Towards the identification of spatially resolved mechanical properties in tissues and materials: State of the art, current challenges and opportunities in the field of flow measurements

Pablo D. RUIZ

Tel.: ++44 (0)1509227660 Email: p.d.ruiz@lboro.ac.uk
Wolfson School of Mechanical and Manufacturing Engineering
Loughborough University, Loughborough, LE11 3TU, UK

Abstract This work is focused on optical methods that provide tomographic reconstructions of the structure of materials and tissues. Phase information can also be used to measure 3-D displacement and strain fields with interferometric sensitivity. Different approaches are presented, including recent developments in phase contrast wavelength scanning interferometry and a combination of optical coherence tomography and digital volume correlation to estimate elastic properties of synthetic phantoms and porcine corneas. Inversion algorithms based on finite elements and the Virtual Fields Method (VFM) are used to extract mechanical properties from the knowledge of the applied loads, geometry and measured deformation fields. Current efforts into extending these methods into single shot techniques have the potential of expanding the range of applications to study dynamic events such as micro-flows in engineering and biological systems in which scattering particles are transported in a flow, e.g. tribology, microfluidic devices, cell migration or multi-phase flows.

Keywords: depth-resolved displacements, volume strain, identification, PIV, OCT, WSI, HSI

1. Introduction

Our ability to predict the mechanical behaviour of materials, components and even tissues and organs is not only as good as the models that we build to do so, but also as the knowledge we have about their internal structure, the spatial distribution of their properties, and the boundary conditions that constrain and drive their deformation. Adhesives, polymers, composites, multi layered films and tissues such as skin, cartilages and cornea are examples of materials whose function is often critical in the performance and structural integrity of the system they belong to. There has been a long history of research into the mechanical and functional behaviour of this type of materials but they are not in many cases fully understood due to their inherent complexity, which cannot always be modelled with continuum mechanics assumptions. Moreover, current experimental techniques are unable to establish the distribution of the constitutive parameters of the materials 'within' their volume with enough sensitivity and spatial resolution in order to relax the need for simplistic bulk properties assumptions.

One way to determine the constitutive parameters of an anisotropic material is to measure how it deforms under known loads and solve an inverse problem. The inverse problem is usually posed as the identification of the most representative components of the material constitutive matrix. However, the measurement of surface deformation is not adequate enough to understand what happens below it. For instance, a big and deep delamination in a composite layered material can give rise to the same surface response than a smaller but shallow one. This 'degeneracy' (multiple solutions) problem is eliminated by directly measuring the 3-D displacement field, removing the need for any a priori assumptions (Maranon et al., 2007). Neutron diffraction (ND), X-ray computed tomography (X-ray CT), Magnetic Resonance

Elastography (MRE) and Optical Coherence Tomography (OCT) are examples of tomographic techniques with the ability to map 3-D strain in the bulk of solid materials –some of them requiring Digital Volume Correlation (DVC). As they exploit different physical principles, they are used specifically for certain types of materials (e.g. crystalline materials for ND, high fat and water content for MRE and semi-transparent scattering materials for OCT). They also differ in spatial and temporal resolution, displacement and strain sensitivity, and cost per data point.

Section 2 presents some of the OCT related techniques that have recently extended the ability of surface interferometry to measure several components of the displacement field inside semi-transparent scattering materials and tissues. Section 3 presents recent advances in the VFM method, aimed at identifying material parameters from 3-D displacement and strain data. Section 4 gives an overview of the challenges ahead and finally Section 5 explores potential applications of 3-D optical tomography and the VFM in the field of flow measurements.

2. Depth-resolved displacement fields

OCT is an exciting technique that provides depth-resolved microstructure images, primarily for medical applications (Drexler and Fujimoto, 2008). It is based on an interferometer and a low temporal coherence broadband source. When implemented in the 'time domain' a reference mirror is scanned to provide cross-sections of the sample by moving the 'coherence gate' through it. When implemented in the 'spectral domain' (SOCT), all the information of a 'slice' inside the material is acquired simultaneously by using a spectrometer and an area photodetector array, without the need of scanning devices. A 2-D interferogram is recorded with depth encoded as spatial frequency along the wavelength axis of the spectrometer, rather than as a function of time. The microstructure is then extracted from the spectral magnitude of the Fourier transform along the wave-number axis. The transverse spatial resolution depends mainly on the NA of the imaging lens and the axial resolution on the spectral bandwidth of the light source. Resolutions of ~1μm (both lateral and axial) can be achieved in tissue, but most commercial systems provide ~10μm. Though usually disregarded, the phase of the Fourier transform is readily available and can be used to evaluate displacements through-the-thickness of the sample. Several related techniques have been proposed to do this, effectively extending surface interferometry to 3-D. Wavelength Scanning Interferometry relies on a tunable laser and the acquisition of a sequence of images which are post process as in SOCT to reconstruct internal sample structure and phase information. 3-D displacements can be evaluated from the phase difference measured between a loaded and an unloaded state of the sample, each state requiring one full spectral scan (Ruiz et al., 2005; Ruiz et al., 2004). This takes several minutes of scanning and a few more for data processing, meaning that only static deformations can be studied. The system provides out-of-plane displacement maps within a 2-D slice into the sample with a sensitivity of order 10 nm. Visco-elastic materials require a faster approach, either using fast tuning broadband sources or by using spectral domain OCT in which the data required to reconstruct a slice through the material is recorded in a single exposure of a CCD photodetector array. Mechanical creep has been observed in porcine corneas after increasing the intraocular pressure (De la Torre-Ibarra et al., 2006), even though with only out-of-plane sensitivity. Further efforts lead to a dual-sensitivity system to measure two orthogonal displacement components. This was achieved by combining sequential oblique illumination of the object and recording two interferograms before and two after the deformation. Depth-resolved out-of-plane and in-plane sensitivities of 0.14 and 4.2 μm per fringe were demonstrated up to a depth of 400 μm in a water-based polymer under static shear (De la Torre-Ibarra et al., 2009). Three years later, by using a WSI setup with multiple illumination directions with different optical paths, all three components of the displacement field could be finally measured simultaneously in the 3-D volume of an epoxy resin. The setup required the acquisition of thousands of images and was therefore only appropriate for measuring static deformation fields (Chakraborty and Ruiz, 2012). The system has a potential to provide up to 512×640×3600 independent voxels in a material volume of 16×28×62mm^3, currently limited to a fraction of it due to 3-D unwrapping errors, see Fig. 1.

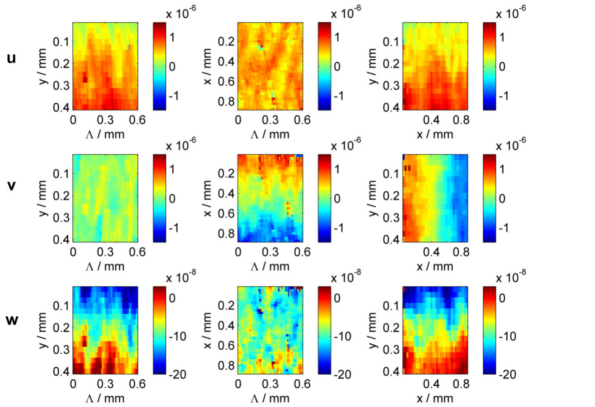

Fig. 1. Cross sections of the measured 3-D displacement field corresponding to a sample under in-plane rotation and out of plane tilt. The rows indicate the displacement components *u*, *v* and *w* along the *x*, *y* and Λ (optical depth) axes, respectively. The columns show sections of the data volume on planes *y*Λ, *x*Λ and *xy*. Displacements units: mm.

Parallel efforts were made to develop a proof-of-principle single-shot system to acquire all the required data in a single image, getting a step closer to snapshot, 3-D, time resolved optical tomography. Instead of relying on a tunable laser as in WSI, a broadband source was used and a series of monochromatic sub-images of the object were arranged across a CCD array by using a diffraction grating and an etalon, and acquired simultaneously (Huntley et al., 2010; Widjanarko et al., 2012). Their system was used to perform single-shot areal surface shape measurement.

What all these techniques have in common is that they rely on phase information to evaluate displacement fields. This makes them very sensitive to small deformations. Specific illumination and observation configurations lead to sensitivity to different components of the displacement vectors. Their spatial resolution (both transverse and axial) and the displacement sensitivity can be fully described in a framework of linear systems theory, a powerful tool to guide the design of new instruments and to compare seemingly different system realizations (Coupland and Lobera, 2008; Ruiz et al., 2011).

More recently, OCT has been used in conjunction with digital volume correlation to extract displacement fields by locally tracking the microstructure of the sample, rather than using phase information. This has the advantage of not relying on 3-D phase unwrapping, a challenging post-processing stage required in the phase based techniques described above. The OCT+DVC approach is equivalent to particle image velocimetry applied in solids. Strains of up to ~1.5% can be measured, with a strain noise floor of ~3×10^{-4} (Fu et al., 2013). Figure 2 shows u_x displacement maps measured with DVC for different *z*-slices of a porcine cornea under inflation (an increase of intra-ocular pressure). All strain components can be evaluated from similar data volumes of u_y and u_z displacements.

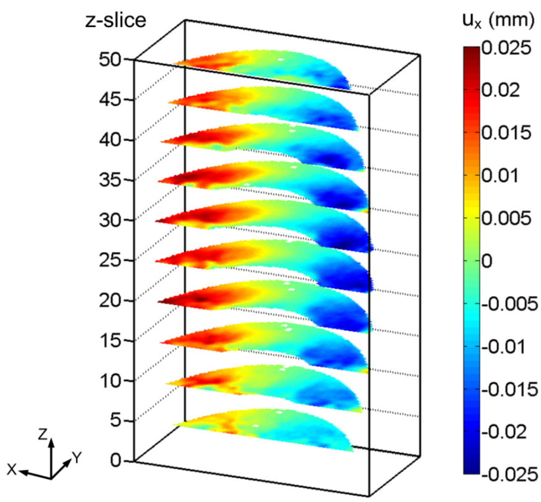

Fig. 3 u_x displacement in cross sections of a porcine cornea under inflation.

This approach is currently being used to great advantage in biology to track cell paths during an embryo's development, not using OCT but light-sheet microscopy.

3. Inverse problem solution

Traditional approaches for the identification of the material constitutive parameters involve simple mechanical tests such as tensile or bending tests. They allow the identification of constitutive parameters based on analytical relations between these parameters and measured data such as local strains and applied load. However, these approaches are usually based on some stringent assumptions such as uniform or linear stress distributions, which are difficult to ensure experimentally. Therefore, alternative methods are currently being developed to relax some of the limitations of these simple tests. The most promising rely on displacement/strain fields obtained from suitable full-field measurement techniques coupled with numerical processing and a robust identification strategy that can extract the parameters from the strain field.

The classical '*direct*' problem consists in determining the displacement, strain and stress fields assuming that the constitutive equations, the constitutive parameters, the load, the boundary conditions and the geometry are known. The '*inverse*' problem aims at finding the parameters that govern the constitutive equations using measured displacement/strain fields and the applied load (sample geometry and the constitutive equations are known a priori). Several methods have been developed to solve the inverse problem including finite element model updating (FEMU) and the virtual fields method (VFM), a review of which is offered in (Avril et al., 2008).

3.1 Finite element model updating

FEMU is the most widely accepted approach for inverse problem solution. It compares the experimental measurements with numerical predictions obtained from a finite element (FE) model. A cost function is built up using the difference between the numerical and the experimental values in terms of force and/or displacement or strain. The cost function is then minimized iteratively with respect to the sought constitutive parameters. FEMU, however, exhibits some shortcomings. In order to start this iterative approach, initial values must be provided, which generally affect the convergence rate and the quality of the results as local minima of the cost function may appear. In addition, at least one FE model needs to be run for each cost-function evaluation, resulting in computationally intensive routines, particularly for highly non-linear problems, e.g. large deformation, hyper-elasticity or visco-plasticity.

The VFM has been developed specifically to solve this inverse problem when full-field measurements are available. It avoids the drawbacks of FEMU by taking maximum advantage of the availability of full-field deformation measurements.

3.2 The Virtual Fields Method

The VFM is based on the principle of virtual work (Grediac et al., 2006). This principle can be expressed using the following equilibrium equation for a body with a volume V, as follows:

$$-\int_V \boldsymbol{\sigma} : \boldsymbol{\varepsilon}^* dV + \int_{\partial V} \overline{\boldsymbol{T}} \cdot \boldsymbol{u}^* dS + \int_V \boldsymbol{b} \cdot \boldsymbol{u}^* dV = \int_V \rho \boldsymbol{a} \cdot \boldsymbol{u}^* dV \qquad (1)$$

σ is the actual stress tensor, $\boldsymbol{\varepsilon}^*$ is the virtual strain tensor, $\boldsymbol{u}^*$ is the virtual displacement vector, $\overline{\boldsymbol{T}}$ is the applied stress vector on the boundary ∂V, *b* is the volume force vector acting in its bulk, *a* is the acceleration vector and ρ is the mass per unit volume, " . " is the vector dot product and " : " is the contracted product for second order tensors (or matrix dot product). The actual strain field, the load and the acceleration are provided by the experiment. The stress components can be expressed as a function of the material constitutive parameters and the strain components through an appropriate constitutive equation. Any new virtual field in the equilibrium equation leads to an equation involving the material constitutive parameters. Therefore, a proper choice of virtual displacement fields enables the extraction of the unknown constitutive parameters. For linear elastic materials, Eqn. (1) depends linearly on the sought parameters. Therefore, a direct identification of the constitutive parameters is available by choosing as many independent virtual fields as the number of unknowns and solving the corresponding linear equation system. For other materials which constitutive equations are not linear functions of the constitutive parameters such as anisotropic hyper-elasticity or elasto-plasticity, direct identification is not feasible. Therefore, an iterative procedure minimizing the residual of the equilibrium equation is applied. It has been used to identify elasto-plastic constitutive parameters of and has been verified to be feasible to identify the anisotropic hyper-elastic constitutive parameters of soft and biological materials such as arteries.

Recent efforts to extend the VFM to 3-D in cases with a general spatial distribution of properties within the material lead to an alternative approach. The material properties are parameterized in the spatial 'spatial frequency', rather than 'spatial', domain by performing a 2-D or 3-D Fourier series expansion of the stiffness distribution over the region of interest. The virtual fields are selected from a set of simple cosine or sine functions of different spatial frequencies rather than as polynomials of spatial variables as in the previous VFM literature. The VFM with a Fourier series for the material property parameterization and cosine/sine functions for the virtual fields is referred to as F-VFM (Nguyen et al., 2014).

4. Challenges ahead

The measurement of 3-D displacement and strain using OCT faces a number of challenges, including:

1) Dispersion, i.e. variation of refractive index with wavelength, which impairs the depth resolution as a wavelength dependent phase delay is introduced in the data, which distorts the reconstructions and introduces phase errors that can lead to spurious displacements in phase-contrast implementations as PC-SOCT (De la Torre-Ibarra et al., 2006, 2009);

2) Refraction effects due to non-perpendicular illumination and observation at the object surface, which lead to spurious strains and require refraction compensation to re-map the internal structure in a regular grid from which strain can be evaluated from displacements;

3) 3-D phase unwrapping, i.e. the process of finding the right multiple of 2π in the wrapped phase distributions obtained with phase-contrast techniques. This is a difficult problem in 3-D as phase singularity loops, which are common in experimental data, lead to phase unwrapping errors that propagate through the data volume;

4) Validation of the 3-D VFM, the study of its applicability to identification and the effect of noise in the measured displacements;

5) There is an upper bound to the strain (~1.5%) that can be measured by combining OCT & DVC, which is due to speckle decorrelation. This could be overcome by using a re-referencing approach (measuring small incremental deformations and then adding up the results) in the case of larger deformations.

5. Potential applications in flow measurements

The literature on PIV and holographic PIV (film based and digital) of micro flows is vast, and this paper will not attempt to review it or suggest that the techniques described above for experimental mechanics applications could replace current methodologies. They certainly share a major feature, i.e. they strive to provide as many components of the displacement or velocity field in as big a field as possible and with high spatial and temporal resolutions. Current efforts into extending OCT methods into single-shot techniques have the potential of expanding the range of applications to study dynamic events such as micro-flows in engineering and biological systems in which scattering particles are transported in a flow (e.g. tribology, microfluidic devices, cell migration, multi-phase flows, etc.). Proof of principle systems are currently limited to a spatial resolution of ~40×40×40 independent measurements but systems with ~200×200×200 voxels are feasible at a few hundred fps using large area S-CMOS photodetector arrays.

5.1 Mapping viscosity in non-Newtonian fluids

In terms of material characterization, would it be possible to use inversion methods to, for instance, measure viscosity in a non-contact way? As the VFM relies on the equilibrium equation from solid mechanics, it does not hold for liquids which do not see stress and it would require some important adaptations. It should be possible to measure the viscosity of fluids from a velocity field but it would be necessary to know also the pressure field, or the pressure boundary conditions. In the case of slow viscous flows, only the pressures and the viscous forces would be involved in the equilibrium as the weight is usually neglected. In faster flows, inertial forces may become non-negligible but these could be evaluated from a 3D acceleration field obtained from a measured 3D velocity field acquired with sufficient time resolution. Even though all these measurements are commonplace using PIV, I am not aware that the VFM has been used on fluids with PIV data.

5.2 Flows close to a boundary

OCT can measure up to a depth of around 2-3 mm. Broadband sources and tunable lasers are in the near-infrared region of the spectrum (~750-1500nm), meaning that materials that look opaque for visible wavelengths appear semi-transparent. Examples are skin and other soft tissues, polymers, rubbers, glass-fibre composites, etc. This means that flows at the boundary layer could be characterized, to measure for instance velocity gradient and wall shear stress. The study of multi-phase flows in which particles of different sizes segregate at different distances to the wall should also be feasible.

5.3 Solid/liquid interactions

Blood-artery interactions are another area in which a marriage between inversion techniques in solids and fluid mechanics could lead to fundamental breakthroughs. These are systems in which a non-Newtonian fluid finds its way in a network of vessels whose walls are made up of multiple layers of tissue, each with different mechanical properties (visco-elasticity, hyper-elasticity) and in which adhesion between layers also plays a role. A better understanding of such systems may lead to a reduction in fatal aneurysms.

6. Conclusion

Some recent techniques for displacement measurements in solid mechanics, which provide multiple component (up to 3D-3C) data, are presented. Either based on phase detection or relying on digital volume correlation, these have been used to feed inversion models with the aim of characterizing the spatial distribution of properties in a multi-material object (such as the elastic modulus). Single shot OCT techniques now open the possibility to extend their application to the study of flows or solid/flow interactions, especially close to solid/liquid interfaces. Current time resolutions seem adequate to study steady state cases, and fast S-CMOS detectors and tunable lasers are opening new possibilities in flows with low Reynolds number, where turbulence does not play a role and models can be simplified. There is much to learn from both fields and it seems that problems that involve solid and liquid phases are getting within grasp. Substantial effort will have to be invested in inversion methods, to really make the most of the data that current techniques can provide.

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