OPTIMAL INCENTIVE CONTRACTS AND THE PRINCIPAL-AGENT PROBLEM

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Contents

1. Introduction

The principal-agent relationship is a relationship in which one party agrees with another to carry out some type of action on his or her behalf. The former party is called the principal, and the latter is called the agent. We could take, as an example, the relationship between the employer and employee. Considering such a relationship is concerned with delegated decision-making. If the principal has complete information about the agent's decisions and their consequences, or if there is no divergence of interests between them, there is no problem in a delegated decision-making relationship. The principal-agent problem arises in situations in which these conditions do not apply.

Moral hazard often occurs in principal-agent relationships when the principal can not observe the behavior of the agent after the contract has been signed, or at least, the principal cannot maintain total control of the action, and where there is some potential divergence of interests between them. For example, in the employer-employee relationship, it is impossible for the employer to observe completely the employee's effort in most real situations. The employer can only infer the employee's effort from his performance ex-post. Moreover, the employee's interests can be in conflict with those of the employer because a cost for one is revenue for the other[1]. Consequently, we have to design the contract as the means by which the employer and employee can be made compatible and moral hazard can be avoided.

The remedies that are suggested by these two conditions (an ex-post informational asymmetry and the conflict of interests) are explicit monitoring of the employee's effort and the use of incentive contract. Monitoring the actions of an employee may make it possible to prevent inappropriate behavior before it occurs, but it may be, in some situations, too expensive to be worthwhile, or it may be impossible to observe the efforts. When the employer can observe outcomes even if monitoring is not cost-effective and the employee's efforts are unobservable, the employer can provide incentives to encourage appropriate behavior of the employee through rewarding favorable outcome. However, very rare cases may occur in which there is a perfect correlation between unobservable efforts and resulting outcomes[2], so the employee has to decide his behavior under conditions of uncertainty. A decision under conditions of uncertainty is risky because there are a number of possible outcomes. Hence, the incentive problem of motivating the employee to act on behalf of the employer has two important aspects: what risks the employee takes, and how hard the employee works.

The purpose of this paper is to consider what strategies are available for the principal to induce high effort levels from the agent. We will focus our discussion on the employer-employee relationship, particularly on the issues surrounding incentive pay.

In Section 2, we first set out a precise formation of the decision-making situation in the principal-agent relationship, and examine the risk attitude of the participants. Secondly, the optimal risk sharing is considered. In Section 3, we discuss the linear compensation contract and derive the optimal choice of parameters. In Section 4, we consider a long-term contract in which the information about the employee's effort is modified by past performance. Finally, we make

1) There are three elements of the conflict of interests between the employer and employee. First, the employer is interested in the outcome, whereas the employee is not directly responsible for this aspect. Secondly, the employer is not directly interested in effort, but the employee is keenly conscious of his effort because it is costly to him. Finally, it is considered that greater effort would more likely lead to a better outcome.

2) For example, a salesperson's outcome depends not only on his effort but also on other uncontrollable factors, such as the price, advertizing, and market conditions.

some comments on the limitation of incentive contract.

2. Delegated Decision-making

The employment relationship is, in general, embodied a form of contract, in which the obligations of each are specified. In particular, the contract signed by both parties stipulates the payments that the employer makes to the employee. We assume that the employer always designs the contract, and then it is offered to the employee. After having considered the terms of the contract, the employee must decide whether or not to sign it. The employee will accept the contract only if the utility obtained from it is greater than the utility obtained from not signing [3].

2.1. Observability of Effort

The employee chooses the level of effort, which is costly to him and which mainly, but not completely determines the output of a production process. The employer receives the output and he pays the wage to the employee under the contract. Output and the payment to the employee are measured in monetary terms. The employer cannot directly observe the employee's effort and can only observe the performance ex-post. Suppose that the performance of business activities is:

$$p = z + y, \tag{2.1}$$

where p is the firm's profit, z is an indicator of effort and y is an observable random variable, such as the industry trend. The indicator of effort, z is divided into two parts:

$$z = e + x, \tag{2.2}$$

where e is the employee's effort and x is a random variable, which is unobservable by the participants. Note that the employer cannot separately observe e and x, but can observe only their sum, z. The same level of observed z is created by many different combinations of e and x. Hence, the performance of business activities is:

$$p = e + x + y. \tag{2.3}$$

For simplicity of keeping our discussion, assume that the two random variables x and y are each

3) We exclude the case of bilateral bargaining that agent may be make a counter offer to the principal.

adjusted to have mean zero. Consequently, the profit can be assumed to depend on the employee's effort:

$$p = p(e). \tag{2.4}$$

Next, the employer receives the output of the employee's effort, that is, the firm's profit, and pays the wage to the employee. The employer's utility function is:

$$U_P = U_P(\, p(e) - w\,). \tag{2.5}$$

On the other hand, the employee receives the wage and offers an effort, which implies some cost to him. We assume that the employee obtains utility from his wage, while greater effort means greater disutility for him. The employee's utility function is:

$$U_A = U_A(w) - C(e), \tag{2.6}$$

additively separable in the components wage, w and effort, e. Describing the employee's preferences by an additively separable function implies that his risk attitude does not vary with the effort he supplies.

2.2. Risk Attitudes

When the participants face a risky decision-making, we must examine how they not only react to return but also risk. The way of dealing with uncertainty is to introduce the statistical concept of probability into a theory of choice. The return and risk are defined as the expected value, and the variance of possible outcome, which can be measured financially.

Risk preferences are expressed by the utility function introducing the concept of expected utility. For the simplicity of analysis, we make two assumptions about the utility function. First, the utility function is assumed to be at least twice differentiable at all income levels, which implies continuity of the utility function. The first derivative of the utility function (the marginal utility of income) is always positive; $U' > 0$ because of non-satiation assumption [4]. The second assumption concerns the risk attitude surrounding uncertain incomes. Now suppose that the employee's income only consists of his wage and all the wage values which appears in them lie on the interval between the greatest wage value, w_{max} and the smallest wage value, w_{min}, with probabilities, p and

4) It implies that more is preferred to less. See Gravelle-Rees (1992, pp.68-78.) for the assumptions that give the desired properties to the individual's preference ordering.

1–p, respectively, where $1 \geq p \geq 0$. The expected value of wage, $\overline{w}$ is then:

$$E\,[w] = p\,w_{max} + (1 - p)\,w_{min} = \overline{w}. \tag{2.7}$$

Figure 1 illustrates three kinds of utility function, whose shapes are drawn by strictly concave (U" < 0), linear (U" = 0) and strictly convex (U" > 0). Corresponding to the wage level, w_{max} and w_{min} are the utility values, $U_A(w_{max})$ and $U_A(w_{min})$, at points A and B respectively. The expected utility of $\overline{w}$ is the weighted average of the utility of risky wages, w_{max} and w_{min} at point C, then:

$$E\,[U_A(\overline{w})] = p\,[\,U_A(w_{max})\,] + (1 - p)\,[\,U_A(w_{min})\,]. \tag{2.8}$$

If the employee will prefer receiving a certain wage of $\overline{w}$ to receiving a random wage with expected value of $\overline{w}$, the utility of the certain wage of $\overline{w}$ would be greater than the expected utility of $\overline{w}$:

$$U_A(\overline{w}) > E\,[U_A(\overline{w})] \tag{2.9}$$

This kind of risk attitude is called as risk averse and depicted by a strict concave utility function as shown in **Figure** 1–(1).

Conversely, if the employee will prefer receiving a risky wage with expected value of $\overline{w}$ to receiving a certain wage of $\overline{w}$, the utility of the certain wage of $\overline{w}$ would be smaller than the expected utility of $\overline{w}$:

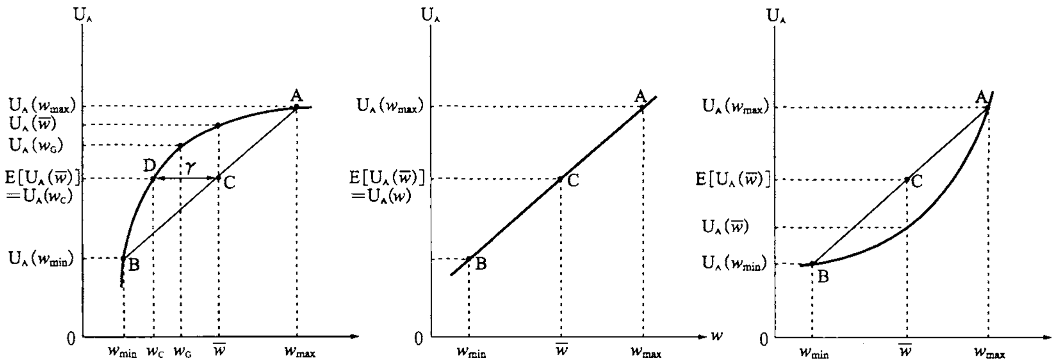
$$U_A(\overline{w}) < E\,[U_A(\overline{w})]\,. \tag{2.10}$$

This kind of risk attitude is called as risk loving and depicted by a strict convex utility function as shown in **Figure** 1–(3).

Finally, if the employee will be equivalent receiving a certain wage of $\overline{w}$ to receiving a random wage with expected value of $\overline{w}$, the utility of the certain wage of $\overline{w}$ would be equal to the expected utility of $\overline{w}$:

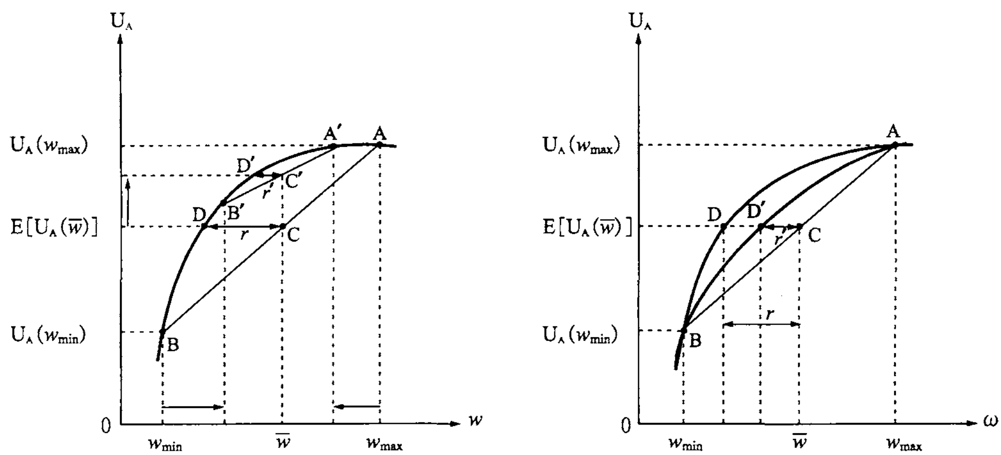
$$U_A(\overline{w}) = E\,[U_A(\overline{w})]\,. \tag{2.11}$$

This kind of risk attitude is called as risk neutral and depicted by a linear utility function as shown in **Figure** 1–(2).



(1) Risk averse (2) Risk neutral (3) Risk loving

Figure 1 : Risk Attitudes



(1) Variability of possible wage (2) Concavity of utility function

Figure 2 : The Risk Premium

Certainty equivalent wage, w_c is defined as a certain wage whose utility is equivalent to the expected utility of $\overline{w}$, that is:

$$U_A(w_c) = E\left[U_A(\overline{w})\right] . \qquad (2.12)$$

Therefore, certainty equivalent wage, w_c of the risk-averter is smaller than the expected wage $\overline{w}$:

$$w_c < \overline{w}, \qquad (2.13)$$

where w_c is found at a point D in **Figure** 1–(1). The distance between $\overline{w}$ and w_c shows how much of the certain wage the employee would be willing to give up rather than face risky wages, w_{max} and w_{min}. It expresses the cost of risk, which is called the risk premium, r.

The risk premium depends on both the variability of possible wage and the agent's degree of risk aversion. Larger variability of possible wage leads to a greater risk premium (**Figure** 2–(1)). Larger concavity of utility function leads to a greater risk premium (**Figure** 2–(2)).

The variability of possible wage is measured by Var (w), which is the squared deviation of a random wage, w from its expected wage, $\overline{w}$. The degree of risk aversion is reflected in a form of concavity of utility function. A measure of the concavity of U_A at the wage, w is shown in:

$$A(\overline{w}) = -U''_A(w)/U'_A(w), \tag{2.14}$$

where $U''_A(w)$ is the rate at which marginal utility of wage changes with wage; $U''_A(w) < 0$. $A(\overline{w})$ is a parameter of the employee's preferences, which is called the coefficient of absolute risk aversion for gambles with mean, $\overline{w}$. Through mathematical approximation by using Taylor's theorem, the risk premium[5] is equal to one-half times the coefficient of absolute risk aversion times the variance of the wage:

$$r = \frac{1}{2} A(\overline{w}) \, Var(w). \tag{2.15}$$

If the employee is risk averse ($U''_A < 0$), $A(\overline{w})$ will be positive. Larger $A(\overline{w})$ or larger Var (w) lead to a larger risk premium. Thus, certainty equivalent income, w_c is rewritten by:

$$w_c = \overline{w} - r = \overline{w} - \frac{1}{2} A(\overline{w}) \, Var(w). \tag{2.16}$$

If the employee is risk neutral ($U''_A = 0$), $A(\overline{w})$ will be zero. The certainty equivalent wage w_c is found at a point C in Figure 1–(2), where $w_c = \overline{w}$. Therefore, there is no cost of risk and the risk premium is zero.

2.3. Optimal Risk Sharing Ignoring Incentives

It is generally assumed that the employee is risk averse with respect to his income and the employer is risk neutral. This is because the employee's income mainly consists of his wage, while the employer who hires a large number of employees is able to offset the risk by pooling. In a case where risks are to be shared between a risk-neutral employer with many employees and a

5) For a more discussion on a measure of risk aversion and the risk premium, see Pratt (1964) and mathematical appendix in Milgrom and Roberts (1992), pp.246-247.

risk-averse employee, the optimal risk sharing is to shift all the risk of the employee onto the employer, who suffers no cost in bearing the risk.

Suppose that there are only two economic situations, boom and bust, in which wages and their probabilities are w_{max} and w_{min}, p and (1 − p), respectively. The expected value of wage is expressed in equation (2.7).

If the risk-averse employee is guaranteed receiving a certain wage, w_G in **Figure 1−(1)**, whose amount is above the certainty equivalent wage, w_c and below the expected wage, $\overline{w}$; $w_c < w_G < \overline{w}$, his utility U_A would be better off:

$$U_A(w_c) - C(e) < U_A(w_G) - C(e). \tag{2.17}$$

On the other hand, for the risk-neutral employer, paying the guaranteed wage w_G is more beneficial than random wages with the expected value of wage, $\overline{w}$, since his utility U_P would be also better off:

$$U_P(p(e) - w_c) < U_P(p(e) - w_G). \tag{2.18}$$

However, this contract ignores the incentive problem because it makes the employee's compensation absolutely risk free and unrelated to outcomes. The optimal contract must balance the need for risk sharing against the need to provide incentives as there is a risk-incentive trade-off. There are many types of incentive contracts in which pay is linked to performance for eliciting higher efforts from the employee. In the next section we discuss a pay-for-performance contract.

3. Linear Incentive Scheme

The relatively simple compensation contract that includes the provision of incentives is often found in actual employment contracts. For example, the employee may be paid a constant salary plus a fixed proportion for his performance. We consider here the pay-for-performance contract with linear incentive provision [6].

It is assumed that the payment function is linear and depends on z and y. It is formulated as follows.

$$w = w(z, y) = \alpha + \beta(z + \gamma y) = \alpha + \beta(e + x + \gamma y) \tag{3.1}$$

Compensation consists of a constant amount, α plus a portion varying with the observable elements, z and y. The parameter β determines the intensity of the incentives provided to the employee. The parameter γ shows how much relative weight is given to the information variable y, comparing with the unobservable variable z (= e + x) to determine compensation.

In designing the optimal incentive contract, first we have to define the objective function to be maximized and the constraints under which the contract makes this feasible, then examine the optimal choice of parameters.

The objective function is the total utility of the employer and employee which is expressed by the total certain equivalent income. The employee's certain equivalent income, T_A is the expected wage minus the cost of his effort minus any risk premium:

$$T_A = \overline{w} - C(e) - \frac{1}{2} A(\overline{w})\, Var(w)$$

$$= \alpha + \beta(e + \overline{x} + \overline{y}) - C(e) - \frac{1}{2} A(\overline{w})\, Var[\alpha + \beta(e + x + \gamma y)],$$

where $\overline{x}$ and $\overline{y}$ are the mean levels of x and y and $A(\overline{w})$ is the employee's coefficient of absolute risk aversion. Assuming the sum of $\overline{x}$ and $\overline{y}$ to be zero [7],

$$T_A = \alpha + \beta e - C(e) - \frac{1}{2} A(\overline{w}) \beta^2 Var(x + \gamma y). \tag{3.2}$$

The risk-neutral employer's certain equivalent income, T_P is:

$$T_P = p(e) - w = p(e) - (\alpha + \beta e). \tag{3.3}$$

Hence, the total certain equivalent income, T_T is:

$$T_T = T_A + T_P = p(e) - C(e) - \frac{1}{2} A(\overline{w}) \beta^2 Var(x + \gamma y). \tag{3.4}$$

Next we specify the feasible set in this contract. Although the employee's choice of effort, e will depend on the other parameters (α, β, γ), he will choose the level of e that maximizes his certain equivalent income, T_A. By differentiating equation (3.2) with respect to e and setting that derivative equal to zero, we obtain the feasible condition of the contract:

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6) The model that we discuss here is based on the model of incentive compensation in Milgrom and Roberts (1992), pp.215-231.

7) According to properties of variance, when α, β and e are assumed to be constant, then Var $[\alpha + \beta(e + x + \gamma y)] = \beta^2 Var(x + \gamma y)$. The formula for the variance of two random variables can be seen in Gujarati (1992), pp.41-46.

$$\beta - C'(e) = 0, \text{ equivalently, } \beta = C'(e), \tag{3.5}$$

which is called an incentive constraint.

The employment contract is efficient if the parameters (α, β, γ) take values that maximize the total certain equivalent subject to the incentive constraint,

$$\max\ p(e) - C(e) - \frac{1}{2} A(\overline{w})\beta^2 \mathrm{Var}(x + \gamma y) \qquad \text{s.t. } \beta - C'(e) = 0.$$

Therefore, we must examine how the employee's choice of effort, e will depend on the parameters (α, β, γ).

As far as α is concerned, the total certain equivalent income is not affected by α because it is not included in the objective function of equation (3.4). Hence, the efficiency of the contract does not depend on the value of α. This part of compensation partly satisfies the employee's risk preference (risk averse) because it can partly isolate the employee from risk by guaranteed compensation, α.

3.1. The Intensity of Incentives

The most central part of designing incentive contracts is to determine the optimal value of β, which expresses the intensity of the incentives. If we wish to fix the information weighting parameter γ, then let $V = \mathrm{Var}(x + \gamma y)$. The determinant factor of β can be specified using the Lagrange method.

$$L = p(e) - C(e) - \frac{1}{2} A(\overline{w})\beta^2 V + \lambda[\beta - C'(e)] \tag{3.6}$$

To find the stationary points of L that satisfy the first-order conditions for a maximum,

$$\frac{\partial L}{\partial e} = p'(e) - C'(e) - \lambda C''(e) = 0 \tag{3.7}$$

$$\frac{\partial L}{\partial \beta} = -\beta A(\overline{w})V + \lambda = 0 \tag{3.8}$$

Therefore,

$$\beta A(\overline{w})V = \lambda \tag{3.9}$$

Substituting equation (3.9), (3.5) into equation (3.7),

$$p'(e) - \beta - \beta A(\overline{w})VC''(e) = 0$$

$$\beta = \frac{p'(e)}{1 + A(\overline{w})VC''(e)} \qquad (3.10)$$

Equation (3.10) shows that the optimal value of β depends on four variables: the marginal gain of effort, $p'(e)$, the accuracy of assessing performance, V, the agent's risk aversion, $A(\overline{w})$, and the agent's responsiveness to incentives, $C''(e)$.

The first factor determining the optimal value of β is the profitability of additional effort, $p'(e)$. Since making extra effort is costly to the employee, he will put in the higher level of effort toward his task only if its additional profit is greater than its marginal cost. According to equation (3.10), the optimal intensity, β is proportional to the marginal gain of effort, $p'(e)$ provided the other three factors remain unchanged.

The second factor is the precision of measuring the employee's performance, V. High precision corresponds to low values of the variance, V, strong incentives thus should be used according to equation (3.10). Conversely, when the precision of performance measurement is low, only weak incentives should be used.

To measure performance highly precisely, the variance of $(x + \gamma y)$ must be decreased. The two random variables x and y cause the variability of wage which determines the risk premium. Thus, the optimal value of γ, which determines the relative weight of observable variable, γ should be chosen to minimize the variance of $(x + \gamma y)$.

$$Var(x + \gamma y) = Var(x) + \gamma^2 Var(y) + 2\gamma Cov(x, y). \qquad (3.11)$$

By differentiating equation (3.11) with respect to γ, we get the optimal value of γ:

$$\frac{\partial[Var(x + \gamma y)}{\partial \gamma} = 2\gamma Var(y) + 2 Cov(x, y) = 0.$$

Therefore,

$$\gamma = -\frac{Cov(x, y)}{Var(y)}. \qquad (3.12)$$

If x and y are independent, Cov (x, y) is zero. Thus, γ is optimally chosen to be zero. If x and y are positively related, Cov(x,y) is positive. Thus, γ should be negative. Conversely, if x and y are negatively related, Cov(x,y) is negative. Then, γ should be positive.

The third factor is the risk aversion of the employee, the level of which is expressed by the coefficient of absolute risk aversion, $A(\overline{w})$. A small value of the coefficient means the low cost of risk bearing, provided the variance remain unchanged. In equation (3.10), as $A(\overline{w})$ decreases, β

increases. Hence, a less risk averse employee should to be provided with more intense incentives, and vice versa.

The final factor is the employee's responsiveness to incentives, C"(e), which depends mainly on the ability of the employee's effort level to affect his observed performance. If the employee is working as a small part of a large group, incentives have little effect of eliciting higher effort because his ability to affect their ex-post observable performance is very small. Thus, incentives should be most intense in cases where the most responsible employee is involved in the task.

3.2. Monitoring

Monitoring the employee's action may make it possible to prevent inappropriate behavior before it occurs. This should be considered in relation to the intensity of incentives. That is, if monitoring the employee's action by the employer reduces the level of variance, then it makes sense to choose a high value of β. But monitoring requires devoting the allocation of resources by the employer.

To specify what level of resources should be spent on monitoring, we suppose that the variance of the performance measure can be reduced at cost. This is achieved by denoting the minimum amount of monitoring cost achieving an error variance as low as the variance, V which is consistent with optimal contracts, by M(V). It is supposed here that M(V) is a decreasing function, which implies that a larger V entails lower monitoring costs. In addition, M′(V) is assumed to be increasing, that is, the marginal cost of variance reduction is a rising function.

Rewriting equation (3.4) to include the cost of monitoring:

$$T_T = p(e) - C(e) - \frac{1}{2} A(\overline{w})\beta^2 V - M(V). \tag{3.13}$$

Differentiating of equation (3.13) with respect to V and setting that derivative equal to zero, we obtain the optimal level of monitoring:

$$\frac{\partial T_T}{\partial V} = \frac{1}{2} - A(\overline{w})\beta^2 - M'(V) = 0. \tag{3.14}$$

Therefore,

$$- M'(V) = \frac{1}{2} A(\overline{w})\beta^2. \tag{3.15}$$

Equation (3.15) demonstrates that the marginal cost of reducing V, which is $-$ M′(V), must be equal to $\frac{1}{2}A(\overline{w})\beta^2$ at the efficient condition.

In designing optimal incentive contracts, setting intense incentives and measuring performance are complementary activities. Comparing two situations with the low and high levels of incentive intensity, β, if β is low, the chosen level of V is high, and if β is high, V is low. In other words, when β is reduced, fewer resources are spent on monitoring, and vice versa. Therefore, as β increases, more resources should be spent on monitoring.

3.3. The Equal Compensation Principle

When an employee is conducting several activities as part of his job, the problem of providing incentives becomes complicated. We suppose that the employee is given two tasks (task 1 and task 2) which require the levels of e^1 and e^2. The employer can measure performance of the two tasks by observing the indicators of effort z^1 ($= e^1 + x^1$) and z^2 ($= e^2 + x^2$), where x^1 and x^2 have expected values of $\overline{x^1}$ and $\overline{x^2}$. It is assumed that the wage is paid to the employee according to a linear compensation scheme based on the two indicators of effort:

$$w = \alpha + \beta^1(e^1 + x^1) + \beta^2(e^2 + x^2). \tag{3.16}$$

The employee will choose e_1 and e_2 to maximize his or her certain equivalent income:

$$T_A = \alpha + \beta^1(e^1 + \overline{x}^1) + \beta^2(e^2 + \overline{x}^2) - C(e^1 + e^2) - \frac{1}{2}A(\overline{w})\mathrm{Var}(\beta^1 x^1 + \beta^2 x^2) \tag{3.17}$$

By differentiating equation (3.17) with respect to e^1 and setting that derivative equal to zero,

$$\frac{\partial T_A}{\partial e^1} = \beta^1 - C'(e^1 + e^2) = 0. \tag{3.18}$$

Similarly,

$$\frac{\partial T_A}{\partial e^2} = \beta^2 - C'(e^1 + e^2) = 0. \tag{3.19}$$

Therefore,

$$\beta^1 = \beta^2, \tag{3.20}$$

where β^1 is marginal gain of effort e^1 and β^2 is marginal gain of effort e^2. Therefore, equation (3.20) represents that marginal gain of effort Spending in each activity must be equal, which is called the equal compensation principle.

If the task 1 is totally unmeasurable by the employer, its parameter of incentives, β^1 cannot play any role in eliciting higher level of effort to be spent the task 1. In other words, β^1 may as well as be set to zero. According to this principle, the incentive pay does not work well and the fixed salary is preferred for the employee who has to conduct multiple tasks including unobservable ones. Thus, the equal compensation principle imposes a serious difficulty on the incentive contract.

4. The Long-term Relationship

When the employer-employee relationship is repeated during several periods, we must examine whether repetition will affect the nature of the contract. Such long-term relationships in which the information about the employee's effort is modified by his past performance in each period have positive and negative implications for designing the incentive contract. The positive side is the possibility of acquiring reputations, and the negative side is known as the ratchet effect.

4.1. The Ratchet Effect

Using information about past performance as the base on current standard can reduce the variance in the measurement of the performance of the next period in case that the same random variable operates over the extended periods. The performance standard tends to increase after a period of good performance and decrease after bad performance. This tendency is called the ratchet effect.

Suppose that an employee works for two periods and his effort in each period is denoted by e_1 and e_2, but that the information about the employee's effort used in the contract in the second period is modified by the performance of the first period.

The employer can only observe the employee's performance z_1 (= e_1+x_1) in the first period and z_2 (= e_2 + x_2) in the second period, where random variables in the two periods, x_1 and x_2 are assumed to have equal variances and to have means equal to zero. The employee's incentive compensation in the first period is:

$$w_1 = \alpha_1 + \beta_1 (e_1 + x_1 + \gamma\ y_1). \qquad (4.1)$$

If random variables in each period, x_1 and x_2 are attributed to the same factors, it is appropriate for there to be a positive correlation between x_1 and x_2. The observed performance of the employee's effort in the first period, z_1 can be used to get an estimate $\hat{x}_2$ of x_2, which may be expressed as a function of z_1:

$$\hat{x}_2 = \delta(z_1) = \delta(e_1 + x_1). \tag{4.2}$$

In turn, the estimate $\hat{x}_2$ can be used to get a better estimate of the employee's actual effort e_2 in the second period. Thus, an estimate of the employee's second-period performance z_2 adjusted by the first-period performance z_1 is:

$$\hat{z}_2 = z_2 - \hat{x}_2 = z_2 - \delta(z_1) = e_2 + x_2 - \delta(e_1 + x_1). \tag{4.3}$$

Part of the performance variation is $x_2 - \delta(e_1 + x_1)$. If δ is carefully selected, then $Var(x_2 - \delta(e_1 + x_1))$ is smaller than $Var(x_2)$. This adjusted estimate, $\hat{z}_2$ should be used in the contract because reducing the error around the employee's choices are estimated to increase the total certainty equivalent.

When the employee's second-period pay, w_2 is given by the same kind of function as in the first period, the employee's total compensation over the two periods is:

$$w_1 + w_2 = \alpha_1 + \alpha_2 + \beta_1(e_1 + x_1 + \gamma_1 y_1) + \beta_2(e_2 + x_2 + \gamma_2 y_2). \tag{4.4}$$

If the adjusted employee's second-period pay, w_2^* is determined by the basis on accounting the first-period, z_1, then the employee's total compensation over the two periods is:

$$\begin{aligned} w_1 + w_2^* = {} & \alpha_1 + \alpha_2 + \beta_1(e_1 + x_1 + \gamma_1 y_1) + \\ & \beta_2(e_2 + x_2 - \delta(e_1 + x_1) + \gamma_2 y_2) \\ = {} & \alpha_1 + \alpha_2 + (\beta_1 - \delta\beta_2)(e_1 + x_1) + \beta_2(e_2 + x_2) + \beta_1\gamma_1 y_1 \\ & + \beta_2\gamma_2 y_2. \end{aligned} \tag{4.5}$$

It is important to note that the coefficient of e_1 in equation (4.5) is not β_1 but the smaller amount $(\beta_1 - \delta\beta_2)$.

When the second-period's bonus is given in proportion to the difference between actual performance in the second period, z_2 $(= e_2 + x_2)$ and the plan target, $\delta(e_1 + x_1)$, which is based on the first-period's performance, z_1 $(= e_1 + x_1)$, higher first-period's effort, e_1 increases the plan target in the second period. That is called the ratchet effect, which reduces the compensation accruing in the second period by $\delta\beta_2$. If the employee engages in the job in both periods and foresees this possibility ex-ante, he would refuse to expend higher effort in the first period, since the ratchet effect unfairly penalizes good performance by decreasing the next period's compensation through setting higher performance standard. Therefore, the ratchet effect is a kind of inefficiency in the linear incentive contract because it diminishes performance in each period in the repeated contracts.

4.2. Efficiency Wages

Reputation accumulated by repeated good behavior is highly valuable for the employee because it is very hard to build and maintain, however, it is very easy to lose by only one occurrence of inappropriate behavior. When the employee's effort is difficult to observe, the employer may plausibly use the performance of the employee in the past as an indicator of present or future effort. The idea of reputation makes sense only under conditions of asymmetric information. If the employer effectively introduces reputation of the employee as contract enforcers, the employer can elicit higher effort from the employee without highly monitoring cost, or the use of costly and complicated legal contracts. The point is that the value of reputation has to exceed the gain from shirking for the employee. This is formalized as the efficiency-wage theory [8]. It argues that the employee will be offered rent in order to increase productivity. The effective wage theory does not emphasize that increased productivity leads to higher wages, but higher wages lead to increased productivity.

For simplicity of the analysis, we assume that the employee can choose between only two possible effort levels: minimal effort ($e = 0$) and some fixed positive level of effort ($e > 0$). There are only two states for the employee at any point in time: employed or unemployed. When he is unemployed, he receives unemployment benefits of w_u and he expends minimal effort ($e = 0$). The period of unemployment during the search for a new job is denoted by a probability b per unit time, which will be taken as exogenous. If the employee expends his effort at some fixed positive level ($e > 0$), he receives a wage of w. On the other hand, if he expends his effort at minimal level ($e = 0$), and only if he is caught shirking, he will be fired. The probability of being caught shirking is denoted by q per unit of time.

The employee would decide his effort's level to maximize his discounted utility stream. This involves his comparing the utility from shirking with the utility from not shirking. We denote the

8) The efficiency wage model discussed here is based on Shapiro and Stiglitz (1984).

expected lifetime utility of an employed shirker by V_e^S, the expected lifetime utility of an employed nonshirker by V_e^N, and the expected lifetime utility of an unemployed person by V_u.

The fundamental asset equation [9] for a shirker is given by:

$$r V_e^S = w + (b+q)(V_U - V_e^S), \tag{4.6}$$

where interest rate times asset value equals flow benefits, which is wage, w plus expected capital gains (or losses), that is, the probability of unemployment, (b + q) times unemployment compensation, V_u minus the opportunity cost of gain from shirking, $(b+q)V_e^S$. Equation (4.6) can be rearranged to give:

$$r V_e^S + (b+q)V_e^S = w + (b+q)V_U.$$

Therefore,

$$V_e^S = \frac{w+(b+q)V_u}{r+b+q}. \tag{4.7}$$

While for a nonshirker, the fundamental equation is:

$$r V_e^N = w - e + b(V_U - V_e^N), \tag{4.8}$$

where the annual payment is wage, w minus the positive level of effort, e plus the gain of unemployment, bV_u minus the opportunity cost of gain from nonshirking, bV_e^N. Equation (4.8) can be solved for V_e^N:

$$V_e^N = \frac{(w-e)+bV_u}{r+b}. \tag{4.9}$$

9) We assume that the employee is infinitely lived, and has a pure rate of time preference of r, which is represented by the interest rate. The expected present value of perpetual utility (the expected lifetime utility), Ve is equal to the annual payment P divided by the interest rate r. By using the present value formula:

$$V_e = \frac{P}{1+r} + \frac{P}{(1+r)^2} + \frac{P}{(1+r)^3} + \cdots.$$

Now let P/(1+r) = a and 1/(1+r) = x. Then we have

[1] $V_e = a(1 + x + x^2 + \cdots).$

Multiplying both sides by x, we have

[2] $V_e x = a(x + x^2 + \cdots).$

Substracting [2] from [1] gives us.

$V_e(1-x) = a$

Therefore, substituting for a and x,

$$V_e\left(1-\frac{1}{1+r}\right) = \frac{P}{1+r}$$

Multiplying both side by (1+r) and rearranging gives

[3] $r V_e = P$

See Brealey and Myers (1996), p.38.

The employee will choose not to shirk if $V_e^N \geq V_e^S$. By using equation (4.7) and (4.9), the no-shirking condition can be written as:

$$\frac{(w-e)+bV_u}{r+b} \geq \frac{w+(b+q)V_u}{r+b+q}.$$

Therefore,

$$w \geq rV_U + (r+b+q)\,e/q \equiv \tilde{w}. \qquad (4.10)$$

Equation (4.10) highlights the basic implication of the no-shirking condition. If the employer pays a sufficiently higher wage, w than the critical wage, $\tilde{w}$, then the employee will not shirk. The difference between w and $\tilde{w}$ is called the rent, which is the excess of earnings in the current job over opportunities elsewhere, in order to prevent the employee from shirking. This rent makes the job valuable and makes the prospect of being fired in the future one to be avoided to the employee. The greater the rent, the greater the penalty from being fired because the employee suffers a big income loss. Therefore, a higher wage motivates the employee and leads to increased productivity. Moreover, it reduces labor turnover, enabling the firm to attract more productive labor.

5. Concluding Remarks

In this paper we have discussed the employment contracts using the principal-agent model. In particular, how to avoid moral hazard in delegated decision-making is considered by designing optimal incentive contract. In cases in which the employer is risk neutral and the employee is risk averse, efficient incentive contracts should balance the cost of risk bearing and the incentive gains. The optimal contract is governed by the values of parameters (α, β, γ) which maximize the total certain equivalent income subject to the incentive constraint. To sum up, incentive intensity should be strengthened,

(a) as the marginal gain of effort increases

(b) as the employee's performance starts to be precisely measured

(c) as the employee becomes less risk averse

(d) as the employee becomes more responsible for the performance

When we consider the long-run employment contract, the ratchet effect and reputation have to be taken into account to design an efficient contract.

The incentive contract is effective, but it has limitations as a device to elicit higher employee efforts because the equal compensation principle and the ratchet effect impose serious constraints on the incentive compensation formulas. The existence of reputation shows a preference for an alternative type of contract, such as efficiency wage contract. Efficiency wage contracts would be more effective in eliciting higher effort levels of the employees when monitoring the performance is very difficult in such situations where the equal compensation principle can be operated. It can be considered that a higher wage is a substitute for close monitoring. To conclude, we have to carefully design the optimal employment contract considering the type of job, the employee's risk attitude, number of periods and general circumstances.

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