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International transmission of instability and synchronization of inventory cycles

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1 Introduction

Synchronization of business cycles is one of the hot topics of macroeconomics. There are many works to capture one or some factors to explain the phenomenon in both theoretical and empirical literature. Relationship between business cycle synchronization and trade intensity among G7 countries is empirically analyzed in Stock and Watson (2005). Correlations between two regional economics simulated by using Kaldorian business cycle model with interregional trade under fixed exchange rates are exemplified in Asada, Inaba and Misawa (2001). It is also recognized that not only international trade but also fiscal policies and labor market regulation should be related to business cycle synchronization (Artis, Fidrmuc and Scharler (2008).

These days propagation of international cycles is widely known. However, it should be noted that both the definition of business cycle and that of synchronization are often ambiguous or not uniquely determined in the studies of international business cycles. The purpose of this paper is to analyze the effect of dynamic coupling on peaks and troughs of business cycles in connected countries. We focus on the Metzlerian short-run business cycle, that is, the persistent oscillation of national income generated by the interaction between inventory and sales expectation in a country. In addition, we define phase on Metzlerian business cycle by a linear function of time for simplicity. Synchronization means that the phase difference between countries is fixed in this paper.

It is well known, *e.g.*, as the story of Huygens' clocks, in physics that weakly coupled oscillating systems tend to synchronize. In other words, the phase difference between them converges to zero. It is called mode-locking phenomenon. Selover and Jensen (1999) investigated whether the world business cycle could be

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explained by the same story. The result of the practical study suggests mode-locking phenomenon also may be one of the major factors that generate international business synchronization and the world business cycle. They took so simple oscillator as to avoid the difficulty of verification. Therefore it was difficult to derive many economically meaningful implications from the model. However, in case of a generalization of the van der Pol oscillator equation as a summarized system of the traditional IS-LM model with a nonlinear investment function and an expectations-augmented Phillips curve, similar results are also obtained (Selover, Jensen and Kroll (2005)).

It is interesting that coupling of endogenous cycles can cause synchronization of the cycles even if the coupling intensity is small. In particular, developed countries positively trade each other. Is the similarity of time series of GDP in G7 illustrated in Stock and Watson (2005) the result of mode-locking? To consider this problem, we must construct an appropriate model that represents the length between successive peaks in the time series of national income observed in the real data. Thus we focus on shorter-run business cycle model than Selover, Jensen and Kroll (2005) as mentioned above. We rule out capital depreciation, technical progress, population growth, and so on.

Selover and Jensen became aware of the fact that business cycles between the United States and Canada appear to follow each other very closely by analyzing annual growth rates of industrial production from January in 1985 to December in 1995 (Selover and Jensen (1999)). Figure 1 shows recent time series data of nominal GDP in the United States and Canada. Trend term of each indicator has been eliminated by a linear polynomial.[1] In this figure, we can see the existence of business cycle of about 15-18 month in length. Furthermore the figure implies that the cyclical factor in nominal GDP of Japan or Canada can synchronize temporarily that of the United States. In this paper, we generalize a nonlinear inventory cycle model into multi-country model with international trade to explain such a phenomenon like as a pioneering work of Lorenz (1987).

The organization of the remainder of the paper is as follows. In the next section we review the classical inventory cycle model briefly. In Section 3 we take

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1   This method is valid because the data period is short and any exponential growth is unlikely to be seen within the period. In general, a band-pass filter is useful for the elimination of seasonal fluctuations or longer cyclical components, *e.g.*, investment cycles.
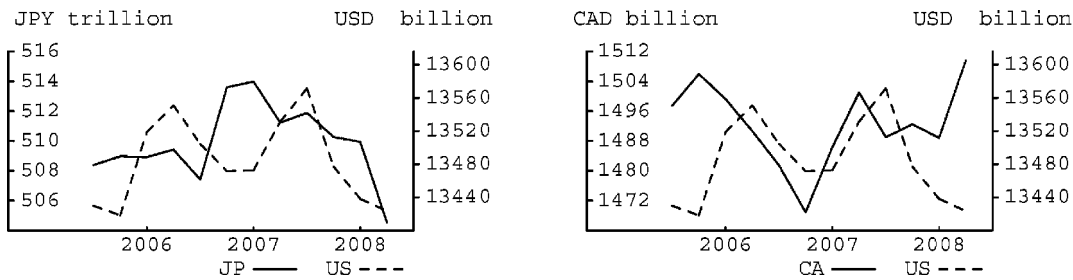
Figure 1: Nominal GDP without trend and seasonal terms, 2005. Q3-2008. Q2
The cyclical components in nominal GDP of Japan (solid line (left)), Canada (solid line (right)) and the United States (dashed line) are plotted. Data before eliminating trend are obtained from *Main Economic Indicators December 2007 and October 2008* (www.oecd.org/publishing).

a nonlinear inventory cycle model proposed by Franke and Lux (1993). We obtain numerical solutions other than a unique fixed point by specifying functions and parameters in the model. In section 4 the effects of international trade on business cycle in each country are examined in various aspects. Final section concludes the paper.

2　Metzler Model

The classical Metzlerian inventory cycle model is a linear dynamical system. Let us review the notations and assumptions of Metzler (1941). Our review begins with assumption of no savings and no capital depreciation. Physical capital stock is fixed. Then net investment means change in the level of inventory. That is,

$$\frac{dV}{dt} = Y - S^a, \tag{1}$$

where V is inventory, S^a denotes actual sales, and Y is output. Income is equal to output, and actual sales S^a depends on income as follows as below.

$$S^a = c_1 + c_2 Y, \ c_1 > 0, \ 0 < c_2 < 1. \tag{2}$$

Positive constant c_1 denotes the intercept of demand function.[2] Its level depends on the unit of variables. Marginal propensity to consume is denoted by c_2. The level of output is determined based on the expectation of sales S^e and adjustment to attain desirable inventory V^d, however, realization of demand always must have priority over the realization of desirable inventory.

2　The intercept is considered to mean the minimum expenditure to survive.

$$Y = S^e + \beta(V^d - V), \quad \beta > 0, \tag{3}$$

where β is a parameter of stock adjustment. As to expectation formation we consider adaptive expectation as follows.

$$\frac{dS^e}{dt} = \sigma(S^a - S^e), \quad \sigma > 0, \tag{4}$$

where σ means a speed of adjustment. To close the model, we assume desired inventory is proportional to sales expectation.

$$V^d = kS^e. \tag{5}$$

Parameter k is fixed, and it stands for the desired inventory-sales ratio. Hence we obtain the classical Metzlerian model as summarized dynamics of inventory and expected sales.

System 1 :

$$\frac{dV}{dt} = -c_1 + (1 - c_2)\{(1 + \beta k) S^e - \beta V\},$$

$$\frac{dS^e}{dt} = \sigma\{c_1 - (1 - c_2 - \beta k c_2) S^e - \beta c_2 V\}. \tag{6}$$

This system has a unique equilibrium in the first orthant. The values of main variables at the equilibrium are

$$V^* = V^{d*} = \frac{kc_1}{1 - c_2}, \quad S^{e*} = S^{a*} = Y^* = \frac{c_1}{1 - c_2}, \tag{7}$$

respectively. It is clear in (7) that parameters c_1 and c_2 determine the position of the steady state. On the other hand, β and σ do not appear there. We can imagine that they are related with the stability of the system. In addition the levels of β and σ would depend on the unit of time variable. It is natural that the value of k becomes smaller over time through improvement of information literacy, we ruled out such a progress so as to concentrate on analyses of short-run dynamics. The Jacobian matrix J of System 1 is

$$J = \begin{pmatrix} -(1 - c_2)\beta & (1 - c_2)(1 + \beta k) \\ -\sigma\beta c_2 & -\sigma(1 - c_2 - \beta k c_2) \end{pmatrix}. \tag{8}$$

Stability condition of the equilibrium is expressed by simultaneous inequalities :

$$\det J = \sigma\beta(1 - c_2) > 0, \tag{9}$$

$$\operatorname{tr} J = -\beta(1 - c_2) - \sigma(1 - c_2 - \beta k c_2) < 0. \tag{10}$$

Since the marginal propensity to consume is between zero and one, the stability is determined by the sign of the trace of J. The stability of the equilibrium mainly depends on the producer who decides based on backward looking expectation. For

example, the equilibrium is stable if the inventory adjustment is sufficiently sluggish.

3 Nonlinear Metzlerian Model

Franke and Lux (1993) introduced flexible stock adjustment function into System 1. It represents more realistic entrepreneur's behavior. According to them let a function $\beta(S^e) > 0$ take the place of the constant β in System 1. It is no wonder that the decision maker of output level has information about the equilibrium value of sales to some extent from his experience. When the expected level of sales is very low and it becomes lower, he may adjust his production more cautiously than usual because he knows sales would recover before long. The similar behavior may be also observed if the expected value of sales is extremely high. Therefore the graph of function $\beta = \beta(S^e)$ has the only peak around S^{e*} and it intercepts with $S^e = 0$ at a point $\beta > 0$. In this paper the function is specialized as follows.[3]

$$\beta(S^e) = \frac{1-c_2}{kc_2} \exp\left(-c_3\left(S^e - \frac{1-c_1}{c_2}\right)^2\right) + c_4, \tag{11}$$

where c_3 and c_4 are positive constants and c_4 is so small that $1-c_2-\beta kc_2 > 0$ is satisfied at $\beta_0 = \beta(0)$ and for sufficient large S^e. We consider intuitively that this function plays the role of a stabilizer. Figure 2 shows the shape of the function for a set of parameters. We call the system with $\beta(S^e)$ System 2. Since $\beta(S^e)$ is always positive, System 2 has the same equilibrium as the previous system.
System 2 :

$$\frac{dV}{dt} = -c_1 + (1-c_2)\{(1+\beta(S^e)k)S^e - \beta(S^e)V\},$$

$$\frac{dS^e}{dt} = \sigma\{c_1 - (1-c_2-\beta(S^e)kc_2)S^e - \beta(S^e)c_2V\}. \tag{12}$$

Numerical simulations[4] find limit cycles for many parameter settings.[5] They have various sizes and periods. Some of the settings are picked up in Table 1 as

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3   The function expresses an ideal case in the closed economy. It is the reason parameters of demand function appear in the right hand side of (11).

4   In this paper, we use Mathematica 4.2 for numerical calculations. Some results are verified by Fortran programs, in which fourth order Runge-Kutta method is adopted for time integrations of the model in double precision.

5   Franke and Lux (1993) discussed the dynamical property of the nonlinear inventory cycle model by sketching a phase diagram, and implied the existence of a closed orbit for economically meaningful conditions applying Poincaré-Bendixson theorem.
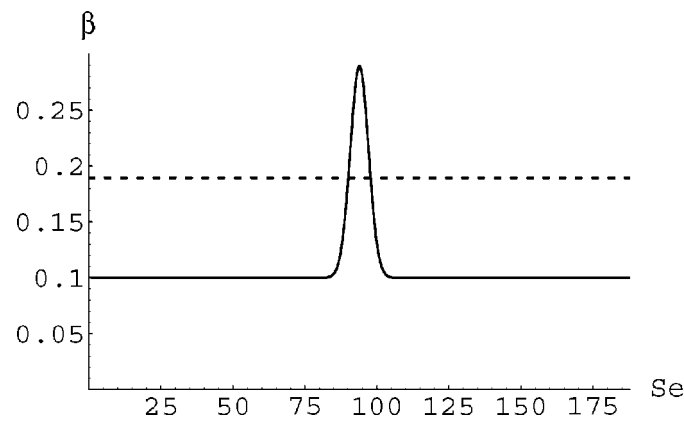
Figure 2: Graph of a specified nonlinear function of stock adjustment

The dashed line is $1-c_2-\beta kc_2=0$. As long as $\beta(S^{e*})$ is below the line, an economy tends to go to the equilibrium.

Table 1: Parameter settings and periods of periodic solutions

	σ	κ	c_1	c_2	c_3	c_4	V^*	S^{e*}	*period*
(a)	0.216	2.519	17.108	0.861	0.047	0.533	26.473	122.727	9.67
(b)	0.623	4.057	52.076	0.607	0.022	0.512	82.449	132.409	4.07
(c)	0.226	19.547	5.546	0.609	0.119	0.498	3.205	14.181	1.24
(d)	0.388	3.732	9.282	0.695	0.051	1.003	11.779	30.389	4.25
(e)	0.505	6.425	28.254	0.724	0.028	0.136	51.608	102.292	5.03
(f)	0.485	3.344	3.009	0.590	0.113	1.106	3.558	7.338	3.37
(g)	0.215	9.865	14.014	0.620	0.058	2.172	7.925	36.892	1.51
(h)	0.468	5.513	42.658	0.543	0.024	1.576	43.669	93.297	2.25
(i)	0.357	5.892	4.543	0.649	0.127	1.157	4.609	12.921	2.81

Averaged values along periodic orbits, which are not described in this table, are almost equal to the levels of corresponding variables at the equilibrium.
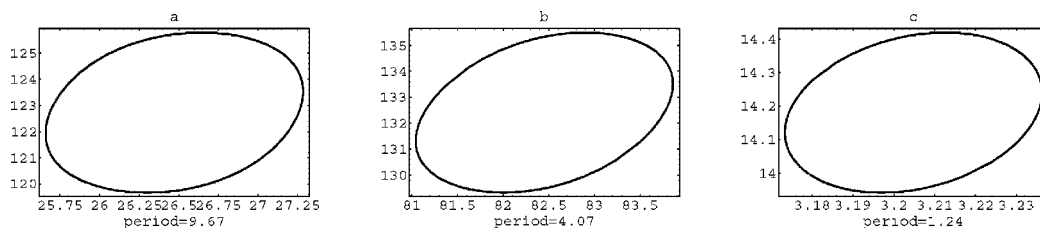

Figure 3: Closed orbits with various sizes and periods

The horizontal axis denotes that of V and the vertical axis is that of S^e in this figure. These orbits are obtained after transition. As to the middle, large, small scale of economies imagine Japan, the United States and Canada, respectively.

representative cases. Limit cycles for settings (a), (b) and (c) in the table are depicted in Figure 3.

4 Coupling Inventory Cycles

4.1 Introduction of international trade

Now we propose an idea to extend the inventory cycle model into multi-country model. We assume that a fixed ratio of expenditure is directed toward goods imported from a foreign country. In other words, we suppose m_{ij}, the degree of dependence of country i upon imports from country j, are constant in our time span. Then we can simply construct three-country international trade model like as Lorenz (1987) by coupling inventory cycles.   The actual sales of goods produced in country i is rewritten as

$$S_i^a = (1 - m_{ij} - m_{ik})(c_{i1} + c_{i2} Y_i) + m_{ji}(c_{j1} + c_{j2} Y_j) + m_{ki}(c_{k1} + c_{k2} Y_k),$$
$$i, j, k = 1, 2, 3; \quad i \neq j, \ j \neq k, \ k \neq i, \tag{13}$$

where the subscript of an marcoeconomic variable denotes country index. The second and the third terms in (13) mean exports from country i to countries j and k, respectively.   The national income of country i is determined by

$$Y_i = S_i^e + \left[\left(\frac{1 - c_{i2}}{k c_{i2}} \right) \exp \left[-c_{i3} \left(S_i^3 - \frac{c_{i1}}{1 - c_{i2}} \right)^2 \right] + c_{i4} \right] (k_i S_i^e - V_i). \tag{14}$$

Hence we obtain the following system.
System 3 :

$$\frac{dV_i}{dt} = Y_i - S_i^e,$$
$$\qquad\qquad i = 1, 2, 3. \tag{15}$$
$$\frac{dS_i^e}{dt} = \sigma_i(S_i^a - S_i^e),$$

The equilibrium of System 3 is different from the above systems. While the inventory adjustment remains expressed by (11) with subscripts, the equilibrium levels of variables changes depending on import shares.[6]   As a result, international trade may stabilize the equilibrium through the process in which the adjustment β becomes sufficiently small at the equilibrium. Otherwise, propagation of instability over countries may occur. Anyway, it is likely that extreme change in trade intensity causes situation producers have not experienced yet.

4.2 Synchronization of oscillations

In this subsection, we study the characteristics of System 3 through numerical

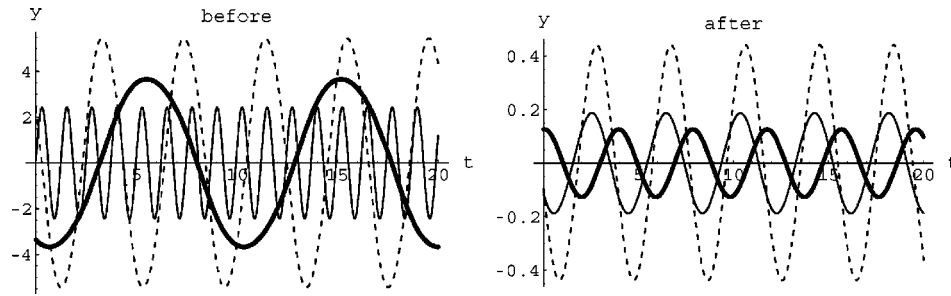---

6   Later, we will confirm it in Figure 9.

Figure 4 :　Time series of GDP in three countries (Case 1)

　　　　$m=0$ (left) ; $m=1$ (right).

　　　　Variable $y=Y-Y^*$ stands for deviation from GDP at the equilibrium. For convenience of comparison, the amplitude of y in each country is artificially adjusted. The setting of parameters of thick curve (country 1) is (a), that of dashed curve (country 2) is (b) and the other (country 3) is （c） in Table 1.
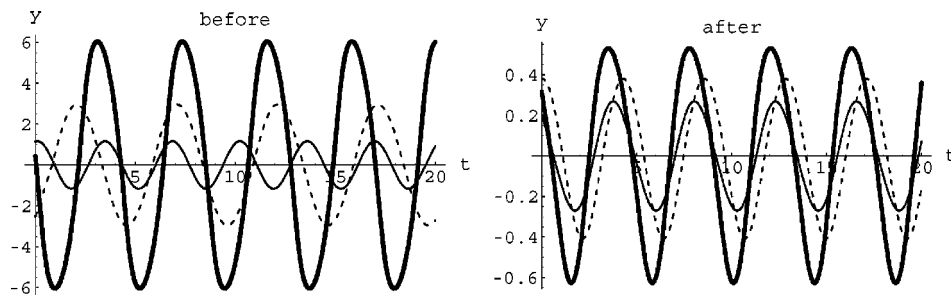

Figure 5 :　Time series of GDP in three countries (Case 2)

　　　　$m=0$ (left) ; $m=1$ (right).

　　　　The setting of parameters of thick curve (country 1) is (d), that of dashed curve (country 2) is (e) and the other (country 3) is (f) in Table 1.

simulations. Figures 4, 5, 6 and 7 exemplify that synchronization can be observed when business cycles with various periods are weakly linked. We suppose country 1, 2 and 3 as Japan, the United States and Canada, respectively. The import ratios in those cases are set as follows as below.

$$m_{12}=0.0155m,\ m_{13}=0.0022m,$$
$$m_{23}=0.0221m,\ m_{21}=0.0115m, \tag{16}$$
$$m_{31}=0.0106m,\ m_{32}=0.1511m,$$

where m is a common parameter among countries, which denotes the trade intensity. Recent import shares in reality are roughly equal to the values determined by (16) when m is unity. Then, time series of national income in three countries are illustrated in Figures 4, 5 and 6. We can see synchronization of all countries in Figures 4 and 5. On the other hand, in Figure 6, the fluctuation of national income
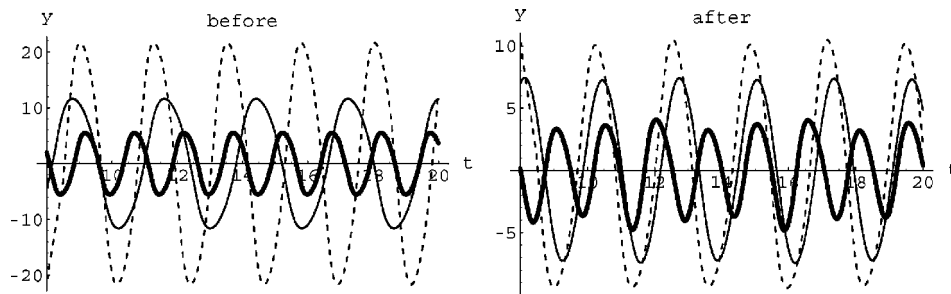
Figure 6 : Time series of GDP in three countries (Case 3)

$m=0$ (left) ; $m=1$ (right).

The setting of parameters of thick curve (country 1) is (g), that of dashed curve (country 2) is (h) and the other (country 3) is (i) in Table 1.
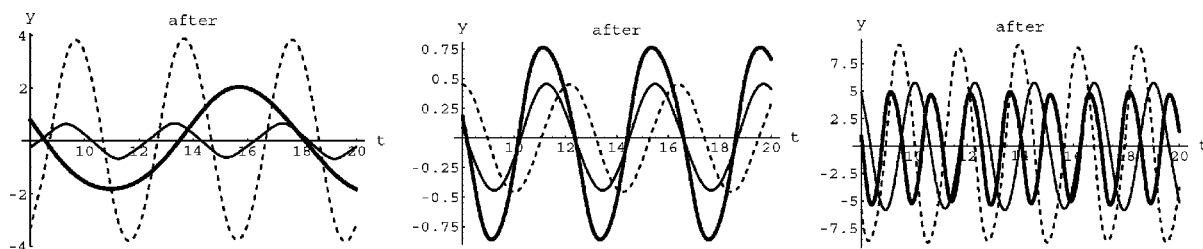

Figure 7 : Time series of GDP in three cases ($m=0.5$)

Case 1 (left), Case 2 (center), Case 3 (right)

The thick line, the dashed line and the solid line represent time series in countries 1, 2 and 3, respectively.

of Japan seems to be independent from the other countries though Canada and the United States synchronize. Under the weaker linkages among countries independent movement in Japan is observed in cases 1 and 3. Depending on marginal propensity to import or other parameters such as the adjustment speed of inventory in each country, economies in two or three countries seem to synchronize. The process toward the synchronization is illustrated in detail in Figure 8. It also shows in the figure that the lengths of successive peaks in national income time series in synchronizing countries are same. Phase of business cycle in a country is defined by a linear function of time satisfying the condition : phase at a local maximum of Y is equal to zero, and it becomes unity at the next local maximum of Y. We notice that the phase difference between Canada and the United States is especially prone to be fixed in Figure 8. The linkage between Canada and the United States is the strongest in our settings. There may be some relationship between these two facts.

Now let us investigate the local stability of the equilibrium when changing the
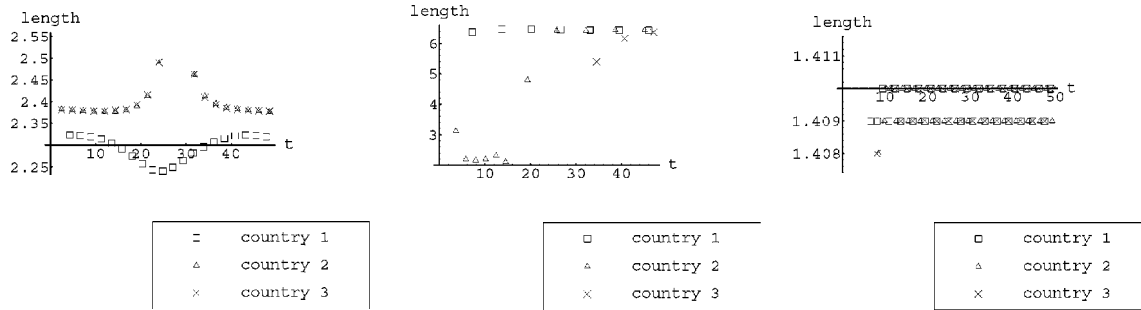
Figure 8 : Length between successive peaks in business cycles ($m=1$)
Case 1 (left), Case 2 (center), Case 3 (right)
The length of successive peaks in business cycle of each country converges to a
level through transition process except for case 1.   In case 1, the length can also
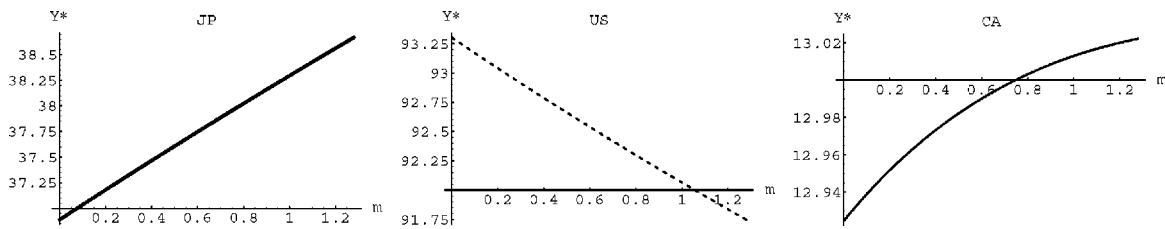fluctuate after the transition.

Figure 9 : The equilibrium national income (Case 1)
This figure shows the levels of the equiliburium national income in countries 1
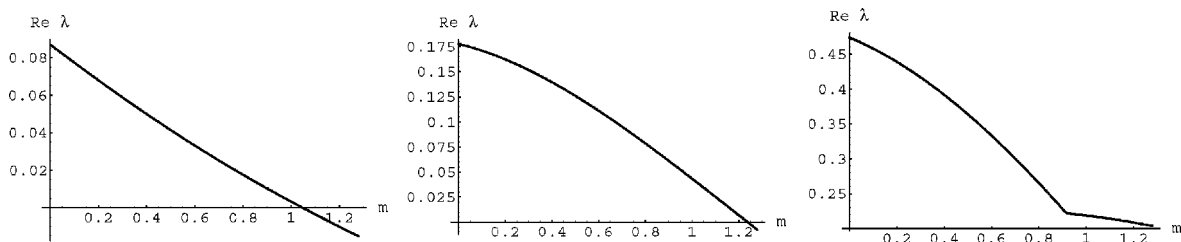(left), 2 (center), and 3 (right) for various levels of trade intensity in Case 1.

Figure 10 : The stability of the equilibrium
This figure shows The maximum real part of eigenvalues of System 3 evaluated
at the equilibrium (Cases 1 (left), 2 (center) and 3 (right)).

linkage of countries.   The equilibrium national income and the maximum value of
the real parts of eigenvalues of Jacobian matrix evaluated at the equilibrium of
System 3 are shown in Figures 9 and 10, respectively.   Figure 10 implies negative
correlation between the degree of coupling of countries and the local instability at
the equilibrium.   In all cases the equilibrium point is locally unstable for $0 \leq m \leq$
1.   However, the equilibrium can be stable when m exceeds unity.   In fact inven-
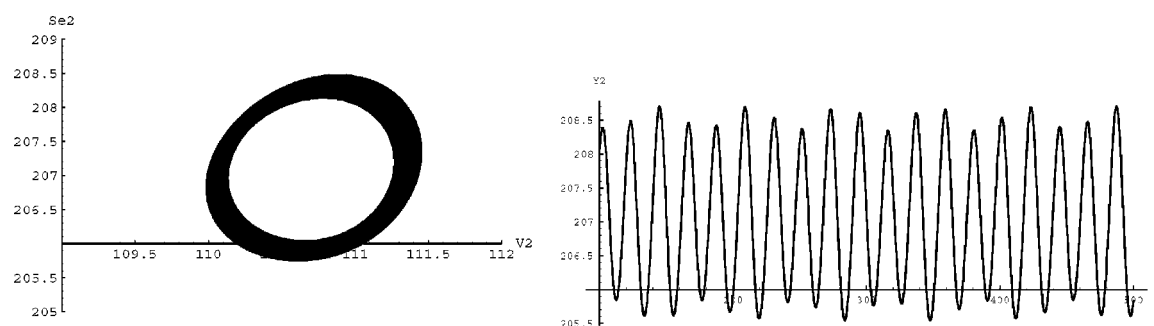tory cycle in a country disappears for some parameter.

Figure 11 : Thick attractor and a little complicated fluctuation
This shows a result of coupling if another setting of parameters are adopted. (Attractor projected onto V- S^e plane (left) and time series of national income in country 2 (right))

Figure 11 also shows a result of weaker coupling oscillators of System 2. We easily notice the thickness of the attractor projected onto V_2-S_2^e plane. From the time series of S_2^e in the figure we cannot know whether the fluctuation is periodic or not. From the results obtained in this section we understand the coupled model, System 3, can represent various behavior which is interesting from the view point of macroeconomics. The diversity of solutions hidden in System 3 may be clarified by making bifurcation diagrams. But it would be difficult for numerical simulation to capture precise model's dynamics under an extremely weak linkage, though it is an attractive work.

5　Concluding Remarks

Stimulated by Selover and Jensen (1999), we investigate whether coupled inventory dynamics can generate synchronization or the world business cycles. The coupling parameters we focus on represent the dependence of a country on consumption good sectors in other countries. If the linkage between countries is sufficiently strong, then inventory cycle in each country can disappear. This is because decision makers behave cowardly under the situation they have not experienced yet. In addition we exemplify the phase difference between countries becomes constant by numerical simulations. Whether such phenomena occur or not depend on the marginal propensity to import or other parameters in each country.

We also find a thick attractor for a low level of coupling. It suggests the possibility of occurrence of chaos in the three-country international trade model as Lorenz (1987) pointed out citing Newhouse, Ruelle and Takens (1978). It is very

interesting in terms of the representation of temporary mode-locking phenomena by a simple nonlinear macroeconomic model.　One of the future's works is to detect unstable periodic orbits embedded in the chaotic attractor in order to clarify the mechanism of synchronization and desynchronization of the coupled nonlinear dynamics and verify the explanation power of the model in detail.

Acknowledgments

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