On a robust modeling of piezo-systems

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This paper propose a new modeling approach which is experimentaly validated on piezoelectric systems in order to provide a robust Black-box model for complex systems control. Industrial applications such as vibration control in machining and active suspension in transportation should be concerned by the results presented here. Generally one uses physical based approaches. These are interesting as long as the user cares about the nature of the system. However, sometimes complex phenomena occur in the system while there is not sufficient expertise to explain them. Therefore, we adopt identification methods to achieve the modeling task. Since, the micro-displacements of the piezo-system sometimes generate corrupted data named observation outliers leading to large estimation errors, we propose a parameterized robust estimation criterion based on a mixed $L_2 - L_1$ norm with an extended range of a scaling factor to tackle efficiently these outliers. This choice is motivated by the high sensitivity of least-squares methods to the large estimation errors. Therefore, the role of the L_1-norm is to make the L_2-estimator more robust. Experimental results are presented and discussed.

1 Introduction

Systems modeling tasks imply choosing an approach which mainly depends on the final purpose. In this paper, we deal with piezoelectric systems models for complex systems control. Therefore, the modeling approach here chosen must provide specifically oriented control models. The methodology and results exposed in this paper are intended to any industrial design involving piezoelectric actuators such as active suspensions and vibration assisted drilling. Recently, an interesting paper on the piezoelectric multilayer-stacked hybrid actuation/transduction system [46] has been proposed. The exposed modeling procedure is possible and useful only afterwards all physical elements have been chosen. As a matter of fact, during the physical choices step, other kinds of models are more suitable. The reader interested in piezoelectric materials could refer to [3] [28]. The literature shows that in such a multi-physical device modeling, the common approaches consists of analogical representation of the physical phenomena or finite elements analysis. In [12] [35] [37] [38] [44] the authors developed electrical and mechanical components that are analogous to the concerned system under some conditions. Assumptions and approximations are made to minimize computation efforts while achieving good accuracy as long as the assumptions made are satisfied. This makes it possible to simulate the response of the system and quantify the influence of each parameter and tradeoffs to be made [36]. This is an interesting approach as one needs to understand the nature of the system and modify or improve their physical behaviour. On the other hand, for high level detailed study the finite-element method is often found to be the most appropriate [42]. Both approaches often lead to models for which it could be difficult to find the appropriate parameters. For example in the case of piezoelectric systems, where one has to deal with micro-displacements, the parameters estimation is very sensitive to outliers. Remember [25] [22] that outliers mean significantly large values of the estimation errors. In this paper, the term outlier refers

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to any big deviation in comparison to the usual observations. Instead of physical modeling approach, we suggest the use a robust identification method. Usually, the model's parameters estimation is based on least squares methods based on a Black-box approach [31]. The main drawback in these techniques comes from their high statistical sensibility to large errors. Generally, two methods are used. The first one consists in simply deleting (filtering) the influencing outliers before the fitting process. This is often an efficient approach when expert knowledge assists this task, ensuring that the removed information are not relevant ones. However, because of its complexity, it is not always feasible. This is the case of the piezo-system we shall deal with in this paper (Section 2). Sometimes, data delation could lead to losing crucial information, since they often provide valuable information about the system's dynamics [43]. The second method therefore consists in treating these outliers, in order to capture relevant information about the system behavior they may contain. It is up to the user to interpret then the identified model and conclude on the origin of the system's behaviour. Since the underlying error estimation distribution presents a heavy tail [21] [33] [25] due to the presence of these outliers, alternative solutions are brought. The LSAD (Least Sum Absolute Deviation) techniques leading to linear programming minimization problems with or without contraints proposes a family of robustly convergent algorithm's seems to be more efficient than the classical least squares criterion in the case of an estimation errors with Laplacian distribution. Another method uses a mixed $L_2 - L_1$ norm based on the parameterized robust estimation criterion according the Huber's M-estimates [25]. A simple insight on the estimation errors characteristics fitted by the least squares criterion, provides an idea on the convenient scaling factor which automatically determines the balance between L_2 and L_1 contributions of the estimation procedure. Moreover, by varying this scaling factor, we can modify the rate of L_1 in comparaison with L_2. A huge scaling factor corresponds to pure L_2-estimator. Our work aims to use a parameterized robust estimation criterion based on the $L_2 - L_1$ Huber's norm with an extended range of a scaling factor, treating the innovation outliers [13]. Moreover, we propose a new decisional tool for the models validation, named L_1-contribution function of the estimation errors to choice the estimated pseudo-linear models.

This paper is organized as follows: Section 2 describes the context of complex systems control. In Section 3, we shall resume the mathematical background related to the Huber's M-estimates problem. The following Section 4, the parameterized robust estimation criterion, the extended range of the scaling factor and the L_1-contribution are proposed and discussed. In section 5 we shall comment these experimental results. Finally, we shall give some conclusions and perspective to this work.

2 Context of complex systems control: drilling case

A piezoelectric material changes its shape when subjected to an electric field. Conversely, it induces electrical charges when subjected to mechanical stress. This makes it possible to use piezo-materials both in actuators and sensors elaboration.

Fig.1 illustrates the macroscopical principle of piezo-actuation. With respect to voltage V input and resistive force F, the device generates micro-vibrations u and consumes a electric charge q. In this paper we shall alternatively use electric charge and electric current $I = \dot{q}$. Piezoelectric devices are useful in advanced and complex mechanical structures design as well as in some of their manufacturing process. This requires the participation of scientists and engineers from diverse fields, mechanical, electrical, control, computing, etc. Some of industrial assembling processes (like aircraft structure assembling processes) may require drilling thousands of holes specified as to be of diameter much smaller than the depth. The main difficulty with such a process concerns breaking and evacuating the chips. There are many risks coming from this problems. The drilling tool could be damaged. A technique for dealing with this problem consists of drilling with vibration assistance. Micro-vibrations are generated and transmitted either to the work-piece either to the drill bit. This offers a controllable solution for chips breaking and eliminates stripping tasks. The major stake in vibrational drilling is about generating with precision the required piezo-vibrations (in terms of frequency and amplitude). The user should therefore elaborate the command. For this purpose, efficient models are required. However not all models could serve in command elaboration. The common approaches consists of analogical and/or phenomenological representation of the physical phenomena or finite elements analysis (FEA). However, as mentioned in [5], FEA methods are useful when high level of detail is required. This is not the case in command elaboration. In previous papers [4], we adopted lumped-parameters approach in order to establish user-oriented models in 20-Sim [10] and Matlab-Simulink [34]. Then, these models were improved in [5] in order to better account for nonlinear phenomena occurring in piezoelectric devices. We obtained good agreements between the models and the experiments. For smooth (sinusoidal) voltage profiles, the proposal models were able to predict the output displacement. Now, the inverse question arises. Which voltage shall we generate in order to obtain a certain vibrations profile? Several techniques exist that are extensively used. Commonly, the system's outputs are measured or estimated in order to track the referee command via a synthesized feedback. In this category one could enumerate adaptive control [32] [9], state

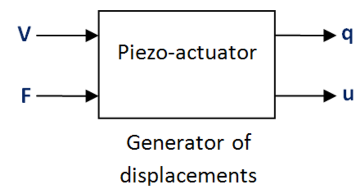

Fig. 1. Piezo-actuation principle

feedback control [26] [20], sliding model control [14] [19] etc. All provide satisfaction despite different sources of perturbations. These methods are classified as global control methods. On the other hand, other techniques (named local techniques) consist of interconnected subparts associated with the different parts of the system. This category includes nested control loops [16], backstepping control [48] and inverse model control. If a model is available, control based on reverse model is prefered. Indeed, inverse model control offers an organized methodology using a decomposition of the systems' organs functionality with respect to exchange energy. It consists in synthesizing the input according to the desired output profile. Therefrom, the task is to determine the physical reverse function of the system (piezo), so, the control loop could be resumed in Fig.2.

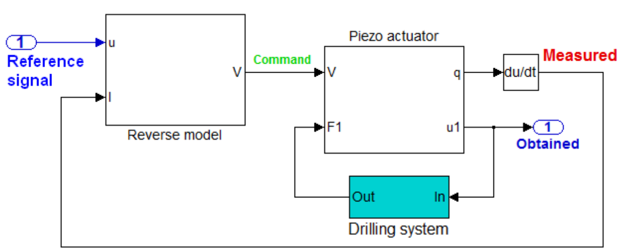

Fig. 2. Control loop proposition

As it appears in Fig.2, in addition to the referee displacement, the proposal control loop requires the real time electric current. However, it is difficult to get this feedback with good precision. Indeed, only small electric current crosses piezoelectric devices. To measure it, a high precision device is required. Unfortunetly, this is not easy and important noises are usually present in the signal. Experiments showed us that the effect of the current loop could be modelised by a noise and the displacement. We can therefore justify Fig.3. In this configuration, we replace the electric current feedback by noises added to the displacement.

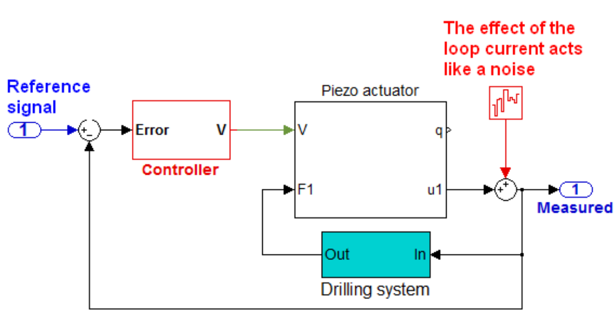

Fig. 3. Alternative control loop

This requires the elaboration of a new controller. However, the model obtained from analogical approaches could be non-robust and lead to bad control loop. In the literature, [47] [17] have proposed an optimal input design for system identification. This optimality approach is worthwhile, only when good prior knowledge is available about the system. In practice, it is suitable to decide upon an important and interesting frequency band to identify the system, and then select a signal with a more or less flat spectrum over this band. Among these signals, there exists the Chirp signals or Swept sinusoids [41] [23] and the Pseudo Random Binary Sequence (PRBS) which is a periodic, deterministic signal with white-noise-like properties, that is a Band-limited Gaussian. For this purpose, we shall excite the system by a PRBS, sufficiently exciting and persisting [27], with small amplitude ($\pm 10V$). This choice is motivated by two arguments. The first and trivial one concerns the fullness (in term of excitation frequencies) of such a signal in comparison with steps and sinusoidal signals. The second reason is related to the operating condition of the concerned system. We are dealing with piezo-systems applied to vibrational drilling. As a matter of fact, during the drilling process, the system is subjected to some random impacts. A PRBS allows therefore to reproduce such this environment. In this case (non-smooth solicitation, eg. sawtooth signal), even with low amplitude, the system could have a behaviour difficult to characterize. Indeed, assuming the physical system presented in Fig.6, sawtooth solicitation could lead to losing contact between element 4 and element 1. Classical mechanics can no longer allow to describe efficiently such a behaviour. Therefore, the models presented in Fig.2 and Fig.3 are no longer valid.

3 The Huber's *M*-estimates problem

In the general way, we denote in the sequel $X_t(\theta) = X(t,\theta)$ and $X_t = X(t)$, for a parameterized time varying signal. Let us consider a discrete-time SISO system with input signal U_t and output signal Y_t described as follows

$$Y_t = G\left(q^{-1},\theta\right) U_t + H\left(q^{-1},\theta\right) e_t \tag{1}$$

See [31] for more details. Here, $G\left(q^{-1},\theta\right)$ and $H\left(q^{-1},\theta\right)$ are the transfer functions of the system, respectively from U to Y and e to Y. The backward shift operator q^{-1} is defined by $U_{t-1} = q^{-1}U_t$. U_t is an exogenous and deterministic input signal and e_t a random variable with mean zero and variance λ. Consider the general parameterized pseudo-linear models set $\mathrm{M}(\theta)$, with the parameters vector $\theta = [\theta_1 \ldots \theta_d]^T \in I\!R^d$ where $\hat{Y}_t(\theta) = \varphi_t^T(\theta)\,\theta$, $t = 1,2,\ldots$, represents the prediction model output and a pseudo-linear regression on the base of a data set $\{U_1, Y_1, \ldots, U_N, Y_N, \ldots\}$. Here $\varphi_t^T(\theta)$, $t = 1,2,\ldots$ denote the $t-th$ observations vector and

$$\varepsilon_t(\theta) = Y_t - \hat{Y}_t(\theta) \tag{2}$$

the prediction error signal also named residuals. The Huber's M-estimation is a minimum problem of the form

$$N^{-1}\Sigma_{t=1}^{N}\rho_\eta\left(\varepsilon_t(\hat{\theta}_{N,\eta})\right) \cong N^{-1}\inf_{\theta\in\Theta}\Sigma_{t=1}^{N}\rho_\eta\left(\varepsilon_t(\theta)\right) \quad (3)$$

or by an implicit equation $N^{-1}\Sigma_{t=1}^{N}\Psi_{t,\eta}\left(\varepsilon;\hat{\theta}_{N,\eta}\right)=0$, where $\hat{\theta}_{N,\eta}$ is the robust estimator of θ, Θ is a subset of $\mathbb{R}^d$ and $\rho_\eta:\mathcal{S}\times\Theta\rightarrow\mathbb{R}$ a nonnegative, convex, piecewise function such as $\rho_\eta(\varepsilon_t(\theta)):\mathcal{S}\rightarrow\mathbb{R}$ is measurable for each $\theta\in\Theta$, with $\mathcal{S}$ a probability space. The constant η, named scaling factor, regulates the amount of robustness and may depend on the observations Y_t. In the literature, [24] [45] [8] [25](p.19) [33](pp.27,61) choose the scaling factor $\eta=k\sigma$ with $1\leq k\leq 2$ named tuning constant, only for the linear models, where σ is the standard deviation. More precisely, the ρ_η-norm is

$$\rho_\eta(X)=\begin{cases}\frac{1}{2}X^2 & \text{if } |X|\leq\eta\\ \eta\,|X|-\frac{1}{2}\eta^2 & \text{if } |X|>\eta\end{cases} \quad (4)$$

Here, ρ_η is chosen to render the estimation more robust than the classical least squares estimation with respect to the innovation outliers [13] supposed to be present in the residuals. The least informative probability density function [25] is defined by $f(X)=Ce^{-\rho_\eta(X)}$. Moreover $\Psi_{t,\eta}(\varepsilon;\theta)=-\frac{\partial}{\partial\theta}\rho_\eta(\varepsilon_t(\theta))$ is the gradient of the ρ_η-norm with respect to θ, named Ψ-function.

4 The robust estimation context based on a L_2-L_1 mixed norm

Now, let us deal with the main contribution of the paper. The goal is to propose a robust criterion based on the Huber's norm and justify the choice of the scaling factor. The new decisional tool for the models validation, named L_1-contribution is proposed and discussed.

4.1 The parameterized robust estimation criterion

Let us introduce two index sets defined by $\nu_2(\theta)=\{t:|\varepsilon_t(\theta)|\leq\eta\}$ and $\nu_1(\theta)=\{t:|\varepsilon_t(\theta)|>\eta\}$. ν_2 and ν_1 are respectively the L_2-contribution and L_1-contribution of the residuals. We define the *sign* function by $s_t(\theta)=0$ if $|\varepsilon_t(\theta)|\leq\eta$, $s_t(\theta)=-1$ if $\varepsilon_t(\theta)<-\eta$ and $s_t(\theta)=1$ if $\varepsilon_t(\theta)>\eta$. From this, the parameterized robust estimation criterion to be minimized can be written as follows

$$W_{N,\eta}(\theta)=\frac{1}{N}\Sigma_{t\in\nu_2(\theta)}\frac{\varepsilon_t^2(\theta)}{2}+\frac{\eta}{N}\Sigma_{t\in\nu_1(\theta)}(|\varepsilon_t(\theta)|-\frac{\eta s_t^2(\theta)}{2}) \quad (5)$$

From the derivative with respect to θ of (5), we deduce the Ψ-function by $\Psi_{t,\eta}(\varepsilon;\theta)=-\psi_{\nu_2,t}(\theta)\varepsilon_{\nu_2,t}(\theta)-\eta\psi_{\nu_1,t}(\theta)s_{\nu_1,t}(\theta)$, where in the general case $f_{\nu_i,t}=f_t$ if $t\in\nu_i(\theta)$, $f_{\nu_i,t}=0$ otherwise, $i=1,2$ and $\psi_t(\theta)$ is the derivative with respect to θ of (2).

4.2 *M*-estimation procedure

Fig.4 describes the estimation phase in this prediction error framework. During this procedure, the prediction errors are treated in a Parametric Adaptative Algorithm (PAA) which include both the solver and the parameterized robust estimation criterion to be minimized. The presence of outliers in the data set induces large values of the prediction errors. Thanks to a convenient choice of the scaling factor, the estimator robustification reduce the effects of these large deviations and the estimated residuals correspondingly decrease. These residuals are built following the described rule in the previous section: $\varepsilon_{\nu_i,t}=\varepsilon_t$ if $t\in\nu_i(\theta)$, $\varepsilon_{\nu_i,t}=0$ otherwise, $i=1,2$. Therefore, let us denote $\mathcal{E}(\theta)\in\mathbb{R}^N$ and $\mathcal{S}(\theta)\in\mathbb{R}^N$, respectively the vector of the residuals and the *sign* function such as $\mathcal{E}(\theta)=[\varepsilon_1(\theta)...\varepsilon_N(\theta)]$ and $\mathcal{S}(\theta)=[s_1(\theta)...s_N(\theta)]$ with $s_t(\theta)$ defined in the previous section. Moreover, let us define the weight matrix $\mathcal{W}(\theta)\in\mathbb{R}^{N\times N}$, $\mathcal{W}(\theta)=diag\,(w_1(\theta)...w_N(\theta))$ where $w_t(\theta)=1-s_t^2(\theta)$. The parameterized robust estimation criterion in Eq.(5) to insert in the PAA can then be rewritten as

$$W_{N,\eta}(\theta)=\frac{1}{2N}\mathcal{E}^T(\theta)\mathcal{W}(\theta)\mathcal{E}(\theta)+\frac{\eta}{N}\mathcal{S}^T(\theta)\left[\mathcal{E}(\theta)-\frac{\eta}{2}\mathcal{S}(\theta)\right] \quad (6)$$

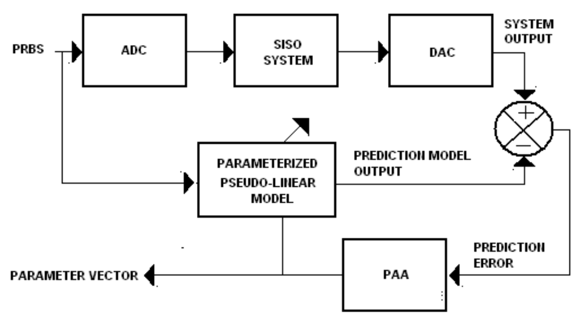

Fig. 4. Description of the estimation phase in the prediction error framework.

4.3 Choice of the scaling factor

In the literature, the tuning constant is chosen in the interval range $[1,2]$ [33] [25], only for the linear models, in order to regulate the amount of robustness. However, this choice of k does not ensure the convergence of the parameterized robust estimation criterion given by (5) and the estimator remains sensitive to the large and numerous innovation outliers. Maybe, because nonlinear models obviously do not match with this type of application? In this case, the distribution of the residuals is strongly disturbed and presents a heavy tail. Thus, formally, it is used a s-corrupted model to induce a topological neighborhood around the target normal distribution F_0, yielding a probability distribution $F_s=(1-s)F_0+sH$, $s\in[0,1[$ where H is an unknown distribution. The breakdown point (BP) of an estimator is the largest amount of contamination that the data contain such as $\hat{\theta}_{N,\eta}$ still gives some information about θ. The asymptotic contamination BP of $\hat{\theta}_{N,\eta}$ denoted s^* is the largest $s^* \in (0,1)$ such as for $s < s^*$, $\hat{\theta}_{N,\eta}$ remains bounded. Moreover, in the outliers detection methods, the leverage points (LP), namely points with highly influence of position in factor space, are increased interest. It is shown that the *M*-estimator is not always robust to LP when the tuning constant belongs in $[1,2]$ [33] [25]. In the case of piezo-systems where the micro-vibrations sometimes generate outliers and disturb the robust estimation, it is interesting, to reduce the influence of the LP on the pseudo-linear models, that we deliberately investigate the smaller values of the tuning constant, that is $k \in [0.05, 2]$. In the other hand, the improvement of the estimated model with the extended tuning constant, may be a somehow favourable effect on the inner feedback (see Fig.4), precisely due to the dependence of $\varphi_t(\theta)$ upon the parameter vector.

4.4 L_1-contribution function: a decisional tool for the models validation

In the sequel, the reader should make the difference between the degree of corruption of the data measured by the engineer and the level of the related corruption of the estimation error (i.e. outliers) only considered by the statisticians.
In the robust estimation procedure describes in Fig.4, the estimated prediction errors larger than the scaling factor are treated by the L_1-norm in order to rend more robust the least squares estimator. In the previous section, we defined two index sets, denoted $\nu_2(\theta)$ and $\nu_1(\theta)$, such as $card\,[\nu_2(\theta)] + card\,[\nu_1(\theta)] = N$ for all $\theta \in \Theta$ and $N \in \mathbb{N}$. Let us denote $card\,[\nu_1(\theta)] = N_{out}(\theta)$ the number of innovation outliers and $\frac{N_{out}(\theta)}{N}$ the fraction of "bad" values of the corrupted samples. The small values of η involves a robust estimation procedure mainly L_1 and $\frac{N_{out}(\theta)}{N}$ increases. In [25], the author's defined this fraction as a *s*-replacement model in the general context of the *s*-corrupted distributions. For any robust estimator $\hat{\theta}_{N,\eta}$, there is a $\hat{s}_N$-corrupted empirical distribution $F_{\hat{s}_N} = (1-\hat{s}_N) F_N + \hat{s}_N H_N$, where $\hat{s}_N = s\left(\hat{\theta}_{N,\eta}\right)$. In our work, we extend the role of the fraction $\frac{N_{out}(\theta)}{N}$ as a decisional tool for the models validation named L_1-contribution function denoted $L_1C(\theta)$. The minima of this function can confirm the robust estimator derived by $\arg\min_{\theta\in\Theta} W_{N,\eta}(\theta)$ or give another estimator. In this case, this tool emphasizes the decision on the choice of the estimated model. In the sequel, we show that the L_1-contribution function has a minimum, therefore, formally, we define the L_1-contribution function as

$$L_1C(\theta) = \frac{1}{N}\Sigma_{t\in\nu_1(\theta)} |s_t(\theta)| \tag{7}$$

The derivative of $L_1C(\theta)$ with respect to θ necessities the derivative of the *sign* function. Therefore, we use an approximation function [6] of $s_t(\theta)$ given by $s_t(\theta) \approx \zeta_t(\theta) = \frac{1-e^{-2K\varepsilon_t(\theta)}}{1+e^{-2K\varepsilon_t(\theta)}}$, where K is a real sufficiently large to ensure the approximation. The L_1-contribution function can then be rewritten as $L_1C(\theta) = \frac{1}{N}\Sigma_{t\in\nu_1(\theta)}\left|\frac{1-e^{-2K\varepsilon_t(\theta)}}{1+e^{-2K\varepsilon_t(\theta)}}\right|$. Now let us define two index subsets of $\nu_1(\theta)$ as $\nu_1^L(\theta) = \{t : |\varepsilon_t(\theta)| < -\eta\}$ and $\nu_1^H(\theta) = \{t : |\varepsilon_t(\theta)| > \eta\}$ such as $\nu_1(\theta) = \nu_1^L(\theta) \cup \nu_1^H(\theta)$ and $\nu_1^L(\theta) \cap \nu_1^H(\theta) = 0$. We then have $L_1C(\theta) = \frac{1}{N}\Sigma_{t\in\nu_1^H(\theta)}\frac{1-e^{-2K\varepsilon_t(\theta)}}{1+e^{-2K\varepsilon_t(\theta)}} + \frac{1}{N}\Sigma_{t\in\nu_1^L(\theta)}\frac{e^{2K\varepsilon_t(\theta)}-1}{e^{2K\varepsilon_t(\theta)}+1}$. After straightforward calculations and using a Taylor's expansion, the derivative with respect to θ of $L_1C(\theta)$ leads to

$$\frac{\partial}{\partial\theta}L_1C(\theta) \approx \frac{-4K}{N}\Sigma_{t\in\nu_1(\theta)}\psi_t(\theta)\,e^{-2K|\varepsilon_t(\theta)|} \tag{8}$$

where $\psi_t(\theta) = -\frac{\partial}{\partial\theta}\varepsilon_t(\theta)$ [31]. Since $|\varepsilon_t(\theta)| > \eta$, we obtain $\left|\frac{\partial}{\partial\theta}L_1C(\theta)\right| \le \frac{4Ke^{-2K\eta}}{N}\Sigma_{t\in\nu_1(\theta)}|\psi_t(\theta)|$. From [30], Ljung and Caines showed that $|\psi_t(\theta)|$ is bounded for all $\theta \in \Theta$. Therefore, $|\psi_{\nu_1,t}(\theta)| \le |\psi_t(\theta)| = C_\psi$ and there exists an estimator $\hat{\theta}_{N,\eta} \in \Theta$ such that $\left|\frac{\partial}{\partial\theta}L_1C\left(\hat{\theta}_{N,\eta}\right)\right| \le 4KC_{\hat{\psi}}e^{-2K\eta} \to 0$ for K sufficiently large.

5 Experimental results

5.1 Experimental setup

The experiments and simulations are performed with a HPSt 1000/35-25/80 piezo-actuator from Piezomechanik. In Fig.6 we show the mechanical assembly used for the experiments. This special setup firstly aims to prevent the piezoelectric device from damaging. Indeed, ceramic made devices are brittle under stretching solicitations. Therefore mechanisms are used to apply a pre-stress on the device. In our design, the pre-stress value depends on the gap between elements 1 and 2. This gap is set via thin metallic films between elements 1 and 2. In our setup, we applied 3000*N*. Element 3 is fixed to a table. The vibrations of the piezoelectric device (yellow element) are transmitted to element 1. This is why the measuring sensor is about element 1 as shown in Fig.5. We consider the whole assembly as a SISO system (Single Input Single Output) because we are only interested in the piezoelectric device displacement. Fig.5 shows the measurements chain. Following are details on instrumentations and signal processing.

Piezoelectric actuator	
Reference	HPSt 1000/35-25/80
Manufacturer	Piezomechanik
Accelerometer	
Reference	DYTRAN 3225F1
Sensitivity	10 mV/G
Frequency response	±10%: 1.6 to 10,000 Hz
Linearity	2% F.S max
National Instruments Cards	
	Channels: 4 for each card

In: NI-9215, Out: NI-9263

	Simultaneous sampling
	Output resolution: 16-bit
	Input resolution: 24-bit
	Output rate: 50 kHz
	Input rate: 50 kHz
Power Operational Amplifier	
Reference	PA-0103

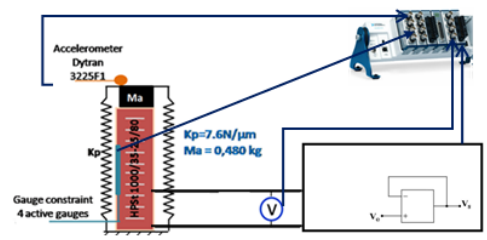

Fig. 5. Experimental setup

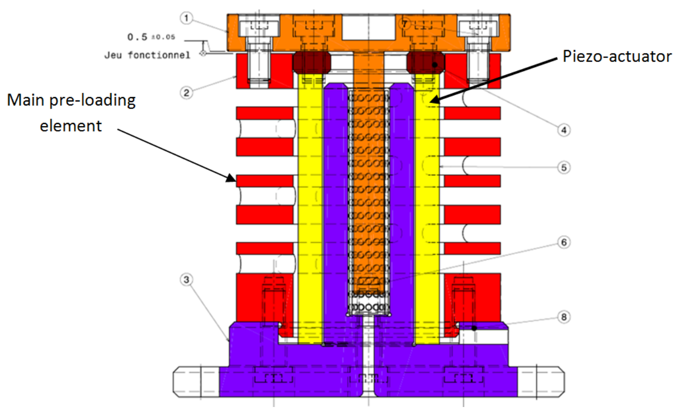

Fig. 6. Experimental piezo-system

In the sequel, we only consider measurements from the gauges constraint. The accelerometer is used in order to check the efficiency of the active gauges.

5.2 Identification procedure

In order to provide a Black-box model of the piezo-system and more particularly a robust model from the output signal of micro-displacements, we applie for the exogenous input a pseudo random binary sequence (PRBS) with a length $\mathcal{L} = 2^{10} - 1$ and level $\pm 10V$, sufficiently exciting and persisting [27]. The sampling period is $T_S = 100\mu s$ and the number of data is $N = 5000$. Fig.7 and Fig.8 respectively show the excitation input and the output signal of the piezo-system. This last signal presents some large values which may be considered as an observation outliers. Therefore, these large samples implie innovations outliers in the estimated residuals.

Since the piezoelectric ceramic system is not a linear experimental device, the adopted model is the classical Output Error (OE(n_B, n_F)) pseudo-linear model given by

$$M(\theta) : Y_t = q^{-d} \frac{B(q^{-1}, \theta)}{F(q^{-1}, \theta)} U_t + e_t$$

where d is the pure plant time delay and $F(q^{-1}, \theta)$ a monic polynomial. In our case $d = 1$ meaning the time delay of the sample and hold in the discritization. The parameters vector is $\theta = [b_1 ... b_{n_B} f_1 ... f_{n_F}]^T$. The observations vector is $\varphi_t^T(\theta) = \left[U_{t-1} ... U_{t-n_B} - \hat{Y}_{t-1}(\theta) ... - \hat{Y}_{t-n_F}(\theta)\right]$. As explain in Section4.3, the tuning constant is chosen in the interval range $[0.05, 2]$, the scaling factor is $\eta = k\sigma$ where σ is the standard deviation obtained from a least squares estimation in the initialization phase. For the polynomials $B(q^{-1}, \theta)$ and $F(q^{-1}, \theta)$, $7 \leq n_B \leq 14$ and $4 \leq n_F \leq 15$ respectively. In the L_1-contribution function, we experimented different values of K. We choosed $K = 15$ since great values do not improve significantly the approximation.

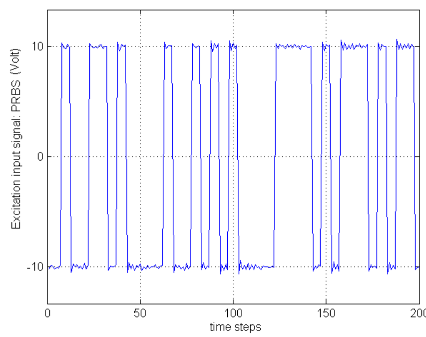

Fig. 7. Excitation input signal: PRBS

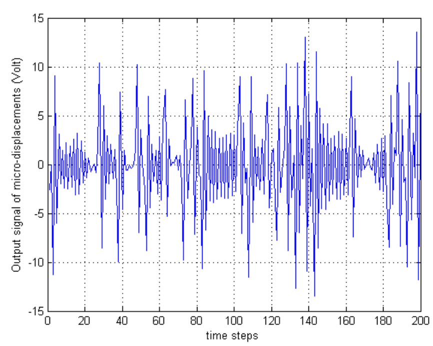

Fig. 8. Output signal of Piezoelectric.

5.3 Distribution of the prediction errors in the least squares estimation

As expected, the distribution of the prediction errors of an estimated model in the classical least squares, is strongly disturbed (see Fig.9). This non-trivial distribution is zero between -2 and $+2$ and presents two distributions around -3 and $+3$. These different results show firstly, the necessity to use a parameterized robust estimation criterion with a scaling factor and secondly, to choose this parameter and reinforce the robustness of the least squares estimation. It seems reasonable to investigate the variations of this scaling factor given in the previous section.

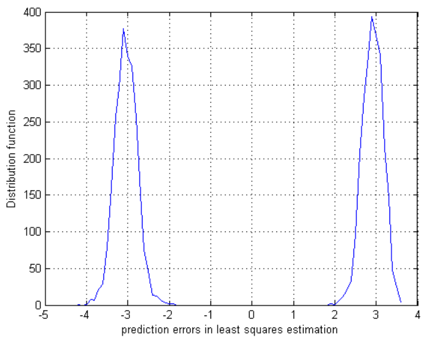

Fig. 9. Probability density function of L_2 estimation for an OE(12,12) model.

5.4 Estimation/Validation results

In a first step, an estimation campaign has led to derive n_B. $n_B = 9$ for the first model denoted M_1 and $n_B = 12$ for the second, denoted M_2. Fig.10 shows the parameterized robust estimation criterion $W_{N,\eta}$ as a function of n_F with $4 \leq n_F \leq 15$ at $n_B = 9$, when the tuning constant is equal to 0.0625. The minima of $W_{N,\eta}$ yields two models at $n_F = 8$ and $n_F = 12$. For the first model at $n_F = 8$, the fit less than 40%. Let us denote the first model at $n_B = 9$ and $n_F = 12$, M_1 : OE(9,12). The scaling factor is $\eta = 0.0625\sigma = 0.2255$ and the fit equal to 82.5% in the frequency interval $[0; 500Hz]$, used for the control. In Fig.12, the frequency response of M_1 is compared to the spectral estimation of the piezo-system. For the model validation, we use the results of the L_1-contribution function. Fig.11 shows L_1C as a function of n_F with $4 \leq n_F \leq 15$ at $n_B = 12$, when the tuning constant is equal to 0.0875. This decisional tool provides two robust models at $n_F = 9$ with a fit equal to 87.2% and $n_F = 12$ with a fit equal to 95.22%. The second selected model is M_2 : OE(12,12) for its very good fit. Even though the L_1-contribution function yields a robust model with a dimentional $d = n_B + n_F = 24$, the choice has been made only on the fit criterion, since in the case of the piezo-system, the robust model must have a relevant characteristics for the sensitivity of the control. For this model, the scaling factor is $\eta = 0.0875\sigma = 0.2619$ and the L_1-contribution function equal to 94.2%. This value shows that the robust estimation has been mainly L_1. The reader shall note in Fig.13 the good frequency response in $[0; 500Hz]$ of M_2 versus the spectral estimation of the piezo-system. In Table 1 and Table 2 the estimated parameters of M_1 and M_2 are respectively shown.

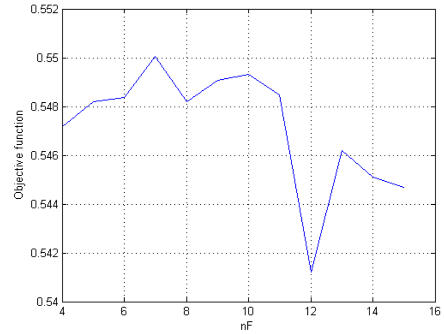

Fig. 10. Parameterized robust estimation criterion as a function of n_F at $n_B = 9$ when $\eta = 0.0625\sigma$.

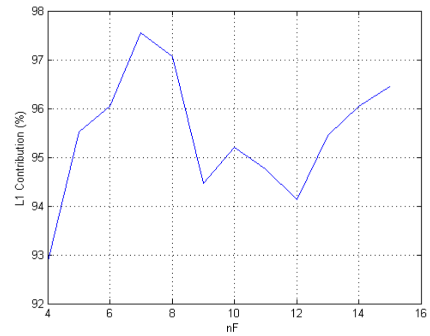

Fig. 11. L_1-contribution function as a function of n_F at $n_B = 12$ when $\eta = 0.0875\sigma$.

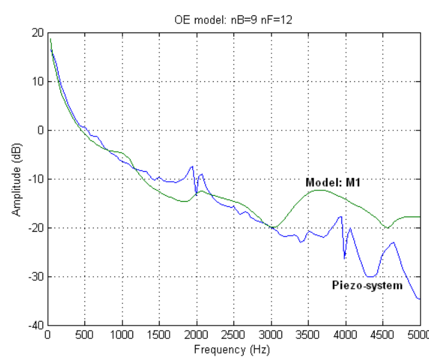

Fig. 12. Robust model M_1 : OE(9,12) compared to the spectral estimation of the piezo-system at $\eta = 0.0625\sigma = 0.2255$.

In order to provide *a reference case*, Fig.14 shows the estimated model in least squares estimation. The great sensitivity with respect to large estimation errors is clearly illustrated.

n	1	2	3	4	5	6	7	8	9	10	11	12
b_n	−0.119	−0.133	−0.115	−0.098	−0.107	−0.236	−0.104	−0.172	−0.234	0	0	0
f_n	−0.470	0.068	0.280	−0.291	−0.068	−0.007	−0.068	0.013	−0.261	0.239	−0.108	0.089

Table 1. Estimated parameters of M_1 : OE(9,12).

n	1	2	3	4	5	6	7	8	9	10	11	12
b_n	−0.042	0.033	0.038	0.054	−0.206	−0.368	−0.254	−0.137	−0.159	−0.201	−0.186	−0.110
f_n	−0.212	−0.178	−0.005	−0.016	0.002	−0.060	−0.048	−0.144	−0.060	−0.200	4.3×10^{-7}	0.0065

Table 2. Estimated parameters of M_2 : OE(12,12).

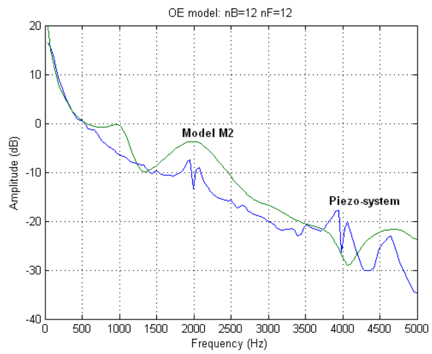

Fig. 13. Robust model M_2 : OE(12,12) compared to the spectral estimation of the piezo-system at $\eta = 0.0875 \sigma = 0.2619$.

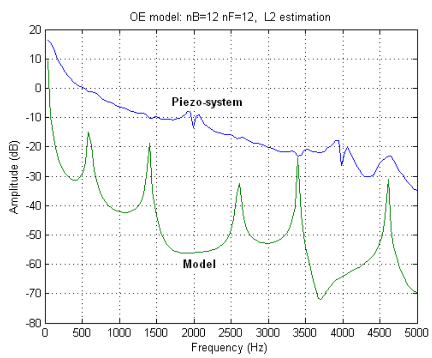

Fig. 14. The model OE(12,12) in least squares estimation. The great sensitivity to the large deviations is clearly shown.

6 Conclusion

In this paper, we showed a piezoelectric ceramic process identification method, based on the Huber's M-estimates, with an extended interval range of a scaling factor to deal with both the large estimated prediction errors and the pseudo-linear Black-box models. We used a parameterized robust estimation criterion composed both of a L_2 part for the small prediction errors and a L_1 part for the innovation outliers. For the models validation, we presented and discussed a new decisional tool, the L_1-contribution function and we extended the role of this term in order to determine the s-corrupted models distribution of the prediction errors. Moreover, we showed that this validation tool provided relevant models with good characteristics in the frequency interval range for the control. Many aspects of these studies are open to further research. It should be interesting to analyse the properties of the L_1-contribution function and an extansion of this method will be proposed for non-linear models.

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