MLE method. Let $\hat{\Lambda}_\bullet$ be an estimate of the cumulative intensity function $\Lambda_\bullet$ based on the observation $(T_1, .., T_n, N_T)$, defined as:

$$\hat{\Lambda}_t = \Lambda_t(\hat{a}, \hat{b}, \hat{\rho}). \tag{5.6}$$

We consider the estimated transformed times defined as follows: $\hat{V}_i = \frac{\hat{\Lambda}_{T_i}}{\hat{\Lambda}_T}$. The distribution of the last sample $\hat{V}_1, ..., \hat{V}_n$ is very close to order statistics of uniforms. We can use the classical GoF tests for the uniform distribution to suggest GoF tests for the tested model based on the $\hat{V}_i$. This was already the approach of Lindqvist-Rannestad [13] and before him of Baker [2]. For $\text{ARA}_1 - LLP$ model, we check the uniformity of the variables $\hat{V}_i$, $i = 1, ...n$:

$$\hat{V}_i = \frac{\sum_{m=1}^{m_{T_i}} \exp(-\hat{b}\hat{\rho}\tau_{m-1})\Big[\exp(\hat{b}\tau_m) - \exp(\hat{b}\tau_{m-1})\Big] + \exp(-\hat{b}\hat{\rho}\tau_{m_{T_i}})\Big[\exp(\hat{b}T_i) - \exp(\hat{b}\tau_{m_{T_i}})\Big]}{\sum_{m=1}^{m_T+1} \exp(-\hat{b}\hat{\rho}\tau_{m-1})\Big[\exp(\hat{b}\tau_m) - \exp(\hat{b}\tau_{m-1})\Big]} \tag{5.7}$$

where $(\hat{b}, \hat{\rho})$ are the maximum likelihood estimators of parameters (b, ρ).

5.5 Test statistics

In all the following simulations, we apply to the transformed samples $\big(\hat{V}_j\big)_{j=1,...n}$ the classical test statistics for the uniform distribution as it was done in [13].

- Laplace statistic:

$$L = \sqrt{\frac{12}{n}} \sum_{j=1}^{n} \left(\hat{V}_j - \frac{1}{2}\right) \tag{5.8}$$

- Modified Cramer-Von Mises statistic:

$$CM = \sum_{j=1}^{n} \left[\hat{V}_j - \frac{(2j-1)}{2n}\right]^2 + \frac{1}{12n} \tag{5.9}$$

- Modified Anderson-Darling statistic:

$$AD = -\frac{1}{n}\left[\sum_{j=1}^{n} (2j-1)\ln(\hat{V}_j) + \ln(1 - \hat{V}_{n+1-j})\right] - n \tag{5.10}$$

- Modified Kolmogorov-Smirnov statistic:

$$KS = \max\left[\max_{1\le j\le n}\left(\frac{j}{n} - \hat{V}_j\right), \max_{1\le j\le n}\left(\hat{V}_j - \frac{j-1}{n}\right)\right] \tag{5.11}$$

In order to sum up the whole approach, if we have failure time $T_1, ..., T_n$ and we wish to test if these times are from ARA$_1$-LLP model, we apply the following algorithm. Let K be a large number to guarantee the computation of the p-value estimated by:

$$\hat{p}_{obs} = \frac{1}{K} \sum_{k=1}^{K} 1_{\{Z_k^{\star} \geq z_{obs}\}} \tag{5.12}$$

Algorithm 7: Computation of the exact p-value

Compute the observed sufficient statistic $\tilde{S} = \tilde{s_{obs}}$.
Compute the MLEs $(\hat{a}, \hat{b}, \hat{\rho})$.
Compute the test statistic z_{obs} from the observation.
for *all* $k \in \{1, ..., K\}$ **do**
    apply algorithm 6 to simulate $T_1, ..., T_n | (\tilde{S} = \tilde{s_{obs}})$
    compute the transformation to the uniforms given in (5.7)
    compute the test statistic Z_k given section 5.5
end for
Compute the p-value $\sum_{k=1}^{K} 1_{\{Z_k^{\star} \geq z_{obs}\}} / K$

5.6 Simulation study

In this section, we will study the power of test for ARA$_1$-LLP model. We will also simulate Brown-Proschan models as alternative models in order to study the power of test. We set 4 PM times ($\tau_1 = 1.833$, $\tau_2 = 2.404$, $\tau_3 = 2.985$, $\tau_4 = 3.538$). The power of test is assessed by the percentage of rejection H_0 over the total number of simulated samples. We set the number of the simulated samples from each tested alternative to 500. We apply the approach presented in algorithm 7: we simulate $K = 100$ samples of $\hat{V}_i$, $i = 1, ..., n$. The 'burn in' period is set to 200.

The GoF tests used here are Laplace L, Cramer-Von Mises CM, Anderson-Darling AD, and Kolmogorov-Smirnov KS tests. We first simulate samples from the ARA$_1$-LLP model, in order to check that the percentage of rejection is close to the significance level. We set the significance level at 5%. All the simulation of the models are done using inverse transform sampling in algorithm 1.

The result of the simulation is presented on the table 5.1. The first column presents the type of the model and the second column presents the parameter values of the simulated models.

Table 5.1: Power of tests

Model	**Parameters**	**L**	**CM**	**AD**	**KS**
ARA_1-LLP	(1,2,0.9)	0.068	0.058	0	0.054
	(1,2.5,0.9)	0.092	0.102	0	0.036
	(1,3,0.9)	0.236	0.186	0	0.058
	(1,2.3,0.5)	0	0	0.012	0.054
	(1,2.3,0.7)	0.042	0.03	0	0.054
	(1,2.3,0.9)	0.114	0.078	0	0.068
Brown-Proschan (log-linear intensity)	(1,0.9,0)	0.056	0.038	0.98	0
	(1,0.9,1)	0.04	0.052	0.018	0.056
	(1,0.9,0.5)	0.06	0.036	0.246	0.034
	(1,0.9,0.7)	0.056	0.044	0.086	0.05
	(1,0.9,0.3)	0.064	0.052	0.458	0.032
	(3,0.5,0.5)	0.05	0.048	0.158	0.058

From the result of the table above, for ARA_1-LLP models, only KS test statistic gives the 5% significance level. The L, CM, and AD test statistics are very biased. For the Brown-Proschan models, only AD test statistic has good performance, while the other test statistics have bad performance. The result we get may be due to the bugs of the program. Since the program is quiet complicated, there might be some computation error which is still unknown up to now.

Chapter 6

Conclusion

There are many imperfect maintenance models which have been proposed. Therefore, the goodness-of-fit test is very important in order to choose which model is more suitable to the dataset. Hence, the study of the power of test becomes essential.

The simulation and estimation play an important role in order to do the test. We have shown that the simulation of imperfect maintenance models can be done by using inverse transform sampling method. Then, the parameter estimation is done by using the MLE method as well as the BFGS method for optimizing the log-likelihood function.

In order to simulate, estimate, and compute the desired model or dataset, we need a high computation programming. We also have shown that the Rcpp is much faster than R. Rcpp could be 21 times faster. Hence, we can save much time.

From the result of the power of test, we see that there are still some errors in which the test performance becomes bad. It is also known that the computation of the test is quiet complicated. Hence, debugging the program is very necessary.

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