Non-linear time series models with applications to financial data

A thesis submitted for the degree of Doctor of Philosophy

by

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Abstract

The purpose of this thesis is to investigate the financial volatility dynamics through the GARCH modelling framework. We use univariate and multivariate GARCH-type models enriched with long memory, asymmetries and power transformations. We study the financial time series volatility and co-volatility taking into account the structural breaks detected and focusing on the effects of the corresponding financial crisis events. We conclude to provide a complete framework for the analysis of volatility with major policy implications and benefits for the current risk management practices.

We first investigate the volume-volatility link for different investor categories and orders, around the Asian crisis applying a univariate dual long memory model. Our analysis suggests that the behaviour of volatility depends upon volume, but also that the nature of this dependence varies with time and the source of volume. We further apply the vector AR-DCC-FIAPARCH and the UEDCC-AGARCH models to several stock indices daily returns, taking into account the structural breaks of the time series linked to major economic events including crisis shocks. We find significant cross effects, time-varying shock and volatility spillovers, time-varying persistence in the conditional variances, as well as long range volatility dependence, asymmetric volatility response to positive and negative shocks and the power of returns that best fits the volatility pattern. We observe higher dynamic correlations of the stock markets after a crisis event, which means increased contagion effects between the markets, a continuous herding investors' behaviour, as the in-crisis correlations remain high, and a higher level of correlations during the recent financial crisis than during the Asian. Finally, we study the High-frEquency-bAsed VolatilitY (HEAVY) models that combine daily returns with realised volatility. We enrich the HEAVY equations through the HYAPARCH formulation to propose the HYDAP-HEAVY

(HYperbolic Double Asymmetric Power) and provide a complete framework to analyse the volatility process.

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Declaration

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Introduction

This thesis aims to study the econometric modelling of volatility in financial markets through the GARCH framework. We examine the applicability of several modelling techniques available in the broad family of GARCH models and their performance in volatility level-shifts observed during financial crises. We focus on the Asian financial crisis of 1997 and the recent global turmoil starting from 2007-08 in order to model adequately the volatility dynamics of the stock markets. Our study aims to provide results and conclusions with major policy implications and impact on the current risk management practices. Policy makers, risk management practitioners and also academics should consider the stochastic properties of the financial time series and especially their volatility dynamics, in the development of the necessary policies and risk tools to better perform during both periods of economic tranquillity and crisis.

We first apply the univariate GARCH modelling framework with long memory features in order to investigate the volume-volatility link in the Korean stock market for different investor categories and orders (buy and sell), before and after the Asian financial crisis (Chapter 1). The volume-volatility relationship has critical policy implications for stock market regulators, since the investors trading activity is proved to affect considerably financial returns' volatility. We complement the literature about the impact of domestic and foreign investors on emerging stock markets by examining the effect of the trading volume on the stock market volatility, taking into consideration for each volume series its sell and buy side as well as its total separately and also investigating the effect of each of the eight different domestic investor groups, that compromise the total domestic trading volume.

We estimate the two main parameters driving the degree of persistence in volatility and its uncertainty using a univariate GARCH model that is fractionally integrated in both the autoregressive mean and variance specifications. Our model provides a general and flexible framework with which to study complicated processes like volume and volatility. In order to be able to examine the volume-volatility relationship, we estimate the dual long memory model with lagged values of the trading volume included in the mean equation of the Garman-Klass volatility series. We further study the volume effect on volatility during different periods of the economic cycle. Our results support the causal effect from volume to volatility, which is found sensitive to the economic period and to the various investors' behaviour. Stock sales are found to affect volatility positively regardless of the period or the investor category, while the buy orders and the total trading activity effect on volatility vary across time and investor. Foreign investors' volume is negatively related to volatility in the pre-crisis period and turns to a positive link after the financial crisis and during the recession period. In the contrary, domestic investors' aggregated volume tends to give a positive effect on volatility in all times, while the more informed players separately have a negative impact on the pre-crisis volatility.

We further investigate the financial volatility and co-volatility dynamics using the multivariate GARCH modelling framework (Chapter 2). The study of the linkages between volatilities and co-volatilities of the financial markets is a critical issue in risk management practice. The multivariate GARCH framework provides the tools to understand how financial volatilities move together over time and across markets. Conrad et al. (2011) applied a multivariate fractionally integrated asymmetric power ARCH (FIAPARCH) model that combines long memory, power transformations of the conditional variances and leverage effects with constant conditional correlations (CCC) on eight national stock market indices returns. The long-range volatility dependence, the power transformation of returns and the asymmetric response of volatility to positive and negative shocks are three features that improve the modelling of the volatility process of asset returns and its implications for the various risk management practices. We extend their model by allowing for cross effects between the markets in the mean of returns and by estimating time-varying conditional correlations. We also study the effect of financial crisis events on the dynamic conditional correlations as well as on the three key features of the conditional variance nested in the model. Therefore, the contribution of our study is that our model provides a complete framework for the analysis of financial markets' co-volatility processes.

The empirical analysis of our model applied on eight stock indices daily returns in a bivariate and trivariate framework provides evidence that confirms the importance of long memory in the conditional variance, of the power transformations of returns to best fit the volatility process and of the asymmetric response of volatility to positive and negative shocks. We extend the existing empirical evidence on the dynamic conditional correlations (DCC) models by adding all cross effects in the mean equation, that is we estimate a full vector autoregressive (VAR) model, to reveal the relationship amongst the returns of each multivariate specification. In the previous studies the researchers have added as regressor in the mean for all stock market indices a prevailing global index return, such as S&P 500 or an index of particular interest for the region and the period investigated. Our cross effects are found significant in most cases. Moreover, another of our main findings regards the DCC analysis with structural breaks. In line with the literature, our model estimates always highly persistent conditional correlations. The correlations increase during crisis events, indicating contagion effects between the markets and remain on a high level after the crisis break, showing the investors' herding behaviour. Finally, we contribute to the existing literature findings by comparing two different financial crises, the Asian (1997) and the recent Global (2007-08) crisis, in terms of their effects on the correlations, where we observe much more heightened conditional correlation estimates for the recent Global crisis than for the Asian crisis. This is reasonable since the international financial integration followed by the financial liberalisation and deregulation in capital controls has reached its peak nowadays compared to its evolution during the Asian financial crisis in 1997.

In the third part of the thesis (Chapter 3) we focus again on the recent financial crises and examine how the mean and volatility dynamics, including the underlying volatility persistence and volatility spillovers structure, have been affected by these crises (the Asian and the recent Global crisis). With this aim we make use of several modern econometric approaches for univariate and multivariate time series modelling, which we also condition on the possibility of breaks in the mean and/or volatility dynamics taking place. Moreover, we unify these approaches by introducing a set of theoretical considerations for time-varying (TV) AR-GARCH models, which are also of independent interest. We use a battery of tests to identify the number and estimate the timing of breaks both in the mean and volatility dynamics. We, finally, employ the bivariate unrestricted extended dynamic conditional correlation (UEDCC) AGARCH process to analyse the volatility transmission structure, applied to stock market returns. The model is based on the dynamic conditional correlation of Engle (2002a) allowing for volatility spillovers effects by imposing the unrestricted extended conditional correlation (dynamic or constant) GARCH specification of Conrad and Karanasos (2010). We extend it by allowing shock and volatility spillovers parameters to shift across abrupt breaks as well as across two regimes of stock returns, positive (increases in the stock market) and negative (declines in the stock market). Our model is flexible enough to capture contagion effects as well as to identify the volatility spillovers associated with the structural changes and exact movements of each market (e.g., upward or downward) to the other and vice versa. Knowledge of this mechanism can provide important insights to investors by focusing their attention on structural changes in the markets as well as their trends and movements (e.g., upward or downward) in order to set appropriate portfolio management strategies. Overall, our results suggest that stock market returns exhibit time-varying persistence in their corresponding conditional variances. The results of the bivariate

UEDCC-AGARCH$(1, 1)$ model applied to FTSE and DAX returns and NIKKEI and Hang Seng returns also show the existence of dynamic correlations as well as time-varying shock and volatility spillovers between the two variables in each pair.

Lastly, the fourth part of this thesis (Chapter 4) applies and extends the univariate high-frequency-based volatility (HEAVY) model of Shephard and Sheppard (2010). The HEAVY framework models financial volatility based on both daily and intra-daily data, so that the system of equations estimated adopts to information arrival more rapidly than the classic daily GARCH models. The HEAVY model is based on the classic GARCH model of Bollerslev (1986), the GARCHX model and the Multiplicative Error Model (MEM) of Engle (2002b) in order to model realised volatility on high-frequency data associated with daily returns GARCH conditional volatility. Its main advantage is the robustness to structural breaks, especially during crisis periods, since the mean reversion and short-run momentum effects result to higher quality performance in volatility level shifts and more reliable forecasts. Our main contribution is the enrichment of the HEAVY model with long memory structure, volatility asymmetries and power transformations through the HYAPARCH specification of Schoffer (2003) and Dark (2005) and the relevant GARCH models nested in the HYAPARCH structure. We compare the results of stock market data modelling with the several long memory, power and asymmetric specifications and conclude to prefer the most comprehensive one which we define as HYDAP-HEAVY (HYperbolic Double Asymmetric Power) for the realised measure models (realised kernel models are presented) and the FIAP-HEAVY (Fractional Integrated Asymmetric Power) for the squared returns models.

Moreover, we follow the GARCH literature that combines trading volume with the conditional variance of returns (Lamoureux and Lastrapes, 1990, Gallo and Pacini, 2000) and test whether the standard HEAVY equations adopt further to the volume increment. We add the overnight trading activity indicator as additional regressor in the benchmark HEAVY model to evaluate the effect of volume on volatility and the adjustment of volatility to the additional information from the trading volume proxy. As expected from the existing empirical evidence, the overnight indicator gives a positive feedback to the volatility of returns. Our main finding is that the HEAVY equations exhibit lower persistence, when the overnight surprise is used for the squared returns. In the realised measure modelling the overnight indicator has immaterial effect on the volatility process. We further study the Garman-Klass (GK) volatility measure in the HEAVY framework in comparison with the other two variables (the squared returns and the realised kernel). We observe that the realised measure shows stronger effects than the GK measure when added as regressor and the GK-models seem to share characteristics with both the other two models (the squared returns and the realised kernel equations), but with more similarities to the realised measure process. Finally, we re-estimate the benchmark HEAVY equations taking into account the structural breaks apparent in the squared returns series and estimate the time-varying behaviour of the arch, garch-x and heavy coefficients. Focusing on the recent Global financial crisis, we observe a positive increment on the volatility process generated by the aforementioned coefficients after the crisis break.

Chapter 1 Trader type effects on the volume-volatility relationship: Evidence from the Korean stock exchange

1.1 Introduction

The volume-volatility relationship has attracted major interest of the financial econometrics research with critical policy implications for stock market regulators, since the investors trading activity is proved to affect considerably financial returns' volatility. The empirical evidence on emerging markets has focused particularly on foreign investors' behaviour. Following Karanasos and Kartsaklas (2009), we investigate the volume-volatility link in the Korean stock market for different investor categories and orders (buy and sell), before and after the Asian financial crisis.

In particular, we complement the literature about the impact of domestic and foreign investors on emerging stock markets by examining the effect of the trading volume on the stock market volatility, taking into consideration for each volume series its sell and buy side as well as its total separately and, also, investigating the effect of each of the eight different domestic investor groups, that compromise the total domestic trading volume. We estimate the two main parameters driving the degree of persistence in volatility and its uncertainty using a univariate Generalised ARCH (GARCH) model that is Fractionally Integrated (FI) in both the Autoregressive (AR) mean and variance specifications. We refer to this model as the ARFI-FIGARCH. It provides a general and flexible framework with which to study complicated processes like volume and volatility. In order to be able to examine the volume-volatility relationship, we estimate the dual long memory model with lagged values of the trading volume included in the mean equation of volatility. We further study the volume effect on volatility during different periods of the economic cycle (tranquil, crisis and recession periods).

Our empirical analysis strongly supports the causal effect from volume to volatility, which is found sensitive to the economic period and to the various investors' behaviour. Stock sales are found to affect volatility positively regardless of the period or the investor category, while the buy orders and the total trading activity effect on volatility vary across time and investor. Foreign investors' volume is negatively related to volatility in the pre-crisis period and turns to a positive link after the financial crisis and during the recession period. In the contrary, domestic investors' aggregated volume tends to give a positive effect on volatility in all times, while the more informed players separately have a negative impact on the pre-crisis volatility.

The remainder of the chapter is structured as follows. In Section 1.2 we illustrate the theoretical background on the trading behaviour of different investor groups. In Section 1.3 we refer to the data used and the structural breaks identified in volatility. In Section 1.4 we detail the econometric models applied. Section 1.5 presents our empirical results and in Section 1.6 we check their robustness. Finally, Section 1.7 concludes the analysis.

1.2 Theoretical background

1.2.1 The trading behaviour of institutional investors

Empirical research in finance documents that buyer- and seller-initiated (institutional/block) trades have an asymmetric impact on prices. Holthausen et al. (1987, 1990) find permanent price effects that increase with block size, whether the block is buyer- or seller-initiated. As regards temporary price effects, they are not related to the size of the block for buy trades as they do for sell trades. Temporary and permanent price effects of block trades have been explained in terms of liquidity costs, inelastic demand curves and information effects [1]. Additionally, the identity of the management firm behind the trade (Chan and Lakonishok, 1993, 1995) [2] and the underlying

[1] Liquidity costs result in a temporary price effect, if it is costly to identify potential buyers or sellers of a large block. The seller of a large block gives the purchaser a price concession as compensation for inventory and search costs. Permanent price effects may arise because of inelastic demand and supply conditions and/or information effects. If there are insufficient close substitutes for a particular stock, the excess-demand (supply) curve faced by sellers (buyers) is not perfectly elastic. This will induce a permanent price effect that will vary with the size of the block for seller- (buyer-) initiated transactions. Block trades which convey information about a firm's prospects, will have a permanent price effect even if there are sufficient close substitutes to produce perfectly elastic excess demand (See Kraus and Stoll, 1972; Scholes, 1972; Mikkelson and Partch, 1985).

[2] Money managers with high demands for immediacy tend to be associated with larger market impact. Some price pressure is also evident but the average effect is small. Saar (2001) provides an institutional trading explanation about the price impact asymmetry of block trades. The main implication of the model is that the history of price

market (bullish or bearish) condition (Chiyachantana et al., 2004) [3] are important contributors to the asymmetric price impact of institutional buy and sell orders.

Institutional investors have different investment styles (active or passive, value or growth) and order-placement strategies (market or limit orders) when they buy or sell stocks in the securities markets. Keim and Madhavan (1995, 1996) find considerable heterogeneity in investment style (buy-sell decision and past excess returns) across institutions. Surprisingly, the motivation for the trade decision is often not symmetric for buys versus sells. For example, some institutions that buy stocks, after they decline in price, do not follow the same trading rule when they sell. Additionally, institutional traders tend to spread buy orders over longer periods than equivalent sell orders. We also find significant differences in the choice of order type across institutional styles. Gompers and Metrick (2001) find that institutions invest in stocks that are larger, more liquid and have had relatively low returns during the previous year. Barber and Odean (2008) find that professional investors are less prone to indulge in attention-driven purchases. With more time and resources, professionals can monitor a wider range of stocks and eventually concentrate on stocks that have passed an initial screen.

Actively managed equity mutual funds buy and sell stocks based on valuation beliefs. The structure of open-end funds also leads them to trade for liquidity, tax and window-dressing purposes [4]. Alexander et al. (2007) relate the performance of mutual fund trades to their motivation. They find that managers making purely valuation-motivated purchases substantially

performance influences the information content of buy and sell trades: the longer the run-up in a stock's price, the less the asymmetry. The intensity of institutional trading and the frequency of information events affect the asymmetry differently depending on recent price performance. The model even predicts negative price asymmetry, that is, sells have greater price impact than buys following a long period of price run-ups.

3 In bullish markets, institutional purchases have a bigger price impact than sells, but in bearish markets sells have a higher price impact. They also find that the price impact varies depending on order characteristics, firm-specific factors and cross-country differences.

4 Unanticipated investor flows force managers to continually rebalance their portfolios to control liquidity. This provision of liquidity, however, imposes significant indirect trading costs on open-end fund. Also, a desire to minimise taxable distributions creates incentives for them to sell losers heading into the tax year-end. Finally, aspiring to impress investors, managers may window-dress their portfolios by buying recent winners and selling recent losers just before reporting dates.

beat the market, but are unable to do so when compelled to invest excess cash from investor inflows (liquidity-motivated trading result in significant trading losses) [5]. A similar, but weaker, pattern is found for stocks that are sold. Grinblatt and Keloharju (2000) using buy and sell trades of individuals and institutions in the Finnish stock market find evidence that investors are reluctant to realise losses (disposition effect), they engage in tax-loss selling activity and that past returns and historical price patterns, such as being at a monthly high or low, affect trading behaviour. There is also modest evidence that life-cycle trading plays a role in the pattern of buys and sells. Griffin et al. (2003) find that the 5-minute intervals with the largest institutional buying (selling) activity are preceded by large positive (negative) abnormal stock returns in the previous 30-minute period. Furthermore, these periods of extreme institutional trading activity are associated with flat contemporaneous and future returns. Barber et al. (2009) construct portfolios that mimic the purchases and sales of each investor group in order to analyse who gains and loses from trade. Individual investors incur substantial losses while institutional ones (corporations, dealers, foreigners and mutual funds) gain from trade [6].

Moreover, investor overconfidence and biased self-attribution is likely to motivate aggressive trading over time (see Odean, 1998; Daniel et al., 1998). Gervais and Odean (2001) find that volatility is increasing in a trader's number of past successes and that both volume and volatility increase with the degree of a trader's learning bias. Chuang and Susmel (2011) investigate the trading behaviour of individual vs. institutional investors in Taiwan in an attempt to identify who is the more overconfident trader. Their findings provide evidence that individual investors are more overconfident traders than institutional investors. Kelley and Tetlock (2013) show that overconfidence (not hedging) explains nearly all uninformed trading, while rational informed

5 For example, a fund manager who buys stocks when there are heavy investor outflows is likely to be motivated by the belief that the stocks are significantly undervalued. In contrast, when there are heavy inflows, the manager is likely to be motivated to work off excess liquidity by buying stocks.

6 The trading and market-timing losses of individual investors represent gains for institutional investors. The institutional gains are eroded, but not eliminated by the commissions and transaction taxes that they pay.

speculation accounts for most overall trading.

Herding and feedback trading have the potential to explain destabilising stock prices or excess volatility. Though, they have also been used to explain momentum and reversals in stock prices depending on who trades and on what type of information [7]. Lakonishok et al. (1992) use data on the holdings of tax-exempt (predominantly pension) funds to evaluate the potential effect of their trading on stock prices. Their evidence suggests that institutional herding moves prices but not necessarily in a destabilising way. For example, if all investors react to the same fundamental information, prices will adjust faster to new fundamentals. De Long et al. (1990) argue that in the presence of positive feedback traders, rational speculation (or trading by institutional investors) can be destabilising. The opposite view is that positive feedback trading will bring prices closer to fundamentals, if stocks underreact to news. There is also a view that institutional traders use different portfolio strategies (herding, positive or negative feedback) which by and large offset each other (resulting in zero excess demand). For example, trading does not destabilise asset prices, if there are enough negative-feedback traders to offset the positive-feedback traders. A substantial trading volume by institutions does not destabilise stock prices.

1.2.2 The trading behaviour of individual investors

Empirical evidence indicates that the average individual investor underperforms the market (see Barber and Odean, 2011). Part of the poor performance borne by individual investors can be attributed to transaction costs (e.g. commissions and bid–ask spread). However, individual investors also seem to lose money on their trades before costs. Barber and Odean (2000) find that households significantly underperform a value-weighted market index, after a reasonable accounting for transaction costs[8]. Grinblatt and Keloharju (2000) analyse two years

[7] Griffin et al. (2003) also find that institutional trading largely follows past stock returns and that price movements ahead of large institutional trades are not caused by market makers accumulating inventory for their institutional clients. Institutional buy (and individual sell) orders are generally executed in the same direction as past daily and intra-daily price movements. These patterns could be driven by institutional and individual investors trading on different information and/or perceiving past stock return movements differently.

[8] After accounting for the fact that the average household tilts its common stock investments toward small value stocks

of trading in Finland and provide supportive evidence regarding the poor gross returns earned by individual investors. Additionally, individual investors are net buyers of stocks with weak future performance, while financial firms and foreigners are net buyers of stocks with strong future performance. Barber et al. (2009), using a complete trading history of all investors in Taiwan, document that the aggregate portfolio of individuals performs poorly and almost all individual trading losses can be traced to their aggressive orders [9].

To address the puzzle of why so much trading occurs, it would be useful to understand what motivates trades. Rational motivations such as liquidity, rebalancing or tax management generate a high volume of individual trading. However, it is difficult to justify the very high annual turnover rates of individual investors with non-speculative trading needs. With such high level of trading individual investors increase the chances of trading with better informed investors and most often to their detriment. Investing in a low cost mutual or index fund could significantly lower their asymmetric information and transaction costs.

Behavioural motivations (or biases) can possibly explain why retail investors trade so much and self-manage their portfolios. Overconfidence can explain the relatively high turnover rates (increased trading) and poor performance of individual investors (see Daniel et al. 1998; Gervais and Odean, 2001; Odean, 1998; Kelley and Tetlock, 2013). Attention can also affect the trading behaviour of individual investors [10]. Barber and Odean (2008) find that individual investors

with high market risk, the underperformance is even worse. The average household turns over approximately 75 percent of its common stock portfolio annually. The poor performance of the average household can be traced to the costs associated with this high level of trading. High levels of trading can partly be explained by overconfidence (and partly by liquidity, risk based rebalancing and taxes). Overconfident investors will overestimate the value of their private information, causing them to trade too actively and, consequently, to earn below-average returns (Odean, 1998, 1999; Daniel et al., 1998; Gervais and Odean, 2001).

9 Three factors contribute (roughly) equally to the shortfall: perverse stock selection ability, commissions and the transaction tax, with a somewhat smaller role being relegated to poor market timing choices. In contrast, institutions enjoy positive abnormal returns (even after commisions and transactions costs) and both the aggressive and passive trades of institutions are profitable. Bae et al. (2006) also find that individual investors have poor market timing ability, but potentially gain during short-run trading intervals as their average sell price is consistently higher than the average purchase price.

10 Barber and Odean (2008) argue that many investors who want to buy stocks may consider only stocks that first catch their attention (e.g. stocks that are in the news or stocks with large price moves) to avoid the huge search problem. This will lead individual investors to buy attention-grabbing stocks heavily. When they want to sell though, most investors consider only stocks they already own and, as a result, selling poses less of a search problem and is less

underperform standard benchmarks (e.g. a low cost index fund) and sell winning investments, while holding losing investments (the "disposition effect"). Moreover, individual investors are heavily influenced by limited attention and past return performance in their purchase decisions. They engage in naïve reinforcement learning by repeating past behaviours that coincided with pleasure, while avoiding past behaviours that generated pain. Others also argue that individual traders overinvest in stocks because they are familiar to them (or love gambling), leading to under-diversification (Goetzmann and Kumar, 2008) and average or even below-par returns (Anderson, 2013). Under-diversification is costly to most investors, but a small subset of investors under-diversify because of superior information (Ivkovic et al., 2008).

Barber et al. (2009a, 2009b) provide evidence that the trading of individuals is highly correlated and persistent. This systematic trading of individual investors is not primarily driven by passive reactions to institutional herding, by systematic changes in risk-aversion or by taxes. Psychological biases more likely contribute to the correlated trading of individuals which lead investors to systematically buy stocks with strong recent performance, to refrain from selling stocks held for a loss and to be net buyers of stocks with unusually high trading volume. Kaniel et al. (2008) provide evidence that individuals tend to buy stocks following declines in the previous month and sell following price increases [11]. Several authors characterise the trading behaviour of individual investors as contrarian (Choe et al., 1999; Griffin et al., 2003; Barber and Odean, 2000; Grinblatt and Keloharju, 2000, 2001). Shapira and Venezia (2001) show that both professional and independent investors exhibit the disposition effect [12], although the effect is

sensitive to attention effects.

11 They also document positive excess returns in the month following intense buying by individuals and negative excess returns after individuals sell, which is distinct from the previously shown past returns or volume effects. The patterns are consistent with the notion that risk-averse individuals provide liquidity (through their contrarian trades) to institutions that require immediacy. Finally, they do not find strong evidence of correlated (systematic) actions of individuals across stocks.

12 Individual investors have a strong preference for selling winner stocks too early and hold on to loser stocks for too long (Shefrin and Statman, 1985). Grinblatt and Keloharju (2001) find that investors have a tendency to hold onto losers and sell stocks with high past returns or trading near their monthly high. Chen et al. (2007) and Choe and Eom (2009) suggest that institutions suffer from the disposition effect but to a lesser extent than individual investors.

stronger for independent investors. They demonstrate that professionally managed accounts were more diversified and that round trips were both less correlated with the market and slightly more profitable than those of independent accounts [13]. There is also intriguing evidence that investors learn to avoid the disposition effect over time [14].

1.2.3 The trading behaviour of foreign investors

Brennan and Cao (1997) present a theoretical model and empirical evidence that supports the view that foreign investors should pursue momentum strategies and achieve inferior performance because they are less informed than domestic investors. Froot et al. (2001) and Choe et al. (1999) find that foreign investors tend to be momentum investors [15]. Wang (2007) documents a strong contemporaneous relationship between foreign equity trading and market volatility in Indonesia and Thailand [16]. Bae et al. (2006) find that foreign investors consistently generate gains from trade due to good market timing, although their average sell price is lower than the average purchase price. Specifically, foreign investors extract significant portion of their gains by trading against Japanese institutional investors, when Japanese investors trade before their fiscal-year end. Barber et al. (2009) find that foreigners earn nearly half of all institutional profits, when profits are tracked over six months (and one-fourth at shorter horizons). The profits

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[13] Consistent with this investment behaviour being a mistake that has its origins in cognitive ability or financial literacy, the disposition effect is most pronounced for financially unsophisticated investors.

[14] Amongst the Chinese individual investors they study, Feng and Seasholes (2005) document that the disposition effect dissipates with trading experience (time since first trade) and various measures of financial sophistication measured early in a trader's history. Yao and Li (2013) model a market in which investors with prospect theory preferences interact with investors with constant relative risk aversion (CRRA) and find that this interaction commonly generates a negative-feedback trading tendency, which favours the disposition effect and contrarian behaviour for prospect theory investors.

[15] Froot et al. (2001) find that international portfolio flows are strongly infuenced by past domestic returns, a finding consistent with positive feedback trading by international investors. Also, the sensitivity of local stock prices to foreign inflows is positive and large. Choe et al. (1999) using order and trade data find strong evidence of positive feedback trading and herding by foreign investors before Korea's economic crisis. During the crisis period herding falls and positive feedback trading by foreign investors mostly disappears. They find no evidence that trades by foreign investors had a destabilising effect on Korea's stock market over the 1996-1997 sample. In particular, the market adjusted quickly and effciently to large sales by foreign investors and these sales were not followed by negative abnormal returns.

[16] Trading within foreign and local investor groups is often negatively related to market volatility in Indonesia. This is consistent with the view that within each group, investors are relatively homogeneous in terms of capital endowments and information. Moreover, in Thailand foreign net purchase is negatively associated with market volatility. Therefore foreign purchase provided liquidity when local investors were under stress to sell and helped to reduce volatility during the Asian crisis by preventing the local markets from dropping further than they actually did.

of foreigners represent an unambiguous wealth transfer from Taiwanese individual investors to foreigners. Grinblatt and Keloharju (2001) also find that foreign investors, often professionally managed funds or investment banking houses, pursue momentum strategies and achieve superior performance. After removing momentum investing's contribution to performance, they find that the momentum-adjusted performance of foreigners is still highly significant.

1.2.4 Informed vs uninformed investors/trades and volatility

Much of the empirical research in finance views individuals and institutions differently. In particular, while institutions are viewed as informed investors, individuals are believed to have psychological biases and are often characterised as noise traders (Black, 1986). In most theoretical models, trading arises because of new information signals. Institutional or large block trades are more informative than small trades and more likely to cause permanent price changes (Easley and O'Hara, 1987, 1992). However, any relation between information effects and the size of the block is attenuated, if informed traders make numerous smaller trades and information gradually incorporates into prices (Kyle, 1985). Easley et al. (2008) find that it is the presence of information, rather than the variation in the intensity of uninformed trade, that determines the arrival rate of informed traders [17]. Cai et al. (2010) using a unique dataset of Chinese Stock Market document that higher proportions of trades initiated by institutional investors can actually be considered as informed compared to trades initiated by individuals. This result is consistent with the argument that institutional investors are better informed and the fact that institutional investors can gain much more profits than individuals. Avramov et al. (2006) decompose sell trades into contrarian and herding trades and conjecture that herding trades are uninformed and contrarian trades are informed using serial correlation tests. They find that contrarian trades decrease volatility, while herding trades increase volatility. They demonstrate that when stock

[17] The interaction of liquidity and information flows provide an insight into the price formation process (Easley et al., 2002; O'Hara, 2003; Acharya and Pedersen, 2005).

price declines, herding (sell) trades govern the increase in next period volatility and when stock price rises, contrarian trades lead to a decrease in next period volatility. Hence, trading activity of contrarian and herding investors seems to explain the relation between daily volatility and lagged returns. Daigler and Wiley (1999) find empirical evidence indicating that the positive volume-volatility relation is driven by the (uninformed) general public, whereas the activity of informed traders such as clearing members and floor traders is often inversely related to volatility.

In our study we associate the trading of institutional and individual investors with those of informed and uninformed traders respectively. We assume that active institutional traders use market orders to assure rapid execution (at the cost of large price impacts) and engage in herding and positive feedback trades (based on shortlived information), which exacerbate short-run volatility. We also assume that passive institutional traders use limit orders [18] and engage in more contrarian trades (based on longer term information), which reduce short-run volatility. Although for some institutions the buy-sell decision has no association with prior excess returns [19], for other institutions there is a significant relation between trades and past excess returns. However, the overall effect of these strategies may be offsetting, because some traders pursue contrarian strategies while others follow trends. As regards the individual investors, recent studies find that their trading patterns are significantly affected by psychological biases which lead to increased levels of trading, systematic behaviour and high trading costs. For example, individual investors tend to hold on to losing common stock positions and sell their winners (disposition effect rather than contrarian trades), buy stocks that catch their attention or are familiar with and under-diversify their stock portfolios. As a result, the buy and sell decisions of individual traders are likely to take place within a broader range of prices unless the extra liquidity provided

[18] Passive trades (using limit orders) offer an opportunity for price improvement, but impose opportunity costs because trade execution is not assured.

[19] Further, for some institutions, trades are determined primarily by pre-determined investment objectives (index tracking, value, growth), liquidity needs and tax-management purposes.

by individual traders is accompanied by increased levels of informed trading by institutional investors.

1.3 Data description and sub-periods

The data set used in this study comprises 2850 daily trading volumes and prices of the Korean Composite Stock Price Index (KOSPI), running from 3rd of January 1995 to 26th of October 2005. The data were obtained from the Korean Stock Exchange (KSE). The KOSPI is a market value weighted index for all listed common stocks in the KSE since 1980.

1.3.1 Price volatility

Using data on the daily high, low, opening and closing prices in the index we generate a daily measure of price volatility. We can choose from amongst several alternative measures, each of which uses different information from the available daily price data. To avoid the microstructure biases introduced by high frequency data and based on the conclusion of Chen et al. (2006) that the range-based and high-frequency integrated volatility provide essentially equivalent results, we employ the classic range-based estimator of Garman and Klass (1980) to construct the daily volatility (VL_t) as follows

$$VL_t = \frac{1}{2}u^2 - (2\ln 2 - 1)c^2, \quad t \in \mathbb{N},$$

where u and c are the differences in the natural logarithms of the high and low and of the closing and opening prices respectively. Figure 1.1 plots the GK volatility from 1995 to 2005.

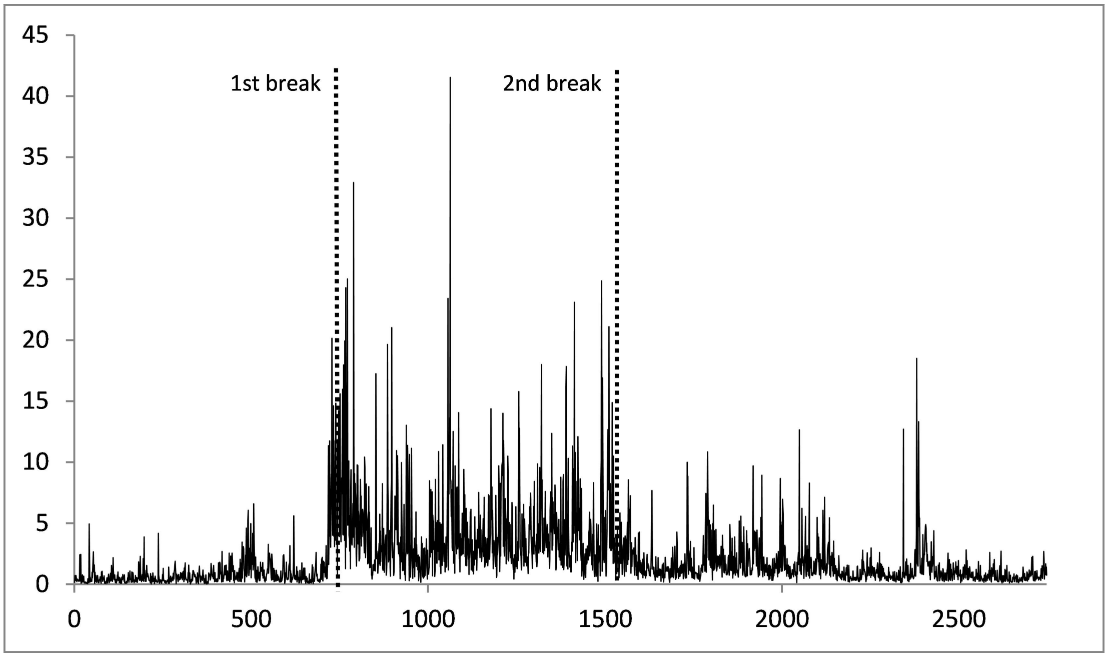


Figure 1.1: Garman Klass volatility

Various measures of GK volatility have been employed by, amongst others, Daigler and Wiley (1999), Fung and Patterson (1999), Wang (2000), Kawaller et al. (2001), Wang (2002b) and Chen and Daigler (2004)[20].

1.3.2 Trading activity

We use the daily trading volume of foreign investors and eight different domestic investors, that is individual investors, securities companies, insurance companies, mutual funds, investment banks, commercial banks, savings banks and other companies. The eight domestic investors are added to construct the domestic volume. We study each volume series from its buy and sell side as well as its total *(=(buy+sell)/2)*. We use the volume series to form the turnover and include it as a measure of volume in our model. This is the ratio of the value of shares traded to the value of shares outstanding (see, Campbell et al., 1993; Bollerlsev and Jubinski, 1999). Because

[20] Chou (2005) propose a Conditional Autoregressive Range (CARR) model for the range (defined as the difference between the high and low prices). In order to be in line with previous research (Daigler and Wiley, 1999; Fung and Patterson, 1999; Kawaller et al., 2001; Wang, 2002a; Wang, 2007) in what follows we model GK volatility as an autoregressive type of process taking into account the feedback from volume to volatility, dual-long memory characteristics and GARCH effects.

trading volume is nonstationary several detrending procedures for the volume data have been considered in the empirical finance literature (see, for details, Lobato and Velasco, 2000). We form a trend-stationary time series of turnover (TV_t) by incorporating the procedure used by Campbell et al. (1993) that uses a 100-day backward moving average $TV_t = \frac{VLM_t}{\frac{1}{100}\sum_{i=1}^{100} VLM_{t-i}}$, where VLM denotes volume. This metric produces a time series that captures the change in the long run movement in trading volume (see, Brooks, 1998; Fung and Patterson, 1999). The moving average procedure is deemed to provide a reasonable compromise between computational ease and effectiveness[21]. Figure 1.2 plots the total turnover volume from January 1995 to October 2005.

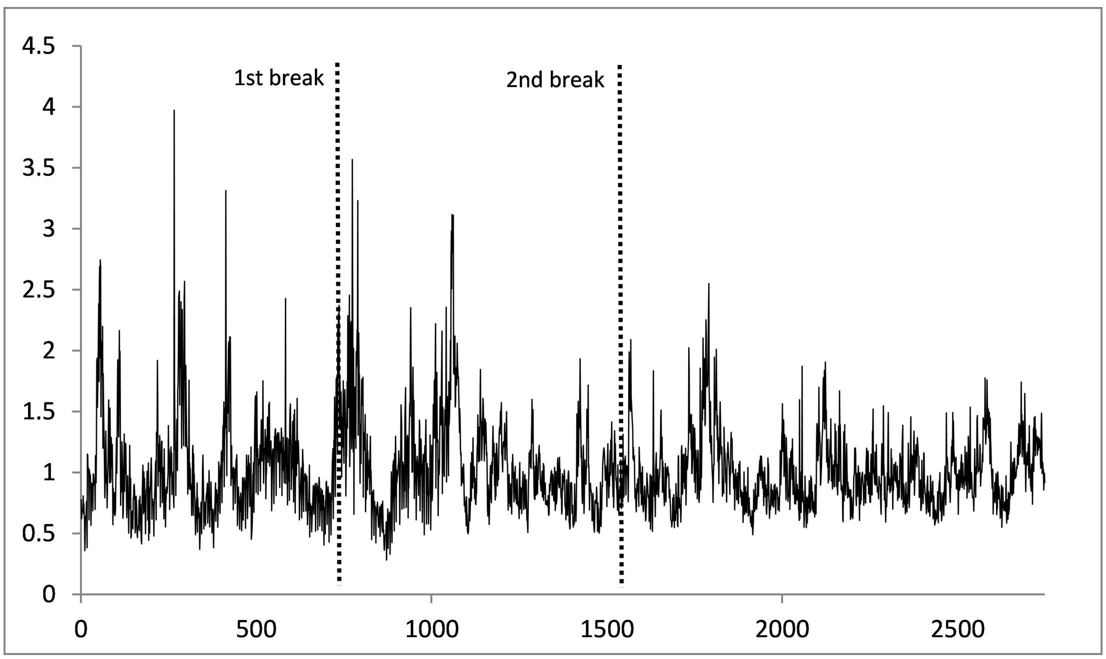


Figure 1.2: Total Turnover volume

[21] We needed (in order to reach any result) to use an outlier reduced series for Savings banks Sell Turnover and Other companies Sell Turnover: the variance of the detrended data is estimated and any value outside four standard deviations is replaced by four standard deviations. Chebyshev's inequality is used as it i) gives a bound of what percentage ($1/k^2$) of the data falls outside of k standard deviations from the mean, ii) holds no assumption about the distribution of the data and iii) provides a good description of the closeness to the mean, especially when the data are known to be unimodal as in our case.

1.3.3 Structural breaks

We also examine whether there are any structural breaks in volatility. We test for structural breaks by employing the methodology in Bai and Perron (1998, 2003a,b), who address the problem of testing for multiple structural changes in a least squares context and under very general conditions on the data and the errors. In addition to testing for the presence of breaks, these statistics identify the number and location of multiple breaks.

Moreover, Bai and Perron (1998, 2003a,b) form confidence intervals for the break dates under various hypotheses about the structure of the data and the errors across segments. This allows us to estimate models for different break dates within the 95 percent confidence interval and also evaluate whether our inferences are robust to these alternative break dates. Our results (not reported) seem to be invariant to break dates around the one which minimises the sum of squared residuals.

The overall picture dates two change points for volatility. The first is detected in October 1997 and the next one is in November 2000. Accordingly, we break our entire sample into three sub-periods. 1st period (the pre-crisis period, sample A hereafter): 3rd January 1995 - 15th October 1997; 2nd: 16th October 1997 - 26th October 2005 (the post-crisis period including the in-crisis period and the economic recovery of Korea, sample B hereafter); the 3rd period: 7th November 2000 - 26th October 2005 (the post-crisis period characterised by the world recession period, which starts with the second change-point in volatility, sample B1 hereafter).

The first change point in volatility is associated with the financial crisis in 1997. As mentioned earlier on, we break our entire sample into three sub-periods: 1st) 3rd January 1995– 15th October 1997 (the first break in volatility): the tranquil and pre-(currency) crisis period. This was the time when Korea was regarded as one of the miracle economies in East Asia and foreign investors were enthusiastic about investing in Korea. While Korea's own currency crisis would come later in November of that year, the currency of Thailand, Baht (and maybe other currencies in Asia) was under several speculative attacks in June. The Thai Baht collapsed at the beginning of July, marking the beginning of what we now call the Asian financial crisis (AFC). The Thai crisis sent repercussions throughout the region. 2nd) 16th October 1997- 26th October 2005: the post-crisis period including the in-crisis period and the economic recovery. On November 18 1997, the Bank of Korea gave up defending the Korean Won. On November 21, the Korean government asked the International Monetary Fund (IMF) for a bail-out. There were also some instances of labour unrest and major bankruptcies during the period. The end of the crisis in Korea is set at the end of 1998. Even though in October 1998 there was significant uncertainty related to emerging markets in Russia and South America as well as in Asia, the worst of the Asian crisis was clearly over, the markets and the economies had begun to recover. In 1999-2000 the Korean economy achieved an early and strong recovery from the severe recession. 3rd) 7th November 2000 - 26th October 2005: the world recession period. Since the end of 2000 the Korean economy faced many challenges, economically and politically, compounded by a global economic slowdown with hesitant recovery, terrorist attacks, regional wars, avian flu outbreaks in Asia and domestic and global uncertainty ahead. A 2005 World Bank research paper on Korea concluded that “the national economy is now suffering from weak investment, slow growth and slow job creation and rising unemployment” (Crotty and Lee, 2006).

The share of foreign trading activity in total stock market volume increased tremendously during the last few years. The internationalisation of capital markets is reflected not only in the addition of foreign securities to otherwise domestic portfolios, but also in active trading in foreign markets (Dvorak, 2001). There is surprisingly little evidence, however, on the impact of foreign trading activity on local equity markets. In Korea foreign stock ownership increased dramatically in the post-crisis period. The share of foreign ownership of Korea’s publicly held stock increased from 15% in 1997 to 22% in 1999, 37% in 2001 and 43% in early 2004 (see Chung, 2005). The foreign ownership share of the eight large urban banks grew from 12% in 1998 to 64% in late 2004. By mid-2005, Korea had higher foreign bank ownership than almost all Latin American and Asian countries. Korea's central bank issued a report underscoring a growing wariness in the country about the role of foreign investors.

1.4 Estimation procedures

1.4.1 Estimation methodology

Tsay and Chung (2000) have shown that regressions involving FI regressors can lead to spurious results. Moreover, in the presence of conditional heteroskedasticity Vilasuso (2001) suggests that causality tests can be carried out in the context of an empirical specification that models both the conditional means and conditional variances.

Furthermore, in many applications the sum of the estimated variance parameters is often close to one, which implies integrated GARCH (IGARCH) behaviour. For example, Chen and Daigler (2004) emphasise that in most cases both variables possess substantial persistence in their conditional variances. In particular, the sum of the variance parameters was at least 0.950. Most importantly, Baillie et al. (1996), using Monte Carlo simulations, show that data generated from a process exhibiting FIGARCH effects may be easily mistaken for IGARCH behaviour. Therefore we focus our attention on the topic of long memory and persistence in terms of the second moments of volatility. Consequently, we utilise a univariate ARFI-FIGARCH model to test for the causal effect of volume on volatility.

1.4.2 Dual long memory

Along these lines we discuss the dual long memory time series model for volatility.

Let us first define the two variables. In the expression below the equation represents the GK volatility (VL_t), where turnover volume (TV_t) is added as regressor. The ARFI($1, d_m$) model for the conditional mean of volatility is given by

$$(1-L)^{d_m}\phi(L)(VL_t - \varphi_s L^s TV_t - \mu) = \varepsilon_t, \tag{1.1}$$

where L is the lag operator, $\phi(L) = 1 - \sum_{i=1}^{p} \phi_i L^i$ is the AR polynomial and $0 \leq d_m \leq 1$. The φ_s coefficient captures the effect from volume on volatility. We assume $\boldsymbol{\varepsilon}_t$ is conditionally normal with mean 0 and variance h_t.

Further, the FIGARCH($1, d_v, 1$) process for the conditional variance of volatility is defined by

$$(1-\beta L)h_t = \omega + [(1-\beta L) - (1-cL)(1-L)^{d_v}]\varepsilon_t^2, \tag{1.2}$$

where $\omega \in (0, \infty)$ and $0 \leq d_v \leq 1$.[22] Note that the FIGARCH model is not covariance stationary. The question whether it is strictly stationary or not is still open at present (see Conrad and Haag, 2006). In the FIGARCH model, conditions on the parameters have to be imposed to ensure the non-negativity of the conditional variances (see Conrad and Haag, 2006 and Conrad, 2010).[23] When $d_v = 0$ the model reduces to the GARCH($1, 1$) model: $(1-\beta L)h_t = \omega + \alpha L \varepsilon_t^2$, where $\alpha = c - \beta$.

1.5 Empirical Analysis

1.5.1 Dual long memory model

Within the framework of the ARFI-FIGARCH model we will analyse the dynamic adjustments of both the conditional mean and variance of volatility for all four sample periods, as well as the implications of these dynamics for the direction of causality from volume to volatility. The estimates of the various formulations were obtained by quasi maximum likelihood estimation (QMLE) as implemented by James Davidson (2009) in Time Series Modelling (TSM). To check

[22] Brandt and Jones (2006) use the approximate result that if log returns are conditionally Gaussian with mean 0 and volatility h_t then the log range is a noisy linear proxy of log volatility. In this study we model the GK volatility as an ARFI-FIGARCH process.

[23] Baillie and Morana (2009) introduce a new long memory volatility process, denoted by Adaptive FIGARCH, which is designed to account for both long memory and structural change in the conditional variance process. One could provide an enrichment of the dual long memory model by allowing the intercepts of the mean and the variance to follow a slowly varying function as in Baillie and Morana (2009). This is undoubtedly a challenging yet worthwhile task.

for the robustness of our estimates we used a range of starting values and hence ensured that the estimation procedure converged to a global maximum.

Table 1.1. Mean Equations: AR lags

Sample:	Total	A	B	B1
VL_t	1	3	1	1

Notes: The table reports the AR lags used in the mean equations.

The best fitting specification (see equation (1.1)) is chosen according to the minimum value of the information criteria (not reported). For the conditional mean of volatility (VL_t), we choose an ARFI($3, d_m$) process for the pre-crisis period and an ARFI($1, d_m$) for the other three samples (see Table 1.1). That is, $\phi(L) = 1 - \phi_3 L^3$ and $\phi(L) = 1 - \phi_1 L$, respectively. We do not report the estimated AR coefficients for space considerations.

Before we discuss the estimation results we want to ensure that the models are well specified. First, we calculate Ljung–Box Q statistics at 12 lags for the levels and squares of the standardised residuals for the estimated dual long memory GARCH models. The results (not reported) show that the time series models for the conditional mean and the conditional variance adequately capture the distribution of the disturbances.

Finally, we employ the diagnostic tests proposed by Engle and Ng (1993), which emphasise the asymmetry of the conditional variance to news. According to the joint test of the size and sign bias, for the entire sample period the sign and the negative size bias test statistics (not reported) for asymmetries in the conditional variance of volatility are significant. For the pre-crisis period (sample A) there is no indication of asymmetry in the conditional variance. In sharp contrast, for the post-crisis period (sample B) the results from the diagnostic tests point to the presence of a leverage effect in the conditional variance. To check the sensitivity of our results to the possible presence of skewness in the conditional variance of volatility in Section 1.6.1 we re-estimate our models using the skewed t density without asymmetries.

1.5.2 Volume-volatility link

To recapitulate, we employ the univariate ARFI-FIGARCH model with lagged values of volume included in the mean equation of volatility to test for causality. The estimated coefficients φ_s, defined in equation (1.1), which capture the possible feedback between the two variables, are reported in Appendix 1B. We also tested the contemporaneous effect of volume on volatility adding the volume series in the volatility equation (1.1) with lag order $s = 0$. The estimated value of φ_0 (not reported) was always positive and significant, indicating a positive contemporaneous effect of volume on volatility.

Regarding the lags used to find the causal effect, we tried to test the first ten lags for significance and in case of reaching no significant lag we extended our search up to the twentieth lag. The first two lags show an immediate causal effect of volume on volatility, lag order five indicates a one-week effect and so on. The twentieth lag can mean a one-month in advance effect of the trading turnover volume on the market's volatility, that we count as a more weak relationship between the two variables (ie. other companies' total volume in sample B and securities companies-members' purchases in sample B). In most cases, we used up to the eight lags, to detect the causal effect. The likelihood ratio tests and the information criteria (not reported) choose the specification for the feedback from volume to volatility.

Table 1.2 gives an overview of the volume-volatility link over the entire sample period and the three different subsamples considered. Panel C shows the effect of the total, domestic and foreign trading volume on volatility. The total and foreign volume have a negative effect on volatility in the total sample, while the domestic volume affects it positively. This volume-volatility link is in line with the results in Karanasos and Kartsaklas (2009), who find that, the negative effect from total volume to volatility reflects the causal relation between foreign volume and volatility. In particular, total and foreign purchases show a negative impact, while the respective sales are related to volatility positively. Moreover, both purchases and sales from domestic investors generate a positive link.

Regarding the structural breaks considered, the results suggest that the causal effect from volume on volatility is sensitive to structural changes. We always find a positive and significant link between the two variables in the post-crisis sample periods B and B1 for all volume series. In the pre-crisis period (sample A) total (domestic) volume affects volatility negatively (positively), contrary to Karanasos and Kartsaklas (2009), where no link was detected.

Foreign investors' purchases show a negative link to volatility in the pre-crisis period. This behaviour of the foreign purchases seems to define the effect of the total purchases and the total trading activity, which shows the same sign. In sharp contrast, all investors' sales have a positive impact on volatility. These findings are in accordance with Wang (2007), where it is found that foreign purchases tend to stabilise stock markets - by increasing the investor base in emerging markets - especially in the first few years after market liberalisation, when foreigners are buying into local markets. Moreover, it is noteworthy to highlight the theoretical arguments of Daigler and Wiley (1999) and Wang (2007). The former argue that the positive relation between the two variables is driven by the uninformed general public, whereas the latter states that foreign sales reduce investor base and destabilise the stock markets. Note that after the financial crisis the Korean stock market experienced large foreign outflows (see Chung, 2005).

Panel A of Table 1.2 gives the results of the volume-volatility link from 6 different domestic investor groups that are regarded as non-members of the market. Commercial banks', savings banks' and other companies' turnover volume have a positive effect on volatility across all samples, in total and in both buy and sell sides. Insurance companies, mutual funds and investment banks (similar to total and foreign volume) affect the market's volatility negatively with their purchases in the pre-crisis period. This finding is justified by the fact that the latter three investors are more informed than the former three ones, as they participate in the stock markets more actively. Insurance companies, mutual funds and investment banks are investors oriented towards trading and investing in stock markets. On the contrary, commercial and savings banks participate in markets as a residual portfolio activity rather than as a core business operation, which is acceptance of deposits and loan supply. So, insurance companies, mutual funds and investment banks are specialised in trading and, therefore, more informed to stabilise the markets than the other non-members. The more informed traders are the less noisy ones in the markets as evidenced by previous studies (Black, 1986; Easley and O'Hara, 1987, 1992; Easley et al., 1997).

In Panel B of Table 1.2 non-members' volumes are aggregated and presented with the other two domestic investors, namely the market members (securities companies) and the individual investors. The aggregated non-members and the individual investors affect volatility positively across all samples, in total and in both buy and sell sides. The individual investors' turnover impact on volatility is in accordance with, amongst others, Barber and Odean (2008) and Barber et al. (2009) results. The attention effects and the psychological biases, in general, for individuals are depicted on higher price impact and, consequently, on higher market volatility. In sharp contrast, the securities companies, which are the most informed amongst the domestic investors and the main liquidity providers, show a negative impact on volatility through their purchases in the total sample and both their purchases and sales in the pre-crisis period. This is the only case that an investor's sales affects volatility negatively. This is in line with the literature on institutional investors trading activity linked to their superior information. The institutions' trading volume does not destabilise the markets in most cases even with herding and feedback trading conditions (see Lakonishok et al., 1992; De Long et al., 1990). This result is consistent with the views that (i) the activity of informed traders is often inversely related to volatility and (ii) a marketplace with a larger population of liquidity providers will be less volatile than one with a smaller population.

Table 1.2a. The Volume-Volatility link

Panel A. The effect of Non members' trading volume on volatility					
Sample:		Total	A	B	B1
Insurance Companies	total	**negative**	**negative**	positive	positive
	buy	**negative**	**negative**	positive	positive
	sell	positive	positive	positive	positive
Mutual Funds	total	**negative**	**negative**	positive	positive
	buy	**negative**	**negative**	positive	positive
	sell	positive	positive	positive	positive
Investment Banks	total	**negative**	**negative**	positive	positive
	buy	**negative**	**negative**	positive	positive
	sell	positive	positive	positive	positive
Commercial Banks	total	positive	positive	positive	positive
	buy	positive	positive	positive	positive
	sell	positive	positive	positive	positive
Savings Banks	total	positive	positive	positive	positive
	buy	positive	positive	positive	positive
	sell	positive	positive	positive	positive
Other Companies	total	positive	positive	positive	positive
	buy	positive	positive	positive	positive
	sell	positive	positive	positive	positive

Table 1.2b. The Volume-Volatility link

Panel B. The effect of Domestic Investors' trading volume on volatility					
Sample:		Total	A	B	B1
Members (Securities Companies)	total	**negative**	**negative**	positive	positive
	buy	**negative**	**negative**	positive	positive
	sell	positive	negative	positive	positive
Non-members	total	positive	positive	positive	positive
	buy	positive	positive	positive	positive
	sell	positive	positive	positive	positive
Individual Investors	total	positive	positive	positive	positive
	buy	positive	positive	positive	positive
	sell	positive	positive	positive	positive
Panel C. The effect of Total trading volume on volatility					
Sample:		Total	A	B	B1
Total	total	**negative**	**negative**	positive	positive
	buy	**negative**	**negative**	positive	positive
	sell	positive	positive	positive	positive
Domestic	total	positive	positive	positive	positive
	buy	positive	positive	positive	positive
	sell	positive	positive	positive	positive
Foreign	total	**negative**	**negative**	positive	positive
	buy	**negative**	**negative**	positive	positive
	sell	positive	positive	positive	positive

To sum up the results of Table 1.2, our main findings are drawed on Chart 1.1 and refer to the sign of the volume effect on volatility with focus on the total trading volume and its buy side regarding the total sample and the pre-crisis period (sample A). We focus on these aspects as the sell side of the trading activity and the post-crisis samples (B, B1) in all volumes always result to a positive sign. Domestic non-members affect the market's volatility positively, while the more informed ones amongst them show a negative effect, which is overridden by the less informed investors' positive impact. Domestic members have a negative effect on volatility in contrast to individuals that show a positive impact, same as the non-members. The positive link is the prevailing result for the domestic investors' trading activity, when all domestic investor groups are aggregated. On the other hand, foreign investors affect volatility negatively, which is reflected also on the total volume, when all investors are included. Foreign investors are the ones that determinate, eventually, the impact of the total trading activity on volatility, which is found negative.

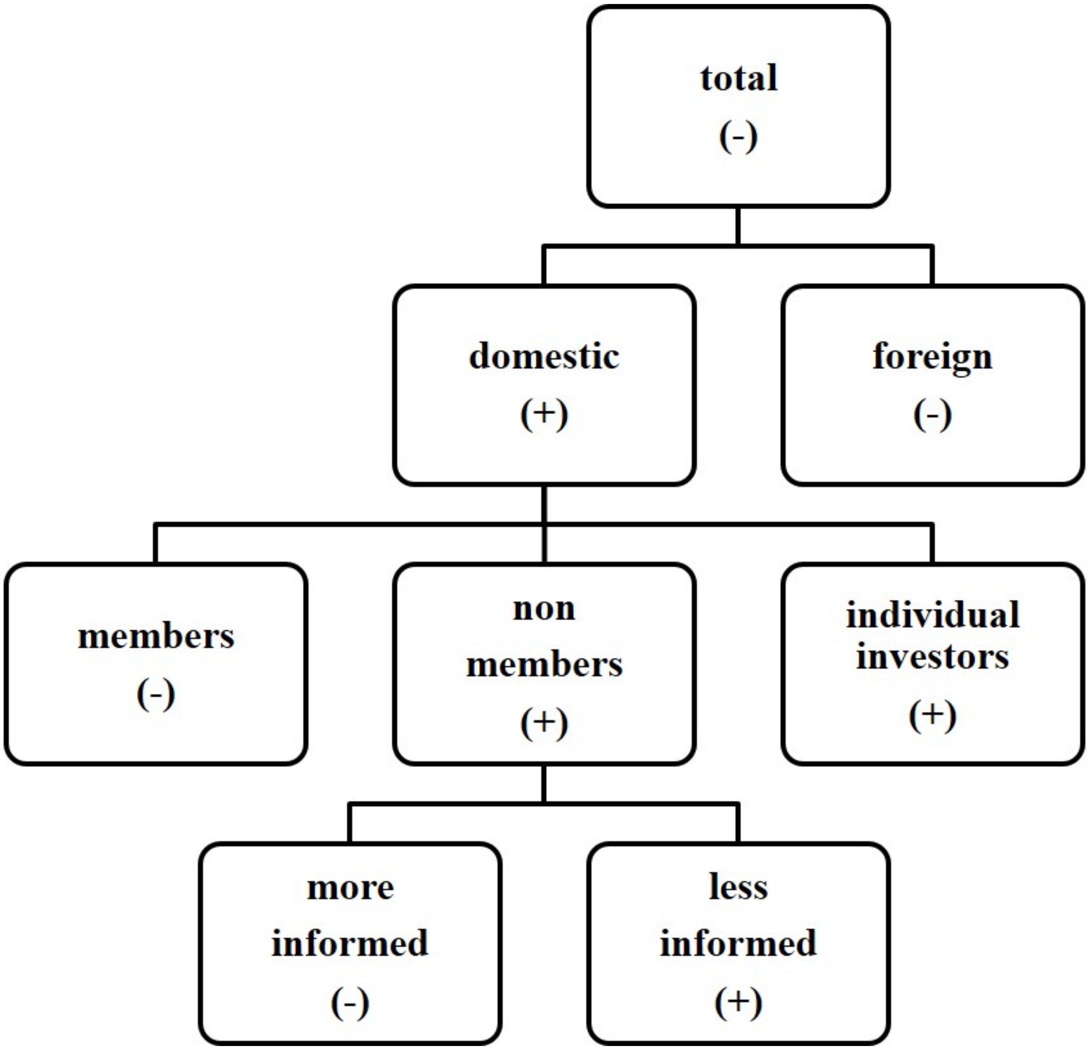


Chart 1.C1: Trading volume (total & buy) link to volatility, Total sample & sample A

1.5.3 Fractional mean parameters

Estimates of the fractional mean parameters are shown in Table 1.3. Several findings emerge from this table. In all cases the estimated value of d_m is robust to the measures of volume used. In other words, all ARFI models across each sample period generated very similar estimates of d_m. For example, in the total sample the twelve long memory mean parameters are between 0.40 and 0.44. For the post-crisis period (sample B) the estimated values of d_m ($0.38 - 0.42$) are similar to the total sample's estimates, but higher than the corresponding values for the pre-crisis period (sample A): $0.23 - 0.27$. Generally speaking, we find that the apparent long memory in volatility is quite resistant to 'mean shifts'.

Table 1.3. Mean Equations: Fractional parameters (d_m)						
Panel A. Non-members domestic investors						
	Insurance Companies	Mutual Funds	Investment Banks	Commercial Banks	Savings Banks	Other Companies
Total Sample	0.43**** (0.06)	0.43**** (0.05)	0.42**** (0.05)	0.40**** (0.11)	0.44**** (0.05)	0.42**** (0.05)
Sample A	0.24**** (0.06)	0.25**** (0.07)	0.27**** (0.08)	0.24**** (0.06)	0.25**** (0.08)	0.23**** (0.08)
Sample B	0.41**** (0.03)	0.42**** (0.04)	0.41**** (0.04)	0.38**** (0.04)	0.42**** (0.04)	0.42**** (0.04)
Panel B. Total trading volume - Domestic investors						
	Total	Domestic	Foreign	Members	Non-members	Individual Investors
Total Sample	0.43**** (0.05)	0.41**** (0.05)	0.42**** (0.08)	0.42**** (0.05)	0.41**** (0.05)	0.41**** (0.05)
Sample A	0.25**** (0.06)	0.24**** (0.06)	0.25**** (0.06)	0.25**** (0.06)	0.23**** (0.06)	0.24**** (0.06)
Sample B	0.41**** (0.04)	0.42**** (0.04)	0.40**** (0.04)	0.41**** (0.04)	0.41**** (0.04)	0.42**** (0.04)

Notes: The table reports the fractional parameter estimates of the long memory in the mean equations.
d_m is defined in equation (1).
The estimates are reported only for the case when total TV_t is added as regressor
and not for the buy and sell side of each series, due to space considerations.
The estimates of the sample B1 are not reported for space considerations.
**** denotes significance at the 0.01 level.
The numbers in parentheses are standard errors.

1.5.4 FIGARCH specifications

Table 1.4 presents estimates of the d_v of the FIGARCH model.[24] d_v's govern the long-run dynamics of the conditional heteroscedasticity of volatility. The fractional parameter d_v is robust to the measures of volume used. In other words, all FIGARCH models across each sample period generated very similar fractional variance parameters. For example, in the post-crisis period the fractional variance parameters ($0.55 - 0.59$) are higher than the corresponding parameters of the total sample: $0.40 - 0.43$, except for the case when the commercial banks' turnover volume is added where d_v is 0.46 in sample B, lower than the 0.49 of the total sample. In the pre-crisis period d_v's are close to and not significantly different from zero. In other words, the conditional variances are characterised by a GARCH behaviour. Overall, when allowing for 'structural

[24] Various tests for long memory in volatility have been proposed in the literature (see, for details, Hurvich and Soulier, 2002).

breaks' the order of integration of the variance series decreases considerably, as in the pre-crisis period the long memory in variance disappears.

Finally, the estimated values of the GARCH coefficients in the conditional variance are robust to the different volumes added as regressors (see Appendix 1C). Note that in all cases the necessary and sufficient conditions for the non-negativitiy of the conditional variances are satisfied (see Conrad and Haag, 2006).

Table 1.4. Variance Equations: Fractional parameters (d_v)

Panel A. Non-members domestic investors	Insurance Companies	Mutual Funds	Investment Banks	Commercial Banks	Savings Banks	Other Companies
Total Sample	0.42**** (0.16)	0.42**** (0.16)	0.42**** (0.16)	0.49**** (0.10)	0.40**** (0.14)	0.42**** (0.15)
Sample A	–	–	–	–	–	–
Sample B	0.59**** (0.17)	0.57**** (0.18)	0.56**** (0.16)	0.46**** (0.08)	0.57**** (0.18)	0.55**** (0.17)
Panel B. Total trading volume - Domestic investors	**Total**	**Domestic**	**Foreign**	**Members**	**Non-members**	**Individual Investors**
Total Sample	0.42**** (0.16)	0.43**** (0.16)	0.43**** (0.17)	0.42**** (0.16)	0.42**** (0.15)	0.43**** (0.16)
Sample A	–	–	–	–	–	–
Sample B	0.56**** (0.17)	0.56**** (0.17)	0.58**** (0.18)	0.57**** (0.19)	0.56**** (0.17)	0.57**** (0.17)

Notes: The table reports the fractional parameter estimates of the long memory in the variance equations.

d_v is defined in equation (2).

The estimates are reported only for the case when total TV_t is added as regressor and not for the buy and sell side of each series, due to space considerations.

The estimates of the sample B1 are not reported for space considerations.

**** denotes significance at the 0.01 level.

The numbers in parentheses are standard errors.

1.6 Sensitivity analysis

1.6.1 Distributional assumptions

To check the sensitivity of our results to different error distributions we re-estimate the ARFI-FIGARCH models using the skewed t density without asymmetries. We do not report the estimated results for space considerations.

A comparison of the results with those obtained when the normal distribution is used reveals that the results are qualitatively very similar. The sign of the volume effect on volatility remains in most cases the same. This similarity disappears in the case of securities companies' trading activity, which is positively related to volatility as a total and in its buy side in the total sample, contrary to the link found with the QMLE that is negative. Moreover, a major difference between the two distributional assumptions is detected in the foreign volume, that is the foreign investors' total turnover has a positive impact on volatility using the skewed t density contrary to the QMLE case where the respective link is negative. However, foreign purchases are robust to the distributional choice and remain negative in both cases, confirming the view that foreign purchases tend to stabilise emerging stock markets. Finally, in the entire sample period the total turnover as a total and from its buy side has a positive effect on volatility in the skewed t density, whereas in the normal distribution the link is negative. In the former case, the total purchases seem to reflect most the domestic investors' activity, in contrast with the latter case, where the total purchases' link to volatility is determined by the negative link of the foreign investors' purchases.

Comparing the quantitative measures, we observe that the same specifications are chosen in the AR lags of the mean equations and the FIGARCH coefficients of the variance equations. In particular, the ARCH and GARCH coefficients [$\alpha(= c - \beta)$, β] are higher in the normal distribution than in the skewed t in most cases. The estimated values of the fractional variance parameters (d_v) are lower in the skewed t density than in the normal case and remain constant across the different volume series added in the mean equations. The same conclusion can be derived comparing the fractional mean parameters (d_m). Finally, we observe that the further lag order s chosen for the turnover series added as regressors in the volatility mean equation in the skewed t density is lower ($s \leq 12$) in comparison with the QMLE case where the further lag order s reached the seventeenth and the twentiethth lag in two cases.

Overall the results appear very robust and are generally insensitive to the presence of skewness.

1.6.2 Structural dynamics

Furthermore, we check the robustness of our results given by the specification in equation (1.1), where the lagged values of TV_t exhibit ‘error dynamics’, since a transformation allows it to be rewritten with only the error terms entering in the infinite moving average representation. So, we also estimate a model, where the lagged values of TV_t exhibit ‘structural dynamics’, since they have a distributed lag representation. Overall the new results (not reported) are in broad agreement with those presented above.

1.7 Conclusions

In this chapter we have investigated the issue of temporal ordering of the range-based volatility and turnover volume in the Korean market for the period 1995–2005. We examined the long-run dynamics of volatility and its uncertainty using a dual long memory model. We also studied the nature of the volume-volatility link, focusing on the one-side effect of trading volume on volatility, by adding the volume as regressor to the volatility model. The volume effect was examined separately for the purchases and the sales of each investor, including eight different domestic investor groups as well as the foreign investors. We further distinguished volume trading before the Asian financial crisis from trading after the crisis, taking into account the structural breaks in volatility. Our results suggest the following:

First, we find that the apparent long memory in volatility is quite resistant to ‘mean shifts’. However, when we take into account structural breaks the order of integration of the conditional variance series decreases considerably.

Second, the causality effects are found to be sensitive to the sample period used in terms of their sign. Thus our analysis suggests that the behaviour of volatility depends upon volume, but also that the nature of this dependence varies with time and the measure of volume used. In particular, in the pre-crisis period foreign investors’ volume as a total and from its buy side affect volatility negatively, while in the post-crisis period this effect turns to positive. This behaviour is reflected also in the total volume's respective effects. This is consistent with the view that foreign purchases tend to lower volatility in emerging markets-especially in the first few years after market liberalisation when foreigners are buying into local markets, whereas foreign sales increase volatility. Total domestic investors affect volatility positively across all samples, while the most informed 'market players' (securities companies, investment banks, mutual funds and insurance companies), when examined separately, are proved to have a negative impact on volatility in the pre-crisis period. This result is in line with the theoretical argument that the activity of informed traders tends to stabilise the market, while the positive impact of volume on volatility is driven by the uninformed general public. In sharp contrast, in the post-crisis period increased volume leads always to higher volatility. Finally, almost all investors' sales are found to affect volatility positively regardless of the sample period.

Third, most of the effects found in our study are quite robust to the distributional assumptions concerning our models' error distribution, as the estimates from the normal and the skewed t density gave similar results.

Finally, our findings reinforce and extend the conclusions of Karanasos and Kartsaklas (2009).

1.8 APPENDIX 1A: Turnover volume graphs

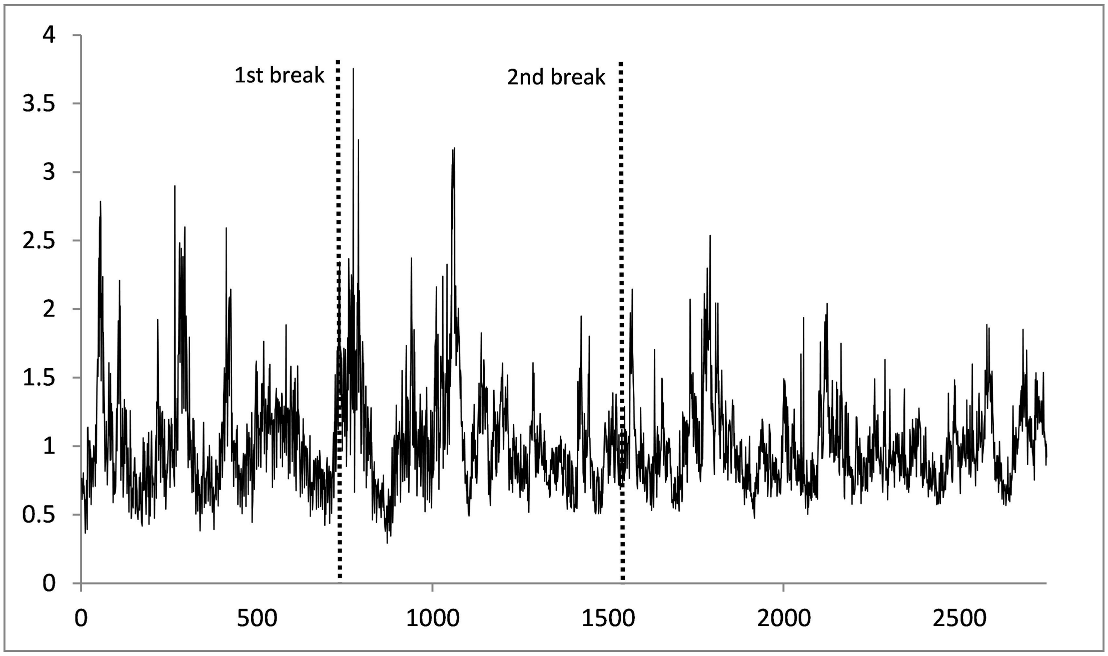


Figure 1A.1: Total Domestic Turnover volume

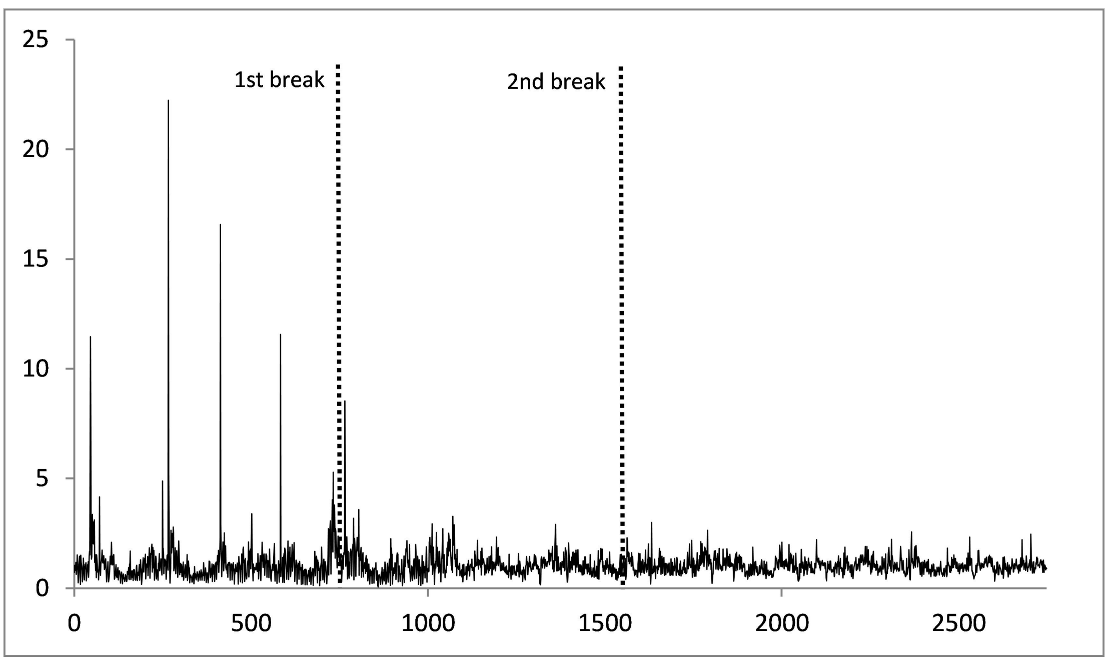


Figure 1A.2: Total Foreign Turnover volume

1.9 APPENDIX 1B: Mean equations cross effects

Table 1B.1a: Mean Equations: Cross effects

Panel A. Non-members domestic investors a

	Insurance Companies			Mutual Funds		
Sample	Total	Buy	Sell	Total	Buy	Sell
Total	-0.06^{***}	-0.08^{***}	0.06^{**}	-0.03^{***}	-0.06^{**}	0.02^{****}
	(0.03)	(0.03)	(0.03)	(0.01)	(0.03)	(0.01)
	[8]	[8]	[6]	[7]	[2]	[6]
A	-0.08^{***}	-0.08^{***}	0.05^{***}	-0.05^{**}	-0.08^{*}	0.02^{*}
	(0.03)	(0.04)	(0.02)	(0.03)	(0.05)	(0.01)
	[8]	[8]	[6]	[8]	[8]	[6]
B	0.34^{**}	0.22^{*}	0.29^{**}	0.03^{**}	0.23^{*}	0.02^{***}
	(0.18)	(0.14)	(0.18)	(0.02)	(0.15)	(0.01)
	[1]	[7]	[1]	[6]	[1]	[6]

	Investment Banks		
Sample	Total	Buy	Sell
Total	-0.08^{***}	-0.11^{***}	0.07^{****}
	(0.03)	(0.05)	(0.03)
	[2]	[2]	[5]
A	-0.14^{***}	-0.11^{***}	0.09^{***}
	(0.07)	(0.05)	(0.04)
	[1]	[1]	[6]
B	0.53^{***}	0.34^{**}	0.38^{***}
	(0.25)	(0.18)	(0.19)
	[1]	[1]	[1]

Notes: The table reports parameter estimates of the cross effects φ_s in the mean equations (as defined in equation (1)). The estimates of the sample B1 are not reported for space considerations.

****, ***, **, * denote significance at the $0.01, 0.05, 0.10, 0.15$ level respectively.

The numbers in parentheses are standard errors. The numbers in brackets are the lag order S of the regressor.

Table 1B.1b: Mean Equations: Cross effects

Panel B. Non-members domestic investors b.

	Commercial Banks			Savings Banks			Other Companies		
Sample	Total	Buy	Sell	Total	Buy	Sell	Total	Buy	Sell
Total	0.10^{***}	0.07^{**}	0.15^{***}	0.03^{**}	0.04^{*}	0.05^{**}	0.04^{*}	0.06^{**}	0.05^{***}
	(0.05)	(0.04)	(0.07)	(0.01)	(0.03)	(0.03)	(0.03)	(0.04)	(0.02)
	[4]	[6]	[4]	[3]	[6]	[4]	[6]	[6]	[5]
A	0.13^{***}	0.10^{**}	0.12^{**}	0.03^{***}	0.04^{**}	0.08^{*}	0.16^{***}	0.06^{*}	0.06^{*}
	(0.06)	(0.05)	(0.06)	(0.02)	(0.02)	(0.05)	(0.08)	(0.04)	(0.04)
	[5]	[5]	[5]	[3]	[3]	[4]	[6]	[1]	[5]
B	0.07^{***}	0.15^{**}	0.20^{**}	0.07^{*}	0.05^{***}	0.07^{****}	0.04^{*}	0.10^{*}	0.10^{**}
	(0.04)	(0.08)	(0.11)	(0.05)	(0.02)	(0.02)	(0.03)	(0.07)	(0.06)
	[4]	[1]	[1]	[1]	[10]	[11]	[17]	[12]	[12]

See Notes in Table 1B.1a

Table 1B.1c: Mean Equations: Cross effects

Panel C. Domestic investors

	Members			Non-members		
Sample	Total	Buy	Sell	Total	Buy	Sell
Total	-0.06^{*}	-0.05^{*}	0.04^{*}	0.07^{*}	0.15^{***}	0.07^{**}
	(0.04)	(0.03)	(0.03)	(0.05)	(0.07)	(0.05)
	[2]	[2]	[5]	[5]	[6]	[4]
A	-0.09^{****}	-0.07^{**}	-0.08^{***}	0.12^{**}	0.13^{**}	0.09^{**}
	(0.03)	(0.04)	(0.04)	(0.07)	(0.07)	(0.05)
	[8]	[8]	[8]	[5]	[5]	[5]
B	0.25^{****}	0.15^{*}	0.20^{****}	0.34^{***}	0.26^{**}	0.33^{***}
	(0.10)	(0.10)	(0.08)	(0.17)	(0.14)	(0.16)
	[1]	[20]	[1]	[1]	[1]	[1]

	Individual Investors		
Sample	Total	Buy	Sell
Total	0.12^{*}	0.23^{***}	0.12^{**}
	(0.07)	(0.10)	(0.07)
	[1]	[6]	[5]
A	0.14^{**}	0.13^{**}	0.12^{**}
	(0.08)	(0.08)	(0.07)
	[5]	[5]	[5]
B	0.63^{****}	0.71^{****}	0.50^{****}
	(0.21)	(0.23)	(0.20)
	[1]	[1]	[1]

See Notes in Table 1B.1a

Table 1B.1d: Mean Equations: Cross effects

Panel D. Total trading volume

	Domestic			Foreign		
Sample	Total	Buy	Sell	Total	Buy	Sell
Total	0.13^{**}	0.16^{**}	0.12^{***}	-0.03^{***}	-0.02^{****}	0.12^{****}
	(0.08)	(0.09)	(0.06)	(0.01)	(0.01)	(0.04)
	[5]	[1]	[5]	[2]	[2]	[6]
A	0.15^{***}	0.17^{***}	0.13^{**}	-0.02^{****}	-0.01^{***}	0.08^{***}
	(0.08)	(0.08)	(0.07)	(0.01)	(0.00)	(0.04)
	[5]	[5]	[5]	[2]	[2]	[6]
B	0.78^{****}	0.84^{****}	0.71^{****}	0.37^{**}	0.22^{*}	0.35^{**}
	(0.26)	(0.27)	(0.26)	(0.21)	(0.15)	(0.21)
	[1]	[1]	[1]	[1]	[1]	[1]

	Total		
Sample	Total	Buy	Sell
Total	-0.16^{****}	-0.16^{****}	0.11^{*}
	(0.05)	(0.05)	(0.07)
	[8]	[8]	[5]
A	-0.15^{****}	-0.15^{****}	0.12^{**}
	(0.06)	(0.06)	(0.07)
	[8]	[8]	[5]
B	0.79^{****}	0.79^{****}	0.79^{****}
	(0.29)	(0.28)	(0.28)
	[1]	[1]	[1]

See Notes in Table 1B.1a

1.10 APPENDIX 1C: Variance equations GARCH coefficients

Table 1C.1a: Variance Equations: GARCH coefficients

Panel A. Non-members domestic investors a.						
	Insurance Comp.		Mutual Funds		Investment Banks	
Sample	α	β	α	β	α	β
Total	-0.16 (0.15)	0.59^{****} (0.22)	-0.16 (0.15)	0.59^{****} (0.23)	-0.16 (0.15)	0.59^{****} (0.23)
A	0.15 (0.16)	0.72^{****} (0.22)	0.14 (0.22)	0.73^{***} (0.32)	0.23 (0.28)	0.61^{**} (0.33)
B	-0.29^{**} (0.17)	0.70^{****} (0.16)	-0.26^{**} (0.16)	0.71^{****} (0.21)	-0.25^{**} (0.14)	0.71^{****} (0.20)
Panel B. Non-members domestic investors b.						
	Commercial Banks		Savings Banks		Other Companies	
Sample	α	β	α	β	α	β
Total	-0.15 (0.14)	0.55^{****} (0.21)	-0.17 (0.14)	0.52^{**} (0.27)	-0.16 (0.15)	0.60^{****} (0.21)
A	0.16 (0.26)	0.73^{***} (0.35)	0.16 (0.25)	0.71^{***} (0.35)	0.17 (0.15)	0.74^{****} (0.18)
B	-0.11 (0.11)	0.59^{****} (0.16)	-0.27^{**} (0.16)	0.71^{****} (0.19)	-0.25^{**} (0.15)	0.69^{****} (0.23)

Notes: The table reports estimates of the ARCH (α) and GARCH (β) parameters in the variance equations. α, β are defined in equation (2).
The estimates are reported only for the case when total TV_t is added as regressor and not for the buy and sell side of each series, due to space considerations.
The estimates of the sample B1 are not reported for space considerations.
**** , *** , ** , * denote significance at the $0.01, 0.05, 0.10, 0.15$ level respectively. The numbers in parentheses are standard errors.

Table 1C.1b: Variance Equations: GARCH coefficients

Panel C. Domestic investors						
	Members		Non-members		Individual Investors	
Sample	α	β	α	β	α	β
Total	-0.16 (0.15)	0.59^{***} (0.24)	-0.16 (0.15)	0.60^{****} (0.21)	-0.16 (0.15)	0.60^{****} (0.23)
A	0.13 (0.12)	0.76^{****} (0.18)	0.16 (0.28)	0.71^{**} (0.38)	0.14 (0.17)	0.75^{****} (0.26)
B	-0.26^{*} (0.16)	0.72^{****} (0.22)	-0.25^{**} (0.15)	0.72^{****} (0.20)	-0.26^{**} (0.15)	0.71^{****} (0.22)
Panel D. Total trading volume						
	Total		Domestic		Foreign	
Sample	α	β	α	β	α	β
Total	-0.16 (0.15)	0.60^{****} (0.21)	-0.16 (0.15)	0.61^{****} (0.22)	-0.16 (0.15)	0.61^{****} (0.24)
A	0.14 (0.15)	0.74^{****} (0.22)	0.13 (0.16)	0.76^{****} (0.24)	0.11 (0.10)	0.78^{****} (0.16)
B	-0.25^{**} (0.15)	0.72^{****} (0.21)	-0.25^{**} (0.15)	0.71^{****} (0.22)	-0.25^{**} (0.16)	0.73^{****} (0.21)

See Notes in Table 1C.1a

Chapter 2 Multivariate FIAPARCH modelling of financial markets with dynamic correlations in times of crisis

2.1 Introduction

The intrinsic informational content that financial crises provide to the research community is certainly one of the key reasons they remain in the spotlight of the finance and broader economic literature long after they are resolved. The 1997 Asian financial crisis, the Global financial crisis of 2007-08 and the ongoing European sovereign-debt crisis are evidently amongst the most important events that stirred universal fear of a worldwide economic meltdown due to financial contagion amongst investors, financial market practitioners and policy makers alike. And inevitably, what our modelling tools can tell us about the period around those times is, amongst other things, the channel through which our existing risk management paradigms and decision-making processes will evolve to better address similar episodes in the future.

In this spirit, the availability of data and processing power capacity together with the recent developments in econometrics allow us to pinpoint better than ever before, properties of the underlying stochastic processes that are crucial albeit hard to uncover (i) in constructively challenging long-established assumptions of the financial practice such as the benefits of international portfolio diversification, especially during periods of economic turmoil or (ii) in shedding light on how the properties of our modelling efforts of the underlying stochastic processes project the impact of these crises. Our study introduces a unified approach and demonstrates how it can be used to determine key aspects of modelling around periods of economic turmoil, such as changes in the linkages between financial markets, in long memory and power effects amongst others. In particular, we focus on stock market volatilities and co-volatilities and how they have changed due to the Asian and the recent Global financial crises.

The study of the linkages between volatilities and co-volatilities of the financial markets is a critical issue in risk management practice. The multivariate GARCH framework provides the tools to understand how financial volatilities move together over time and across markets. For thorough surveys of the available Multivariate GARCH models and their use in various fields of risk management such as option pricing, hedging and portfolio selection see Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2009).

Conrad et al. (2011) applied a multivariate fractionally integrated asymmetric power ARCH (FIAPARCH) model that combines long memory, power transformations of the conditional variances and leverage effects with constant conditional correlations (CCC) on eight national stock market indices returns. The long-range volatility dependence, the power transformation of returns and the asymmetric response of volatility to positive and negative shocks are three features that improve the modelling of the volatility process of asset returns and its implications for the various risk management practices. We extend their model by allowing for cross effects between the markets in the mean of returns and by estimating time-varying conditional correlations. We also study the effect of financial crisis events on the dynamic conditional correlations as well as on the three key features of the conditional variance nested in the model. Therefore, the contribution of the present study is that our model provides a complete framework for the analysis of financial markets' co-volatility processes.

The empirical analysis of our model applied to eight stock indices daily returns in a bivariate and trivariate framework provides evidence that confirms the importance of long memory in the conditional variance, of the power transformations of returns to best fit the volatility process and of the asymmetric response of volatility to positive and negative shocks. A Wald testing procedure strongly supports our results. We extend the existing empirical evidence on the dynamic conditional correlations (DCC) models by adding all cross effects in the mean equation, that is we estimate a full vector autoregressive (VAR) model, to reveal the relationship amongst the returns of each multivariate specification. In the previous studies the researchers have added as regressor in the mean for all stock market indices a prevailing global index return, such as S&P 500 or an index of particular interest for the region and the period investigated. Our cross effects are found significant in most cases.

Moreover, another of our main findings regards the DCC analysis with structural breaks. In line with the literature, our model estimates always highly persistent conditional correlations. The correlations increase during crisis events, indicating contagion effects between the markets and remain on a high level after the crisis break, showing the investors' herding behaviour. Finally, we contribute to the existing literature findings by comparing two different financial crises, the Asian (1997) and the recent Global (2007-08) crisis, in terms of their effects on the correlations, where we observe much more heightened conditional correlation estimates for the recent Global crisis than for the Asian crisis. This is reasonable since the international financial integration followed by the financial liberalisation and deregulation in capital controls has reached its peak nowadays compared to its evolution during the Asian financial crisis in 1997.

The remainder of the chapter is structured as follows. Section 2.2 discusses the existing empirical literature on the financial crises, the contagion effects amongst the financial markets and the investors' herding behaviour. In Section 2.3 we detail the multivariate FIAPARCH model with DCC and the methodology for detecting structural breaks. Section 2.4 discusses the data and presents the empirical results. Quasi Maximum likelihood parameter estimates for the various specifications and results of the Wald testing procedures are presented. We also evaluate the different specifications, taking into account the structural breaks of each time series linked with two financial crisis events. Each multivariate specification is re-estimated under three subsamples defined by the break dates detected for each country combination. In addition, two contagion tests are performed in Section 2.5. The final Section concludes the analysis.

2.2 Literature review

2.2.1 Financial crises and the DCC model

There are several studies that investigate the two crises (the Asian and the recent Global one) using the DCC model. Cho and Parhizgari (2008) study the Asian financial crisis effects on correlations between eight East Asian stock markets. Using the AR(1)-DCC-GARCH(1,1) model on daily returns they find an upward trend in DCCs after the break date of the crisis. They observe a shift in the mean and the median of the DCCs computed by the model. Chiang et al. (2007) also use an AR(1)-DCC-GARCH(1,1) on nine Asian stock markets plus the US market (as explanatory variable in the mean equation) to investigate the effects of the Asian crisis. They conclude that there are higher correlations during the crisis, where volatility is also increased. They also observe two phases in the crisis period. In the first phase the correlations increase, which means contagion effect and in the second phase the correlations remain high, which means investors' herding behaviour.

Syllignakis and Kouretas (2011) use the AR(1)-DCC-GARCH(1,1) model to investigate the correlation pattern (before and after the current financial crisis) between the US, the Russian and seven emerging markets of Central and East Europe. They consider cross effects in the mean caused only by either the US, the German or the Russian index returns but not by the other dependent variables of each multivariate model. They find an increase in conditional correlations between the stock market returns during the crisis (2007-2009). They use weekly returns and then dummy variables for the crisis periods as regressors in a separate regression of the generated DCC. Kenourgios and Samitas (2011) apply the asymmetric generalised (AG) DCC-GARCH$(1,1)$ model of Cappiello et al. (2006) to confirm the increased dynamic correlations between five emerging Balkan stock markets, the US and three developed European markets during the current financial crisis, also considering asymmetries in correlation dynamics. They conclude that the higher stock market interdependence is due to herding behaviour during the crisis period.

Kenourgios et al. (2011) extend their paper to investigate the conditional correlations over five financial crisis events from 1995 to 2006 for the BRICs, the US and the UK using various DCC models like the original one of Engle (2002a) and the AG-DCC as well. More recently, Kenourgios and Padhi (2012) again estimated AG-DCC models to study correlations during crisis periods between 1994 and 2008 on nine emerging markets and the US.

Kazi et al. (2011) use a multivariate DCC-GARCH(1,1) model to investigate the correlations between seventeen OECD stock market returns before and during the current Global financial crisis. They use the Bai-Perron (2003a) structural break test and apply the DCC model for the whole period (2002-2009) and the two sub-periods, defined by the structural break detected (1-10-2007), which corresponds to the beginning of the crisis. They observe a significant increase in DCC during the crisis (after October 2007) compared to the pre-crisis period (before October 2007), which confirms the finding of previous studies of a higher contagion effect during financial crisis periods. Kotkatvuori-Ornberg et al. (2013) also focus on the current financial crisis with data from fifty stock market indices for the period 2007 to 2009, accounting for two major events: JP Morgan's acquisition of Bear Stearns and the Lehman Brothers' collapse with dummy variables for the unconditional variance in the multivariate GARCH(1,1) equation. Then the DCC model is applied in six multivariate specifications for each region and the correlations generated are further used to run multivariate GARCH(1,1) with the same intercept dummies in the mean and the variance. The impact of the crisis is found significant on stock markets' comovements and especially the effect of the Lehman Brothers' collapse is prominent across all regions.

The advantage of our analysis in comparison with the above studies is the FIAPARCH specification of the conditional variance, while the existing studies use the simple GARCH model. We also assume t-distributed innovations, since daily financial data exhibit excess kurtosis, while all the above mentioned papers assume Gaussian innovations. Moreover, we add in the mean equation the cross effects between all the dependent variables and not a common regressor for all the returns, such as the US stock index in Chiang et al. (2007) and thus we estimate a full VAR model. We also apply the complete methodology of Karoglou (2010) to identify the structural breaks in the mean and the volatility dynamics of the stock returns, using a comprehensive set of data-driven methods of structural change detection and not only a single statistical test. We finally use a very large sample period from 1988 to 2010 of daily stock returns, the widest amongst the studies considered under our literature review.

2.2.2 Long Memory and Power Transformed returns

There are some recent studies that use the DCC models of either Engle (2002a) or Tse and Tsui (2002) with the FIAPARCH specification in the variance equation. Aloui (2011) uses daily stock index returns from Latin American markets for the period 1995-2009 and runs the multivariate FIAPARCH with Engle's DCC, assuming t-distributed innovations following Conrad et al. (2011). The DCCs generated are modelled separately with an AR(p)-GARCH(1,1) with intercept dummies for the crisis events in the mean and the variance equation. The breaks are defined from the economic approach of each crisis timing, the Asian financial crisis (AFC), the Global financial crisis (GFC) and the regional Latin American crises. They prove that the correlations are much higher during periods of financial crises and especially the regional crises and the GFC. Ho and Zhang (2012) apply amongst other models the multivariate FIAPARCH framework with the DCC of Tse and Tsui (2002) with the normality assumption for the errors on daily Chinese stock index returns from 1992-2006. They focus on the key features of the variance specification, the asymmetries and the long memory and on the time-varying behaviour of the conditional correlations. They do not use breaks and do not investigate the effect of crisis events.

Dimitriou and Kenourgios (2013) apply the multivariate FIAPARCH framework of Tse (1998) with the DCC of Engle (2002a) on foreign exchange rates daily data from 2004 to 2011 with t-distributed errors, in order to identify the effect of the recent financial crisis. They detect the structural breaks according to an economic approach defining the exact timing of the major crisis events and a statistical approach applying the Markov Switching Dynamic Regression model. They run the multivariate DCC-FIAPARCH on the whole sample without cross effects for the five currency series and with the DCCs generated they run an AR(p)-GJR-GARCH(1,1) with intercept dummies for the crisis breaks in the mean and the variance equation of the DCCs to measure the crisis effects. They conclude that there are lower exchange rate correlations during turbulent times. Dimitriou et al. (2013) also use the same FIAPARCH specification in a bivariate framework for stock returns of the US and the BRICs markets pairwise for the period 1997-2012. They assume again t-distributed errors but they use the DCC of Tse and Tsui (2002) instead of Engle's (2002a) specification. They model the DCCs extracted from the whole sample and detect the breaks in the same way in order to investigate the correlation dynamics during the several phases of the recent financial crisis. Stock market correlations are found to be increased after early 2009.

In the light of the more recent DCC-FIAPARCH studies, our modelling still provides a comprehensive analysis of the volatility and correlation processes for three main reasons: we use an outstanding breaks methodology, we apply the mean cross effects (that is a full VAR model) and our data cover the longest sample period, which is split into subsamples for the crisis periods in order to re-estimate the same model specifications and analyse the time-varying behaviour of the parameters and the effects of the financial crises.

2.2.3 Contagion effects

Our empirical results below (see Section 2.5) are in line with the existing empirical evidence that supports the increase in conditional correlations during crisis and justifies the contagion effects amongst the financial markets and the investors' herding behaviour. As a brief review of the studies on the markets' interdependence during crisis events we first refer to Lin et al. (1994), who report the link between higher correlations and higher volatility periods in equity market returns as an ‘empirical regularity’ to start their research on intra-daily stock prices across markets. Ang and Bekaert (1999) and Longin and Solnik (2001) observe higher volatility periods associated with higher correlations between different stock index returns in bear markets. Bartram and Wang (2005) provide evidence that contagion effects exist during crises with higher correlation estimates. Boyer et al. (2006) show that correlation estimates increase during crisis periods and investigate the transmission mechanisms across different markets. Increased herding behaviour during crisis is proved in Chiang and Zheng (2010) for some of the countries under study. Sandoval and Franca (2012) use various techniques to measure the correlation between the markets during crises and find that in turbulent times markets exhibit higher degrees of comovement.

Corsetti et al. (2001) show that although the stock markets’ volatilities and covariances increase during crises, the correlations are not necessarily higher. Forbes and Rigobon (2002) give the definition of contagion as “the significant increase in cross-market linkages after a shock to one country”. They develop tests on the contagion effect during a crisis and show that the correlation coefficients are conditional on market volatilities. During a crisis the market volatilities are higher, so the correlations are biased upwards. They find no contagion effect during crises by estimating the unconditional correlations, but they accept that there is interdependence (high level of market comovement) across the markets in any state of the economy. Billio and Pelizzon (2003) investigate the tests proposed by the two above mentioned studies to detect contagion or interdependence across markets during financial crisis events. Chakrabarti and Roll (2002) observe higher covariances, correlations and volatilities, after the Asian financial crisis arose, in both Asian and European markets. Yang and Lim (2004) find that during the Asian financial crisis a contagion effect is apparent across the stock markets with a higher degree of interdependence in the whole region. Khan and Park (2009) find herding contagion across Asian markets during the

Asian financial crisis, measuring the cross-country correlations. Finally, Moldovan (2011) proves that correlations between the three major financial markets (US, UK and Japan) are higher after the recent financial crash of 2007 than before.

In the financial crisis literature review we find no study that compares the AFC and the GFC, except for Aloui (2011), who investigates the effects of the two crises only on Latin American markets with a narrower sample. Our extended sample from 1988 to 2010 gave us the chance to compare the conditional correlations after each crisis. We find higher correlation estimates after the GFC break than after the AFC. This is absolutely expected since the international financial integration is more apparent in recent years. The evident risk transmission across markets as well as the key characteristics of volatility (co-persistence and asymmetry) during crises should be of primary interest for the market players (all sorts of investors and risk managers) and the regulators. The market participants must take into account the market's stylised facts captured by our model. For example, the volatility persistence affects the investment horizon and the higher correlations reduce the portfolio diversification gains. The financial authorities have to consider such findings in order to establish the appropriate market control measures and protect the investors from extreme risk exposures.

2.3 Methodology

2.3.1 Multivariate FIAPARCH-DCC model

The most common model in finance to describe a time series of daily stock index returns is the VAR of order 1 process. Let us define the N-dimensional column vector of the returns $\mathbf{r}_t$ as $\mathbf{r}_t = [r_{it}]_{i=1,\ldots,N}$ and the corresponding residual vector $\boldsymbol{\varepsilon}_t$ as $\boldsymbol{\varepsilon}_t = [\varepsilon_{it}]_{i=1,\ldots,N}$. The structure of the VAR (1) mean equation with cross effects is given by

$$\mathbf{r}_t = \boldsymbol{\phi} + \boldsymbol{\Phi}\mathbf{r}_{t-1} + \boldsymbol{\varepsilon}_t \tag{2.1}$$

where $\boldsymbol{\phi} = [\phi_i]_{i=1,\ldots,N}$ is an $N \times 1$ vector of constants; the $N \times N$ coefficient matrix $\boldsymbol{\Phi} = [\phi_{ij}]_{i,j=1,\ldots,N}$ can be expressed as $\boldsymbol{\Phi} = \boldsymbol{\Phi}^{(d)} + \boldsymbol{\Phi}^{(od)}$, with $\boldsymbol{\Phi}^{(d)}$ =diag$(\phi_{11}, \ldots, \phi_{NN})$, that is to allow for cross effects we allow $\mathbf{\Phi}^{(od)} \neq 0$ (matrices and vectors are denoted by upper and lower case boldface symbols, respectively.). For example, the bivariate AR(1) model is given by

$$\begin{bmatrix} r_{1t} \\ r_{2t} \end{bmatrix} = \begin{bmatrix} \phi_{01} \\ \phi_{02} \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \text{ or}$$
$$\begin{bmatrix} r_{1t} \\ r_{2t} \end{bmatrix} = \begin{bmatrix} \phi_{01} \\ \phi_{02} \end{bmatrix} + \left\{ \begin{bmatrix} \phi_{11} & 0 \\ 0 & \phi_{22} \end{bmatrix} + \begin{bmatrix} 0 & \phi_{12} \\ \phi_{21} & 0 \end{bmatrix} \right\} \begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}.$$

Regarding $\boldsymbol{\varepsilon}_t$ we assume that it is conditionally student-t distributed with mean vector $\mathbf{0}$, covariance matrix $\mathbf{\Sigma}_t = \mathbb{E}(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' \,|\mathcal{F}_{t-1}) = [\sigma_{ij,t}]_{i,j=1,\dots,N}$ and variance vector $\boldsymbol{\sigma}_t = \mathbb{E}(\boldsymbol{\varepsilon}_t^{\wedge 2} \,|\mathcal{F}_{t-1}) = [\sigma_{ii,t}]_{i=1,\dots,N}$ or $\boldsymbol{\sigma}_t = (\mathbf{I}_N \odot \mathbf{\Sigma}_t)\mathbf{i}$ with $\mathbf{i}$ being an $N \times 1$ vector of ones (the symbol $\odot$ denotes element wise multiplication); $\boldsymbol{\sigma}_t$ follows a multivariate FIAPARCH$(1, d, 1)$ model (see below).

Notice that $\boldsymbol{\varepsilon}_t$ can be written as $(\mathbf{e}_t \odot \mathbf{q}_t^{\wedge -1/2}) \odot \boldsymbol{\sigma}_t^{\wedge 1/2}$ (the symbol $^\wedge$ denotes element wise exponentiation) where $\mathbf{e}_t = [e_{it}]_{i=1,\dots N}$ is conditionally student-t distributed with mean vector $\mathbf{0}$, time-varying covariance (symmetric positive definite) matrix $\mathbf{Q}_t$=$[q_{ij,t}]_{i,j=1,\dots,N}$ (the so called quasi-correlations, see Engle, 2009) and variance vector $\mathbf{q}_t = (\mathbf{I}_N \odot \mathbf{Q}_t)\mathbf{i}$. It follows that

$$\begin{aligned}\sigma_{ij,t} &= \mathbb{E}(\varepsilon_{it}\varepsilon_{jt} \,|\mathcal{F}_{t-1}) = \mathbb{E}(\frac{e_{it}e_{jt}}{\sqrt{q_{ii,t}q_{jj,t}}}\sqrt{\sigma_{ii,t}\sigma_{jj,t}} \,|\mathcal{F}_{t-1}) \\ &= \frac{\sqrt{\sigma_{ii,t}\sigma_{jj,t}}}{\sqrt{q_{ii,t}q_{jj,t}}}\mathbb{E}(\, e_{it}e_{jt}|\, \mathcal{F}_{t-1}) = \sqrt{\sigma_{ii,t}\sigma_{jj,t}}\frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}} = \sqrt{\sigma_{ii,t}\sigma_{jj,t}}\rho_{ij,t}.\end{aligned}$$

Most importantly, we allow for DCC, $\rho_{ij,t} = \sigma_{ij,t}/\sqrt{\sigma_{ii,t}\sigma_{jj,t}}$, $|\rho_{ij,t}| \leq 1$ $(i, j = 1, \dots, N)\forall\, t$, instead of the constant ones, ρ_{ij}, used by Conrad et al. (2011) (see below).

The covariance matrix $\mathbf{\Sigma}_t$ can be expressed as

$$\mathbf{\Sigma}_t = (\mathbf{I}_N \odot \mathbf{\Sigma}_t^{\wedge 1/2})\mathbf{R}_t(\mathbf{I}_N \odot \mathbf{\Sigma}_t^{\wedge 1/2}), \tag{2.2}$$

where $\mathbf{R}_t = [\rho_{ij,t}]_{i,j=1,\dots,N}$ is the $N \times N$ symmetric positive semi-definite time-varying correlation matrix with ones on the diagonal ($\rho_{ii,t} = 1$) and the off-diagonal elements less than one in absolute value.

Next, the structure of the conditional variance is specified as in Tse (1998), who combines the FIGARCH formulation of Baillie et al. (1996) with the APARCH model of Ding et al. (1993). The multivariate FIAPARCH$(1, d, 1)$ we estimate is specified as follows:

$$\boldsymbol{\beta}(L) \odot \boldsymbol{\sigma}_t^{\bar{\wedge}\delta_i/2} = \boldsymbol{\omega} + [\boldsymbol{\beta}(L) - \mathbf{c}(L) \odot \mathbf{d}(L)] \odot f(\boldsymbol{\varepsilon}_t), \quad (3)$$

$$f(\boldsymbol{\varepsilon}_t) = (|\boldsymbol{\varepsilon}_t| - \boldsymbol{\gamma}\boldsymbol{\varepsilon}_t)^{\bar{\wedge}\delta_i},$$

where $\boldsymbol{\beta}(L) = [1 - \beta_i L]_{i=1,\dots,N}$, $\boldsymbol{\omega} = [\omega_i]_{i=1,\dots,N}$, $\omega_i \in (0, \infty)$; $\mathbf{c}(L) = [1 - c_i L]_{i=1,\dots,N}$, $|c_i| < 1$ and $\mathbf{d}(L) = [(1 - L)^{d_i}]_{i=1,\dots N}$, $0 \leq d_i \leq 1$ are all $N \times 1$ vectors; $|\boldsymbol{\varepsilon}_t|$ is the vector $\boldsymbol{\varepsilon}_t$ with elements stripped of negative values and $\boldsymbol{\gamma} = [\gamma_i]_{i=1,\dots,N}$ is the vector of the leverage coefficients, $|\gamma_i| < 1$; the power terms, δ_i, take finite positive values and are used in elementwise exponentiation, that is $\boldsymbol{\sigma}_t^{\bar{\wedge}\delta_i/2}$ raises the ith standard deviation to the power of δ_i. In other words, each conditional variance follows a FIAPARCH$(1, d, 1)$ model:

$$(1 - \beta_i L)\sigma_{ii,t}^{\delta_i/2} = \omega_i + [(1 - \beta_i L) - (1 - c_i L)(1 - L)^{d_i}](|\varepsilon_{it}| - \gamma_i \varepsilon_{it})^{\delta_i}, i = 1, \dots, N. \quad (2.4)$$

The sufficient conditions of Bollerslev and Mikkelsen (1996) for the positivity of the conditional variance of a FIGARCH $(1, d, 1)$ model: $\omega_i > 0$, $\beta_i - d_i \leq c_i \leq \frac{2-d_i}{3}$ and $d_i(c_i - \frac{1-d_i}{2}) \leq \beta_i(c_i - \beta_i + d_i)$, should be satisfied $\forall\ i$ (see also Conrad and Haag (2006) and Conrad (2010)). Of course when $d_i = 0$ the model reduces to the APARCH$(1, 1)$: $(1 - \beta_i L)\sigma_{ii,t}^{\delta_i/2} = \omega_i + \alpha_i L(|\varepsilon_{it}| - \gamma_i \varepsilon_{it})^{\delta_i}$, $\alpha_i = c_i - \beta_i$; in addition, when $\delta_i = 2$, $\gamma_i = 0$ it reduces to the GARCH$(1, 1)$: $(1 - \beta_i L)\sigma_{ii,t} = \omega_i + \alpha_i L\varepsilon_{it}^2$.

Finally, the structure of $\mathbf{R}_t$ according to Engle (2002a) is given by

$$\mathbf{R}_t = (I_N \odot \mathbf{Q}_t{}^{\wedge-1/2})\mathbf{Q}_t(I_N \odot \mathbf{Q}_t{}^{\wedge-1/2}), \quad (5)$$

$$\mathbf{Q}_t = (1 - a - b)\mathbf{Q} + a\mathbf{e}_{t-1}\mathbf{e}_{t-1}^{'} + b\mathbf{Q}_{t-1}, \quad (6)$$

where $\mathbf{Q} = \mathbb{E}(\mathbf{Q}_t) = [q_{ij}]_{i,j=1,\dots,N}$, a and b are nonnegative scalar parameters satisfying $a + b < 1$. It is clear that Engle (2002a) specifies the conditional correlations as a weighted sum of past correlations, since the matrix of the quasi correlations, $\mathbf{Q}_t$, is written as a GARCH process and then transformed to a correlation matrix. Engle (2002a, 2009) used the estimator $\widehat{\mathbf{Q}} = \frac{1}{T}\sum_{t=1}^{T} \widehat{\mathbf{e}}_t \widehat{\mathbf{e}}_t'$.

In the bivariate case the conditional correlation coefficient $\rho_{12,t}$ is expressed as follows:

$$\rho_{12,t} = \frac{q_{12,t}}{\sqrt{q_{11,t}q_{22,t}}}, \tag{7}$$

$$q_{12,t} = (1-a-b)q_{12} + ae_{1,t-1}e_{2,t-1} + bq_{12,t-1},$$

$$q_{11,t} = (1-a-b)q_{11} + ae_{1,t-1}^2 + bq_{11,t-1},$$

$$q_{22,t} = (1-a-b)q_{22} + ae_{2,t-1}^2 + bq_{22,t-1}.$$

2.3.2 Structural breaks

In order to identify the number and timing of the potential structural breaks we employ the Awarding-Nominating procedure of Karoglou (2010). This procedure involves two stages: the “Nominating breakdates” stage and the “Awarding breakdates” stage.

The “Nominating breakdates” stage involves the use of one or more statistical tests to identify some dates as possible breakdates. In recent years, a number of statistical tests have been developed for that reason and for the purposes of this study, we use the following ones:

(a) I&T (Inclán and Tiao, 1994)

(b) SAC1 (The first test of Sansó et al., 2003)

(c) SAC2BT, SAC2QS, SAC2VH (The second test of Sansó et al., 2003, with the Bartlett kernel, the Quadratic Spectral kernel and the Vector Autoregressive HAC or VARHAC kernel of den Haan and Levin, 1998 respectively)

(d) K&LBT, K&LQS, K&LVH (The version of the Kokoszka and Leipus, 2000 test refined by Andreou and Ghysels, 2002 with the Bartlett kernel, the Quadratic Spectral kernel and the VARHAC kernel respectively).

These tests are designed to detect a structural change in the volatility dynamics, but in fact they do not discriminate between shifts in the mean and shifts in the variance. For the purpose of this study, this is a plausible feature since all types of breaks need to be considered in order to determine if and to what extent the distributional properties change when moving from one regime to another. Furthermore, their properties for strongly dependent series have been extensively investigated (e.g. Andreou and Ghysels, 2002, Sansó et al., 2003, Karoglou, 2006) and there is evidence that they perform satisfactorily under the most common ARCH-type processes.

To identify multiple breaks in a series we incorporate the aforementioned test in the following iterative scheme (algorithm):

1. Calculate the test statistic under consideration using the available data.

2. If the statistic is above the critical value split the particular sample into two parts at the date at which the value of a test statistic is maximised.

3. Repeat steps 1 and 2 for the first segment until no more (earlier) change-points are found.

4. Mark this point as an estimated change-point of the whole series.

5. Remove the observations that precede this point (i.e. those that constitute the first segment).

6. Consider the remaining observations as the new sample and repeat steps 1 to 5 until no more change-points are found.

The above algorithm is implemented with each of the (single breakdate CUSUM-type) test statistics described above (i.e. I&T, SAC1, SAC2BT, SAC2QS, SAC2VH, K&LBT, K&LQS, K&LVH).

What differentiates this scheme from a simple binary division procedure is that it forces the existing breaks to be detected in a time-orderly fashion, which makes it more robust when transitional periods exist - in which case a simple binary division procedure is likely to produce more breaks in the interim period. In the absence of transitional periods both procedures will produce the same breaks.

The nominated breakdates for each series are simply all those which have been detected in each case. Note that at this stage we are not much concerned with detecting more breaks than those that actually exist because whichever is not an actual breakdate will be picked up in the Awarding breakdates stage.

The "Awarding breakdates" stage is a procedure which, in essence, is about uniting contiguous nominated segments (i.e. segments that are defined by the nominated breakdates) unless one of the following two conditions is satisfied:

(I) the means of the contiguous segments are statistically different (as suggested by the t-test)

(II) the variances of the contiguous segments are statistically different (as suggested by the battery of tests which is described below)

This testing procedure is repeated until no more segments can be united, that is, until no condition of the two above is satisfied for any pair of contiguous segments.

The battery of tests mentioned in (II) constitute a different approach to the CUSUM-type tests described previously in that they test for the homogeneity of variances of contiguous segments without encompassing the time series dimension of the data . They include the standard F-test, the Siegel-Tukey test with continuity correction (Siegel and Tukey, 1960 and Sheskin, 2004), the adjusted Bartlett test (see Sokal and Rohlf, 1995 and Judge, et al., 1988), the Levene test (1960) and the Brown-Forsythe (1974) test.

Overall, we find that the stochastic behaviour of all indices yields about three to seven breaks during the sample period, roughly one every two to four years on average. The resulting break dates for each series are in the Appendix 2B, Table 2B.1. The predominant feature of the underlying segments is that mainly changes in variance are found statistically significant. Finally, there are several breakdates that are identical in all series and others that are very close to one another, which apparently signify economic events with a global impact.

Table 2B.2 in the Appendix 2B provides a detailed account of the possible associations that can be drawn between each breakdate and a major economic event that took place at or around the breakdate period, either in the world or in each respective economy. It appears that dates for the extraordinary events of the AFC of 1997, the GFC of 2007–08 and the European sovereign-debt crisis that followed are very clearly identified in all stock return series and with very little or no variability. Other less spectacular events, such as the Russian financial crisis of 1998, the Japanese asset price bubble of 1986-1991 or the UK's withdrawal from the European Exchange Rate Mechanism (ERM), can also be associated with the breakdates that have been identified in some series. Table 2B.3 presents some of the descriptive statistics of the stock returns of each segment between the breakdates. The variability of the mean returns becomes particularly prominent for all countries at the end of our sample i.e. after the 2007-08 financial crisis. In exactly the same period, the stock market uncertainty as proxied by the standard deviation rises dramatically.

We selected amongst the breaks detected (for each series' combination for the respective bivariate and trivariate models) the two dates that correspond to the two financial crisis events, on which we will focus in our analysis. These dates are also the most common breaks of each series' combination. We intend to study the impact of the AFC of 1997 and the recent GFC of 2007-08 on the volatility and correlation dynamics of the eight stock markets. As seen in Table 2.1 we break the whole sample into three subsamples and rerun all the models under the same specifications. The first subsample (A) starts from our first observation of 1988 and ends on the break date near the AFC. This is the pre-AFC period. The second subsample (B) starts from the AFC and ends on our last observation of 2010. This is called the post-AFC period, which also includes the current crisis. Finally, the third subsample (C) starts from the AFC break point and ends on the GFC break. This is the period between the two crises.

Table 2.1: Break dates and subsamples

Panel A: Break dates

	1st break	2nd break
CAC-DAX	17/03/1997	15/01/2008
CAC-FTSE	17/03/1997	24/07/2007
DAX-FTSE	21/07/1997	24/07/2007
HS-NIKKEI	24/10/2001	27/07/2007
HS-STRAITS	28/08/1997	26/07/2007
NIKKEI-STRAITS	28/08/1997	26/07/2007
SP-TSE	27/03/1997	15/01/2008
ASIA	28/08/1997	26/07/2007
EUROPE	17/03/1997	24/07/2007

Panel B: Subsamples

	subsample A	subsample B	subsample C
CAC-DAX	01/01/1988 - 17/03/1997	18/03/1997 - 30/06/2010	18/03/1997 - 15/01/2008
CAC-FTSE	01/01/1988 - 17/03/1997	18/03/1997 - 30/06/2010	18/03/1997 - 24/07/2007
DAX-FTSE	01/01/1988 - 21/07/1997	22/07/1997 - 30/06/2010	22/07/1997 - 24/07/2007
HS-NIKKEI	01/01/1988 - 24/10/2001	25/10/2001 - 30/06/2010	25/10/2001 - 27/07/2007
HS-STRAITS	01/01/1988 - 28/08/1997	29/08/1997 - 30/06/2010	29/08/1997 - 26/07/2007
NIKKEI-STRAITS	01/01/1988 - 28/08/1997	29/08/1997 - 30/06/2010	29/08/1997 - 26/07/2007
SP-TSE	01/01/1988 - 27/03/1997	28/03/1997 - 30/06/2010	28/03/1997 - 15/01/2008
ASIA	01/01/1988 - 28/08/1997	29/08/1997 - 30/06/2010	29/08/1997 - 26/07/2007
EUROPE	01/01/1988 - 17/03/1997	18/03/1997 - 30/06/2010	18/03/1997 - 24/07/2007

2.4 Empirical analysis

2.4.1 Data

Daily stock price index data for eight countries were sourced from the Datastream database for the period 1st January 1988 to 30th June 2010, giving a total of $5,869$ observations. The eight countries and their respective price indices are: UK: FTSE 100 (FTSE), US: S&P 500 (SP), Germany: DAX 30 (DAX), France: CAC 40 (CAC), Japan: Nikkei 225 (NIKKEI), Singapore: Straits Times (STRAITS), Hong Kong: Hang Seng (HS) and Canada: TSE 300 (TSE). We selected the most representative indices for the European, Asian and American stock markets. Our sample is large enough to include various crisis events like the Asian (1997), the Russian (1998) and the recent Global crisis, which is still an on-going process beginning from 2007. For each national index, the continuously compounded return was estimated as $r_t = (\log p_t - \log p_{t-1}) \times 100$ where p_t is the price on day t.

The descriptive statistics of each return series and the series correlations pairwise are reported in Table 2.2. The mean of all returns is positive except for NIKKEI. The Asian returns show greater standard deviation on average than the European and the American. FTSE from Europe and the two American series have the lowest values of unconditional volatility, between 44% and 49%. HS and NIKKEI exhibit the highest volatility, 73% and 64%, respectively and DAX follows with 62%. CAC and STRAITS volatility is calculated in the middle, 59% and 57%, respectively. It is obvious that the normality hypothesis for our daily returns is rejected. All series exhibit skewness with negative values of the relevant measure, indicating that the data are skewed left (long left tail) and excess kurtosis, far above the benchmark of 3 of the normality case, which means a more 'peaked' data distribution (leptokurtosis). The higher correlations are computed for the European returns (CAC-DAX-FTSE) and the American pair (SP-TSE). Moreover, the American variables correlation to the Asian variables is lower than their correlation to the European. See in the Appendix 2A the graphs of each return series.

Table 2.2: Descriptive Statistics

Panel A: Returns descriptive statistics

	CAC	DAX	FTSE	HS	NIKKEI	STRAITS	SP	TSE
Minimum	-4.1134	-5.9525	-4.0240	-10.649	-5.2598	-4.43287	-4.1126	-4.2509
Maximum	4.6011	4.6893	4.0756	7.4903	5.7477	6.4573	4.7587	4.0695
Mean	0.0092	0.0132	0.0078	0.0160	-0.0062	0.0105	0.0106	0.0094
Median	0.0000	0.0188	0.0028	0.0000	0.0000	0.0000	0.0113	0.0153
Standard deviation	0.5948	0.6240	0.4812	0.7292	0.6403	0.5686	0.4946	0.4351
Skewness	-0.0369	-0.2220	-0.1276	-0.5687	-0.0384	-0.0362	-0.2635	-0.7959
Kurtosis	8.2136	9.3199	9.8214	19.9725	9.2271	12.6490	12.4805	15.2183
Jarque-Bera statistic	6647.14	9813.85	11393	70748.6	9482.4	22764.9	22043.6	37119.9

Panel B: Returns correlations

	CAC	DAX	FTSE	HS	NIKKEI	STRAITS	SP	TSE
CAC	1.0000							
DAX	0.7869	1.0000						
FTSE	0.7950	0.7004	1.0000					
HS	0.3110	0.3343	0.3286	1.0000				
NIKKEI	0.2775	0.2591	0.2820	0.4310	1.0000			
STRAITS	0.3203	0.3360	0.3291	0.6251	0.4100	1.0000		
SP	0.4550	0.4674	0.4598	0.1550	0.1136	0.1723	1.0000	
TSE	0.4600	0.4491	0.4785	0.2286	0.1968	0.2261	0.6986	1.0000

2.4.2 Multivariate models

Multivariate GARCH models with time-varying correlations are essential for enhancing our understanding of the relationships between the (co-)volatilities of economic and financial time series. Thus in this Section, within the framework of the multivariate DCC model, we will analyse the dynamic adjustments of the variances and the correlations for the various indices. Overall we estimate seven bivariate specifications: three for the European countries: CAC 40-DAX 30 (CAC-DAX), CAC 40-FTSE 100 (CAC-FTSE) and DAX 30-FTSE 100 (DAX-FTSE); three for the Asian countries: Hang Seng-Nikkei 225 (HS-NIKKEI), Hang Seng-Straits Times (HS-STRAITS) and Nikkei 225-Straits Times (NIKKEI-STRAITS); one for the S&P 500 and TSE 300 indices (SP-TSE). Moreover, we estimate two trivariate models: one for the three European countries (CAC-DAX-FTSE) and one for the three Asian countries (NIKKEI-HS-STRAITS). We have also performed the test of Engle and Sheppard (2001) for DCC against constant conditional correlations in all models. Table 2.3 shows that the CCC hypothesis is always rejected at 100% significance level.

Table 2.3: Engle and Sheppard Test for DCC
E-S Test(j)~$\chi^2(j+1)$ under H_0: CCC model

	E-S Test(12)	p-values
CAC-DAX	375.73	[0.00]
CAC-FTSE	414.24	[0.00]
DAX-FTSE	305.11	[0.00]
HS-NIKKEI	99.54	[0.00]
HS-STRAITS	128.76	[0.00]
NIKKEI-STRAITS	53.43	[0.00]
SP-TSE	56.62	[0.00]
ASIA	211.63	[0.00]
EUROPE	533.58	[0.00]

We estimate the various specifications using the approximate Quasi Maximum Likelihood Estimation (QMLE) method as implemented in the OxMetrics module G@rch 5.0 by Laurent (2007). The existence of outliers, particularly in daily data, causes the distribution of returns to exhibit excess kurtosis (Table 2.2, Panel A with descriptive statistics). To accommodate the presence of such leptokurtosis, we estimate the models using student-t distributed innovations.

2.4.2.1 Bivariate Processes

For the mean equation we choose a VAR(1) process whereas in the variance equation a $(1, d, 1)$ order is chosen for the FIAPARCH formulation with DCC.

Table 2.4 gives the mean equation coefficients estimates. In the majority of the models (nine out of fourteen) the AR(1) coefficients (ϕ_{ii}) are significant at the 10% level or better. The mean equation of diagonal elements of Φ (ϕ_{ij}), which capture the cross effects between the series, are also significant in most of the cases (eight out of the fourteen cases). In the European stock markets we see that DAX is positively affected by the other two European indices while the German index has a negative impact on FTSE. In the Asian markets there is a mixed bidirectional feedback between HS and NIKKEI, where the latter affects the former negatively and the effect in the opposite direction is positive. STRAITS affects both HS and NIKKEI positively, but it is independent of changes from the other two Asian indices. Finally, there is a unidirectional positive feedback from SP to TSE.

Table 2.4: Bivariate AR(1)-DCC-FIAPARCH$(1, d, 1)$ Models
Mean equation

		ϕ_{ii}	ϕ_{ij} $(i \neq j)$
CAC-DAX	CAC	0.02 (0.97)	-0.01 (-0.53)
	DAX	-0.10 $(-5.18)^{***}$	0.11 $(6.32)^{***}$
CAC-FTSE	CAC	-0.01 (-0.74)	0.03 (1.27)
	FTSE	0.02 (0.85)	-0.01 (-0.88)
DAX-FTSE	DAX	-0.07 $(-4.45)^{***}$	0.11 $(5.40)^{***}$
	FTSE	0.03 $(1.81)^{**}$	-0.02 $(-1.84)^{**}$
HS-NIKKEI	HS	0.04 $(2.50)^{***}$	-0.02 $(-1.93)^{***}$
	NIKKEI	-0.03 $(-2.53)^{***}$	0.03 $(3.15)^{***}$
HS-STRAITS	HS	0.01 (0.53)	0.06 $(3.24)^{***}$
	STRAITS	0.08 $(4.83)^{***}$	0.02 (1.28)
NIKKEI-STRAITS	NIKKEI	-0.04 $(-2.84)^{***}$	0.07 $(5.03)^{***}$
	STRAITS	0.08 $(5.66)^{***}$	-0.003 (-0.31)
SP-TSE	SP	-0.02 (-1.34)	0.01 (0.35)
	TSE	0.06 $(3.63)^{***}$	0.07 $(5.57)^{***}$

Notes: The numbers in parentheses are t-statistics.
***, **, * denote significance at the $0.05, 0.10, 0.15$ level respectively.

Table 2.5 summarises the variance equation results. In all cases the fractional differencing parameter (d_i), the power term parameter (δ_i) and the asymmetry parameter (γ_i) are highly significant. The estimates for the two GARCH parameters (β_i, c_i) are also significant except for one case. The fractional parameters are very similar in the three European models with values between 0.36 and 0.43, while in the Asian models we get similar but slightly lower values of long-range volatility dependence ($0.30 - 0.38$). The SP-TSE process generated significant estimates (0.37 and 0.41), similar to the other six models. The power terms are also similar, with the values from the Asian pairs being higher than in the other four bivariate formulations. The three Asian processes gave powers between 1.58 to 1.89, while in the rest of the models we obtained power terms between 1.47 to 1.66. It is worth mentioning that STRAITS exhibits the highest power terms (1.89 and 1.81) and the lowest degree of (long memory) persistence (0.30 and 0.35) in the two bivariate formulations that are included (NIKKEI-STRAITS and HS-STRAITS, respectively). Finally, the asymmetric response of volatility to positive and negative shocks is strong in all cases. The value of the corresponding parameter γ_i is between 0.18 (STRAITS) and 0.56 (SP).

Table 2.5: Bivariate AR(1)-DCC-FIAPARCH$(1, d, 1)$ Models - Variance equation

		β_i	c_i	γ_i	δ_i	d_i
CAC-DAX	CAC	0.61 (13.26)***	0.26 (10.31)***	0.41 (6.02)***	1.52 (17.54)***	0.41 (8.33)***
	DAX	0.57 (10.70)***	0.24 (7.64)***	0.31 (5.10)***	1.64 (19.70)***	0.39 (8.64)***
CAC-FTSE	CAC	0.59 (11.62)***	0.26 (8.99)***	0.40 (5.62)***	1.63 (18.64)***	0.36 (7.78)***
	FTSE	0.63 (16.71)***	0.28 (10.80)***	0.38 (5.88)***	1.55 (17.60)***	0.41 (10.43)***
DAX-FTSE	DAX	0.55 (9.27)***	0.20 (5.73)***	0.31 (5.25)***	1.62 (17.97)***	0.39 (8.83)***
	FTSE	0.62 (14.75)***	0.24 (8.89)***	0.40 (6.20)***	1.47 (16.27)***	0.43 (10.58)***
HS-NIKKEI	HS	0.54 (7.21)***	0.23 (4.86)***	0.31 (4.61)***	1.58 (19.68)***	0.38 (7.76)***
	NIKKEI	0.54 (9.06)***	0.19 (5.05)***	0.47 (4.47)***	1.70 (15.33)***	0.38 (7.21)***
HS-STRAITS	HS	0.53 (7.36)***	0.26 (5.46)***	0.31 (4.75)***	1.58 (21.15)***	0.35 (7.63)***
	STRAITS	0.46 (6.25)***	0.22 (4.19)***	0.18 (4.66)***	1.81 (19.92)***	0.35 (7.80)***
NIKKEI-STRAITS	NIKKEI	0.50 (7.68)***	0.19 (4.18)***	0.48 (4.46)***	1.75 (15.25)***	0.36 (7.23)***
	STRAITS	0.27 (1.94)***	0.09 (0.72)	0.20 (4.67)***	1.89 (19.52)***	0.30 (7.74)***
SP-TSE	SP	0.59 (10.40)***	0.27 (8.54)***	0.56 (5.60)***	1.52 (16.78)***	0.37 (6.99)***
	TSE	0.57 (10.29)***	0.24 (6.16)***	0.23 (4.52)***	1.66 (21.62)***	0.41 (10.40)***

Notes: See Notes in Table 2.4

The unconditional correlation coefficient ρ_{ij} is highly significant in most cases (five out of the seven cases, see the first column of Table 2.6). CAC-FTSE and DAX-FTSE generated insignificant coefficients. It is interesting that CAC-FTSE also gave insignificant cross effects in the mean of returns (see Table 2.4). Amongst the other models, SP-TSE gave the highest unconditional correlation parameter, 0.64, whereas the lowest significant value is obtained from NIKKEI-STRAITS, that is 0.30. The DCC parameters a and b are also highly significant, indicating a considerable time-varying comovement. The persistence of the conditional correlations, measured by the sum of a and b, is always high and close to unity, that is between 0.9661 and 0.9999. b is always above 0.90 and a is below 0.05, revealing slight response to innovations and major persistence.

Table 2.6: BivariateAR(1)-DCC-FIAPARCH$(1, d, 1)$ Models
Equation for Quasi Correlations

	ρ_{ij}	a	b
CAC-DAX	0.42 (1.60)*	0.0159 (3.52)***	0.9840 (213.3)***
CAC-FTSE	0.25 (1.25)	0.0241 (2.81)***	0.9758 (112.1)***
DAX-FTSE	0.26 (1.37)	0.0228 (2.79)***	0.9771 (117.4)***
HS-NIKKEI	0.37 (5.09)***	0.0119 (2.14)***	0.9861 (134.9)***
HS-STRAITS	0.52 (20.95)***	0.0523 (5.09)***	0.9138 (43.00)***
NIKKEI-STRAITS	0.30 (4.41)***	0.0117 (1.56)*	0.9860 (92.24)***
SP-TSE	0.64 (28.31)***	0.0261 (3.59)***	0.9589 (61.74)***

Notes: See Notes in Table 2.4

The degrees of freedom (υ) parameters are highly significant and fluctuate around 7 for the Asian and American models and around 9 for the European processes. In the majority of the cases the hypothesis of uncorrelated standardised and squared standardised residuals is well supported (see the last two columns of Table 2.7).

Table 2.7: Bivariate AR(1)-DCC-FIAPARCH$(1, d, 1)$ Models
Degrees of freedom - Ljung-Box test statistics

		υ	Q_{12}	Q^2_{12}
CAC-DAX	CAC	8.03 (12.61)***	13.93 [0.31]	34.99 [0.00]
	DAX		21.37 [0.05]	15.06 [0.24]
CAC-FTSE	CAC	9.63 (11.41)***	18.64 [0.10]	11.23 [0.51]
	FTSE		12.11 [0.44]	19.94 [0.07]
DAX-FTSE	DAX	9.20 (11.50)***	19.74 [0.07]	8.40 [0.75]
	FTSE		11.03 [0.53]	24.94 [0.02]
HS-NIKKEI	HS	7.02 (15.22)***	32.52 [0.00]	57.59 [0.00]
	NIKKEI		11.67 [0.47]	7.15 [0.85]
HS-STRAITS	HS	6.23 (15.75)***	21.39 [0.04]	76.45 [0.00]
	STRAITS		16.69 [0.16]	1.58 [1.00]
NIKKEI-STRAITS	NIKKEI	6.92 (14.87)***	8.33 [0.76]	10.46 [0.58]
	STRAITS		23.24 [0.03]	1.56 [1.00]
SP-TSE	SP	7.33 (14.31)***	34.23 [0.00]	8.75 [0.72]
	TSE		19.33 [0.08]	5.21 [0.95]

Notes: The numbers in parentheses are t-statistics.
The numbers in brackets are p-values.
***, **, * denote significance at the $0.05, 0.10, 0.15$ level respectively.

Next, the Wald testing procedure applied on the estimated models provides support for the consideration of long memory and power features in our modelling. We examine the Wald statistics for the linear constraints $d_i\,'s = 0$ (stable APARCH) and $d_i\,'s = 1$ (IAPARCH). As seen in Panel A of Table 2.8, the Wald tests clearly reject both the stable and the integrated null hypotheses against the FIAPARCH one. We also test whether the estimated power terms are significantly different from unity or two using Wald tests. All the estimated power coefficients are significantly different from either unity or two (see Table 2.8, Panel B). We observe in all cases higher Wald statistics for the $d_i\,'s = 0$ and the $\delta_i\,'s = 1$ hypotheses in comparison with their alternatives: $d_i\,'s = 1$ and $\delta_i\,'s = 2$, which means that the former hypotheses are more 'rejectable' than the latter ones.

Table 2.8: Wald tests - $\chi^2(1)$ - Bivariate models

Panel A: Tests for restrictions on fractional differencing parameters

H_0	$d_i' s$	$d_i\,'s = 0$	$d_i\,'s = 1$
CAC-DAX	0.41 {0.05} - 0.39 {0.04}	81.17 [0.00]	5.65 [0.02]
CAC-FTSE	0.36 {0.05} - 0.41 {0.04}	95.13 [0.00]	8.75 [0.00]
DAX-FTSE	0.39 {0.04} - 0.43 {0.04}	120.75 [0.00]	5.38 [0.02]
HS-NIKKEI	0.38 {0.05} - 0.38 {0.05}	98.47 [0.00]	9.25 [0.00]
HS-STRAITS	0.35 {0.05} - 0.35 {0.04}	93.60 [0.00]	17.17 [0.00]
NIKKEI-STRAITS	0.36 {0.05} - 0.30 {0.04}	100.74 [0.00]	27.10 [0.00]
SP-TSE	0.37 {0.05} - 0.41 {0.04}	100.95 [0.00]	8.58 [0.00]

Panel B: Tests for restrictions on power term parameters

H_0	$\delta_i\,'s$	$\delta_i\,'s = 1$	$\delta_i\,'s = 2$
CAC-DAX	1.52 {0.09} - 1.64 {0.08}	199.12 [0.00]	57.29 [0.00]
CAC-FTSE	1.63 {0.09} - 1.55 {0.09}	203.89 [0.00]	59.48 [0.00]
DAX-FTSE	1.62 {0.09} - 1.47 {0.09}	199.66 [0.00]	54.28 [0.00]
HS-NIKKEI	1.58 {0.08} - 1.70 {0.11}	260.08 [0.00]	82.24 [0.00]
HS-STRAITS	1.58 {0.07} - 1.81 {0.09}	348.13 [0.00]	118.02 [0.00]
NIKKEI-STRAITS	1.75 {0.11} - 1.89 {0.10}	285.20 [0.00]	109.83 [0.00]
SP-TSE	1.52 {0.09} - 1.66 {0.08}	241.94 [0.00]	70.86 [0.00]

Notes: For each of the seven pairs of indices, Table 2.8 reports the values of the Wald statistics of the unrestricted bivariate DCC-FIAPARCH$(1, d, 1)$ and the restricted ($d_i = 0, 1$; $\delta = 1, 2$) models respectively.
The numbers in curly brackets are standard errors.
The numbers in square brackets are p-values.

2.4.2.2 Trivariate processes

Table 2.9 reports the parameters of interest for the two trivariate AR(1)-DCC-FIAPARCH$(1, d, 1)$ models for the three European and the three Asian indices. The cross effects in the mean equation are similar to the bivariate results. DAX is positively affected by both CAC and FTSE as in the bivariate processes, while FTSE is independent of changes from the other two markets in the trivariate model. In the trivariate model of the Asian countries we obtain the same results for the cross effects as in the bivariate ones. The ARCH and GARCH parameters (β_i, c_i) are highly significant in all cases. The fractional parameters (d_i) are all significant and similar to the ones obtained from the bivariate models. FTSE gives the highest value for d_i amongst the three European series as in the bivariate case and the same stands for NIKKEI (0.40) in the Asian countries. The power terms δ_i are also significant and in accordance with the corresponding results from the bivariate models. The Asian indices give higher power terms on average in comparison with the European indices. The asymmetry parameter γ_i is strong in both models and similar to the bivariate cases. STRAITS again gives the lowest value of d_i (0.32), the highest value of δ_i (1.83) and the lowest value of γ_i (0.16). Both trivariate models generate strong unconditional correlation coefficients ρ_{ij}, which are all highly significant unlike the bivariate cases of the European countries. In Europe the highest unconditional correlation is between CAC and DAX (0.45). The highest correlation between the French and the German financial markets is justified since they are both Continental European markets. FTSE is the Anglo-Saxon market with characteristics that differ traditionally from the Continental European markets because of more advanced financial liberalisation and deregulation. So, the correlation of FTSE to CAC or DAX is found to be lower. In Asia the highest unconditional correlation is between HS and STRAITS (0.50), the same as in the bivariate models. The conditional correlations' persistence is again high (close to unity) and significant in both models. Finally, the degrees of freedom (υ) parameters are highly significant and lower in Asia than in Europe, which also confirms the bivariate results.

Table 2.9: Trivariate AR(1)-DCC-FIAPARCH($1, d, 1$) Models

		EUROPE			ASIA	
	CAC	DAX	FTSE	NIKKEI	HS	STRAITS
ϕ_{ii}	-0.02 (-0.78)	-0.10 $(-5.14)^{***}$	0.03 $(1.52)^{*}$	-0.04 $(-2.98)^{***}$	0.01 (0.68)	0.07 $(4.28)^{***}$
ϕ_{ij}	D: -0.01 (-0.46)	C: 0.08 $(4.03)^{***}$	C: -0.01 (-0.46)	HS: 0.02 $(2.05)^{***}$	N: -0.02 $(-1.99)^{***}$	N: -0.002 (-0.19)
	F: 0.05 $(2.24)^{***}$	F: 0.05 $(2.31)^{***}$	D: -0.01 (-0.93)	S: 0.05 $(3.37)^{***}$	S: 0.05 $(2.82)^{***}$	HS: 0.01 (0.95)
β_i	0.59 $(14.91)^{***}$	0.56 $(10.94)^{***}$	0.62 $(18.64)^{***}$	0.55 $(9.10)^{***}$	0.56 $(8.67)^{***}$	0.45 $(6.15)^{***}$
c_i	0.29 $(11.37)^{***}$	0.26 $(7.75)^{***}$	0.29 $(11.83)^{***}$	0.19 $(5.05)^{***}$	0.27 $(6.54)^{***}$	0.23 $(4.14)^{***}$
γ_i	0.35 $(6.04)^{***}$	0.25 $(4.73)^{***}$	0.38 $(5.73)^{***}$	0.41 $(4.91)^{***}$	0.25 $(4.26)^{***}$	0.16 $(4.05)^{***}$
δ_i	1.59 $(19.86)^{***}$	1.70 $(20.68)^{***}$	1.52 $(16.72)^{***}$	1.77 $(16.48)^{***}$	1.61 $(21.04)^{***}$	1.83 $(20.25)^{***}$
d_i	0.35 $(10.21)^{***}$	0.35 $(9.72)^{***}$	0.38 $(12.12)^{***}$	0.40 $(7.61)^{***}$	0.36 $(7.89)^{***}$	0.32 $(7.67)^{***}$
ρ_{ij}	$C{-}D$: 0.45 $(2.65)^{***}$	$C{-}F$: 0.27 $(1.74)^{**}$	$D{-}F$: 0.33 $(2.38)^{***}$	$HS{-}N$: 0.38 $(13.68)^{***}$	$N{-}S$: 0.33 $(11.43)^{***}$	$S{-}HS$: 0.50 $(18.50)^{***}$
a		0.0129 $(5.61)^{***}$			0.0326 $(2.89)^{***}$	
b		0.9870 $(425.2)^{***}$			0.9449 $(36.48)^{***}$	
υ		8.57 $(15.42)^{***}$			7.42 $(17.09)^{***}$	
Q_{12}	15.89 [0.20]	20.01 [0.07]	12.96 [0.37]	9.55 [0.66]	26.81 [0.01]	19.40 [0.08]
Q_{12}^2	46.76 [0.00]	23.57 [0.02]	24.78 [0.02]	9.82 [0.63]	95.19 [0.00]	1.55 [1.00]

Notes: See Notes in Table 2.7

Next, again we examine the Wald statistics for the linear constraints $d_i\,'s = 0$ (stable APARCH) and $d_i\,'s = 1$ (IAPARCH). As seen in Table 2.10 the Wald tests reject the stable null hypothesis but not the integrated one, unlike the bivariate results, where both hypotheses are rejected against the FIAPARCH one. Regarding the Wald tests of the power terms, all the estimated power coefficients are significantly different from either unity or two as in the bivariate models.

Table 2.10: Wald tests - $\chi^2(1)$ - Trivariate models

H_0	EUROPE	ASIA
$d_i's$	0.35{0.03}-0.35{0.04}-0.38{0.03}	0.40{0.05}-0.36{0.05}-0.32{0.04}
$d_i\ 's = 0$	157.89 [0.00]	137.46 [0.00]
$d_i\ 's = 1$	0.92 [0.34]	0.73 [0.39]
$\delta's$	1.59{0.08}-1.70{0.08}-1.52{0.09}	1.77{0.11}-1.61{0.08}-1.83{0.09}
$\delta_i\ 's = 1$	358.65 [0.00]	593.48 [0.00]
$\delta_i\ 's = 2$	194.97 [0.00]	345.16 [0.00]

Notes: The numbers in curly brackets are standard errors.

The numbers in square brackets are p-values.

2.4.3 Subsamples

2.4.3.1 Bivariate processes

All bivariate models run for the whole sample period are re-estimated for each subsample period under the same specification, that is the AR(1)-DCC-FIAPARCH$(1, d, 1)$ with student-t distributed errors. Only the model for SP-TSE did not converge for subsamples B and C. The leverage parameter γ_i is significant in most models in the three subsamples and the estimated values are similar to those for the whole sample (see in the Appendix 2C, Tables 2C.3-2C.5).

The fractional parameter results in Table 2.11 show that all estimates are significant except for one. In most cases the subsample models' values of d_i fluctuate around the respective value of the original model (for the whole sample). We cannot conclude on a certain direction of this fluctuation. The degree of the series' long memory 'persistence' across the different subperiods remains at the same level for the majority of the models. Table 2.12 reports the Wald statistics for the linear constraints $d_i\ 's = 0$ and $d_i\ 's = 1$ across the sub-periods. Both hypotheses are rejected against the FIAPARCH in most cases.

Table 2.11: Bivariate AR(1)-DCC–FIAPARCH(1, d, 1) Models
Variance equation: Fractional parameter d_i

	whole sample		subsample A		subsample B		subsample C	
	cac	dax	cac	dax	cac	dax	cac	dax
CAC-DAX	0.41 (8.33)***	0.39 (8.64)***	0.29 (3.76)***	0.43 (3.64)***	0.42 (5.17)***	0.39 (5.78)***	0.29 (3.61)***	0.29 (5.06)***
	cac	ftse	cac	ftse	cac	ftse	cac	ftse
CAC-FTSE	0.36 (7.78)***	0.41 (10.43)***	0.40 (4.46)***	0.45 (2.68)***	0.31 (6.19)***	0.36 (9.01)***	0.37 (5.66)***	0.37 (8.30)***
	dax	ftse	dax	ftse	dax	ftse	dax	ftse
DAX-FTSE	0.39 (8.83)***	0.43 (10.58)***	0.40 (5.11)***	0.31 (3.80)***	0.35 (6.97)***	0.39 (8.76)***	0.40 (6.51)***	0.39 (7.93)***
	hs	nikkei	hs	nikkei	hs	nikkei	hs	nikkei
HS-NIKKEI	0.38 (7.76)***	0.38 (7.21)***	0.39 (5.29)***	0.34 (4.90)***	0.41 (5.32)***	0.43 (5.61)***	0.48 (2.43)***	0.39 (2.64)***
	hs	straits	hs	straits	hs	straits	hs	straits
HS-STRAITS	0.35 (7.63)***	0.35 (7.80)***	0.27 (5.54)***	0.05 (1.02)	0.19 (2.80)***	0.29 (4.10)***	0.19 (2.77)***	0.31 (4.64)***
	nikkei	straits	nikkei	straits	nikkei	straits	nikkei	straits
NIKKEI-STRAITS	0.36 (7.23)***	0.30 (7.74)***	0.34 (3.78)***	0.22 (7.27)***	0.37 (6.44)***	0.32 (5.44)***	0.28 (4.55)***	0.26 (3.96)***
	sp	tse	sp	tse	sp	tse	sp	tse
SP-TSE	0.37 (6.99)***	0.41 (10.40)***	0.32 (5.01)***	0.12 (5.06)***	—	—	—	—

Notes: See Notes in Table 2.4

Table 2.12: Tests for restrictions on fractional differencing parameters - Wald tests - $\chi^2(1)$ - Bivariate models

	whole sample		subsample A	
H_0	d_i 's= 0	d_i 's= 1	d_i 's= 0	d_i 's= 1
C-D	81.17 [0.00]	5.65 [0.02]	22.53 [0.00]	3.46 [0.06]
C-F	95.13 [0.00]	8.75 [0.00]	14.72 [0.00]	0.50 [0.48]
D-F	120.75 [0.00]	5.38 [0.02]	35.58 [0.00]	5.88 [0.02]
HS-N	98.47 [0.00]	9.25 [0.00]	49.17 [0.00]	7.23 [0.01]
HS-S	93.60 [0.00]	17.17 [0.00]	21.81 [0.00]	105.70 [0.00]
N-S	100.74 [0.00]	27.10 [0.00]	35.87 [0.00]	22.39 [0.00]
SP-T	100.95 [0.00]	8.58 [0.00]	41.01 [0.00]	64.98 [0.00]
	subsample B		subsample C	
H_0	d_i 's= 0	d_i 's= 1	d_i 's= 0	d_i 's= 1
C-D	31.85 [0.00]	1.62 [0.20]	20.97 [0.00]	11.21 [0.00]
C-F	64.91 [0.00]	15.60 [0.00]	57.71 [0.00]	7.66 [0.01]
D-F	78.46 [0.00]	9.71 [0.00]	69.90 [0.00]	4.68 [0.03]
HS-N	48.55 [0.00]	1.76 [0.18]	11.09 [0.00]	0.23 [0.63]
HS-S	15.11 [0.00]	17.39 [0.00]	22.86 [0.00]	24.01 [0.00]
N-S	59.74 [0.00]	12.41 [0.00]	30.79 [0.00]	22.38 [0.00]
SP-T	—	—	—	—

Notes: The numbers in brackets are p-values.

The power term parameter δ_i is highly significant across all subsamples' estimates (see Table 2.13). As in the case of the fractional parameter, the power terms for the sub-periods' models also fluctuate around the level of the value in the corresponding model for the entire period. Interestingly, for most cases the power term estimates of the period between the two crises (subsample C) are higher than the estimates in the other two subsamples (A and B) and the whole sample's values. The Wald tests (Table 2.14) show that δ_i is significantly different from either unity or two for all the cases across the three subsamples.

Table 2.13: Bivariate AR(1)-DCC-FIAPARCH(1, d, 1) Models
Variance equation: Power term parameter δ_i

	whole sample		subsample A		subsample B		subsample C	
	cac	dax	cac	dax	cac	dax	cac	dax
CAC-DAX	1.52 (17.54)***	1.64 (19.70)***	1.65 (8.16)***	1.53 (5.81)***	1.51 (13.60)***	1.61 (14.54)***	2.17 (8.13)***	2.14 (8.94)***
	cac	ftse	cac	ftse	cac	ftse	cac	ftse
CAC-FTSE	1.63 (18.64)***	1.55 (17.60)***	1.42 (6.22)***	1.36 (4.28)***	1.63 (14.90)***	1.47 (15.02)***	1.73 (12.65)***	1.48 (10.88)***
	dax	ftse	dax	ftse	dax	ftse	dax	ftse
DAX-FTSE	1.62 (17.97)***	1.47 (16.27)***	1.37 (6.74)***	1.34 (4.21)***	1.62 (14.16)***	1.43 (14.54)***	1.64 (11.08)***	1.55 (11.01)***
	hs	nikkei	hs	nikkei	hs	nikkei	hs	nikkei
HS-NIKKEI	1.58 (19.68)***	1.70 (15.33)***	1.51 (14.84)***	1.91 (10.14)***	1.83 (11.61)***	1.79 (9.74)***	1.81 (3.24)***	2.04 (5.49)***
	hs	straits	hs	straits	hs	straits	hs	straits
HS-STRAITS	1.58 (21.15)***	1.81 (19.92)***	1.39 (16.01)***	2.18 (6.42)***	2.07 (11.30)***	1.98 (14.31)***	2.27 (11.11)***	2.09 (12.68)***
	nikkei	straits	nikkei	straits	nikkei	straits	nikkei	straits
NIKKEI-STRAITS	1.75 (15.25)***	1.89 (19.52)***	2.10 (8.32)***	1.87 (10.93)***	1.85 (9.05)***	1.98 (16.46)***	2.23 (7.09)***	2.05 (13.51)***
	sp	tse	sp	tse	sp	tse	sp	tse
SP-TSE	1.52 (16.78)***	1.66 (21.62)***	1.89 (7.17)***	2.22 (6.34)***	–	–	–	–

Notes: See Notes in Table 2.4

Table 2.14: Tests for restrictions on power term parameters
Wald tests - $\chi^2(1)$ - Bivariate models

	whole sample		subsample A	
H_0	$\delta_i' s = 1$	$\delta_i' s = 2$	$\delta_i' s = 1$	$\delta_i' s = 2$
C-D	199.12 [0.00]	57.29 [0.00]	43.01 [0.00]	12.66 [0.00]
C-F	203.89 [0.00]	59.48 [0.00]	14.11 [0.00]	2.70 [0.10]
D-F	199.66 [0.00]	54.28 [0.00]	19.14 [0.00]	3.32 [0.07]
HS-N	260.08 [0.00]	82.24 [0.00]	126.62 [0.00]	43.41 [0.00]
HS-S	348.13 [0.00]	118.02 [0.00]	47.15 [0.00]	17.59 [0.00]
N-S	285.20 [0.00]	109.83 [0.00]	92.78 [0.00]	40.88 [0.00]
SP-T	241.94 [0.00]	70.86 [0.00]	37.37 [0.00]	17.20 [0.00]
	subsample B		subsample C	
H_0	$\delta_i' s = 1$	$\delta_i' s = 2$	$\delta_i' s = 1$	$\delta_i' s = 2$
C-D	103.78 [0.00]	29.06 [0.00]	49.67 [0.00]	24.19 [0.00]
C-F	129.00 [0.00]	35.45 [0.00]	88.42 [0.00]	26.55 [0.00]
D-F	126.57 [0.00]	33.18 [0.00]	85.87 [0.00]	25.46 [0.00]
HS-N	94.59 [0.00]	36.13 [0.00]	16.13 [0.00]	6.80 [0.01]
HS-S	125.28 [0.00]	56.59 [0.00]	134.03 [0.00]	66.07 [0.00]
N-S	125.11 [0.00]	52.24 [0.00]	77.70 [0.00]	37.53 [0.00]
SP-T	—	—	—	—

Notes: The numbers in brackets are p-values.

The dynamic correlation estimates follow the predictable pattern according to the financial crisis literature. They are always lower before the crisis. After the crisis break they are much higher and remain on a higher level. These findings are depicted on the graphs of the dynamic conditional correlations for each bivariate model presented in the Appendix 2A. It is obvious that the DCCs estimated after the second break for the GFC period are much higher than those after the AFC break, revealing that the recent crisis has caused stronger contagion effects in the market and leads the investors to exhibit more evident herding behaviour. During the GFC the international financial integration is complete in comparison with the AFC in 1997, where the financial liberalisation and deregulation was still in process. As seen in Table 2.15, the correlation coefficient ρ_{ij}, which is significant in most cases, in the pre-AFC period (subsample A) always receives lower values than in the post-AFC period and the period between the two crises (subsamples B and C, respectively). For the majority of the models, we also observe that the ρ_{ij} value of the whole period model approaches mostly the level of the pre-crisis model.

Table 2.15: Bivariate AR(1)-DCC-FIAPARCH$(1, d, 1)$ Models
Unconditional Correlations ρ_{ij}

	whole sample	subsample A	subsample B	subsample C
CAC-DAX	0.42 (1.60)*	0.53 (21.82)***	0.66 (1.98)***	0.68 (1.97)***
CAC-FTSE	0.25 (1.25)	0.24 (1.57)*	0.88 (41.51)***	0.84 (34.38)***
DAX-FTSE	0.26 (1.37)	0.29 (1.39)	0.82 (23.70)***	0.77 (25.75)***
HS-NIKKEI	0.37 (5.09)***	0.30 (12.40)***	0.55 (20.81)***	0.50 (13.73)***
HS-STRAITS	0.52 (20.95)***	0.38 (10.66)***	0.63 (37.52)***	0.57 (22.50)***
NIKKEI-STRAITS	0.30 (4.41)***	0.20 (5.50)***	0.46 (18.75)***	0.22 (2.23)***
SP-TSE	0.64 (28.31)***	0.54 (9.56)***	–	–

Notes: See Notes in Table 2.4

Finally, in the Appendix 2C with all the parameters' estimations we observe that the AR(1) coefficients (ϕ_{ii}) are significant at the 15% level or better for the majority of the models in the subsamples. The cross effects are significant in many cases (see also Panel A in Table 2.18). DAX, as with the whole sample, is affected positively by the other two European indices before the AFC (subsample A) and between the two crises (subsample C). Interestingly, these two effects disappear in subsample B, that is in the period after the AFC until the end of the sample. Similarly, the negative effect of the German index on FTSE disappears in the three subsamples.

For the HS-NIKKEI pair, there is still a mixed bidirectional feedback in the periods after the AFC and in between the two crises. However, the negative effect of NIKKEI on HS disappears in the pre AFC period. In the other two Asian pairs with STRAITS the ϕ_{ij} coefficients indicate a positive effect from STRAITS to HS and NIKKEI for all three subsamples, as with the whole sample. The higher values of the cross effect coefficients in the period with the two crises taking place indicate a more sound market integration in Asia during the turbulent times. For the American pair in the pre-AFC period, as in the whole period, SP affects TSE positively.

2.4.3.2 Trivariate processes

Finally, we re-estimate the two trivariate models, one for the Asian indices and one for the European, for the three subsamples. The Asian model did not converge for the third sub-period and the European for the second one. Our findings are very similar to the ones for the bivariate processes. The fractional parameters and the power terms (Table 2.16, Panels A and B) fluctuate around the values of the whole sample and are always significant. The Wald tests show that δ_i is significantly different from either unity or two and they also reject the $d_i\,'s = 0$ hypothesis, but do not reject the $d_i\,'s = 1$ (see Panels A and B in Table 2.17). The correlation coefficients (Table 2.16, Panel C) are again higher in the post-AFC periods (subsamples B and C) than in the pre-AFC period (subsample A). See also the graphs of the conditional correlations for the two trivariate models in the Appendix 2A. The asymmetric response of volatility to positive and negative shocks is strong in most subsamples' models, with γ_i fluctuating around the respective estimated values of the whole sample (see Tables 2C.6-2C.8 in the Appendix 2C).

Table 2.16: Trivariate AR(1)-DCC-FIAPARCH$(1, d, 1)$ Models

Panel A: Variance equation: Fractional parameter d_i

	EUROPE			ASIA		
	cac	dax	ftse	nikkei	hs	straits
whole sample	0.35 (10.21)***	0.35 (9.72)***	0.38 (12.12)***	0.40 (7.61)***	0.36 (7.89)***	0.32 (7.67)***
subsample A	0.32 (4.61)***	0.41 (4.15)***	0.42 (3.77)***	0.43 (3.06)***	0.26 (3.00)***	0.19 (7.36)***
subsample B	—	—	—	0.36 (6.80)***	0.22 (3.30)***	0.32 (5.90)***
subsample C	0.36 (2.70)***	0.39 (3.28)***	0.34 (3.80)***	—	—	—

Panel B: Variance equation: Power term parameter δ_i

	EUROPE			ASIA		
	cac	dax	ftse	nikkei	hs	straits
whole sample	1.59 (19.86)***	1.70 (20.68)***	1.52 (16.72)***	1.77 (16.48)***	1.61 (21.04)***	1.83 (20.25)***
subsample A	1.72 (9.58)***	1.57 (6.29)***	1.48 (5.47)***	2.19 (6.65)***	1.61 (10.09)***	2.01 (11.95)***
subsample B	—	—	—	1.95 (9.40)***	2.07 (12.72)***	1.94 (17.43)***
subsample C	1.70 (6.98)***	1.63 (6.00)***	1.45 (5.69)***	—	—	—

Panel C: Unconditional Correlations ρ_{ij}

	EUROPE			ASIA		
	cac-dac	cac-ftse	dax-ftse	nikkei-hs	nikkei-straits	hs-straits
whole sample	0.45 (2.65)***	0.27 (1.74)**	0.33 (2.38)***	0.38 (13.68)***	0.33 (11.43)***	0.50 (18.50)***
subsample A	0.54 (19.38)***	0.58 (23.08)***	0.44 (14.94)***	0.22 (6.77)***	0.20 (5.97)***	0.37 (11.29)***
subsample B	—	—	—	0.51 (25.07)***	0.47 (21.22)***	0.62 (36.10)***
subsample C	0.62 (0.84)	0.57 (1.18)	0.54 (1.62)*	—	—	—

Notes: See Notes in Table 2.4

Table 2.17: Wald tests - $\chi^2(1)$ - Trivariate models

Panel A: Tests for restrictions on fractional differencing parameters

	EUROPE		ASIA	
H_0	$d_i{}'s = 0$	$d_i{}'s = 1$	$d_i{}'s = 0$	$d_i{}'s = 1$
whole sample	157.89 [0.00]	0.92 [0.34]	137.46 [0.00]	0.73 [0.39]
subsample A	37.98 [0.00]	0.62 [0.43]	27.92 [0.00]	0.49 [0.48]
subsample B	—	—	53.55 [0.00]	0.65 [0.42]
subsample C	10.76 [0.00]	0.07 [0.79]	—	—

Panel B: Tests for restrictions on power term parameters

	EUROPE		ASIA	
H_0	$\delta's = 1$	$\delta's = 2$	$\delta_i{}'s = 1$	$\delta_i{}'s = 2$
whole sample	358.65 [0.00]	194.97 [0.00]	593.48 [0.00]	345.16 [0.00]
subsample A	74.46 [0.00]	40.16 [0.00]	137.19 [0.00]	86.16 [0.00]
subsample B	—	—	230.53 [0.00]	146.89 [0.00]
subsample C	27.05 [0.00]	14.63 [0.00]	—	—

Notes: The numbers in brackets are p-values.

Regarding the cross effects in the Appendix 2C (see also Table 2.18), DAX, similarly to the whole sample, is positively affected by both CAC and FTSE in the pre-AFC period but only by CAC in the period between the two crises, where the FTSE index affects the French index positively as in the model for the whole sample. In the Asian case, HS positively affects both NIKKEI and STRAITS before the AFC, while STRAITS has a positive impact on the other two indices in the post-AFC period, including also the GFC, as in the whole sample. During this period NIKKEI affects HS negatively, as in the whole sample.

Table 2.18: Cross Effects (ϕ_{ij}, $i \neq j$, coefficients)

Whole Sample	Pre-AFC Period	Post-AFC Period	Subsample C
Panel A: Bivariate Models			
CAC, FTSE$\xrightarrow{+}$DAX	CAC, FTSE$\xrightarrow{+}$DAX	-	CAC, FTSE$\xrightarrow{+}$DAX
DAX$\xrightarrow{-}$FTSE	-	-	-
STRAITS$\xrightarrow{+}$NIKKEI,HS	STRAITS$\xrightarrow{+}$NIKKEI,HS	STRAITS$\xrightarrow{+}$NIKKEI,HS	STRAITS$\xrightarrow{+}$NIKKEI,HS
HS$\overset{+}{\underset{-}{\leftrightarrows}}$NIKKEI	HS$\xrightarrow{+}$NIKKEI	HS$\overset{+}{\underset{-}{\leftrightarrows}}$NIKKEI	HS$\overset{+}{\underset{-}{\leftrightarrows}}$NIKKEI
SP$\xrightarrow{+}$TSE	SP$\xrightarrow{+}$TSE	NC	NC
Panel B: Trivariate Models			
CAC, FTSE$\xrightarrow{+}$DAX	CAC, FTSE$\xrightarrow{+}$DAX	NC	CAC$\xrightarrow{+}$DAX
FTSE$\xrightarrow{+}$CAC	-	NC	FTSE$\xrightarrow{+}$CAC
STRAITS$\xrightarrow{+}$NIKKEI,HS	-	STRAITS$\xrightarrow{+}$NIKKEI,HS	NC
HS$\overset{+}{\underset{-}{\leftrightarrows}}$NIKKEI	HS$\xrightarrow{+}$NIKKEI, STRAITS	NIKKEI$\xrightarrow{-}$HS	NC

Notes: CAC, FTSE$\xrightarrow{+}$DAX: CAC and FTSE affect DAX positively. HS$\overset{+}{\underset{-}{\leftrightarrows}}$NIKKEI: there is a mixed bidirectional feedback between HS and NIKKEI, where the latter affects the former negatively. NC: No Convergence.

2.4.4 Discussion

Our analysis gives strong evidence that conditional volatility is best modelled with the FIAPARCH specification, which combines long memory, leverage effects and power transformations of the conditional variances. These three features augment the traditional GARCH model in a suitable way to adequately fit the volatility process. The Wald tests applied support the particular augmented model and are in line with the results of Conrad et al. (2011). The corresponding parameters are found robust to the structural breaks in the returns' and volatilities' series, since their estimated values in the subsamples are similar to those of the whole sample. The volatility 'persistence', as measured by the long memory parameter d_i, is significant in almost all cases and different from either zero or unity. In the whole sample it hovers around the same level for the eight stock markets, which indicates that a common factor of 'persistence' may affect the markets and due to the financial integration their co-persistence is apparent. The asymmetry parameter γ_i is always significant and positive, meaning a leverage for negative returns. That is, negative shocks have stronger influence on the volatility of returns than the positive shocks of the same level. The power term δ_i allows us to increase the flexibility of our modelling. The power transformation of returns, which is significantly different from one and two, gives the appropriate formulation to model the volatility process. One or more cross effects between the dependent variables in the majority of the multivariate specifications are also significant for the mean of returns and show a time-varying behaviour across the subsamples. Finally, the implementation of the DCC model of Engle (2002a) provides a thorough insight into the time-varying pattern of conditional correlations, which accounts for structural breaks that correspond to major financial crisis events.

2.5 Contagion effect

In order to complete our empirical modelling of the main equity markets during the two crisis periods we perform two contagion tests. We intend to clarify whether the higher correlations observed in the post crisis periods are due to the contagion between the financial markets or their interdependence. Following Forbes and Rigobon (2002), contagion is characterised by the increased spillovers between different markets after a crisis shock in one market and interdependence is their high inter-linkages during all states of the economy. The higher volatilities after a shock result in higher correlation coefficients calculations due to heteroskedasticity and omitted variables. This can mislead the analysis in favour of contagion, while the interdependence is the actual spillover phenomenon. Forbes and Rigobon (2002) proposed an adjustment to the correlation coefficient calculation in order to test it during crisis events. We will use the DCC coefficients generated by (the estimated) Engle's model in order to overcome the limitations of the classic correlations coefficients. Cho and Parhizgari (2008) point out the superiority of the DCCs in comparison to the Forbes and Rigobon (2002) modified coefficients, since Engle's model estimates not only volatility-adjusted correlations but also correlations that consider the time-varying behaviour of the volatility pattern.

Our model's DCCs computed from the multivariate framework (with cross effects in the mean equation and long memory, asymmetries and power transformations in the variance equation) are suitable to test the contagion effect during both crises (AFC and GFC). We perform two contagion tests used broadly in the empirical literature: the t-test in the difference of the means of DCCs across the subsamples to detect the significant increase after crisis episodes (see for example Cho and Parhizgari, 2008) and the DCCs regression analysis with crisis intercept dummies to observe the upward shift of the correlations' mean (see, for example, Chiang et al., 2007). The DCCs from the whole sample's bivariate models are used for both tests. The two crisis breaks (see Table 2.1) are applied to determine the pre- and post-crisis periods of the t-test and to form the dummies for the regressions.

The t-test is calculated for the difference of the dynamic correlations means of each period before and after both crises. Tables 2.19 and 2.20 report the main statistical properties of the correlations for the whole sample and each subsample around the crises, as well as the t-test's p-value for the means' difference. For both crises, we always reject the null hypothesis that the means are equal (two-sided test). We conclude that their difference is statistically significant and their increase after the crisis event denotes sound contagion effects due to the financial shocks of the AFC and the GFC. For the AFC shock, in particular, we also confirm the contagion effect by excluding the GFC period from the post-AFC subsample. It is interesting that the lowest correlation shift after both crises is observed between the US and the Canadian stock indices. We recalculate the t-statistics for shorter periods around the crisis breaks (500 observations before and after each crisis) and again the DCCs mean difference is statistically significant (results not reported due to space considerations). Our empirical results confirm the contagion phenomenon for all the main financial markets under study for both crises using the t-test irrespective of the sample size.

Table 2.19: DCC mean difference t-tests for the Asian Financial Crisis

		C-D	C-F	D-F	HS-N	HS-S	N-S	SP-T
whole sample	mean	0.7277	0.7090	0.6217	0.3972	0.5297	0.3543	0.6505
	median	0.7527	0.7425	0.6494	0.4172	0.5439	0.3655	0.6658
	std dev	0.1877	0.1803	0.2060	0.1478	0.1467	0.1426	0.0975
	N	5867	5867	5867	5867	5867	5867	5867
pre-AFC	mean	0.5538	0.5532	0.4327	0.3220	0.4666	0.2648	0.6198
	median	0.5476	0.5858	0.4367	0.3504	0.4793	0.2615	0.6430
	std dev	0.1222	0.1500	0.1372	0.1317	0.1593	0.1449	0.1126
	N	2400	2400	2490	3602	2518	2518	2408
post AFC	mean	0.8481	0.8168	0.7611	0.5167	0.5772	0.4216	0.6720
	median	0.8818	0.8382	0.7772	0.5228	0.5863	0.4386	0.6863
	std dev	0.1178	0.1048	0.1185	0.0758	0.1157	0.0964	0.0786
	N	3467	3467	3377	2265	3349	3349	3459
post AFC excl. GFC	mean	0.8250	0.7852	0.7224	0.4817	0.5579	0.4008	0.6663
	median	0.8645	0.8060	0.7413	0.4904	0.5596	0.4030	0.6783
	std dev	0.1187	0.0970	0.1065	0.0665	0.1121	0.0939	0.0763
	N	2826	2701	2611	1502	2585	2585	2818
AFC mean difference	increase	0.2943	0.2637	0.3284	0.1947	0.1107	0.1568	0.0522
	(%) increase	53.15	47.67	75.88	60.47	23.72	59.22	8.43
	t-test p value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AFC mean difference excl. GFC	increase	0.2712	0.2320	0.2897	0.1597	0.0914	0.1360	0.0466
	(%) increase	48.97	41.94	66.94	49.59	19.59	51.35	7.51
	t-test, p value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 2.20: DCC mean difference t-tests for the recent Global Financial Crisis								
		C-D	C-F	D-F	HS-N	HS-S	N-S	SP-T
whole	mean	0.7277	0.7090	0.6217	0.3972	0.5297	0.3543	0.6505
sample	median	0.7527	0.7425	0.6494	0.4172	0.5439	0.3655	0.6658
	std dev	0.1877	0.1803	0.2060	0.1478	0.1467	0.1426	0.0975
	N	5867	5867	5867	5867	5867	5867	5867
pre-GFC	mean	0.7004	0.6760	0.5810	0.3690	0.5128	0.3337	0.6449
	median	0.7120	0.7088	0.5902	0.3892	0.5247	0.3432	0.6601
	std dev	0.1809	0.1702	0.1896	0.1372	0.1448	0.1394	0.0976
	N	5226	5101	5101	5104	5103	5103	5226
post GFC	mean	0.9501	0.9285	0.8930	0.5857	0.6424	0.4921	0.6969
	median	0.9530	0.9324	0.8947	0.5866	0.6630	0.4872	0.7182
	std dev	0.0137	0.0223	0.0272	0.0342	0.1032	0.0668	0.0834
	N	641	766	766	763	764	764	641
GFC mean	increase	0.2497	0.2525	0.3120	0.2167	0.1296	0.1584	0.0521
difference	(%) increase	35.64	37.36	53.70	58.73	25.27	47.46	8.07
	t-test, p value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

In the regression analysis we run the DCCs ($\rho_{ij,t}$) on a constant (ψ_0), the two crisis intercept dummies DUM_1 for the AFC and DUM_2 for the GFC (with coefficients ψ_1 and ψ_2, respectively) and the AR(1) lag with the coefficient χ_1 to remove any serial correlation:

$$\rho_{ij,t} = \psi_0 + \psi_1 DUM_1 + \psi_2 DUM_2 + \chi_1 \rho_{ij,t-1} + u_{ij,t}$$

We limit our correlation model to the mean equation without conditional variance estimation, since no ARCH effect is neglected. Table 2.21 presents the regression results. The AR(1) coefficient is always above 0.95, denoting very high correlation persistence. The intercept dummies are always positive and significant confirming the significant correlations' increase, which means contagion effects after both crises. For the SP-TSE pair the GFC dummy is insignificant when both dummies are included, so we run two regressions for each crisis dummy separately. We observe the lowest dummy coefficients with the smallest t-statistic for the US and Canada, which is in accordance with the t-test procedure for the DCCs mean difference. Our dynamic correlations analysis proves that both contagion tests are in favour of contagion rather than simple interdependence after the crisis shocks.

Table 2.21: DCC AR(1) mean equation with crisis dummies

$\rho_{ij,t} = \psi_0 + \psi_1 DUM_1 + \psi_2 DUM_2 + \chi_1 \rho_{ij,t-1} + \varepsilon_{ij,t}$

	ψ_0	ψ_1	ψ_2	χ_1
CAC-DAX	0.0030 $(2.43)^{***}$	0.0015 $(3.03)^{***}$	0.0006 $(2.54)^{***}$	0.9946 $(526.3)^{***}$
CAC-FTSE	0.0046 $(2.84)^{***}$	0.0017 $(2.56)^{***}$	0.0012 $(2.81)^{***}$	0.9921 $(386.0)^{***}$
DAX-FTSE	0.0046 $(3.99)^{***}$	0.0032 $(4.32)^{***}$	0.0017 $(3.79)^{***}$	0.9895 $(453.9)^{***}$
HS-NIKKEI	0.0111 $(7.95)^{***}$	0.0035 $(3.77)^{***}$	0.0025 $(2.22)^{***}$	0.9673 $(282.0)^{***}$
HS-STRAITS	0.0200 $(8.67)^{***}$	0.0041 $(3.72)^{***}$	0.0036 $(2.81)^{***}$	0.9569 $(246.6)^{***}$
NIKKEI-STRAITS	0.0085 $(7.10)^{***}$	0.0029 $(3.06)^{***}$	0.0020 $(1.77)^{**}$	0.9703 $(297.6)^{***}$
SP-TSE	0.0100 $(4.86)^{***}$	0.0008 $(1.83)^{**}$		0.9840 $(324.7)^{***}$
SP-TSE	0.0100 $(4.81)^{***}$		0.0010 $(1.49)^{*}$	0.9846 $(323.9)^{***}$

Notes: See Notes in Table 2.4

2.6 Conclusions

The purpose of the current analysis was to investigate the applicability of the multivariate FIAPARCH model with DCC to eight stock market indices returns, also taking into account the structural breaks corresponding to financial crisis events. The VAR-DCC-FIAPARCH model is proved to capture thoroughly the volatility and correlation processes compared to simpler specifications, like the multivariate GARCH with CCC.

We have provided strong evidence that conditional volatilities are better modelled incorporating long memory, power effects and leverage features. We further prove that time-varying conditional correlations across markets, estimated by the DCC model, are highly persistent and follow a sound upward pattern during financial crises. The cross-border contagion effects depicted on the increasing correlations and the herding behaviour amongst investors as the correlations remain high confirm the existing empirical evidence. We also compare two different crises in terms of correlations to observe higher correlations in the recent Global financial crisis than in the Asian one. The financial liberalisation, deregulation and integration of the markets has led to more apparent market interdependence nowadays. Such a conclusion has major policy implications and a substantial impact on the current risk management practices.

2.7 APPENDIX 2A: Graphs

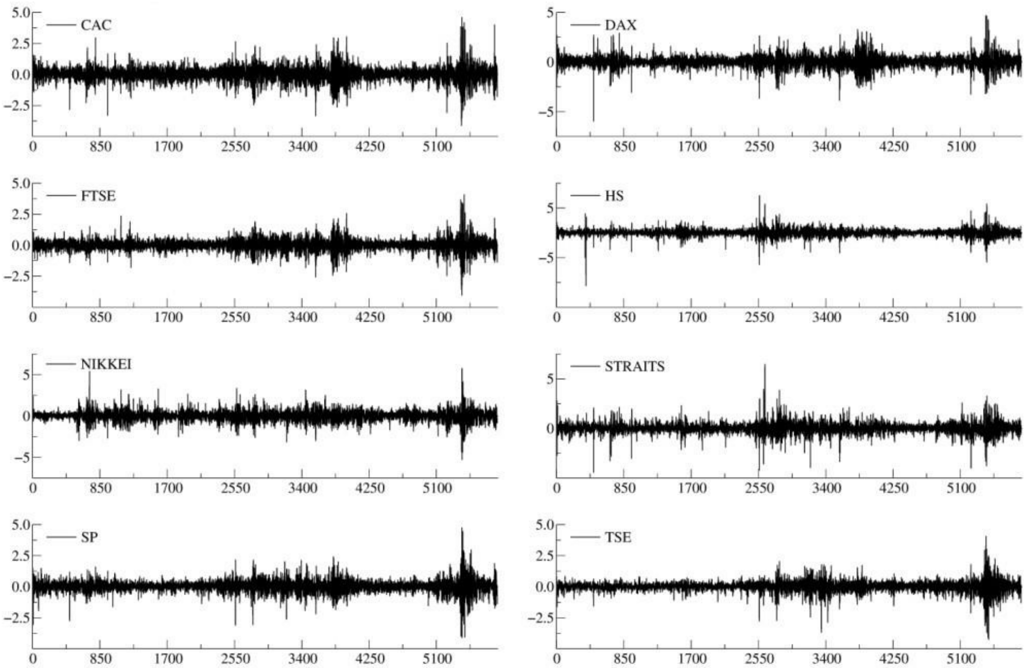
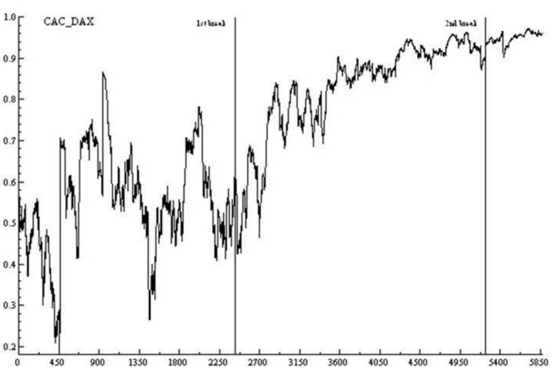
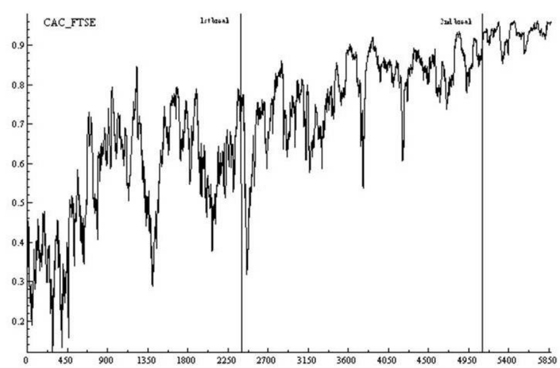


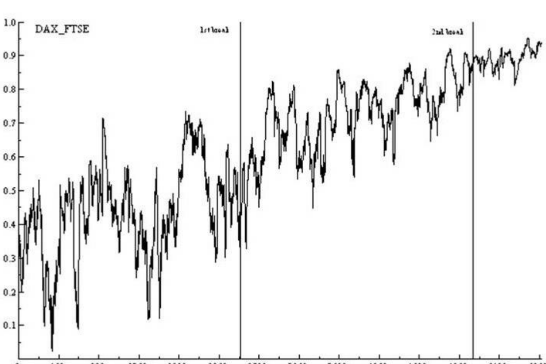
Figure 2A.1: Returns graphs



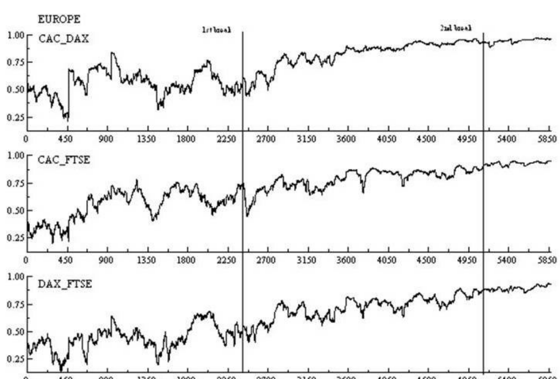
CAC_DAX



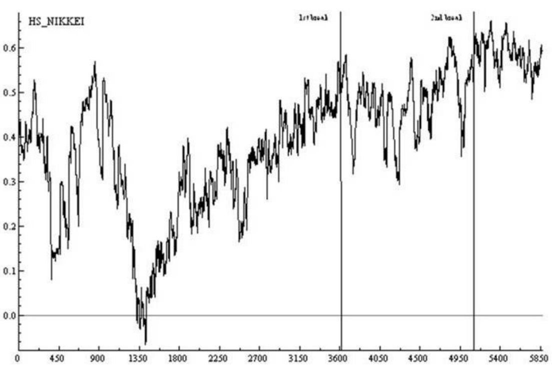
CAC_FTSE



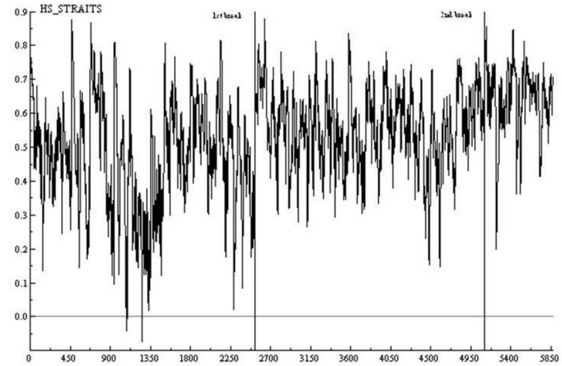
DAX_FTSE



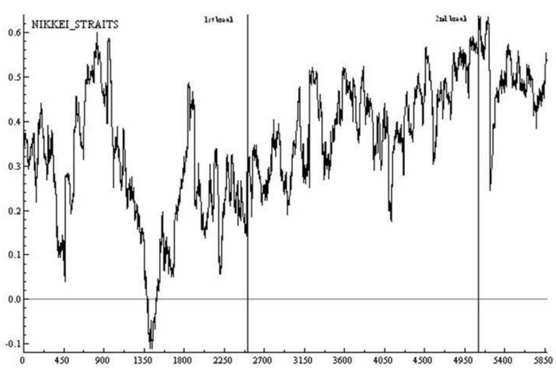
EUROPE



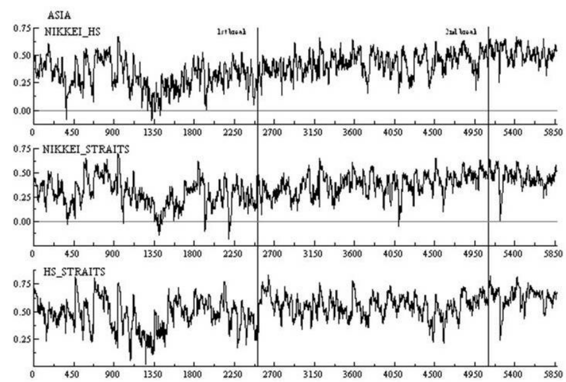
HS_NIKKEI



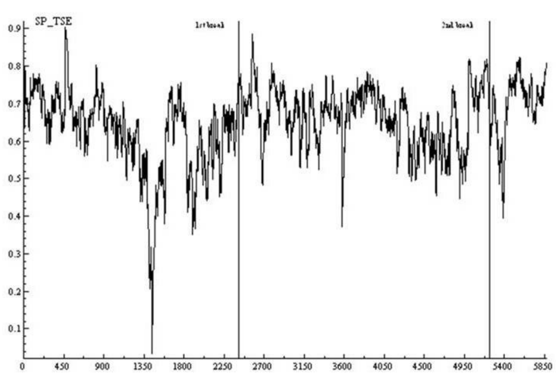
HS_STRAITS



NIKKEI_STRAITS

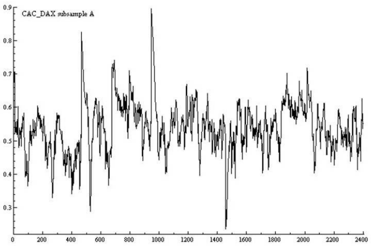


ASIA

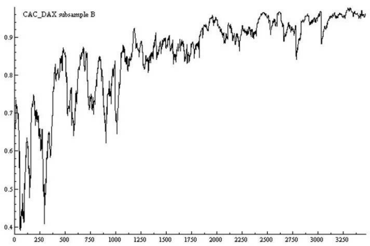


SP_TSE

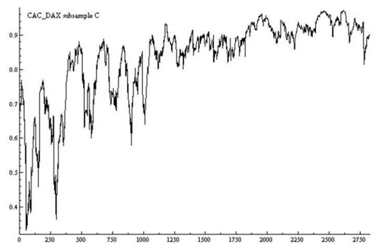
Figure 2A.2: Dynamic conditional correlations graphs whole sample



cac_dax A

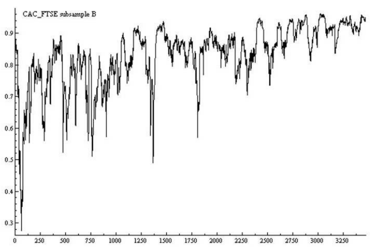


cac_dax B

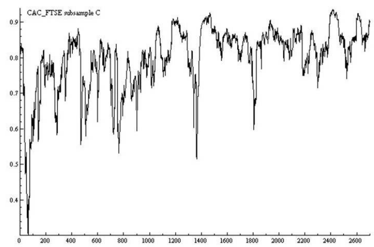


cac_dax C

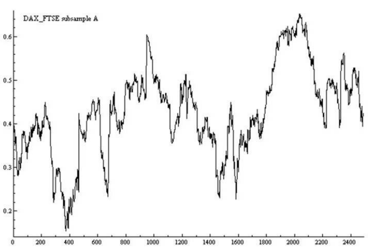
cac_ftse A



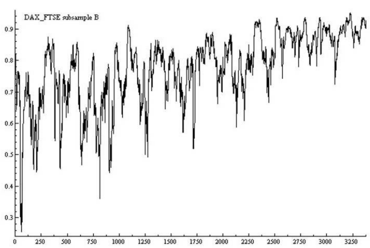
cac_ftse B



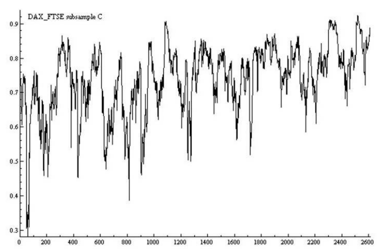
cac_ftse C



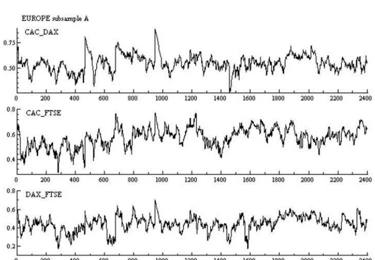
dax_ftse A



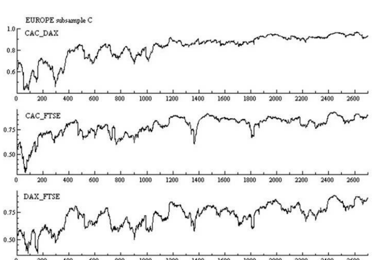
dax_ftse B



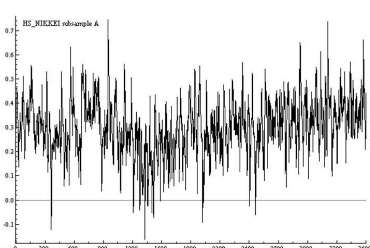
dax_ftse C



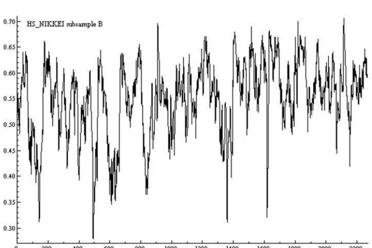
Europe A



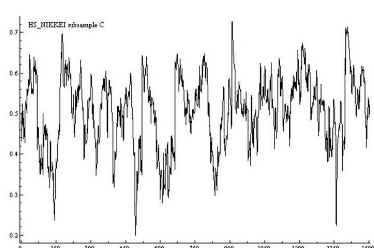
Europe B



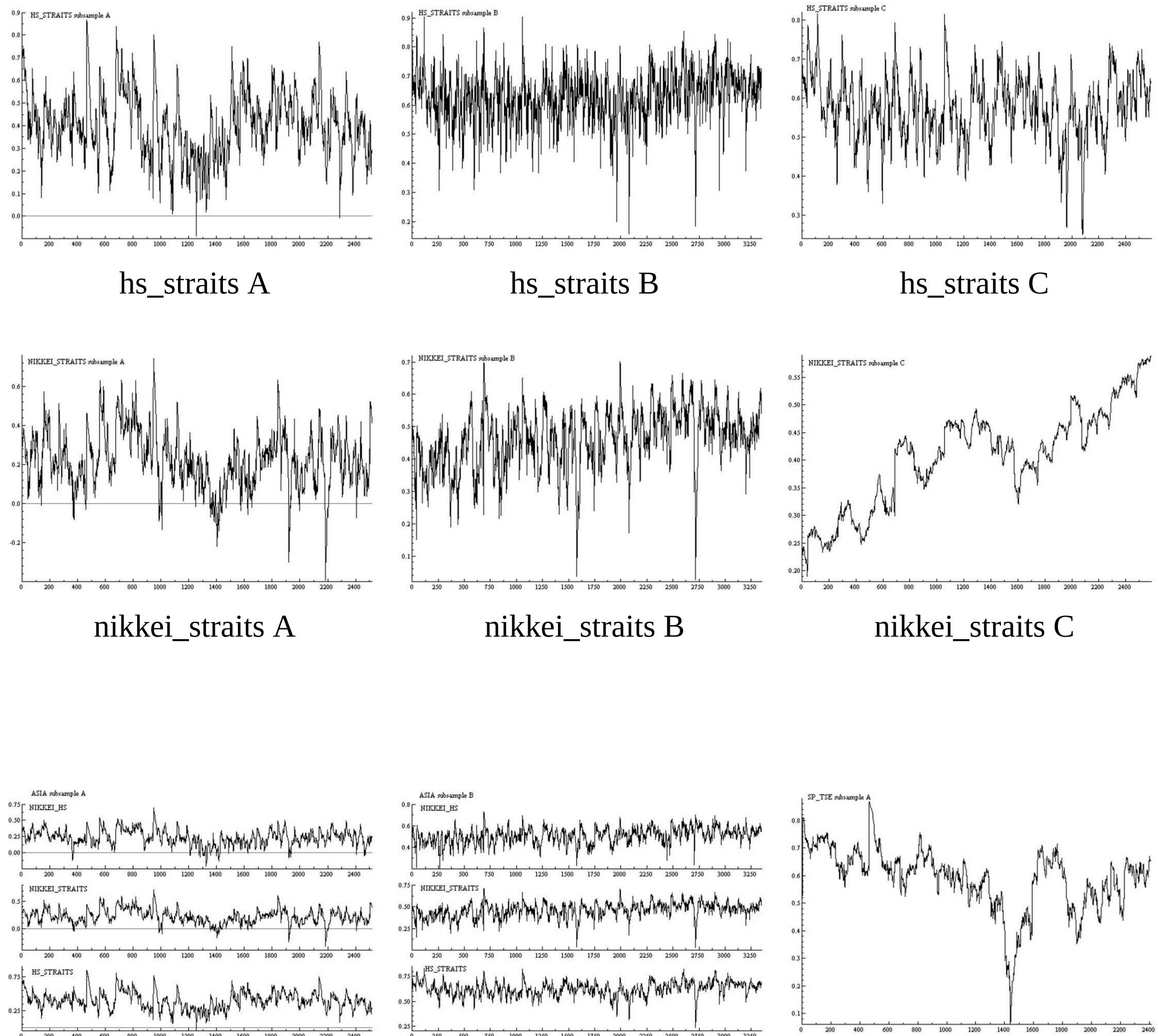
hs_nikkei A



hs_nikkei B



hs_nikkei C



hs_straits A | hs_straits B | hs_straits C

nikkei_straits A | nikkei_straits B | nikkei_straits C

asia A | asia B | sp_tse A

Figure 2A.3: Dynamic conditional correlations graphs subsamples

2.8 APPENDIX 2B: Breaks

Table 2B.1: Break dates

Break	CAC	DAX	FTSE	HS	NIKKEI	STRAITS	SP	TSE
1	17/03/1997	27/08/1991	22/10/1992	24/10/2001	21/02/1990	26/08/1991	27/03/1997	05/11/1996
2	31/07/1998	21/07/1997	13/07/1998	27/07/2007	04/01/2008	28/08/1997	04/09/2008	15/01/2008
3	15/01/2008	17/06/2003	24/07/2007	05/05/2009	03/04/2009	06/06/2000	31/03/2009	02/04/2009
4	03/04/2009	15/01/2008	06/04/2009	01/12/2009		26/07/2007	16/07/2009	19/08/2009
5	27/04/2010	03/04/2009	27/04/2010			28/05/2009	27/04/2010	
6						25/08/2009		
7						28/04/2010		

Table 2B.2: Possible explanations of each identified break

	Breakdate	Major economic event that may be associated with the breakdate
CAC	17/03/1997	Asian financial crisis of 1997
	31/07/1998	Russian financial crisis of 1998
	15/01/2008	Financial crisis of 2007–08
	03/04/2009	European sovereign-debt crisis
	27/04/2010	European sovereign-debt crisis
DAX	27/08/1991	German reunification
	21/07/1997	Asian financial crisis of 1997
	17/06/2003	German Chancellor announces (29/06/2003) a plan to bring forward about €18bn tax cuts
	15/01/2008	Financial crisis of 2007–08
	03/04/2009	European sovereign-debt crisis
FTSE	22/10/1992	The UK withdraws the pound sterling from the European Exchange Rate Mechanism (ERM)
	13/07/1998	Russian financial crisis of 1998
	24/07/2007	Financial crisis of 2007–08
	06/04/2009	European sovereign-debt crisis
	27/04/2010	European sovereign-debt crisis

Table 2B.2 (Continued): Possible explanations of each identified break

	Breakdate	Major economic event that may be associated with the breakdate
HS	24/10/2001	The interest rate caps on demand and savings deposits are removed in July 2001
	27/07/2007	Financial crisis of 2007–08
	05/05/2009	European sovereign-debt crisis
	01/12/2009	European sovereign-debt crisis
NIKKEI	21/02/1990	The Japanese asset price bubble of 1986-1991
	04/01/2008	Financial crisis of 2007–08
	03/04/2009	European sovereign-debt crisis
STRAITS	26/08/1991	
	28/08/1997	Asian financial crisis of 1997
	06/06/2000	
	26/07/2007	Financial crisis of 2007–08
	28/05/2009	European sovereign-debt crisis
	25/08/2009	European sovereign-debt crisis
	28/04/2010	European sovereign-debt crisis
SP	27/03/1997	Asian financial crisis of 1997
	04/09/2008	Financial crisis of 2007–08
	31/03/2009	Stimulus package and FED's quantitative easing
	16/07/2009	European sovereign-debt crisis
	27/04/2010	European sovereign-debt crisis
TSE	05/11/1996	
	15/01/2008	Financial crisis of 2007–08
	02/04/2009	European sovereign-debt crisis
	19/08/2009	European sovereign-debt crisis

Table 2B.3: Returns descriptive statistics for segments

	Break	Mean	Std. Dev.	Skewness	Kurtosis	JB	Obs.
CAC	0	0.0172	0.4680	−0.1940	6.1543	1010.43	2401
	1	0.0579	0.5758	−0.0530	4.4944	33.57	359
	2	0.0040	0.6090	−0.1173	5.8423	836.04	2467
	3	−0.0783	1.0846	0.3216	6.3276	152.20	318
	4	0.0411	0.5730	−0.3648	3.2788	7.04	277
	5	−0.1042	1.0373	1.0082	6.8047	35.54	46
DAX	0	0.0228	0.5885	−0.9455	17.4267	8397.65	952
	1	0.0258	0.3866	−0.2593	4.7893	222.54	1539
	2	−0.0063	0.8239	−0.1423	4.6356	176.96	1541
	3	0.0303	0.4243	−0.2741	3.9821	62.99	1195
	4	−0.0745	1.0370	0.4527	7.0541	228.63	318
	5	0.0414	0.6113	−0.1452	3.5569	5.31	323
FTSE	0	0.0152	0.3787	0.1734	5.6049	360.84	1254
	1	0.0235	0.3239	−0.1352	3.9077	55.76	1492
	2	0.0016	0.4947	−0.1739	5.8543	811.61	2356
	3	−0.0476	0.9041	0.0647	6.6606	248.20	444
	4	0.0533	0.4660	−0.2996	3.3883	5.86	276
	5	−0.1234	0.6864	0.5729	4.5836	7.32	46

Notes: Break 0 covers the period preceding all breaks and so on so forth

Table 2B.3 (Continued): Returns descriptive statistics for segments

	Break	Mean	Std. Dev.	Skewness	Kurtosis	JB	Obs.
HS	0	0.0180	0.7484	−0.9064	23.2083	61800.70	3603
	1	0.0228	0.4359	−0.0875	4.3076	108.93	1502
	2	−0.0298	1.2230	0.1870	6.3446	218.04	462
	3	0.0860	0.7488	−0.0433	2.9288	0.08	150
	4	−0.0270	0.5276	−0.1819	3.0258	0.84	151
NIKKEI	0	0.0393	0.2903	0.5437	11.5241	1716.86	558
	1	−0.0083	0.6177	0.1443	6.4670	2351.10	4662
	2	−0.0693	1.1933	−0.2458	6.8361	202.54	325
	3	0.0094	0.6000	−0.0446	3.3719	1.97	323
STRAITS	0	0.0220	0.5357	−0.8734	13.5488	4530.31	951
	1	0.0132	0.3957	−0.1909	6.4644	793.66	1568
	2	0.0065	0.8843	0.5168	9.7091	1388.15	723
	3	0.0136	0.4517	−0.3967	6.7701	1151.59	1862
	4	−0.0403	0.8820	−0.1018	5.5337	129.22	480
	5	0.0916	0.6268	−0.2433	2.4451	1.43	63
	6	0.0279	0.3721	−0.2435	3.23	2.11	176
	7	−0.0323	0.5077	−0.1169	2.7358	0.23	45
SP	0	0.0206	0.3357	−0.6297	9.6680	4622.07	2409
	1	0.0068	0.4926	−0.0803	5.9488	1084.68	2985
	2	−0.1286	1.5148	0.0923	3.7766	3.93	148
	3	0.0929	0.7025	−0.2396	3.0389	0.74	77
	4	0.0492	0.4087	−0.6348	4.1249	24.34	203
	5	−0.1307	0.7609	0.3152	3.2404	0.87	46
TSE	0	0.0110	0.2373	−0.6003	6.3146	1194.65	2307
	1	0.0127	0.4310	−0.6714	8.4297	3806.26	2920
	2	−0.0526	1.0517	−0.3814	5.6312	99.13	317
	3	0.0718	0.7116	−0.4300	2.9335	3.07	99
	4	0.0107	0.4099	−0.2839	3.5215	5.57	225

Notes: See Notes in Table 2B.3

2.9 APPENDIX 2C: The estimated models in the whole sample and the subsamples

Table 2C.1: Bivariate AR(1)-FIAPARCH($1, d, 1$)-DCC($1, 1$) models

Whole Sample

	CAC-DAX		CAC-FTSE		DAX-FTSE	
	CAC	DAX	CAC	FTSE	DAX	FTSE
c_i	0.019 (3.53)***	0.030 (5.37)***	0.017 (3.02)***	0.016 (3.42)***	0.027 (4.99)***	0.016 (3.62)***
ζ_i	0.019 (0.97)	−0.098 (−5.18)***	−0.014 (−0.74)	0.016 (0.85)	−0.073 (−4.45)***	0.031 (1.81)**
η_i	−0.010 (−0.53)	0.114 (6.32)***	0.031 (1.27)	−0.013 (−0.88)	0.111 (5.40)***	−0.024 (−1.84)**
ω_i	0.022 (4.32)***	0.018 (3.88)***	0.017 (3.66)***	0.013 (3.94)***	0.019 (3.74)***	0.016 (4.21)***
ϕ_i	0.262 (10.31)***	0.242 (7.64)***	0.265 (8.99)***	0.277 (10.80)***	0.199 (5.73)***	0.235 (8.89)***
β_i	0.611 (13.26)***	0.574 (10.70)***	0.586 (11.62)***	0.632 (16.71)***	0.548 (9.27)***	0.615 (14.75)***
d_i	0.405 (8.33)***	0.386 (8.64)***	0.360 (7.78)***	0.408 (10.43)***	0.393 (8.83)***	0.432 (10.58)***
γ_i	0.413 (6.02)***	0.314 (5.10)***	0.403 (5.62)***	0.377 (5.88)***	0.310 (5.25)***	0.404 (6.20)***
δ_i	1.515 (17.54)***	1.642 (19.70)***	1.627 (18.64)***	1.547 (17.60)***	1.616 (17.97)***	1.474 (16.27)***
ρ_{ij}	0.421 (1.60)*		0.251 (1.25)		0.255 (1.37)	
a	0.0159 (3.52)***		0.0241 (2.81)***		0.0228 (2.79)***	
b	0.9840 (213.3)***		0.9758 (112.1)***		0.9771 (117.4)***	
υ	8.035 (12.61)***		9.633 (11.41)***		9.205 (11.50)***	
$Loglik$	−5391.32		−4408.54		−5152.36	
Q_{12}	13.93 [0.31]	21.37 [0.05]	18.64 [0.10]	12.11 [0.44]	19.74 [0.07]	11.03 [0.53]
Q^2_{12}	34.99 [0.00]	15.06 [0.24]	11.23 [0.51]	19.94 [0.07]	8.40 [0.75]	24.94 [0.02]

Notes:

Robust-standard errors are used. Q_{12} and Q^2_{12} are Ljung-Box tests for serial correlation of 12 lags on the standardised and standardised squared residuals, respectively. The numbers in parentheses are t-statistics.

The numbers in brackets are p-values.

$^{***}, ^{**}, ^{*}$ denote significance at the $0.05, 0.10, 0.15$ level respectively.

Table 2C.1 (Continued): Bivariate AR(1)-FIAPARCH($1, d, 1$)-DCC($1, 1$) models

Whole Sample

	HS-NIKKEI		HS-STRAITS		NIKKEI-STRAITS		SP-TSE	
	HS	NIKKEI	HS	STRAITS	NIKKEI	STRAITS	SP	TSE
c_i	0.031 (5.10)***	0.009 (1.63)**	0.027 (4.48)***	0.015 (3.11)***	0.008 (1.30)	0.018 (3.57)***	0.019 (4.41)***	0.017 (5.15)***
ζ_i	0.036 (2.50)***	−0.033 (−2.53)***	0.008 (0.53)	0.077 (4.83)***	−0.038 (−2.84)***	0.082 (5.66)***	−0.021 (−1.34)	0.061 (3.63)***
η_i	−0.025 (−1.93)***	0.034 (3.15)***	0.061 (3.24)***	0.017 (1.28)	0.073 (5.03)***	−0.003 (−0.31)	0.007 (0.35)	0.070 (5.57)***
ω_i	0.030 (3.99)***	0.012 (2.03)***	0.034 (4.26)***	0.018 (4.11)***	0.009 (1.60)*	0.018 (3.12)***	0.014 (3.52)***	0.009 (4.04)***
ϕ_i	0.232 (4.86)***	0.195 (5.05)***	0.256 (5.46)***	0.222 (4.19)***	0.186 (4.18)***	0.085 (0.72)	0.265 (8.54)***	0.241 (6.16)***
β_i	0.540 (7.21)***	0.538 (9.06)***	0.533 (7.36)***	0.459 (6.25)***	0.500 (7.68)***	0.272 (1.94)***	0.592 (10.40)***	0.567 (10.29)***
d_i	0.384 (7.76)***	0.381 (7.21)***	0.353 (7.63)***	0.348 (7.80)***	0.357 (7.23)***	0.302 (7.74)***	0.368 (6.99)***	0.406 (10.40)***
γ_i	0.305 (4.61)***	0.471 (4.47)***	0.314 (4.75)***	0.185 (4.66)***	0.481 (4.46)***	0.196 (4.67)***	0.557 (5.60)***	0.227 (4.52)***
δ_i	1.580 (19.68)***	1.705 (15.33)***	1.582 (21.15)***	1.812 (19.92)***	1.749 (15.25)***	1.887 (19.52)***	1.519 (16.78)***	1.660 (21.62)***
ρ_{ij}	0.369 (5.09)***		0.519 (20.95)***		0.301 (4.41)***		0.644 (28.31)***	
a	0.0119 (2.14)***		0.0523 (5.09)***		0.0117 (1.56)*		0.0261 (3.59)***	
b	0.9861 (134.9)***		0.9138 (43.00)***		0.9860 (92.24)***		0.9589 (61.74)***	
υ	7.018 (15.22)***		6.225 (15.75)***		6.917 (14.87)***		7.329 (14.31)***	
$Loglik$	−9078.62		−7545.80		−7850.93		−2670.31	
Q_{12}	32.52 [0.00]	11.67 [0.47]	21.39 [0.04]	16.69 [0.16]	8.33 [0.76]	23.24 [0.03]	34.23 [0.00]	19.33 [0.08]
Q^2_{12}	57.59 [0.00]	7.15 [0.85]	76.45 [0.00]	1.58 [1.00]	10.46 [0.58]	1.56 [1.00]	8.75 [0.72]	5.21 [0.95]

Notes: See Notes in Table 2C.1

Table 2C.2: Trivariate AR(1)-FIAPARCH$(1, d, 1)$-DCC$(1, 1)$ models

Whole sample

	EUROPE			ASIA		
	CAC	DAX	FTSE	NIKKEI	HS	STRAITS
c_i	0.019 (3.64)***	0.030 (5.47)***	0.017 (3.91)***	0.008 (1.44)*	0.032 (5.28)***	0.019 (3.94)***
ζ_i	−0.017 (−0.78)	−0.102 (−5.14)***	0.029 (1.52)*	−0.040 (−2.98)***	0.010 (0.68)	0.068 (4.28)***
η_i	D: −0.009 (−0.46)	C: 0.084 (4.03)***	C: −0.008 (−0.46)	HS: 0.021 (2.05)***	N: −0.025 (−1.99)***	N: −0.002 (−0.19)
	F: 0.053 (2.24)***	F: 0.053 (2.31)***	D: −0.015 (−0.93)	S: 0.053 (3.37)***	S: 0.054 (2.82)***	HS: 0.013 (0.95)
ω_i	0.021 (4.84)***	0.017 (4.12)***	0.015 (4.40)***	0.010 (1.93)***	0.028 (4.27)***	0.016 (3.77)***
ϕ_i	0.288 (11.37)***	0.256 (7.75)***	0.295 (11.83)***	0.189 (5.05)***	0.274 (6.54)***	0.226 (4.14)***
β_i	0.593 (14.91)***	0.561 (10.94)***	0.624 (18.64)***	0.548 (9.10)***	0.561 (8.67)***	0.449 (6.15)***
d_i	0.351 (10.21)***	0.352 (9.72)***	0.380 (12.12)***	0.397 (7.61)***	0.357 (7.89)***	0.324 (7.67)***
γ_i	0.348 (6.04)***	0.251 (4.73)***	0.382 (5.73)***	0.411 (4.91)***	0.251 (4.26)***	0.156 (4.05)***
δ_i	1.594 (19.86)***	1.696 (20.68)***	1.517 (16.72)***	1.771 (16.48)***	1.608 (21.04)***	1.833 (20.25)***
ρ_{ij}	$C{-}D$: 0.446 (2.65)***	$C{-}F$: 0.273 (1.74)**	$D{-}F$: 0.331 (2.38)***	$HS{-}N$: 0.378 (13.68)***	$N{-}S$: 0.333 (11.43)***	$S{-}HS$: 0.504 (18.50)***
a		0.0129 (5.61)***			0.0326 (2.89)***	
b		0.9870 (425.2)***			0.9449 (36.48)***	
υ		8.571 (15.42)***			7.417 (17.09)***	
$Loglik$		−5400.20			−11496.10	
Q_{12}	15.89 [0.20]	20.01 [0.07]	12.96 [0.37]	9.55 [0.66]	26.81 [0.01]	19.40 [0.08]
Q^2_{12}	46.76 [0.00]	23.57 [0.02]	24.78 [0.02]	9.82 [0.63]	95.19 [0.00]	1.55 [1.00]

Notes: See Notes in Table 2C.1

Table 2C.3: Bivariate AR(1)-FIAPARCH(1, d, 1)-DCC(1, 1) models

Subsample A

	CAC-DAX		CAC-FTSE		DAX-FTSE	
	CAC	DAX	CAC	FTSE	DAX	FTSE
c_i	0.018 (2.19)***	0.028 (3.77)***	0.020 (2.27)***	0.018 (2.79)***	0.029 (3.91)***	0.020 (3.24)***
ζ_i	0.043 (1.71)**	−0.086 (−3.47)***	0.038 (1.43)*	0.063 (2.20)***	−0.068 (−3.05)***	0.046 (1.94)***
η_i	−0.023 (−0.82)	0.157 (6.91)***	0.012 (0.34)	−0.027 (−1.21)	0.195 (6.38)***	−0.014 (−0.79)
ω_i	0.060 (2.96)***	0.033 (2.57)***	0.052 (2.04)***	0.029 (1.25)	0.032 (2.58)***	0.035 (1.66)**
ϕ_i	0.258 (3.66)***	0.290 (4.73)***	0.231 (5.01)***	0.239 (3.75)***	0.214 (3.30)***	0.184 (1.70)**
β_i	0.502 (4.38)***	0.640 (5.13)***	0.602 (6.69)***	0.671 (4.91)***	0.546 (5.15)***	0.485 (2.98)***
d_i	0.288 (3.76)***	0.431 (3.64)***	0.399 (4.46)***	0.445 (2.68)***	0.402 (5.11)***	0.309 (3.80)***
γ_i	0.333 (3.05)***	0.180 (2.17)***	0.304 (2.70)***	0.172 (1.88)**	0.172 (2.03)***	0.235 (2.09)***
δ_i	1.652 (8.16)***	1.535 (5.81)***	1.417 (6.22)***	1.360 (4.28)***	1.369 (6.74)***	1.344 (4.21)***
ρ_{ij}	0.534 (21.82)***		0.242 (1.57)*		0.289 (1.39)	
a	0.0286 (2.81)***		0.0092 (0.52)		0.0103 (2.92)***	
b	0.9184 (29.53)***		0.9907 (51.78)***		0.9889 (260.7)***	
υ	7.916 (7.74)***		9.560 (6.92)***		8.350 (7.73)***	
$Loglik$	−2203.12		−1613.55		−1751.71	
Q_{12}	7.31 [0.84]	11.06 [0.52]	6.22 [0.90]	10.03 [0.61]	12.09 [0.44]	12.94 [0.37]
Q^2_{12}	15.21 [0.23]	7.01 [0.86]	5.65 [0.93]	13.80 [0.31]	4.29 [0.98]	20.42 [0.06]

Notes: See Notes in Table 2C.1

Table 2C.3 (Continued): Bivariate AR(1)-FIAPARCH($1, d, 1$)-DCC($1, 1$) models

Subsample A

	HS-NIKKEI		HS-STRAITS		NIKKEI-STRAITS		SP-TSE	
	HS	NIKKEI	HS	STRAITS	NIKKEI	STRAITS	SP	TSE
c_i	0.034 (4.01)***	0.006 (0.93)	0.037 (4.08)***	0.010 (1.43)*	0.013 (1.73)**	0.015 (2.17)***	0.024 (4.45)***	0.012 (3.05)***
ζ_i	0.067 (3.58)***	−0.025 (−1.50)*	0.069 (3.02)***	0.164 (7.16)***	0.019 (1.71)**	0.166 (7.60)***	0.009 (0.41)	0.189 (8.04)***
η_i	−0.011 (−0.69)	0.021 (2.07)***	0.007 (0.29)	0.040 (2.20)***	−0.011 (−0.50)	0.043 (2.66)***	0.014 (0.47)	0.037 (2.27)***
ω_i	0.072 (2.67)***	0.020 (2.43)***	0.103 (2.73)***	0.016 (3.69)***	0.017 (2.17)***	0.150 (4.45)***	0.012 (2.01)***	0.031 (1.02)
ϕ_i	0.177 (1.49)*	0.211 (3.94)***	−0.008 (−0.04)	0.829 (21.76)***	0.233 (3.04)***	−0.986 (−57.20)***	0.372 (6.13)***	−0.771 (−1.07)
β_i	0.459 (2.74)***	0.499 (6.05)***	0.144 (0.59)	0.710 (12.81)***	0.515 (4.08)***	−0.979 (−40.13)***	0.659 (8.68)***	−0.745 (−0.95)
d_i	0.386 (5.29)***	0.337 (4.90)***	0.267 (5.54)***	0.045 (1.02)	0.335 (3.78)***	0.224 (7.27)***	0.324 (5.01)***	0.118 (5.06)***
γ_i	0.350 (4.49)***	0.554 (3.40)***	0.300 (3.11)***	0.134 (2.45)***	0.503 (3.43)***	0.096 (1.45)*	0.201 (1.50)*	0.110 (1.16)
δ_i	1.506 (14.84)***	1.907 (10.14)***	1.386 (16.01)***	2.183 (6.42)***	2.104 (8.32)***	1.870 (10.93)***	1.888 (7.17)***	2.221 (6.34)***
ρ_{ij}	0.296 (12.40)***		0.379 (10.66)***		0.199 (5.50)***		0.541 (9.56)***	
a	0.0550 (3.46)***		0.0545 (4.06)***		0.0485 (3.55)***		0.0192 (2.39)***	
b	0.8548 (15.57)***		0.8925 (29.27)***		0.9016 (26.51)***		0.9704 (67.29)***	
v	6.436 (13.28)***		5.264 (11.71)***		6.289 (10.92)***		5.805 (10.97)***	
$Loglik$	−5731.41		−2650.80		−2686.69		312.22	
Q_{12}	25.81 [0.01]	12.60 [0.40]	14.82 [0.25]	14.41 [0.28]	12.70 [0.39]	15.25 [0.23]	20.75 [0.05]	23.41 [0.02]
Q^2_{12}	32.42 [0.00]	9.58 [0.65]	53.77 [0.00]	0.86 [1.00]	8.13 [0.77]	0.44 [1.00]	4.37 [0.98]	4.94 [0.96]

Notes: See Notes in Table 2C.1

Table 2C.4: Bivariate AR(1)-FIAPARCH($1, d, 1$)-DCC($1, 1$) models

Subsample B

	CAC-DAX		CAC-FTSE		DAX-FTSE	
	CAC	DAX	CAC	FTSE	DAX	FTSE
c_i	0.021 $(2.32)^{***}$	0.031 $(3.37)^{***}$	0.015 $(1.84)^{**}$	0.013 $(2.03)^{***}$	0.022 $(2.66)^{***}$	0.011 $(1.73)^{**}$
ζ_i	-0.055 $(-1.51)^{*}$	-0.053 $(-1.44)^{*}$	-0.045 (-1.41)	-0.023 (-0.73)	-0.046 $(-1.78)^{**}$	-0.010 (-0.37)
η_i	0.035 (0.98)	0.036 (0.95)	0.036 (0.93)	-0.001 (-0.04)	0.036 (1.12)	-0.016 (-0.76)
ω_i	0.018 $(2.09)^{***}$	0.015 $(1.80)^{**}$	0.012 $(1.95)^{***}$	0.012 $(2.67)^{***}$	0.012 $(1.73)^{**}$	0.013 $(2.77)^{***}$
ϕ_i	0.270 $(7.90)^{***}$	0.249 $(6.31)^{***}$	0.300 $(7.19)^{***}$	0.313 $(10.20)^{***}$	0.219 $(4.61)^{***}$	0.267 $(8.89)^{***}$
β_i	0.628 $(9.92)^{***}$	0.586 $(8.54)^{***}$	0.561 $(9.20)^{***}$	0.608 $(14.77)^{***}$	0.524 $(8.25)^{***}$	0.598 $(14.12)^{***}$
d_i	0.421 $(5.17)^{***}$	0.395 $(5.78)^{***}$	0.313 $(6.19)^{***}$	0.358 $(9.01)^{***}$	0.350 $(6.97)^{***}$	0.390 $(8.76)^{***}$
γ_i	0.497 $(4.51)^{***}$	0.407 $(3.96)^{***}$	0.597 $(4.37)^{***}$	0.650 $(5.25)^{***}$	0.513 $(3.93)^{***}$	0.683 $(5.22)^{***}$
δ_i	1.510 $(13.60)^{***}$	1.614 $(14.54)^{***}$	1.628 $(14.90)^{***}$	1.474 $(15.02)^{***}$	1.623 $(14.16)^{***}$	1.426 $(14.54)^{***}$
ρ_{ij}	0.658 $(1.98)^{***}$		0.881 $(41.51)^{***}$		0.818 $(23.70)^{***}$	
a	0.0246 (1.13)		0.0400 $(5.02)^{***}$		0.0439 $(4.39)^{***}$	
b	0.9753 $(43.36)^{***}$		0.9510 $(89.79)^{***}$		0.9468 $(68.51)^{***}$	
υ	8.276 $(9.92)^{***}$		10.248 $(8.82)^{***}$		10.494 $(8.46)^{***}$	
$Loglik$	-3158.20		-2757.32		-3353.53	
Q_{12}	20.30 [0.06]	10.66 [0.56]	19.01 [0.09]	22.39 [0.03]	9.44 [0.66]	23.49 [0.02]
Q^2_{12}	21.56 [0.04]	36.74 [0.00]	11.65 [0.47]	11.64 [0.48]	26.17 [0.01]	19.21 [0.08]

Notes: See Notes in Table 2C.1

Table 2C.4 (Continued): Bivariate AR(1)-FIAPARCH(1, d, 1)-DCC(1, 1) models

Subsample B

	HS-NIKKEI		HS-STRAITS		NIKKEI-STRAITS		SP-TSE	
	HS	NIKKEI	HS	STRAITS	NIKKEI	STRAITS	SP	TSE
c_i	0.028 (2.99)***	0.011 (1.10)	0.016 (1.84)**	0.016 (2.13)***	0.003 (0.38)	0.018 (2.46)***	–	–
ζ_i	0.029 (1.23)	−0.087 (−3.55)***	−0.054 (−2.52)***	0.038 (1.70)**	−0.018 (−0.86)	0.042 (1.88)**	–	–
η_i	−0.046 (−2.11)***	0.114 (4.43)***	0.111 (4.13)***	−0.009 (−0.46)	0.103 (4.43)***	−0.024 (−1.28)	–	–
ω_i	0.011 (1.82)**	0.018 (1.77)**	0.017 (2.37)***	0.013 (2.26)***	0.016 (1.35)	0.008 (1.61)*	–	–
ϕ_i	0.217 (4.73)***	0.113 (2.13)***	0.198 (2.41)***	0.147 (1.17)	0.130 (2.29)***	0.099 (0.73)	–	–
β_i	0.611 (8.07)***	0.549 (5.94)***	0.343 (2.60)***	0.368 (1.99)***	0.479 (5.71)***	0.347 (1.98)***	–	–
d_i	0.411 (5.32)***	0.429 (5.61)***	0.192 (2.80)***	0.291 (4.10)***	0.370 (6.44)***	0.317 (5.44)***	–	–
γ_i	0.189 (1.97)***	0.203 (2.33)***	0.373 (3.80)***	0.212 (3.99)***	0.324 (2.82)***	0.234 (3.91)***	–	–
δ_i	1.832 (11.61)***	1.786 (9.74)***	2.071 (11.30)***	1.979 (14.31)***	1.850 (9.05)***	1.976 (16.46)***	–	–
ρ_{ij}	0.551 (20.81)***		0.632 (37.52)***		0.465 (18.75)***		–	
a	0.0287 (3.37)***		0.0787 (1.85)**		0.0341 (3.33)***		–	
b	0.9315 (42.85)***		0.7670 (3.77)***		0.9210 (33.67)***		–	
υ	8.870 (7.47)***		7.514 (9.95)***		7.818 (9.94)***		–	
$Loglik$	−3303.53		−4801.84		−5092.13		–	
Q_{12}	8.91 [0.71]	13.16 [0.36]	16.97 [0.15]	11.78 [0.46]	15.38 [0.22]	13.61 [0.33]	–	–
Q^2_{12}	9.64 [0.65]	4.89 [0.96]	23.74 [0.02]	3.18 [0.99]	11.52 [0.49]	1.65 [1.00]	–	–

Notes: See Notes in Table 2C.1

Table 2C.5: Bivariate AR(1)-FIAPARCH$(1, d, 1)$-DCC$(1, 1)$ models

Subsample C

	CAC-DAX		CAC-FTSE		DAX-FTSE	
	CAC	DAX	CAC	FTSE	DAX	FTSE
c_i	0.025 (2.74)***	0.035 (3.64)***	0.024 (2.70)***	0.015 (2.16)***	0.027 (2.78)***	0.012 (1.68)**
ζ_i	−0.058 (−1.54)*	−0.084 (−2.22)***	−0.046 (−1.35)	−0.013 (−0.38)	−0.074 (−2.47)***	0.001 (0.04)
η_i	0.039 (1.10)	0.073 (1.85)**	0.050 (1.13)	0.005 (0.17)	0.086 (2.01)***	−0.014 (−0.63)
ω_i	0.017 (1.93)***	0.019 (1.90)**	0.007 (1.24)	0.010 (2.00)***	0.008 (1.21)	0.007 (1.70)**
ϕ_i	0.298 (6.37)***	0.304 (5.62)***	0.322 (7.83)***	0.337 (10.31)***	0.263 (5.49)***	0.287 (8.13)***
β_i	0.539 (6.71)***	0.547 (7.49)***	0.634 (9.76)***	0.639 (15.10)***	0.610 (9.49)***	0.614 (12.30)***
d_i	0.285 (3.61)***	0.292 (5.06)***	0.366 (5.66)***	0.367 (8.30)***	0.401 (6.51)***	0.393 (7.93)***
γ_i	0.336 (3.21)***	0.308 (2.78)***	0.416 (3.58)***	0.611 (4.08)***	0.423 (3.19)***	0.551 (3.97)***
δ_i	2.169 (8.13)***	2.142 (8.94)***	1.731 (12.65)***	1.482 (10.88)***	1.645 (11.08)***	1.551 (11.01)***
ρ_{ij}	0.682 (1.97)***		0.844 (34.38)***		0.768 (25.75)***	
a	0.0311 (1.96)***		0.0348 (4.99)***		0.0439 (4.56)***	
b	0.9688 (56.99)***		0.9545 (91.17)***		0.9400 (59.82)***	
υ	9.492 (8.64)***		10.650 (7.44)***		12.279 (6.58)***	
$Loglik$	−2532.62		−2005.71		−2512.15	
Q_{12}	29.44 [0.00]	8.35 [0.76]	16.97 [0.15]	18.64 [0.10]	6.07 [0.91]	18.59 [0.10]
Q^2_{12}	12.65 [0.40]	20.62 [0.06]	12.06 [0.44]	6.58 [0.88]	19.34 [0.08]	8.66 [0.73]

Notes: See Notes in Table 2C.1

Table 2C.5 (Continued): Bivariate AR(1)-FIAPARCH(1, d, 1)-DCC(1, 1) models

Subsample C

	HS-NIKKEI		HS-STRAITS		NIKKEI-STRAITS		SP-TSE	
	HS	NIKKEI	HS	STRAITS	NIKKEI	STRAITS	SP	TSE
c_i	0.034 (3.46)***	0.024 (2.07)***	0.019 (2.11)***	0.019 (2.44)***	0.010 (1.01)	0.021 (2.65)***	–	–
ζ_i	0.046 (1.59)*	−0.062 (−2.10)***	−0.029 (−1.23)	0.045 (1.81)**	−0.009 (−0.39)***	0.033 (1.39)	–	–
η_i	−0.043 (−1.86)**	0.095 (2.69)***	0.097 (3.32)***	0.018 (0.86)	0.071 (2.89)***	0.006 (0.29)	–	–
ω_i	0.007 (0.67)	0.007 (0.50)	0.019 (2.47)***	0.014 (2.62)***	−0.002 (−0.11)	0.008 (1.27)	–	–
ϕ_i	0.238 (2.67)***	0.119 (1.14)	0.297 (3.91)***	0.196 (1.88)**	0.114 (1.41)	0.004 (0.01)	–	–
β_i	0.748 (5.51)***	0.538 (2.47)***	0.460 (4.43)***	0.431 (3.07)***	0.392 (3.46)***	0.185 (0.58)	–	–
d_i	0.479 (2.43)***	0.395 (2.64)***	0.188 (2.77)***	0.305 (4.64)***	0.284 (4.55)***	0.256 (3.96)***	–	–
γ_i	0.028 (0.22)	0.094 (1.08)	0.377 (2.24)***	0.203 (3.31)***	0.189 (1.66)**	0.199 (3.07)***	–	–
δ_i	1.814 (3.24)***	2.037 (5.49)***	2.266 (11.11)***	2.091 (12.68)***	2.231 (7.09)***	2.047 (13.51)***	–	–
ρ_{ij}	0.502 (13.73)***		0.567 (22.50)***		0.222 (2.23)***		–	
a	0.0381 (3.21)***		0.0376 (2.63)***		0.0047 (1.58)*		–	
b	0.9154 (31.99)***		0.9133 (18.08)***		0.9952 (291.7)***		–	
υ	8.345 (6.46)***		8.062 (8.88)***		7.885 (9.20)***		–	
$Loglik$	−1650.20		−3398.31		−3650.21		–	
Q_{12}	5.83 [0.92]	13.40 [0.34]	11.86 [0.46]	7.46 [0.83]	10.61 [0.56]	12.50 [0.41]	–	–
Q^2_{12}	7.39 [0.83]	5.33 [0.95]	8.46 [0.75]	5.69 [0.93]	11.05 [0.52]	3.03 [1.00]	–	–

Notes: See Notes in Table 2C.1

Table 2C.6: Trivariate AR(1)-FIAPARCH(1, d, 1)-DCC(1, 1)

Subsample A

	EUROPE			ASIA		
	CAC	DAX	FTSE	NIKKEI	HS	STRAITS
c_i	0.021 (2.46)***	0.029 (3.85)***	0.018 (2.95)***	0.013 (1.66)**	0.041 (4.72)***	0.016 (2.41)***
ζ_i	0.053 (1.88)**	−0.103 (−3.92)***	0.041 (1.59)*	−0.011 (−0.56)	0.075 (3.28)***	0.154 (6.96)***
η_i	D −0.018 (−0.64)	C 0.116 (4.31)***	C −0.018 (−0.85)	HS 0.022 (2.05)***	N −0.010 (−0.62)	N 0.016 (1.21)
	F −0.003 (−0.09)	F 0.129 (3.78)***	D 0.005 (0.24)	S −0.012 (−0.53)	S 0.010 (0.40)	HS 0.040 (2.26)***
ω_i	0.050 (3.33)***	0.035 (2.80)***	0.029 (2.15)***	0.013 (2.37)***	0.118 (1.37)	0.140 (5.00)***
ϕ_i	0.251 (4.46)***	0.292 (4.47)***	0.237 (3.80)***	0.232 (4.09)***	0.096 (0.22)	−0.988 (−116.4)***
β_i	0.532 (6.22)***	0.623 (5.34)***	0.640 (6.21)***	0.601 (4.37)***	0.249 (0.49)	−0.985 (−94.47)***
d_i	0.320 (4.61)***	0.411 (4.15)***	0.415 (3.77)***	0.426 (3.06)***	0.263 (3.00)***	0.194 (7.36)***
γ_i	0.225 (2.83)***	0.163 (2.05)***	0.181 (1.99)***	0.392 (4.45)***	0.241 (2.25)***	0.079 (1.26)
δ_i	1.721 (9.58)***	1.566 (6.29)***	1.478 (5.47)***	2.193 (6.65)***	1.612 (10.09)***	2.013 (11.95)***
ρ_{ij}	$C{-}D$ 0.542 (19.38)***	$C{-}F$ 0.582 (23.08)***	$D{-}F$ 0.445 (14.94)***	$HS{-}N$ 0.220 (6.77)***	$N{-}S$ 0.197 (5.97)***	$S{-}HS$ 0.370 (11.29)***
a		0.0261 (3.36)***			0.0411 (4.85)***	
b		0.9393 (37.31)***			0.9081 (37.30)***	
υ		8.842 (8.96)***			6.604 (12.48)***	
$Loglik$		−2339.36			−4192.18	
Q_{12}	4.75 [0.97]	11.27 [0.51]	13.23 [0.35]	15.61 [0.21]	17.21 [0.14]	14.44 [0.27]
Q^2_{12}	16.33 [0.18]	6.97 [0.86]	16.12 [0.19]	7.15 [0.85]	45.95 [0.00]	0.60 [1.00]

Notes: See Notes in Table 2C.1

Table 2C.7: Trivariate AR(1)-FIAPARCH(1, d, 1)-DCC(1, 1)
Subsample B

	EUROPE			ASIA		
	CAC	DAX	FTSE	NIKKEI	HS	STRAITS
c_i	–	–	–	0.002 (0.25)	0.019 (2.20)***	0.017 (2.38)***
ζ_i	–	–	–	−0.081 (−4.22)***	−0.042 (−1.82)**	0.036 (1.57)*
η_i	–	–	–	HS 0.011 (0.49)	N −0.042 (−2.14)***	N −0.017 (−1.02)
	–	–	–	S 0.120 (5.09)***	S 0.110 (4.03)***	HS −0.009 (−0.45)
ω_i	–	–	–	0.014 (1.27)	0.013 (2.22)***	0.010 (2.25)***
ϕ_i	–	–	–	0.116 (1.95)***	0.251 (4.07)***	0.198 (2.65)***
β_i	–	–	–	0.480 (5.90)***	0.431 (4.12)***	0.451 (4.26)***
d_i	–	–	–	0.364 (6.80)***	0.221 (3.30)***	0.315 (5.90)***
γ_i	–	–	–	0.237 (2.56)***	0.282 (3.56)***	0.184 (3.49)***
δ_i	–	–	–	1.945 (9.40)***	2.071 (12.72)***	1.941 (17.43)***
ρ_{ij}	–	–	–	$HS-N$ 0.509 (25.07)***	$N-S$ 0.466 (21.22)***	$S-HS$ 0.625 (36.10)***
a		–			0.0363 (3.16)***	
b		–			0.8979 (17.71)***	
v		–			8.556 (11.27)***	
$Loglik$		–			−7176.49	
Q_{12}	–	–	–	6.95 [0.86]	17.98 [0.12]	11.66 [0.47]
Q_{12}^2	–	–	–	9.70 [0.64]	23.38 [0.02]	4.33 [0.98]

Notes: See Notes in Table 2C.1

Table 2C.8: Trivariate AR(1)-FIAPARCH($1, d, 1$)-DCC($1, 1$)

Subsample C

	EUROPE			ASIA		
	CAC	DAX	FTSE	NIKKEI	HS	STRAITS
c_i	0.027 (2.69)***	0.037 (3.45)***	0.018 (2.28)***	–	–	–
ζ_i	−0.085 (−1.92)**	−0.116 (−2.78)***	0.013 (0.36)	–	–	–
η_i	D: 0.015 (0.38)	C: 0.073 (1.56)*	C: 0.008 (0.22)	–	–	–
	F: 0.079 (1.68)**	F: 0.058 (1.18)	D: −0.028 (−0.91)	–	–	–
ω_i	0.011 (0.51)	0.014 (0.55)	0.013 (0.73)	–	–	–
ϕ_i	0.326 (7.83)***	0.304 (7.01)***	0.350 (9.91)***	–	–	–
β_i	0.635 (5.84)***	0.637 (6.07)***	0.632 (9.24)***	–	–	–
d_i	0.359 (2.70)***	0.389 (3.28)***	0.339 (3.80)***	–	–	–
γ_i	0.291 (1.71)**	0.274 (1.72)**	0.558 (2.23)***	–	–	–
δ_i	1.698 (6.98)***	1.627 (6.00)***	1.454 (5.69)***	–	–	–
ρ_{ij}	$C-D$: 0.619 (0.84)	$C-F$: 0.565 (1.18)	$D-F$: 0.542 (1.62)**	–	–	–
a		0.0181 (0.71)			–	
b		0.9818 (35.87)***			–	
υ		9.707 (9.68)***			–	
$Loglik$		−2392.47			–	
Q_{12}	26.38 [0.01]	6.57 [0.88]	18.26 [0.11]	–	–	–
Q^2_{12}	10.91 [0.54]	31.51 [0.00]	4.65 [0.97]	–	–	–

Notes: See Notes in Table 2C.1

Chapter 3 Modelling returns and volatilities during financial crises: A time-varying coefficient approach

3.1 Introduction

The Global financial crisis of 2007-08 and the European sovereign-debt crisis that took place immediately afterwards are at the heart of the research interests of practitioners, academics and policy makers alike. Given the widespread fear of an international systemic financial collapse at the time it is no wonder that the currently on-going heated discussion on the actual causes and effects of these crises is the precursor to the development of the necessary tools and policies for dealing with similar phenomena in the future.

The inevitable step in undertaking such an enormous task is to map, as accurately as possible, the 'impact' of these crises onto what are currently considered the main stochastic properties of the underlying financial time series. In this way, informed discussions on the causes and effects of these crises can take place and thus more accurately specify the set of features that have to characterise the necessary tools and policies to address them. This study aspires to provide a platform upon which changes in the main statistical properties of financial time series due to economic crises can be measured.

In particular, we focus on the recent financial crises and examine how the mean and volatility dynamics, including the underlying volatility persistence and volatility spillovers structure, have been affected by these crises. With this aim we make use of several modern econometric approaches for univariate and multivariate time series modelling, which we also condition on the possibility of breaks in the mean and/or volatility dynamics taking place. Moreover, we unify these approaches by introducing a set of theoretical considerations for time-varying (TV) AR-GARCH models, which are also of independent interest. In particular, we make three broad contributions to the existing literature.

First, we present and utilise some new theoretical results on time-varying AR and/or asymmetric GARCH (AGARCH) models. We limit our analysis to low order specifications to save space and also since it is well documented that low order AR models for stock returns often emerge in practice. We show the applicability of these general results to one important case: that of abrupt breaks, which we make particular use of in our empirical investigation. Our models produce time-varying unconditional variances in the spirit of Engle and Rangel (2008) and Baillie and Morana (2009). TV-GARCH specifications have recently gained popularity for modelling structural breaks in the volatility process (see, for example, Frijns et al., 2011 and Bauwens, et al., 2014). Despite nearly half a century of research work and the widely recognised importance of time-varying models, until recently there was a lack of a general theory that can be employed to explore their time series properties systematically. Granger in some of his last contributions highlighted the importance of the topic (see, Granger 2007 and 2008). The stumbling block to the development of such a theory was the lack of a method that can be used to solve time-varying difference equations of order two or higher. Paraskevopoulos et al. (2013) have developed such a general theory (see also Paraskevopoulos and Karanasos, 2013). The starting point of the solution method that we present below is to represent the linear time-varying difference equation of order two as an infinite system of linear equations. The coefficient matrix of such an infinite system is row finite. The solution to such infinite systems is based on an extension of the classic Gauss elimination, called Infinite Gaussian elimination (see Paraskevopoulos, 2012, 2014). Our method is a natural extension of the first order solution formula. It also includes the linear difference equation with constant coefficients (see, for example, Karanasos, 1998, 2001) as a special case. We simultaneously compute not only the general solution but also its homogeneous and particular parts as well. The coefficients in these solutions are expressed as determinants of tridiagonal matrices. This allows us to provide a thorough description of time-varying models by deriving, first, multistep ahead forecasts, the associated forecast error and the mean square error and, second, the first two time-varying unconditional moments of the process and its covariance structure.

Second, we use a battery of tests to identify the number and estimate the timing of breaks both in the mean and volatility dynamics. Following our theoretical results and prompted by Morana and Beltratti (2004) amongst others who acknowledge that misleading inference on the persistence of the volatility process may be caused by unaccounted structural breaks, we implement these break tests in the univariate context also to determine changes in the persistence of volatility. The special attention we pay to this issue is well justified, especially within the finance literature given that it is well-established that the proper detection of breaks is pivotal for a variety of financial applications, particularly in risk measurement, asset allocation and option pricing. Kim and Kon (1999) emphasise the importance of incorporating some break detection procedure into the existing financial modelling paradigms when they call attention to the fact that "...*Public announcements of corporate investment and financial decisions that imply a change in the firm's expected return and risk will be impounded in stock prices immediately in an efficient market. The announcements of relevant macroeconomic information will affect the return and risk of all securities and hence, portfolios (indexes). Since relevant information that changes the risk structure is randomly released with some time interval (not at every moment) in sequence, these information events translate into sequential discrete structural shifts (or change-points) for the mean and/or variance parameter(s) in the time series of security returns.*"

Third, we employ the bivariate unrestricted extended dynamic conditional correlation (UEDCC) AGARCH process to analyse the volatility transmission structure, applied to stock market returns. The model is based on the dynamic conditional correlation of Engle (2002a) allowing for volatility spillovers effects by imposing the unrestricted extended conditional correlation (dynamic or constant) GARCH specification of Conrad and Karanasos (2010). The most recent applications of the model can be found in Conrad et al. (2010), Rittler (2012), Karanasos and Zeng (2013) and Conrad and Karanasos (2013). However, we extend it by allowing shock and volatility spillovers parameters to shift across abrupt breaks as well as across two regimes of stock returns, positive (increases in the stock market) and negative (declines in the stock market) (see also Karanasos et al., 2013). Recently, following our work, Caporale et al. (2014) adopted our UEDCC framework but they do not allow for breaks in the shock and volatility spillovers. The extant literature on modelling returns and volatilities is extensive and it has evolved in several directions. One line of literature has focused on return correlations and comovements or what is known as contagion amongst different markets or assets (e.g., Caporale et. al., 2005; Rodriguez, 2007, amongst others), while another line of the literature has focused on volatility spillovers amongst the markets (e.g., Baele, 2005; Asgharian and Nossman, 2011, amongst others). The model adopted in this study is flexible enough to capture contagion effects as well as to identify the volatility spillovers associated with the structural changes and exact movements of each market (e.g., upward or downward) to the other and vice versa. Knowledge of this mechanism can provide important insights to investors by focusing their attention on structural changes in the markets as well as their trends and movements (e.g., upward or downward) in order to set appropriate portfolio management strategies.

Overall, our results suggest that stock market returns exhibit time-varying persistence in their corresponding conditional variances. The results of the bivariate UEDCC-AGARCH$(1,1)$ model applied to FTSE and DAX returns and NIKKEI and Hang Seng returns also show the existence of dynamic correlations as well as time-varying shock and volatility spillovers between the two variables in each pair. For example, the results of the bivariate FTSE and DAX returns show that the transmission of volatility from DAX to FTSE exhibited a time-varying pattern across the Asian financial crisis and the announcement of the €18bn German tax cuts plan as well as the Global financial crisis. As far as the NIKKEI and Hang Seng pair is concerned, the results provide evidence that these two financial markets have only been integrated during the different phases of the recent financial crisis. With regard to the regime-dependent volatility spillovers, the results suggest that declines in FTSE and DAX generate shock spillovers to each other, whereas increases in each of these market generate negative volatility spillovers to the other. Furthermore, the results show that declines in NIKKEI generate shock spillovers to Hang Seng, whilst increases in NIKKEI generate negative volatility spillovers to Hang Seng.

The remainder of this chapter is as follows. Section 3.2 considers the AR-GARCH model with abrupt breaks in the first two conditional moments and the time-varying process, which are our two main objects of inquiry. Section 3.3 introduces the theoretical considerations on the time-varying AR and AGARCH models. In Section 3.3.1 we represent the former as an infinite linear system and concentrate on the associated coefficient matrix. This representation enables us to establish an explicit formula for the general solution in terms of the determinants of tridiagonal matrices. We also obtain the statistical properties of the aforementioned models, e.g., multi-step-ahead predictors and their forecast error variances. Section 3.4 describes our methodology and data. Section 3.5 presents our empirical univariate results and the next Section discusses the results from various bivariate models. The final Section contains the summary and our concluding remarks.

3.2 Abrupt breaks

First, we introduce the notation and the AR-AGARCH model with abrupt breaks both in the conditional mean and variance. Throughout the chapter we will adhere to the conventions: ($\mathbb{Z}^+$) $\mathbb{Z}$ and ($\mathbb{R}^+$) $\mathbb{R}$ stand for the sets of (positive) integers and (positive) real numbers, respectively. To simplify our exposition we also introduce the following notation. Let $t \in \mathbb{Z}$ represent present time and $k \in \mathbb{Z}^+$ the prediction horizon.

3.2.1 The conditional mean

In this study we will examine an AR(2) model[25] with n abrupt breaks, $0 \leq n \leq k-1$, at times $t-k_1$, $t-k_2$, ..., $t-k_n$, where $0 = k_0 < k_1 < k_2 < \cdots < k_n < k_{n+1} = k$, $k_l \in \mathbb{Z}^+$ and k_n is finite. That is, between $t-k = t-k_{n+1}$ and the present time $t = t-k_0$ the AR process contains n structural breaks and the switch from one set of parameters to another is abrupt. In particular,

$$y_\tau = \phi_{0,l} + \phi_{1,l} y_{\tau-1} + \phi_{2,l} y_{\tau-2} + \varepsilon_\tau, \tag{3.1}$$

for $l = 1, \ldots, n+1$ and $\tau = t-k_{l-1}, \ldots, t-k_l+1$, where[26] $\mathbb{E}[\varepsilon_\tau \,|\mathcal{F}_{\tau-1}] = 0$ and ε_τ follows a time-varying AGARCH type of process with finite variance σ_τ^2 (see the next Section).[27] Within the class of AR(2) processes, this specification is quite general and allows for intercept and slope shifts as well as errors with time-varying variances (see also Pesaran and Timmermann, 2005 and Pesaran et al. 2006). Each regime l is characterised by a vector of regression coefficients, $\boldsymbol{\phi}_l = (\phi_{0,l}, \phi_{1,l}, \phi_{2,l})'$ and positive and finite time-varying variances, σ_τ^2, $\tau = t-k_{l-1}, \ldots, t-k_l+1$. We will term the AR(2) model with n abrupt breaks: abrupt breaks autoregressive process of order $(2; n)$, AB-AR$(2; n)$.

3.2.2 The conditional variance

We assume that the noise term is characterised by the relation $\varepsilon_\tau = e_\tau \sqrt{h_\tau}$, where h_τ is positive with probability one and it is a measurable function of $\mathcal{F}_{t-1}$; e_τ is an i.i.d sequence with zero mean and finite second and fourth moments: $\varkappa^{(i)} = \mathbb{E}(e_\tau^{2i})$, $i = 1, 2$. In other words the conditional (on time $\tau - 1$) variance of y_τ is $\mathbb{V}ar(y_\tau \,|\mathcal{F}_{\tau-1}) = \varkappa^{(1)} h_\tau$. In what follows, without loss of generality, we will assume that $\varkappa^{(1)} = 1$.

Moreover, we specify the parametric structure of h_τ as an AGARCH$(1,1)$ model with m abrupt breaks, $0 \leq m \leq k-1$, at times $t-\kappa_1$, $t-\kappa_2$, ..., $t-\kappa_m$, where

[25] To keep the exposition tractable and reveal its practical significance we work with low order specifications.
[26] $\{\mathcal{F}_t\}$ is a non-decreasing sequence of σ-fields $\mathcal{F}_{t-1} \subseteq \mathcal{F}_t \subseteq \mathcal{F}$.
[27] Without loss of generality we will assume that outside the prediction horizon there are no breaks. That is: regime one $(l = 1)$ extends to time $\tau = \ldots, t+2, t+1$ and the $(n+1)$th regime extends to time $\tau = t-k, t-k-1, \ldots$.

$0 = \kappa_0 < \kappa_1 < \kappa_2 < \cdots < \kappa_m < \kappa_{m+1} = k$, $\kappa_m \in \mathbb{Z}^+$ and κ_m is finite. That is, between $t - k = t - \kappa_{m+1}$ and the present time $t = t - \kappa_0$ the AGARCH process contains m structural breaks and the switch from one set of parameters to another is abrupt:

$$h_\tau = \omega_\ell + \alpha^*_\ell \varepsilon^2_{\tau-1} + \beta_\ell h_{\tau-1}, \tag{3.2}$$

for $\ell = 1, \ldots, m+1$ and $\tau = t - \kappa_{\ell-1}, \ldots, t - \kappa_\ell + 1$; where $\alpha^*_\ell \triangleq \alpha_\ell + \gamma_\ell S^-_{\tau-1}$, with $S^-_{\tau-1} = 1$ if $e_{\tau-1} < 0$, 0 otherwise.[28] As with the AR process we will assume that outside the prediction horizon there are no breaks. Obviously, the above process nests the simple AGARCH$(1,1)$ specification if we assume that the four coefficients are constant.

In what follows we provide a complete characterisation of the main time series properties of this model. Although in this work we will focus our attention on the AB-AR$(2; n)$-AGARCH$(1,1; m)$ process[29] our results can easily be extended to models of higher orders (see Paraskevopoulos et al., 2013).

3.2.3 Time-varying model

In the current Section we face the non-stationarity of processes with abrupt breaks head on by employing a time-varying treatment. In particular, we put forward a framework for examining the AR-AGARCH specification with n and m abrupt breaks in the conditional mean and variance respectively. We begin by expressing the model as a TV-AR(2)-AGARCH$(1,1)$ process:

$$y_t = \phi_0(t) + \phi_1(t) y_{t-1} + \phi_2(t) y_{t-2} + \varepsilon_t, \tag{3.3}$$

where for $l = 1, \ldots, n+1$ and $\tau = t - k_{l-1}, \ldots, t - k_l + 1$, $\phi_i(\tau) \triangleq \phi_{i,l}$, $i = 0, 1, 2$, are the time-varying drift and AR parameters; as before $\{\varepsilon_t, t \in \mathbb{Z}\}$ is a sequence of zero mean serially uncorrelated random variables with positive and finite time-varying variances $\sigma^2_t \; \forall \, t$. Recall that we have relaxed the assumption of homoscedasticity that is likely to be violated in practice and

---

28 This type of asymmetry is the so called GJR-GARCH model (named for Glosten et al., 1993). The asymmetric power ARCH process (see, amongst others, Karanasos and Kim, 2006; Margaronis et al., 2013) is yet another asymmetric variant. For other asymmetric GARCH models see Francq and Zakoïan (2010, chapter 10) and the references therein.

29 That is an AR(2)-AGARCH$(1,1)$ model with n and m abrupt breaks in the conditional mean and variance respectively.

allow ε_t to follow a TV-AGARCH$(1,1)$ type of process:

$$h_t = \omega(t) + \alpha^*(t)\varepsilon_{t-1}^2 + \beta(t)h_{t-1}, \tag{3.4}$$

where for $\ell = 1, \ldots, m+1$ and $\tau = t - \kappa_{\ell-1}, \ldots, t - \kappa_\ell + 1$, $\omega(\tau) \triangleq \omega_\ell$, $\alpha^*(\tau) \triangleq \alpha(\tau) + \gamma(\tau)S_{t-1}^- \triangleq \alpha_\ell^*$ and $\beta(\tau) \triangleq \beta_\ell$ are the time-varying parameters of the conditional variance equation.

The TV-AGARCH$(1,1)$ formulation in eq. (3.4) can readily be seen to have the following representation

$$h_t = \omega(t) + c(t)h_{t-1} + \alpha^*(t)v_{t-1}, \tag{3.5}$$

with $c(t) \triangleq \alpha^*(t) + \beta(t) = \alpha(t) + \gamma(t)S_{t-1}^- + \beta(t)$ and for $\ell = 1, \ldots, m+1$ and $\tau = t - \kappa_{\ell-1}, \ldots, t - \kappa_\ell + 1$, $c(\tau) \triangleq c_\ell$; the 'innovation' of the conditional variance $v_t = \varepsilon_t^2 - h_t$ is, by construction, an uncorrelated term with expected value 0 and $\mathbb{E}(v_t^2) = \sigma_{vt}^2 = \widetilde{\varkappa}\mathbb{E}(h_t^2)$, with $\widetilde{\varkappa} = \mathbb{V}ar(e_t^2) = \varkappa^{(2)} - 1$. The above equation has the linear structure of a TV-ARMA model allowing for simple computations of the linear predictions (see Section 3.3.2.1 below).[30]

Although in the next Section we will focus our attention on the TV-AR(2)-AGARCH$(1,1)$ model our results can easily be extended to time-varying models of higher orders (see Paraskevopoulos et al., 2013).

3.3 Theoretical considerations

The current Section presents some new theoretical findings for time-varying models which also provide the platform upon which we unify the results we obtain from the different econometric tools. That is, we put forward a framework for examining AR models with abrupt breaks, like eq.(3.1), based on a workable closed form solution of stochastic time-varying difference equations. In other words, we exemplify how our theoretical methodology can be used to incorporate structural changes, which in this study we view as abrupt breaks. We also explain how

30 As pointed out, amongst others, by Francq and Zakoïan (2010, p. 20) under additional assumptions (implying the second-order of h_t or ε_t^2), we can state that if ε_t follows a TV-AGARCH model then h_t or ε_t^2 are TV-ARMA processes as well.

we can extend our approach to the AGARCH specification with abrupt breaks in the conditional variance.

3.3.1 The mean

In the context of eq. (3.3), the second-order homogeneous difference equation with time-varying coefficients is written as

$$\phi_2(t)y_{t-2} + \phi_1(t)y_{t-1} - y_t = 0,\ t \geq \tau + 1 = t - k + 1. \tag{3.6}$$

The infinite set of equations in the above equation is equivalent to the infinite linear system whose coefficient matrix is row-finite (row-finite matrices are infinite $\mathbb{N} \times \mathbb{N}$ matrices whose rows have a finite number of nonzero elements)

$$\begin{pmatrix} \phi_2(\tau+1) & \phi_1(\tau+1) & -1 & & & & \cdots \\ & \phi_2(\tau+2) & \phi_1(\tau+2) & -1 & & & \cdots \\ & & \phi_2(\tau+3) & \phi_1(\tau+3) & -1 & & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\vdots\vdots \end{pmatrix} \begin{pmatrix} y_{\tau-1} \\ y_\tau \\ y_{\tau+1} \\ y_{\tau+2} \\ y_{\tau+3} \\ y_{\tau+4} \\ \vdots \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \tag{3.7}$$

(here and in what follows empty spaces in a matrix[31] have to be replaced by zeros) or in a compact form: $\mathbf{\Phi} \cdot \mathbf{y} = \mathbf{0}$. The equivalence of eqs. (3.6) and (3.7) follows from the fact that for an arbitrary i in $\{1, 2, 3, \ldots\}$ the ith equation of (3.7), as a result of the multiplication of the ith row of $\mathbf{\Phi}$ by the column of ys equated to zero, is equivalent to eq. (3.6), as of time $t = \tau + i$. By deleting the first column of the $\mathbf{\Phi}$ matrix and then keeping only the first k rows and columns we obtain the following square matrix:

[31] Matrices and vectors are denoted by upper and lower case boldface symbols, respectively. For square matrices $\mathbf{X} = [x_{ij}]_{i,j=1,\ldots,k} \in \mathbb{R}^{k \times k}$ using standard notation, det($\mathbf{X}$) or $|\mathbf{X}|$ denotes the determinant of matrix $\mathbf{X}$.

$$
\mathbf{\Phi}_{t,k} = \begin{pmatrix} \phi_1(\tau+1) & -1 & & & & \\ \phi_2(\tau+2) & \phi_1(\tau+2) & -1 & & & \\ & \phi_2(\tau+3) & \phi_1(\tau+3) & -1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & \phi_2(t-1) & \phi_1(t-1) & -1 \\ & & & & \phi_2(t) & \phi_1(t) \end{pmatrix} \tag{3.8}
$$

(where $\tau = t-k$). Formally $\mathbf{\Phi}_{t,k}$ is a square $k \times k$ matrix whose (i,j) entry $1 \le i,j \le k$ is given by

$$
\begin{cases} -1 & \text{if} & & i=j-1, & \text{and} & 2 \le j \le k, \\ \phi_{1+d}(t-k+i) & \text{if} & d=0,1, & i=j+d, & \text{and} & 1 \le j \le k-d, \\ 0 & & \text{otherwise.} & & & \end{cases}
$$

This is a tridiagonal or continuant matrix, that is a matrix that is both upper and lower Hessenberg matrix. We next define the bivariate function $\xi : \mathbb{Z} \times \mathbb{Z}^+ \longmapsto \mathbb{R}$ by

$$
\xi_{t,k} = \det(\mathbf{\Phi}_{t,k}) \tag{3.9}
$$

coupled with the initial values $\xi_{t,0} = 1$ and $\xi_{t,-1} = 0$. $\xi_{t,k}$ for $k \ge 2$, is a determinant of a $k \times k$ matrix; each two nonzero diagonals (below the superdiagonal) of this matrix consists of the time-varying coefficients $\phi_i(\cdot)$, $i = 1,2$, from $t-k+i$ to t. That is, the number of elements of $\phi_i(\cdot)$ in the diagonals below the superdiagonal is $k-i+1$. In other words, $\xi_{t,k}$ is a k-order tridiagonal determinant. For the AB-AR($2;n$) process, $\xi_{t,k}$ is given by

$$
\xi_{t,k} = \det(\mathbf{\Phi}_{t,k}) = \begin{vmatrix} \phi_{1,n+1} & -1 & & & & & & \\ \phi_{2,n+1} & \phi_{1,n+1} & -1 & & & & & \\ & \ddots & \ddots & \ddots & & & & \\ & & \phi_{2,l} & \phi_{1,l} & -1 & & & \\ & & & \phi_{2,l} & \phi_{1,l} & -1 & & \\ & & & & \ddots & \ddots & \ddots & \\ & & & & & \phi_{2,1} & \phi_{1,1} & -1 \\ & & & & & & \phi_{2,1} & \phi_{1,1} \end{vmatrix}, \tag{3.10}
$$

that is, the $(i, i-1)$ and (i,i) elements in rows $i = k - k_{l-1}, \dots, k-(k_l - 1)$, $l = 1, \dots, n+1$, of the matrix $\mathbf{\Phi}_{t,k}$ are given by $\phi_{2,l}$ and $\phi_{1,l}$, respectively.

The general term of the general homogeneous solution of eq. (3.6) with two free constants (initial condition values), y_{t-k} and y_{t-k-1}, is given by

$$y_{t,k}^{hom} = \xi_{t,k} y_{t-k} + \phi_2(t-k+1)\xi_{t,k-1} y_{t-k-1}. \tag{3.11}$$

Similarly, the general particular solution, $y_{t,k}^{par}$, can be expressed as

$$y_{t,k}^{par} = \sum_{r=0}^{k-1} \xi_{t,r}\phi_0(t-r) + \sum_{r=0}^{k-1} \xi_{t,r}\varepsilon_{t-r}. \tag{3.12}$$

The general solution of eq. (3.3) with free parameters y_{t-k}, y_{t-k-1} is given by the sum of the homogeneous solution plus the particular solution:

$$y_{t,k}^{gen} = y_{t,k}^{hom} + y_{t,k}^{par} = \xi_{t,k} y_{t-k} + \phi_2(t-k+1)\xi_{t,k-1} y_{t-k-1} + \sum_{r=0}^{k-1} \xi_{t,r}\phi_0(t-r) + \sum_{r=0}^{k-1} \xi_{t,r}\varepsilon_{t-r}. \tag{3.13}$$

(see the Appendix and also Paraskevopoulos et al., 2013 and Karanasos et al., 2014a). In the above expression $y_{t,k}^{gen}$ is decomposed into two parts: the $y_{t,k}^{hom}$ part, which is written in terms of the two free constants (y_{t-k-i}, $i = 0, 1$); and, the $y_{t,k}^{par}$ part, which contains the time-varying drift terms ($\phi_0(\cdot)$) and the error terms (εs) from time $t-k+1$ to time t. When $k = 1$, since $\xi_{t,0} = 1$ and $\xi_{t,1} = \phi_1(t)$, the above expression reduces to eq. (3.3). Notice also that for the model with n abrupt breaks, we have

$$\sum_{r=0}^{k-1} \xi_{t,r}\phi_0(t-r) = \sum_{l=1}^{n+1} \phi_{0,l} \sum_{r=k_{l-1}}^{k_l-1} \xi_{t,r} \text{ and } \phi_2(t-k+1) = \phi_{2,n+1},$$

where $\xi_{t,r}$ is given in eq. (3.10). The main advantage of our TV model/methodology is that we suppose that the law of evolution of the parameters is unknown, in particular they may be stochastic (i.e., we can either have a stationary or non-stationary process) or non stochastic (e.g., periodic models serve as an example, see Karanasos et al., 2014a,b). Therefore, no restrictions are imposed on the functional forms of the time-varying AR parameters. In the non stochastic case the model allows for (past/known) abrupt breaks.

3.3.1.1 First moments

We turn our attention to a consideration of the time series properties of the TV-AR(2)-AGARCH$(1,1)$ process. Let the triplet $(\Omega, \{\mathcal{F}_t, t \in \mathbb{Z}\}, P)$ denote a complete probability space with a filtration, $\{\mathcal{F}_t\}$. L_p stands for the space of P-equivalence classes of finite complex random variables with finite p-order. Finally, $H = L_2(\Omega, \mathcal{F}_t, P)$ stands for a Hilbert space of random variables with finite first and second moments. Assuming that the drift and the two AR time-varying coefficients $\phi_i(t)$, $i = 0, 1, 2$, are non stochastic and taking the conditional expectation of eq. (3.13) with respect to the σ field $\mathcal{F}_{t-k}$ yields the k-step-ahead optimal (in L_2-sense) linear predictor of y_t

$$\mathbb{E}(y_t \,|\mathcal{F}_{t-k}) = \sum_{r=0}^{k-1} \xi_{t,r}\phi_0(t-r) + \xi_{t,k}y_{t-k} + \phi_2(t-k+1)\xi_{t,k-1}y_{t-k-1}. \tag{3.14}$$

In addition, the forecast error for the above k-step-ahead predictor, $\mathbb{FE}(y_t \,|\mathcal{F}_{t-k}) = y_t - \mathbb{E}[y_t \,|\mathcal{F}_{t-k}]$, is given by

$$\mathbb{FE}(y_t \,|\mathcal{F}_{t-k}) = \sum_{r=0}^{k-1} \xi_{t,r}\varepsilon_{t-r}, \tag{3.15}$$

which is a linear combination of k error terms from time $t-k+1$ to time t, where the time-varying coefficients, $\xi_{t,r}$, are (for $r \geq 2$) the determinants of an $r \times r$ tridiagonal matrix ($\Phi_{t,r}$); each nonzero variable diagonal of this matrix consists of the AR time-varying coefficients $\phi_i(\cdot)$, $i = 1, 2$ from time $t - r + i$ to t.

The Assumption below provides conditions that are used to obtain the equivalent of the Wold decomposition for non-stationary time-varying processes with non stochastic coefficients.

Assumption 1. $\sum_{r=0}^{k} \xi_{t,r}\phi_0(t-r)$ as $k \to \infty$ converges for all t and $\sum_{r=0}^{\infty} \sup_t(\xi_{t,r}^2 \sigma_{t-r}^2) < M < \infty$, $M \in \mathbb{Z}^+$.

The challenge we face is that in the time-varying models we cannot invert the AR polynomial due to the presence of time-dependent coefficients. We overcome this difficulty and formulate a type of time-varying Wold decomposition theorem (see also Singh and Peiris, 1987; Kowalski and Szynal, 1991).

Under Assumption 1 the model in eq. (3.3) with non stochastic coefficients admits a second-order MA(∞) representation:

$$y_t \stackrel{L_2}{=} \lim_{k\to\infty} y_{t,k}^{par} \stackrel{L_2}{=} \sum_{r=0}^{\infty} \xi_{t,r}[\phi_0(t-r) + \varepsilon_{t-r}], \tag{3.16}$$

which is a unique solution of the TV-AR(2)-AGARCH$(1,1)$ model (3.3). In other words y_t is decomposed into a non random part and a zero mean random part. In particular, the time-dependent first moment:

$$\mathbb{E}(y_t) = \lim_{k\to\infty} \mathbb{E}(y_t \,|\mathcal{F}_{t-k}) = \sum_{r=0}^{\infty} \xi_{t,r}\phi_0(t-r) \tag{3.17}$$

is the non random part of y_t while $\lim_{k\to\infty} \mathbb{FE}(y_t \,|\mathcal{F}_{t-k}) = \sum_{r=0}^{\infty} \xi_{t,r}\varepsilon_{t-r}$ is the zero mean random part.

The time-varying expected value of y_t is an infinite sum of the time-varying drifts where the time-varying coefficients are expressed as determinants of continuant matrices (the ξs).

3.3.1.2 Second moments

The current Section and Section 3.3.2.1 below discusses the second-order properties of the TV-AR(2)-AGARCH$(1,1)$ model. Next we state the results for the second moment structure.[32]

The mean square error

$$\mathbb{V}ar[\mathbb{FE}(y_t \,|\mathcal{F}_{t-k})] = \sum_{r=0}^{k-1} \xi_{t,r}^2\sigma_{t-r}^2 \tag{3.18}$$

is a linear combination of k variances from time $t-k+1$ to time t, with time-varying coefficients (the squared ξs).

Moreover, under Assumption 1 the second time-varying unconditional moment of y_t exists and it is given by

$$\mathbb{E}(y_t^2) = [\mathbb{E}(y_t)]^2 + \sum_{r=0}^{\infty} \xi_{t,r}^2\sigma_{t-r}^2, \tag{3.19}$$

which is an infinite sum of the time-varying unconditional variances of the errors, σ_{t-r}^2, (see Section 3.3.2.1 below) with time-varying 'coefficients' or weights (the squared values of the ξs).

In addition, the time-varying autocovariance function $\gamma_{t,k}$ is given by

$$\begin{aligned}\gamma_{t,k} &= \mathbb{C}ov(y_t, y_{t-k}) = \sum_{r=0}^{\infty} \xi_{t,k+r}\xi_{t-k,r}\sigma_{t-k-r}^2 \\ &= \xi_{t,k}\mathbb{V}ar(y_{t-k}) + \phi_2(t-k+1)\xi_{t,k-1}\mathbb{C}ov(y_{t-k}, y_{t-k-1}),\end{aligned} \tag{3.20}$$

---

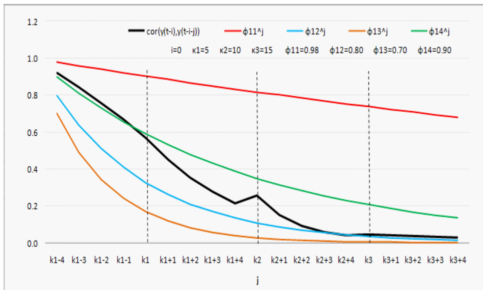
32 Estimating the time-varying parameters of forecasting models is beyond the scope of this study (see Elliott and Timmermann, 2008, for an excellent survey on forecasting methodologies available to the applied economist).

where the second equality follows from the MA(∞) representation of y_t in eq. (3.16) and the third one from the general solution in eq. (3.13) and

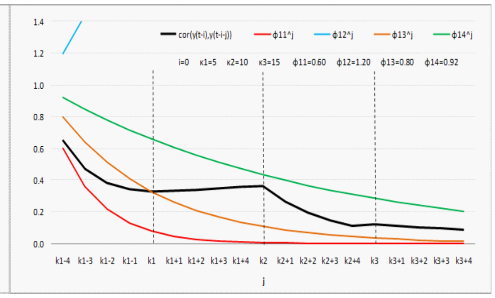
$$\mathbb{C}ov(y_{t-k}, y_{t-k-1}) = \sum_{r=0}^{\infty} \xi_{t-k,r+1}\xi_{t-k-1,r}\sigma^2_{t-k-1-r}.$$

For any fixed t, $\lim_{k\to\infty} \gamma_{t,k} \to 0$ when $\lim_{k\to\infty} \xi_{t,k} = 0\ \forall\ t$. For the process with n abrupt breaks in eq. (3.1) $\xi_{t,k}$ is given by eq. (3.10).

Panel A: AR(1) Model; 3 Breaks at: $t-5$, $t-10$ and $t-15$;

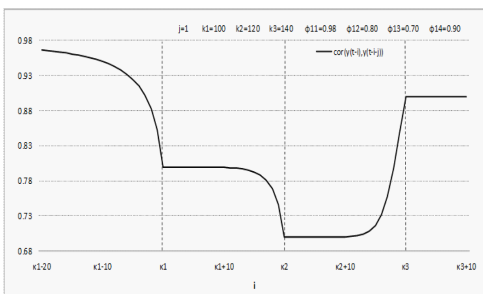


$\mathbb{C}or(y_t, y_{t-j}), j = 1, \ldots, 19$

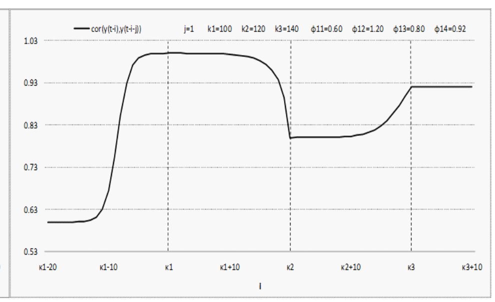


$\mathbb{C}or(y_t, y_{t-j}), j = 1, \ldots, 19$

Panel B: AR(1) Model; 3 Breaks at: $t-100$, $t-120$ and $t-140$;

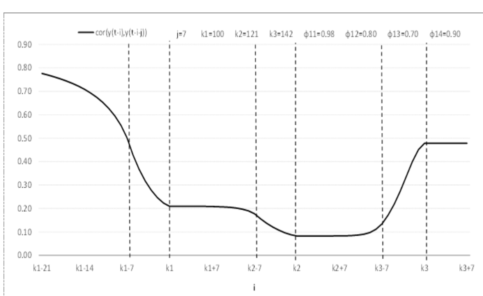


$\mathbb{C}or(y_{t-i}, y_{t-i-1}), i = 80, \ldots, 150$

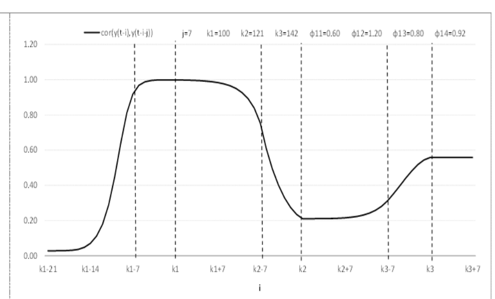


$\mathbb{C}or(y_{t-i}, y_{t-i-1}), i = 80, \ldots, 150$

Panel C: AR(1) Model; 3 Breaks at: $t-100$, $t-121$ and $t-142$;



$\mathbb{C}or(y_{t-i}, y_{t-i-7}), i = 79, \ldots, 149$



$\mathbb{C}or(y_{t-i}, y_{t-i-7}), i = 79, \ldots, 149$

Figure 3.1: Time-varying Autocorrelations

As an illustrative example Figure 3.1 shows the autocorrelations (ACR) of an AR(1) model with three breaks and homoscedastic/independent innovations. The left graph in Panel B shows the first order ACR, $\mathbb{C}or(y_{t-i}, y_{t-i-1})$, for an AR(1) model with three breaks at times $t - k_1(= 100)$, $t - k_2(= 120)$ and $t - k_3(= 140)$ and autoregressive coefficients $\phi_{1,1} = 0.98$, $\phi_{1,2} = 0.80$, $\phi_{1,3} = 0.70$ and $\phi_{1,4} = 0.90$. The first part of the graph shows the ACR when $i < k_1 = 100$, that is, when y_{t-i} is after all three breaks: $t - i > t - k_1$ (the construction of the autocorrelations is based on eq. (3.20)). As i increases, that is, as we are going back in time, the first order ACR decrease at an increasing rate. The second part of the graph shows the ACR when $k_1 \leq i \leq k_2 - 1$, that is, when y_{t-i} is between the first and the second break. The third part of the graph shows the ACR when $k_2 \leq i \leq k_3 - 1$. The ACR increase since after the third break the autoregressive coefficient increases from 0.70 to 0.90. Finally, for $i \geq k_3$, the first order ACR are not affected by the three breaks and therefore are equal to $\phi_{1,4} = 0.90$, whereas when $i \to -\infty$, the ACR converge to $\phi_{1,1} = 0.98$.

Moreover, the right graph in Panel C shows the seventh order ACR (y_{t-i}, y_{t-i-7}) for an AR(1) model with three breaks at times $t - k_1(= 100)$, $t - k_2(= 121)$ and $t - k_3(= 142)$, autoregressive coefficients $\phi_{1,1} = 0.60$, $\phi_{1,2} = 1.20$, $\phi_{1,3} = 0.80$ and $\phi_{1,4} = 0.92$ and homoscedastic/independent innovations. The second part of the graph shows the ACR when $i \leq k_1 - 1$ and $k_1 + 1 \leq i + 7 \leq k_2$. The fourth part of the graph shows the ACR when $k_1 \leq i \leq k_2 - 1$ and $k_2 + 1 \leq i + 7 \leq k_3$. The sixth part of the graph shows the ACR when $k_2 \leq i \leq k_3 - 1$ and $k_3 + 1 \leq i + 7$. Notice that when $i \leq k_1 - 1$ or $k_2 \leq i \leq k_3 - 1$ the seventh order ACR increase with i whereas when $k_1 \leq i \leq k_2 - 1$ they decrease as i increases. Finally, for $i \geq k_3$, the ACR are equal to $\phi_{1,4}^7 = 0.56$, whereas when $i \to -\infty$, the ACR converge to $\phi_{1,1}^7 = 0.03$.

3.3.2 The conditional variance

In order to simplify the description of the analysis of this Section we will introduce the following notation. As before t represents the present time and k the prediction horizon. We define the bivariate function $\varsigma : \mathbb{Z} \times \mathbb{Z}^{+} \longmapsto \mathbb{R}$ by

$$\varsigma_{t,k} = \prod\nolimits_{j=0}^{k-1} c(t-j), \tag{3.21}$$

coupled with the initial values $\varsigma_{t,0} = 1$ and $\varsigma_{t,-1} = 0$ where $c(\cdot)$ has been defined above (see eq. (3.5)). In other words $\varsigma_{t,1} = c(t)$ and $\varsigma_{t,k}$ for $k \geq 2$ is a product of k terms which consist of the time-varying coefficients $c(\cdot)$ from time $t-k+1$ to time t. For the GARCH process with m abrupt breaks in eq. (3.2) we have

$$\varsigma_{t,k} = \prod\nolimits_{\ell=0}^{m} c_{\ell+1}^{\kappa_{\ell+1}-\kappa_{\ell}}. \tag{3.22}$$

Next, we define

$$g_{t,r+1} = \varsigma_{t,r}\alpha^{*}(t-r),\ r \geq 0, \tag{3.23}$$

where $\alpha^{*}(t)$ has been defined in eq. (3.4). Notice that when $r = 0$, $g_{t,1} = \alpha^{*}(t)$, since $\varsigma_{t,0} = 1$.

Since the TV-AGARCH$(1,1)$ model can be interpreted as a 'TV-ARMA$(1,1)$' process, it follows directly from the results in Section 3.3.1 that the general solution of eq. (3.5) with free constant (initial condition value) h_{t-k}, is given by

$$h_{t,k}^{gen} = h_{t,k}^{hom} + h_{t,k}^{par} = \varsigma_{t,k}h_{t-k} + \sum_{r=0}^{k-1} \varsigma_{t,r}\omega(t-r) + \sum_{r=1}^{k} g_{t,r}v_{t-r}, \tag{3.24}$$

where $\varsigma_{t,r}$ and $g_{t,r}$ have been defined in eqs. (3.21) and (3.23) respectively. In the above expression h_t^{gen} is decomposed into two parts: the $h_{t,k}^{hom}$ part, which is written in terms of the free constant (h_{t-k}); and the $h_{t,k}^{par}$ part, which contains the time-varying drift terms, $\omega(\cdot)$ and the uncorrelated terms (vs). Notice that in eq. (3.24) $h_{t,k}^{gen}$ is expressed in terms of diagonal determinants (the ςs and therefore the gs).

Next consider the case of a GARCH$(1,1)$ model with constant coefficients. Since for this model $\alpha(t) \triangleq a$ and $c(t) \triangleq c \triangleq \alpha + b$, for all t, then $\varsigma_{t,k}$ reduces to c^k and $g_{t,k}$ becomes $c^{k-1}a$, for $k \in \mathbb{Z}^{+}$ (see, for example, Karanasos, 1999).

3.3.2.1 Time-varying unconditional variances

In this Section in order to provide a thorough description of the TV-AGARCH$(1,1)$ process given by eq. (3.4) we derive, first its multistep ahead predictor, the associated forecast error and the mean square error and, second, the first unconditional moment of this process.

The k-step-ahead predictor of h_t, $\mathbb{E}(h_t\,|\mathcal{F}_{t-k-1})$, is readily seen to be[33]

$$\mathbb{E}(h_t\,|\mathcal{F}_{t-k-1}) = \sum_{r=0}^{k-1}\overline{\varsigma}_{t,r}\omega(t-r) + \overline{\varsigma}_{t,k}h_{t-k}, \tag{3.25}$$

where, for $r \geq 1$, $\overline{\varsigma}_{t,r} = \mathbb{E}(\varsigma_{t,r})$.[34] In addition, the forecast error for the above k-step-ahead predictor (for the symmetric case), $\mathbb{FE}(h_t\,|\mathcal{F}_{t-k-1})$, is given by

$$\mathbb{FE}(h_t\,|\mathcal{F}_{t-k-1}) = \sum_{r=1}^{k} g_{t,r}v_{t-r}. \tag{3.26}$$

Notice that this predictor is expressed in terms of k uncorrelated terms (the vs) from time $t-k$ to time $t-1$, where the 'coefficients' have the form of diagonal determinants (the ςs). The mean square error is given by

$$\mathbb{V}ar(h_t\,|\mathcal{F}_{t-k-1}) = \mathbb{V}ar[\mathbb{FE}(h_t\,|\mathcal{F}_{t-k-1})] = \widetilde{\varkappa}\sum_{r=1}^{k} g_{t,r}^2\mathbb{E}(h_{t-r}^2). \tag{3.27}$$

This is expressed in terms of k second moments, $\mathbb{E}(h_{t-r}^2)$, from time $t-k$ to time $t-1$, where the coefficients are the expectations of the squared coefficients of the multistep ahead predictor multiplied by $\widetilde{\varkappa}$. Moreover, the definition of the uncorrelated term v_t implies that $\mathbb{E}(\varepsilon_t^2\,|\mathcal{F}_{t-k-1}) = \mathbb{E}(h_t\,|\mathcal{F}_{t-k-1})$, $\mathbb{FE}(\varepsilon_t^2\,|\mathcal{F}_{t-k-1}) = v_t + \mathbb{FE}(h_t\,|\mathcal{F}_{t-k-1})$. The associated mean squared error is given by $\mathbb{V}ar[\mathbb{FE}(\varepsilon_t^2\,|\mathcal{F}_{t-k-1})] = \widetilde{\varkappa}\mathbb{E}(h_t^2) + \mathbb{V}ar[\mathbb{FE}(h_t\,|\mathcal{F}_{t-k-1})] = \widetilde{\varkappa}\sum_{r=0}^{k} g_{t,r}^2\mathbb{E}(h_{t-r}^2)$.

Next to obtain the first unconditional moment of h_t, for all t, we impose the conditions that: $\sum_{r=0}^{k}\overline{\varsigma}_{t,r}\omega(t-r)$ as $k \to \infty$ is positive and converges and

$$\widetilde{\varkappa}\sum_{r=1}^{\infty}\sup_t[\overline{g}_{t,r}^2\mathbb{E}(h_{t-r}^2)] < M < \infty,\ M \in \mathbb{Z}^+, \tag{3.28}$$

[33] For the issue of temporal aggregation and a discussion of the wider class of weak GARCH processes see Bollerslev and Ghysels (1996) and Ghysels and Osborn (2001, pp. 195-197).

[34] $\mathbb{E}(\varsigma_{t,r}) = \mathbb{E}[\prod_{j=0}^{r-1} c(t-j)] = \prod_{j=0}^{r-1}\overline{c}(t-j)$ with $\overline{c}(t) \triangleq \mathbb{E}[c(t)] = \alpha(t) + \beta(t) + \frac{\gamma(t)}{2}$. For the process with m abrupt breaks: $\mathbb{E}(\varsigma_{t,r}) = \prod_{\ell=0}^{m}\overline{c}_{\ell+1}^{\kappa_{\ell+1}-\kappa_\ell}$.

where $\overline{g}^2_{t,r} = \mathbb{E}(g^2_{t,r})$ for $r \geq 1$[35]. This guarantees that, for all t, the model in eq. (3.5) admits the second-order MA(∞) representation:

$$h^{gen}_{t,\infty} = \lim_{k\to\infty} h^{par}_{t,k} \overset{L_2}{=} \sum_{r=0}^{\infty} \overline{\varsigma}_{t,r}\omega(t-r) + \sum_{r=1}^{\infty} g_{t,r}v_{t-r}, \tag{3.29}$$

which is a unique solution of the TV-AGARCH$(1,1)$ model in eq. (3.4). The above result states that $\{h^{par}_{t,k}, t \in \mathbb{Z}^+\}$ (defined in eq. (3.24)) L_2 converges as $k \to \infty$ if and only if $\sum_{r=0}^{k} \overline{\varsigma}_{t,r}\omega(t-r)$ as $k \to \infty$ converges and $\sum_{r=1}^{k} g_{t,r}v_{t-r}$ converges a.s. and thus under the aforementioned conditions $h^{gen}_{t,\infty} \overset{L_2}{=} \lim_{k\to\infty} h^{par}_{t,k}$ satisfies eq. (3.24).

Moreover, the first time-varying unconditional moment of h_t, $\mathbb{E}(h_t) = \sigma^2_t$, is the limit of the $(k+1)$-step-ahead predictor of h_t, $\mathbb{E}(h_t\,|\mathcal{F}_{t-k-1})$, as $k \to \infty$:

$$\mathbb{E}(h_t) = \lim_{k\to\infty} \mathbb{E}(h_t\,|\mathcal{F}_{t-k-1}) = \sum_{r=0}^{\infty} \overline{\varsigma}_{t,r}\omega(t-r). \tag{3.30}$$

Notice of course that the first moment is time-varying. The expected value of the conditional variance, that is the unconditional variance of the error, is an infinite sum of the time-varying drifts where the coefficients (the $\overline{\varsigma}$s) are expressed as expectations of diagonal determinants. Finally, for the process with m abrupt breaks in eq. (3.2), for $i \leq \kappa_1$ we have (if and only if $\overline{c}_{m+1} < 1$):

$$\mathbb{E}(h_{t-i}) = \frac{1-\overline{c}_1^{\kappa_1 - i}}{1-\overline{c}_1}\omega_1 + \sum_{\ell=2}^{m} \widetilde{c}_\ell \frac{1-\overline{c}_\ell^{\kappa_\ell - \kappa_{\ell-1}}}{1-\overline{c}_\ell}\omega_\ell + \widetilde{c}_{m+1}\frac{1}{1-\overline{c}_{m+1}}\omega_{m+1}, \tag{3.31}$$

with

$$\widetilde{c}_\ell = \overline{c}_1^{\kappa_1 - i} \prod_{j=2}^{\ell-1} (\overline{c}_j^{\kappa_j - \kappa_{j-1}}),$$

where we use the convention $\prod_{r=i}^{j}(\cdot) = 1$ for $j < i$ and the ωs and the cs are defined in eqs. (3.4) and (3.5) respectively. Notice that if and only if $\overline{c}_1 < 1$ the above expression as $i \to -\infty$ becomes: $\mathbb{E}(h_{t-i}) = \frac{\omega_1}{1-\overline{c}_1}$ since $\widetilde{c}_\ell = \overline{c}_1^{\kappa_1 - i} = 0$ for all ℓ. Finally, when $i > \kappa_m$, that is when we are before all the breaks, then if and only if $\overline{c}_{m+1} < 1$: $\mathbb{E}(h_{t-i}) = \frac{\omega_{m+1}}{1-\overline{c}_{m+1}}$.

3.4 Methodology and data

This Section outlines the methodology we have employed to study the different properties of

[35] $\mathbb{E}(g^2_{t,r+1}) = \mathbb{E}(\varsigma^2_{t,r})[\alpha^2(t-r) + \gamma^2(t-r)/2 + \alpha(t-r)\gamma(t-r)]$ and, for $r \geq 1$, $\mathbb{E}(\varsigma^2_{t,r}) = \prod_{j=0}^{r-1} \mathbb{E}[c^2(t-j)]$, with $\mathbb{E}[c^2(t)] = [\alpha(t)+\beta(t)]^2 + \gamma^2(t)/2 + [\alpha(t)+\beta(t)]\gamma(t)$.

the stochastic processes around the 2007-08 crisis and offers an overview of the data employed. First, we describe the univariate models we have estimated. Then we describe the break identification method which we have adopted. Finally, we provide a brief discussion of our data.

3.4.1 Univariate modelling

Let stock returns be denoted by $r_t = (\log p_t - \log p_{t-1}) \times 100$, where p_t is the stock price index and define its mean equation as:

$$r_t = \mu + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \varepsilon_t, \tag{3.32}$$

where $\varepsilon_t \mid \mathcal{F}_{t-1} \sim N(0,\, h_t)$, that is the innovation is conditionally normal with zero mean and variance h_t.[36] Next, the dynamic structure of the conditional variance is specified as an AGARCH$(1,1)$ process of Glosten et al. (1993) (the asymmetric power ARCH could also be employed, as in Karanasos and Kim, 2006). In order to examine the impact of the breaks on the persistence of the conditional variances, the following equation is specified as follows:

$$\begin{aligned} h_t \;&=\; \omega + \sum_{i=1}^{7} \omega_i D_i + \alpha \varepsilon_{t-1}^2 + \sum_{i=1}^{7} \alpha_i D_i \varepsilon_{t-1}^2 + \gamma S_{t-1}^{-} \varepsilon_{t-1}^2 + \sum_{i=1}^{7} \gamma_i D_i S_{t-1}^{-} \varepsilon_{t-1}^2 \\ &\quad + \beta h_{t-1} + \sum_{i=1}^{7} \beta_i D_i h_{t-1}, \end{aligned} \tag{3.33}$$

where $S_{t-1}^{-} = 1$ if $e_{t-1} < 0$ and 0 otherwise. Note that failure to reject $H_0 : \gamma = 0$ and $\gamma_i = 0$, $i = 1, \ldots, 7$, implies that the conditional variance follows a symmetric GARCH$(1,1)$ process. Furthermore, the second order conditions require that $\overline{c} < 1$ and $\overline{c} + \sum_{i=1}^{7} \overline{c}_i < 1$.[37] The breakdates $i = 1, ..., 7$ are given in Table 3.1 and D_i are dummy variables defined as 0 in the period before each break and one after the break.[38] We also consider a simple GARCH$(1,1)$ model which allows

36 Since mainly structural breaks in the variance are found statistically significant (see Section 3.5.1 below) we do not include any dummies in the mean. Moreover, low order AR specifications capture the serial correlation in stock returns.

37 $\overline{c} \triangleq \alpha + \beta + \frac{\gamma}{2}$ and $\overline{c}_i \triangleq \alpha_i + \beta_i + \gamma_i/2$.

38 The relation between the parameters in eq. (3.33) and the ones in eq. (3.2) is given by, i.e., for the ωs: $\omega + \sum_{i=1}^{m+1-\ell} \omega_i = \omega_\ell$, $\ell = 1, \ldots, m+1$, where the ωs in the right hand side are the ones in eq. (3.2).

the dynamics of the conditional variances to switch across positive and negative stock returns. This is given by

$$h_t = \omega + \omega^- D_{t-1}^- + \alpha\varepsilon_{t-1}^2 + \alpha^- D_{t-1}^- \varepsilon_{t-1}^2 + \beta h_{t-1} + \beta^- D_{t-1}^- h_{t-1}. \tag{3.34}$$

where $D_{t-1}^- = 1$ if $r_{t-1} < 0$, 0 otherwise.[39] This is an example of a TV-AGARCH model with stochastic coefficients.

3.4.2 Data and breaks overview

We use daily data that span the period 1/1/1988 - 30/6/2010 for the stock market indices, obtained from Thomson DataStream (same as in Chapter 2). To account for the possibility of breaks in the mean and/or volatility dynamics we use a set of non-parametric data-driven methods to identify the number and timing of the potential structural breaks. In particular, we adopt the two-stage Nominating-Awarding procedure of Karoglou (2010) to identify breaks that might be associated either to structural changes in the mean and/or volatility dynamics or to latent non-linearities that may manifest themselves as dramatic changes in the mean and/or volatility dynamics and might bias our analysis.[40] Alternatively, we could choose the break points by employing the methodologies in Kim and Kon (1999), Bai and Perron (2003a) and Lavielle and Moulines (2000) (see, for example, Karanasos and Kartsaklas, 2009 and Campos et al., 2012).

3.5 Empirical analysis

This Section presents the empirical results we obtain from the different econometric tools. First, we present the breaks that we have identified and discuss the possible economic events that may be associated with them. Then we focus on the stock market returns and condition our analysis based on these breaks to discuss first the findings from the univariate modelling and then from the bivariate one (presented in Section 3.6).

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39 We estimate another specification with $\alpha^+ D_{t-1}^+$, $\beta^+ D_{t-1}^+$, and $\omega^+ D_{t-1}^+$, instead of $\alpha^- D_{t-1}^-$, $\beta^- D_{t-1}^-$, and $\omega^- D_{t-1}^-$, where $D_{t-1}^+ = 1$ if $r_{t-1} > 0$, 0 otherwise. The results (not reported) are very similar.

40 The details of the two stages in the Nominating-Awarding procedure and a summary of the statistical properties of stock market returns are presented in the second Chapter.

3.5.1 Estimated breaks

After applying the Nominating-Awarding procedure on stock market returns we find that the stochastic behaviour of all indices yields about three to seven breaks during the sample period, roughly one every two to four years on average. The predominant feature of the underlying segments is that mainly changes in variance are found statistically significant. Finally, there are several breakdates that are either identical in all series or very close to one another, which apparently signify economic events with a global impact.

It appears that dates for the extraordinary events of the Asian financial crisis of 1997, the Global financial crisis of 2007–08 and the European sovereign-debt crisis that followed are clearly identified in all stock return series and with very little or no variability (see Table 3.1). Other less spectacular events, such as the Russian financial crisis of 1998 or the Japanese asset price bubble of 1986-1991 or the UK's withdrawal from the European Exchange Rate Mechanism (ERM), can also be associated with the breakdates that have been identified in some series.[41]

Table 3.1: The break points (Stock Returns)

Break	S&P	TSE	CAC	DAX	FTSE	Hang Seng	NIKKEI	STRAITS
1	**27/03/97**	**05/11/96**	**17/03/97**	27/08/91	**22/10/92**	**24/10/01**	**21/02/90**	**26/08/91**
2	**04/09/08**	15/01/08	**31/07/98**	**21/07/97**	**13/07/98**	**27/07/07**	04/01/08	**28/08/97**
3	**31/03/09**	**02/04/09**	**15/01/08**	**17/06/03**	**24/07/07**	05/05/09	03/04/09	**06/06/00**
4	16/07/09	**19/08/09**	03/04/09	**15/01/08**	06/04/09	**01/12/09**		**26/07/07**
5	27/04/10		27/04/10	03/04/09	27/04/10			**28/05/09**
6								**25/08/09**
7								28/04/10

Notes: The dates in bold indicate breakdates for which, in the univariate estimation (see Table 3.2), at least one dummy variable is significant, i.e, for the S&P index for the $04/09/08$ breakdate β_2 and γ_2 are significant. The underlined dates indicate breakdates for which, in the bivariate estimation (see Tables 3.6 and 3.8), at least one dummy variable is significant, i.e., for the NIKKEI-HS bivariate model, for the $01/12/09$ breakdate α_{12}^{4} is significant.

3.5.2 Univariate results

The quasi-maximum likelihood estimates of the AGARCH$(1,1)$ model allowing the drifts (the ω_s) as well as the 'dynamics of the conditional variance' (the α_s, β_s and γ_s) to switch across the

41 A detailed account of the possible associations that can be drawn between each breakdate for stock returns and a major economic event that took place at or around the breakdate period either in the world or in each respective economy is presented in the second Chapter, as is a summary of the descriptive statistics of each segment.

considered breaks, as in eq. (3.33), are reported in Table 3.2. The estimated models are shown to be well-specified: there is no linear or nonlinear dependence in the residuals in all cases, at the 5% level. Note that the insignificant parameters are excluded. The impact of the breaks on the ω is insignificant in all eight cases. However, there exists a significant impact of the breaks on the 'dynamic structure of the conditional variance' for all stock returns (irrespective of whether a symmetric GARCH$(1, 1)$ or an AGARCH $(1, 1)$ model is considered). More specifically, while the ARCH parameter shows time-varying features across a single break in the cases of S&P and DAX, for CAC and Hang Seng it is shifted across two breaks and for STRAITS it is shifted across three breaks (see the α_i coefficients). With regard to the GARCH parameter, CAC and NIKKEI show time-varying parameters for only one break, but S&P, TSE and FTSE across two breaks. Furthermore, the GARCH parameter shows a time-varying pattern across three breaks in the case of DAX and across five breaks in the case of STRAITS.

Table 3.2: The estimated univariate AGARCH (1,1) allowing for breaks in the variance

	S&P	TSE	CAC	DAX	FTSE	Hang Seng	NIKKEI	STRAITS
μ	0.012^a	0.011^a	0.010^c	0.019^a	0.009^b	0.019^a	0.006	0.010^b
	(0.004)	(0.003)	(0.006)	(0.005)	(0.004)	(0.005)	(0.005)	(0.005)
ϕ_1		0.129^a				0.079^a		0.124^a
		(0.013)				(0.014)		(0.016)
ω	0.001^c	0.003^a	0.005^a	0.011^a	0.002^a	0.015^a	0.007^a	0.018^a
	(0.0002)	(0.0007)	(0.0004)	(0.0006)	(0.0003)	(0.003)	(0.001)	(0.004)
α	0.018^a	0.012^c	0.006^b	0.031^a	0.013^a	0.039^a	0.019^a	0.018^c
	(0.006)	(0.007)	(0.003)	(0.006)	(0.004)	(0.007)	(0.005)	(0.010)
α_1	-0.039^a					-0.050^a		0.059^a
	(0.008)					(0.011)		(0.013)
α_2			0.011^c			0.068^a		
			(0.006)			(0.014)		
α_3			-0.044^a	-0.050^a				
			(0.016)	(0.011)				
β	0.954^a	0.906^a	0.936^a	0.861^a	0.952^a	0.866^a	0.820^a	0.854^a
	(0.002)	(0.016)	(0.003)	(0.002)	(0.001)	(0.013)	(0.026)	(0.011)
β_1					-0.019^a		0.081^a	-0.112^a
					(0.002)		(0.021)	(0.029)
β_2	-0.048^a		-0.031^a	0.029^a	-0.019^a			0.115^a
	(0.009)		(0.003)	(0.007)	(0.006)			(0.029)
β_3	0.039^a	0.017^c		-0.029^b				-0.076^a
	(0.015)	(0.009)		(0.012)				(0.018)
β_4		-0.025^c		0.038^a				0.137^a
		(0.013)		(0.006)				(0.029)
γ	0.023^c	0.028^a	0.056^a	0.117^a	0.029^a	0.130^a	0.117^a	0.105^a
	(0.012)	(0.009)	(0.004)	(0.023)	(0.006)	(0.021)	(0.013)	(0.017)
γ_1	0.092^a	0.097^a	0.035^a		0.028^a			
	(0.014)	(0.023)	(0.007)		(0.005)			
γ_2	0.113^a		0.019^b		0.055^a			
	(0.027)		(0.009)		(0.016)			
γ_3	-0.094^a		0.117^a	0.075^c	0.026^b			
	(0.029)		(0.038)	(0.043)	(0.012)			
$LogL$	-2921.3	-1837.5	-4374.3	-4469.8	-2904.1	-5231.4	-4764.1	-3957.7
$LB(5)$	8.343	2.316	10.870	5.170	9.745	2.928	2.555	3.303
	[0.138]	[0.128]	[0.054]	[0.395]	[0.082]	[0.231]	[0.768]	[0.069]
$LB^2(5)$	1.947	0.759	3.953	5.524	4.192	4.105	8.992	1.635
	[0.856]	[0.979]	[0.556]	[0.354]	[0.522]	[0.534]	[0.109]	[0.897]

Notes: Robust-standard errors are used in parentheses. $LB(5)$ and $LB^2(5)$ are Ljung-Box tests for serial correlations of five lags on the standardised and squared standardised residuals, respectively (p-values reported in brackets). Insignificant parameters are excluded. ^{a}, ^{b} and ^{c} indicate significance at the 1%, 5% and 10% levels, respectively. For the Hang Seng index ϕ_3 and γ_4 are significant and for the STRAITS index $\alpha_4, \alpha_6, \beta_6, \gamma_5$ and γ_6 are also significant.

Interestingly, the asymmetry parameter also displays significant time-variation over the considered breaks. Specifically, the TSE, DAX and Hang Seng cases are significantly shifted for one break, whereas S&P, CAC and FTSE show a time-varying pattern across three breaks and STRAITS for two breaks (see the γ_i coefficients in Table 3.2). Furthermore, the results are shown to be robust by considering the dynamics of a GARCH$(1,1)$ process to switch across positive and negative stock returns (see Table 3.3). Clearly, the ARCH and GARCH parameters show time-dependence across positive and negative returns in all cases (see the α^- and β^- coefficients).

Table 3.3: The estimated univariate GARCH (1, 1) models allowing for different persistence across positive and negative returns: $h_t = \omega + \omega^- D^-_{t-1} + \alpha\varepsilon^2_{t-1} + \alpha^- D^-_{t-1}\varepsilon^2_{t-1} + \beta h_{t-1} + \beta^- D^-_{t-1} h_{t-1}$

	S&P	TSE	CAC	DAX	FTSE	Hang Seng	NIKKEI	STRAITS
μ	0.036^a (0.005)	0.023^a (0.004)	0.044^a (0.007)	0.054^a (0.008)	0.032^a (0.004)	0.051^a (0.007)	0.034^a (0.007)	0.027^a (0.004)
ϕ_1		0.114^a (0.012)				0.069^a (0.013)		0.112^a (0.011)
ω	0.002^a (0.0008)	0.002^a (0.0006)	0.007^a (0.001)	0.008^a (0.002)	0.002^a (0.0005)	0.009^a (0.002)	0.004^a (0.0008)	0.006^a (0.002)
α	0.054^a (0.005)	0.062^a (0.012)	0.070^a (0.008)	0.091^a (0.018)	0.066^a (0.006)	0.088^a (0.011)	0.065^a (0.008)	0.051^a (0.015)
α^-		0.033^c (0.017)				0.033^c (0.020)	0.025^c (0.015)	0.104^a (0.021)
β	0.837^a (0.023)	0.861^a (0.027)	0.822^a (0.023)	0.779^a (0.039)	0.832^a (0.014)	0.815^a (0.025)	0.842^a (0.016)	0.883^a (0.023)
β^-	0.208^a (0.034)	0.106^a (0.024)	0.181^a (0.029)	0.233^a (0.043)	0.187^a (0.023)	0.141^a (0.037)	0.157^a (0.027)	
$LogL$	-2941.2	-1865.7	-4388.4	-4478.8	-2903.4	-5260.7	-4799.1	-4048.6
$LB(5)$	9.526 [0.089]	1.674 [0.195]	3.256 [0.071]	4.464 [0.484]	8.031 [0.154]	4.521 [0.104]	2.180 [0.823]	3.650 [0.056]
$LB^2(5)$	2.398 [0.791]	0.573 [0.989]	4.237 [0.515]	5.340 [0.375]	5.428 [0.365]	4.998 [0.416]	8.430 [0.134]	2.385 [0.793]

Notes: See notes of Table 3.2. The ϕ_3 coefficient was significant for the CAC and Hang Seng indices.

Overall, Table 3.4 shows that the persistence of the conditional variances of stock returns varies over the considered breaks in all cases by considering the AGARCH $(1, 1)$ models. The persistence is measured by $\bar{c}_\ell = \alpha_\ell + \beta_\ell + \gamma_\ell/2$, $\ell = 1, \ldots, m+1$ (these are the $\bar{c}$s used in eq. (3.31) as well) and, for example, $\beta_\ell = \underbrace{\beta + \sum_{i=1}^{m+1-\ell}}_{\text{Eq. (3.33)}} \beta_i$.

Table 3.4: The persistence of the AGARCH (1,1) models

Panel A: The persistence of the standard AGARCH (1,1) models

	S&P	TSE	CAC	DAX	FTSE	Hang Seng	NIKKEI	STRAITS
	0.986	0.986	0.978	0.979	0.985	0.976	0.990	0.990

Panel B: The persistence of the AGARCH (1,1) models allowing for breaks in the variance

Break	S&P	TSE	CAC	DAX	FTSE	Hang Seng	NIKKEI	STRAITS
0	$(\bar{c}_4 =)0.983$	0.932	0.970	$(\bar{c}_4 =)0.950$	0.979	0.970	0.897	0.924
1	$(\bar{c}_3 =)0.990$	0.980	0.987		0.974	0.920	0.978	0.871
2	$(\bar{c}_2 =)0.998$		0.976	$(\bar{c}_3 =)0.979$	0.982	0.988		0.986
3	$(\bar{c}_1 =)0.990$	0.997	0.990	$(\bar{c}_2 =)0.937$	0.995			0.910
4		0.972		$(\bar{c}_1 =)0.976$		0.945		0.974
5								0.948
6								0.884

Notes: Break 0 covers the period preceding all breaks, while break 1 covers the period between break 1 and 2 and break 2 covers the period between break 2 and 3 and so on (see Table 3.1 for the dates of the breaks). When the value of the persistence is left blank for a break, it indicates that such persistence has not changed during the period covered by such a break. The persistence is measured by $\bar{c}_\ell = \alpha_\ell + \beta_\ell + \gamma_\ell/2$, $\ell = 1, \ldots, m+1$ and, for example, $\beta_\ell = \underbrace{\beta + \sum_{i=1}^{m+1-\ell}}_{\text{Eq. (3.33)}} \beta_i$. That is $\bar{c}_{m+1}$ is the persistence before all breaks and $\bar{c}_1$ is the persistence after all the breaks.

The cases which are shown to have been impacted strongly by the breaks are those of TSE, DAX, Hang Seng, NIKKEI and STRAITS. In particular, the persistence of the conditional variance of TSE increases from 0.93 to 0.98 after the break in 1996, remains 0.98 during the recent financial crisis and then increases to near unity after the European sovereign-debt crisis. With regard to the persistence of the conditional variance of DAX, it appears to be unaffected by German reunification, its highest value is 0.98 during the Asian financial crisis, its lowest value is 0.94 after the break associated with the announcement of the €18bn tax cuts plan in Germany ($17/06/03$), it increases to 0.97 on the onset of the recent financial crisis and it remains there during the sovereign-debt crisis. The results also suggest that the persistence of the conditional variance of Hang Seng declines from 0.97 to 0.92 (its lowest value) after the savings deposits were removed in July 2001, it increases to 0.99 during the recent financial crisis in 2007/2008 and finally it declines to 0.94 after the European sovereign-debt crisis. Furthermore, the corresponding persistence of STRAITS increases from 0.87 to near unity (0.99) after the Asian financial crisis.

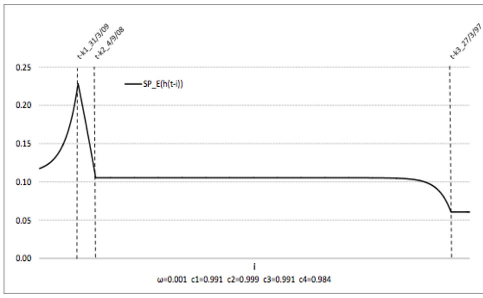
However, such persistence declines after the break in June 2000 to 0.91, remains the same through the unexpected economic recession in Singapore in 2001 before bounding back to 0.97 at the onset of the Global financial crisis and then exhibits a sharp decline to 0.88 during the European sovereign-debt crisis. Surprisingly, the persistence of the conditional variance of NIKKEI increases from 0.90 to approximately 0.98 during the asset price bubble in Japan over the period 1986-1991 and remains unaffected afterwards. For example, the impact of the Asian financial crisis as well as that of the recent financial crisis are shown to be limited, which may be due to the fact that Japan has been immune to such crises.

The persistence of the conditional variances by allowing the GARCH $(1, 1)$ process to switch across positive and negative returns also shows a time-varying pattern (see Table 3.5). In particular, it is shown that the persistence of the conditional variances stemming from positive returns is lower than those of the negative counterparts. More specifically, positive returns are shown to lower the persistence of the conditional variances in most of the cases to around 0.90 whereas the persistence of the negative returns is close to unity (0.99).

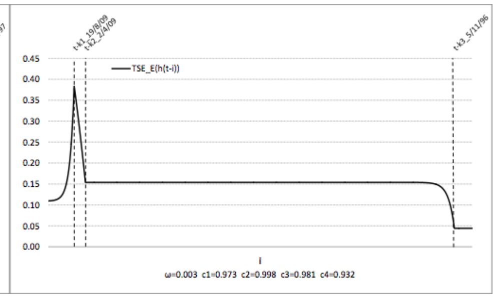
Table 3.5: The persistence of the GARCH (1,1) allowing for different persistence across positive and negative returns

	S&P	TSE	CAC	DAX	FTSE	Hang Seng	NIKKEI	STRAITS
r	0.986	0.986	0.978	0.979	0.985	0.976	0.990	0.990
r^+	0.891	0.923	0.892	0.870	0.898	0.903	0.907	0.934
r^-	0.995	0.992	0.982	0.986	0.991	0.990	0.998	0.986

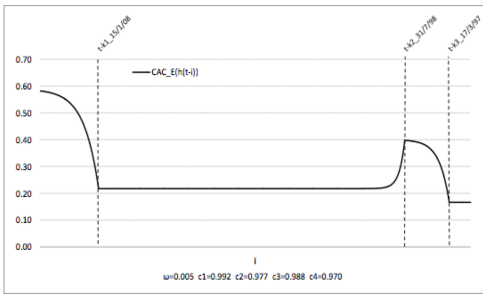
Notes: r denotes the persistence generated from returns, that is from the standard AGARCH model whilst $r^+(r^-)$ corresponds to the persistence generated from positive (negative) returns.



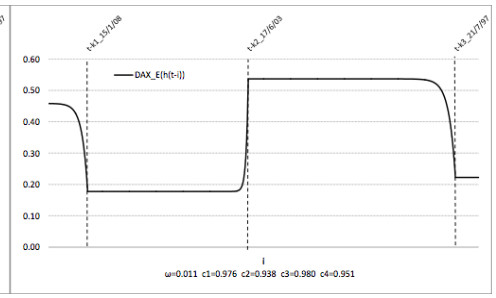
S&P Index



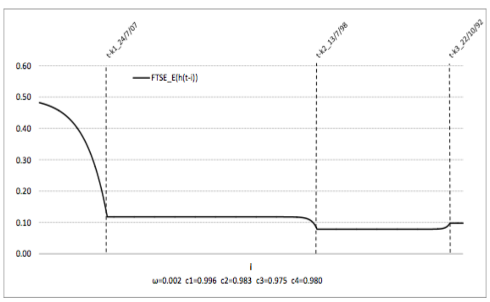
TSE Index



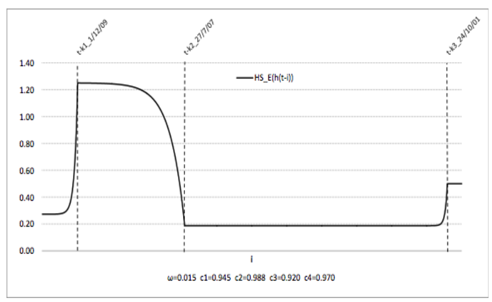
CAC Index



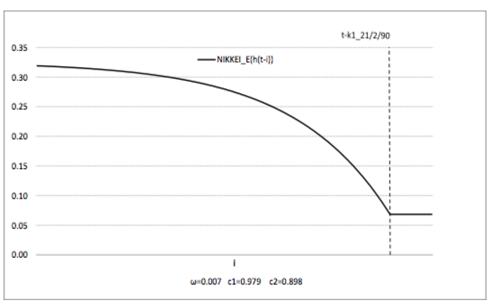
DAX Index



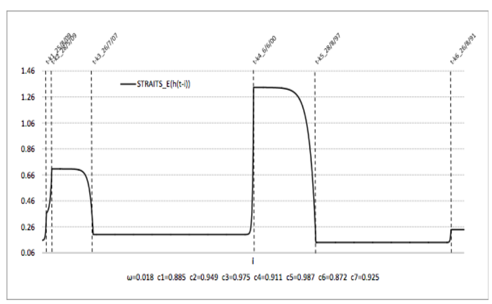
FTSE Index



HS Index



NIKKEI Index



STRAITS Index

Figure 3.2. Unconditional Variances (Stock Returns)

AGARCH(1, 1) model allowing for abrupt breaks in the variance

Figure 3.2 shows the estimated time-varying unconditional variances for the eight stock index returns. For the S&P the first part of the graph shows the unconditional variances when $i < k_1$, that is, when h_{t-i} is after all three breaks ($t-k_3$(=03/97), $t-k_2$(=09/08) and $t-k_1$(=03/09)) (we construct the time-varying unconditional variances using the formula in eq. (3.31)). When $i \to -\infty$, the unconditional variances converge to $\omega/(1-\bar{c}_1) = 0.001/(1-0.990) = 0.100$. As i increases, that is, as we are going back in time, the unconditional variances increase at an increasing rate. The second part of the graph shows the unconditional variances when $k_1 \leq i \leq k_2 - 1$, that is, when h_{t-i} is between the first and the second break. Higher values of i are associated with lower unconditional variances. When $i = k_1$, the unconditional variance is $[(1-\bar{c}_2^{k_2-k_1})/(1-\bar{c}_2) + \bar{c}_2^{k_2-k_1}(1-\bar{c}_3^{k_3-k_2})/(1-\bar{c}_3) + \bar{c}_2^{k_2-k_1}\bar{c}_3^{k_3-k_2}/(1-\bar{c}_4)]\omega = 0.228$ (see eq. (3.31) and the $\bar{c}$s in the first column of Table 3.4). The third part of the graph shows the unconditional variances when $k_2 \leq i \leq k_3 - 1$. When $i = k_2$, the unconditional variance is $[(1-\bar{c}_3^{k_3-k_2})/(1-\bar{c}_3) + \bar{c}_3^{k_3-k_2}/(1-\bar{c}_4)]\omega = 0.105$. Finally, for $i \geq k_3$, the unconditional variances are not affected by the three breaks and therefore are equal to $\omega/(1-\bar{c}_4) = 0.061$.

Similarly, for the DAX the first part of the graph shows the unconditional variances when $i < k_1$, that is, when h_{t-i} is after all three breaks ($t-k_3$(=07/97), $t-k_2$(=06/03) and $t-k_3$(=01/08)). When $i \to -\infty$, the unconditional variances converge to $\omega/(1-\bar{c}_1) = 0.011/(1-0.976) = 0.458$. As i increases, that is, as we are going back in time, the unconditional variances decrease at an increasing rate. The second part of the graph shows the unconditional variances when $k_1 \leq i \leq k_2 - 1$ ($\mathbb{E}(h_{t-k_1}) = 0.177$). Higher values of i are associated with higher unconditional variances. The third part of the graph shows the unconditional variances when $k_2 \leq i \leq k_3 - 1$. They are decreasing with i. Finally, for $i \geq k_3$, the unconditional variances are not affected by the three breaks and therefore are equal to $\omega/(1-\bar{c}_4) = 0.222$.

For the NIKKEI the first part of the graph shows the unconditional variances when $i < k_1$, that is, when h_{t-i} is after the only break ($t-k_1$(=02/90)). When $i \to -\infty$, the unconditional variances converge to $\omega/(1-\bar{c}_1) = 0.326$. As i increases the unconditional variances decrease at an increasing rate. In addition, for $i \geq k_1$, the unconditional variances are not affected by the break and therefore are equal to $\omega/(1-\bar{c}_2) = 0.068$.

Finally, STRAITS exhibits the highest number of breaks, that is six. The first part of the graph shows the unconditional variances when $i < k_1$, that is, when h_{t-i} is after all six breaks ($t-k_6$(=08/91), $t-k_5$(=08/97), $t-k_4$(=06/00), $t-k_3$(=07/07), $t-k_2$(=05/09), $t-k_1$(=08/09)). As i increases, that is, as we are going back in time, the unconditional variances increase at an increasing rate. When $i \to -\infty$, the unconditional variances converge to $\omega/(1-\bar{c}_1) = 0.157$. The second part of the graph shows the unconditional variances when $k_1 \leq i \leq k_2 - 1$. Higher values of i are associated with higher unconditional variances. The third part of the graph shows the unconditional variances when $k_2 \leq i \leq k_3 - 1$. They are decreasing with i. For the fourth and sixth part the unconditional variances increase with i whereas for the fifth part they decrease with i. Finally, for $i \geq k_6$, the unconditional variances are not affected by the six breaks and therefore are equal to $\omega/(1-\bar{c}_7) = 0.238$.

3.6 Bivariate models

In this Section we use a bivariate extension of the univariate formulation of Section 3.4.1 In particular, we use a bivariate model to simultaneously estimate the conditional means, variances and covariances of stock returns. Let $\mathbf{y}_t = (y_{1,t}, y_{2,t})'$ represent the 2×1 vector with the two returns. $\mathcal{F}_{t-1} = \sigma(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots)$ is the filtration generated by the information available up through time $t-1$. We estimate the following bivariate AR(2)-AGARCH$(1,1)$ model

$$\mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\Phi}_1 \mathbf{y}_{t-1} + \boldsymbol{\Phi}_2 \mathbf{y}_{t-2} + \boldsymbol{\varepsilon}_t, \tag{3.35}$$

where $\boldsymbol{\mu} = [\mu_i]_{i=1,2}$ is a 2×1 vector of drifts and $\boldsymbol{\Phi}_l = [\phi_{ij}^{(l)}]_{i,j=1,2}$, $l = 1, 2$, is a 2×2 matrix of autoregressive parameters. We assume that the roots of $\left|\mathbf{I} - \sum_{l=1}^{2} \boldsymbol{\Phi}_l L^l\right|$ (where $\mathbf{I}$ is the 2×2 identity matrix) lie outside the unit circle.

Let $\mathbf{h}_t = (h_{1,t}, h_{2,t})'$ denote the 2×1 vector of $\mathcal{F}_{t-1}$ measurable conditional variances. The residual vector is defined as $\boldsymbol{\varepsilon}_t = (\varepsilon_{1,t}, \varepsilon_{2,t})' = [\mathbf{e}_t \odot \mathbf{q}_t^{\wedge -1/2}] \odot \mathbf{h}_t^{\wedge 1/2}$, where the symbols $\odot$ and $^\wedge$ denote the Hadamard product and the elementwise exponentiation respectively. The stochastic vector $\mathbf{e}_t = (e_{1,t}, e_{2,t})'$ is assumed to be independently and identically distributed ($i.i.d.$) with mean zero, conditional variance vector $\mathbf{q}_t = (q_{11,t}, q_{22,t})'$ and 2×2 conditional correlation matrix $\mathbf{R}_t = diag\{\mathbf{Q}_t\}^{-1/2}\mathbf{Q}_t diag\{\mathbf{Q}_t\}^{-1/2}$ with diagonal elements equal to one and off-diagonal elements absolutely less than one**.** A typical element of $\mathbf{R}_t$ takes the form $\rho_{ij,t} = q_{ij,t} / \sqrt{q_{ii,t}\, q_{jj,t}}$ for $i, j = 1, 2$. The conditional covariance matrix $\mathbf{Q}_t = [q_{ij,t}]_{i,j=1,2}$ is specified as in Engle (2002a)

$$\mathbf{Q}_t = (1 - \alpha_D - \beta_D)\bar{Q} + \alpha_D \mathbf{e}_{t-1}\mathbf{e}'_{t-1} + \beta_D \mathbf{Q}_{t-1}, \tag{3.36}$$

where $\bar{Q}$ is the unconditional covariance matrix of $\mathbf{e}_t$ and α_D and β_D are non-negative scalars fulfilling $\alpha_D + \beta_D < 1$.

Following Conrad and Karanasos (2010) and Rittler (2012), we impose the UEDCC-AGARCH$(1,1)$ structure on the conditional variances (multivariate fractionally integrated APARCH models could also be used, as in Conrad et al., 2011 or Karanasos et al., 2014) and we also amend it by allowing the shock and volatility spillovers parameters to be time-varying:

$$\mathbf{h}_t = \boldsymbol{\omega} + \mathbf{A}^* \boldsymbol{\varepsilon}_{t-1}^{\wedge 2} + \sum_{l=1}^{n} \mathbf{A}_l D_l \boldsymbol{\varepsilon}_{t-1}^{\wedge 2} + \mathbf{B}\mathbf{h}_{t-1} + \sum_{l=1}^{n} \mathbf{B}_l D_l \mathbf{h}_{t-1}, \tag{3.37}$$

where $\boldsymbol{\omega} = [\omega_i]_{i=1,2}$, $\mathbf{A} = [\alpha_{ij}]_{i,j=1,2}$, $\mathbf{B} = [\beta_{ij}]_{i,j=1,2}$; $\mathbf{A}_l$, $l = 1, \ldots, n$ (and $n = 0, 1, \ldots, 7$) is a cross diagonal matrix with nonzero elements α_{ij}^l, $i, j = 1, 2$, $i \neq j$ and $\mathbf{B}_l$, is a cross diagonal matrix with nonzero elements β_{ij}^l, $i, j = 1, 2$, $i \neq j$; $\mathbf{A}^* = \mathbf{A} + \boldsymbol{\Gamma}\mathbf{S}_{t-1}$, $\boldsymbol{\Gamma}$ is a diagonal matrix with elements γ_{ii}, $i = 1, 2$ and $\mathbf{S}_{t-1}$ is a diagonal matrix with elements $S_{i,t-1}^- = 1$ if $e_{i,t-1} < 0$, 0 otherwise. The model without the breaks for the shock and volatility spillovers, that is $\mathbf{h}_t = \boldsymbol{\omega} + \mathbf{A}^{*}\boldsymbol{\varepsilon}_{t-1}^{\wedge 2} + \mathbf{B}\mathbf{h}_{t-1}$, is minimal in the sense of Jeantheau (1998, Definition 3.3) and invertible (see Assumption 2 in Conrad and Karanasos, 2010). The invertibility condition implies that the inverse roots of $|\mathbf{I} - \mathbf{B}L|$, denoted by φ_1 and φ_2, lie inside the unit circle. Following Conrad and Karanasos (2010) we also impose the four conditions which are necessary and sufficient for $\mathbf{h}_t \succ 0$ for all t: (i) $(1 - b_{22})\omega_1 + b_{12}\omega_2 > 0$ and $(1 - b_{11})\omega_2 + b_{21}\omega_1 > 0$, (ii) φ_1 is real and $\varphi_1 > |\varphi_2|$, (iii) $\mathbf{A}^{*} \succeq 0$ and (iv) $[\mathbf{B}{-}\max(\varphi_2, 0)\mathbf{I}]\mathbf{A}^{*} \succ 0$, where the symbol $\succ$ denotes the elementwise inequality operator. Note that these constraints do not place any *a priori* restrictions on the signs of the coefficients in the $\mathbf{B}$ matrix. In particular, these constraints imply that negative volatility spillovers are possible. Finally, if conditional correlations are constant, the model reduces to the UECCC-GARCH$(1, 1)$ specification of Conrad and Karanasos (2010).

Finally, we also amend the UEDCC-AGARCH$(1, 1)$ model by allowing shocks and volatility spillovers to vary across positive and negative returns:

$$\mathbf{h}_t = \boldsymbol{\omega} + \mathbf{A}^{*}\boldsymbol{\varepsilon}_{t-1}^{\wedge 2} + \mathbf{B}^{*}\mathbf{h}_{t-1}, \tag{3.38}$$

where $\mathbf{A}^{*} = \mathbf{A} + \boldsymbol{\Gamma}\mathbf{S}_{t-1} + \mathbf{A}^{-}\mathbf{D}_{t-1}^{-}$ and $\mathbf{B}^{*} = \mathbf{B} + \mathbf{B}^{+}\mathbf{D}_{t-1}^{+}$; $\mathbf{A}^{-}(\mathbf{B}^{+})$ is a cross diagonal matrix with nonzero elements $\alpha_{ij}^{-}(\beta_{ij}^{+})$, i, $j = 1$, 2, $i \neq j$; $\mathbf{D}_t^{-}(\mathbf{D}_t^{+})$ are 2 x 1 vectors with elements $d_{it}^{-}(d_{it}^{+})$, $i = 1$, 2 where $d_{it}^{-}(d_{it}^{+})$ is one if $r_{jt} < 0$ ($r_{jt} > 0$) and zero otherwise, $j = 1$, 2, $j \neq i$.

3.6.1 Bivariate results

Example 1: FTSE-DAX

Table 3.6 reports the results of the UEDCC-AGARCH$(1, 1)$ model between the returns on FTSE and DAX allowing shock and volatility spillover parameters to shift across the breaks in order to analyse the time-varying volatility transmission structure between the two variables.[42] As

[42] For an application on the returns of commodity metal futures see Karanasos et al. (2013).

is evident from Table 3.6, the results suggest the existence of strong conditional heteroscedasticity in the two variables. The ARCH as well as the asymmetry parameters of the two variables are positive and significant, indicating the existence of asymmetric responses in the two variables. In addition, rejection of the model with constant conditional correlations, using Tse's (2000) test, indicates the time-varying conditional correlation between the two financial markets. Figure 3.3 displays the evolution of the time-varying conditional correlation between the two variables over the sample period.

Table 3.6: Coefficient estimates of bivariate UEDCC-AGARCH models allowing for shifts in shock and volatility spillovers between FTSE and DAX

Conditional Variance Equation					
ω_1	0.003^a (0.0006)	γ_{11}	0.078^a (0.016)	β_{12}^3	-0.007^a (0.002)
ω_2	0.004^a (0.001)	γ_{22}	0.082^a (0.022)	α_D	0.044^a (0.010)
α_{11}	0.016^b (0.007)	α_{12}	0.010^a (0.003)	β_D	0.952^a (0.011)
α_{22}	0.033^a (0.009)	α_{12}^4	0.011^a (0.004)		
β_{11}	0.921^a (0.014)	β_{12}	-0.007^c (0.003)		
β_{22}	0.912^a (0.015)	β_{12}^2	0.003^a (0.001)		
$LogL$	-5427.03				
$Q(5)$	27.970 [0.110]	$Q^2(5)$	9.427 [0.977]		

Notes: Robust-standard errors are used in parentheses, 1= FTSE, 2=DAX. $Q(5)$ and $Q^2(5)$ are the multivariate Hosking (1981) tests for serial correlation of five lags on the standardised and squared standardised residuals, respectively (p-values are reported in brackets). $\alpha_{12}(\beta_{12})$ indicates shock (volatility) spillovers from DAX to FTSE, while $\alpha_{12}^l(\beta_{12}^l)$ indicates the shift in shock (volatility) spillovers for the break l (see Table 3.1) from DAX to FTSE. Insignificant parameters are excluded. ^{a}, ^{b} and ^{c} indicate significance at the 1%, 5% and 10% levels, respectively. Tse's (2000) test for constant conditional correlations: 20.41.

Furthermore, the results suggest that there is evidence of shock spillovers as well as negative volatility spillovers from DAX to FTSE (the α_{12} and β_{12} coefficients are significant at the 1% and 10% levels, respectively). With regard to the impact of the breaks on the volatility transmission structure, it is shown that both shock and volatility spillovers between the two variables change over time. The most significant changes include the impact of the fourth break in DAX (15/01/2008), which corresponds to the Global financial crisis, in which it shifts the shock spillovers parameter from DAX to FTSE (the α_{12}^4 coefficient is significant at the 1% level). Also, volatility spillovers from DAX to FTSE are shown to be shifted after the second (21/07/1997) and the third break (17/06/2003), corresponding to the Asian financial crisis and the announcement of the €18bn German tax cuts plan, respectively (see the β_{12}^2 and β_{12}^3 coefficients in Table 3.6).

These results are consistent with the time-varying conditional correlations. The average time-varying conditional correlation for the period before the break 15/01/2008 is 0.58 compared to the period after the break of 0.89. This also applies for the break 21/07/1997 (17/06/2003) with an average time-varying correlation of 0.43 (0.52) for the period before the break and 0.75 (0.82) for the period after the break. Overall these findings are indicative of the existence of contagion between DAX and FTSE during the turbulent periods of the two financial crises.

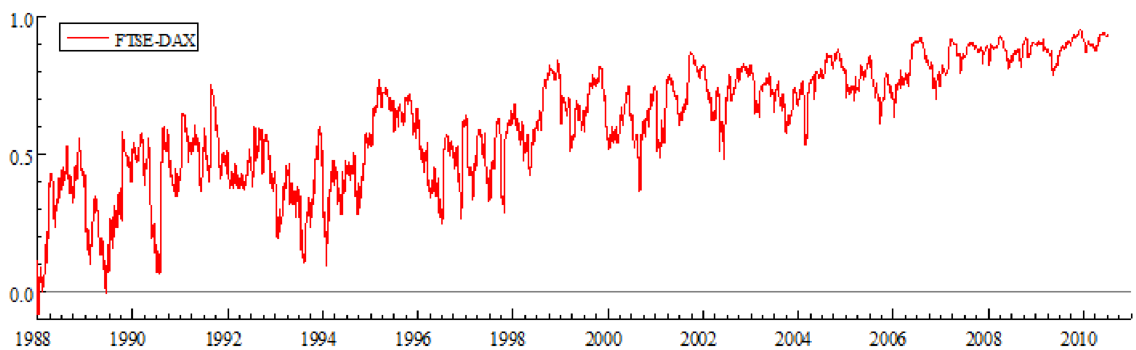


Figure 3.3. Evolution of the dynamic conditional correlation between FTSE and DAX returns.

Another way to look at the structure of the volatility spillovers between DAX and FTSE is to allow volatility (and shock) spillover parameters to shift across two regimes of stock returns: positive (increases in the stock market) and negative (declines in the stock market) returns. The results, displayed in Table 3.7, suggest that declines in each market generate shock spillovers to the other (the coefficients α_{12}^- and α_{21}^- are positive and significant), whilst increases in each market generate negative volatility spillovers to the other (the coefficients β_{12}^+ and β_{21}^+ are negative and significant).

Table 3.7: Coefficient estimates of bivariate UEDCC-AGARCH models allowing for different spillovers across positive and negative returns (FTSE-DAX)

Conditional Variance Equation					
ω_1	0.002^a (0.0005)	γ_{11}	0.058^a (0.012)	α_D	0.043^a (0.010)
ω_2	0.004^a (0.001)	γ_{22}	0.060^a (0.016)	β_D	0.954^a (0.011)
α_{11}	0.030^a (0.008)	α_{12}^-	0.019^a (0.005)		
α_{22}	0.027^a (0.008)	β_{12}^+	-0.014^a (0.004)		
β_{11}	0.926^a (0.012)	α_{21}^-	0.042^a (0.015)		
β_{22}	0.928^a (0.012)	β_{21}^+	-0.036^a (0.016)		
$LogL$	-5430.26				
$Q(5)$	26.965 [0.136]	$Q^2(5)$	9.533 [0.975]		

Notes: Robust-standard errors are used in parentheses, 1= FTSE, 2=DAX. $Q(5)$ and $Q^2(5)$ are the multivariate Hosking (1981) tests for serial correlation of five lags on the standardised and squared standardised residuals, respectively (p-values reported in brackets). $\alpha_{12}^-(\beta_{12}^+)$ indicates the shock (volatility) spillovers from DAX to FTSE generated by negative (positive) returns in DAX. $\alpha_{21}^-(\beta_{21}^+)$ reports the shock (volatility) spillovers from FTSE to DAX generated by negative (positive) returns in FTSE. Insignificant parameters are excluded.
^{a} indicates significance at the 1% level.

Example 2: NIKKEI-Hang Seng

Next, we consider the structure of the volatility spillovers between the returns on NIKKEI and Hang Seng to provide an example about the dynamic linkages between the Asian financial markets. The estimated bivariate model, reported in Table 3.8, suggests the existence of strong conditional heteroscedasticity. There is evidence of asymmetric effects of the two variables as the ARCH and asymmetry parameters (the α and the γ coefficients) are positive and significant. Furthermore, the model with constant conditional correlations is rejected according to Tse's (2000) test, hence the correlation between the two variables is time-varying. This is also confirmed by Figure 3.4, which shows the evolution of the time-varying correlation between the two variables.

Table 3.8: Coefficient estimates of bivariate UEDCC-AGARCH models allowing for shifts in shock and volatility spillovers between NIKKEI and Hang Seng

Conditional Variance Equation					
ω_1	0.003^a (0.0008)	γ_{11}	0.094^a (0.012)	α_D	0.015^a (0.005)
ω_2	0.009^a (0.002)	γ_{22}	0.081^a (0.021)	β_D	0.982^a (0.006)
α_{11}	0.024^a (0.004)	α_{12}^3	0.050^a (0.017)		
α_{22}	0.050^a (0.007)	α_{12}^4	0.025^b (0.011)		
β_{11}	0.920^a (0.007)	β_{12}^3	-0.046^a (0.015)		
β_{22}	0.885^a (0.015)	β_{21}^2	0.016^c (0.009)		
$LogL$	-9413.42	Tse's test:	10.10		
$Q(5)$	22.122 [0.333]	$Q^2(5)$	13.594 [0.850]		

Notes: Robust-standard errors are used in the parentheses, 1= NIKKEI, 2=Hang Seng. $Q(5)$ and $Q^2(5)$ are the multivariate Hosking (1981) tests for serial correlation of five lags on the standardised and squared standardised residuals, respectively (p-values are reported in brackets). $\alpha_{12}^l(\beta_{12}^l)$ indicates shift in shock (volatility) spillovers for the break l (see Table 3.1) from Hang Seng to NIKKEI, whilst β_{21}^l reports the shift in volatility spillovers for the break l in the reverse direction. Insignificant parameters are excluded. ^{a}, ^{b} and ^{c} indicate significance at the 1%, 5% and 10% levels, respectively.

With regard to the linkages between the two variables, the results show the existence of shock spillovers from Hang Seng to NIKKEI after the third (05/05/2009) and the fourth break (01/12/2009), which correspond to the different phases of the European sovereign-debt crisis. Also, while Hang Seng generates negative volatility spillovers to NIKKEI after the third break in the former (05/05/2009), there are positive volatility spillovers from NIKKEI to Hang Seng after the second break (04/01/2008) in the former, which corresponds to the Global financial crisis. These findings indicate the superiority of the time-varying spillover model over the conventional one. In contrast to the conventional model, allowing for breaks shows that the two financial markets have been integrated during the Global financial crisis.[43]

With regard to the time-varying conditional correlations, the average time-varying conditional correlation for the period before the breaks 04/01/2008, 05/05/2009 and 01/12/2009 are respectively $0.40, 0.41$ and 0.415 compared to the period after the breaks of $0.60, 0.58$ and 0.585,

[43] The results from the conventional bivariate UEDCC-AGARCH$(1,1)$ process indicate that there is no evidence of volatility spillovers between the two financial markets. For this model the stationarity condition of Engle (2002) is fulfilled.

respectively. These results are consistent with those of volatility spillovers in which these two types of markets have become more dependent during the recent financial crisis.

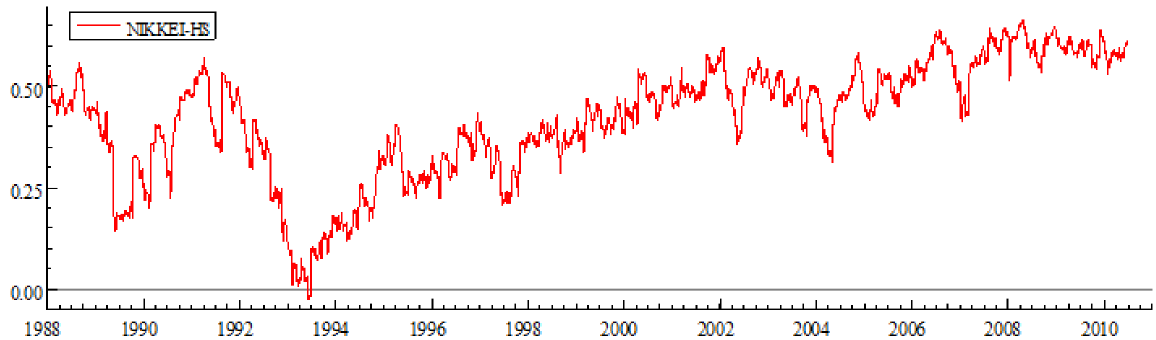


Figure 3.4. Evolution of the dynamic conditional correlation between NIKKEI and HS returns.

Finally, allowing the volatility spillover structure to shift across two different regimes, that is, positive and negative returns, also shows the existence of time-varying volatility spillovers between the two variables. Specifically, the results, displayed in Table 3.9, suggest that declines in NIKKEI generate shock spillovers to Hang Seng (the estimated α_{21}^{-} coefficient is positive and significant), whilst increases in NIKKEI generate negative volatility spillovers to Hang Seng (the estimated β_{21}^{+} coefficient is negative and significant).

Table 3.9: Coefficient estimates of bivariate UEDCC-AGARCH models allowing for different spillovers across positive and negative returns (NIKKEI-Hang Seng)

Conditional Variance Equation					
ω_1	0.003^a (0.0009)	β_{11}	0.917^a (0.007)	α_{21}^{-}	0.017^a (0.009)
ω_2	0.008^a (0.002)	β_{22}	0.897^a (0.013)	β_{21}^{+}	-0.018^a (0.008)
α_{11}	0.027^a (0.005)	γ_{11}	0.099^a (0.015)	α_D	0.016^a (0.007)
α_{22}	0.052^a (0.007)	γ_{22}	0.065^a (0.019)	β_D	0.980^a (0.010)
$LogL$	-9414.61				
$Q(5)$	22.918 [0.292]	$Q^2(5)$	9.534 [0.975]		

Notes: Robust-standard errors are used in parentheses, 1= NIKKEI, 2=Hang Seng. $Q(5)$ and $Q^2(5)$ are the multivariate Hosking (1981) tests for serial correlation of five lags on the standardised and squared standardised residuals, respectively (p-values are reported in brackets). $\alpha_{21}^{-}(\beta_{21}^{+})$ reports the shock (volatility) spillovers from NIKKEI to Hang Seng generated by negative (positive) returns in NIKKEI. Insignificant parameters are excluded. ^{a} indicates significance at the 1% level.

3.7 Conclusions

In this chapter, we have introduced a platform to examine empirically the link between financial crises and the principal time series properties of the underlying series. We have also adopted several models, both univariate and bivariate, to examine how the mean and volatility dynamics, including the volatility persistence and volatility spillovers structure of stock market returns have changed due to the recent financial crises and conditioned our analysis on non-parametrically identified breaks. Overall, our findings are consistent with the intuitively familiar albeit empirically hard-to-prove time-varying nature of asset market linkages induced by economic events and suggest the existence of limited diversification opportunities, especially during turbulent periods.

In particular, with respect to the mean and volatility dynamics our findings suggest that in general the financial crises clearly affect more the (un)conditional variances. Also, the results of the volatility persistence are clear-cut and suggest that they exhibit substantial time-variation. This time-variation applies to all stock market returns irrespective of whether we allow for structural changes or positive and negative changes in the underlying market. As far as the direction of this time-variation during financial crises is concerned the jury is still out, but there is little doubt that the financial crises are the primary driving force behind the profound changes in the unconditional variances.

Finally, with respect to the existence of dynamic correlations as well as time-varying shock and volatility spillovers our findings are also conclusive. Specifically, they suggest that in the cases we examine there is an increase in conditional correlations, occurring at different phases of the various financial crises, hence providing evidence as to the existence of contagion during this period. Such a finding is comparable to those of other studies using only conditional correlation analysis to examine the existence of contagion during the various financial crises. The results also suggest the existence of regime dependent volatility spillovers in all cases we examine by using two regimes of returns, positive and negative. Given that this is to our knowledge the first attempt to take into account the joint effect of dynamic correlations, volatility spillovers and structural breaks in the mean and/or volatility dynamics, these findings are of particular interest to those seeking refuge from financial crises.

3.8 APPENDIX

In this Appendix we will prove eq. (3.13) by mathematical induction. For $k = 1$ the result is trivial since eq. (3.13) reduces to eq. (3.3). If we assume that eq. (3.13) holds for k then it will be sufficient to prove that it holds for $k + 1$ as well. Combining eqs. (3.13) and (3.3, at time $t - k$) yields

$$\begin{aligned}
y_{t,k}^{gen} &= \xi_{t,k} y_{t-k} + \phi_2(t-k+1)\xi_{t,k-1} y_{t-k-1} + \sum_{r=0}^{k-1} \xi_{t,r}\phi_0(t-r) + \sum_{r=0}^{k-1} \xi_{t,r}\varepsilon_{t-r} \Rightarrow \\
y_{t,k+1}^{gen} &= \xi_{t,k}[\phi_0(t-k) + \phi_1(t-k)y_{t-k-1} + \phi_2(t-k)y_{t-k-2} + \varepsilon_{t-k}] + \phi_2(t-k+1)\xi_{t,k-1} y_{t-k-1} \\
&\quad + \sum_{r=0}^{k-1} \xi_{t,r}\phi_0(t-r) + \sum_{r=0}^{k-1} \xi_{t,r}\varepsilon_{t-r} \\
&= [\xi_{t,k}\phi_1(t-k) + \phi_2(t-k+1)\xi_{t,k-1}] y_{t-k-1} + \phi_2(t-k)\xi_{t,k} y_{t-k-2} \\
&\quad + \sum_{r=0}^{k-1} \xi_{t,r}\phi_0(t-r) + \phi_0(t-k) + \sum_{r=0}^{k-1} \xi_{t,r}\varepsilon_{t-r} + \varepsilon_{t-k}. \qquad \text{(A.1)}
\end{aligned}$$

Expanding the determinant $\xi_{t,k+1}$ in eq. (3.9) along the first column we have: $\xi_{t,k+1} = \xi_{t,k}\phi_1(t-k) + \phi_2(t-k+1)\xi_{t,k-1}$. Substituting this expression into eq. (A.1) gives

$$y_{t,k+1}^{gen} = \xi_{t,k+1} y_{t-k-1} + \phi_2(t-k)\xi_{t,k} y_{t-k-2} + \sum_{r=0}^{k} \xi_{t,r}\phi_0(t-r) + \sum_{r=0}^{k} \xi_{t,r}\varepsilon_{t-r},$$

which is eq. (3.13), at time t, when the prediction horizon is $k + 1$.

Chapter 4 Stylised facts for extended HEAVY models: the importance of asymmetries, power transformations and long memory, the use of Garman-Klass volatility and structural breaks

4.1 Introduction

In the final chapter we apply the high-frequency-based volatility (HEAVY) model of Shephard and Sheppard (2010), SS10 hereafter. We estimate this new class of models using financial data from the Oxford-Man Institute's (OMI) realised library, version 0.2, Heber et al. (2009). The library provides realised measures calculated on high-frequency data. The HEAVY framework models financial volatility based on both daily and intra-daily data, so that the system of equations estimated adopts to information arrival more rapidly than the classic daily GARCH models. The HEAVY model is based on the classic GARCH model of Bollerslev (1986), the GARCHX model and the Multiplicative Error Model (MEM) of Engle (2002b) in order to model realised volatility on high-frequency data associated with daily returns GARCH conditional volatility. Its main advantage, proved in SS10, is the robustness to structural breaks, especially during crisis periods, since the mean reversion and short-run momentum effects result to higher quality performance in volatility level shifts and more reliable forecasts.

Our main contribution is the enrichment of the HEAVY model with long memory structure, volatility asymmetries and power transformations through the HYAPARCH specification of Schoffer (2003) and Dark (2005) and the relevant GARCH models nested in the HYAPARCH structure. We compare the results of stock market data modelling with the several long memory, power and asymmetric specifications and conclude to prefer the most comprehensive one which we define as HYDAP-HEAVY (HYperbolic Double Asymmetric Power) for the realised measure models and the FIAP-HEAVY (Fractional Integrated Asymmetric Power) for the returns models.

Moreover, we follow the GARCH literature that combines trading volume with the conditional variance of returns (Lamoureux and Lastrapes, 1990, Gallo and Pacini, 2000) and test whether the standard HEAVY equations adopt further to the volume increment. We add the overnight trading activity indicator as additional regressor in the benchmark HEAVY equations to evaluate the effect of volume on volatility and the adjustment of volatility to the additional information from the trading volume proxy. As expected from the existing empirical evidence, the overnight indicator gives a positive feedback to the volatility of returns. Our main finding is that the HEAVY equations exhibit lower persistence, when the overnight surprise is used for the squared returns. In the realised measure modelling the overnight indicator has immaterial effect on the volatility process. So, the benchmark HEAVY framework is proved adequate to capture most of the eligible information needed for volatility modelling.

We further study the Garman-Klass (GK) volatility measure in the HEAVY framework in comparison with the other two variables (the squared returns and the realised kernel). We observe that the realised measure shows stronger effects than the GK measure when added as regressor and the GK-models seem to share characteristics with both the other two models (the squared returns and the realised kernel equations), but with more similarities to the realised measure process. Finally, we re-estimate the benchmark HEAVY equations taking into account the structural breaks apparent in the squared returns series and estimate the time-varying behaviour of the arch, garch-x and heavy coefficients. Focusing on the recent Global financial crisis, we observe a positive increment on the volatility process generated by the aforementioned coefficients after the crisis break.

The remainder of the chapter is structured as follows. In Section 4.2 we refer to the literature on available models for realised volatility and high-frequency data in general, the GARCH and MEM frameworks used in the HEAVY models. In Section 4.3 we detail the benchmark HEAVY models and the extended HEAVY with long memory, volatility asymmetries and power transformations.

Section 4.4 presents and discusses our empirical results of the HEAVY framework. In Section 4.5 we extend the standard HEAVY with the overnight trading activity indicator and we test the Garman-Klass volatility measure in the HEAVY framework. Section 4.6 presents our empirical results taking into account the structural breaks of the squared returns series. Finally, Section 4.7 concludes the analysis.

4.2 A review of the literature

4.2.1 Realised volatility modelling

The asset return volatility has attracted major interest of the financial econometrics research. We focus on the realised volatility measurement, modelling and forecasting. Several studies have introduced non-parametric estimators of realised volatility using high-frequency market data and trying to overcome the market microstructure noise contained in the dataset. Andersen and Bollerslev (1998), Andersen et al. (2001b) and Barndorff-Nielsen and Shephard (2002) were the first studies that formalised econometrically the realised variance with quadratic variation-like measures. Hansen and Lunde (2006) studied, amongst others, the effect of market frictions on the measurement of realised volatility and proved the superiority of kernel-based estimators. Finally, Barndorff-Nielsen et al. (2008, 2009) focus on the realised kernel estimation as the realised measure the more robust to noise. Thorough reviews of realised measures calculation and modelling are written by Hansen and Lunde (2011), Andersen and Benzoni (2009), Andersen et al. (2009), McAleer and Medeiros (2008) and Barndorff-Nielsen and Shephard (2007).

Moreover, voluminous empirical evidence on modelling and forecasting the realised volatility is developed. A popular approach broadly used is the ARFIMA time series model of realised variance in its original or logarithmic form. Dettling and Buhlmann (2004) apply the ARFIMA model for the log-realised volatility, recognising the slow hyperbolic decay in its autocorrelation function. Andersen et al. (2003) follow the multivariate approach with the fractionally integrated VAR model for exchange rate realised volatilities and compare its performance to the daily

GARCH and FIEGARCH models. Chiriac and Voev (2011) also propose, amongst other methodologies, the VARFIMA model for realised volatilities in order to forecast realised covariance matrices. Koopman et al. (2005) estimate univariate ARFIMA models of realised volatility and compare them with their simpler ARMA counterparts and the Stochastic Volatility and GARCH models. Oomen (2001) enrich the long memory model of realised variance with two exogenous variables: the lagged positive and negative returns, to measure the leverage effect and the (log) contemporaneous trading volume as in Lamoureux and Lastrapes (1990). Martens et al. (2009), apart from studying solely the long memory characteristics of the realised measures, improve the classic ARFIMA specification incorporating level-shifts, day-of-the-week, leverage and volatility level effects. Asai et al. (2012) also associate the long memory process with asymmetries of positive and negative shocks as well as the size effect of the shocks on realised volatility. Allen et al. (2013) is the more recent study to propose a fractionally integrated model with asymmetries named Dually Asymmetric Realised Volatility model (DARV-FI), where the ARFIMA model incorporates leverage effect parameters to measure the higher volatility risk in periods of negative returns.

Another popular approach to model the temporal aggregation of realised volatility is the Heterogeneous Autoregressive (HAR-RV) model introduced by Corsi (2004). Focusing on the persistence of the volatility time series Corsi (2004, 2005, 2009) build a long memory autoregressive model that captures the hyperbolic autocorrelation of the realised volatility process. The model is estimated as a restricted AR(22) process with the 1st, 5th and 22nd autoregressive lag. The realised volatility is related to its values back in the previous day, week and month. Andersen et al. (2007) extend the HAR model to include jump and non-jump components and result to the HAR-CJ, which is further enriched by Huang et al. (2013) with a momentum parameter producing the HAR-CJ-M. Liu and Maheu (2008) use the HAR model to investigate the structural breaks in realised volatility. Bollerslev et al. (2009) also include the HAR model for bipower variation in their discrete-time daily stochastic volatility model. The DARV model of Allen et al. (2013) is specified additionally with the HAR-RV structure (DARV-HAR) with the same leverage terms as the DARV-FI. Celik and Ergin (2014) estimate the HAR-CJ and compare its performance with simpler HAR models, GARCH and MIDAS. The HAR-CJ and the MIDAS are found to better fit the volatility process, while MIDAS is proved the best in crisis periods. Finally, Soucek and Todorova (2014) recently estimated a multivariate LHAR-CJ specification, introduced in its univariate form by Corsi and Reno (2012), who extended the HAR-RV with leverage effects and jump components.

4.2.2 GARCH modelling with realised volatility and high frequency data

Regarding the GARCH volatility modelling technique, there is plenty of empirical evidence relating the realised volatility with the conditional variance of asset returns. Engle (2002b) introduced the GARCHX model of daily returns, where the realised volatility is included as exogenous variable in the conditional variance equation. Martens (2002) also incorporated realised volatility measures of intra-daily returns in the daily GARCH variance. Corsi et al. (2008) extended the HAR model of realised volatility with a GARCH error process (HAR-GARCH) to model the volatility of realised volatility, in order to account for the time-varying conditional heteroscedasticity of the normally distributed HAR errors and improve its predictive power. Louzis et al. (2011) include the lagged realised variance in the GARCH equation of daily returns after being estimated first with ARFIMA and HAR models, in order to generate forecasts of Value at Risk. Chen et al. (2012) select to include the after-hours realised variance in the daily GARCH equation as regressor and produce significantly better forecasts for the following day's volatility than without including it.

Amongst the models that combine realised volatility with GARCH modelling, Hansen et al. (2012a) introduce the Realised GARCH model, which is the most close specification to the

HEAVY model. They estimate two equations: firstly, the GARCH $(1, 0)$ with a realised measure as regressor replacing the ARCH term, same as the HEAVY-r equation of SS10 and secondly, the measurement equation of the realised measure. Unlike the MEM$(1, 1)$ structure of SS10 HEAVY-RM, the second equation relates the realised measure to the contemporaneous GARCH conditional variance of returns estimated in the first equation and a leverage function to allow for the asymmetric response of volatility to different signed or sized return shocks. The Realised GARCH is also presented with a log-linear specification with both the conditional variance and the realised measure in logarithmic form. Hansen and Huang (2012) propose further the Realised EGARCH extending the Realised GARCH with the EGARCH framework. Finally, Hansen et al. (2012b) build the multivariate version of Realised GARCH, the Realised Beta GARCH to model additionally realised co-volatilities and spillover effects.

In many studies researchers use directly high frequency returns in the GARCH models instead of incorporating daily realised volatility measures in the daily returns GARCH equation. Martens (2001) estimated a continuous time GARCH process with intra-daily foreign exchange rates returns of different frequencies to compare the forecasts of daily volatility from the various return frequencies. Hashimoto (2005) use intra-daily Japanese exchange rates to estimate an EGARCH and a TGARCH and detect the Japanese crisis effects in 1997. Giot (2005) incorporate intra-daily data in a GARCH model assuming alternatively normal and student-t error distribution. Gau (2005) and Haniff and Pok (2010) prefer the Periodic GARCH to model high frequency returns. Kang and Yoon (2008) estimate a FIAPARCH process with high frequency data and the student-t distribution assumption and study the asymmetric long memory property of returns in different frequencies. Chen et al. (2008) investigate the time series dynamics of hourly DJ returns with an exponential asymmetric AR–GARCH model assuming a generalised error distribution to account for fat tails apparent in the data. They employ in the mean equation the exponential

AR (EAR) and in the variance the GJR-GARCH. Xie and Li (2010) use tick data from S&P500 with GARCH-in-mean amongst other GARCH models. Chortareas et al. (2011) calculate the 15min returns of Euro exchange rates, estimate intra-daily GARCH and FIGARCH processes and compare them to daily returns GARCH and FIGARCH, as well as to the daily realised volatility ARFIMA model. The intra-daily GARCH models and the ARFIMA realised volatility model perform better than the daily data processes. Chen et al. (2011) introduce the HYBRID (High FrequencY Data-Based PRojectIon-Driven)-GARCH class of models with various parametrisations based on intra-daily returns. Kitamura (2010) use intra-daily data to measure interdependencies between foreign currency markets in the multivariate GARCH framework with time-varying correlations of Tse and Tsui (2002), while Chiang et al. (2009) apply the Dynamic Conditional Correlation (DCC) model of Engle (2002a) with high frequency stock index returns. The intra-daily GARCH framework is applied also on commodity data. In Hickey et al. (2012) hourly electricity price data are used to estimate the simple GARCH specification and compare it with the EGARCH, the APARCH and the Component GARCH models. Finally, Engle (2000) move the attention from high-frequency data to ultra-high-frequency (UHF) data, that are irregularly spaced in time, introducing the UHF-GARCH after incorporating the conditional duration from the Autoregressive Conditional Duration (ACD) model (Engle and Russell, 1998) into the GARCH specification. Park and Kim (2011) extend Engle's UHF-GARCH building the two-state Markov-Switching MS-GARCH with UHF futures data.

4.2.3 Multiplicative Error Models for realised volatility and the HEAVY specification

Engle (2002b) first introduced the MEM specification for the conditional expectation of non-negative valued time series. MEM nests the GARCH structure with the squared returns series being replaced by any non-negative process. The MEM structure also nests several GARCH-type models for positive valued processes like the ACD model of Engle and Russell (1998) for durations, the Conditional Autoregressive Range (CARR) of Chou (2005) for the price range and the Autoregressive Conditional Volume (ACV) of Manganelli (2005) for the transaction volume. Engle and Gallo (2006) estimate a trivariate MEM for three non-negative series: the squared returns, the high-low range and the realised variance. They also include in their multivariate specification cross effects to measure the volatility spillovers and asymmetries to detect the asymmetric volatility response to positive and negative shocks. Lanne (2006) extend the MEM for exchange rates realised volatility estimating time-varying coefficients, that vary along with the parameters of the error distribution or the values of an exogenous variable with two distinct probability regimes. Cipollini et al. (2007, 2013) estimate multivariate MEMs allowing for interdependence across the terms of the vector representation of the model and formalise the joint probability density function of the vector error term with a copula approach (Cipollini et al., 2007) and a semiparametric approach (Cipollini et al., 2013). Brownlees et al. (2011) propose a further MEM extension, the Component MEM, which incorporates both daily and intra-daily components in the non-negative process modelling. Finally, Gallo and Otranto (2012), following Lanne (2006), focus on the time-varying behaviour of the MEM's parameters in the realised volatility modelling and propose Markov Switching parameters in order to capture the volatility regimes with different dynamics.

Following the MEM framework, SS10 model the realised volatility with a MEM$(1,1)$ equation, the HEAVY-RM. They model also the returns with a GARCH process, the HEAVY-r, where the ARCH term is replaced by the lagged realised volatility. The two-equation system, the HEAVY-r and the HEAVY-RM, defines the HEAVY model, which is extended to its multivariate specification by Noureldin et al. (2012). Cipollini et al. (2013) refer to the HEAVY model by simply restricting the bivariate Vector MEM representation for squared returns and realised variance. Lastly, Borovkova and Mahakena (2013) are the first to apply the univariate HEAVY model with different error distributions (student-t and skewed-t). They also extend the HEAVY-r equation with a leverage term, a news sentiment proxy and a time to maturity variable alternatively.

In the present study, we extend the univariate HEAVY model with long memory, asymmetries and power transformations through the HYAPARCH framework.

4.3 The HEAVY framework: models description

4.3.1 The benchmark HEAVY/GARCH/MEM models

The HEAVY-i, $i = r, R$, GARCH type of models[44], introduced by SS10, use two variables: the close-to-close return r_t and the open-to-close variation proxied by the realised measure, RM_t. We first form the signed square rooted (SSR) realised measure as follows: $\widetilde{RM}_t =$*sign*$(r_t)\sqrt{RM_t}$, where *sign*$(r_t) = 1$, if $r_t \geqslant 0$ and *sign*$(r_t) = -1$, if $r_t < 0$. We assume that the returns and the SSR realised measure follow zero conditional mean equations:

$$\begin{aligned} r_t &= \varepsilon_{rt}, \widetilde{RM}_t = \varepsilon_{Rt}, \text{ or} \\ r_t^2 &= \varepsilon_{rt}^2, RM_t = \widetilde{RM}_t^2 = \varepsilon_{Rt}^2, \end{aligned}$$

where $\varepsilon_{it} = e_{it}\sigma_{it}$, $i = r, R$ and $e_{it} \overset{i.i.d.}{\sim} N(0, 1)$; σ_{it}^2 is positive with probability one for all t and it is a measurable function of $\mathcal{F}_{t-1}^{HF}$, the high frequency past data for the case of the realised measure (or $\mathcal{F}_{t-1}^{LF}$, low frequency past data for the case of the close-to-close return). That is, the conditional variance of ε_{it} (or the conditional mean of ε_{it}^2) is σ_{it}^2: $\mathbb{E}(\varepsilon_{it}^2 \left| \mathcal{F}_{t-1}^{HF} \right.) \triangleq \sigma_{it}^2$.

The HEAVY-i, $i = r, R$, models consist of the following GARCH$(1, 1)$-type equations[45]:

$$(1 - \beta_i L)\sigma_{it}^2 = \omega_i + \alpha_i\varepsilon_{i,t-1}^2 + \gamma_i\varepsilon_{j,t-1}^2,\ i, j = r, R, j \neq i, \tag{4.1}$$

where L is the lag operator, $\omega_i \in (0, \infty)$, $\alpha_i, \beta_i, \gamma_i \geq 0$ and $(\alpha_i + \beta_i) \in [0, 1)$.

It will be convenient to have labels for the six different models that we estimate (see aso below Panel A in Table 4.1). The abbreviations HEAVY-E-r or GARCH-X-r stand for the model for

[44] The acronym HEAVY stands for High-frEquency-bAsed VolatilitY models (see SS10).

[45] This is the way to run the Multiplicative Error Model (MEM) of Engle (2002b) for the conditional mean of a non-negative time series process with the GARCH packages already available. Assuming zero conditional mean equations we obtain the squared series, r_t^2 or $\widetilde{RM}_t^2 = RM_t$ and run the MEM model. In other words, the GARCH model for the conditional variance of the returns or the SSR realised measure, is identical to the MEM model for the conditional mean of the squared returns or the realised measure.

stock returns (where $i = r$ and $j = R$) with $\alpha_r, \gamma_r \neq 0$:

$$(1 - \beta_r L)\sigma_{rt}^2 = \omega_r + \alpha_r r_{t-1}^2 + \gamma_r RM_{t-1}. \tag{4.2}$$

The benchmark conditional volatility standard GARCH$(1,1)$ process is the one with $\gamma_r = 0$, while the so called Heavy-r process is the one with $\alpha_r = 0$: $(1 - \beta_r L)\sigma_{rt}^2 = \omega_r + \gamma_r RM_{t-1}$.[46] The γ_r coefficient will be called the Heavy coefficient. The general model in eq. (4.2) can be thought of as an extended HEAVY-r process with the lagged squared returns included as an additional regressor. The name suggests that it is the lagged realised measure which does almost all the work at moving around the conditional variance of returns (see SS10). Alternatively it can be considered as a GARCH-X-r process, that is the realised measure is used as a regressor in the GARCH$(1,1)$ process (see also Engle, 2002b). As pointed out by SS10, the GARCH-X terminology suggests that it is the squared returns which drive the model.

Similarly, the HEAVY-E-RM or GARCH-X-RM model for the SSR realised measure, where $i = R$, $j = r$ and $\alpha_R, \gamma_R \neq 0$, is given by

$$(1 - \beta_R L)\sigma_{Rt}^2 = \omega_R + \alpha_R RM_{t-1} + \gamma_R r_{t-1}^2. \tag{4.3}$$

The α_R coefficient will be called the Heavy coefficient. The GARCH$(1,1)$ process for $\widetilde{RM}_t$ with $\gamma_R = 0$ is also called HEAVY-RM, while the GARCH$(1,0)$-X model is obtained by setting $\alpha_R = 0$: $(1 - \beta_R L)\sigma_{Rt}^2 = \omega_R + \gamma_R r_{t-1}^2$. That is, γ_R is the GARCH-X coefficient.

4.3.1.1 Bivariate representation

The two HEAVY-E or GARCH$(1,1)$-X processes in eqs. (4.2) and (4.3), can be expressed/interpreted as a bivariate GARCH$(1,1)$ process with shocks spillovers:

$$(\mathbf{I} - \mathbf{B}L)\boldsymbol{\sigma}_t^{\wedge 2} = \boldsymbol{\omega} + \mathbf{A}L\boldsymbol{\varepsilon}_t^{\wedge 2}, \tag{4.4}$$

where $\mathbf{B}$ is a 2×2 diagonal matrix with nonzero elements β_i, $i = r, R$ and $\boldsymbol{\omega} = [\omega_r \ , \omega_R]'$;

[46] That is, the HEAVY-r model is identical to the GARCH $(1,0)$-X model. Thus for the HEAVY-r process, we run a zero mean return process with variance equation GARCH $(1,0)$ and adding as a regressor the lagged realised measure.

$\boldsymbol{\sigma}_t^{\wedge 2} = [\sigma_{rt}^2\,,\,\sigma_{Rt}^2]'$ and $\varepsilon_t^{\wedge 2} = [\varepsilon_{rt}^2\,,\,\varepsilon_{Rt}^2]$; $\mathbf{A}$ is a 2×2 full matrix with (cross)diagonal elements $\alpha_i(\gamma_i)$. The above bivariate GARCH model is also identical to a bivariate HEAVY-E model since it can be written as

$$(\mathbf{I} - \mathbf{B}L)\boldsymbol{\sigma}_t^{\wedge 2} = \boldsymbol{\omega} + \boldsymbol{\alpha}_2 L\varepsilon_{Rt}^2 + \boldsymbol{\alpha}_1 L\varepsilon_{rt}^2 = \boldsymbol{\alpha}_2 L \cdot RM_t + \boldsymbol{\alpha}_1 L r_t^2, \tag{4.5}$$

where $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ are the two columns of $\mathbf{A}$. If $\boldsymbol{\alpha}_1 = 0$ then we have the simple bivariate HEAVY model. In other words $\boldsymbol{\alpha}_2$ is the column with the two heavy coefficients. If $\mathbf{A}$ is a diagonal matrix the model is equivalent to two univariate GARCH$(1, 1)$ or MEM$(1, 1)$ processes.

4.3.2 Extended HEAVY/GARCH/MEM specifications

The benchmark specification of the HEAVY/GARCH/MEM models in eq. (4.1) can be extended in many directions. We allow for power transformations of the volatilities, leverage effects and long memory in the conditional variance process. We re-run the six aforementioned models, estimated in the simple specification, enriched with the three key features to improve further the HEAVY/GARCH volatility modelling.

4.3.2.1 Double Asymmetric Power formulations

First we estimate two alternative double asymmetric power (DAP) HEAVY-E specifications:

$$(1-\beta_i L)\sigma_{it}^{\delta_i} = \omega_i + \alpha_i(1+\mu_i s_{t-1})|\varepsilon_{i,t-1}|^{\delta_i} + (\gamma_i + \pi_i s_{t-1})\,|\varepsilon_{j,t-1}|^{\delta_j}\,, \quad \text{or} \tag{4.6}$$

$$(1-\beta_i L)\sigma_{it}^{\delta_i} = \omega_i + (\alpha_i + \mu_i s_{t-1})|\varepsilon_{i,t-1}|^{\delta_i} + (\gamma_i + \pi_i s_{t-1})\,|\varepsilon_{j,t-1}|^{\delta_j}\,,\ i,j = r, R, \text{ and } j \neq i \tag{4.7}$$

where $s_t = [1 - \mathit{sign}(r_t)]/2$, that is, $s_t = 1$ if $r_t < 0$ and 0 otherwise; μ_i, π_i are the own and cross leverage coefficients respectively (positive μ_i, π_i means larger contribution of negative 'shocks' in the volatility process[47]); δ_i is the parameter of the power transformed variance, that takes (finite) positive values and as before $\gamma_i \geq 0$ is the coefficient of the lagged exogenous variable ε_{jt}^2, which in our case is either the lagged squared returns ($j = r$: r_{t-1}^2) or the lagged realised measure, $j = R$: RM_{t-1}. The exogenous variable allows the conditional variance to exhibit also

[47] They capture the possible 'double' asymmetry in the two conditional variances. That is, both the own (μ_i) and cross (π_i) asymmetries.

structural dynamics and is always a non-negative time series, in order to ensure the positivity of the conditional variance.

Bivariate formulation Equations (4.6) and (4.7) can be written as a bivariate system:

$$(\mathbf{I} - \mathbf{B}L)\boldsymbol{\sigma}_t^{(\delta)} = \boldsymbol{\omega} + \mathbf{A}^* L \left|\boldsymbol{\varepsilon}_t\right|^{(\delta)}, \tag{4.8}$$

where $\boldsymbol{\sigma}_t^{(\delta)} = [\sigma_{rt}^{\delta_r}, \sigma_{Rt}^{\delta_R}]'$ and $|\boldsymbol{\varepsilon}_t|^{(\delta)} = [|\varepsilon_{rt}|^{\delta_r}, |\varepsilon_{Rt}|^{\delta_R}]$; $\mathbf{A}^* = \mathbf{A} + \mathbf{G}s_{t-1}$ and $\mathbf{G}$ is a 2×2 matrix with diagonal elements $\alpha_i \mu_i$ (or just μ_i), $i = r, R$ and off diagonal elements π_i.

4.3.2.2 Long Memory formulations

In this Section we estimate the most general specification, that is the hyperbolic double asymmetric power (HYDAP) HEAVY-E or GARCH-X process (see, for example, Schoffer, 2003 and Dark, 2005):

$$(1 - \beta_i L)(\sigma_{it}^{\delta_i} - \omega_i) = A_i(L)(1 + \mu_i s_t)|\varepsilon_{it}|^{\delta_i} + (\gamma_i + \pi_i s_{t-1})\left|\varepsilon_{j,t-1}\right|^{\delta_j}, \tag{4.9}$$

with

$$A_i(L) = (1 - \beta_i L) - (1 - \phi_i L)[(1 - \zeta_i) + \zeta_i(1 - L)^{d_i}], \tag{4.10}$$

where $i, j = r, R$ and $j \neq i$; β_i, $|\phi_i| < 1$; d_i is the long memory parameter: $0 \leq d_i \leq 1$ and ζ_i is the amplitude parameter: $0 \leq \zeta_i \leq 1$. For the HEAVY-E-R model the HYDAP specification has five Heavy coefficients: ζ_R, d_R, β_R, ϕ_R and μ_R and two Heavy extended coefficients: γ_i and π_i. If $\zeta_i = 0$ and $\phi_i - \beta_i = \alpha_i$, the HYDAP specification reduces to the DAP one in eq. (4.6) since in this case we have: $A_i(L) = \alpha_i L$.

The HYDAP-HEAVY-E specification also nests the fractional integrated (FI) one by imposing the restriction $\zeta_i = 1$. In this case $A_i(L)$ in eq. (4.10) becomes

$$A_i(L) = (1 - \beta_i L) - (1 - \phi_i L)(1 - L)^{d_i}. \tag{4.11}$$

It also nests the HYP specification by imposing the restriction $\mu_i = \pi_i = 0$ which if, in addition $\delta_i = 2$, reduces to the HY one.

Overall we estimate six HEAVY/GARCH models and nine specifications (see Table 4.1 below).[48] For the HEAVY-r or GARCH$(1,0)$-X-r and the GARCH$(1,0)$-X-RM models we only estimate the three specifications with $A_i(L) = 0$:

$$(1 - \beta_i L)(\sigma_{it}^{\delta_i} - \omega) = (\gamma_i + \pi_i s_{t-1}) \left|\varepsilon_{j,t-1}\right|^{\delta_j}.$$

Table 4.1: Models (HEAVY/GARCH/MEM) and Specifications (HYDAP/FIDAP/DAP)

Panel A: Six alternative models: $\underset{(i,j=r,R,j\neq i)}{\sigma_{it}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i \varepsilon_{j,t-1}^2}$

Returns (r_t): $\underset{(i=r)}{\varepsilon_{i,t-1}^2} = r_{t-1}^2$, $\underset{(j=R)}{\varepsilon_{j,t-1}^2} = RM_{t-1}$, $\sigma_{it}^2 = \sigma_{rt}^2 = \mathbb{E}(r_t^2 \| \mathcal{F}_{t-1})$	$\underbrace{\gamma_r = 0}$ GARCH(1,1) or MEM(1,1)	$\underbrace{\underset{(\gamma_r \neq 0)}{\alpha_r = 0}}$ HEAVY or GARCH(1,0)-X or MEM(1,0)-X	$\underbrace{\alpha_r, \gamma_r \neq 0}$ HEAVY-E or GARCH(1,1)-X or MEM(1,1)-X
SSR Realised Measure ($\widetilde{RM}_t$): $\underset{(i=R)}{\varepsilon_{i,t-1}^2} = RM_{t-1}$, $\underset{(j=r)}{\varepsilon_{j,t-1}^2} = r_{t-1}$, $\sigma_{it}^2 = \sigma_{Rt}^2 = \mathbb{E}(\widetilde{RM}_t^2 \| \mathcal{F}_{t-1})$	$\underbrace{\gamma_R = 0}$ HEAVY or GARCH(1,1) MEM(1,1)	$\underbrace{\underset{(\gamma_R \neq 0)}{\alpha_R = 0}}$ GARCH(1,0)-X or MEM(1,0)-X	$\underbrace{\alpha_R, \gamma_R \neq 0}$ HEAVY-E or GARCH(1,1)-X or MEM(1,1)-X

Panel B: HYDAPGARCH specification and eight alternative restricted ones

$$(1 - \beta_i L)(\sigma_{it}^{\delta} - \omega_i) = A_i(L)(1 + \mu_i s_{it})|\varepsilon_{it}|^{\delta} + (\gamma_i + \pi_i s_{t-1}) \left|\varepsilon\right|_{j,t-1}^{\delta_j},$$
$$A_i(L) = (1 - \beta_i L) - (1 - \phi_i L)[(1 - \zeta_i) + \zeta_i (1 - L)^{d_i}].$$

Restrictions↓ →:	$\zeta_i = 0$	FI: $\zeta_i = 1$	HY: $\zeta_i \in (0,1)$
$\delta_i = 2$ and $\mu_i = \pi_i = 0$	GARCH	FIGARCH	HYGARCH
P : $\mu_i = \pi_i = 0$	PGARCH	FIPGARCH	HYPGARCH
AP: no restrictions	DAPGARCH	FIDAPGARCH	HYDAPGARCH

Notes: In the case of the three HY specifications the condition $\zeta_i < 1$ ensures their stationarity. For the HEAVY-r model we estimate only the three specifications with $A_i(L) = 0$. Recall that the HYDAP–HEAVY-E model is identical to the HYDAPGARCH-X and HYDAP-MEM-E models.

The power transformation (δ_i), leverage effects (μ_i, π_i) and long memory (d_i, ζ_i) are our main contribution to the HEAVY-E models of SS10 as well as to the GARCH-X and MEM models of Engle (2002b).[49]

For the simple, fractionally integrated (FI) or hyperbolic (HY) specifications we provide results with and without (double) asymmetries and/or power transformations (DAP formulations) in

[48] SS10 propose as an extension of the HEAVY-RM model a fractional process with leverage effects or Corsi's (2009) long memory HAR structure. They also suggest the use of realised semivariances in the HEAVY formulations, to capture leverage effects or the inclusion of a leverage parameter multiplied with the realised measure as in Engle and Gallo (2006).

[49] Engle (2002b) first proposed the MEM model using the various GARCH family specifications to estimate the volatility of volatility, which is a non-negative process. He uses the Asymmetric Power MEM (AP-MEM) model in his Volatility Laboratory (V-Lab) amongst other processes for real-time financial volatility modelling.

order to study thoroughly their effects on the conditional variances of either the stock returns or the SSR realised measures. The sufficient conditions of Dark (2005) for the positivity of the conditional variance of a HYGARCH $(1, d_i, 1)$ specification are: $\omega_i > 0$, $\beta_i - \zeta_i d_i \leq \phi_i \leq \frac{2-d_i}{3}$ and $\zeta_i d_i(\phi_i - \frac{1-d_i}{2}) \leq \beta_i(\phi_i - \beta_i + \zeta_i d_i)$, $i = r, R$ (see also Conrad, 2010). When $\zeta_i = 1$ they reduce to the ones for the FIGARCH $(1, d_i, 1)$ specification given in Bollerslev and Mikkelsen (1996).

Bivariate formulation The two HYDAP-HEAVY-E models in eq. (4.9) can be written in a matrix form as

$$(\mathbf{I} - \mathbf{B}L)(\boldsymbol{\sigma}_t^{(\delta)} - \boldsymbol{\omega}) = \mathbf{A}(L)[(\mathbf{I} + \mathbf{M}s_t) + (\boldsymbol{\Gamma} + \boldsymbol{\Pi}s_{t-1})]L\,|\boldsymbol{\varepsilon}_t|^{(\delta)},$$

with

$$\mathbf{A}(L) = (\mathbf{I} - \mathbf{B}L) - (\mathbf{I} - \boldsymbol{\Phi}L)(\mathbf{I} - \mathbf{Z} + \mathbf{Z}\cdot\mathbf{D}),$$

where $\boldsymbol{\Phi}$ is a 2×2 diagonal matrix with nonzero elements ϕ_i, $i = r, R$; $\mathbf{Z}$ and $\mathbf{D}$ are 2×2 diagonal matrices with nonzero elements ζ_i and $(1-L)^{d_i}$ respectively; $\mathbf{M}$, $\boldsymbol{\Pi}(\boldsymbol{\Gamma})$ are 2×2 (cross)diagonal matrices with nonzero elements μ_i and $\pi_i(\gamma_i)$ respectively. The above formulation is identical to the bivariate HYDAPGARCH$(1,1)$-X model. When $\mathbf{Z} = \mathbf{0}$ it reduces to the bivariate specification in eq. (4.8) with $\mathbf{A} = \boldsymbol{\Phi} - \mathbf{B} + \boldsymbol{\Gamma}$ and $\mathbf{G} = \mathbf{A}\mathbf{M} + \boldsymbol{\Pi}$.

If in addition, $\mathbf{M} = \boldsymbol{\Pi} = \mathbf{0}$ and $\delta_i = 2$, $i = r, R$, then it becomes the bivariate specification in eq. (4.4).

4.4 Empirical analysis

4.4.1 Data description

The various GARCH/HEAVY models are estimated for twenty one stock indices returns and realised volatilities. According to the analysis in SS10, the HEAVY formulations improve considerably the volatility modelling by allowing momentum and mean reversion effects and adjusting quickly to the structural breaks of volatility. We first run the simple specifications for the twenty one assets and then we extend them by adding the features of long memory, power transformation of the conditional variances and leverage effects in the volatility process. We finally run the benchmark HEAVY models with the Overnight trading indicator, the Garman Klass volatility measure and dummies for the structural breaks of the squared returns to identify the recent Global financial crisis effects on the volatility process.

We use daily data for twenty one stock market indices extracted from the Oxford-Man Institute's (OMI) realised library version 0.2 of Heber et al. (2009). Our sample covers the period from 03/01/2000 to 01/03/2013 for most indices. For the Canadian stock market index TSE the data begin from 2002. The Indian index NIFTY has too many missing observations despite its wide sample dates' range. The OMI's realised library includes daily stock market indices' prices, returns and several realised volatility measures calculated on high-frequency data from the Reuters DataScope Tick History database. The high-frequency data are first cleaned and then used in the realised measures calculations. According to the library's documentation, the data cleaning applied on stock index data consists of deleting records outside the time interval that the stock exchange is open. Some minor manual changes are also needed when results are ineligible due to the rebasing of indices. We use the daily closing prices to form the daily returns as follows: $r_t = \ln(P_t^C) - \ln(P_{t-1}^C)$, P_t^C is the stock market index closing price and two realised measures as drawn from the library: the realised kernel and the 5-minute realised variance. The estimation results using the two realised measures alternatively are very similar, so we present only the ones with the realised kernels (the results for the 5-minute realised variances are available upon request). We also choose to present the results from the six indices of the more developped countries (due to space considerations), that is S&P 500 from the US, Nikkei 225 from Japan, TSE from Canada, FTSE from the UK, DAX from Germany and Eustoxx 50 from the Eurozone.

4.4.1.1 Realised Measures

The library's realised measures are calculated in the way described in SS10. The 5-minute realised variance, RV_t, which we also employ as an alternative realised measure, is calculated with the formula: $RV_t = \sum x_{j,t}^2$, where $x_{j,t} = X_{t_{j,t}} - X_{t_{j-1,t}}$, $x_{j,t}$ are the 5-minute intra-daily returns, $X_{t_{j,t}}$ are the intra-daily prices and $t_{j,t}$ are the times of trades on the t-th day. Heber et al. (2009) implement additionally a subsampling procedure from the data to the most feasible level in order to eliminate the stock market noise effects. The subsampling involves averaging across many realised variance estimations from different data subsets (see also the references in SS10 for realised measures surveys, noise effects and subsampling procedures). The realised kernel, which we present in our analysis here, is chosen as a measure more robust to noise, as in SS10, where the exact calculation with a Parzen weight function is described as follows: $RK_t = \sum_{k=-H}^{H} k(h/(H+1))\gamma_h$, where $\gamma_h = \sum_{j=|h|+1}^{n} x_{j,t}x_{j-|h|,t}$ and $k(x)$ is the Parzen kernel function. They declare that they select the bandwidth of H as in Barndorff-Nielsen et al. (2009).

Table 4.2 presents the stock indices extracted from the database and provides volatility estimations for each one's squared returns and realised kernels time series for the respective sample period. We calculate the standard deviation (sd) of the series and the annualised volatility (Avol). Avol is the square rooted mean of 252 times the squared return or the realised kernel. The standard deviations are always lower than the annualised volatilities. The realised kernels have lower Avol and sd than the squared returns since they ignore the overnight effects and are affected by less noise. The returns represent the close-to-close yield and the realised kernels the open-to-close variation. The annualised volatility of the realised measure is between 11% and 26%, while the squared returns show figures from 16% to 30%.

Table 4.2: Data Description								
		sample period			r_t^2		RK_t	
Index symbol	Index name (Country)	Strart date	End date	Obs.	Avol	sd	Avol	sd
sp	S&P 500 (US)	03/01/2000	01/03/2013	3281	0.212	0.054	0.183	0.029
dj	DJIA (US)	03/01/2000	01/03/2013	3283	0.198	0.047	0.178	0.028
nasdaq	NASDAQ 100 (US)	03/01/2000	01/03/2013	3286	0.285	0.086	0.204	0.029
russell	RUSSELL 2000 (US)	03/01/2000	01/03/2013	3284	0.262	0.067	0.180	0.024
tse	S&P/TSX Comp. Index (Canada)	02/05/2002	01/03/2013	2701	0.183	0.041	0.128	0.016
ipc	IPC Mexico (Mexico)	03/01/2000	01/03/2013	3288	0.228	0.053	0.133	0.011
bvsp	Bovespa Index (Brazil)	03/01/2000	28/02/2013	3207	0.304	0.094	0.258	0.043
aord	All Ordinaries (Australia)	04/01/2000	01/03/2013	3297	0.155	0.027	0.113	0.009
nikkei	NIKKEI 225 (Japan)	04/01/2000	01/03/2013	3184	0.250	0.073	0.174	0.020
hs	HANG SENG (China)	03/01/2000	01/03/2013	2978	0.279	0.217	0.158	0.018
straits	FT Straits Times Index (Singapore)	03/01/2000	01/03/2013	3242	0.200	0.065	0.127	0.009
kospi	KOSPI Comp. Index (South Korea)	04/01/2000	28/02/2013	3242	0.275	0.081	0.205	0.026
nifty	S&P CNX Nifty (India)	06/01/2000	01/03/2013	2732	0.289	0.168	0.215	0.038
cac	CAC 40 (France)	03/01/2000	01/03/2013	3350	0.244	0.059	0.203	0.026
dax	DAX (Germany)	03/01/2000	01/03/2013	3333	0.253	0.070	0.227	0.035
ftse	FTSE 100 (UK)	04/01/2000	01/03/2013	3301	0.197	0.044	0.159	0.017
aex	AEX (Netherlands)	03/01/2000	01/03/2013	3349	0.243	0.065	0.191	0.024
ssmi	Swiss Market Index (Switzerland)	04/01/2000	01/03/2013	3295	0.200	0.047	0.154	0.015
mib	FTSE MIB (Italy)	03/01/2000	28/02/2013	3316	0.248	0.065	0.188	0.022
ibex	IBEX 35 (Spain)	03/01/2000	01/03/2013	3315	0.245	0.061	0.199	0.022
eustoxx	EUROSTOXX 50 (Eurozone)	03/01/2000	01/03/2013	3325	0.248	0.062	0.216	0.035

Notes: Avol is the annualised volatility and sd is the standard deviation.

4.4.2 The benchmark HEAVY results

We first estimate the original HEAVY models, as introduced in SS10 and described in the six equations of Table 4.1, Panel A. Table 4.3 presents the results for the six stock indices chosen to be reported as more representative. We obtain similar results as in SS10 and we observe the following stylised facts:

Firstly, for the squared returns equations the preferred model is the HEAVY-r since the ARCH coefficient, α_r, of the HEAVY-E-r is insignificant in all cases but two, where it is very low. Additionally, the Heavy coefficient, γ_r, of the HEAVY-E-r is significant and around 0.30 to 0.55, which means that the lagged realised measure does all the work at moving around the conditional variance of returns and it entirely crowds out the lagged squared returns. So, we exclude the ARCH coefficient and prefer the simpler HEAVY-r equation with the momentum or GARCH coefficient, β_r, to be estimated around 0.60 to 0.70.

Secondly, for the SSR realised kernel equations we prefer the HEAVY-RM model, where the results are again similar to the SS10 analysis and we can also compare them to HEAVY-E-RM and the GARCH($1,0$)-X-RM, not estimated by SS10. In the HEAVY-E-RM model the Heavy-E or GARCH-X coefficient, γ_R, although significant, it is very close to zero, around $0.02 - 0.04$ and the Heavy coefficient, α_R, is significant and around 0.35 to 0.45. It is obvious, that the lagged realised measure (α_R) drives the model of its conditional mean and not the squared returns (γ_R). So, we select the HEAVY-RM equation as more preferred, where the Heavy coefficient is estimated around 0.40 to 0.50.

So, the benchmark HEAVY models estimated result to the HEAVY-r and the HEAVY-RM as the equations that best describe the volatility process. These are exactly the two equations proposed also by SS10 to constitute the HEAVY system of equations.

Table 4.3: HEAVY/GARCH Models; Specification: $\zeta_i = \mu_i = 0, \delta_i = 2.$

		SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
		Panel A: Squared Returns					
GARCH$(1,1)-r$	α_r	0.09 (7.84)***	0.10 (6.60)***	0.09 (6.32)***	0.09 (7.33)***	0.09 (6.90)***	0.09 (6.46)***
	β_r	0.90 (80.08)***	0.89 (59.16)***	0.90 (52.70)***	0.90 (70.98)***	0.90 (70.98)***	0.90 (63.68)***
HEAVY-E-r or GARCH(1,1)-X-r	α_r	0.000 (0.00)	0.04 (1.72)**	0.05 (2.66)***	0.003 (0.12)	0.000 (0.00)	0.000 (0.00)
	β_r	0.71 (15.44)***	0.76 (11.30)***	0.77 (9.97)***	0.60 (8.62)***	0.60 (9.67)***	0.65 (14.01)***
	γ_r	0.37 (6.70)***	0.37 (2.54)***	0.29 (1.80)**	0.56 (4.38)***	0.47 (5.80)***	0.46 (6.71)***
HEAVY-r or GARCH(1,0)-X-r	β_r	0.71 (16.70)***	0.71 (8.66)***	0.69 (16.21)***	0.60 (11.11)***	0.60 (9.69)***	0.65 (14.01)***
	γ_r	0.37 (6.61)***	0.53 (3.55)***	0.52 (6.50)***	0.55 (6.69)***	0.47 (5.89)***	0.46 (7.08)***
		Panel B: Realised Measures					
HEAVY-RM or GARCH(1,1)-RM	α_R	0.41 (10.75)***	0.41 (9.41)***	0.40 (10.97)***	0.48 (11.26)***	0.50 (10.78)***	0.46 (11.92)***
	β_R	0.58 (15.97)***	0.58 (13.28)***	0.59 (16.18)***	0.51 (12.31)***	0.48 (10.69)***	0.52 (13.35)***
HEAVY-E-RM or GARCH(1,1)-X-RM	α_R	0.37 (9.83)***	0.35 (9.34)***	0.37 (11.04)***	0.40 (9.10)***	0.45 (9.61)***	0.40 (10.38)***
	β_R	0.59 (17.19)***	0.59 (14.89)***	0.58 (16.54)***	0.54 (10.65)***	0.50 (11.06)***	0.54 (14.03)***
	γ_R	0.02 (2.89)***	0.02 (4.19)***	0.02 (4.55)***	0.04 (4.34)***	0.03 (4.08)***	0.03 (5.01)***
GARCH$(1,0)$-X-RM	β_R	0.85 (90.61)***	0.86 (70.07)***	0.84 (128.7)***	0.87 (74.47)***	0.87 (93.89)***	0.84 (77.67)***
	γ_R	0.10 (13.47)***	0.06 (11.57)***	0.06 (12.98)***	0.09 (10.66)***	0.10 (12.88)***	0.11 (12.84)***

Notes: The numbers in parentheses are t-statistics.
***, **, * denote significance at the $0.05, 0.10, 0.15$ level respectively.

4.4.3 The extended HEAVY results

4.4.3.1 Stylised facts for Asymmetric Power (AP) specifications

After running the six benchmark HEAVY equations, we add asymmetries and power transformations to enrich our volatility modelling by extending the original HEAVY models. From the estimated results we choose to present in Table 4.4, we conclude to the stylised facts of the Asymmetric Power Specifications:

For the squared returns we prefer the AP-HEAVY-E-r model since the power term δ_r is very close to two in all cases, $\delta_r \in [1.93, 2.05]$ (see also the Wald tests of the power terms, where the hypothesis of $\delta = 2$ is not rejected) and the Heavy coefficient, γ_r, is significant and around 0.15 to 0.30. Although α_r is insignificant and excluded in most cases, the own asymmetry coefficient

(μ_r) is significant and around 0.10 for four out of the six cases. In other words, not only the lagged realised measure but also the lagged squares of the negative returns drive the model of the conditional variance of returns. Moreover, the momentum coefficient, β_r, is estimated to be around 0.70 to 0.85.

Regarding the realised measure equations we present the most preferred AP-HEAVY-E-RM formulation, where we model the conditional standard deviation of the SSR realised measure, as δ_R is estimated around 1.00 to 1.10 in all cases but one. The Wald tests of the power terms do not reject the hypothesis of $\delta = 1$. The Heavy coefficient, α_R, is significant and around 0.30 to 0.35, while the Heavy-E or GARCH-X coefficient, γ_R, is between 0.50 and 1.00. This means that both the SSR realised measure and the lagged squared returns affect significantly the conditional standard deviation of the SSR realised measure. Lastly, the own asymmetry, μ_R, is significant and around 0.10 to 0.20.

To sum up, in our first HEAVY extension with the inclusion of power transformations and asymmetries in the equations, we estimate the HEAVY-E models with $\delta_i \neq 2$ and $\mu_i \neq 0$, where both asymmetric coefficients are proved significant and positive. In the AP-HEAVY-E-r model, γ_r is significant and around $0.20 - 0.30$ in all cases but one; α_r is insignificant in all cases (except for NIKKEI for which we estimate an APGARCH$(1,1)$-X specification); μ_r is significant and around 0.10 in most cases[50] and δ_r is close to two. In the AP-HEAVY-E-RM model, γ_R is significant (around 0.50 to 1.00) and not close to zero as in the benchmark HEAVY, δ_R is close to one, which means that the squared returns have a significant effect on the conditional standard deviation of the SSR realised measure and μ_R is around 0.10 to 0.20[51]. (See also in the Appendix Table 4A.1 with the two preferred AP-HEAVY-E equations estimated linearly with fixed powers).

[50] When we estimate models with μ_r, γ_r and π_r, γ_r becomes insignificant.

[51] When we estimate a model with μ_R, γ_R and π_R, π_R is around 0.55 to 1.00 and γ_R becomes insignificant (results not reported).

Table 4.4: AP-HEAVY-E Models; Specifications with $\zeta_i = 0$ and
Wald tests for restrictions on power terms - ChiSq(1)

Specification↓		SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
Panel A: Power Transformed Squared Returns							
P-HEAVY-E-r or PGARCH(1,1)-X-r	β_r	0.72 (16.37)***	0.85 (17.67)***	0.70 (13.17)***	0.61 (8.98)***	0.60 (9.66)***	0.65 (14.01)***
	α_r	0.000 (0.00)	0.06 (2.50)***	0.000 (0.00)	0.01 (0.22)	0.001 (0.00)	0.001 (0.00)
	γ_r	0.60 (2.30)***	0.15 (14.72)***	0.54 (1.69)**	0.69 (1.77)**	0.59 (2.12)***	0.48 (2.25)***
	δ_r	1.88 (16.94)***	2.00 (9.15)***	2.01 (14.77)***	1.94 (13.44)***	1.94 (15.64)***	1.98 (17.56)***
Wald tests	$\delta = 1$	62.10 [0.00]	26.14 [0.00]	51.49 [0.00]	42.30 [0.00]	57.34 [0.00]	81.00 [0.00]
	$\delta = 2$	1.21 [0.27]	0.00 [1.00]	0.004 [0.95]	0.19 [0.67]	0.23 [0.63]	0.01 [0.91]
AP-HEAVY-E-r or APGARCH(1,0)-X-r°	β_r	0.77 (19.72)***	0.85 (29.92)***	0.80 (16.41)***	0.80 (21.69)***	0.71 (10.90)***	0.76 (14.33)***
	μ_r	0.08 (1.88)**	0.50 (5.66)***	0.11 (1.61)**	0.19 (1.86)***	0.11 (1.90)**	0.10 (2.23)***
	γ_r	0.28 (2.26)***	0.13 (7.00)***	0.31 (1.62)**	0.21 (1.69)**	0.32 (1.66)**	0.19 (1.62)*
	δ_r	1.95 (20.84)***	2.00 (12.90)***	1.96 (13.18)***	1.93 (14.46)***	1.97 (15.71)***	2.05 (17.14)***
Wald tests	$\delta = 1$	103.2 [0.00]	39.80 [0.00]	41.50 [0.00]	48.81 [0.00]	60.21 [0.00]	77.57 [0.00]
	$\delta = 2$	0.27 [0.60]	0.00 [1.00]	0.09 [0.77]	0.29 [0.59]	0.07 [0.80]	0.17 [0.68]
Panel B: Power Transfomed Realised Measures							
AP-HEAVY-E-RM or APGARCH(1,1)-X-RM	β_R	0.66 (24.79)***	0.60 (16.31)***	0.69 (15.11)***	0.58 (14.88)***	0.59 (15.27)***	0.63 (19.91)***
	α_R	0.30 (11.30)***	0.34 (10.79)***	0.29 (9.16)***	0.33 (10.26)***	0.35 (9.68)***	0.31 (11.01)***
	μ_R	0.17 (8.52)***	0.11 (7.78)***	0.14 (5.81)***	0.20 (3.71)***	0.11 (7.21)***	0.15 (8.76)***
	γ_R	0.56 (1.76)**	0.50 (2.14)***	0.74 (1.67)**	0.59 (1.33)	0.78 (1.80)**	0.94 (2.02)***
	δ_R	1.10 (6.55)***	1.10 (5.57)***	1.00 (64.51)***	1.28 (5.07)***	1.11 (6.52)***	1.08 (6.71)***
Wald tests	$\delta = 1$	0.34 [0.56]	0.26 [0.61]	0.002 [0.97]	1.21 [0.27]	0.41 [0.52]	0.25 [0.62]
	$\delta = 2$	28.84 [0.00]	20.66 [0.00]	34.66 [0.00]	8.23 [0.00]	27.43 [0.00]	32.72 [0.00]

Notes: See Notes in Table 4.3. The numbers in square brackets are p-values.
°For the NIKKEI we estimate a APGARCH(1, 1)-X-r model with α_r : $\underset{(4.24)^{***}}{0.05}$.

4.4.3.2 Stylised facts for Long Memory Asymmetric Power specifications

We further extend the asymmetric power transformations with long memory through the HYAPARCH framework and present the preferred models for each volatility process. For the squared returns the chosen equation is the FIAP-HEAVY-E-r and for the SSR realised kernel we select the HYDAP-HEAVY-E-RM.

In the FIAP-HEAVY-E-r specification for the power transformed absolute returns (Table 4.5)

δ_r is close to 1.50 (around 1.30 to 1.70) with d_r close to 0.50 (around 0.40 to 0.55). In most cases the Wald tests reject the null hypotheses of $d = 0$ or 1 and $\delta = 1$ or 2. The HEAVY coefficient, γ_r, is significant and around 0.40 to 0.60. In other words, both the lagged realised measure and the lagged power transformed absolute returns drive the model of the power transformed conditional variance of returns. Furthermore, the own asymmetry coefficient (μ_r) is significant and around 0.30 to 0.60, while the α_r is insignificant and excluded.

Table 4.5: FIAP-HEAVY-E-r Specifications with $\zeta_r = 1, \phi_r = 0$
Power Transformed Squared Returns and Wald tests for restrictions
on power terms and fractional differencing parameters - ChiSq(1) and ChiSq(2)

		SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
FIAP-HEAVY-E-r or FIAPGARCH(1,0)-X-r	β_r	0.46 (6.13)***	0.46 (4.17)***	0.49 (8.89)***	0.36 (4.88)***	0.46 (4.57)***	0.52 (1.79)**
	d_r	0.49 (7.34)***	0.51 (5.74)***	0.52 (6.94)***	0.40 (6.07)***	0.49 (5.71)***	0.54 (3.07)***
	μ_r	0.58 (3.73)***	0.29 (1.92)**	0.40 (1.87)**	0.60 (4.13)***	0.53 (3.88)***	0.48 (2.42)***
	γ_r	0.38 (1.52)*	0.42 (1.50)*	0.59 (1.63)*	0.62 (1.45)*	0.63 (2.41)***	0.49 (2.15)***
	δ_r	1.40 (10.64)***	1.55 (5.92)***	1.37 (5.84)**	1.52 (11.83)***	1.35 (12.15)***	1.66 (6.57)***
Wald tests	$d = 0$	53.88 [0.00]	32.93 [0.00]	48.17 [0.00]	36.83 [0.00]	32.70 [0.00]	8.78 [0.00]
	$d = 1$	58.37 [0.00]	30.40 [0.00]	41.04 [0.00]	82.87 [0.00]	34.57 [0.00]	6.37 [0.01]
	$\delta = 1$	9.13 [0.00]	4.36 [0.04]	2.46 [0.12]	16.58 [0.00]	9.67 [0.00]	7.07 [0.01]
	$\delta = 2$	21.10 [0.00]	3.01 [0.08]	7.29 [0.01]	13.61 [0.00]	35.10 [0.00]	1.77 [0.18]
	$d = 0$ and $\delta = 1$	58.20 [0.00]	33.38 [0.00]	63.56 [0.00]	36.84 [0.00]	39.65 [0.00]	9.39 [0.01]
	$d = 0$ and $\delta = 2$	85.26 [0.00]	43.65 [0.00]	139.7 [0.00]	152.0 [0.00]	74.06 [0.00]	32.83 [0.00]
	$d = 1$ and $\delta = 1$	74.81 [0.00]	43.22 [0.00]	98.97 [0.00]	281.5 [0.00]	47.67 [0.00]	46.73 [0.00]
	$d = 1$ and $\delta = 2$	71.51 [0.00]	30.53 [0.00]	44.91 [0.00]	94.88 [0.00]	64.24 [0.00]	6.82 [0.03]

Notes: See Notes in Table 4.3. The numbers in square brackets are p-values.

In the HYDAP-HEAVY-E-RM of the power transformed SSR realised measure (Tables 4.6a and 4.6b) we model the power transformed conditional variance of the SSR realised measure since δ_R is estimated around 1.25 to 1.40. The Wald tests do not reject the null of $\delta = 1$ at 5% significance level for two out of six cases. There is also strong evidence of hyperbolic memory as ζ_R and d_R are around $0.80 - 0.90$ and $0.50 - 0.70$ respectively, with the Wald tests always rejecting the null of both equal to 0 or 1. We further include the two GARCH-X coefficients, γ_R

and π_R. The former is always insignificant and excluded and the latter, which captures the cross asymmetries, is significant and around $0.80 - 1.10$ in all but one case. So, both lagged power transformations of the SSR realised measure and the lagged squares of negative returns affect significantly the power transformed conditional variances of the SSR realised measure. The own asymmetry is significant in all but one case and around 0.20 to 0.70 and the other two Heavy coefficients, β_R and ϕ_R, are around $0.45 - 0.65$ and $0.15 - 0.35$ respectively. It seems that the HYDAP-HEAVY-E-RM specification with $\delta_R \sim 1.30$, $\zeta_R \sim 0.85$ and $d_R \sim 0.70$ is the preferred model.

Table 4.6a: HYDAP-HEAVY-E-RM Specification with no restrictions
Power Transformed Realised Measures

		SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
HYDAP-HEAVY-E-RM or HYDAPGARCH(1,1)-X-RM	β_R	0.63 (13.64)***	0.50 (7.17)***	0.46 (7.03)***	0.47 (6.43)***	0.65 (12.14)***	0.63 (12.16)***
	ϕ_R	0.20 (8.14)***	0.20 (5.42)***	0.28 (4.44)***	0.16 (7.36)***	0.35 (6.04)***	0.28 (7.03)***
	d_R	0.72 (14.39)***	0.69 (13.10)***	0.53 (15.04)***	0.67 (16.00)***	0.69 (15.78)***	0.71 (15.30)***
	ζ_R	0.81 (29.85)***	0.89 (45.59)***	0.85 (40.10)***	0.92 (68.07)***	0.87 (51.63)***	0.85 (41.75)***
	μ_R	0.71 (4.36)***	0.17 (3.45)***	0.20 (2.95)***	0.03 (0.33)	0.18 (3.31)***	0.27 (3.60)***
	π_R	0.95 (1.65)**	0.78 (1.45)*	1.11 (2.30)***	1.86 (2.24)***	0.84 (1.81)**	1.07 (1.47)*
	δ_R	1.42 (7.52)***	1.25 (5.06)***	1.33 (12.79)***	1.24 (10.61)***	1.39 (9.04)***	1.35 (6.76)***

Notes: See Notes in Table 4.3

Table 4.6b: HYDAP-HEAVY-E-RM Specification with no restrictions

Wald tests for restrictions on power terms, fractional differencing and amplitude parameters

ChiSq(1), ChiSq(2) and ChiSq(3)

	SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
$d = 0$	207.3 [0.00]	171.8 [0.00]	225.8 [0.00]	255.9 [0.00]	249.1 [0.00]	234.2 [0.00]
$d = 1$	30.22 [0.00]	35.39 [0.00]	184.8 [0.00]	63.57 [0.00]	49.78 [0.00]	40.27 [0.00]
$\zeta = 0$	890.5 [0.00]	2077.3 [0.00]	1604.7 [0.00]	4622.7 [0.00]	2662.1 [0.00]	1735.3 [0.00]
$\zeta = 1$	45.97 [0.00]	29.47 [0.00]	53.06 [0.00]	36.03 [0.00]	56.85 [0.00]	52.53 [0.00]
$\delta = 1$	4.93 [0.03]	1.04 [0.31]	9.93 [0.00]	4.20 [0.04]	6.39 [0.01]	3.05 [0.08]
$\delta = 2$	9.48 [0.00]	9.10 [0.00]	42.15 [0.00]	42.50 [0.00]	15.90 [0.00]	10.67 [0.00]
$d = 1$ and $\zeta = 1$	50.53 [0.00]	51.22 [0.00]	210.8 [0.00]	90.02 [0.00]	70.65 [0.00]	59.10 [0.00]
$d = 0$ and $\zeta = 1$	513.8 [0.00]	258.1 [0.00]	324.3 [0.00]	317.3 [0.00]	580.7 [0.00]	641.1 [0.00]
$d = 1$ and $\zeta = 0$	1569.7 [0.00]	2433.4 [0.00]	2027.1 [0.00]	4868.4 [0.00]	4186.3 [0.00]	3204.2 [0.00]
$d = 0$ and $\zeta = 0$	895.8 [0.00]	2078.1 [0.00]	1676.5 [0.00]	4693.9 [0.00]	2816.8 [0.00]	1867.7 [0.00]
$d = 1$ and $\delta = 1$	36.75 [0.00]	38.55 [0.00]	194.2 [0.00]	63.99 [0.00]	51.61 [0.00]	40.27 [0.00]
$d = 1$ and $\delta = 2$	37.80 [0.00]	74.75 [0.00]	228.2 [0.00]	128.4 [0.00]	77.42 [0.00]	67.25 [0.00]
$d = 0$ and $\delta = 1$	209.2 [0.00]	229.0 [0.00]	236.4 [0.00]	280.4 [0.00]	276.9 [0.00]	272.1 [0.00]
$d = 0$ and $\delta = 2$	223.0 [0.00]	181.2 [0.00]	266.7 [0.00]	269.9 [0.00]	250.9 [0.00]	235.1 [0.00]
$\zeta = 1$ and $\delta = 1$	47.07 [0.00]	33.95 [0.00]	53.67 [0.00]	36.03 [0.00]	65.60 [0.00]	70.43 [0.00]
$\zeta = 1$ and $\delta = 2$	95.51 [0.00]	76.33 [0.00]	142.1 [0.00]	117.3 [0.00]	187.1 [0.00]	173.7 [0.00]
$\zeta = 0$ and $\delta = 1$	1220.9 [0.00]	2918.6 [0.00]	1907.5 [0.00]	5300.5 [0.00]	4775.5 [0.00]	3355.4 [0.00]
$\zeta = 0$ and $\delta = 2$	1039.1 [0.00]	2666.6 [0.00]	1656.3 [0.00]	4910.2 [0.00]	4069.4 [0.00]	2855.8 [0.00]
$d = 1$ and $\zeta = 1$ and $\delta = 1$	50.56 [0.00]	395.5 [0.00]	212.8 [0.00]	91.11 [0.00]	75.20 [0.00]	73.38 [0.00]
$d = 1$ and $\zeta = 1$ and $\delta = 2$	96.39 [0.00]	586.5 [0.00]	288.1 [0.00]	194.6 [0.00]	188.2 [0.00]	173.8 [0.00]

Notes: The numbers in square brackets are p-values.

4.4.3.3 Power Terms and Long Memory parameters

Power terms ($\widehat{\delta}_i$)

We further focus on the behaviour of the power terms across the different specifications estimated (see also in the Appendix Tables 4A.2 and 4A.3 with the esimated γ_R and π_R). In Table 4.7 we present analytically the powers estimated ($\widehat{\delta}_i$). Regarding the returns equations, for the short memory specifications $\widehat{\delta}_r$ is very close to 2 (with or without asymmetries), while for the symmetric fractionally integrated ones in three cases the estimated power is between 1.60 and 1.70. When we include asymmetries as well, the estimated power terms are decreasing further around 1.40 to 1.65. In the case of the realised measure formulations, for the symmetric short memory specifications $\widehat{\delta}_R$ is around 1.10 to 1.30. When we include own asymmetries the estimated power decreases a bit and in all cases but one, is around $1.00 - 1.10$. For the symmetric hyperbolic specifications $\widehat{\delta}_R$ is higher in the majority of the cases; it is between 1.00 and 1.40 (for four out of the six cases it is around 1.30 to 1.40). When we include both own and cross asymmetries it slightly increases to $1.25 - 1.40$.

Table 4.7: HEAVY-E Models; Power Parameters δ

Spec.↓	SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
Panel A: Power Transformed Squared Returns						
P°	1.88 (17.28)***	2.00 (12.71)***	2.00 (14.17)***	1.94 (13.95)***	1.94 (16.30)***	1.98 (18.00)***
AP°	1.95 (20.84)***	2.00 (12.90)***	1.96 (13.18)***	1.93 (14.46)***	1.97 (15.71)***	2.05 (17.14)***
FIP	1.67 (27.77)***	1.90 (27.74)***	1.64 (22.27)***	1.98 (28.10)***	1.62 (20.04)***	1.94 (19.48)***
$FIAP$	1.40 (10.64)***	1.55 (5.92)***	1.37 (5.84)***	1.52 (11.83)***	1.35 (12.15)***	1.66 (6.57)***
Panel B: Power Transformed Realised Kernels						
P	1.14 (6.38)***	1.12 (4.77)***	1.17 (71.45)***	1.30 (6.57)***	1.18 (6.45)***	1.25 (6.93)***
AP	1.10 (6.55)***	1.10 (5.57)***	1.00 (64.51)***	1.28 (5.07)***	1.11 (6.52)***	1.08 (6.71)***
HYP	1.01 (4.68)***	1.14 (4.68)***	1.35 (9.04)***	1.26 (6.52)***	1.29 (6.19)***	1.38 (6.31)***
$HYAP$	1.14 (6.14)***	1.06 (4.74)***	1.33 (9.40)***	1.27 (5.54)***	1.21 (7.18)***	1.26 (7.14)***
$HYDAP$	1.42 (7.52)***	1.25 (5.06)***	1.33 (12.79)***	1.24 (10.61)***	1.39 (9.04)***	1.35 (6.76)***

Notes: See Notes in Table 4.3

$^{\circ}$Estimated models without α_r except for NIKKEI (AP).

Long memory parameters ($\widehat{d}_i$ and $\widehat{\zeta}_i$)

In Table 4.8 we present the long memory parameters estimated under each long memory extension we run. For the returns equations the fractionally integrated models are the preferred ones, that is $\zeta_r = 1$. In the symmetric FIGARCH model $\widehat{d_r}$ is between 0.50 and 0.70 and reduces to $0.40 - 0.55$ when power transformations and asymmetries are added (FIAPARCH). In the realised measure HYAPARCH models chosen, $\widehat{d}_R$ is between 0.60 and 0.80 for the hyperbolic models without asymmetries and power transformations. When we include them the estimated long memory parameter slightly reduces and hovers around 0.55 to 0.70. The estimated amplitude parameter, $\widehat{\zeta}_R$, is between 0.90 and 1.00 for the hyperbolic models without asymmetries and power transformations. When we include them the estimated hyperbolic parameter is smaller, around 0.80 to 0.90.

Table 4.8: HEAVY-E Models; Long memory Parameters d, ζ

	SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
Panel A: Power Transformed Squared Returns						
FI	0.62 (6.13)***	0.53 (3.94)***	0.69 (9.40)***	0.47 (4.13)***	0.56 (9.09)***	0.66 (3.86)***
FIP	0.56 (5.59)***	0.52 (5.21)***	0.61 (4.51)***	0.47 (5.40)***	0.48 (2.16)***	0.59 (7.12)***
$FIAP$	0.49 (7.34)***	0.51 (5.74)***	0.52 (6.94)***	0.40 (6.07)***	0.49 (5.71)***	0.54 (3.07)***
Panel B: Power Transformed Realised Kernels						
d						
HY	0.76 (11.05)***	0.72 (12.38)***	0.62 (13.29)***	0.62 (9.22)***	0.72 (10.83)***	0.74 (10.25)***
HYP	0.77 (9.50)***	0.72 (11.55)***	0.55 (11.75)***	0.63 (11.84)***	0.72 (10.89)***	0.75 (10.07)***
$HYAP$	0.71 (10.76)***	0.74 (11.81)***	0.55 (12.36)***	0.63 (13.10)***	0.70 (12.19)***	0.70 (10.57)***
$HYDAP$	0.72 (14.39)***	0.69 (13.10)***	0.53 (15.04)***	0.67 (16.00)***	0.69 (15.78)***	0.71 (15.30)***
ζ						
HY	0.95 (52.95)***	0.93 (54.94)***	0.98 (89.48)***	0.95 (47.40)***	0.93 (48.54)***	0.91 (41.98)***
HYP	0.97 (70.58)***	0.94 (59.74)***	0.92 (51.18)***	0.94 (30.28)***	0.93 (56.72)***	0.92 (48.79)***
$HYAP$	0.83 (18.99)***	0.87 (31.13)***	0.79 (27.79)***	0.85 (23.44)***	0.85 (32.76)***	0.81 (21.52)***
$HYDAP$	0.81 (29.85)***	0.89 (45.59)***	0.85 (40.10)***	0.92 (68.07)***	0.87 (51.63)***	0.85 (41.75)***

Notes: See Notes in Table 4.3

Finally, in the Appendix Tables 4A.2 and 4A.3 we present the esimated γ_R and π_R of the HEAVY-E-RM formulations. We see that the Garch-x coefficient (γ_R) is small in the specifications without powers and asymmetries and becomes large when power and asymmetries are added. The cross asymmetry coefficient (π_R) is mostly higher when the hyperbolic memory is added.

4.5 HEAVY extended with the Overnight indicator and the Garman-Klass volatility

4.5.1 The Overnight trading activity indicator

We further extend the HEAVY models adding a trading activity proxy for volume to study the volume-volatility relationship. Lamoureux and Lastrapes (1990) added a volume variable in the GARCH $(1,1)$ model to explain the conditional variance of daily returns and especially the persistence of the model. We follow Gallo and Pacini (2000) who test two alternative proxies for trading activity, apart from volume, in the GARCH equation of daily returns. The first variable is the intra-day volatility IDV_t, which is calculated as the difference between the highest (P_t^H) and the lowest (P_t^L) price divided by the closing price (P_t^C) on day t.: $IDV_t = \frac{P_t^H - P_t^L}{P_t^C}$, which we do not test with the HEAVY models, as it is a close (but more simplistic) specification to the Garman-Klass volatility that we intend to study in the following part of the chapter. The second trading activity proxy suggested by Gallo and Pacini (2000) is the overnight indicator $ONI_t = \left|\log \frac{P_t^O}{P_{t-1}^C}\right|$, where P_t^O is the opening price of day t. The overnight indicator ONI_t, which represents the overnight surprise and we choose to study with the HEAVY models, is proved to contain information for the conditional variance of the close-to-close returns and the conditional mean of the realised kernel (open-to-close variation).

We calculate ONI_t from the prices available in the OMI's realised library and add it with the coefficient ϑ_i ($i = r, R$) as variance regressors in the three returns equations as well as in the three realised kernel equations to detect the effect on the original models of Table 4.3. Table 4.11 presents the overnight information effect on the HEAVY models. ONI_t captures the trading activity information of the end-of-day traders of the previous day's closing to the following day's opening. We add the contemporaneous ONI_t as in Gallo and Pacini (2000). All $\vartheta's$ are positive with sound influence on the coefficients of the returns equations estimated. In the GARCH$(1,1)$-r α_r is mostly higher and β_r lower with their sum lower. The overnight surprise absorbs some of the previous day's conditional variance effect (β_r) and a part of the whole model's persistence ($\alpha_r + \beta_r$). In the HEAVY-E-r with ONI_t γ_r becomes higher, all α_r coefficients become zero and β_r is much lower, as ONI_t receives again a lot of the previous day's conditional variance and the whole persistence, increasing the effect of the lagged realised measure. In the HEAVY-r model we observe material differences in γ_r and β_r comparing to the original results without ONI_t. Their movement is similar to the HEAVY-E-r equations with lower β_r and higher γ_r. All $\vartheta_r' s$ are significant in the returns models, while in the realised kernel models we have two times insignificant ϑ_R for the GARCH$(1,0)$-X-RM. The realised kernel equations are almost identical to the benchmark equations without ONI_t and $\vartheta_R' s$ are close to zero. The realised measure models receive immaterial contribution from the overnight trading activity indicator.

Table 4.9: HEAVY/GARCH models with ONI_t coefficient ϑ_i
Equ. (1) becomes: $(1-\beta_i L)\sigma_{it}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \gamma_i \varepsilon_{j,t-1}^2 + \vartheta_i ONI_t,\ i,j = r, R, j \neq i$

		SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
GARCH$(1,1)$-r	α_r	0.09 (7.86)***	0.10 (6.21)***	0.10 (5.02)***	0.11 (6.36)***	0.10 (5.66)***	0.10 (5.28)***
	β_r	0.88 (60.39)***	0.89 (58.82)***	0.86 (22.44)***	0.85 (28.16)***	0.86 (32.34)***	0.83 (20.61)***
	ϑ_r	0.007 (2.78)***	0.000 (0.33)	0.002 (1.93)***	0.003 (2.85)***	0.005 (2.99)***	0.013 (2.53)***
HEAVY-E-r or GARCH(1,1)-X-r	α_r	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)
	β_r	0.62 (4.81)***	0.75 (8.31)***	0.73 (9.70)***	0.45 (4.88)***	0.33 (3.44)***	0.53 (7.40)***
	γ_r	0.44 (4.12)***	0.41 (2.33)***	0.31 (4.87)***	0.49 (6.47)***	0.62 (6.83)***	0.44 (6.86)***
	ϑ_r	0.017 (1.68)**	0.004 (2.76)***	0.007 (2.90)***	0.012 (4.44)***	0.019 (5.01)***	0.023 (4.03)***
HEAVY-r or GARCH(1,0)-X-r	β_r	0.62 (6.49)***	0.77 (13.21)***	0.70 (13.06)***	0.46 (7.03)***	0.34 (3.58)***	0.53 (7.75)***
	γ_r	0.44 (4.32)***	0.39 (3.55)***	0.49 (4.93)***	0.48 (8.43)***	0.62 (7.17)***	0.45 (5.97)***
	ϑ_r	0.017 (2.10)***	0.005 (4.00)***	0.005 (5.01)***	0.012 (7.87)***	0.019 (5.08)***	0.028 (4.47)***
HEAVY-RM or GARCH(1,1)-RM	α_R	0.41 (11.21)***	0.40 (10.17)***	0.41 (11.07)***	0.48 (11.44)***	0.49 (10.61)***	0.46 (11.41)***
	β_R	0.56 (15.07)***	0.57 (13.46)***	0.58 (15.60)***	0.50 (11.81)***	0.48 (10.11)***	0.49 (11.55)***
	ϑ_R	0.006 (3.03)***	0.001 (3.31)***	0.001 (2.75)***	0.001 (3.62)***	0.003 (3.40)***	0.005 (4.82)***
HEAVY-E-RM or GARCH(1,1)-X-RM	α_R	0.37 (9.99)***	0.37 (9.41)***	0.39 (7.61)***	0.42 (9.83)***	0.44 (9.40)***	0.40 (9.99)***
	β_R	0.57 (15.95)***	0.57 (13.43)***	0.57 (8.73)***	0.52 (12.13)***	0.49 (10.28)***	0.51 (11.94)***
	γ_R	0.02 (2.70)***	0.01 (2.94)***	0.02 (3.74)***	0.03 (3.77)***	0.02 (3.83)***	0.03 (4.51)***
	ϑ_R	0.005 (2.97)***	0.001 (2.29)***	0.000 (1.55)*	0.001 (2.40)***	0.002 (3.09)***	0.005 (4.38)***
GARCH$(1,0)$-X-RM	β_R	0.84 (81.52)***	0.87 (67.63)***	0.84 (76.78)***	0.87 (55.45)***	0.86 (85.38)***	0.82 (67.70)***
	γ_R	0.10 (14.13)***	0.06 (10.72)***	0.06 (11.73)***	0.09 (9.10)***	0.10 (12.92)***	0.11 (13.04)***
	ϑ_R	0.003 (3.60)***	0.001 (4.83)***	0.000 (0.00)	0.000 (0.00)	0.002 (4.06)***	0.004 (6.03)***

Notes: See Notes in Table 4.3

4.5.2 The Garman-Klass volatility

In this part of our study, we test the inclusion of an alternative measure of volatility (apart from squared returns and realised kernel) to the HEAVY framework already analysed. Using data on the daily high, low, opening and closing prices of each index in the OMI's realised library we generate an alternative daily measure of price volatility. To avoid the microstructure biases introduced by high frequency data and based on the conclusion of Chen et al. (2006) that the range-based and high-frequency integrated volatility provide essentially equivalent results, we employ the classic range-based estimator of Garman and Klass (1980) - GK - to construct the daily volatility (GK_t) as follows:

$$GK_t = \frac{1}{2}u^2 - (2\ln 2 - 1)c^2, \quad t \in \mathbb{N},$$

where u and c are the differences in the natural logarithms of the high and low and of the closing and opening prices respectively. We further form the SSR of the GK_t in order to use it as dependent variable in the HEAVY/GARCH/MEM models.

We run for SP all benchmark equations of Table 4.3 applying all possible combinations with GK_t as dependent variable and as regressor. Table 4.10 presents the new results with SP's GK_t. The boxed area contains the combinations already shown in Table 4.3. It is obvious from the results below that the realised kernel has a stronger effect on the GK volatility than the opposite when we estimate a significant non zero arch coefficient and add the lagged squared returns as second regressor. The reverse effect is observed when the arch effect is omitted. In the regressions with GK_t as dependent variable the arch coefficient is zero when the lagged RK_t is added, same as in the benchmark returns equations and when the lagged r_t^2 is added, the α_{GK} remains significant and non zero, same as in the benchmark realised measure equations. Comparing the standard GARCH$(1, 1)$ models of the three dependent variables, we see that the arch and garch coefficients of the GK volatility take values between the other two variables' coefficients' values, respectively. For the RK_t models, we result to a more sound impact of GK_t than the impact of the lagged r_t^2, with a lower arch coefficient when GK_t is added. Regarding the returns equations, the effect of RK_t dominates GK_t, which becomes zero when both regressors are used and both also absorb the arch effect (of the lagged r_t^2). Overall, we could deduce from our GK-extended models that the GK volatility by definition is a measure for intra-day volatility a lot more sufficient and 'correct' than the squared returns but less comprehensive and 'suitable' than the realised kernel to contain the intra-daily trading information. Our results show stronger RK_t effects than GK_t

and the GK_t models seem to share characteristics with both the other two dependent variables' equations, but with more similarities to the realised measure models. The same conclusions can be drawn after running the GK_t as dependent variable with asymmetries and power transformations for the six indices presented in this study (see in the Appendix Tables 4A.4 and 4A.5, the values of the power terms are between the respective values of the other two variables on average).

Table 4.10: HEAVY/GARCH models with GK_t, RK_t and r_t^2 for SP

Equ. (1) becomes:

$(1-\beta_i L)\sigma_{it}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \gamma_j \varepsilon_{j,t-1}^2,\ i,j = r, R, GK,\ j \neq i$

dependent variable ⟶		GK_t		RK_t		r_t^2
GARCH$(1,1)$	α_{GK}	0.21 (6.75)***	α_R	0.41 (10.75)***	α_r	0.09 (7.84)***
	β_{GK}	0.78 (26.84)***	β_R	0.58 (15.97)***	β_r	0.90 (80.08)***
GARCH$(1,1)$-X	α_{GK}	0.15 (7.29)***	α_R	0.37 (9.83)***	α_r	0.000 (0.00)
	β_{GK}	0.78 (33.04)***	β_R	0.59 (17.19)***	β_r	0.71 (15.44)***
	γ_{r^2}	0.03 (3.61)***	γ_{r^2}	0.02 (2.89)***	γ_{RK}	0.37 (6.70)***
GARCH$(1,0)$-X	β_{GK}	0.87 (67.47)***	β_R	0.85 (90.61)***	β_r	0.71 (16.70)***
	γ_{r^2}	0.07 (8.86)***	γ_{r^2}	0.10 (13.47)***	γ_{RK}	0.37 (6.61)***
GARCH$(1,1)$-X	α_{GK}	0.000 (0.00)	α_R	0.33 (6.95)***	α_r	0.000 (0.00)
	β_{GK}	0.64 (10.98)***	β_R	0.59 (16.00)***	β_r	0.81 (40.67)***
	γ_{RK}	0.28 (4.19)***	γ_{GK}	0.08 (3.13)***	γ_{GK}	0.30 (7.61)***
GARCH$(1,0)$-X	β_{GK}	0.64 (11.68)***	β_R	0.73 (35.16)***	β_r	0.81 (40.28)***
	γ_{RK}	0.28 (5.95)***	γ_{GK}	0.32 (11.64)***	γ_{GK}	0.30 (9.01)***
GARCH$(1,1)$-X	α_{GK}	0.000 (0.00)	α_R	0.28 (6.08)***	α_r	0.000 (0.00)
	β_{GK}	0.65 (11.30)***	β_R	0.60 (17.39)***	β_r	0.71 (14.90)***
	γ_{r^2}	0.01 (1.15)	γ_{r^2}	0.02 (3.06)***	γ_{RK}	0.37 (4.08)***
	γ_{RK}	0.25 (3.67)***	γ_{GK}	0.08 (3.36)***	γ_{GK}	0.000 (0.00)
GARCH$(1,0)$-X	β_{GK}	0.65 (12.09)***	β_R	0.72 (39.48)***	β_r	0.71 (16.11)***
	γ_{r^2}	0.01 (1.16)	γ_{r^2}	0.05 (6.38)***	γ_{RK}	0.37 (4.00)***
	γ_{RK}	0.25 (5.07)***	γ_{GK}	0.25 (12.69)***	γ_{GK}	0.000 (0.00)

Notes: See Notes in Table 4.3. The γ coefficient's subscript denotes the regressor variable used and not the dependent variable of the equation.

4.6 Structural breaks in the HEAVY framework

In this Section of the study we intend to identify the structural breaks effects on the HEAVY models and focus on the recent Global financial crisis. We test for structural breaks by employing the methodology in Bai and Perron (1998, 2003a,b), who address the problem of testing for multiple structural changes in a least squares context and under very general conditions on the data and the errors. In addition to testing for the presence of breaks, these statistics identify the number and location of multiple breaks. So, for each index squared returns and realised kernel series we identify the structural breaks with the Bai and Perron methodology (Table 4.11, Panels A and B). We select the breaks of the returns series (Table 4.11, Panel C) first to build the slope dummies (for the garch-x and heavy coefficient and the asymmetries in the AP-models) for the benchmark HEAVY models. We observe that for all indices a break date for the current financial crisis of 2007-08 is detected, so that we can focus on the recent crisis effect.

We first run the benchmark HEAVY equations with the selected returns breaks and present the results in Tables 4.12a and 4.12b. Our main finding is that the dummies corresponding to the 2007-08 crisis are mainly positive and give an increment to the coefficient they refer to. We observe always that the heavy and garch-x coefficients in the both the returns and the realised measure equations become higher after the crisis. The two other dummies for 2003 and 2009-10 (sovereign debt crisis) give mainly a negative effect. The garch $\beta's$ remain mostly in the same level as the models without dummies.

We then run the same equations with the realised kernel breaks and the asymmetric power specifications with the returns breaks (results not reported, available upon request). All our estimates are consistent and lead to similar conclusions for the crisis effects as the results reported from the benchmark models in Tables 4.12a,b. We could only notice that the power terms are close, but tend to be a bit higher than the models without break dummies.

Table 4.11: Break dates

Panel A: Squared returns

Break	SP-r	NIKKEI-r	TSE-r	FTSE-r	DAX-r	EUSTOXX-r
1	02/04/2003	17/12/2003	14/01/2008	19/05/2003	07/07/2003	22/05/2003
2	08/09/2006	27/12/2007	03/09/2009	23/07/2007	14/01/2008	18/01/2008
3	03/09/2008	13/01/2010		15/07/2009	11/01/2010	27/05/2010
4	17/08/2010					

Panel B: Realised Kernel

Break	SP-R	NIKKEI*-R	TSE-R	FTSE-R	DAX-R	EUSTOXX-R
1	11/04/2003	17/12/2003	21/09/2005	29/04/2003	21/05/2003	06/06/2003
2	04/01/2008	09/05/2007	04/01/2008	24/07/2007	09/05/2005	15/01/2008
3	28/12/2009	19/05/2009	25/08/2009	16/07/2009	17/01/2008	26/01/2010

Panel C: Selected break dates for each index (from squared returns)

Break	SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
1	02/04/2003	17/12/2003	14/01/2008	19/05/2003	07/07/2003	22/05/2003
2	03/09/2008	27/12/2007	03/09/2009	23/07/2007	14/01/2008	18/01/2008
3	17/08/2010	13/01/2010		15/07/2009	11/01/2010	27/05/2010

Bai & Perron breaks identification:
Results selected from the repartition procedure for 1% significance level
with 5 maximum number of breaks and 0.15 trimming parameter.
* For the realised kernel of NIKKEI the 10% significance level is preferred,
since the 1% level gives only 1 break at 19/05/2009

Table 4.12a: HEAVY/GARCH models with breaks (selected from squared returns)

$(1-\beta_i L)\sigma_{it}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \sum_{z=1}^{3} \alpha_{iz} D_{iz} \varepsilon_{i,t-1}^2 + \gamma_i \varepsilon_{j,t-1}^2 + \sum_{z=1}^{3} \gamma_{iz} D_{iz} \varepsilon_{j,t-1}^2$,

$i, j = r, R$, $j \neq i$, $z = 1, 2, 3$ the break dates and D_{iz} the dummies for the breaks

		SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
GARCH$(1,1)-r$	α_r	0.10 (7.24)***	0.11 (6.28)***	0.074 (6.35)***	0.11 (6.61)***	0.11 (5.88)***	0.11 (6.06)***
	α_{r1}	-0.028 (-2.67)***	-0.026 (-1.94)**	0.027 (2.04)***	-0.048 (-3.52)***	-0.037 (-2.74)***	-0.038 (-3.06)***
	α_{r2}	0.028 (2.37)***	0.030 (1.95)**	-0.024 (-1.92)**	0.066 (3.51)***	0.028 (2.66)**	0.034 (2.97)**
	α_{r3}	-0.018 (-1.47)*	-0.020 (-1.42)		-0.030 (-2.07)***		
	β_r	0.90 (75.79)***	0.88 (54.21)***	0.90 (56.13)***	0.88 (56.63)***	0.89 (60.94)***	0.89 (55.56)***
HEAVY-E-r or GARCH(1,1)-X-r	α_r	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)
	β_r	0.71 (14.44)***	0.72 (9.12)***	0.74 (16.24)***	0.61 (10.52)***	0.60 (9.66)***	0.62 (12.11)***
	γ_r	0.38 (6.43)***	0.44 (2.90)***	0.69 (4.52)***	0.44 (5.13)***	0.45 (5.79)***	0.50 (6.42)***
	γ_{r1}	-0.061 (-1.89)**		-0.14 (-2.01)***			-0.145 (-3.13)***
	γ_{r2}	0.066 (2.24)***	0.22 (2.23)***	-0.22 (-3.29)***	0.18 (3.91)***	0.05 (1.44)*	0.167 (2.90)***
	γ_{r3}						-0.085 (-1.65)**
HEAVY-r or GARCH(1,0)-X-r	β_r	0.71 (16.22)***	0.72 (10.27)***	0.74 (16.93)***	0.61 (11.68)***	0.60 (9.69)***	0.62 (12.12)***
	γ_r	0.38 (6.33)***	0.44 (3.94)***	0.69 (5.60)***	0.44 (6.74)***	0.45 (5.88)***	0.50 (6.71)***
	γ_{r1}	-0.061 (-1.89)**		-0.14 (-2.13)***			-0.145 (-3.13)***
	γ_{r2}	0.066 (2.27)***	0.22 (2.93)***	-0.22 (-3.58)***	0.18 (4.12)***	0.05 (1.44)*	0.167 (2.91)***
	γ_{r3}						-0.085 (-1.65)**

Notes: See Notes in Table 4.3

Table 4.12b: HEAVY/GARCH models with breaks (selected from squared returns)

$(1-\beta_i L)\sigma_{it}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \sum_{z=1}^{3} \alpha_{iz} D_{iz} \varepsilon_{i,t-1}^2 + \gamma_i \varepsilon_{j,t-1}^2 + \sum_{z=1}^{3} \gamma_{iz} D_{iz} \varepsilon_{j,t-1}^2$,

$i, j = r, R$, $j \neq i$, $z = 1, 2, 3$ the break dates and D_{iz} the dummies for the breaks

		SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
HEAVY-RM or GARCH(1,1)-RM	α_R	0.43 $(11.10)^{***}$	0.43 $(9.66)^{***}$	0.39 $(11.04)^{***}$	0.52 $(6.34)^{***}$	0.54 $(10.50)^{***}$	0.50 $(11.49)^{***}$
	α_{R1}	-0.053 $(-3.51)^{***}$	-0.045 $(-2.84)^{***}$	0.077 $(3.15)^{***}$	-0.108 $(-2.38)^{***}$	-0.092 $(-4.04)^{***}$	-0.082 $(-4.15)^{***}$
	α_{R2}	0.065 $(2.37)^{***}$	0.051 $(2.62)^{***}$	-0.029 $(-1.59)^{*}$	0.110 $(2.24)^{***}$	0.082 $(3.59)^{***}$	0.070 $(4.12)^{***}$
	α_{R3}	-0.044 $(-1.58)^{*}$	-0.053 $(-2.36)^{***}$		-0.065 $(-2.73)^{***}$	-0.037 $(-1.94)^{**}$	
	β_R	0.56 $(15.22)^{***}$	0.56 $(12.65)^{***}$	0.55 $(12.38)^{***}$	0.49 $(6.07)^{***}$	0.45 $(9.62)^{***}$	0.49 $(12.01)^{***}$
HEAVY-E-RM or GARCH(1,1)-X-RM	α_R	0.37 $(10.48)^{***}$	0.34 $(8.48)^{***}$	0.35 $(10.32)^{***}$	0.38 $(9.28)^{***}$	0.43 $(9.75)^{***}$	0.39 $(10.32)^{***}$
	β_R	0.58 $(17.38)^{***}$	0.58 $(14.56)^{***}$	0.56 $(12.12)^{***}$	0.52 $(13.08)^{***}$	0.49 $(11.13)^{***}$	0.53 $(13.82)^{***}$
	γ_R	0.037 $(3.74)^{***}$	0.041 $(6.05)^{***}$	0.012 $(3.59)^{***}$	0.094 $(5.41)^{***}$	0.070 $(5.82)^{***}$	0.049 $(5.59)^{***}$
	γ_{R1}	-0.031 $(-3.15)^{***}$	-0.025 $(-3.27)^{***}$	0.035 $(4.18)^{***}$	-0.074 $(-4.52)^{***}$	-0.060 $(-4.35)^{***}$	-0.034 $(-3.02)^{***}$
	γ_{R2}	0.036 $(2.17)^{***}$	0.012 $(1.46)^{*}$		0.038 $(3.38)^{***}$	0.033 $(2.70)^{***}$	0.027 $(2.59)^{***}$
	γ_{R3}	-0.033 $(-1.81)^{**}$	-0.015 $(-1.88)^{**}$		-0.030 $(-3.13)^{**}$	-0.019 $(-1.62)^{*}$	
GARCH$(1,0)$-X-RM	β_R	0.85 $(83.48)^{***}$	0.84 $(59.76)^{***}$	0.81 $(64.65)^{***}$	0.84 $(68.14)^{***}$	0.84 $(78.92)^{***}$	0.83 $(70.50)^{***}$
	γ_R	0.11 $(13.64)^{***}$	0.09 $(10.32)^{***}$	0.042 $(13.90)^{***}$	0.130 $(10.93)^{***}$	0.138 $(11.71)^{***}$	0.120 $(11.95)^{***}$
	γ_{R1}	-0.021 $(-4.08)^{***}$	-0.026 $(-7.22)^{***}$	0.054 $(9.55)^{***}$	-0.068 $(-7.46)^{***}$	-0.057 $(-7.13)^{***}$	-0.028 $(-4.35)^{***}$
	γ_{R2}	0.015 $(1.85)^{**}$			0.035 $(4.97)^{***}$	0.032 $(4.84)^{***}$	0.026 $(4.76)^{***}$
	γ_{R3}	-0.020 $(-2.42)^{***}$	-0.022 $(-6.69)^{***}$		-0.025 $(-4.65)^{***}$	-0.009 $(-1.60)^{*}$	

Notes: See Notes in Table 4.3

4.7 Conclusions

Our study extends the HEAVY models introduced by SS10 and the MEM models of Engle (2002b) through the GARCH framework with long memory, leverage and power transformations. We result to prefer the most comprehensive specification for the realised measure models, the HYAPARCH of Schoffer (2003), after which we name the new model estimated as HYDAP-HEAVY and, respectively, the MEM models introduced as HYDAP-MEM. For the squared returns equations we prefer the FIAP-HEAVY formulation, restricting the hyperbolic parameter to 1. Since the HEAVY class of models with realised volatility on high-frequency data are proved to outperform the simple daily GARCH estimations and forecasts, our extensions can provide a complete framework to analyse the volatility process. The fractional integration of volatility, its asymmetric response to negative and positive shocks and its power transformations ensures the superiority of our contribution, which can be implemented on the areas of asset allocation and portfolio selection as well as on several risk management practices.

Moreover, when adding the overnight trading activity indicator to the original HEAVY equations the positive feedback effect of the overnight surprise to volatility is small but significant for the squared returns and immaterial for the realised measures. So, the HEAVY framework is proved adequate to capture the volatility dynamics without additional trading information. The Garman-Klass volatility included in the HEAVY framework exhibits more similarities to the realised measure behaviour and the structural breaks applied prove the time-varying pattern of the benchmark HEAVY models' parameters with the break corresponding to the financial crisis of 2007-08 giving a positive increment on the garch-x and heavy coefficients.

Further research on the HEAVY model could focus on the multivariate specification applying the HYDAP-HEAVY extensions on the multivariate HEAVY of Noureldin et al. (2012). In the univariate framework we could also test more lags of the coefficients in both HEAVY equations, assume different error distributions like the Student-t or the Skewed-t and perform forecasts using the selected HYDAP-HEAVY model or the simpler fractional, power or asymmetric structures presented.

4.8 APPENDIX

Table 4A.1: Linear A(P)-HEAVY-E Models; Specifications with $\zeta_i = 0$ and δ_i fixed

Specification↓		SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
		Panel A: Power Transformed Squared Returns					
AP-HEAVY-E-r or APGARCH(1,0)-X-r	β_r	0.88 (66.98)***	0.84 (33.70)***	0.88 (65.71)***	0.84 (30.15)***	0.86 (47.45)***	0.88 (50.84)***
	μ_r	0.15 (7.27)***	0.14 (5.06)***	0.12 (7.01)**	0.15 (7.96)***	0.16 (6.63)***	0.17 (6.54)***
	γ_r	0.001 (2.69)***	0.003 (4.99)***	0.001 (3.96)**	0.008 (3.09)***	0.001 (3.55)***	0.001 (2.79)***
	δ_r	2.00	2.00	2.00	1.90	2.00	2.00
		Panel B: Power Transfomed Realised Measures					
AP-HEAVY-E-RM or APGARCH(1,1)-X-RM	β_R	0.66 (28.98)***	0.62 (19.93)***	0.69 (24.55)***	0.62 (19.61)***	0.63 (18.87)***	0.65 (23.21)***
	α_R	0.29 (13.16)***	0.32 (12.20)***	0.26 (11.15)***	0.29 (10.46)***	0.32 (10.69)***	0.28 (11.63)***
	μ_R	0.10 (9.73)***	0.06 (8.01)***	0.06 (7.16)***	0.24 (4.63)***	0.06 (8.04)***	0.08 (9.32)***
	γ_R	0.46 (3.35)***	0.42 (3.69)***	0.84 (4.14)***	0.31 (4.99)***	0.63 (4.66)**	0.72 (5.11)***
	δ_R	1.10	1.10	1.00	1.30	1.10	1.10

Notes: See Notes in Table 4.3

The estimated models with fixed δ_i, $i = r, R$ are:

$$\sigma_{it}^{\delta_i} = \omega_i + (\alpha_i + \mu_i s_{t-1}) |\varepsilon_{i,t-1}|^{\delta_i} + \beta_i \sigma_{i,t-1}^{\delta_i} + (\gamma_i + \pi_i s_{t-1}) |\varepsilon_{j,t-1}|^{\delta_j}$$

Table 4A.2: HEAVY-E-RM Models; The γ_R Parameter

Spec.↓	SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
–	0.02 (2.89)***	0.02 (4.19)***	0.02 (4.55)***	0.04 (4.34)***	0.03 (4.08)***	0.03 (5.01)***
P	0.59 (1.68)**	0.53 (1.57)*	0.74 (6.82)***	0.60 (1.58)*	0.71 (1.60)*	0.62 (1.64)**
AP	0.56 (1.76)**	0.50 (2.14)***	0.74 (1.67)**	0.59 (1.33)	0.78 (1.80)**	0.94 (2.02)***
HY	0.02 (2.92)***	0.02 (3.76)***	0.02 (5.59)***	0.05 (3.59)***	0.02 (3.58)***	0.03 (4.25)***
HYP	0.95 (1.63)**	0.58 (1.34)	0.59 (1.43)*	1.01 (1.68)**	0.44 (1.18)	0.37 (1.19)
$HYAP$	0.46 (1.35)	0.69 (1.56)**	0.48 (1.52)*	0.95 (1.43)*	0.59 (1.55)*	0.57 (1.56)*

Notes: See Notes in Table 4.3

Table 4A.3: HEAVY-E-RM Models; The π_R Parameter

Spec.↓	SP	NIKKEI	TSE	FTDSE	DAX	EUSTOXX
DAP	0.95 (1.90)**	0.56 (1.97)***	0.84 (9.55)***	0.77 (3.08)***	0.87 (1.87)**	0.96 (2.10)***
$HYDAP$	0.95 (1.65)**	0.78 (1.44)*	1.11 (2.30)***	1.86 (2.24)***	0.84 (1.81)**	1.07 (1.47)*

Notes: See Notes in Table 4.3

Table 4A.4: AP-GARCH-X models - dependent variable GK_t with regressor r_t^2
Equ. (1) becomes: $(1-\beta_i L)\sigma_{it}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \gamma_j \varepsilon_{j,t-1}^2$, $i, j = r, R, GK$, $j \neq i$

		SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
PGARCH$(1,1)$-X-GK	α_{GK}	0.16 (7.20)***	0.18 (5.60)***	0.22 (9.19)***	0.17 (7.77)***	0.20 (10.30)***	0.18 (8.66)***
	β_{GK}	0.77 (31.39)***	0.74 (14.06)***	0.71 (22.62)***	0.76 (34.71)***	0.75 (37.42)***	0.75 (35.21)***
	γ_{GK}	0.27 (1.31)	0.33 (1.05)	1.03 (1.55)*	0.19 (1.20)	0.33 (1.59)*	0.34 (1.48)*
	δ_{GK}	1.47 (6.68)***	1.39 (6.18)***	1.17 (7.81)***	1.67 (7.38)***	1.41 (7.71)***	1.46 (7.45)***
APGARCH$(1,0)$-X-GK	β_{GK}	0.83 (44.20)***	0.81 (33.25)***	0.83 (40.78)***	0.83 (50.50)***	0.85 (69.18)***	0.83 (59.69)***
	γ_{GK}	0.03 (2.74)***	0.01 (1.65)**	0.07 (2.47)***	0.04 (2.23)***	0.04 (3.24)***	0.06 (1.97)***
	δ_{GK}	2.01 (26.36)***	2.27 (19.32)***	1.78 (24.32)***	2.05 (25.29)***	1.99 (29.86)***	1.94 (16.80)***
	μ_{GK}	0.17 (2.93)***	0.05 (1.78)**	0.51 (2.94)***	0.13 (2.74)***	0.16 (3.47)***	0.20 (2.10)***
APGARCH$(1,1)$-X-GK	α_{GK}	0.14 (6.49)***	0.18 (6.38)***	0.18 (7.95)***	0.15 (7.78)***	0.17 (10.62)***	0.16 (8.85)***
	β_{GK}	0.80 (36.09)***	0.73 (15.23)***	0.75 (24.65)***	0.78 (34.52)***	0.78 (40.93)***	0.78 (37.36)***
	γ_{GK}	0.55 (1.68)**	0.57 (1.51)*	0.90 (1.58)*	0.15 (1.53)*	0.95 (2.09)***	0.97 (2.19)***
	δ_{GK}	1.21 (6.02)***	1.23 (6.55)***	1.16 (7.70)***	1.68 (7.68)***	1.00 (5.94)***	1.07 (6.36)***
	μ_{GK}	0.38 (6.99)***	0.18 (5.73)***	0.23 (5.99)***	0.20 (4.52)***	0.27 (9.03)***	0.29 (7.72)***

Notes: See Notes in Table 4.3

Table 4A.5: AP-GARCH-X models - dependent variable GK_t with regressor RK_t
Equ. (1) becomes: $(1-\beta_i L)\sigma_{it}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \gamma_j \varepsilon_{j,t-1}^2$, $i, j = r, R, GK$, $j \neq i$

		SP	NIKKEI	TSE	FTSE	DAX	EUSTOXX
PGARCH$(1,1)$-X-GK	α_{GK}	0.000 (0.00)	0.000 (0.00)	0.06 (0.76)	0.04 (1.52)*	0.07 (2.39)***	0.03 (1.08)
	β_{GK}	0.64 (10.90)***	0.60 (7.27)***	0.55 (1.96)***	0.57 (24.27)***	0.60 (12.39)***	0.62 (14.46)***
	γ_{GK}	0.31 (2.55)***	0.09 (1.46)*	0.50 (2.15)***	0.34 (11.02)***	0.28 (2.59)***	0.31 (2.66)***
	δ_{GK}	1.98 (22.70)***	2.29 (17.64)***	1.94 (19.02)***	2.01 (96.59)***	2.00 (24.45)***	1.99 (22.82)***
APGARCH$(1,0)$-X-GK	β_{GK}	0.71 (15.82)***	0.61 (7.64)***	0.61 (11.95)***	0.63 (11.46)***	0.66 (15.48)***	0.67 (16.05)***
	γ_{GK}	0.14 (2.72)***	0.07 (1.45)*	0.41 (2.63)***	0.30 (1.74)**	0.20 (3.20)***	0.22 (2.88)***
	δ_{GK}	2.01 (26.54)***	2.29 (17.10)***	1.92 (23.79)***	1.99 (16.21)***	2.02 (31.53)***	2.01 (27.22)***
	μ_{GK}	0.17 (2.92)***	0.03 (1.49)*	0.22 (2.43)***	0.12 (2.02)***	0.14 (3.47)***	0.12 (3.07)***
APGARCH$(1,1)$-X-GK	α_{GK}	0.05 (6.02)***	0.02 (2.09)***	0.06 (4.97)***	0.04 (3.82)***	0.04 (2.59)***	0.04 (4.18)***
	β_{GK}	0.72 (15.40)***	0.60 (7.68)***	0.61 (11.80)***	0.63 (11.31)***	0.66 (15.35)***	0.67 (16.41)***
	γ_{GK}	0.31 (2.53)***	0.07 (1.61)*	0.48 (7.79)***	0.31 (1.67)**	0.26 (2.84)***	0.32 (2.45)***
	δ_{GK}	1.81 (16.82)***	2.29 (18.00)***	1.88 (68.43)***	1.97 (15.32)***	1.96 (24.33)***	1.91 (19.61)***
	μ_{GK}	0.98 (24.18)***	0.99 (2.93)***	0.61 (4.94)***	0.69 (4.37)***	0.88 (2.57)***	0.90 (4.91)***

Notes: See Notes in Table 4.3

Concluding Remarks

The purpose of this thesis was to investigate the financial volatility dynamics through the GARCH modelling framework. We use univariate and multivariate GARCH-type models enriched with long memory, asymmetries and/or power transformations that best fit the volatility process. We study the financial time series volatility and co-volatility taking into account the structural breaks detected and focusing on the effects of the corresponding financial crisis events. We conclude to provide a complete framework for the analysis of volatility with major policy implications and usefulness on the current risk management practices.

We first investigate the issue of temporal ordering of the range-based volatility and turnover volume in the Korean stock market applying a univariate dual long memory model. In this framework we study the volume-volatility link for different investor categories and orders, before and after the Asian financial crisis. We complement the literature about the impact of domestic and foreign investors on emerging stock markets by examining the effect of the trading volume on the stock market volatility, taking into consideration for each volume series its sell and buy side as well as its total separately and also investigating the effect of each of the eight different domestic investor groups, that compromise the total domestic trading volume. We further study the volume effect on volatility during different periods of the economic cycle including the Asian financial crisis shock. The causality effects are found to be sensitive to the period examined in terms of their sign. Our analysis suggests that the behaviour of volatility depends upon volume, but also that the nature of this dependence varies with time and the measure of volume used. In particular, in the pre-crisis period foreign investors' volume as a total and from its buy side affect volatility negatively, while in the post-crisis period this effect turns to positive. This behaviour is reflected also in the total volume's respective effects. Total domestic investors affect volatility positively across all samples, while the most informed ‘market players’ (securities companies, investment banks, mutual funds and insurance companies), when examined separately, are proved to have a negative impact on volatility in the pre-crisis period. In sharp contrast, in the post-crisis period increased volume leads always to higher volatility. Finally, almost all investors’ sales are found to affect volatility positively regardless of the sample period. Lastly, the apparent long memory in volatility is quite resistant to ‘mean shifts’. However, when we take into account structural breaks the order of integration of the conditional variance series decreases considerably.

In the second part of the thesis we examine the applicability of the multivariate FIAPARCH model with DCC to eight stock market indices returns, also taking into account the structural breaks corresponding to financial crisis events. The VAR-DCC-FIAPARCH model is proved to capture thoroughly the volatility and correlation processes compared to simpler specifications, like the multivariate GARCH with CCC. We provide strong evidence that conditional volatilities are better modelled incorporating long memory, power effects and leverage features. We further prove that time-varying conditional correlations across markets, estimated by the DCC model, are highly persistent and follow a sound upward pattern during financial crises. The cross-border contagion effects depicted on the increasing correlations and the herding behaviour amongst investors as the correlations remain high confirm the existing empirical evidence. We also compare two different crises in terms of correlations to observe higher correlations in the recent Global financial crisis than in the Asian one. The financial liberalisation, deregulation and integration of the markets has led to more apparent market interdependence nowadays.

The third part of the thesis examines how the most prevalent stochastic properties of key financial time series have been affected during the recent financial crises. In particular we focus on changes associated with the remarkable economic events of the last two decades in the mean and volatility dynamics, including the underlying volatility persistence and volatility spillovers structure. Using daily data from several key stock market indices we find that stock market returns exhibit time-varying persistence in their corresponding conditional variances. Furthermore, the results of our bivariate UEDCC-AGARCH models show the existence of time-varying correlations as well as time-varying shock and volatility spillovers between the returns of FTSE and DAX and those of NIKKEI and Hang Seng, which became more prominent during the recent financial crisis. Our theoretical considerations on the time-varying model which provides the platform upon which we integrate our multifaceted empirical approaches are also of independent interest. In particular, we provide the general solution for low order time-varying specifications, which is a long standing research topic. This enables us to characterise these models by deriving, first, their multistep ahead predictors, second, the first two time-varying unconditional moments and, third, their covariance structure.

The final part of the thesis studies and extends the high-frequency-based volatility (HEAVY) model of Shephard and Sheppard (2010). The HEAVY framework models financial volatility based on both daily and intra-daily data, so that the system of equations estimated adopts to information arrival more rapidly than the daily GARCH models. It combines daily returns with realised volatility calculated on high-frequency data using both the GARCH and the MEM class of models. Its mean reversion and short-run momentum effects result to higher quality performance in volatility level shifts and more reliable forecasts. Our main contribution is the enrichment of the HEAVY model with long memory, asymmetries and power transformations through the HYAPARCH specification of Schoffer (2003) and its nested power, fractional and asymmetric models. The most extended model is named HYDAP-HEAVY and is preferred against the nested structures for the realised measure modelling. For the squared returns equations we prefer the FIAP-HEAVY formulation, restricting the hyperbolic parameter to 1. Since the HEAVY class of models with realised volatility on high-frequency data are proved to outperform the simple daily

GARCH estimations and forecasts, our extensions can provide a complete framework to analyse the volatility process. The fractional integration of volatility, its asymmetric response to negative and positive shocks and its power transformations ensures the superiority of our contribution, which can be implemented on the areas of asset allocation and portfolio selection as well as on several risk management practices. Moreover, we extend the original HEAVY specification with the overnight trading activity indicator resulting to a positive feedback to the volatility of returns, but mostly trivial impact on the realised measure. The Garman-Klass volatility included in the HEAVY framework gives results more similar to the realised kernel modelling. Finally, the structural breaks applied capture the time-varying behaviour of the process' parameters in particular after the Global financial crisis of 2007-08.

Further research on the HEAVY models could focus on the multivariate specification applying the HYDAP-HEAVY extensions on the multivariate HEAVY of Noureldin et al. (2012). In the univariate framework we could also test more lags of the coefficients in both HEAVY equations, assume different error distributions like the Student-t or the Skewed-t and perform forecasts using the selected HYDAP-HEAVY model or the simpler fractional, power or asymmetric formulations.

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