A game-theoretical analysis of the efficient disposal of the industrial waste

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Abstract

We define the waste disposal game where the players are local governments. Players are located dispersedly. Each player is endowed with one unit of the industrial waste and has to decide the place where his waste is disposed. We analyze the players' decisions in the equilibrium in three cases, that is, the present Japanese waste-end tax system, the discrimination waste-end tax system, and the auction. We can obtain that the first case does not always lead to the efficient waste disposal and that the second case and the third case lead to the efficient waste disposal.

Keywords: Industrial Waste Disposal, Taxation, Auction, Nash Equilibirium, Efficiency

JEL classification: C72; Q58

1 Introduction

We define the waste disposal game where the players are local governments. The players are located dispersedly. In the game, each player is endowed with the industrial waste to be disposed and the facilities which has the distinct disposal cost. Each player has to decide the place where he dispose his waste. We would like to clarify the relation between the players' decisions in the equilibrium and the efficiency of the resulting disposal. We analyze the relation in three cases.

The first one is the present Japanese waste-end tax system. Each local government sets the undifferentiated waste-end tax rate. Typically, local government i set t_i to the other governments' disposal to i. That is, each player whose waste is disposed by i has to pay the cost to transfer the waste to i and the tax t_i. Indeed, many local governments adopt the same tax rate in Japan.[1] In this case, we can obtain the possibility of the inefficiency of the equilibrium.

Second, we focus on the discrimination waste-end tax system. In this case, each player is allowed to set distinct tax rates to the other players. Typically, player i can set the tax rate t_{ij} to player j. In this case, the equilibrium realizes the efficiency of the disposal.

Third, we consider the players' decision making of the disposal place in an auction. In this case, each player i can execute a second price auction. The other players bid the price at which they are willing to dispose player i's industrial waste. We can obtain that this case also leads to the efficiency of the disposal at the equilibrium.

In the literature, Shapley and Shubik (1969) and Nakayama (2005) deal with the garbage game. In the garbage game, each player has garbage which he must dump someone's yard. They focus on the solution concept of the cooperative games. In their papas, neither disposal cost nor waste-end tax are not considered. Kaneko et al. (2010) discusses the present Japanese waste-end tax system, gives a theoretical model which include the waste-end tax, and executes an experiment in the laboratory to demonstrate the result of the model. In this paper, we follow and develop Kaneko et al. (2010). Using the model in Kaneko et al. (2010), this paper investigates the non-cooperative decision making of the disposal place in the three cases above.

The rest of the paper is composed as follows. The model is defined in Section 2. The results are summarized in Section 3. Section 4 shows the concluding remarks and the future problems.

2 The Model

In this section, we introduce the notations and define the basic model of the waste disposal game. Then, we apply the game to three cases. That is, the present Japanese waste-end tax system, the discrimination waste-end tax system, and the auction.

Let $N = \{1, 2, ..., n\}$ be a set of players, which is interpreted as the set of local governments. We denote typical players as i, j, k, and so on. Each player is given the amount of the industrial waste to dispose which is denoted by e_i. Player i has the facility which can dispose 1 unit of the waste at

[1] In fact, most of the local government in Japan adopt 1,000yen per ton (Kaneko et al., 2010).

average cost c_i. Players are located dispersedly. If player i dispose the waste of player j, the given transfer cost d_{ij} is necessary. The transfer costs reflects the distances among the players. Hence, we assume the following properties.

$$d_{ij} = d_{ji}, \quad \forall i, j \in N$$

$$d_{ii} = 0, \quad \forall i \in N.$$

The situation (player, cost, and distance) is summarized in Figure 1.

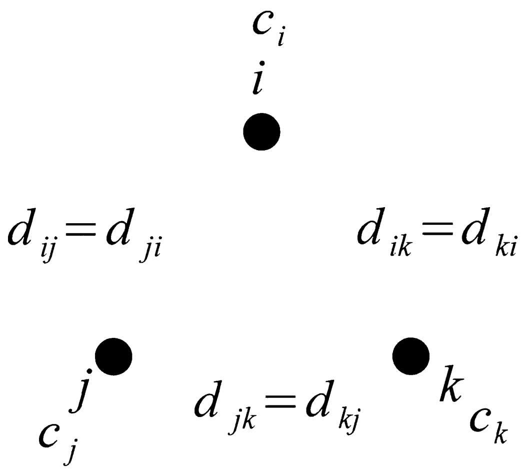

Figure 1: player, cost, and distance

The strategy of each player is to decide the place where to transfer his waste. A strategy of player i is denoted by $x_i = (x_{i1}, x_{i2}, ..., x_{in})$ such that $\sum_{j \in N} x_{ij} = e_i$. holds x_{ij} can be interpreted to expresses the amount of the industrial waste brought from i and disposed by j. Player i's strategy is shown by Figure 2. We denote $(x_1, x_2, ..., x_n)$ as x.

For simplicity, we use the following assumption.

$$e_i = 1, \forall i \in N.$$

This assumption does not affect the decision making of the players below. This is because only the cost and tax rate for one unit does matters. And we regard x_i as the player who disposes i's waste if there is no confusion.

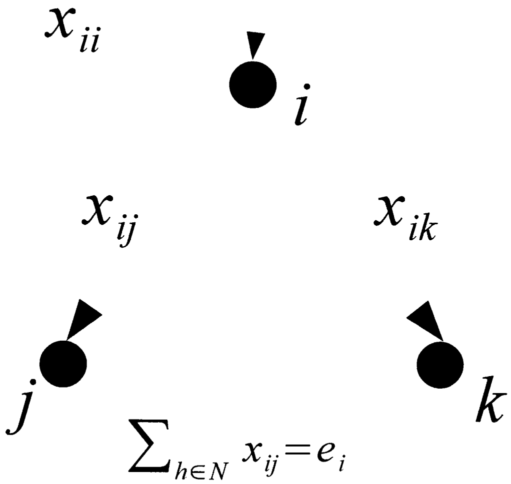

Figure 2: the strategy of player i

2.1 The waste disposal game with tax

This subsection defines the waste disposal game with tax. The present Japanese waste-end tax system is introduced in subsubsection 2.1.1. The discrimination tax system is considered in subsubsection 2.1.2.

In this game, each player can induce the waste-end tax rate before the decision of the disposal place. Player i's tax rate is denoted by $t_i = (t_{i1}, t_{i2}, ..., t_{in})$. t_{ij} is the tax rate player i induces to the industrial waste brought from player j. We assume that t_{ii} is zero for all $i \in N$. We denote $(t_1, t_2), ..., t_n$ as t. We can consider the game as a 2 stage game.

In these settings, each player's payoff can be defined as follows.

Definition 1 (Payoff).

$$u_i(t, x) = \sum_{j \neq i}(t_{ij} - c_i)x_{ji} - \sum_{j \neq i}(t_{ji} + d_{ij})x_{x_{ij}} - c_i x_{ii}, \qquad \forall i, j \in N. \tag{1}$$

Assumption 1 (Tie breaking rule).

If some strategies about the disposal place bring the same payoff to player i, he chooses the place which has the smaller disposal cost (despite of the transfer cost) among them.

Since the waste disposal game with tax is defined as a 2-stage game, we use the subgame perfect

Nash equilibrium (t^*, x^*) as a solution concept of the game. To examine the relationship between the result at the equilibrium and the efficiency, we need to define the efficiency concept.

Definition 2 (Efficiency). The efficient disposal x^{**} is defined by the following equation.

$$x^{**} = \arg\max_{x} \sum_{i \in N} u_i(t, x). \tag{2}$$

On the definition of the efficiency, only the disposal place (x) is essential. Since the tax paid by one player is the reward of another player. Therefore, the tax is not essential to the efficiency.

We can describe the two cases by setting the constraint on the taxation.

2.1.1 The waste disposal game with the present Japanese waste-end tax system

Definition 3 (The present Japanese waste-end tax system). The present- waste-end Japanese tax can be characterized by the following equation.

$$t_{ij} = t_{ik} = t_i \quad \forall i, j, k \in N. \tag{3}$$

More restricted taxation can be characterized by the following equation.

Definition 4 (Unique taxation).

$$t_{ij} = t_{kl} = t \quad \forall i, j, k, l \in N. \tag{4}$$

Indeed, in Japan, almost all local governments induce the same tax rate 1,000yen per ton.

2.1.2 The waste disposal game with the discrimination tax system

In this case, each player is allowed to induce distinct tax rate to different players. (No restriction is required on the basic model.)

2.2 The waste disposal game with auction

In this game, The disposal place and the reward price to the disposer is decided by a second price auction.

Without loss of generality, we focus on the case that the object is the industrial waste of player i. Each player (including i) bids the reward he want if he disposes the object. Based on the second price auction rule, the lowest bidder should be a winner. He disposes the object and he gets the reward as much as the second lowest bid.

3 The results

In this section we can obtain the results: The present Japanese waste-end tax system may lead the inefficiency at the equilibrium. In the case that players can use discriminate tax system or the auction, the equilibrium always realizes the efficiency.

3.1 The waste disposal game with tax

3.1.1 The present Japanese waste-end tax system

Proposition 1 (Kaneko et al., 2010). *In the waste disposal game with the present Japanese waste-end tax system, The equilibrium does not always realize the efficient disposal.*

Proof. The following example shows a counterexample.

Example 1.

Let $N = \{1, 2, 3\}$, $d_{12} = d_{23} = d_{13} = 3$, $c_1 = 1, c_2 = 5, c_3 = 9$. Then, in the equilibrium, the players induces the waste-end tax as follows.

$$t_1 = 5, t_2 = 5, t_3 > 0. \tag{5}$$

Hence, the players choose the disposal place as

$$x_1 = 1, x_2 = 2, and x_3 = 1. \tag{6}$$

In this equilibrium, $\sum_{i \in N} u_i(t, x) = 3 - 5 - 8 = -10$. However, if $x_1 = x_2 = x_3 = 1$ holds, then $\sum_{i \in N} u_i(t, x) = 3 - 4 - 8 = -9$. Therefore, we can obtain that this equilibrium leads to an inefficient disposal. In the equilibrium, player 1 who has the lowest disposal cost aims to get higher tax from player 3 who has the highest disposal cost even if player 1 can not get any tax from player 2. Thus, the waste from player 2 is not disposed efficiently. Therefore, the equilibrium fails to achieve the efficiency.

Figure. 3 summarizes this example.

□

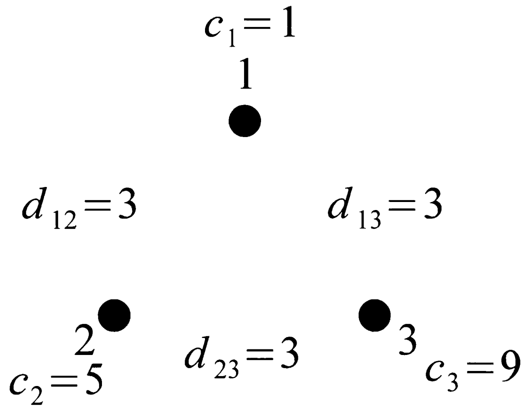

Figure 3: example

Remark 1. In the waste disposal game with unique tax (such as $t_{ij} = t_{ik} = t$ for all $i, j, k \in N$), the efficient disposal can not be always achieved.

Proof. Consider $t_1 = t_2 = t_3 = 5$ in Example 1. □

3.1.2 The discrimination tax

Proposition 2. *In the waste disposal game with the discrimination waste-end tax, the equilibrium leads to the efficient disposal.*

Proof. Without loss of generality, we focus on player i's waste. Define $i^1 = \arg\min_{j \in N} c_j + d_{ij}$ and $i^2 = \arg\min_{k \neq i^1} c_k + d_{ik}$. Then, at the equilibrium, player i^1 induces the tax rate $c_{i^2} + d_{i^2 i} - d_{i^1 i}$ and player i dumps his waste in i^1.

Thus, the waste from i is disposed at the lowest cost. Since i is arbitrary, all the waste are disposed at the lowest cost at the equilibrium. Therefore, the equilibrium can achieve the efficiency in this case. □

The merit of this case is the efficiency. However, in this case, the players' strategy choice is complex. They have to decide n types of the tax rate (Hence, basically, $n \times n$ types of the tax rate are needed.) and it depend on the knowledge of the other players' disposal costs and transfer costs.

3.2 The waste disposal game with auction

In this subsection we use the second price auction and can derive the same result as the discrimination tax case.

Proposition 3. *In the waste disposal game with the auction, an Nash equilibrium can lead the efficient disposal.*

Proof. Without loss of generality, we focus on the case player i's waste is the object. And we denote player j's bid for player i's waste as b_{ji}. By the well known property of the second price auction, each player has a weekly dominant strategy to bid the true cost to dispose the object. That is, $b_{ji} = c_j + d_{ji}$ for all $j \in N$.

As argued before, the amount of the reward does not effect on the efficiency because the payment from i is the revenue of j. The essential element on the efficiency is the disposer of the object. In the equilibrium, all players bid the true cost and the lowest bidder disposes the object. Hence, the efficient disposal is achieved. □

Besides the efficiency, the merit of the second price auction is the simplicity of the strategy. Each player can choose weakly dominant strategy even if he does not know any other players' disposal cost.

However, to achieve the efficient, basically n auctions and $n \times n$ bids are needed. On the execution, the complexity of the second price auction rule maybe matter.

4 Concluding Remarks

We can conclude that (unlike the present Japanese tax-end system,) the waste disposal game with the discrimination tax and with the auction lead to the efficiency. However, in these cases, the (equilibrium) strategies are complex. We had better suggest the more simple system which leads to the efficient waste disposal. Hence, as future problem, we would like to consider two things.

One is to connect our model to NIMBY (not in my back yard) problems such as Sakai (2012). If it is possible, we can make use of the results in the studies of NIMBY problems. The other is to execute the empirical test and understand the behaviors of the players to keep balance between the efficiency and the simplicity of the system to suggest.

By accumulating the researches in the literature, we would like to suggest the appropriate way to dispose the wastes and to cover the cost of the disposal. It would contribute to solve the existing problems such as the radioactive waste from Fukushima Daiichi Nuclear Power Plant in Japan.

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