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# Integrating damping and non-linearities in a vibration design process

**Etienne Balmes**$^{1,2}$

[1] Arts et Metiers ParisTech/PIMM, 151 Bld de l'Hopital, 75013 Paris

[2] SDTools, 44 Rue Vergniaud, 75013 Paris, balmes@sdtools.com

*Abstract: Classical vibration design uses modes and transfer functions generated with the superposition principle to allow the verification of design objectives. If redesign is needed, one optimizes mass and stiffness in order to modify the transfer until the specification is met. Integrating damping and non-linearities in the optimization of detailed industrial models is however still considered a major difficulty, even though the physical mechanisms are well known. Approaches to handle viscoelastic damping and time domain modal damping are thus discussed. Distributed non-linearities, such as contact and friction, are becoming accessible to transient simulation, but lead to responses where modes are no longer defined. It is however illustrated that operational deflection shapes, associated with a singular value decomposition of the response, give similar information. Finally, a fundamental aspect of non-linear vibration simulation is the volume of output and the associated numerical cost. Model reduction is a key ingredient of practical approaches and a perspective on related issues is given.*

***Keywords: damping, non-linearity, model reduction, vibration design***

## NOMENCLATURE

M = mass matrix
K = stiffness matrix
$\{q\}$ = vector of Degree of Freedom (DOF)
$[T]$ = reduction basis (space dependent vectors)
Z = dynamic stiffness matrix
F = applied load

$E_i$ = complex modulus of $i^{th}$ viscoelastic material

**Greek Symbols**

$\omega$ = frequency
$\alpha(T)$ = viscoelastic reduced frequency shift
$[\Phi]$ = matrix of real (elastic) modes

$\omega_j$ = mode frequency
$\zeta_j$ = modal damping ratio
$\alpha_k, \beta_k$ = Rayleigh parameters

**Subscripts**

j = index of mode, singular value, ...
k = index of parameter $p$
e = elastic
v = viscoelastic

## INTRODUCTION

Vibration design is often critical for reasons of safety (flutter, seismic response, high cycle fatigue, ...), performance or comfort (acoustic, squeal, ...). As shown in figure 1, classical vibration design is very closely linked to the notion of transfer function. One specifies an input that characterizes excitation in terms of spatial and frequency/time content. One then describes the relation between inputs and response in the form of computed or measured transfers. Under the assumption of linearity, the superposition principle is used to obtain responses which are checked against specifications. If redesign is needed, one optimizes properties in order to modify the transfer until the specification is met. This standard process has many constraints and limitations

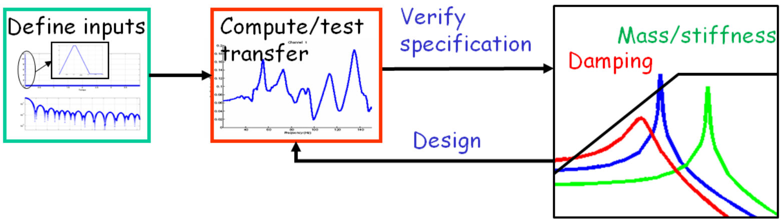

**Figure 1 – Classical vibration design process.**

- The definition of loads is often a problem. Getting from a measured load to a specification load opens the door to overdesign but is necessary to demonstrate the verification of objectives. In the case of multiple random sources, the combination of excitations poses questions as to specific realizations and statistical analysis of responses. Tools such as spectral radii are seldom used. When loads are associated with component coupling as discussed by [Corus, 2010], it is often difficult to measure loads and justify that they are not specific to a given design.

- While mass and stiffness design processes are relatively well mastered, integration of damping is rarely considered, in great part because predictive modeling of damping is often an issue. Advances in this area will be the focus of the first section.
- If the system is non-linear, transfers do not exist and principle of superposition, which drastically reduces the cost of linear system computations, is lost. A safe process is the statistical analysis of many simulations or tests, and model reduction is a key ingredient needed to make the associated cost acceptable. Extracting information from the time responses is another objective that will be discussed.
- Objective functions are generally evaluated at the system level as shown in Figure 2. Redesign of the component based on the system response is often quite costly. Response surface methodologies are not necessarily suited for system behavior showing many modes. Specifically tailored model reduction methods will thus be shown to be of interest.

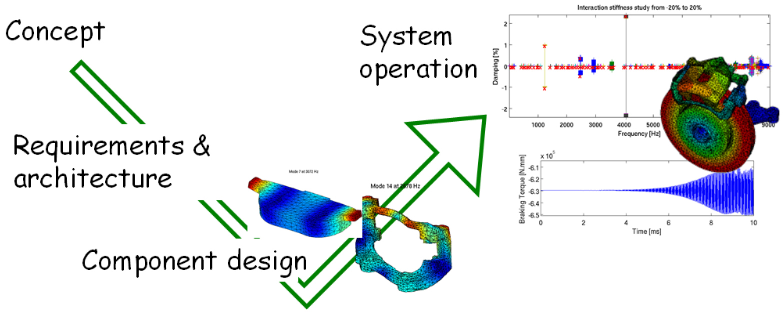

**Figure 2 – V design cycle. Objectives (modes, transients, ...) are evaluated at the system level while design is done at component level.**

## Damping modeling

Physical mechanisms leading to dissipation are well known

- Viscoelasticity corresponds to the case where stresses depend on the history of strains rather than their instant value, see [Nashif et al., 1985]. Delays associated with this behavior is a source of dissipation of many materials and polymers in particular. The highest levels of damping are found for materials close to their glass transition temperature. In damping applications, one thus almost always has a significant dependence of moduli to temperature and frequency. For low amplitudes this dependence is accounted for using frequency/temperature nomograms. For higher loads (suspension blocs, ...), non-linear viscoelastic effects (Paine, Mullins, ...) must be accounted for.
- Friction describes tangential loads occurring when contacting surfaces slide with respect to each other. One can distinguish physically simple but locally detailed models (Signorini/Coulomb, ...) and locally rich models (Dahl, ...) which are often used at the macroscopic level. These mechanisms are non-linear and will be discussed in the next section.
- Fluid viscosity, piezoelectricity, thermo-elasticity, Eddie currents, micromechanical behavior (dislocations, microscopic plastic deformations, ...) are other sources of dissipation that have also been used in vibration damping devices.

All these effects can be modeled and documented cases of good test/analysis correlation exist. The main difficulty limiting their use is their integration into models, with enough detail to allow precise optimization of components, and low enough cost for realistic evaluation.

Taking the case of viscoelastic damping which is commonly used in many applications (see for example the review given in [Rao, 2003]), the behavior is well described by a complex modulus that depends on frequency and temperature

$$E(\omega,T)=E(\omega\alpha(T))=E'(\omega\alpha(T))(1+i\eta(\omega\alpha(T))) \tag{1}$$

where the temperature/frequency equivalence property, states that the dependence on $\omega$ and $T$ is related to a single reduced frequency $\omega\alpha(T)$. $E'$ the real part of the modulus is called the storage modulus and corresponds to the elastic behavior. $\eta$ the ratio between the real and imaginary parts, is called the loss factor and gives a measure of the energy dissipated over one cycle. Integrating such behavior into full order model leads to forced frequency responses of the form

$$[Z(\omega,T)]_{(N\times N)}\{q(\omega,T)\}_{(N)}=[F(\omega)]_{(N)} \tag{2}$$

where the number $N$ of DOF $q$ typically needs to be large to allow a correct representation of geometrical detail of the viscoelastic treatments, and the dynamic stiffness $Z$ can be decomposed as a sum of constant matrices

$$[Z(E_i,\omega)] = \left[-M\omega^2 + K_e + iK_{ei} + i\omega C + \textstyle\sum_k E_k(\omega,T,\sigma_0)\frac{K_{vk}(E_0)}{E_0}\right] \tag{3}$$

For realistic model sizes, in the $10^5$ to $10^7$ DOF range, classical direct frequency response leads to prohibitive computation costs. On the optimistic basis of 1 mn per frequency point for an industrial model, one cannot envision a study with 100 frequencies and 500 design points (300 days). The results shown in figure 3a are thus only accessible through model reduction methods discussed in particular in [Balmes et al., 2006]. The case shown, also illustrates the non-trivial evolution of responses with viscoelastic design parameters. Performing optimization, thus typically requires many detailed evaluations.

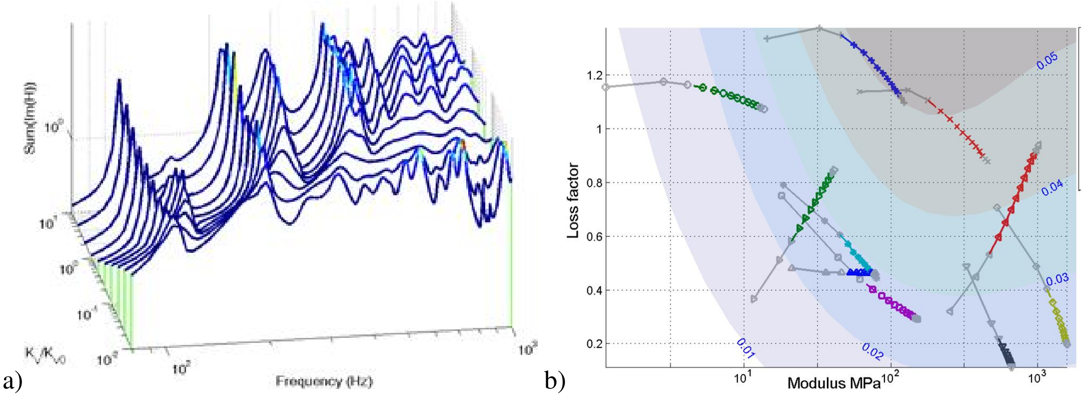

**Figure 3 – Left: evolution of a transfer function a s a function of a design stiffness (damped windshield). Right : iso-performance curve in the modulus/loss factor plane, with material laws for a multiple possible materials.**

Figure 3b, gives another illustration of the need to perform parametric analysis. For the case of an isolated mode, the figure shows iso-damping curves within the modulus/loss factor plane. Within this plane, material selection will define complex moduli, which are shown as colored curves. For each material, temperature changes will correspond to a change of modulus along the curve, thus leading to a performance change. It should be noted that a change of thickness will typically modify the target modulus. For example a shear layer has a stiffness that is inversely proportional to thickness. Thus dividing the thickness by two will double the equivalent stiffness. In the figure, the high loss blue material could thus be used at a lower thickness and thus become the optimal solution.

A major difficulty of the complex modulus approach is that it does not have a time domain equivalent and is thus not compatible with non-linearities that need to be described in the time domain. Typical practice is the use of global or piece-wise Rayleigh models which assume a viscous damping matrix that is a linear combination of mass and stiffness. Using the coordinate system associated with elastic normal modes $\{\phi_j\}$ leads the Modal Strain Energy [Rogers et al., 1981] approximation where the modal damping ratio is given by

$$2\zeta_j\omega_j = \sum_k \alpha_k\left(\phi_j^T[M_k]\phi_j\right) + \beta_e\left(\phi_j^T[K_k]\phi_j\right) \tag{4}$$

with Rayleigh parameters $\alpha_k$ and $\beta_k$ that need to be adjusted by some form of optimization. For global Rayleigh damping, the Rayleigh parameters are constand for all elements and one has a modal damping ratio given by

$$\zeta_j = \alpha\frac{1}{\omega_j} + \beta\omega_j \tag{5}$$

with a mass contribution decreasing with frequency and a stiffness contribution varying linearly with frequency, as shown in figure 4a. The reality is more often a distribution of damping ratio within a band that does not increase with frequency. Rayleigh damping is thus fundamentally only relevant for fairly narrow frequency bands. In piece-wise Rayleigh, multiple linear lines can be created with the damping of each mode depending on the energy fraction in the associated component.

In the modal basis, the traditional practice is to use modal damping where the viscous damping matrix is diagonal of the form $2\zeta_j\omega_j$, where the damping ratio $\zeta_j$ are derived from test or based on design rules. Going back to the physical basis the modal diagonal matrix becomes full and unusable for transient analysis. [Bianchi et al., 2010] proposed to use the mass orthogonality condition, from which one can derive $[\Phi]^{-1} = [\Phi]^T[M]$ and $[\Phi^T]^{-1} = [M][\Phi]$ to obtain a closed form solution of the modal damping matrix in the physical domain

$$[C] = [M\Phi]_{N\times N_m}\left[{}^{\backslash}2\zeta_j\omega_j{}_{\backslash}\right]_{N_m\times N_m}[M\Phi]^T_{N_m\times N} \tag{6}$$

While computing $[C]$ is not acceptable, computing the product $[C]\{u\}$ at each time step as a series of three matrix vector products is quite acceptable. Figure 4, illustrates how this can generate a flat modal damping ratio for a low frequency band and resume with the traditionnal Rayleigh behavior only in the higher range. The difference on associated transfer functions clearly illustrates the major impact of this change.

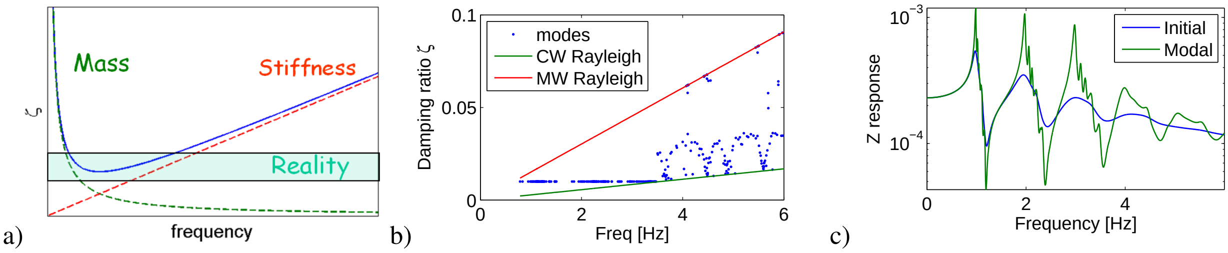

**Figure 4 – a) Global Rayleigh, b) Piece-wise Rayleigh with low frequency modal damping, c) Transfers with and without modal damping [Bianchi et al., 2010]**

## NON-LINEARITIES AND FEATURE EXTRACTION

When considering transient non-linear responses, multiple applications come to mind. Rapid transients (crash, press forming, ...), one then rarely talks about vibration because the characteristics of the system undergo major changes and the total type is typically relatively small compared to the time characteristics of modes of interest.

Multi-body simulations (vehicle dynamics, engine dynamics, global dynamics of rotating machinery, ...) get closer to vibration problems. The need to represent component flexibility is however typically limited (the first few flexible modes of a rotating body) and the representation of interfaces is generally simplified. For example, one may consider a fairly complex hydrodynamic behavior in a bearing but the flexibility of the bearing will generally be ignored. The number of Degree of Freedom in the model is most often below the thousand and distributed non-linearities are rarely treated.

A last class of non-linear problems, which will be the focus of the discussion made here, considers vibration problems without geometric non-linearity but with distributed non-linearities (non-linear contact, friction, squeeze film, ...). Sample applications are the study of brake squeal by [Vermot des Roches et al., 2012], assembled joints by [Gaul et al., 1994], systems with gaps by [Thénint et al., 2012]. The distributed nature of the non-linearity implies that detailed models, model reduction, and adjustments of time integration solvers are often needed, as discussed for example by [Vermot Des Roches, 2010] and [Thénint, 2011]. The state of the art is that such adjustments can be made so that parametric studies based on time transients are becoming accessible for a fairly wide range of problems.

When dealing with linear vibrations, mode shapes and frequencies are typically exploited to orient design. One thus identifies specific resonances in a response and orients the redesign to modify their properties. Extracting shape information from non-linear responses is thus a key question.

A first classical form of feature extraction is the case of non-linear response with coarse frequency content. Non-linear harmonic responses and non-linear modes, see [Kerschen et al., 2006, Sinou, 2008], are thus studied by introducing solvers that can make use of the coarse content. Examples of such solvers are harmonic balance, or MAN [Cochelin et al., 2007]. These approaches are not always relevant (for example if the excitation is broadband) and typically require the implementation of specialized solvers that are not easily manipulated. It thus is quite interesting to obtain similar shape extraction results for standard time response transients. The notion of Operational Deflection Shapes, often used to extract shapes from experiments, can often be useful.

For a nearly periodic response, one can thus extract a block of responses $Q_{ij}$ with DOFs as rows and time steps as columns. The Singular Value Decomposition (SVD)

$$[Q_{ij}]_{N\times Nt} = \sum_j \{U_j\}_{N\times 1}\sigma_j\{V_j\}^T_{1\times Nt} \tag{7}$$

with possible mass or strain energy weighting, then gives, as illustrated in figure 5, left singular vectors $U_i$ that correspond to shapes, singular values that give an amplitude and right singular vectors $V_i^T$ that correspond to a time dependence.

The case of a gap non-linearity under broadband excitation treated by [Thénint, 2011] provides another illustration non-linear feature extraction. In this case, [Thénint et al., 2012] showed that non-linear excitation amplitude and linear variable contact damping/stiffness led to very similar trends in the evolution of Power Spectral Densities (PSD, shown in figure 6). Furthermore, the evolution of first singular vector associated with the main peak in the PSD is clearly similar to that of the first model of the equivalent linear model.

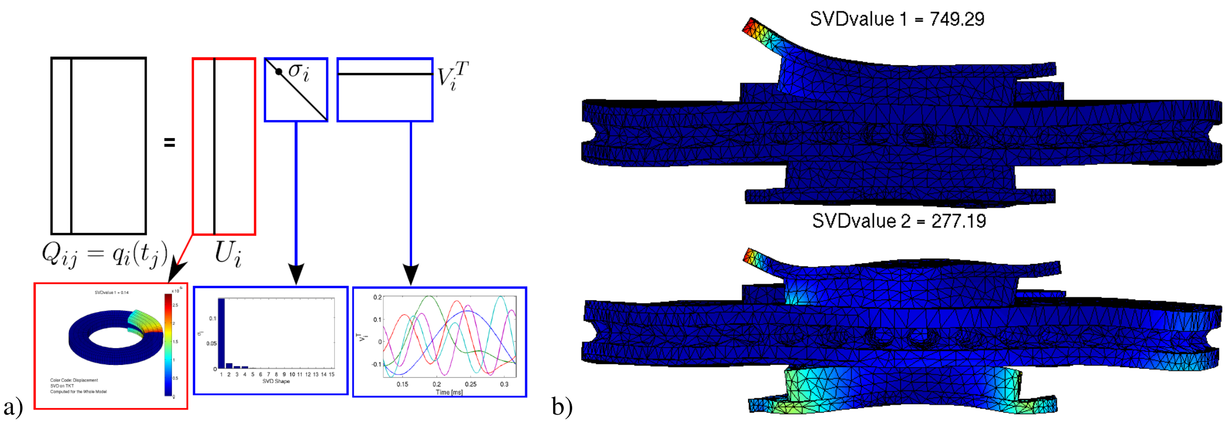

**Figure 5 – SVD of a non-linear squeal cycle. a) principle, b) sample shape for a realistic model**

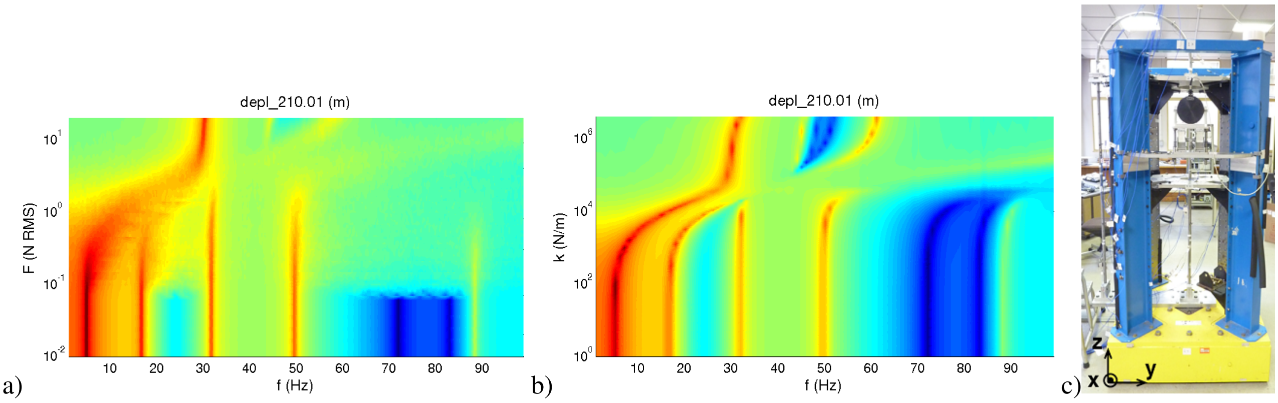

**Figure 6 – Comparison of (a) non-linear PSD as a function of load amplitude and (b) linear PSD as a function of contact stiffness for testbed (c), see [Thénint et al., 2012]**

## PARAMETRIC STUDIES, LONG TRANSIENTS, AND MODEL REDUCTION

When running simulations for problems that require optimization for vibration performance, it is often critical to represent the geometry in significant detail to allow proper reproduction of the modes and associated loads in the areas that are being designed. Model sizes in the 0,1 to 10 million DOF range are thus the norm. Design implies the comparison of multiple configurations. Running tens to thousands of configuration is thus a clear objective. Furthermore, the number of cases of interest tends to grow as a power of the number of parameters. For frequency responses thousands of points are typically needed, for time simulations the number of steps is typically multiple orders of magnitude larger.

Taking a simple case of a 1e6 DOF model, with 100e3 time steps and 5 design points, storing results in double precision requires close to 4 TB. A fundamental issue is thus to optimize feature extraction to reduce the pure volume of data but also possibly to allow the introduction of optimized solvers.

An interesting branch of research calls this issue the *curse of dimensionality* (see [González et al., 2010]). The configuration parameters are then considered as additional dimensions and solutions are searched using variable separation

$$\{u(x,y,z,\omega,p_1,...p_k)\} = \sum_{j=1}^{N} F_1^j(x) \times \ldots \times F_k^j(p_k) \tag{8}$$

leading to Proper Orthogonal Decomposition (POD) or Proper Generalized Decomposition (PGD). The relation with the *a posteriori* decomposition (7) of full responses should be noted.

For vibration problems, one is however interested in spectra, so that some responses need to be known over the full time length, and shapes, so that some responses need to be known over the full spatial detail of the FEM model. Applying general purpose variable separation on space and time/frequency is thus difficult.

Classical Component Mode Synthesis (CMS) methods are however well established methods that perform variable separation on space and time/frequency coordinates. Assuming fixed basis reduction [Balmes, 1996], considers a similar reduction

$$\{u(x,y,z,,\omega,p_1,...p_k)\} = \sum_{j=1}^{N} [T(x,y,z)]\{q_R(\omega,p_1,...p_k)\} \tag{9}$$

and transient or frequency domain resolution of reduced equations of the form

$$\left[T^T[Z(\omega,p_1,...p_k)]T\right]\{q_R\} = [T]^T\{F(\omega,p_1,...p_k)\} \tag{10}$$

When dealing with modal frequencies, curve veering is a critical issue illustrated in figure 7 for the sensitivity of an unstable brake squeal mode to the modulus of the disk. The transition from stable to unstable is a behavior that is typically difficult to represent using smooth representations as the PGD (8). In the CMS, this is not a difficulty because a reduced model (10) is solved. A good way to phrase the difference is to say that PGD generates meta models of the solution, while CMS generates a low cost meta-model of equations to be solved.

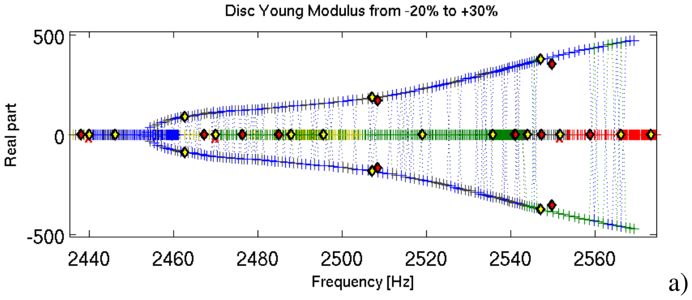

**Figure 7 – Evolution of complex pole with a disk parameter, [Vermot Des Roches, 2010]**

A critical aspect of the costs associated with CMS methods is the fact that the memory needed to store the subspace $T$ can be significantly lowered by considering subspaces for multiple components as illustrated in figure 8a. This principle is the basis of multi-frontal sparse solvers used by all finite element software and Automated Multi-Level Substructuring (AMLS, see [Gao et al., 2007]) which is a generalization to dynamics.

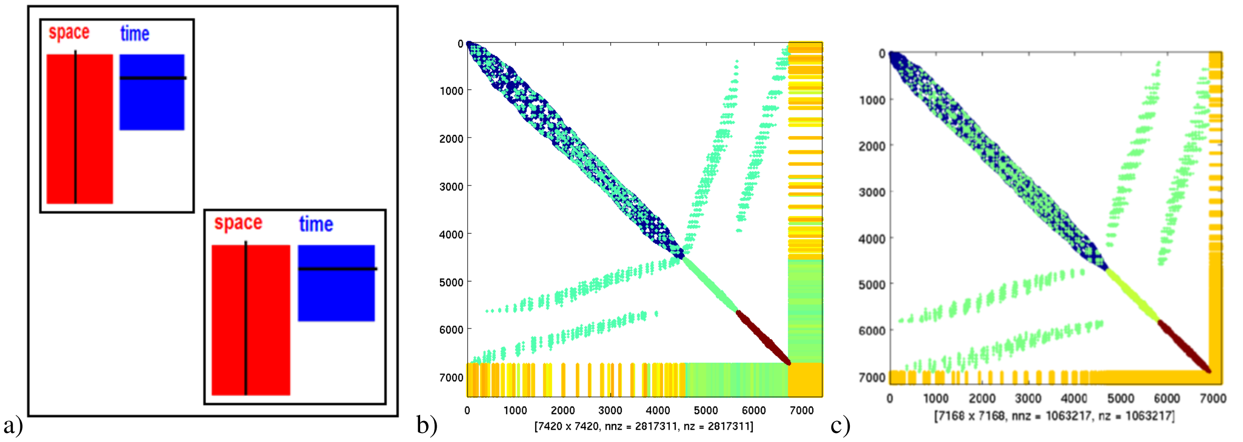

**Figure 8 – a) Decomposition topology for multiple components. b) Stiffness matrix topology for a Craig Bampton reduction. c) Matrix topolology for CMT.**

When CMS methods were developed in the early 80's the need was to lower computational costs and the base proposition was that the reduction subspace for each component should be independent of information on other components. When the number of DOFs at the interface between two components becomes large, the associated matrix topology (illustrated in 8b for a brake model) tends to present large bands of non-zero terms which can drastically limit the interest. [Vermot des Roches et al., 2010] proposed to use traces of system modes as reduction bases. This approach is thus particularly well suited for applications, such as brake squeal, where transient analysis needs to be performed with detailed description of non-linearities. One keeps DOFs $q_i$ close to the non-linearity explicitly (for example the pad shown right in figure 9), and reduces other DOFs $q_c$ (the disk and associated components) by assuming a linear combination of the trace of assembled system modes

$$\{q\} \approx \begin{bmatrix} [\Phi_{Total}]_c & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} q_{Rc} \\ q_i \end{Bmatrix} \tag{11}$$

Figure 8c clearly shows that this significantly lowers the interface band, while preserving exact computations for linear computations in the frequency domain.

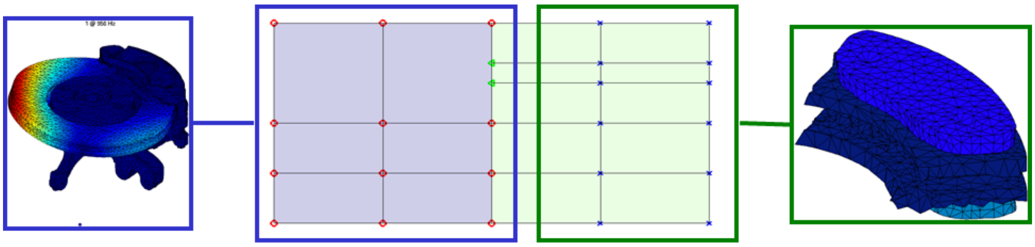

**Figure 9 – Decomposition of DOFs in reduced and kept areas.**

For design studies it is often desirable to be able to analyze the impact of components integrated within a full system model (see figure 2). A simple generalization of the reduction proposed above is to build for each component a basis combining free component modes and the trace of assembled system modes

$$T_C = \left[\ [\Phi_{C,Free}] \quad [\Phi_{Total}]_C\ \right]_{Orth} \tag{12}$$

This reduction, detailed in [Vermot des Roches et al., 2010], provides the clear advantage of using modal amplitudes of components as explicit degrees of freedom. Changing component mode frequencies or damping ratio then becomes a simple modification of the stiffness or viscous damping matrices.

## CONCLUSION

The combination of physically representative models of viscoelastic damping and contact/friction models with appropriate model reduction methods was shown to open the way for accurate and yet accessible numerical simulations of these phenomena. These first steps pave the way for numerically oriented design processes, which will require the choice of features considered for the optimization and processing of enormous volumes of results to orient design. Rethinking computations flows in this light, rather than in the traditional validation process, where only a few computations are used, is thus seen as an interesting challenge for the coming years.

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