Publisher: Institute for Operations Research and the Management Sciences (INFORMS) INFORMS is located in Maryland, USA


## Management Science

Publication details, including instructions for authors and subscription information: http:// pubsonline.informs.org

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## To cite this article:

Ioana Chioveanu, Jidong Zhou (2013) Price Competition with Consumer Confusion. Management Science 59(11):2450-2469. http:// dx. doi.org/ 10.1287/ mnsc. 2013.1716

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# Price Competition with Consumer Confusion 

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#### Abstract

This paper proposes a model in which identical sellers of a homogeneous product compete in both prices and price frames (i.e., ways to present price information). Frame choices affect the comparability of price offers and may cause consumer confusion and lower price sensitivity. In equilibrium, firms randomize their frame choices to obfuscate price comparisons and sustain positive profits. The nature of the equilibrium depends on whether frame differentiation or frame complexity is more confusing. Moreover, an increase in the number of competitors induces firms to rely more on frame complexity, and this may boost industry profits and lower consumer surplus.


Key words: bounded rationality; framing; oligopoly markets; frame dispersion; price dispersion
History: Received January 10, 2012; accepted January 18, 2013, by J. Miguel Villas-Boas, marketing. Published online in Articles in Advance May 7, 2013.

## 1. Introduction

Sellers often use various ways to convey price information to consumers. Retailers use different discount methods to promote products, such as direct price reductions, percentage discounts, volume discounts, or vouchers. ${ }^{1}$ Some restaurants, hotels, and online booksellers offer a single all-inclusive price, whereas others divide the price by quoting table service, breakfast, Internet access, parking, or shipping fees separately. Airlines and travel agencies charge card payment fees in different ways. For instance, Wizz charges a flat $£ 4$ per person, whereas Virgin Atlantic charges $1.3 \%$ of the total booking (see BBC News 2009). Retailers offer store cards with diverse terms such as " $10 \%$ off first shop if opened online or $10 \%$ off for the first week if opened in store," " 500 bonus points on first order," or " $£ 5$ voucher after first purchase." Financial product prices are also often framed distinctively: mortgage arrangement fees may be rolled into the interest rate or not; some loans may specify the monthly interest rate, whereas others the annual interest rate. Many retailers also change their price presentation formats over time. For example, some products are sold at a discounted price (say, $30 \%$ off) in one

[^0]week but are offered with a "buy one get one free" deal in another. ${ }^{2}$

Strategic choice of price presentation formats, or simply, price framing, has received relatively little attention in the economics literature in spite of its prevalence. If firms use different price frames to compete better for consumers, industry-specific pricing schemes whose terms facilitate comparisons should emerge. But persistent variation in price frames in many markets is more likely to confuse consumers and harm competition. In recent years, potentially confusing pricing has attracted the public's and policy makers' attention. A main concern is that sellers deliberately use price framing to obfuscate price comparisons and soften competition. For example, the inconsistent use of unit prices (e.g., price per unit versus price per weight) in British grocery stores was criticized by consumer watchdogs. In utility markets, complex tariff structures make it difficult for consumers to understand what type of deal they are on and how to reduce energy use/costs. ${ }^{3}$

[^1]This paper proposes a model where sellers of a homogeneous product can choose both price frames and prices. We assume that consumers might be confused by price framing and so fail to identify the best available deal in the market. ${ }^{4}$ Specifically, in our model firms can choose one of two frames. Consumers may get confused by two prices in different frames or two prices in a common but complex frame (if it is available). If consumers get confused, they choose one product randomly; otherwise, they behave rationally. ${ }^{5}$ We show that in equilibrium firms adopt mixed strategies that randomize on both price frames and prices and make strictly positive profits in an otherwise homogeneous product market (regardless of the number of firms). Moreover, if the number of firms increases, as it becomes more difficult to obfuscate price comparisons by adopting different frames, using complex price frames becomes more effective in confusing consumers. As a result, more competition induces firms to adopt complex frames more often, and this may actually boost profits and harm consumers. Our results suggest that in the presence of price framing, a standard competition policy approach may have undesired effects on consumer welfare.
Marketing research has provided evidence that consumers have difficulties in comparing prices presented differently or prices presented in similar but involved formats (see Estelami 1997, Morwitz et al. 1998, and Thomas and Morwitz 2009). Economics experiments (see, e.g., Kalayci and Potters 2011 and Kalayci 2011) show that increasing the number of product attributes or price scheme dimensions can create confusion and lead to suboptimal consumer choices. We explore two sources of consumer confusion due to price presentation: frame differentiation (when firms adopt different frames) and frame complexity (when firms use a common but complex frame).
Consider, for instance, the following two frames: "price per unit" and "price per kilogram" (these formats are commonly used in grocery stores for varieties of the same fruit/vegetable). In that case, each price frame is simple, and comparing two prices in the same frame should be easy. But comparing a price per unit with a price per kilogram may be difficult for some consumers. Here, frame differentiation is the

[^2]only confusion source. Other examples where frame differentiation is the main confusion source are "price including VAT" versus "price excluding VAT" (when the same tax rate applies), flat card payment fees versus percentage ones, and monthly interest rate versus annual interest rate quotations on loans.

Now consider the frames "price including shipping fee" and "price plus shipping fee." Ranking allinclusive prices is easy and, as before, comparing prices in different frames might be difficult. But comparing prices that quote the shipping fees separately may also be confusing if the fees vary across sellers. Here, frame complexity related to the use of a two-dimensional format is also a source of consumer confusion. This is true in other settings (e.g., financial services or utility markets) where some frames (e.g., multipart tariffs) are intrinsically complex and it may be difficult for consumers to figure out the true price. ${ }^{6}$ For instance, mortgage deals with the service fee quoted separately are usually harder to compare than are deals with the service fees rolled in the interest rate. When both sources of confusion coexist, it is not obvious which of them is more likely to confuse consumers (i.e., if more consumers get confused when they face prices in different frames or when they face prices in the same complex frame). The answer might depend on the microfoundations of confusion, as we discuss in $\S 2$. In this paper, we consider both possibilities.

Our study predicts both frame and price dispersion in the presence of price framing. When the market is relatively transparent (say, when duopolists use the same simple frame), a firm has an incentive to create more confusion through its price frame choice (by switching to a different frame) and take advantage of confused consumers. But when the market is already confusing enough (say, when duopolists use different frames), a firm has an incentive to reduce confusion by choosing its rival's frame and then undercutting. Because of this conflict, firms mix on price frames, which generates both price aware buyers (who compare prices perfectly) and confused buyers (who shop at random). As a result, firms also randomize on prices in equilibrium. This prediction is consistent with casual observations in many markets. Retailers such as grocery stores and online shops use different price frames and also change their frames over time. In these markets there is, moreover, substantial price dispersion. (Note that the price dispersion literature associates mixed-strategy equilibria with both cross-sectional and intertemporal variations.)

[^3]The nature of equilibrium depends on which source of confusion leads to more confused consumers. The intuition is best illustrated in duopoly. If frame differentiation leads to more confused consumers than does frame complexity, then when a firm switches from one frame to the other, more or fewer consumers get confused depending on its rival's frame. In effect, there is no clear ranking (on average) among prices associated with different frames. For example, with two simple frames "price per unit" versus "price per kg," our model predicts no significant price differences on average across different price formats. By contrast, in markets where frame complexity leads to more confused consumers, when a firm switches from a relatively simple frame to a more complex one, more consumers get confused regardless of the rival's frame choice. Then, the more complex frame is associated with higher prices. Woodward (2003) and Woodward and Hall (2010) provide evidence that mortgage market deals with the arrangement fees rolled into the interest rate are on average better than the deals with separate fees. ${ }^{7}$

In our setting, deliberately choosing different or complex price frames is a way to make price comparisons difficult that ultimately allows suppliers of otherwise homogeneous products to obtain positive profits. We show that a decrease in concentration weakens firms' ability to frame differentiate (if the number of frames is fixed) and makes them overload on frame complexity. In particular, in fragmented oligopolies, firms tend to use the more complex price frame almost surely and industry profits are always bounded away from zero. A decrease in concentration has a positive effect on consumer welfare (depresses the prices) but also a negative one (higher market complexity and less competitive pressure). In the presence of price framing, when the latter effect dominates, an increase in the number of firms boosts industry profits and harms consumers. ${ }^{8}$ Hortaçsu and Syverson (2004) provide evidence that in the S\&P index fund market where multidimensional fee schemes prevail, a decrease in concentration between 1995 and 1999 indeed triggered an increase in the average price.

[^4]
### 1.1. Related Literature

Recent economics research investigates price complexity and firms' intentional attempts to degrade the quality of information to the consumers. Ellison and Ellison (2009) find empirical evidence on obfuscation strategies in online markets where retailers deliberately create more confusing websites to make it harder for the consumers to figure out the total price. Carlin (2009) and Ellison and Wolitzky (2012) address this issue in the information search framework where each firm chooses both a price and a price complexity level. They argue that if it is more costly for consumers to assess complex prices, each firm will individually increase price complexity to reduce consumers' incentives to gather information and weaken price competition. ${ }^{9}$ Our model also considers price complexity, but it incorporates the effect of price frame differentiation and regards it as an important source of market complexity. In particular, in our model, how a firm's frame choice affects price competition also depends on rivals' frame choices. This strategic dependence induces firms to mix frames. So our model predicts that firms tend to adopt different price frames or change their frames over time.

In a closely related paper, Piccione and Spiegler (2012) also examine frame-price competition. They focus on a duopoly model with a more general frame structure and mainly examine the relation between equilibrium properties and the frame structure. Our duopoly example in $\S 2$ can be regarded as a special case of their model. However, we develop an oligopoly model to analyze the impact of greater competition on firms' strategies and market outcomes in the presence of price framing. In addition, our analysis explores the interaction between frame differentiation and frame complexity as sources of consumer confusion.

Our paper is also related to a recent literature on how shrouding a price component or making it less salient affects consumers' price perception and market outcomes. Ellison (2005) studies an add-on pricing model in which consumers have imperfect information about the prices of add-ons and need to pay a search cost to find them out. He shows that if the consumers who have a low valuation for the add-ons are also more price sensitive, the existence of add-ons can

[^5]soften price competition and sustain positive profits. Gabaix and Laibson (2006) endogenize shrouded addon price information by introducing boundedly rational consumers who ignore the add-on price if no firm advertises it. (See Hossain and Morgan 2006, Chetty et al. 2009, and Brown et al. 2010 for related empirical work.) Our model does not explore hidden price information, but while all price information is available, framing affects choice because of consumers' limited cognitive abilities.

Finally, our study contributes to the growing literature on bounded rationality and industrial organization (see the survey by Ellison 2006 and Spiegler 2011). In our model, the inability of boundedly rational consumers to compare framed prices leads to equilibrium frame dispersion. Our study is also related to the literature on consumer search and price dispersion (see Baye et al. 2006 for a survey). But we focus on how firms may confuse consumers by mixing their frame choices, so in our model price dispersion is a by-product of frame dispersion.

## 2. The Duopoly Model

This section illustrates in a duopoly example the coexistence of frame and price dispersion and how the relative importance of frame differentiation and frame complexity affects the nature of equilibrium.

### 2.1. Model

Consider a market for a homogeneous product with two sellers, firms 1 and 2. The constant marginal cost of production is normalized to zero. There is a unit mass of consumers, each demanding at most one unit of the product and willing to pay at most 1 . There are two possible price presentation formats, referred to as frames $A$ and $B$. We assume that frame $A$ is a simple frame (e.g., an all-inclusive price) and that frame $B$ is a different and possibly more complex frame (e.g., a multidimensional price).

Definition 1. A frame is simple if two prices presented in this common frame are perfectly comparable. A frame is complex if not all consumers are able to perfectly compare two prices presented in this common frame.

Note that we allow frame $B$ to be also a simple frame (but different from $A$ ). This is the case, for instance, if $A$ is "price per unit" and $B$ is "price per kilogram." Each firm can choose just one of the two frames, and the firms simultaneously and noncooperatively choose frames and prices $p_{1}$ and $p_{2}$. The timing reflects that in many cases both price frames and prices can be changed relatively easily. If changing price frames takes longer than adjusting prices, a two-stage game where firms first commit to frames and then compete in prices would be more suitable.

| Table 1 | Confused Consumers |  |
| :--- | :---: | :---: |
| $z_{i} \backslash z_{j}$ | $A$ | $B$ |
| $A$ | $\alpha_{0}=0$ | $\alpha_{1}$ |
| $B$ | $\alpha_{1}$ | $\alpha_{2}$ |

We discuss a sequential version of our model in the end of this section.

Price framing affects consumer choice as follows. If both firms choose the same simple frame $A$, nobody gets confused and all consumers buy the cheaper product with a positive net surplus. Formally, in this case, firm $i$ 's demand is

$$
q_{i}\left(p_{i}, p_{j}\right)= \begin{cases}1 & \text { if } p_{i}<p_{j} \text { and } p_{i} \leq 1, \\ \frac{1}{2} & \text { if } p_{i}=p_{j} \leq 1,  \tag{1}\\ 0 & \text { if } p_{i}>p_{j} \text { or } p_{i}>1, \\ & \text { for } i, j \in\{1,2\} \text { and } i \neq j .\end{cases}
$$

If the two firms adopt different frames, a fraction $\alpha_{1}>0$ of consumers get confused and are unable to compare the two prices. The remaining $1-\alpha_{1}$ fraction of consumers can still accurately compare prices. In this duopoly example, for simplicity, we assume that confused consumers shop at random: half of them buy from firm 1 and the other half buy from firm 2. In general, consumers might favor the firm adopting the simpler frame whenever they get confused between different frames. The oligopoly model in $\S 3$ allows for such preferences.

If both firms choose the same frame $B$, a fraction $\alpha_{2} \geq 0$ of consumers get confused and shop randomly. Note that if frame $B$ is also a simple (but different) frame, then $\alpha_{2}=0$. Table 1 shows the fraction of confused consumers for all possible frame profiles, where $z_{i}$ is the frame chosen by firm $i$ and $z_{j}$ is the frame chosen by firm $j$.

Notice that the simple frame $A$ can cause confusion only when it is combined with a different frame $B$, whereas if $\alpha_{2}>0$, frame $B$ is confusing itself and can obfuscate price comparisons even if both firms adopt it. ${ }^{10}$

Firm $i$ 's profit is

$$
\pi_{i}\left(p_{i}, p_{j}, z_{i}, z_{j}\right)=p_{i} \cdot\left[\frac{1}{2} \alpha_{z_{i}, z_{j}}+q_{i}\left(p_{i}, p_{j}\right)\left(1-\alpha_{z_{i}, z_{j}}\right)\right],
$$

where $\alpha_{z_{i}, z_{j}}$ is presented in Table 1 and $q_{i}\left(p_{i}, p_{j}\right)$ is given by (1).

In our model, confused consumers do not pay more than their reservation price, which is normalized to 1 .

[^6]Arguably, if price framing prevents a consumer from comparing competing offers, it may also prevent her from accurately comparing framed prices and her willingness to pay. In this case, one way to justify our assumption is that consumers can figure out at checkout (or after purchase) if a product's price exceeds their valuation and can decline to buy it (or return it). Given such ex post participation constraint, the firms have no incentive to charge prices above $1 .{ }^{11}$ In addition, confused consumers are assumed to be boundedly rational in the sense that they are unable to understand the relationship between price frames and prices. That is, even if a particular frame is always associated with higher prices, confused consumers are unable to infer prices from the price frame. One justification for this assumption is that consumers who get confused by framed prices have limited cognitive abilities and cannot be expected to understand the mixed-strategy market equilibrium. In addition, in our setting, an equilibrium where the more complex frame is associated with higher prices only occurs when $\alpha_{1}<\alpha_{2}$, i.e., when frame complexity leads to more confused consumers than does frame differentiation. But this is more likely in markets where the consumers participate infrequently (e.g., the mortgage market), and so they may not have a chance to learn to infer prices from frames.

Our model explores two sources of consumer confusion: frame differentiation (i.e., prices are presented in incompatible formats) and frame complexity (i.e., prices are presented in a common involved format). If $\alpha_{2}=0$, because frame $B$ is also a simple (but different) frame, frame differentiation is the only source of consumer confusion and it is captured by $\alpha_{1}$. If $\alpha_{2}>0$, frame complexity is also a source of consumer confusion. When consumers face the frame profile $(B, B)$, a fraction $\alpha_{2}$ of consumers get confused solely because of frame complexity. When consumers face the frame profile $(A, B)$, a fraction $\alpha_{1}$ of consumers get confused because of frame incompatibility. (In the latter case, frame incompatibility might also result because the profile involves a complex price and so both sources of confusion may be conceptually related.) The ranking of $\alpha_{1}$ and $\alpha_{2}$ reflects the relative importance of frame differentiation and frame complexity as sources of consumer confusion.

[^7]The relative role of the two confusion sources and their relevance in the marketplace stems from the microfoundations of consumer confusion. We present below two possible interpretations.

Frame differentiation is more confusing than is frame complexity $\left(\alpha_{1}>\alpha_{2}\right)$. When consumers face a simple frame $A$ (e.g., an all-inclusive price) and a complex frame $B$ (e.g., a multidimensional price), to compare the two offers they eventually need to convert the price in frame $B$ into a single all-inclusive price. Imagine that because of differences in numeracy skills, some consumers are able to make a correct conversion, whereas others are not. We assume that those who are unable to convert get confused and end up choosing randomly. When consumers face two offers in frame $B$, those who are able to convert $B$ into an all-inclusive price should still be able to compare. Meanwhile, those with poor numeracy skills may now benefit from format similarity. For example, if frame $B$ is a two-dimensional price and one offer dominates the other in both dimensions, then even those who are unable to convert will make the right choice. ${ }^{12}$ In general, similarity between the price formats may induce consumers to use a different comparison procedure and this might mitigate the confusion caused by frame complexity.

Frame complexity is more confusing than is frame differentiation $\left(\alpha_{2}>\alpha_{1}\right)$. Consumers might be able to convert a price presented in frame $B$ into a simple all-inclusive price in frame $A$, but this requires costly information processing and consumers may decide whether or not to make the conversion. When they give up making the conversion, they end up confused. If confusion stems from this conversion cost, a consumer is more likely to give up the effort when she compares two complex prices than when she compares one complex price with a simple one. Then, the frame profile $(B, B)$ leads to more confused consumers than does the profile $(A, B)$.

We use a reduced-form approach and do not explicitly model the specific comparison procedures that may lead to confusion. In reality, there may be several confusion mechanisms so that both cases of $\alpha_{1}>\alpha_{2}$ and $\alpha_{2}>\alpha_{1}$ are worth exploring. ${ }^{13}$

[^8]Finally, in our setting confused consumers' choices are assumed to be totally independent of firms' prices. This is a tractable way to capture the idea that confusion in price comparisons reduces consumers' price sensitivity and weakens price competition. A more realistic (but less tractable) model might assume that price framing leads to noisy price comparisons. Suppose firm $i$ charges a price $p_{i}$. If it uses the simple frame $A$, consumers will understand its price perfectly. In contrast, if it uses frame $B$, consumers will perceive its price as $p_{i}+\varepsilon_{i}$, where $\varepsilon_{i}$ is a random variable that captures possible misperceptions. Then, for example, if firm $i$ adopts the relatively complex frame $B$ and firm $j$ adopts frame $A$, consumers perceive their prices as $p_{i}+\varepsilon_{i}$ and $p_{j}$, respectively. As a result, demand becomes less elastic compared to the case where both firms use frame $A$. (This is reflected by $\alpha_{1}>0$ in our current setting.) But consumers' choices still depend somewhat on the relative price $p_{i}-p_{j}$. We discuss this alternative model in $\S 4$ and relate the relative ranking of $\alpha_{1}$ and $\alpha_{2}$ to the correlation between the errors. ${ }^{14}$

### 2.2. Analysis

Let us now characterize the duopoly equilibrium. ${ }^{15}$ We first show that there is no pure-strategy framing equilibrium, and then we prove the existence and uniqueness of a symmetric mixed-strategy equilibrium. All proofs missing from the text are relegated to Appendix A.

Lemma 1. If $\alpha_{1} \neq \alpha_{2}$, there is no equilibrium where both firms choose price frames deterministically.

Proof. (a) Suppose both firms choose frame $A$ for sure. Then, the unique candidate equilibrium entails marginal-cost pricing and zero profit. But if

[^9]firm $i$ unilaterally deviates to frame $B$ and a positive price (no greater than 1), it makes a positive profit. A contradiction.
(b) Suppose both firms choose frame $B$ for sure. For clarity, consider two cases. (b1) If $\alpha_{2}=1$ (and so $\alpha_{1}<\alpha_{2}$ ), at the unique candidate equilibrium $p_{i}=1$ and $\pi_{i}=1 / 2$ for all $i$. But if firm $i$ unilaterally deviates to frame $A$ and price $p_{i}=1-\varepsilon$, it earns $(1-\varepsilon)\left[\alpha_{1} / 2+\left(1-\alpha_{1}\right)\right]>1 / 2$ for a small enough $\varepsilon$. (b2) If $\alpha_{2}<1$, the unique candidate equilibrium dictates mixed strategy pricing according to a cumulative distribution function (cdf) on $\left[p_{0}, 1\right]$ as in Varian (1980), and each firm's expected profit is $\alpha_{2} / 2=$ $p_{0}\left(1-\alpha_{2} / 2\right)$. If $\alpha_{1}>\alpha_{2}$, firm $i$ can make a higher profit $\alpha_{1} / 2>\alpha_{2} / 2$ by deviating to frame $A$ and price $p_{i}=1$. If $\alpha_{1}<\alpha_{2}$, firm $i$ can make a higher profit $p_{0}\left(1-\alpha_{1} / 2\right)>p_{0}\left(1-\alpha_{2} / 2\right)$ by deviating to frame $A$ and price $p_{i}=p_{0}$. Both (b1) and (b2) lead to a contradiction.
(c) Suppose firm $i$ chooses frame $A$ and firm $j$ chooses B. Again consider two cases. (c1) If $\alpha_{1}=1$, the unique candidate equilibrium entails $p_{i}=1$ and $\pi_{i}=1 / 2$ for all $i$. But then firm $j$ is better off deviating to frame $A$ and $p_{j}=1-\varepsilon$, in which case its profit is $1-\varepsilon>1 / 2$ for any $\varepsilon<1 / 2$. (c2) If $\alpha_{1}<1$, then the unique candidate equilibrium is again of Varian type and dictates mixed strategy pricing according to a cdf on [ $p_{0}, 1$ ], with each firm earning $\alpha_{1} / 2=p_{0}\left(1-\alpha_{1} / 2\right)$. But if firm $j$ deviates to frame $A$ and price $p_{j}=p_{0}$, it makes a higher profit $p_{0}$. Both (c1) and (c2) lead to a contradiction. ${ }^{16}$

If both firms use the same simple frame (that is, $A$ or, for $\alpha_{2}=0$, also $B$ ), they compete à la Bertrand and make zero profits. A unilateral deviation to the other frame yields positive profits because some consumers are confused by "frame differentiation" and shop at random. For $\alpha_{2}>0$, Lemma 1 also shows that in equilibrium, the firms cannot rely on only one confusion source. Otherwise, a firm using frame $B$ has a unilateral incentive to deviate to the simpler frame $A$ to attract price aware consumers. But if $\alpha_{1}=\alpha_{2}>0$, there is an equilibrium with both firms using frame $B$ because a unilateral deviation to frame $A$ does not change the composition of consumers in the market.

In continuation, we focus on the general case with $\alpha_{1} \neq \alpha_{2}$. By Lemma 1, in any candidate equilibrium at least one firm mixes its frame choice. Therefore, there is a positive probability that firms have bases of fully aware consumers and also a positive probability that they have bases of confused consumers who cannot compare prices at all. The conflict between the

[^10]incentives to fully exploit confused consumers and to vigorously compete for the aware ones leads to the absence of pure-strategy pricing equilibria. The proof of the following result is standard and therefore omitted.

Lemma 2. If $\alpha_{1} \neq \alpha_{2}$, there is no equilibrium where both firms charge prices deterministically.

Lemmas 1 and 2 show that any duopoly equilibria must exhibit dispersion in both price frames and prices. Let us now focus on the symmetric mixedstrategy equilibrium $\left(\lambda, F_{A}, F_{B}\right)$ where each firm assigns probability $\lambda \in(0,1)$ to frame $A$ and $1-\lambda$ to frame $B$, and when a firm uses frame $z \in\{A, B\}$, it chooses its price randomly according to a cdf $F_{z}$ that is strictly increasing on its connected support $S_{z}=\left[p_{0}^{z}, p_{1}^{z}\right]$. We first show that $F_{z}$ is continuous (except when $\alpha_{2}=1$ ) 。

Lemma 3. In the symmetric mixed-strategy equilibrium $\left(\lambda, F_{A}, F_{B}\right)$, the price distribution associated with frame $A$ $\left(F_{A}\right)$ is always atomless, and the one associated with frame $B\left(F_{B}\right)$ is atomless whenever $\alpha_{2}<1$.

Denote by

$$
x_{z}(p) \equiv 1-F_{z}(p)
$$

the probability that a firm using frame $z$ charges a price higher than $p$. Suppose firm $j$ is employing the equilibrium strategy. Then, if firm $i$ uses frame $A$ and charges a price $p \in\left[p_{0}^{A}, p_{1}^{A}\right]$, its expected profit is

$$
\begin{equation*}
\pi(A, p)=p\left\{\lambda x_{A}(p)+(1-\lambda)\left[\alpha_{1} / 2+\left(1-\alpha_{1}\right) x_{B}(p)\right]\right\} \tag{2}
\end{equation*}
$$

With probability $\lambda$, the rival is also using $A$ so that the firms compete à la Bertrand. With probability $1-\lambda$, the rival is using $B$, so that a fraction $\alpha_{1}$ of consumers are confused (by frame differentiation) and shop randomly, and the firms compete à la Bertrand for the remaining $1-\alpha_{1}$ fully aware consumers.

If instead firm $i$ uses $B$ and charges $p \in\left[p_{0}^{B}, p_{1}^{B}\right]$, its expected profit is

$$
\begin{align*}
\pi(B, p)=p\{ & \lambda\left[\alpha_{1} / 2+\left(1-\alpha_{1}\right) x_{A}(p)\right] \\
& \left.+(1-\lambda)\left[\alpha_{2} / 2+\left(1-\alpha_{2}\right) x_{B}(p)\right]\right\} \tag{3}
\end{align*}
$$

With probability $\lambda$, the rival uses $A$ so that a fraction $\alpha_{1}$ of consumers are confused (by frame differentiation) and shop randomly. With probability $1-\lambda$, the rival also uses $B$ so that a fraction $\alpha_{2}$ of consumers are confused (by frame complexity) and shop randomly. (Note that the profit functions apply for any price $p$ because $x_{z}(p)=1$ for $p<p_{0}^{z}$ and $x_{z}(p)=0$ for $p>p_{1}^{z}$.)

The nature of the equilibrium depends on which source leads to more confusion. Intuitively, when $\alpha_{1}<\alpha_{2}$, if a firm shifts from frame $A$ to $B$, more consumers get confused regardless of its rival's frame choice. Thus, each firm charges higher prices when it
uses frame $B$ than when it uses frame $A$. For $\alpha_{1}>\alpha_{2}$, when a firm shifts from frame $A$ to $B$, more consumers get confused if its rival uses $A$, whereas fewer consumers get confused if its rival uses $B$. Hence, there is no obvious monotonic relationship between the prices associated with $A$ and $B$. Below we analyze these two cases separately.

- Frame differentiation is more confusing than is frame complexity: $0 \leq \alpha_{2}<\alpha_{1}$.

The unique symmetric equilibrium in this case dictates $F_{A}(p)=F_{B}(p)$ and $S_{A}=S_{B}=\left[p_{0}, 1\right]$ (see Appendix A for the proof). That is, a firm's price is independent of its frame. Let $F(p)$ be the common price cdf and $x(p) \equiv 1-F(p)$. Then, using the profit functions (2) and (3) and the frame indifference condition $\pi(A, p)=\pi(B, p)$, we obtain

$$
\begin{equation*}
\lambda=1-\frac{\alpha_{1}}{2 \alpha_{1}-\alpha_{2}} \tag{4}
\end{equation*}
$$

When $\alpha_{2}=0$ (i.e., frame differentiation is the sole confusion source), firms are equally likely to adopt each frame (i.e., $\lambda=1 / 2$ ). When $\alpha_{2}>0$ (i.e., frame complexity is also a confusion source), firms adopt it more often (i.e., $1-\lambda$ increases with $\alpha_{2}$ ).

Note that (4) can be rewritten as $(1-\lambda) \alpha_{1}=\lambda \alpha_{1}+$ $(1-\lambda) \alpha_{2}$ and it actually requires the expected number of confused consumers to be the same when a firm uses frame $A$ (the left-hand side) and when it uses frame $B$ (the right-hand side). As in duopoly, there are only two types of consumers (the confused and the fully aware), and then the expected market composition along the equilibrium path does not depend on a firm's frame choice. Since the pricing balances the incentives to extract surplus from the confused and to compete for the fully aware, a frame-independent market composition implies frame-independent pricing. This explains why $F_{A}=F_{B}$. (This result may not hold if the confused are biased toward the simple frame as formally shown in the oligopoly model.)

Let $\pi$ be a firm's equilibrium profit. Since all prices on $\left[p_{0}, 1\right]$ should result in the same profit, we obtain (e.g., from $\pi(A, 1)$ by using $x(1)=0$ )

$$
\begin{equation*}
\pi=\frac{\alpha_{1}^{2}}{2\left(2 \alpha_{1}-\alpha_{2}\right)} \tag{5}
\end{equation*}
$$

Note that $\pi$ increases with both $\alpha_{1}$ and $\alpha_{2}$. That is, confusion (regardless of its source) always boosts firms' payoffs and harms consumers.

Finally, the common price cdf $F(p)$ can be derived from the mixed-strategy equilibrium constant profit condition, $\pi(A, p)=\pi$. Explicitly, $x(p)=1-F(p)$ solves

$$
\begin{equation*}
\lambda x(p)+(1-\lambda)\left[\alpha_{1} / 2+\left(1-\alpha_{1}\right) x(p)\right]=\frac{\pi}{p} \tag{6}
\end{equation*}
$$

Then the boundary price $p_{0}$ is defined by $x\left(p_{0}\right)=1$ and one can check that $p_{0} \in(0,1)$. The price cdf for a higher $\alpha_{1}\left(\alpha_{2}\right)$ first-order stochastically dominates that for a lower $\alpha_{1}\left(\alpha_{2}\right)$. This is consistent with the observation that confusion benefits firms and harms consumers. We summarize these findings below.

Proposition 1. In the duopoly model with $0 \leq \alpha_{2}$ $<\alpha_{1}$, there is a unique symmetric mixed-strategy equilibrium where each firm adopts frame $A$ with probability $\lambda$ and frame $B$ with probability $1-\lambda$, and $\lambda$ is given in (4). Regardless of its frame choice, each firm chooses its price randomly according to a cdf F, which is defined by (6) on $\left[p_{0}, 1\right]$. Each firm's equilibrium profit is $\pi$ given in (5).

Notice that the equilibrium price dispersion is driven by firms' obfuscation effort through random framing but not necessarily by the coexistence of price aware and confused consumers. This is best seen in the polar case with $\alpha_{1}=1$ and $\alpha_{2}=0$, where consumers are always homogeneous both ex ante and ex post (i.e., once a pair of frames is realized, either all consumers are confused or all of them are fully aware), but price dispersion still persists.

- Frame complexity is more confusing than is frame differentiation: $0<\alpha_{1}<\alpha_{2}$.

In this case, the unique symmetric equilibrium dictates adjacent supports $S_{A}=\left[p_{0}^{A}, \hat{p}\right]$ and $S_{B}=[\hat{p}, 1]$ (see Appendix A for the proof). In particular, if $\alpha_{2}=1$, then $S_{A}=\left[p_{0}^{A}, 1\right)$ and $S_{B}=\{1\}$. That is, frame $B$ is always associated with higher prices in frame $A$. This happens because when a firm shifts from frame $A$ to frame $B$, regardless of the rival's frame, more consumers get confused given that $\alpha_{1}<\alpha_{2}$.

With adjacent price supports, in the profit function $\pi(A, p)$ (in expression (2)), $x_{B}(p)=1$ for any $p \in S_{A}$ because frame $B$ is always associated with higher prices. Similarly, in the profit function $\pi(B, p)$ (in expression (3)), $x_{A}(p)=0$ for any $p \in S_{B}$. Then from the indifference condition $\pi(A, \hat{p})=\pi(B, \hat{p})$, we can derive

$$
\begin{equation*}
\lambda=1-\frac{\alpha_{1}}{\alpha_{2}} . \tag{7}
\end{equation*}
$$

Note that the probability of using the complex frame $B(1-\lambda)$ decreases with the complexity index $\alpha_{2}$, unlike the previous case (with $\alpha_{1}>\alpha_{2}$ ). This happens because when confusion from frame complexity dominates, the prices associated with frame $B$ are already high (so a rival using frame $B$ is a softer competitor). This makes more attractive the use of frame $A$ together with a relatively high price (but still lower than $\hat{p}$ ). Hence, for fixed $\alpha_{1}$, the overall relationship between $1-\lambda$ and $\alpha_{2}$ is nonmonotonic: when $\alpha_{2}<\alpha_{1}$, the probability of using frame $B$ rises with $\alpha_{2}$, and when $\alpha_{2}>\alpha_{1}$, it decreases with $\alpha_{2}$.

Each firm's equilibrium profit $\pi$ is given by $\pi(B, 1)$ :

$$
\begin{equation*}
\pi=\alpha_{1}\left(1-\frac{\alpha_{1}}{2 \alpha_{2}}\right) \tag{8}
\end{equation*}
$$

As before, it can be verified that this equilibrium profit increases (and so consumer surplus decreases) with both $\alpha_{1}$ and $\alpha_{2}$.

Finally, $F_{z}(p)$ is determined by $\pi(z, p)=\pi$. Explicitly, we have

$$
\begin{equation*}
\lambda x_{A}(p)+(1-\lambda)\left(1-\alpha_{1} / 2\right)=\frac{\pi}{p} \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
\lambda \alpha_{1} / 2+(1-\lambda)\left[\alpha_{2} / 2+\left(1-\alpha_{2}\right) x_{B}(p)\right]=\frac{\pi}{p} . \tag{10}
\end{equation*}
$$

The boundary prices $p_{0}^{A}$ and $\hat{p}$ are defined by $x_{A}\left(p_{0}^{A}\right)=1$ and $x_{A}(\hat{p})=0$, respectively. Both of them are well defined with $p_{0}^{A}<\hat{p}$. We summarize these results below.

Proposition 2. Consider the duopoly model.
(i) If $\alpha_{1}<\alpha_{2}<1$, there is a unique symmetric mixedstrategy equilibrium where each firm adopts frame $A$ with probability $\lambda$ and frame $B$ with probability $1-\lambda$, and $\lambda$ is given in (7). When a firm uses frame $A$, it chooses its price randomly according to the $c d f F_{A}$ defined on $\left[p_{0}^{A}, \hat{p}\right]$, which solves (9); when a firm uses frame $B$, the price cdf is $F_{B}$ defined on $[\hat{p}, 1]$, which solves (10). Each firm's equilibrium profit $\pi$ is given in (8).
(ii) If $\alpha_{1}<\alpha_{2}=1$, the equilibrium has the same form except that $F_{B}$ is a degenerate distribution on $\{1\}$ and $F_{A}$ is defined on $\left[p_{0}^{A}, 1\right)$.

When $\alpha_{2} \rightarrow \alpha_{1}$, Propositions 1 and 2 indicate that the firms use frame $B$ almost surely $(\lambda \rightarrow 0)$, and the price cdf's associated with $B$ in the two cases tend to coincide. So when $\alpha_{1}=\alpha_{2}>0$, there is a unique symmetric equilibrium in which both firms use frame $B .{ }^{17}$

Our analysis focuses on cases where price framing is a short term decision. However, there are also cases where changes in price frames might take time or be costly (say, they require to redesign the contract form), whereas firms still can adjust prices frequently. In this scenario, it is more appropriate to consider a two-stage game where firms commit to frames before competing in prices. Below we discuss the equilibria in such a sequential-move variant of our duopoly model.

When $\alpha_{1}>\alpha_{2}$, there are two pure-strategy equilibria where firms choose different frames (if they can coordinate successfully). In the second stage, firms mix on prices (as in part (c) in the proof of Lemma 1) and each firm earns $\alpha_{1} / 2$. There is also an equilibrium where firms mix their frame choices in the first stage. This equilibrium is more likely when frame

[^11]coordination is hard to achieve. More specifically, it can be shown that in the mixed-strategy equilibrium each firm adopts frame $A$ with probability equal to (4) and makes profit equal to (5). That is, in the mixedstrategy equilibrium of the sequential-move game, the frequency of using each frame and the welfare are the same as in the simultaneous-move game.

When $\alpha_{1}<\alpha_{2}$, there is a unique equilibrium in which both firms adopt the complex frame $B$. This is qualitatively different from the simultaneous-move setting. The outcome in the pricing stage echoes part (b) in the proof of Lemma 1, and each firm makes $\alpha_{2} / 2$ (which is greater than (8), the profit in our simultaneous-move model). In sum, in a twostage game, a pure-strategy equilibrium is more likely to happen and firms tend to refrain from mixing on frames. But there is still consumer confusion in the market either because firms adopt different frames or because they use complex frames.

## 3. The Oligopoly Model

In this section, we develop a general oligopoly version of the model to analyze the impact of competition on market outcomes in the presence of price framing.
Consider a homogeneous product market with $n \geq 2$ identical sellers and, as before, two frames$A$ and $B: A$ is a simple frame so that all prices in this frame are comparable; $B$ is possibly complex so that with probability $\alpha_{2} \geq 0$, the consumers cannot compare prices in this frame. Consumers can also be confused by frame differentiation and so unable to compare prices in different frames with probability $\alpha_{1}>0$. In continuation, we focus on the case where confusion due to frame differentiation is independent of confusion due to frame complexity. However, depending on the microfoundations, the two types of confusion may be correlated. We argue in $\S 4$ that our analysis and its main insights carry over to the case where the two confusion sources are dependent.
In duopoly, for any realized frame profile, there is at most one confusion source, and so there are at most two types of consumers: fully aware (who buy the cheaper product) and totally confused (who shop randomly). With more than two firms, for a realized frame profile (e.g., $(A, B, B)$ ), both confusion sources might be present. Thus, there are up to four groups of consumers: $\left(1-\alpha_{1}\right)\left(1-\alpha_{2}\right)$ fully aware ones, $\alpha_{1}\left(1-\alpha_{2}\right)$ consumers confused only by frame differentiation, $\left(1-\alpha_{1}\right) \alpha_{2}$ consumers confused only by frame complexity, and $\alpha_{1} \alpha_{2}$ consumers confused by both confusion sources. ${ }^{18}$ Below is an illustrative example.

[^12]Example 1. Consider a case with three firms. Suppose firm 1 uses frame $A$, and firms 2 and 3 use frame $B$, respectively. The following graphs show the comparability among options for the four types of consumers. If two offers are comparable, they are connected; if they are not comparable, there is no link between them.
Fully

aware $\quad$\begin{tabular}{c}
Confused by frame <br>
differentiation

$\quad$

Confused by frame <br>
complexity

$\quad$

Fully <br>
confused
\end{tabular}

Moreover, with more than two firms, even if there is only one confusion source, a consumer may be only partially confused, as the following example shows.

Example 2. Consider a case with three firms. Firm 1 uses frame $A$ and charges price $p_{1}$, and firms 2 and 3 use frame $B$ and charge $p_{2}$ and $p_{3}$, respectively. If $\alpha_{1}=1$ and $\alpha_{2}=0$, then only frame differentiation causes confusion. All consumers can accurately compare $p_{2}$ with $p_{3}$ since they are presented in the same frame but cannot compare $p_{1}$ with either $p_{2}$ or $p_{3}$. Thus, consumers are neither fully aware nor totally confused.

A major question is, How does a consumer choose from a "partially ordered" set in which some pairs of alternatives are comparable but others are not? Note that this is not an issue in the duopoly model. To address this consumer choice issue, following the literature on incomplete preferences, we adopt a dominance-based consumer choice rule. The basic idea is that consumers only choose, according to some stochastic rule, from the "maximal" alternatives that are not dominated by any other comparable alternative. From now on, we use "dominated" in the following sense.

Definition 2. For a consumer, firm $i$ 's offer $\left(z_{i}, p_{i}\right) \in\{A, B\} \times[0,1]$ is dominated if there exists firm $j \neq i$ that offers alternative $\left(z_{j}, p_{j}<p_{i}\right)$ and the two offers are comparable.

Notice that for any consumer, the set of maximal or undominated alternatives is well defined and nonempty (for example, the firm that charges the lowest price in the market is never dominated), and it can be constructed, for example, by conducting pairwise comparisons among all alternatives. ${ }^{19}$ The example below illustrates our consumer choice rule.

[^13]Example 3. Consider Example 2 and let $p_{1}<$ $p_{2}<p_{3}$.


Then firm 3's offer is dominated by firm 2's offer given that $p_{2}<p_{3}$. Offers in different frames are not comparable since $\alpha_{1}=1$, so both firm 1's offer and firm 2's offer survive. Then consumers buy from firm 1 with some probability and from firm 2 with the complementary probability.

In this example, there is only one consumer group (all buyers are confused only by frame differentiation) so that all consumers face the same set of undominated offers. In general, when $\alpha_{1}, \alpha_{2} \in(0,1)$ the set of undominated alternatives varies across consumer groups.
Now we can formally state our dominance-based consumer choice rule as follows:

1. Consumers first eliminate all dominated offers in the market.
2. They then buy from the undominated firms according to the following stochastic purchase rule (which is independent of prices): (i) if all these firms use the same frame, they share the market equally; (ii) if among them $n_{A} \geq 1$ firms use frame $A$ and $n_{B} \geq 1$ firms use frame $B$, then each undominated $A$ firm is chosen with probability $\phi\left(n_{A}, n_{B}\right) / n_{A}$ and each undominated $B$ firm is chosen with probability $\left[1-\phi\left(n_{A}, n_{B}\right)\right] / n_{B}$, where $\phi(\cdot) \in(0,1)$ is nondecreasing in $n_{A}$ and nonincreasing in $n_{B}$ and $\phi\left(n_{A}, n_{B}\right) \geq n_{A} /\left(n_{A}+n_{B}\right)$.

Note that $\phi\left(n_{A}, n_{B}\right) \geq n_{A} /\left(n_{A}+n_{B}\right)$ in 2(ii) allows the consumers to favor the simple frame $A .{ }^{20}$ This generalizes the random purchase assumption in our duopoly example. $\phi(\cdot)<1$ excludes the possibility that all consumers favor the simple frame. (For example, some consumers might be overconfident in their ability to compare offers, so they do not favor any particular frame but may actually make mistakes.) The monotonicity assumption in 2(ii) means that the presence of more undominated firms with one frame increases the overall probability that consumers buy

[^14]from them. Note that the uniformly random purchase rule $\phi\left(n_{A}, n_{B}\right)=n_{A} /\left(n_{A}+n_{B}\right)$ satisfies all the conditions.

For the rest of the paper, let

$$
\phi_{k} \equiv \phi(1, k)
$$

denote the probability that a consumer buys from the $A$ firm when there are other $k$ undominated $B$ firms to choose from. Then, 2(ii) implies that $\left\{\phi_{k}\right\}_{k=1}^{n-1}$ is a nonincreasing sequence: when more $B$ firms survive, the undominated $A$ firm has less demand, and $\phi_{k} \in$ $[1 /(1+k), 1)$. Note that $\phi_{k}=1 /(1+k)$ is the uniformly random purchase rule when consumers have no bias toward the simple frame.

Recall that in duopoly the type of market equilibrium depends on whether frame differentiation or frame complexity is more confusing. The same is true in the general case. Subsections 3.1 and 3.2 analyze the corresponding symmetric equilibrium and the impact of greater competition for $\alpha_{1}<\alpha_{2}$ and $\alpha_{1}>\alpha_{2}$, respectively.

Before we proceed, let us summarize two main findings. First, when $\alpha_{2}>0$ (i.e., when frame $B$ is complex), greater competition tends to induce firms to use frame $B$ more often. In particular, when there is a large number of firms, they use frame $B$ almost surely. Intuitively, with more firms it becomes harder for them to frame differentiate (given the fixed number of frames), so firms rely more on frame complexity to soften price competition. Second, when $\alpha_{2}>0$, industry profit is bounded away from zero even when there are an infinite number of firms, and greater competition can increase industry profit and harm consumers (i.e., consumers may actually pay more in a more competitive market).

### 3.1. Frame Differentiation Is More Confusing Than Frame Complexity ( $\alpha_{1}>\alpha_{2}$ )

We analyze now the case where consumers are more likely to be confused by frame differentiation than by frame complexity (that is, $\alpha_{1}>\alpha_{2}$ ). For simplicity, we first focus on the polar case in which prices in different frames are always incomparable (i.e., $\alpha_{1}=1$ ). We then discuss how the main results can be extended to the case with $\alpha_{1}<1$. All proofs missing from the text are relegated to Appendix B.1.

As shown in the online supplementary document (https://sites.google.com/site/jidongzhou77/research/ Confusion_Supplement_Jan2013.pdf), there is no pure-strategy equilibrium when $\alpha_{2}>0$. If $\alpha_{2}=0$ (i.e., if both frames are simple) and $n \geq 4$, there are always asymmetric pure-strategy equilibria in which each frame is used by more than one firm and all firms price at marginal cost. However, for any $n \geq 2$, there is a symmetric mixed-strategy equilibrium in which firms make positive profits.
3.1.1. A Symmetric Mixed-Strategy Equilibrium. Let $\left(\lambda, F_{A}, F_{B}\right)$ be a symmetric mixed-strategy equilibrium, where $\lambda$ is the probability of using frame $A$ and $F_{z}$ is a price cdf associated with frame $z \in\{A, B\}$. Let [ $p_{0}^{z}, p_{1}^{z}$ ] be the support of $F_{z}$. As in Lemma 3, it is clear that $F_{z}$ is atomless everywhere (as now $\alpha_{2}<1$ ). For the rest of the paper,

$$
P_{n-1}^{k} \equiv C_{n-1}^{k} \lambda^{k}(1-\lambda)^{n-k-1}
$$

denotes the probability that $k$ firms among $n-1$ ones adopt frame $A$ at equilibrium, where $C_{n-1}^{k}$ stands for combinations of $n-1$ taken $k$. Recall that $x_{z}(p)=1-F_{z}(p)$.

Along the equilibrium path, if firm $i$ uses frame $A$ and charges price $p$, its profit is

$$
\begin{align*}
\pi(A, p)= & p \lambda^{n-1} x_{A}(p)^{n-1} \\
& +p \sum_{k=0}^{n-2} P_{n-1}^{k} x_{A}(p)^{k}\left[\alpha_{2} \phi_{n-k-1}+\left(1-\alpha_{2}\right) \phi_{1}\right] \tag{11}
\end{align*}
$$

If $k$ other firms also use frame $A$, firm $i$ has a positive demand only if all other $A$ firms price higher than $p$. This happens with probability $x_{A}(p)^{k}$. Conditional on that, if there are no $B$ firms in the market (if $k=n-1$ ), then firm $i$ serves the whole market. The first term in $\pi(A, p)$ follows from this. Otherwise, firm $i$ 's demand depends on whether the consumer can compare the $B$ firms' offers. If she is confused by frame complexity and unable to compare (which happens with probability $\alpha_{2}$ ), all $B$ firms are undominated (since no comparison between $A$ and $B$ is possible), so firm $i^{\prime}$ s demand is $\phi_{n-k-1}$. If she is not confused by frame complexity and, therefore, can compare prices in frame $B$ (this happens with probability $1-\alpha_{2}$ ), only one $B$ firm is undominated, so firm $i$ 's demand is $\phi_{1}$.

If instead, along the equilibrium path, firm $i$ uses $B$ and charges price $p$, its profit is

$$
\begin{align*}
& \pi(B, p)= p(1-\lambda)^{n-1}\left[\frac{\alpha_{2}}{n}+\left(1-\alpha_{2}\right) x_{B}(p)^{n-1}\right] \\
&+p \sum_{k=1}^{n-1} P_{n-1}^{k}\left[\alpha_{2} \frac{1-\phi_{n-k}}{n-k}\right. \\
&\left.+\left(1-\alpha_{2}\right)\left(1-\phi_{1}\right) x_{B}(p)^{n-k-1}\right] \tag{12}
\end{align*}
$$

The first term gives the expected profit when there are no $A$ firms in the market: the consumers who are confused by frame complexity purchase randomly among all $B$ firms, whereas those who are not confused buy from firm $i$ only if it offers the lowest price. When $k \geq 1$ firms use frame $A$ (note that only one of them will be undominated), if the consumer is confused by frame complexity (i.e., unable to compare prices in frame $B$ ), all $B$ firms are undominated and have demand $1-\phi_{n-k}$ in total. Firm $i$ shares equally this
residual demand with the other $B$ firms. If the consumer is not confused by frame complexity, to face a positive demand, firm $i$ must charge the lowest price in group $B$ (this happens with probability $x_{B}(p)^{n-k-1}$ ), in which case it gets the residual demand $1-\phi_{1}$.

Note that for $\alpha_{1}=1$ price competition can only take place among firms that use the same frame, so $x_{A}(p)$ does not appear in $\pi(B, p)$ and $x_{B}(p)$ does not appear in $\pi(A, p)$. This also implies that both profit functions are valid even if firm $i$ charges an off-equilibrium price. Thus, the upper bounds of the price cdf's are frame-independent: $p_{1}^{A}=p_{1}^{B}=1$. Otherwise any price greater than $p_{1}^{z}$ would lead to a higher profit. Then the frame-indifference condition $\pi(A, 1)=\pi(B, 1)$ pins down a unique well-defined $\lambda \in(0,1)$ (see Equation (B1) in Appendix B.1). Each firm's equilibrium profit is

$$
\begin{equation*}
\pi=\pi(A, 1)=(1-\lambda)^{n-1}\left[\alpha_{2} \phi_{n-1}+\left(1-\alpha_{2}\right) \phi_{1}\right] . \tag{13}
\end{equation*}
$$

The price distributions $F_{A}$ and $F_{B}$ are implicitly determined by $\pi(z, p)=\pi$ since any price in the support of $F_{z}$ should lead to the same profit in a mixed-strategy equilibrium. Both $F_{z}$ are uniquely defined. The boundary prices $p_{0}^{z}<1$ are determined by $\pi\left(z, p_{0}^{z}\right)=\pi$. Deviations to prices lower than $p_{0}^{z}$ are not profitable because they only result in a price loss and no demand increase. We characterize the symmetric equilibrium below.

Proposition 3. For $n \geq 2$ and $\alpha_{2}<\alpha_{1}=1$, there is a symmetric mixed-strategy equilibrium in which each firm adopts frame $A$ with probability $\lambda$ and frame $B$ with probability $1-\lambda$. When a firm uses frame $z \in\{A, B\}$, it chooses its price randomly according to a cdf $F_{z}$ defined on $\left[p_{0}^{z}, 1\right]$ and implicitly determined by $\pi(z, p)=\pi$ with $\pi(z, p)$ given in (11) and (12) and $\pi$ given in (13).

Figure 1 shows the equilibrium price distributions $F_{A}(p)$ (the solid line) and $F_{B}(p)$ (the dashed line) in the case with $n=3, \alpha_{1}=1, \alpha_{2}=0.5$, and $\phi_{k}=1 /(1+k)$.

Recall that in the duopoly equilibrium in Proposition 1 pricing is frame independent (i.e., $F_{A}(p)=$ $\left.F_{B}(p)\right)$ if confused consumers have no exogenous bias toward a specific frame. However, this is no longer

Figure 1 Price Distributions with $n=3, \alpha_{1}=1$, and $\alpha_{2}=0.5$

true in the case with more than two firms, as the above example indicates. (See the online supplementary document for a rigorous treatment of this issue.) When $F_{A} \neq F_{B}$, there is no straightforward way to analytically rank the prices associated with the two frames.
3.1.2. The Impact of Greater Competition. We now study the impact of an increase in the number of firms on the equilibrium framing strategies and on profits and consumer surplus. Our analysis is based on the equilibrium characterized in Proposition 3. We first consider a market with many sellers, which provides the key insight for our main result.

Proposition 4. When there are a large number of firms in the market,

$$
\lim _{n \rightarrow \infty} \lambda=\left\{\begin{array}{ll}
1 / 2 & \text { if } \alpha_{2}=0, \\
0 & \text { if } \alpha_{2}>0 ;
\end{array} \quad \lim _{n \rightarrow \infty} n \pi= \begin{cases}0 & \text { if } \alpha_{2}=0 \\
>0 & \text { if } \alpha_{2}>0\end{cases}\right.
$$

When frame $B$ is also a simple frame different from $A$ (i.e., when $\alpha_{2}=0$ ), the only way to reduce price competition is by frame differentiation. This is why in a sufficiently competitive market $\lambda$ tends to $1 / 2$, which maximizes frame differentiation. However, the ability of frame differentiation alone to weaken price competition is limited. In fragmented markets, each frame is adopted by more than one firm almost surely (as long as $\lambda$ is bounded away from zero and one), so price competition becomes extremely intense and the market price tends to marginal cost.

When frame $B$ is complex (i.e., when $\alpha_{2}>0$ ), the impact of greater competition on firms' framing strategies changes completely. In a sufficiently competitive market, firms use frame $B$ almost surely: they rely on frame complexity to soften price competition. (This is true even if frame $B$ is only slightly more complex than is frame $A$.) The reason is that in a large market, the role of frame differentiation in reducing price competition becomes negligible, but the effect of frame complexity is still significant. For example, if all firms employ frame $B$ for sure, industry profit is always $\alpha_{2}$, regardless of the number of firms in the market. Hence, when frame $B$ is complex, competition does not drive the market price to marginal cost. Recall that our oligopoly model allows consumers to have a bias toward the simple frame when facing both simple and complex frames. Note that our results hold even if some (but not all) confused consumers favor the simple frame. More precisely, they hold for any $\phi_{k} \in(1 /(1+k), 1)$, where $\phi_{k}$ captures the likelihood that a confused consumer buys from the frame $A$ firm when there are other $k$ undominated frame $B$ firms to choose from. ${ }^{21}$

[^15]

The analysis for large $n$ suggests that when the number of firms increases, frame B's complexity becomes a more important anticompetitive device. In effect, as we show below, $\lambda$ tends to decrease in the number of firms. That is, greater competition tends to induce firms to use the complex frame more frequently. Is it then possible that in the presence of a complex frame $B$, greater competition raises market prices by increasing market complexity? The answer, in general, depends on the parameter values. But at least for sufficiently large $\alpha_{2}$, greater competition can actually increase industry profit and harm consumers. Therefore, in the market with price framing, competition policy that focuses exclusively on an increase in the number of competitors might have undesired effects. For tractability, we focus on the uniformly random purchase rule $\phi_{k}=1 /(1+k)$.

Proposition 5. With $0<\alpha_{2}<\alpha_{1}=1$ and the random purchase rule $\phi_{k}=1 /(1+k)$,
(i) when $n$ increases from 2 to 3 , both $\lambda$ and industry profit $n \pi$ decrease;
(ii) for any $n \geq 3$, there exists $\hat{\alpha} \in(0,1)$ such that for $\alpha_{2}>\hat{\alpha}, \lambda$ decreases, but industry profit $n \pi$ increases from $n$ to $n+1$.

Beyond the limit results, numerical simulations suggest that $\lambda$ tends to decrease in $n$, and industry profit can increase in $n$ for a relatively large $\alpha_{2}$. Figure 2 shows how industry profit varies with $n$ when $\alpha_{2}=0.9$.
3.1.3. The Case with $\alpha_{2}<\alpha_{1}<1$. Price competition can now take place between firms using different frames. Then both $x_{A}(p)$ and $x_{B}(p)$ appear in the profit functions $\pi(z, p)$. The more involved related analysis is presented in the online supplementary document. We show there that if a symmetric mixed-strategy equilibrium exists, then it still satisfies $p_{1}^{A}=p_{1}^{B}=1$.

[^16]Numerical simulations suggest that greater competition can still have undesired effects when $\alpha_{1}$ is large and $\alpha_{2}$ is close to $\alpha_{1}$. For example, when $\alpha_{1}=0.98$ and $\alpha_{2}=0.9$, industry profit varies with $n$ in a way similar to Figure 2.

### 3.2. Frame Complexity Is More Confusing Than Frame Differentiation $\left(\alpha_{2}>\alpha_{1}\right)$

Consider the case where consumers are more likely to be confused by frame complexity than by frame differentiation (i.e., $\alpha_{2}>\alpha_{1}$ ). We first analyze the polar case in which prices in frame $B$ are always incomparable ( $\alpha_{2}=1$ ) and then discuss the robustness of our main results to the case with $\alpha_{2}<1$. Like before, there is no pure-strategy equilibrium (see the online supplementary document for details). Below, we report the main results and relegate the technical analysis to Appendix B.2.

Proposition 6. For $n \geq 2$ and $0<\alpha_{1}<\alpha_{2}=1$, there is a symmetric mixed-strategy equilibrium in which each firm adopts frame $A$ with probability $\lambda$ and frame $B$ with probability $1-\lambda$. When a firm uses frame $A$, it chooses its price randomly according to a cdf $F_{A}$ defined on $\left[p_{0}^{A}, 1\right)$; when it uses frame $B$, it charges price $p=1$ for sure.

Using the equilibrium in Proposition 6, we analyze the impact of greater competition on the market outcome. When there are many sellers in the market, the results for $\alpha_{2}>0$ in Proposition 4 still hold. That is, $\lim _{n \rightarrow \infty} \lambda=0$ and $\lim _{n \rightarrow \infty} n \pi>0$. The same intuition applies: in a sufficiently competitive market, the ability of frame differentiation to soften price competition is negligible, so firms resort to the complexity of frame $B$.

The following result shows that in the current case greater competition can also improve industry profit and decrease consumer surplus. In particular, this must happen when $\alpha_{1}$ is small. The reason is that for a small $\alpha_{1}$, the complexity of frame $B$ is more effective in reducing price competition, which makes the frequency of using frame $B$ increase fast enough with the number of firms. The resulting market complexity could then dominate the usual competitive effect of larger $n$. Figure 3 illustrates how industry profit varies with $n$ when $\alpha_{1}=0.05 .{ }^{22}$

Proposition 7. In the case with $0<\alpha_{1}<\alpha_{2}=1$, for any $n \geq 2$, there exists $\hat{\alpha} \in(0,1)$ such that for $\alpha_{1}<\hat{\alpha}$, $\lambda$ decreases while industry profit $n \pi$ increases from $n$ to $n+1$.
3.2.1. The Case with $\alpha_{1}<\alpha_{2}<1$. A symmetric separating equilibrium with $S_{A}=\left[p_{0}^{A}, \hat{p}\right]$ and $S_{B}=$ [ $\hat{p}, p_{1}^{B}$ ], resembling the one in Proposition 6, still

[^17]Figure 3 Industry Profit and $n$ for $\alpha_{1}=0.05$ and $\alpha_{2}=1$

exists under some parameter restrictions (when $\alpha_{1}$ is not too close to $\alpha_{2}<1$ ). Also, for fixed $\alpha_{2}<1$, if $\alpha_{1}$ is sufficiently small, greater competition can still increase industry profit and harm consumers. The more involved analysis of this case is presented in the online supplementary document.

## 4. Discussion

Comparison with the default-bias choice rule in Piccione and Spiegler (2012). The dominance-based choice rule embeds a simultaneous assessment of competing offers, and a consumer's final choice is not affected by the sequence of pairwise comparisons. This "simultaneous search" feature is suitable in markets where the consumers are not influenced by past experiences (or are newcomers). Piccione and Spiegler (2012) consider a default-bias model where consumers are initially randomly attached to one brand (the default option), and they switch to another brand only if it is comparable to and better than their default. There, with sequential comparisons, a consumer's final choice depends on her default option.

In duopoly, the default-bias model is equivalent to the simultaneous assessment one (with the random purchase rule for confused consumers). This is because if the two firms' offers are comparable, in both models the better one attracts all consumers, whereas if they are incomparable, in both models the firms share the market equally. But with more than two firms, the two models diverge. In this case, the default-bias model calls for further structure on the choice rule. To see why, consider the following example.

Example 4. There are three firms in the market. Let $\alpha_{2}=1$ and $\alpha_{1}=0$ (the only confusion source is frame complexity). Firm 1 adopts frame $A$ and prices at $p_{1}$, whereas firms 2 and 3 adopt frame $B$ and price at $p_{2}$ and $p_{3}$, respectively, with $p_{2}<p_{1}<p_{3}$.

The dominance-based rule implies that consumers purchase only from firm 2 because firm 3 is dominated by firm 1 and firm 1 is dominated by firm 2 .

Now consider the default-bias model. A consumer initially attached to firm 2 does not switch. If she is initially attached to firm 1, she switches to firm 2. However, if she is initially attached to firm 3, she switches to firm 1, but whether she further switches to firm 2 depends on what the choice rule of the defaultbiased consumer dictates. The rule should specify if the consumer assesses firm 2's offer using her default option (i.e., firm 3) or using her new choice (i.e., firm 2). By contrast, the dominance-based rule applies regardless of the number of firms in the market. ${ }^{23}$

A default bias adds another type of bounded rationality to confusion caused by framing. In this sense, our framework is a minimal deviation from the rational benchmark.

Noisy price comparisons. Our framework is a tractable way to capture the idea that confusion in price comparisons reduces consumers' price sensitivity. In particular, $\alpha_{1}$ can be regarded as a measure of price elasticity reduction when consumers face prices in two different frames, and $\alpha_{2}$ can be regarded as a measure of price elasticity reduction when consumers face two offers in frame $B$. An alternative (but less tractable) way to model the framing effect is to introduce noisy price comparisons. If firm $i$ adopts the simple frame $A$, consumers understand its price $p_{i}$ perfectly. Instead, if it adopts a complex frame $B$, consumers perceive its price as $p_{i}+\varepsilon_{i}$. This allows consumer choices to depend on the price difference between two products even when consumers get confused. In this alternative model, if both $\varepsilon_{i}$ and $\varepsilon_{j}$ have a symmetric distribution around zero, it can be shown that in the symmetric equilibrium the firms still randomize on both frames and prices (see the online supplementary document for details). However, it is not possible to fully characterize the equilibrium.

Both cases $\alpha_{1}>\alpha_{2}$ and $\alpha_{1}<\alpha_{2}$ can be justified in this setting with noisy price comparisons. To illustrate, suppose $\varepsilon_{i}$ is a random variable with the standard normal distribution $\Phi$. When both firm $i$ and firm $j$ use frame $B$, suppose $\left(\varepsilon_{i}, \varepsilon_{j}\right)$ follow a joint normal distribution with correlation coefficient $\rho \in[0,1]$. Then $\varepsilon_{i}-\varepsilon_{j}$ follows a normal distribution with zero mean and variance $2(1-\rho)$.

When both firms adopt frame $A$, demand is perfectly elastic at $p_{i}=p_{j}$. When only one firm, say, firm $i$, adopts frame $B$, its demand function is

$$
Q_{i}=\operatorname{Pr}\left(p_{i}+\varepsilon_{i}<p_{j}\right)=\Phi\left(p_{j}-p_{i}\right) .
$$

[^18]Thus, demand elasticity at $p_{i}=p_{j}$ is

$$
\begin{equation*}
2 p_{i} \phi(0), \tag{14}
\end{equation*}
$$

where $\phi(\cdot)$ is the standard normal density. When both firms adopt frame $B$, firm $i$ 's demand function is

$$
Q_{i}=\operatorname{Pr}\left(p_{i}+\varepsilon_{i}<p_{j}+\varepsilon_{j}\right)=\Phi\left(\frac{p_{j}-p_{i}}{\sqrt{2(1-\rho)}}\right)
$$

so demand elasticity at $p_{i}=p_{j}$ is

$$
\begin{equation*}
2 p_{i} \frac{\phi(0)}{\sqrt{2(1-\rho)}} \tag{15}
\end{equation*}
$$

If $\rho<1 / 2$ (e.g., if $\varepsilon_{i}$ is independent of $\varepsilon_{j}$ ), (15) is less than (14), so the demand is less elastic when both firms adopt frame $B$ than when only one firm does so. This corresponds to the case of $\alpha_{1}<\alpha_{2}$. In contrast, if $\rho>1 / 2$, the opposite is true. This corresponds to the case of $\alpha_{1}>\alpha_{2}$. In particular, when the two error terms $\varepsilon_{i}$ and $\varepsilon_{j}$ are perfectly correlated, frame $B$ can be regarded as a simple frame and we return to the case with perfectly elastic demand. The correlation between $\varepsilon_{i}$ and $\varepsilon_{j}$ might be affected by how frame similarity influences consumer misperception. If a consumer misperceives two prices in frame $B$ in a similar way (e.g., underestimates them to a similar extent), then the correlation between $\varepsilon_{i}$ and $\varepsilon_{j}$ should be high.

Costly information processing as an alternative interpretation. Our model assumes that there are boundedly rational consumers who are unable to compare framed prices or understand the market equilibrium. But it can also be interpreted as a model with rational consumers and costly information processing. Price comparisons in the presence of framing might require costly information processing, and consumers may differ in their costs. As a result, some consumers who have high information processing costs will opt out of doing so and just behave as the confused consumers do in our model. ${ }^{24}$

However, an interpretation with rational consumers might be inconsistent with the separating equilibrium in Proposition 2 (where the complex frame is always associated with higher prices than the simple one). Rational consumers should be able to infer prices from frames and always choose the simpleframe product. (In this sense, our assumption that

[^19]consumers weakly favor the simple frame partially reflects such sophistication.) In this case, the separating equilibrium would not be valid. (This is not an issue in our model with boundedly rational consumers.) Nevertheless, notice that the separating equilibrium could still make sense if there is always a nontrivial mass of naive consumers who do not try to understand market equilibrium.

Carlin (2009) considers a setting related to our case with $\alpha_{2}>\alpha_{1}$. In his model, if a consumer incurs a cost, she can learn all prices in the market, thereby purchasing the cheapest product; otherwise, she remains uninformed and shops randomly. In equilibrium, higher complexity is associated with higher prices. Consumers in Carlin's model cannot infer prices from a firm's price complexity level because they cannot observe individual firms' complexities; they can only observe the aggregate market complexity.

Dependence between the two sources of confusion. Our oligopoly model in $\S 3$ assumes that confusion due to frame differentiation and confusion due to frame complexity are independent, and it considers up to four types of consumer groups whose sizes are determined by the parameters $\alpha_{1}$ and $\alpha_{2}$. However, the two sources of confusion may be related. Take, for instance, our numeracy-skill example for $\alpha_{1}>\alpha_{2}$ in $\S 2.1$. There, confusion stems from poor numeracy skills and it is mitigated by similarity, so if a consumer is confused by two complex frames, she must also be confused by two different frames.

To allow for dependence between the two sources of confusion, we can regard the four consumer groups as the primitives of the model. A fraction $\alpha_{F D}$ of consumers are confused only by frame differentiation, a fraction $\alpha_{F C}$ of consumers are confused only by frame complexity, a fraction $\alpha_{B}$ of consumers are confused by either source, and the remaining fraction $1-\alpha_{F D}-$ $\alpha_{F C}-\alpha_{B}$ of consumers are fully aware. (Note that the two confusion sources are independent if and only if $\alpha_{F D}=\alpha_{1}\left(1-\alpha_{2}\right), \alpha_{F C}=\alpha_{2}\left(1-\alpha_{1}\right)$ and $\alpha_{B}=\alpha_{1} \alpha_{2}$.) Then, our analysis carries over with some change of notation. (The analysis in the duopoly example also remains unchanged as long as we replace $\alpha_{1}$ by $\alpha_{F D}+\alpha_{B}$ and $\alpha_{2}$ by $\alpha_{F C}+\alpha_{B}$.) In particular, the case with $\alpha_{1}=1>\alpha_{2}$ analyzed in $\S 3.1$ corresponds to $\alpha_{F D}=1-\alpha_{2}$ and $\alpha_{B}=\alpha_{2}$, and the case with $\alpha_{2}=1>$ $\alpha_{1}$ analyzed in $\S 3.2$ corresponds to $\alpha_{F C}=1-\alpha_{1}$ and $\alpha_{B}=\alpha_{1}$. In these two polar cases, (in)dependence of the two confusion sources actually does not play a role.

More frames. For simplicity, this study has focused on the case with only two frames. But it would be interesting to investigate how introducing more frames affects competition and market performance. An oligopoly model with a general frame structure is not tractable, in general. In the online supplementary
document, we explore two tractable cases and show that the impact of a greater number of frames depends on the frame structure. In particular, when there exist both simple and complex frames, greater frame variety may induce lower market prices and so benefit consumers.

In the first case, there are $m \geq 2$ simple frames $\left\{A_{1}, \ldots, A_{m}\right\}$ and $n$ firms. We assume that (i) consumers can perfectly compare prices in the same frame, but are totally confused between different frames, and (ii) consumers use the dominance-based choice rule with the uniformly random purchase rule among undominated firms. (This is a generalization of the two-frame case with $\alpha_{0}=\alpha_{2}=0$ and $\alpha_{1}=1$.) In the mixed-strategy equilibrium, numerical simulations suggest that industry profit decreases with $n$ and increases with $m$. Intuitively, when there are more firms in the market, it becomes more difficult for each firm to differentiate itself from its rivals (and, in this variant of the model, all available frames are simple and so firms cannot rely on frame complexity). However, when there are more frames available, firms can frame differentiate and avoid price competition more easily. In addition, if $m=n$ and both increase (e.g., this would be the case if each new entrant brings a new frame to the market), industry profit goes up (but is bounded from above). This suggests that the frame differentiation effect is stronger than the competition effect.

In the second case, we consider the duopoly scenario with two simple frames $\left\{A_{1}, A_{2}\right\}$ and one complex frame $B$. We assume that a fraction $\alpha_{1}$ of consumers get confused if they see $\left(A_{j}, B\right)$ for $j=1,2$; a fraction $\alpha_{2}$ of consumers get confused if they see $(B, B)$; and a fraction $\beta<\alpha_{1}, \alpha_{2}$ of consumers get confused if they see $\left(A_{1}, A_{2}\right)$. We show that compared to the case with only one simple frame and one complex frame, firms now use simple frames more often. That is, the availability of more simple frames induces firms to rely more on frame differentiation. If frame differentiation is less confusing than frame complexity (i.e., if $\alpha_{1}<\alpha_{2}$ ), market prices will become lower: An increase in the number of (simple) frames lowers industry profits and improves consumer surplus. If frame differentiation is more confusing than frame complexity (i.e., if $\alpha_{1}>\alpha_{2}$ ), the opposite is true.

## 5. Conclusion

This paper presents a model of competition in both prices and price frames. In a homogeneous product market, price framing can obstruct consumers' ability to compare prices and create confusion. Our study shows that in the symmetric equilibrium, firms randomize on both price frames and prices and make positive profits. An increase in the number of firms
reduces firms' ability to frame differentiate and makes them use complex frames more often. As a result, greater competition might increase profits and harm consumers. In our setting, consumer confusion may stem from price format incompatibility or price complexity. The nature of the equilibrium depends on which source of consumer confusion leads to more confused consumers.

This study is motivated by price framing, but it also applies to situations where product framing reduces the comparability of offers. For instance, the way of presenting nutritional information might frame identical food products differently. An "improved recipe" or a "British meal" label might spuriously differentiate a ready meal from its close substitutes. ${ }^{25}$ Package size differences or quantity premia could also make it harder to compare products. On the same supermarket shelf toothpastes come in tubes of 50,75 , or 100 ml , and refreshments, cleaning products, tea boxes occasionally come in larger-"extra $25 \%$ free"containers. This interpretation also relates our paper to the literature on endogenous product differentiation (see, for instance, in Tirole 1988, Chap. 7). A main difference is that in our model firms make product differentiation and price choices simultaneously.

Our model predicts that firms randomize their frame choices in order to obstruct consumer price comparisons and sustain higher profits. But this prediction does not take into account several factors that may weaken firms' incentive to obfuscate consumers. First, in some markets changing frames frequently could be costly and, as a result, firms may refrain from mixing on price frames. Second, price framing and the resulting market complexity may induce some consumers to drop out of the market. ${ }^{26}$ Third, ex post some consumers may be able to discover that they were misled by a firm's price framing strategy, so they may avoid this firm in the future. This reputation concern would also reduce firms' incentive to frame prices.

Finally, in financial, energy, or mortgage markets, among others, information intermediaries could help consumers compare offers and identify the best deals. Their presence should mitigate the confusion caused

[^20]by price framing and thus reduce firms' incentives to frame their prices. However, in spite of this, consumer confusion seems to persist in these environments. There might be several reasons why information intermediaries do not completely solve the issue of price framing and consumer confusion. On the demand side, take-up of such services is not universal. Consumers differ in their opportunity costs of time or search methods and they may rationally opt out of using such services. They may also overestimate their ability to make accurate comparisons without specialized help. In some markets where the best tariff choice depends on individual characteristics, consumers might be reluctant to share the relevant information with intermediaries because of privacy concerns. On the supply side, information intermediaries have incentives to strategically limit sellers' participation to such platforms (e.g., by charging high participation fees) in order to protect their informational value and ability to extract rents.

## Acknowledgments

The authors thank J. Miguel Villas-Boas (the department editor), an associate editor, and three anonymous referees for insightful comments and suggestions that have greatly improved the paper. They are also grateful to Ugur Akgun, Mark Armstrong, V. Bhaskar, Drew Fudenberg, Antonio Guarino, Steffen Huck, Meg Meyer, José Luis MoragaGonzález, Pierre Regibeau, Ran Spiegler, Chris Wilson, Xiaojian Zhao, and to participants at various seminars and conferences for valuable comments. Financial support from the British Academy, the Economic and Social Research Council (UK), and the European Commission [IEF Project 221661] is gratefully acknowledged.

## Appendix A. Proofs in the Duopoly Case

Proof of Lemma 3. Suppose that $F_{A}$ has a mass point at some price $p \in S_{A}$ in the symmetric equilibrium. The equilibrium condition requires that $(A, p)$ generates the same profit as do other frame and price combinations. Given the mass point, in the symmetric equilibrium there is a positive probability that both firms use frame $A$ and tie at $p$. In that event, because all consumers are price aware, reducing price slightly can increase demand discontinuously. This implies that a unilateral deviation to $(A, p-\varepsilon)$ (for a sufficiently small $\varepsilon>0$ ) generates a higher profit than does ( $A, p$ ), which leads to a contradiction.

A similar argument works for frame $B$. Suppose that $F_{B}$ has a mass point at some price $p \in S_{B}$ in the symmetric equilibrium. The equilibrium condition requires that ( $B, p$ ) generates the same profit as other frame and price combinations. Given the mass point, in the symmetric equilibrium there is a positive probability that both firms use frame $B$ and tie at $p$. In that event, because some consumers are price aware when $\alpha_{2}<1$, reducing the price slightly can increase demand discontinuously. This implies that a unilateral deviation to ( $B, p-\varepsilon$ ) (for a sufficiently small $\varepsilon>0$ ) generates a higher profit than $(B, p)$, which also leads to a contradiction. Q.E.D.

Proof of Proposition 1. The proposed configuration is indeed an equilibrium since no deviation to $p<p_{0}$ is profitable. We show now that it is the unique symmetric mixed-strategy equilibrium with $F_{z}$ strictly increasing on its support. Recall that, by Lemma 3, when $\alpha_{2}<1$, in any symmetric mixed-strategy equilibrium $F_{z}$ is continuous on $S_{z}$. The proof entails several steps.

Step 1: $S_{A} \cap S_{B} \neq \varnothing$. Suppose $p_{1}^{A}<p_{0}^{B}$. Then if a firm uses frame $A$ and charges $p_{1}^{A}$, its profit is

$$
\pi\left(A, p_{1}^{A}\right)=p_{1}^{A}(1-\lambda)\left[\left(1-\alpha_{1}\right)+\alpha_{1} / 2\right] .
$$

The firm has positive demand only if the rival uses frame $B$, in which case it sells to all price aware consumers and to half of the confused ones. Clearly, this firm can do better by charging a price slightly higher than $p_{1}^{A}$. A contradiction. Similarly, we can rule out the possibility of $p_{1}^{B}<p_{0}^{A}$.

Step 2: $\max \left\{p_{1}^{A}, p_{1}^{B}\right\}=1$. Suppose $p_{1}^{z}=\max \left\{p_{1}^{A}, p_{1}^{B}\right\}<1$. Then, $p_{1}^{z}$ is dominated by $p_{1}^{z}+\varepsilon$ (for some small $\varepsilon>0$ ).

Step 3: $S_{A}=S_{B}=\left[p_{0}, 1\right]$. Suppose $p_{1}^{A}<p_{1}^{B}=1$. Then, along the equilibrium path, if firm $i$ uses frame $A$ and charges $p \in\left[p_{1}^{A}, 1\right]$, its profit is

$$
\pi(A, p)=p(1-\lambda)\left[\left(1-\alpha_{1}\right) x_{B}(p)+\alpha_{1} / 2\right],
$$

since it faces a positive demand only if firm $j$ uses frame $B$. If firm $i$ uses frame $B$ and charges the same price $p$, its profit is

$$
\pi(B, p)=p\left\{\lambda \alpha_{1} / 2+(1-\lambda)\left[\left(1-\alpha_{2}\right) x_{B}(p)+\alpha_{2} / 2\right]\right\}
$$

that should be equal to the candidate equilibrium profit. Because the supposition that $p_{1}^{A}<p_{1}^{B}=1$ and Step 1 imply that $p_{1}^{A} \in S_{B}$, the indifference condition requires $\pi\left(A, p_{1}^{A}\right)=$ $\pi\left(B, p_{1}^{A}\right)$ or

$$
(1-\lambda)\left(\alpha_{1}-\alpha_{2}\right)-\lambda \alpha_{1}=2(1-\lambda)\left(\alpha_{1}-\alpha_{2}\right) x_{B}\left(p_{1}^{A}\right)
$$

But if this equation holds, $\pi(A, p)>\pi(B, p)$ for $p \in\left(p_{1}^{A}, 1\right]$ as $\alpha_{1}>\alpha_{2}$ and $x_{B}$ is strictly decreasing on $S_{B}$. A contradiction. Similarly, we can exclude the possibility of $p_{1}^{B}<p_{1}^{A}=1$. Hence, it must be that $p_{1}^{A}=p_{1}^{B}=1$.

Then, from $\pi(A, 1)=\pi(B, 1)$, it follows that

$$
\begin{equation*}
\lambda \alpha_{1}=(1-\lambda)\left(\alpha_{1}-\alpha_{2}\right) \tag{A1}
\end{equation*}
$$

Now suppose $p_{0}^{A}<p_{0}^{B}$. Then

$$
\begin{gathered}
\pi\left(A, p_{0}^{B}\right)=p_{0}^{B}\left[\lambda x_{A}\left(p_{0}^{B}\right)+(1-\lambda)\left(1-\alpha_{1} / 2\right)\right] \quad \text { and } \\
\pi\left(B, p_{0}^{B}\right)=p_{0}^{B}\left\{\lambda\left[\left(1-\alpha_{1}\right) x_{A}\left(p_{0}^{B}\right)+\alpha_{1} / 2\right]+(1-\lambda)\left(1-\alpha_{2} / 2\right)\right\} .
\end{gathered}
$$

Since the supposition $p_{0}^{A}<p_{0}^{B}$ and Step 1 imply that $p_{0}^{B} \in S_{A}$, we need $\pi\left(A, p_{0}^{B}\right)=\pi\left(B, p_{0}^{B}\right)$, or

$$
2 x_{A}\left(p_{0}^{B}\right)=1+\frac{1-\lambda}{\lambda} \frac{\alpha_{1}-\alpha_{2}}{\alpha_{1}} .
$$

The left-hand side is strictly lower than 2 given that $x_{A}$ is strictly decreasing on $S_{A}$ and $p_{0}^{A}<p_{0}^{B}$. But (A1) implies that the right-hand side is equal to 2 . A contradiction. Similarly, we can exclude the possibility of $p_{0}^{A}<p_{0}^{B}$. Hence, it must be that $p_{0}^{A}=p_{0}^{B}$.

Step 4: $F_{A}=F_{B}$. For any $p \in\left[p_{0}, 1\right]$, the indifference condition requires $\pi(A, p)=\pi(B, p)$. Using (2) and (3), we get

$$
\lambda \alpha_{1}\left[x_{A}(p)-1 / 2\right]=(1-\lambda)\left(\alpha_{1}-\alpha_{2}\right)\left[x_{B}(p)-1 / 2\right]
$$

for all $p \in\left[p_{0}, 1\right]$. Then (A1) implies $x_{A}=x_{B}\left(\right.$ or $\left.F_{A}=F_{B}\right)$. Q.E.D.

Proof of Proposition 2. (1) Let us first prove the result for $\alpha_{2}<1$.
(1.1) A deviation to $\left(A, p<p_{0}^{A}\right)$ is obviously not profitable. A deviation to $(A, p>\hat{p})$ generates a profit equal to

$$
p(1-\lambda)\left[\left(1-\alpha_{1}\right) x_{B}(p)+\alpha_{1} / 2\right] .
$$

Using (7), one can check that the deviation profit is lower than $\pi(B, p)$ in (3) with $x_{A}(p)=0$. The last possible deviation is $(B, p<\hat{p})$ that results in a profit equal to

$$
p\left\{\lambda\left[\left(1-\alpha_{1}\right) x_{A}(p)+\alpha_{1} / 2\right]+(1-\lambda)\left(1-\alpha_{2} / 2\right)\right\} .
$$

Again, using (7), one can check that the deviation profit is lower than $\pi(A, p)$ in (2) with $x_{B}(p)=1$.
(1.2) We now prove uniqueness. As in the proof of Proposition 1, we can show that $S_{A} \cap S_{B} \neq \varnothing$ and $\max \left\{p_{1}^{A}, p_{1}^{B}\right\}=1$. Then the two steps below complete the proof.

Step 1: $S_{A} \cap S_{B}=\{\hat{p}\}$ for some $\hat{p}$. Suppose to the contrary that $S_{A} \cap S_{B}=\left[p^{\prime}, p^{\prime \prime}\right]$ with $p^{\prime}<p^{\prime \prime}$. Then for any $p \in\left[p^{\prime}, p^{\prime \prime}\right]$, it must be that $\pi(A, p)=\pi(B, p)$, where the profit functions are given by (2) and (3). This indifference condition requires that

$$
\lambda \alpha_{1}\left[x_{A}(p)-1 / 2\right]=(1-\lambda)\left(\alpha_{1}-\alpha_{2}\right)\left[x_{B}(p)-1 / 2\right]
$$

for all $p \in\left[p^{\prime}, p^{\prime \prime}\right]$. Since $\alpha_{1}<\alpha_{2}$ and $F_{z}$ is strictly increasing on $S_{z}$, the left-hand side is a decreasing function of $p$, whereas the right-hand side is an increasing function of $p$. So the condition cannot hold for all $p \in\left[p^{\prime}, p^{\prime \prime}\right]$. A contradiction.

Step 2: $p_{1}^{B}=1$. Suppose $p_{1}^{B}<1$. Then Step 1 and $\max \left\{p_{1}^{A}, p_{1}^{B}\right\}=1$ imply that $p_{1}^{A}=1$ and $p_{1}^{B}=p_{0}^{A}=\hat{p}<1$. Then each firm's equilibrium profit should be equal to $\pi(A, 1)=$ $(1-\lambda) \alpha_{1} / 2$ since the prices associated with $B$ are lower than one. But if a firm chooses frame $B$ and $p=1$, its profit is $\left[\lambda \alpha_{1}+(1-\lambda) \alpha_{2}\right] / 2$ because it sells to half of the confused consumers. This deviation profit is greater than $\pi(A, 1)$ given that $\alpha_{2}>\alpha_{1}$. A contradiction.

Therefore, in equilibrium, it must be that $S_{A}=\left[p_{0}^{A}, \hat{p}\right]$ and $S_{B}=[\hat{p}, 1]$.
(2) The equilibrium when $\alpha_{2}=1$ is just the limit of the equilibrium in (1) as $\alpha_{2} \rightarrow 1$. But now $S_{A}=\left[p_{0}^{A}, 1\right.$ ) and $S_{B}=\{1\} . \quad$ Q.E.D.

## Appendix B. Proofs in the Oligopoly Model

## B.1. The Case with $\alpha_{2}<\alpha_{1}=1$

Equilibrium Condition for $\lambda$ When $0<\alpha_{2}<\alpha_{1}=1$. Since the price distributions for frames $A$ and $B$ share the same upper bound $p=1$, letting $p=1$ in (11) and (12) yields a frame indifference condition $\pi(A, 1)=\pi(B, 1)$.

By dividing each side by $(1-\lambda)^{n-1}$ and rearranging the equation, we obtain

$$
\begin{align*}
& \alpha_{2}\left(\phi_{n-1}-\frac{1}{n}\right)+\left(1-\alpha_{2}\right) \phi_{1} \\
& =\alpha_{2} \sum_{k=1}^{n-2} \frac{C_{n-1}^{k}\left(1-\phi_{n-k}\right)}{n-k}\left(\frac{\lambda}{1-\lambda}\right)^{k}+\left(1-\phi_{1}\right)\left(\frac{\lambda}{1-\lambda}\right)^{n-1} . \tag{B1}
\end{align*}
$$

The right-hand side of (B1) increases in $\lambda \in[0,1]$ from zero to infinity, and the left-hand side is positive for any $\alpha_{2} \in$ $(0,1)$ as $\phi_{n-1} \geq 1 / n$. Hence, (B1) has a unique solution $\lambda$ in $(0,1)$ as we claimed in the main text.

Proof of Proposition 4. When frame $B$ is also a simple frame (i.e., when $\alpha_{2}=0$ ), the equilibrium condition (B1) for $\lambda$ becomes

$$
\frac{\lambda}{1-\lambda}=\left(\frac{\phi_{1}}{1-\phi_{1}}\right)^{1 /(n-1)} .
$$

It follows that $\lambda$ tends to $1 / 2$ as $n \rightarrow \infty$. Then industry profit $n \pi=n \phi_{1}(1-\lambda)^{n-1}$ must converge to zero. ${ }^{27}$

Now consider $\alpha_{2}>0$. Since the left-hand side of (B1) is bounded, it must be that $\lim _{n \rightarrow \infty} \lambda \leq 1 / 2$ (otherwise, the right-hand side would tend to infinity). Since $\left\{\phi_{k}\right\}_{k=1}^{n-1}$ is a nonincreasing sequence, the right-hand side of (B1) is greater than

$$
\frac{\alpha_{2}\left(1-\phi_{1}\right)}{n} \sum_{k=1}^{n-2} C_{n-1}^{k}\left(\frac{\lambda}{1-\lambda}\right)^{k}=\frac{\alpha_{2}\left(1-\phi_{1}\right)}{n}\left[\frac{1-\lambda^{n-1}}{(1-\lambda)^{n-1}}-1\right]
$$

So it must be that $\lim _{n \rightarrow \infty} n(1-\lambda)^{n-1}>0$; otherwise, the right-hand side of ( B 1 ) tends to infinity (given that $\lim _{n \rightarrow \infty} \lambda \leq 1 / 2$ and so $\lim _{n \rightarrow \infty}\left(1-\lambda^{n-1}\right)=1$ ). This result implies that $\lambda$ must converge to zero and industry profit $n \pi=n(1-\lambda)^{n-1}\left[\alpha_{2} \phi_{n-1}+\left(1-\alpha_{2}\right) \phi_{1}\right]$ must be bounded away from zero as $n \rightarrow \infty$. ${ }^{28}$ Q.E.D.

Proof of Proposition 5. Note that with the random purchase rule $\phi_{k}=1 /(1+k)$, (B1) becomes

$$
\begin{equation*}
1-\alpha_{2}=2 \alpha_{2} \sum_{k=1}^{n-2} \frac{C_{n-1}^{k}}{n-k+1}\left(\frac{\lambda}{1-\lambda}\right)^{k}+\left(\frac{\lambda}{1-\lambda}\right)^{n-1}, \tag{B2}
\end{equation*}
$$

and industry profit is

$$
\begin{equation*}
n \pi=n(1-\lambda)^{n-1}\left(\frac{\alpha_{2}}{n}+\frac{1-\alpha_{2}}{2}\right) \tag{B3}
\end{equation*}
$$

(i) For $n=2$, we have $\lambda=\left(1-\alpha_{2}\right) /\left(1+\left(1-\alpha_{2}\right)\right)$; for $n=3$, we have $\lambda=x /(1+x)$ with $x=\sqrt{4 \alpha_{2}^{2} / 9+1-\alpha_{2}}-$ $2 \alpha_{2} / 3$. The latter is smaller if $x<1-\alpha_{2}$, which can be easily verified given that $\alpha_{2}<1$. The industry profit result follows from straightforward algebra calculation by using (B3).

[^21](ii) Consider the limit case $\alpha_{2} \rightarrow 1$. The equilibrium condition (B2) implies that $\lambda$ should then tend to zero. Since $\lambda \approx 0$, then $\lambda /(1-\lambda) \approx \lambda+\lambda^{2}$. For $n \geq 4$, the right-hand side of (B2) can be approximated as
\[

$$
\begin{equation*}
2 \alpha_{2}\left[\frac{n-1}{n}\left(\lambda+\lambda^{2}\right)+\frac{n-2}{2}\left(\lambda+\lambda^{2}\right)^{2}\right] \tag{B4}
\end{equation*}
$$

\]

by discarding all higher-order terms. (For $n=3$, the righthand side of (B2) is approximately $(4 / 3) \alpha_{2}\left(\lambda+\lambda^{2}\right)+$ $\left(\lambda+\lambda^{2}\right)^{2}$. One can check that the approximation result below still applies.)

Let $\alpha_{2}=1-\varepsilon$ with $\varepsilon \approx 0$, and use the second-order (linear) approximation $\lambda \approx k_{1} \varepsilon+k_{2} \varepsilon^{2}$. Substituting them into (B4) and discarding all terms of order higher than $\varepsilon^{2}$, we obtain

$$
\frac{2(n-1)}{n} k_{1} \varepsilon+\left(\frac{2(n-1)}{n}\left(k_{2}-k_{1}\right)+\frac{n^{2}-2}{n} k_{1}^{2}\right) \varepsilon^{2} .
$$

Because the left-hand side of (B2) is $\varepsilon$, we can solve

$$
k_{1}=\frac{n}{2(n-1)}, \quad k_{2}=k_{1}-\frac{n^{2}-2}{2(n-1)} k_{1}^{2} .
$$

Since $k_{1}$ decreases with $n, \lambda$ must decrease with $n$.
Since $\varepsilon \approx 0$ (so that $\lambda \approx 0$ ), industry profit (for $n \geq 3$ ) can be approximated as

$$
\begin{aligned}
n \pi & =(1-\lambda)^{n-1}\left[1+\left(\frac{n}{2}-1\right) \varepsilon\right] \\
& \approx\left[1-(n-1) \lambda+C_{n-1}^{2} \lambda^{2}\right]\left[1+\left(\frac{n}{2}-1\right) \varepsilon\right] \\
& \approx 1-\varepsilon+\frac{(n-2) n^{2}}{8(n-1)^{2}} \varepsilon^{2} .
\end{aligned}
$$

The second step follows from discarding all terms of order higher than $\lambda^{2}$, and the third step comes from substituting $\lambda \approx k_{1} \varepsilon+k_{2} \varepsilon^{2}$ and discarding all terms of order higher than $\varepsilon^{2}$. It is easy to see that the approximated industry profit increases with $n$. (Note that the first-order approximation of $\lambda$ is not sufficient to tell how $n \pi$ varies with $n$.) Q.E.D.

## B.2. The Case with $\alpha_{1}<\alpha_{2}=1$

We first characterize a symmetric mixed-strategy equilibrium $\left(\lambda, F_{A}, F_{B}\right)$, where $\lambda$ is the probability of using frame $A$, $F_{A}$ is defined on $S_{A}=\left[p_{0}^{A}, 1\right)$ and is atomless, and $F_{B}$ is degenerate on $S_{B}=\{1\}$.

Along the equilibrium path, if firm $i$ uses frame $A$ and charges $p \in\left(p_{0}^{A}, 1\right)$, its profit is given by

$$
\pi(A, p)=p \sum_{k=0}^{n-1} P_{n-1}^{k} x_{A}(p)^{k}\left(\alpha_{1} \phi_{n-k-1}+1-\alpha_{1}\right) .
$$

This expression follows because when $k$ other firms also use frame $A$, firm $i$ has a positive demand only if all other $A$ firms charge prices higher than $p$. Conditional on that, with probability $\alpha_{1}$, the consumer is confused by frame differentiation and buys from firm $i$ with probability $\phi_{n-k-1}$ (since all $n-k-1$ firms which use $B$ are undominated); with probability $1-\alpha_{1}$, the consumer can compare $A$ and $B$ and,
because all $B$ firms charge price $p_{B}=1>p$ and consequently are dominated, she only buys from firm $i$.

A firm's equilibrium profit is equal to

$$
\pi=\lim _{p \rightarrow 1} \pi(A, p)=(1-\lambda)^{n-1}\left(\alpha_{1} \phi_{n-1}+1-\alpha_{1}\right) .
$$

Then the expression for $F_{A}(p)$ follows from $\pi(A, p)=\pi$, and $p_{0}^{A}$ satisfies $\pi\left(A, p_{0}^{A}\right)=\pi$. Both of them are well defined.

If firm $i$ uses $B$ and charges $p=1$, then its profit is

$$
\pi(B, 1)=\frac{(1-\lambda)^{n-1}}{n}+\alpha_{1} \sum_{k=1}^{n-1} P_{n-1}^{k} \frac{1-\phi_{n-k}}{n-k} .
$$

Notice that firm $i$ has a positive demand only if all other firms also use frame $B$ or there are $A$ firms but the consumer is unable to compare prices in different frames.

The equilibrium condition $\pi(B, 1)=\lim _{p \rightarrow 1} \pi(A, p)$ pins down a well-defined $\lambda$ :

$$
\begin{equation*}
\frac{1-1 / n}{\alpha_{1}}+\phi_{n-1}-1=\sum_{k=1}^{n-1} \frac{C_{n-1}^{k}\left(1-\phi_{n-k}\right)}{n-k}\left(\frac{\lambda}{1-\lambda}\right)^{k} . \tag{B5}
\end{equation*}
$$

The left-hand side of (B5) is positive given that $\phi_{n-1} \geq 1 / n$, and the right-hand side is increasing in $\lambda$ from zero to infinity. Hence, for any given $n \geq 2$ and $\alpha_{1} \in(0,1)$, Equation (B5) has a unique solution $\lambda$ in $(0,1)$.

To complete the proof of Proposition 6, we only need to rule out profitable deviations from the proposed equilibrium. First, consider two possible deviations with frame $A$ : (i) a deviation to $\left(A, p<p_{0}^{A}\right)$ is not profitable because the firm does not gain market share, but loses on prices; (ii) a deviation $(A, p=1)$ is not profitable either since the deviator's profit is $(1-\lambda)^{n-1} \phi_{n-1}<\pi$.

Let us now consider a deviation to ( $B, p \in\left(p_{0}^{A}, 1\right)$ ). The deviator's profit is

$$
\hat{\pi}(B, p)=p \pi(B, 1)+p\left(1-\alpha_{1}\right) \sum_{k=1}^{n-1} P_{n-1}^{k} x_{A}(p)^{k} .
$$

This expression captures the fact that when $n-1$ other firms also use $B$, or when $k \geq 1$ firms use $A$ and the consumer is confused between $A$ and $B$, firm $i$ 's demand does not depend on its price so that it is equal to $\pi(B, 1)$. When $k \geq 1$ firms use $A$ and the consumer is not confused between $A$ and $B$, all other $B$ firms (which charge price $p=1$ ) are dominated by the cheapest $A$ firm, and the consumer buys from firm $i$ only if the cheapest $A$ firm charges a price greater than $p$. Notice that from $\pi(A, p)=\pi$ for $p \in\left(p_{0}^{A}, 1\right)$, the second term in $\hat{\pi}(B, p)$ is equal to

$$
\pi-p \pi-p \alpha_{1} \sum_{k=1}^{n-1} P_{n-1}^{k} x_{A}(p)^{k} \phi_{n-k-1}
$$

Then, $\hat{\pi}(B, p)<p \pi+\pi-p \pi=\pi$. The deviation to ( $B, p<p_{0}^{A}$ ) will result in a lower profit. This completes the proof.

Proof of Proposition 7. From (B5), it follows that $\lambda \rightarrow 1$ as $\alpha_{1} \rightarrow 0$. Let $\alpha_{1}=\varepsilon$ with $\varepsilon \approx 0$, and $\lambda=1-\delta$ with $\delta \approx 0$. Then the right-hand side of (B5) can be approximated as

$$
\left(1-\phi_{1}\right)\left(\frac{1-\delta}{\delta}\right)^{n-1} \approx \frac{1-\phi_{1}}{\delta^{n-1}}
$$

since only the term with $k=n-1$ matters when $\delta \approx 0$. Hence, from (B5), we can solve

$$
\delta \approx\left(\frac{1-\phi_{1}}{(1 / \varepsilon)(1-1 / n)+\phi_{n-1}-1}\right)^{1 /(n-1)} \approx\left(\frac{n\left(1-\phi_{1}\right) \varepsilon}{n-1}\right)^{1 /(n-1)} .
$$

The second step follows because $\phi_{n-1}-1$ is negligible compared to $(1 / \varepsilon)(1-1 / n)$. Given that $\varepsilon \approx 0$, it is not difficult to see that $\delta$ increases with $n$ (e.g., one can show that $\ln \delta$ increases with $n$ ). Hence, $\lambda$ decreases with $n$. Since $\varepsilon \approx 0$, industry profit is

$$
n \pi=n \delta^{n-1}\left[1+\left(\phi_{n-1}-1\right) \varepsilon\right] \approx \frac{n^{2}\left(1-\phi_{1}\right) \varepsilon}{n-1}
$$

by discarding the term of $\varepsilon^{2}$. Clearly, $n \pi$ increases with $n$. Q.E.D.

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[^0]:    ${ }^{1}$ For example, to buy a 50 ml whitening toothpaste in a grocery store, one can choose a Macleans sold at $£ 2.31$ with a "buy one get one free" offer or an Aquafresh that "was $£ 1.93$ now is $£ 1.28$ saves 65 p."

[^1]:    ${ }^{2}$ Examples of intertemporal variation of price presentation formats are easy to find. For instance, cosmetics retailer the Body Shop offered a " $\$ 10$ off" for any $\$ 20$ purchase on November 27, 2012; it changed to a deal of "up to $50 \%$ off" on October 2, 2012; and it offered a deal of "buy 2 get 2 free" on October 7, 2012.
    ${ }^{3}$ UK's gas and energy market regulator Ofgem has started to address complex tariffs (see Ofgem 2011, BBC News 2012). See also related articles on confusing supermarket and energy market pricing (The Guardian 2011, Which? 2009).

[^2]:    ${ }^{4}$ Psychology research has long recognized framing effects in decision making (see Tversky and Kahneman 1981). Individuals' responses to essentially the same decision problem may differ systematically if the problem is framed differently. Here, we focus on frames as price presentation modes and on their ability to cause confusion in price comparisons.
    ${ }^{5}$ With product differentiation, consumers who are confused by framed prices may also use other heuristics (e.g., rely on brand names) to make a choice.

[^3]:    ${ }^{6}$ See, for instance, the European Commission (2007) documents on the integration of EU mortgage credit markets.

[^4]:    ${ }^{7}$ In this paper, consumers who lack the ability to compare prices are assumed to lack the ability to understand the equilibrium relationship between frames and prices. We discuss this assumption more formally in $\$ 2$.
    ${ }^{8}$ Lower concentration can also lead to higher prices in other price dispersion models (e.g., Rosenthal 1980, Janssen and MoragaGonzález 2004) where competition induces firms to exploit uninformed consumers rather than to fight for shoppers. However, our result stems from a novel effect of lower concentration on firms' framing incentives and the overall market complexity.

[^5]:    ${ }^{9}$ In an all-or-nothing search model, Carlin (2009) assumes that if a firm increases its price complexity, consumers regard the entire market as being more complex. Then, as information gathering becomes more costly, they are more likely to remain uninformed and shop randomly. In a sequential search model, Ellison and Wolitzky (2012) introduce a convex search cost function. If a firm makes its price information more costly to process, consumers are less likely to search further. See also Wilson (2010) for a twostage model of obfuscation with asymmetric equilibrium complexity levels.

[^6]:    ${ }^{10}$ Note that our main results hold qualitatively (though the analysis would be more involved) even if a fraction of consumers also get confused when both firms use frame $A$ (i.e., when frame $A$ also involves some complexity), provided that $\alpha_{0} \leq \alpha_{2}$ and $\alpha_{0} \neq \alpha_{1}$.

[^7]:    ${ }^{11}$ Carlin (2009) and Piccione and Spiegler (2012) make the same assumption that confused consumers do not pay more than their willingness to pay. Alternatively, suppose that a confused consumer cannot perfectly compare prices to her willingness to pay, and as a result she may pay up to some $v>1$. This general setting is less tractable. However, in the special case with $\alpha_{2}=0$, we can show that our main results hold qualitatively if $v<2$. That is, there is no pure-strategy equilibrium and in the symmetric equilibrium firms mix on both frames and prices (and the price distribution has a mass point on $v$ ).

[^8]:    ${ }^{12}$ Even if there is no clear dominance relationship between offers, frame similarity may still facilitate the comparison of prices framed in $B$. Take, for example, two offers in frame B: (1) $£ 32.78$ plus $£ 4.75$ shipping, and (2) $£ 32.97$ plus $£ 4.32$ shipping. When a consumer compares them, she may assess different components separately. The base price in (2) is about 20 p higher than in (1), but the shipping fee in (2) is about 40 p cheaper than in (1), so (2) is a better deal than (1). However, if the consumer needs to compare, say, (1) with a single price $£ 37.25$, it seems plausible that she has to convert (1) into an all-inclusive price first, which is eventually harder and so it may block the comparison.
    ${ }^{13}$ Another justification for the cases $\alpha_{1}>\alpha_{2}$ and $\alpha_{2}>\alpha_{1}$ relates to costly information processing. Suppose that frame differentiation

[^9]:    imposes an information processing cost $d$ and frame complexity imposes a separate cost $c$. Then it costs $d+c$ to compare a price in $A$ with a price in $B$, and it costs $c+c$ to compare two prices in frame $B$. So which frame profile leads to more confused consumers depends on the relative size of $d$ and $c$.
    ${ }^{14}$ Notice that a noisy price comparison model might be related to the quantal response equilibrium (QRE). An interpretation of QRE is that players make random errors when choosing a strategy (even when they know what is the best strategy) but still choose a strategy with a higher expected payoff with a higher probability. Our benchmark model of consumer confusion can be seen as a fully "irrational" version of the QRE. In this case, the fact that consumers do not infer prices from frames is irrelevant: even if consumers know what is the best deal, they may still make choice mistakes.
    ${ }^{15}$ Our duopoly example can be regarded as a reduced-form model of the bi-symmetric graph case in Piccione and Spiegler (2012). All their results apply to our model, but in our setting it is subtler to exclude the possibility of firms adopting deterministic frames. In their model, consumers are always able to perfectly compare prices in the same frame (i.e., frame differentiation is the only confusion source), so it is easy to see that firms never adopt deterministic frames.

[^10]:    ${ }^{16}$ Note that although parts (a) and (c) used the fact that consumers can compare prices perfectly when both firms use frame $A$, our result still holds even if $\alpha_{0}>0$ provided that $\alpha_{0} \neq \alpha_{1}$ (the logic in (b) applies).

[^11]:    ${ }^{17}$ Note that when $\alpha_{1}=\alpha_{2}>0$, the frame profile $(A, B)$ cannot form part of an equilibrium. Otherwise, the firm using frame $B$ would have an incentive to switch to frame $A$ and undercut its rival, as we argued in part (c) in the proof of Lemma 1.

[^12]:    ${ }^{18}$ Note that in our model consumer confusion occurs at the frame level. For example, across all pairs of one $A$ offer and one $B$ offer, a consumer is able to compare either all or none.

[^13]:    ${ }^{19}$ In our model, the comparability of two offers is independent of their comparability with other available offers. This excludes transitivity of comparability. Consider a consumer who can compare

[^14]:    offers in different frames but cannot compare offers in frame $B$. Then the presence of an offer in frame $A$ (which is comparable with any of the $B$ offers) does not help the consumer compare offers in frame $B$ directly. This might be the case when the consumers use different procedures to compare prices in different formats and to compare prices in a complex format.
    ${ }^{20}$ There is evidence that people have preferences for simpler options, especially when they face many alternatives. See, for instance, Iyengar and Kamenica (2010) and the references therein.

[^15]:    ${ }^{21}$ We previously assumed that $\phi_{k}$ weakly decreases with $k$. But, arguably, consumers might opt for the simple frame more often

[^16]:    when they face more options, which would imply that $\phi_{k}$ increases with $k$. In effect, as we discuss in the proof of Proposition 4, our results still hold in that case as long as $\lim _{n \rightarrow \infty} \phi_{n-1}<1$ (i.e., if even
    in the limit, not all consumers favor the simple frame).

[^17]:    ${ }^{22}$ For industry profit to increase at a larger $n, \alpha_{1}$ needs to be smaller. But this is always feasible according to Proposition 7.

[^18]:    ${ }^{23}$ The fact that these two choice rules may lead to different outcomes can also be seen from the following example: consider the frame choices in Example 4, but let $\alpha_{2}=0, \alpha_{1}=1$, and $p_{1}<p_{2}<p_{3}$. Our approach (with the uniform purchase rule) predicts that firms 1 and 2 will share the market equally, whereas the default-bias rule predicts that firm 1 has demand $1 / 3$ and firm 2 has demand $2 / 3$.

[^19]:    ${ }^{24}$ Price framing reduces the comparability of competing offers and increases consumers' search/evaluation costs. In a related vein, Kuksov (2004) presents a consumer search model where firms produce more differentiated products in response to lower search costs. As a result, lower search costs may lead to higher prices in the market. In Kuksov and Villas-Boas (2010), firms' range of alternatives affects consumers' search/evaluation costs, and too many alternatives may induce consumers to leave the market without making a choice.

[^20]:    ${ }^{25}$ The reportage "What's Really in Our Food?" broadcast on BBC One in July 2009 (http://www.bbc.co.uk/programmes/b00lrjk4). stressed this point. Interviewed customers admitted to being misled by a ready food made with imported meat and labeled as "British meal." Also, buyers seem to have a poor understanding of what labels such as "free range" really mean.
    ${ }^{26}$ Market complexity often increases consumer search costs, so a consumer search model without the assumption of full market coverage can capture this effect. For example, Janssen et al. (2005) show in the framework of Stahl (1989) that if the first price quote is also costly, then when search costs become too high consumers start to drop out of the market, and thus industry profits fall.

[^21]:    ${ }^{27}$ However, industry profit $n \pi$ can rise with $n$ when $n$ is small and $\phi_{1}$ takes relatively extreme values. For example, when $\phi_{1}=0.95$ or 0.05 , from $n=2$ to 3 , industry profit $n \pi$ increases from 0.095 to about 0.099 .
    ${ }^{28}$ If $\phi_{k}$ increases with $k$, then we can replace $\phi_{1}$ in the above displayed equation by $\phi_{n-1}$. Hence, if $\lim _{n \rightarrow \infty} \phi_{n-1}<1$, our argument still works.

