On signals compression by EMD

Kais Khaldi[1,2] and Abdel-Ouahab Boudraa[1]
[1]Ecole Navale, IRENav, BCRM Brest CC 600, 29240 Brest, France.
[2]ENIT, U2S, BP 37, Le Belvedère, 1002 Tunis, Tunisia.

Abstract

In this letter a new signals coding framework based on the Empirical Mode Decomposition (EMD) is introduced. The EMD breaks down any signal into a reduced number of oscillating components called Intrinsic Modes Decomposition (IMFs). Based on IMF properties, different coding strategies are presented. No assumptions concerning the linearity or the stationarity are made about the signal to be coded. Results obtained on ECG signals are presented and compared to those of wavelets coding.

1 Introduction

Signal compression is a central topic is many areas such speech and image processing, telemedicine or health monitoring. The aim is to reduce the number of allocated bits in the compact digital representation of the signal while retaining all its relevant information. Most of signal compression strategies use signal expansions such as transform coding or subband coding [1]. The principle of the transform coding is to transform the signal into a domain where it is more amenable to quantization and entropy coding [2]. In subband coding, which is widely used in practice, the signal is decomposed into a set of band-limited components called subbands. Reconstruction of the original signal is accomplished by upsampling, filtering an summing the individual subbands [1]. Even good bit rates are obtained, this class of coding uses pre-determined basis functions. Unfortunately, using fixed basis functions prevents the decomposition from being parsimonious for any kind of signals. As a matter of fact, even if a basis is well suited for a class of signals, in the sense that it yields compact descriptions with only a few significant terms, there are other signals for which the basis under consideration performs poorly. Thus, there is a need for data driven coding approach. Recently, a new expansion, referred to as Empirical Mode Decomposition (EMD) has been introduced for analyzing data from non-stationary derived from linear or non-linear systems in totally adaptive way [3]. Main interest of such decomposition relies on no a priori choice of basis functions. In such decomposition the extracted oscillating modes, called called Intrinsic Mode Functions (IMFs), are fully described by their local extrema [3]. The IMFs are zero-mean AM-FM components with symmetric envelopes. These modes are orthogonal and the original signal can be recovered by superposition of individual IMFs [3]. Compared to classical kernel based approaches, the EMD is fully data driven technique that recursively breaks down any signal, $\mathtt{x}(t)$, into a reduced number of IMFs. These modes are extracted using the sifting process [3] and the decomposition is given by the following expansion:

$$\mathtt{x}(t) = \sum_{k=1}^{K} \mathtt{IMF}_k(t) + \mathtt{r}_K(t) = \sum_{k=1}^{K} \mathtt{a}_k(t) e^{i\phi_k(t)} + \mathtt{r}_K(t) \tag{1}$$

where K is the number of modes and $\mathtt{r}_K(t)$ is the residual [3]. Functions $\mathtt{a}_k(t)$ and $\phi_k(t)$ are Instantaneous Amplitude (IA) and Instantaneous Phase (IP) of the $k^{\mathtt{th}}$ IMF. We have shown that extrema of IMFs can be used for audio coding purpose [4]. We show in this letter that this approach can be extended to encode any signal and from any source. Different coding strategies based on IMFs properties are proposed.

2 Extrema coding

An IMF is a zero-mean component that can be represented by its extrema. Figure 1 shows an example of IMF and its approximate version by a spline interpolation of its extrema with negligible reconstruction error. This example illustrates the interest of encoding extrema. Depending on the application the number of extrema of each IMF can be reduced using an appropriate thresholding. Only extrema saved in the coding are those that extend above a threshold. In application such as speech coding, the threshold can be given by psychoacoustic model. Figure 2 shows an example of EGC signal decomposed by EMD. The associated extrema of extracted IMFs are given in Table 1 where, by reconstruction, the number of extrema is decreased when going from an IMF to next one. Thus, the way this number varies from mode to mode is exploited in the unequal bit allocation strategy. The extrema amplitude of each IMF is scaled by a factor equal to the maximum of the extrema amplitude value. Finally we quantize the position extrema, scaling factor and extrema amplitude value by a scalar quantization. In this strategy both minima and maxima are encoded.

3 Envelope coding

One step of the sifting is to identify local maxima (minima) and then connect them by a spline line to form upper (lower) envelope. The aim is to remove the asymmetry between the upper and lower envelopes in order to transform the original signal into an AM signal (Fig. 3). Thus, compared to first strategy (Sec. 2) one can only encode the minima or the maxima (one envelope). However, the EMD is a numerical method that is prone to numerical errors that may persist in the decomposition as extra IMFs. A source of this error seems to be edge effects due to the construction of envelopes through interpolation. Thus, extracted IMFs are, in general, not all truly symmetric with respect to the time axis ($y = 0$) but they are symmetric about a parallel line $y = \alpha$. An example of offset values obtained for six IMFs extracted from ECG frame signal is presented in Table 1. As expected, IMFs are not all symmetric with respect to $y = 0$. Thus, provided the offset α is encoded, at the decoder the upper (lower) envelope is reconstructed and the lower (upper) envelope is determined by symmetry about the line $y = \alpha$. An advantage of such strategy is that, the bit rate is approximately reduced by half.

4 IA, IP and IF codings

For some classes of signals such as audio signals [6], both IA, $\mathsf{a}_k(t)$, and Instantaneous Frequency (IF), $f_k(t) = \dot{\phi}_k(t)/2\pi$ are correlated while IP values are slowly varying. This fact is well illustrated in the case of an IMF by figure 4. The idea is to encode both IA and IF functions of the signal [5]. Thus, these functions can be coded by linear prediction and extrema of IP encoded using scalar quantization. Remaining IP values can be calculated by linear interpolation. IA and IF components of each IMF can be modeled as follows:

$$a(t) = \sum_{k=1}^{p} c_a(k)a(t-k) + \varepsilon_1(t), \;\; f(t) = \sum_{i=1}^{L} c_f(i)f(t-i) + \varepsilon_2(t) \tag{2}$$

where $[c_a(1), \ldots, c_a(p)]$ and $[c_f(1), \ldots, c_f(L)]$ are the coefficients of the AR model for $a(t)$ and $f(t)$ respectively. $\varepsilon_1(t)$ and $\varepsilon_2(t)$ are zero mean white noise processes. Thus, one can encode $c_a(k)$, $c_f(i)$ and variances of $\varepsilon_1(t)$ and $\varepsilon_2(t)$ using lossless compression such as Lempel-Ziv encoding. For each IMF, the AR order of the associated IF (IA) is calculated. For IA, the order can be calculated, for example for audio signals, using perceptual constraint [6]. Since each IMF contains lower frequency oscillations than the previously extracted one (Fig. 2), the order for IF component varies from one IMF to the next one. This order can be estimated using Partial Autocorrelation

Coefficient (PAC). Table 3 shows the PAC of IF of a IMF 5 extracted from ECG signal. Order from which the PAC values are constant, is identified as the order for IF modeling (Eq. 2). AR order of each IF is presented in Table 4.

5 Results

We illustrate the potential of EMD coding on four ECG records taken from BIOE 415 database and we compare the obtained results to those of wavelets approach. We used Daubechies wavelet of order 8 which, in general, gives good results in comparison to other wavelets. Validation of the EMD coding is done through Compression Ratio (CR) and the Percentage Root mean square Difference (PRD). This measure evaluates the distortion between the original and the reconstructed ECG signals. The increase in PRD value is actually undesirable. Aim in ECG compression is to increase the CR but not at the expense of the quality of the reconstructed signal. Original analyzed ECG signals are depicted in figure 5. This figure shows that ECG signals are not linear in their nature, but rather more curvaneous. This why the EMD coding approaches are illustrated on this kind of signals. Table 5 lists the achieved PRD and CR values for each method. From this Table, it can be seen that the EMD-based codings achive better performance in terms of PRD and CR values compared to Wavelets approach. The highest CR (59 : 1) is achieved for record no. 4 by Envelope coding. Also, the best result from reconstruction error point of view (PRD= 3.4%), obtained for record no. 3, is achieved by Envelope coding. Figures 6(a)-(d) confirm the results reported in Table 5 where record no. 1 is well reconstructed with high fidelity with EMD coding methods. Also the characteristic features of this signal are preserved well in the reconstructed signal (Fig. 6(c)). Even only results of four records are presented, overall the Envelope coding achieves better performance compared to other methods.

This is expected because this approach encodes one out of two envelopes of the IMF and thus higher CRs are obtained (Fig. 3). However, a large class of ECG signals (Arrhythmia,...) or other class of signals (EEG, EMG,...) is necessary to confirm these findings. Also, even the analysed signals (Fig. 5) have different frequency contents, the achieved performances show that these signals are well expanded into a reduced number of IMFs (atoms), mainly due to the adaptive nature of the EMD. No prior assumptions have been made about these signals concerning the number of IMFs for their expansions and their codings.

6 Conclusion

In this letter a new signals coding framework is introduced. Based on IMF properties four strategies are proposed which have shown promising results. Based on EMD these codings are data driven approaches. Further, these methods are computationally simple and no pre or post processing is needed. The proposed framework is not limited to audio or ECG signals, but can be extended to large classes of signals such as EEG, evoked potentials or EMG signals. As future work, we plan to extend the proposed framework to images.

References

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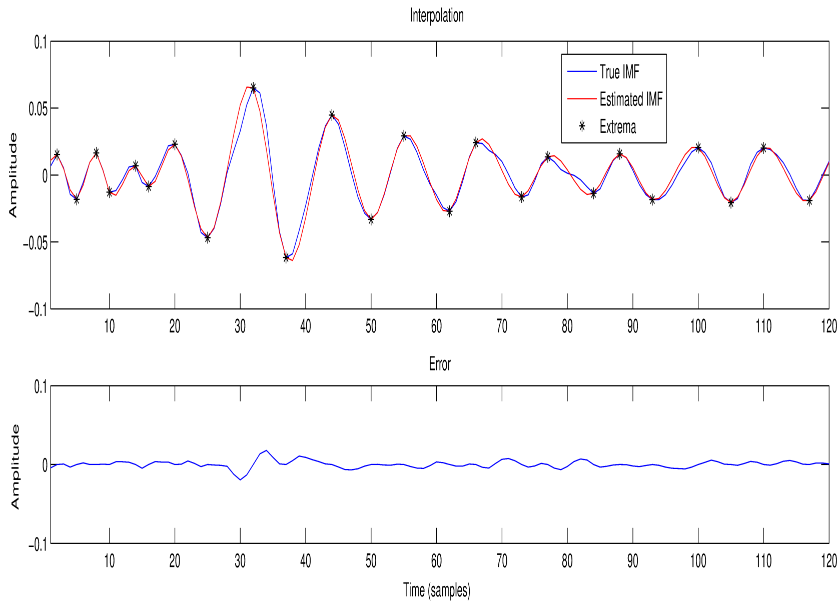

Figure 1: Original IMF and estimated version by spline interpolation.

Table 1: Number of extrema per IMF of ECG signal.

IMF	1	2	3	4	5	6	7	8
Extrema	140	45	20	14	9	6	4	2

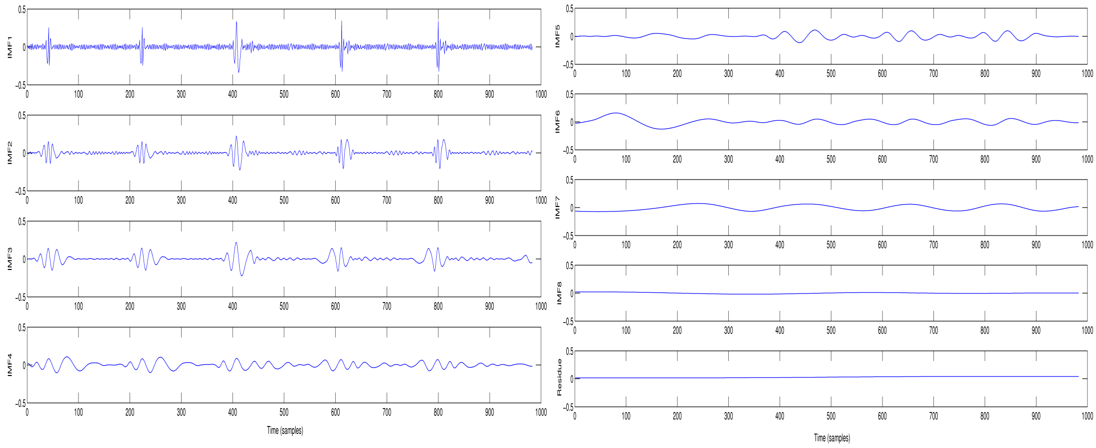

Figure 2: Decomposition of ECG signal by EMD.

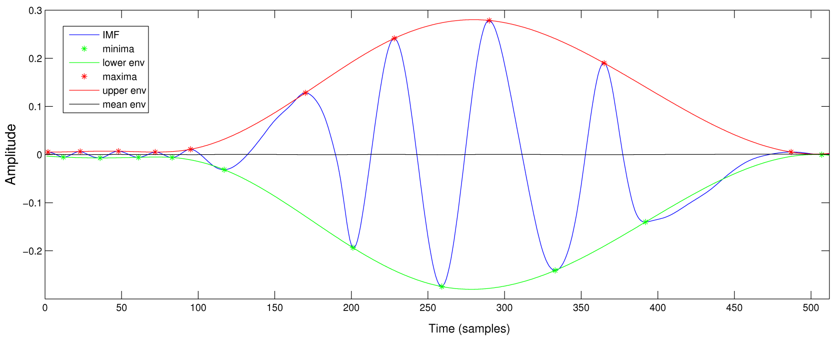

Figure 3: Envelopes of an IMF.

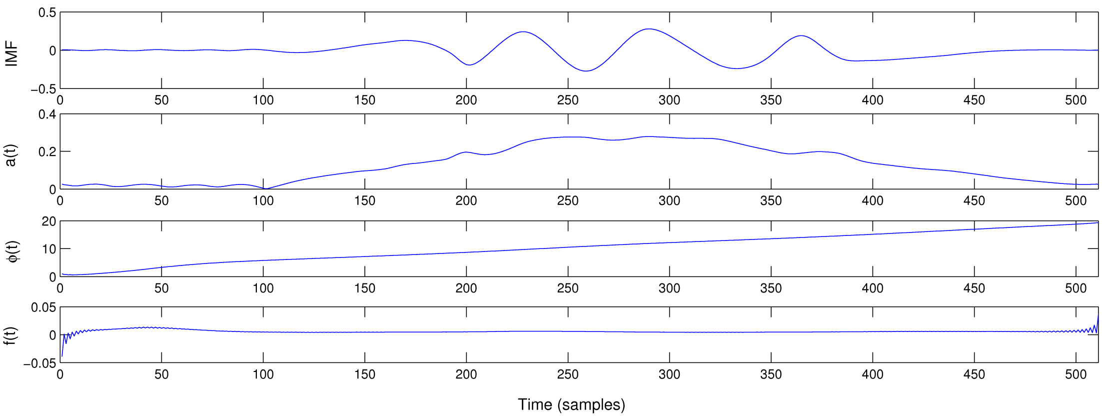

Figure 4: IA, IP and IF of an IMF.

Table 2: Offset values of IMFs extracted from of ECG frame

IMF	1	2	3	4	5
α	-0.0001	-0.0026	-0.0008	-0.0011	0.0014

Table 3: PAC of IF of IMF 5 extracted from ECG signal (Fig. 2).

PAC	0.79	0.55	0.32	0.19	0.063	0.029	0.029	0.029	0.029
L	1	2	3	4	5	6	7	8	9

Table 4: Order of AR model of IF components.

IMF	1	2	3	4	5	6	7	8
L	13	9	8	8	6	6	5	5

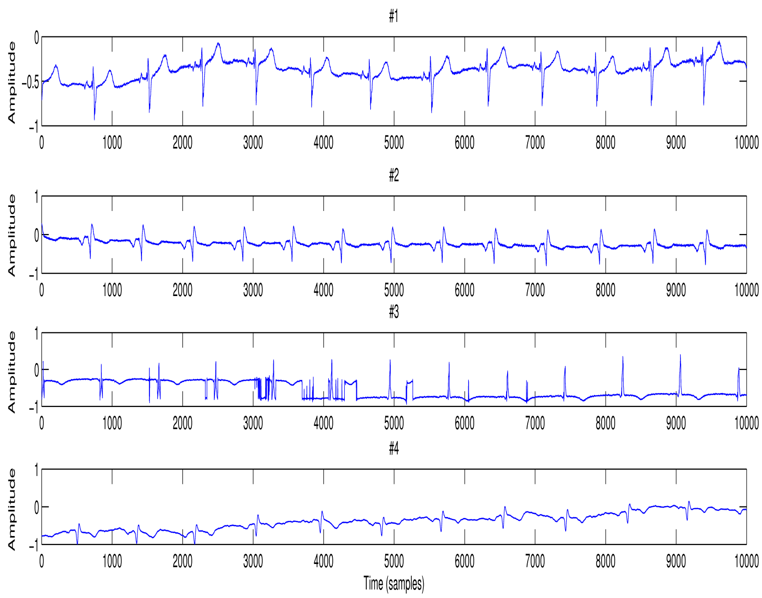

Figure 5: Original ECG signals (records 1,2,3 and 4).

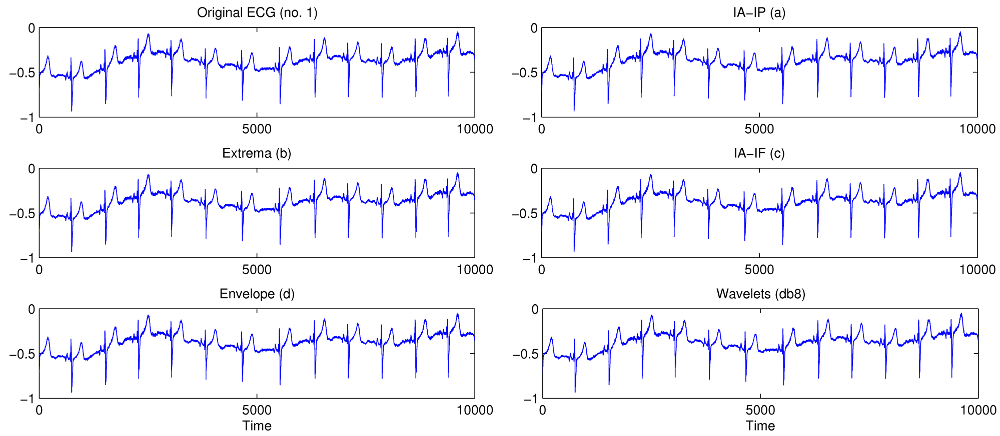

Figure 6: Original and reconstruced ECG signal (record no. 1) by Extrema, Envelope, IA-IP, IA-IF and Wavelets (8db) codings.

Table 5: Compression results of ECG signals (records 1, 2, 3, and 4) by Extrema, Envelope, IA-IP, IA-IF and Wavelet codings.

	Measure	#1	#2	#3	#4
	CR	31:1	22:1	35:1	39:1
Extrema	PRD%	10	**8.3**	4.5	6.3
	CR	**44:1**	30:1	52:1	**59:1**
Envelope	PRD%	10.2	9.4	**3.4**	**6.1**
	CR	34:1	26:1	39:1	42:1
IA-IP	PRD%	9.7	9.8	6.5	7
	CR	39:1	**32:1**	**55:1**	51:1
IA-IF	PRD%	**9.4**	9.7	5.9	6.5
	CR	32:1	20:1	39:1	38:1
Wavelets	PRD%	10.2	9.9	5.4	6.2