Laminar Fluid Flow in Microchannels with Complex Shape

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Abstract Laminar flow of newton incompressible fluid with constant viscosity in system of channels (Fig.1) is considered. It's demonstrated that there are infinitely many stationary solutions with same boundary conditions. Possible flow fields and ways of practical realization are studied. Solutions are investigated numerically, scheme of calculation is described partially in [1, 2].

Keywords: Laminar Flow, Microchannels, Viscosity, Numerical Simulation

Introduction

This paper presents the results of a numerical simulation of fluid flow in microchannels with complex shapes.

Physical formulation of the problem

The steady flow of newton viscous incompressible fluid in system of channels (Fig. 1) is considered.

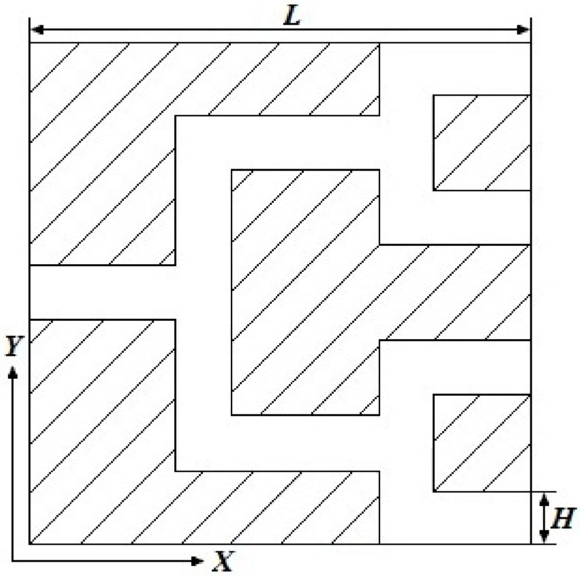


Fig. 1. Geometry of the area

The whole area is a square with a side $L = 10H$. The liquid moves in a channel of height H with velocity u_0 along the horizontal axis.

Assume that the fluid viscosity is constant inside the channels. Outside the channels we simulate solid body as a liquid with a very high viscosity

$$\mu = \begin{cases} \mu_0, & \text{inside the channels} \\ 10^{15}\mu_0, & \text{outside the channels} \end{cases}.$$

Suppose that the specific heat capacity c and density ρ are constant and thermal conductivity is

$$k = \begin{cases} k_0, & \text{inside the channels} \\ 0, & \text{outside the channels} \end{cases}.$$

Mathematical formulation of the problem

Mathematical model of the process is Navier – Stokes equations and energy equation, taking into account a viscous dissipation:

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0,$$

$$\rho u \frac{\partial u}{\partial x} + \rho \upsilon \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial \upsilon}{\partial x}\right),$$

$$\rho u \frac{\partial \upsilon}{\partial x} + \rho \upsilon \frac{\partial \upsilon}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu \frac{\partial \upsilon}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial \upsilon}{\partial y}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial \upsilon}{\partial y}\right) + \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial y}\right),$$

$$\rho c\left(u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+$$
$$+2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial \upsilon}{\partial y}\right)^2+\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial \upsilon}{\partial x}\right)^2\right].$$

Here u, υ are velocities along horizontal and vertical axes, respectively, p is a pressure.

Boundary conditions inside the channels:

$$u\big|_{x=0}=u_0,\ \upsilon\big|_{x=0}=0,\ T\big|_{x=0}=T_0,$$
$$\left.\frac{\partial u}{\partial x}\right|_{x=L}=0,\ \upsilon\big|_{x=L}=0,\ \left.\frac{\partial T}{\partial x}\right|_{x=L}=0,$$

outside the channels:

$$u\big|_{x=0}=u\big|_{x=L}=u\big|_{y=0}=u\big|_{y=L}=0,$$
$$\upsilon\big|_{x=0}=\upsilon\big|_{x=L}=\upsilon\big|_{y=0}=\upsilon\big|_{y=L}=0,$$
$$\left.\frac{\partial T}{\partial x}\right|_{x=0}=\left.\frac{\partial T}{\partial x}\right|_{x=L}=\left.\frac{\partial T}{\partial y}\right|_{y=0}=\left.\frac{\partial T}{\partial y}\right|_{y=L}=0.$$

Introduce the dimensionless variables:

$$X=\frac{x}{H},\ Y=\frac{y}{H},\ U=\frac{u}{u_0},\ V=\frac{\upsilon}{u_0},$$
$$P=\frac{p}{\rho u_0^2},\ \bar{\mu}=\frac{\mu}{\mu_0},\ \bar{k}=\frac{k}{k_0},\ \theta=\frac{T-T_0}{T_0}.$$

The system of equations transforms to

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0,$$
$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\left[\frac{\partial}{\partial X}\left(\bar{\mu}\frac{\partial U}{\partial X}\right)+\right.$$
$$\left.+\frac{\partial}{\partial Y}\left(\bar{\mu}\frac{\partial U}{\partial Y}\right)+\frac{\partial}{\partial X}\left(\bar{\mu}\frac{\partial U}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\bar{\mu}\frac{\partial V}{\partial X}\right)\right],$$
$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\left[\frac{\partial}{\partial X}\left(\bar{\mu}\frac{\partial V}{\partial X}\right)+\right.$$
$$\left.+\frac{\partial}{\partial Y}\left(\bar{\mu}\frac{\partial V}{\partial Y}\right)+\frac{\partial}{\partial Y}\left(\bar{\mu}\frac{\partial V}{\partial Y}\right)+\frac{\partial}{\partial X}\left(\bar{\mu}\frac{\partial U}{\partial Y}\right)\right],$$
$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Re}\cdot\mathrm{Pr}}\left[\frac{\partial}{\partial X}\left(\bar{k}\frac{\partial \theta}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\bar{k}\frac{\partial \theta}{\partial Y}\right)\right]+$$
$$+2\frac{\mathrm{Ec}}{\mathrm{Re}}\cdot\bar{\mu}\left[\left(\frac{\partial U}{\partial X}\right)^2+\left(\frac{\partial V}{\partial Y}\right)^2+\frac{1}{2}\left(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right)^2\right]$$

with the following dimensionless viscosity and dimensionless thermal conductivity:

$$\bar{\mu}=\begin{cases}1, & \text{inside the channels}\\ 10^{15}, & \text{outside the channels}\end{cases},$$
$$\bar{k}=\begin{cases}1, & \text{inside the channels}\\ 0, & \text{outside the channels}\end{cases}.$$

Boundary conditions inside the channels are

$$U\big|_{X=0}=1,\ V\big|_{X=0}=0,\ \theta\big|_{X=0}=0,$$
$$\left.\frac{\partial U}{\partial X}\right|_{X=10}=0,\ V\big|_{X=10}=0,\ \left.\frac{\partial \theta}{\partial X}\right|_{X=10}=0,$$

outside the channels:

$$U\big|_{X=0}=U\big|_{X=10}=U\big|_{Y=0}=U\big|_{Y=10}=0,$$
$$V\big|_{X=0}=V\big|_{X=10}=V\big|_{Y=0}=V\big|_{Y=10}=0,$$
$$\left.\frac{\partial \theta}{\partial X}\right|_{X=0}=\left.\frac{\partial \theta}{\partial X}\right|_{X=10}=\left.\frac{\partial \theta}{\partial Y}\right|_{Y=0}=\left.\frac{\partial \theta}{\partial Y}\right|_{Y=10}=0.$$

Thus, studied process is defined by three dimensionless parameters:

$$\mathrm{Re}=\frac{\rho u_0 H}{\mu_0},\ \mathrm{Pr}=\frac{\mu_0 c}{k_0},\ \mathrm{Ec}=\frac{u_0^2}{cT_0}.$$

The numerical scheme

The explored problem was solved numerically by the method which was described in [1, 2]. Calculations were done on the uniform grid consisting of 400×400 internal nodes. The Reynolds number Re was varied from 10 to 100, the Prandtl number Pr was equaled to 10 and the Eckert number Ec was equaled to 0,008.

Results

In this paper we have obtained infinitely many stationary solutions of the above mentioned system of equations, which have been determined from nonstationary equations with same boundary conditions by changing its initial conditions. For example, consider the following three cases. Note, channels numbering is done from the bottom up.

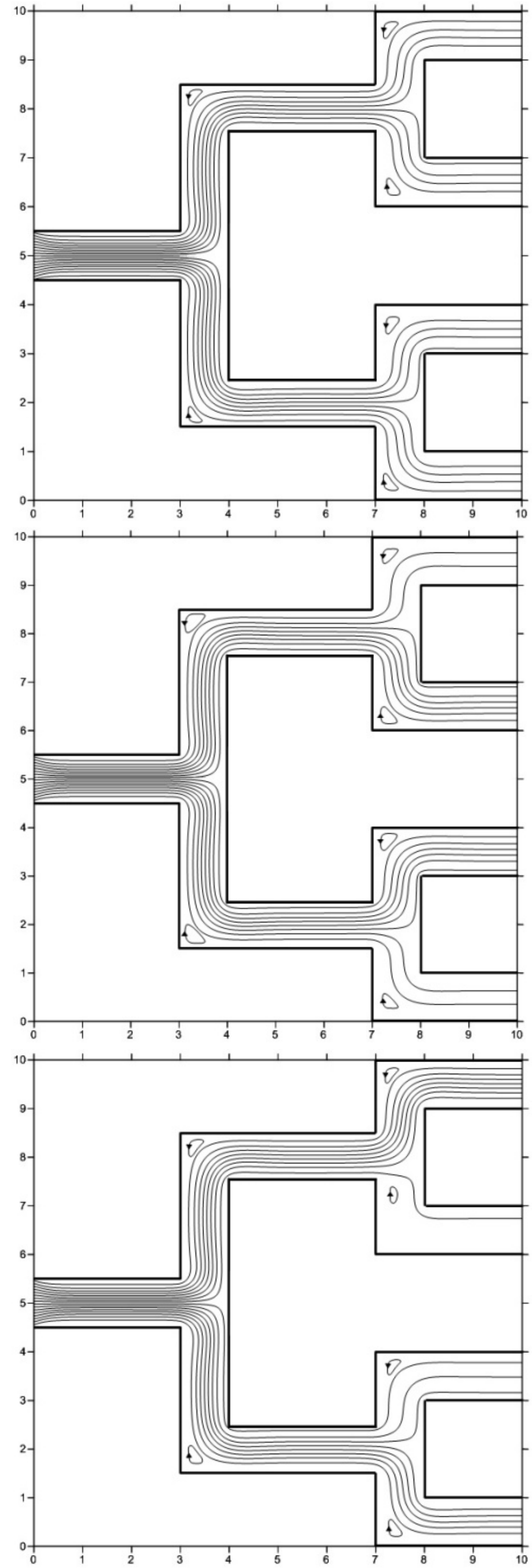
1) The same flow rate $\int UdY$ through all four channels equals to $0,25$

2) The symmetrical flow rate about the axis of symmetry of the whole area ($\int UdY=0,15$ for the first and fourth channels, $\int UdY=0,35$ for the second and third ones)

3) The different flow rate through all

channels ($\int UdY = 0{,}35;\ 0{,}15;\ 0{,}05;\ 0{,}45$ for the channel from 1 to 4, respectively).

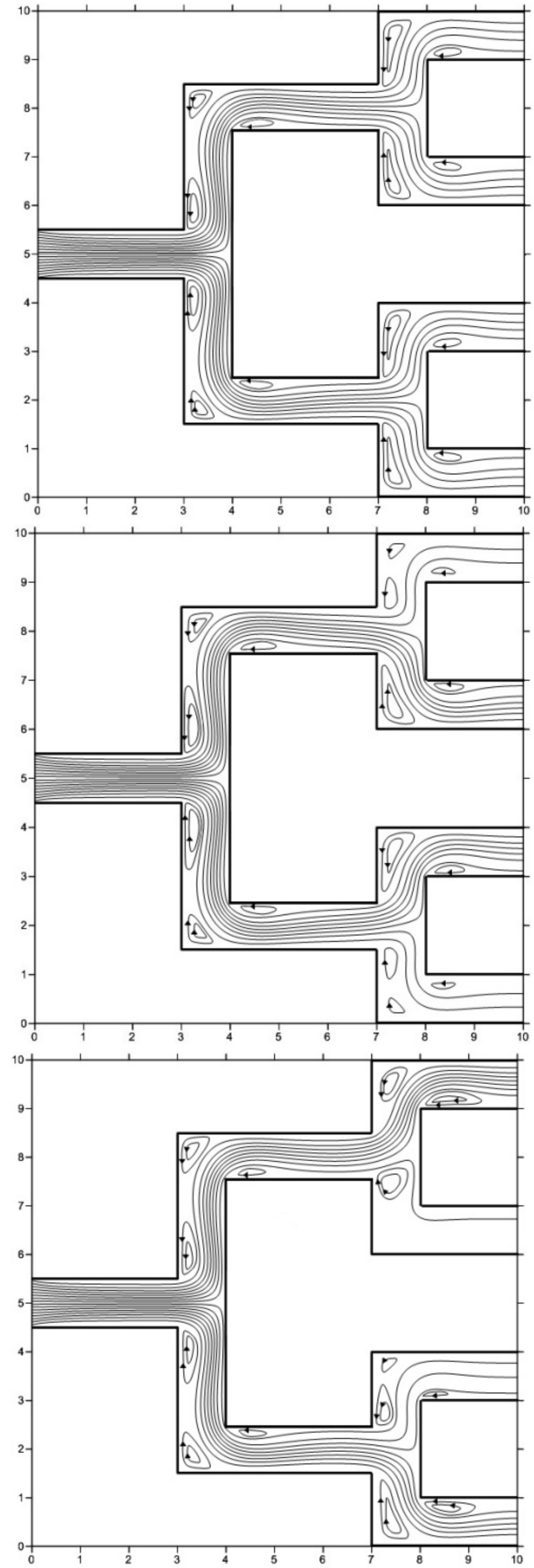
Fig. 2 (a - f) shows a stream function for all these cases.



a)

b)

c)



d)

e)

f)

Fig. 2. Stream function in case
a) 1, b) 2, c) 3 (Re=10), d) 1, e) 2, f) 3(Re=100)

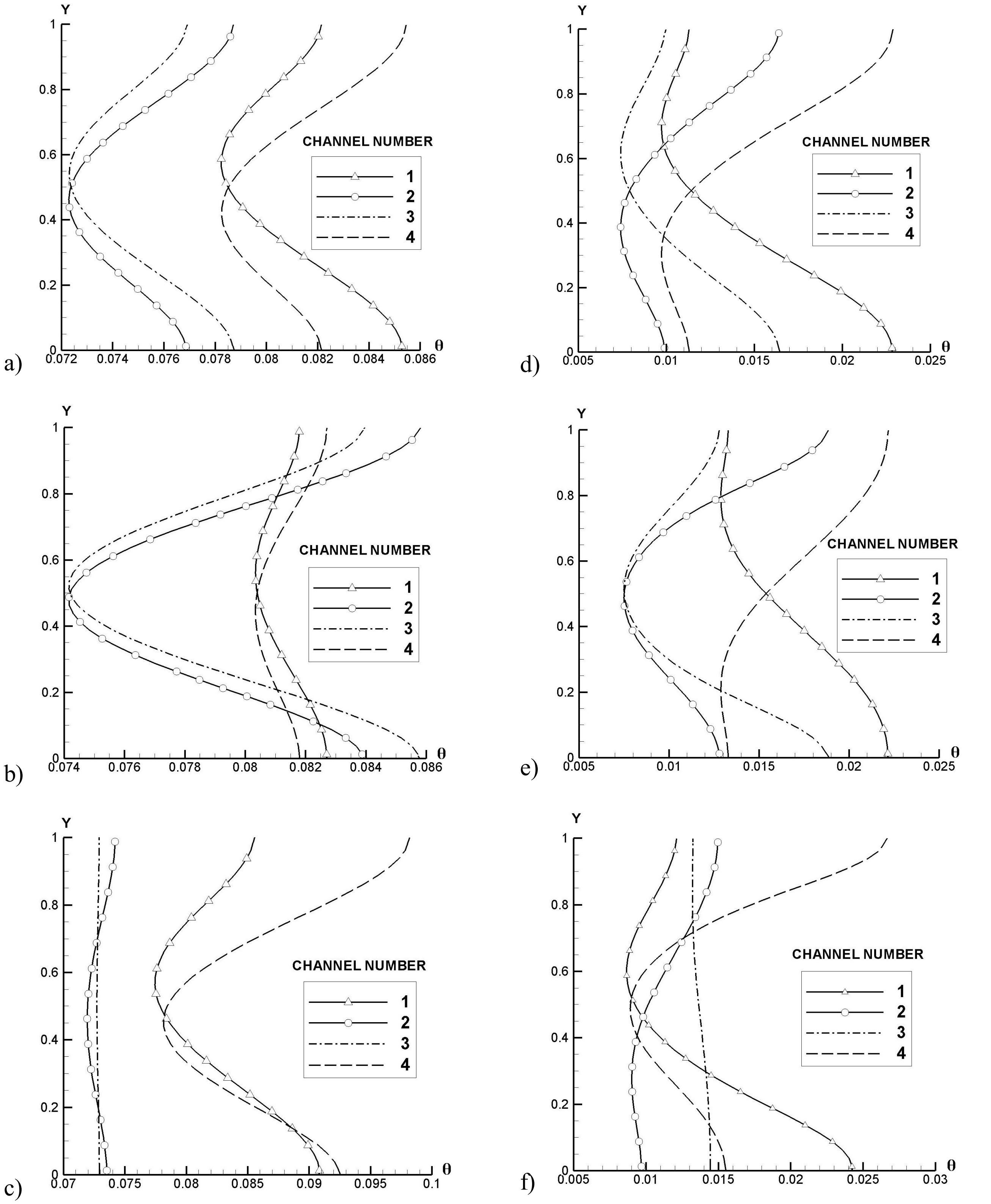


Fig. 3. Dependence of θ on coordinate Y in case a) 1, b) 2, c) 3 (Re=10), d) 1, e) 2, f) 3(Re=100)

Fig. 3 (a - f) demonstrates a temperature at the four channel exits. We see that temperature profiles are symmetric about the axis of symmetry of the area. However, there is different temperature in channels even in case of the same flow rate through all of them (Fig. 3 a). It happens because of heating of fluid near channel walls owing to viscous dissipation. Thus, after the first channel branching heated fluid goes to walls, positioned further from the axis of symmetry. The less heated fluid goes to walls, positioned closer to the axis of symmetry. A similar situation occurs in the next branching points. We obtain the highest temperature in the upper and lower channels and the lowest one in middle channels. Notice, if viscosity were not a constant in channels, but, for example, it were a function of temperature, then the flow rate at channel exits would be different.

Let us study a change of an average pressure in a cross section of the channel $\int PdY$ along the system of channels as a function of the X - coordinate.

Suppose that $P_0 = \int PdY$ at $X = 10$.

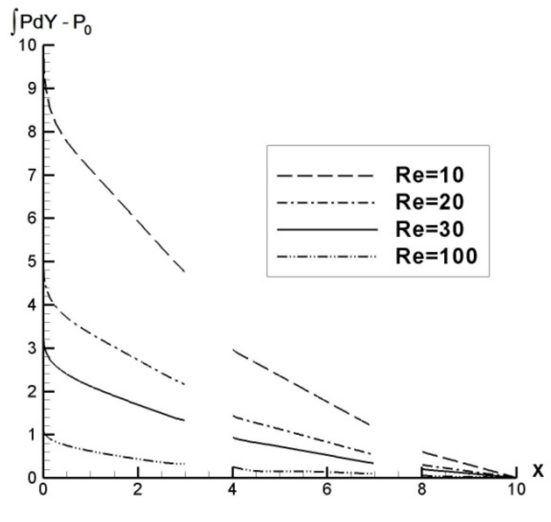


Fig. 4. Dependence of $\int PdY - P_0$ on coordinate X

In the first case the dimensionless pressure drop is the same in all four channels and it decreases with growth of the Reynolds number (Fig. 4). Make some transformation in dimensionless pressure and obtain

$$P = \frac{p}{\rho u_0^2} = C \cdot \frac{p}{\mathrm{Re}^2}, \quad \text{where} \quad C = \rho \cdot \left(\frac{H}{\mu_0}\right)^2 .$$

Thus, the actual pressure value increases with the growth of Re. There are nonlinear parts of plots of the actual pressure with $C = 1$ (Fig. 5) due to development of flow in initial hydrodynamic regions.

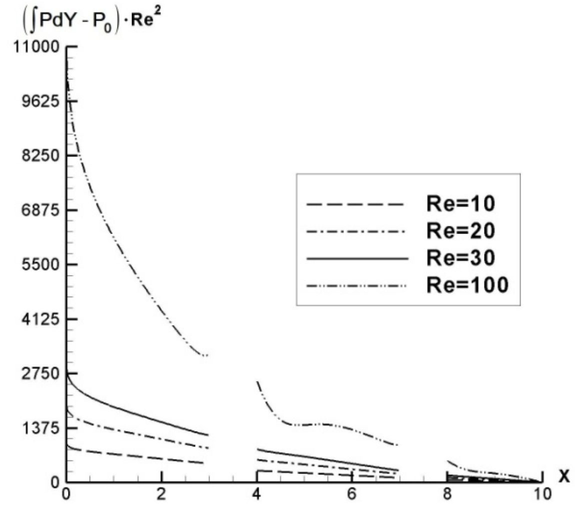


Fig. 5. Dependence of $\left(\int PdY - P_0\right) \cdot \mathrm{Re}^2$ on coordinate X

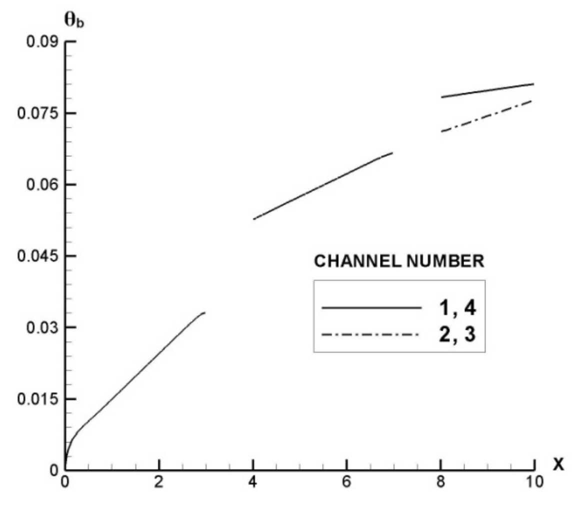


Fig. 6. Dependence of bulk temperature θ_b on coordinate X (Re=10)

In the second case, with symmetric flow rate, bulk temperature that is calculated by the formula $\theta_b = \dfrac{\int \theta \cdot U \, dY}{\int U \, dY}$ is everywhere greater in the upper and the lower channels, despite the fact that the main part of fluid flows through a pair of middle ones (Fig. 6).

In the third case, with the different flow rate through all channels, a greater pressure drop (Fig. 7) and greater bulk temperature θ_b (Fig. 8) corresponds to channel with a greater flow rate.

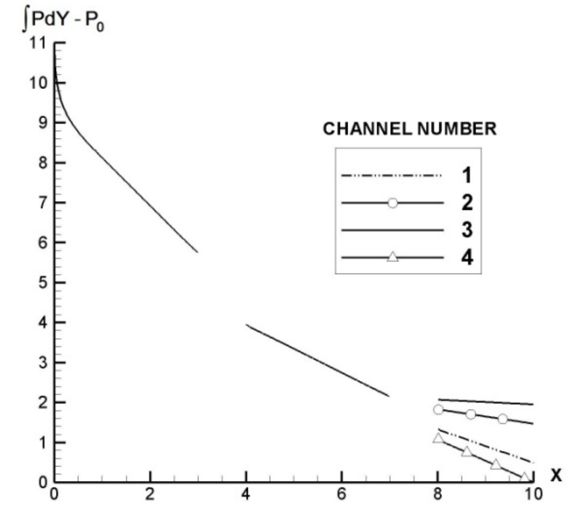


Fig. 7. Dependence of $\int PdY - P_0$ on coordinate X (Re=10)

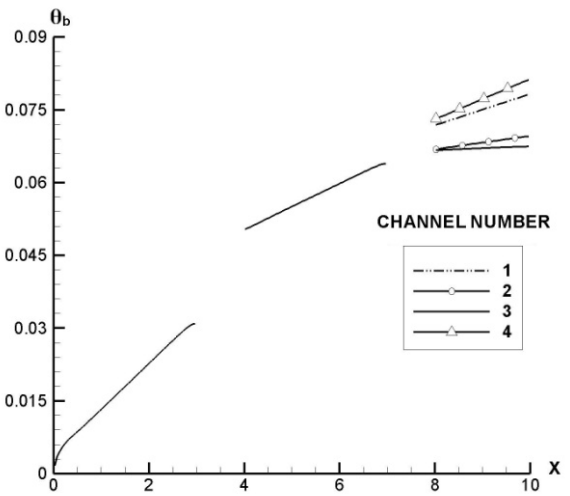


Fig. 8. Dependence of bulk temperature θ_b on coordinate X (Re=10)

References

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