Relationships between phases of business cycles in two large open economies

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雑誌名	国際地域学研究
号	13
ページ	131-145
発行年	2010-03
URL	http://id.nii.ac.jp/1060/00003683/

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1. Introduction

We have observed large increases in trade and capital flows for several decades. Does it mean the emergence of a world business cycle or the increase in possibility of global recession? This problem is closely related whether the government in each country should take a coordinated policy to control domestic business cycles or not. The purpose of this paper is to obtain some implications concerning the problem through analyzing phenomena represented by a nonlinear macroeconomic model.

Some interesting macroeconomic models described by nonlinear equations were proposed about five decades ago (e.g., Kaldor 1940). Since the nonlinearity in time series of main economic indices proposed by many empirical researches (e.g., Brock and Sayers 1988), nonlinear models have been attracting much attention. Generalizations of those models have been studied energetically. Asada et al. (2003, Ch.10) have exemplified that interaction between two large open economies can generate persistent business fluctuations by a high-dimensional nonlinear model. The model is complete in the sense that all markets in the Keynesian macroeconomics are taken into consideration. In this paper, through understanding the dynamics of the model deeply, we try to see what large increases in trade and openness would give rise to.

This paper is organized as follows. In the next section, we review the KWG model proposed in Asada et al. (2003, Ch.10). In Section 3 we consider a unique equilibrium of the model. Typical behavior of the model is discussed in Section 4. The continuous dynamical system is numerically integrated for two parameter settings. One is a case of no interaction between countries and the other corresponds to a case of international trade and capital flows. In Section 5, unstable periodic orbits are detected in order to understand a chaotic solution as typical behavior under the interactions between countries. Final section concludes our results.

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2. The model

In this section we review a variant of the two-country KWG model given in Asada et al. (2003, Ch.10). Let us begin with the definitions of important macroeconomic magnitudes. The symbol ω denotes the real wage, which is measured by the unit of domestic output. The real wage is defined as follows:

$$\omega = \frac{w}{p}, \tag{1}$$

where w and p are the money wage and the price level, respectively. The actual rate of profit ρ is defined by

$$\rho = \frac{Y - \delta K - \omega L^d}{K}, \tag{2}$$

where Y denotes the real output, K the capital stock, L^d the employed labor force, and δ the capital depreciation rate. Here, we employ the following abbreviations:

$$y = \frac{Y}{K}, \; l^d = \frac{L^d}{K}.$$

Equation (2) can be rewritten as

$$\rho = y - \delta - \omega l^d. \tag{3}$$

We assume δ is constant. In addition, assuming a Leontief-type production function, y and l^d are also constant. Economic agents in the two-country KWG model are workers, asset holders, firms, and governments. We consider how excess demand in the goods market is determined through reviewing assumptions with respect to their expenditure.

Households consist of workers and asset holders. Workers spend all their money to consume domestic goods solely. A part of the expenditure of asset holders is directed to imports. The amount is

$$(1 - \gamma_c(\eta))(1 - s_c) Y_c^D, \tag{4}$$

where variable η means the real exchange rate[1], $\gamma_c(\eta)$ is a negative function of η, s_c is the average saving rate of asset holders, and Y_c^D denotes their disposable income. We express the disposable income per unit of capital as

[1] The rate means the amount of foreign goods enable to exchange for a unit of goods made in home country.

$$\rho - t_c, \tag{5}$$

where t_cK means the difference between all taxes they pay and all their interest income. For simplicity, t_c is assumed to be fixed. This simplification removes the effect of the accumulation of domestic and foreign bonds from the model. Thus, for domestic goods,

$$c_1 = \omega l^d + \gamma_c(\eta)(1-s_c)(\rho - t_c) \tag{6}$$

expresses the households' demand per unit of capital.[2] On the other hand, firms determine their investment level according to the following equation:

$$I = i(\rho - (r - \pi))K + nK, \tag{7}$$

where variables r and π are the nominal interest rate and the expected inflation rate, respectively; i is a parameter of this function. Note that capital accumulation (and the growth rate of GDP) in the model is represented by

$$\hat{K} = \frac{I}{K} = i(\rho - (r - \pi)) + n. \tag{8}$$

A hat over a variable denotes the change rate of the variable. The government expenditure is proportional to the capital stock, that is,

$$G = gK. \tag{9}$$

We have already assumed trade between two countries. An asterisk indicates a foreign country variable. Initial levels of labor supply in both countries being equivalent, the export per unit of capital from home country can be expressed by

$$c_1{}^* = \frac{l}{\eta l^*}(1 - \gamma_c^*(\eta))(1-s_c^*)(\rho^* - t_c^*), \tag{10}$$

where l means the labor supply capital ratio in home country. From Equations (6), (7), (9) and (10), excess demand on the goods market in home country is expressed as

$$X^p = c_1 + c_1^* + i(\rho - (r - \pi)) + n + \delta + g - y. \tag{11}$$

Similarly that in foreign country is

$$X^{p*} = c_2^* + c_2 + i^*(\rho^* - (r^* - \pi^*)) + n^* + \delta^* + g^* - y^*, \tag{12}$$

[2] The tax rate on wage income is assumed to be zero.

where $c_2^* K^*$ means foreign households' demand level for foreign goods, and $c_1 K^*$ is the amount of imports from foreign country to home country.

Next, let us consider the labor market. Excess demand on the labor market in home country is indicated by

$$X^w = \frac{l^d}{l} - \bar{V}, \tag{13}$$

where $\bar{V}$ is NAIRU-type normal utilization rate concept of labor. Each growth rate of p and w is assumed to be influenced by both X^p and X^w. Wage deflation is excluded from the model.[3] Hence $\hat{w}$ and $\hat{p}$ are determined by the following simultaneous equations:

$$\begin{cases} \hat{w} = \max[\beta_w X^w + k_w \hat{p}_w + (1-k_w)\pi, 0] \\ \hat{p} = \beta_p X_p + k_p \hat{w} + (1-k_p)\pi. \end{cases} \tag{14}$$

From the definition,

$$\hat{\omega} = \hat{w} - \hat{p}. \tag{15}$$

Labor force is assumed to grow at a constant rate n. Hence,

$$\hat{l} = n - \hat{K} = -i(\rho - (r - \pi)). \tag{16}$$

Expectation formation for the domestic price is expressed as

$$\dot{\pi} = \beta_\pi(\alpha_\pi(\hat{p} - \pi) + (1 - \alpha_\pi)(\hat{p}_o - \pi)), \tag{17}$$

where π is the expected inflation, and $\hat{p}_o$ indicates the long-run inflation rate.

Now, we consider the money market. We assume asset market clearing. The stock demand for real money balances is assumed to depend on output, capital [4] and the nominal interest as follows:

$$\frac{M^d}{p} = h_1 Y + h_2 K(r_o - r), \tag{18}$$

where r_o is the long-run nominal interest rate, and parameters h_1 and h_2 are positive constants. The only rule of monetary policy by the central bank is to keep the domestic money supply M growing a constant rate μ. The nominal interest rate is determined so that the following equation holds.

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[3] Such a wage rigidity contributes to bound fluctuations of state variables in an economically meaningful region (Chiarella et al. 2003).

[4] For simplicity, we think K real wealth.

$$\frac{M}{p}=h_1Y+h_2K(r_o-r). \tag{19}$$

That is,

$$r_o=r+\frac{h_1y-m}{h_2}, \tag{20}$$

where variable m denotes real money balances per unit of capital, therefore, the change rate of m is

$$\hat{m}=\mu-\hat{p}-i(\rho-(r-\pi))-n. \tag{21}$$

Finally, we consider the dynamics of foreign exchange market. By using the nominal exchange rate e, the definition of η is rewritten as

$$\eta=\frac{p}{ep^*}. \tag{22}$$

It follows

$$\hat{\eta}=\hat{p}-\hat{e}-p^*. \tag{23}$$

The way and the extent of international capital flows per unit of capital depend on the interest differential under imperfect capital mobility. The nominal exchange rate is assumed to be adjusted according to the capital flows and net exports as follows:

$$\hat{e}=\beta_e(\beta(r^*+\varepsilon-r)-nx)+\hat{e}_o, \tag{24}$$

where nx denotes net export (per unit of capital) of home country, $\hat{e}_o$ is the growth rate of the nominal exchange at a long-run equilibrium[5], and ε is the expected rate of exchange depreciation. The expectation formation for the nominal exchange rate has analogous to the case of inflation. That is,

$$\dot{\varepsilon}=\beta_\varepsilon(\alpha_\varepsilon(\hat{e}-\varepsilon)+(1-\alpha_\varepsilon)(\hat{e}_o-\varepsilon)). \tag{25}$$

The dynamics of foreign county is modeled analogously. Hence, if the magnitudes of ω, l, m, π, η, ε, ω^*, l^*, m^*, π^* are known, their growth rates can be determined by the right hand sides in the following equations.

$$\hat{\omega}=\hat{w}-\hat{p}, \tag{26}$$

$$\hat{l}=-i(\rho-(r-\pi)), \tag{27}$$

[5] The prices and the nominal exchange rate grow at the equilibrium depending on increases in money supply in both countries.

$$\hat{m} = \mu - \hat{p} - i(\rho - (r - \pi)) - n, \tag{28}$$

$$\dot{\pi} = \beta_\pi(\alpha_\pi(\hat{p} - \pi) + (1 - \alpha_\pi)(\hat{p}_o - \pi)), \tag{29}$$

$$\hat{\eta} = \hat{p} - \hat{e} - \hat{p}^*, \tag{30}$$

$$\dot{\varepsilon} = \beta_\varepsilon(\alpha_\varepsilon(\hat{e} - \varepsilon) + (1 - \alpha_\varepsilon)(\hat{e}_o - \varepsilon)), \tag{31}$$

$$\hat{\omega}^* = \hat{w}^* - \hat{p}^*, \tag{32}$$

$$\hat{l}^* = -i^*(\rho^* - (r^* - \pi^*)), \tag{33}$$

$$\hat{m}^* = \mu^* - \hat{p}^* - i^*(\rho^* - (r^* - \pi^*)) - n^*, \tag{34}$$

$$\dot{\pi}^* = \beta_\pi^*(\alpha_\pi^*(\hat{p}^* - \pi^*) + (1 - \alpha_\pi^*)(\hat{p}_o^* - \pi^*)). \tag{35}$$

This is the 10 dimensional differential equation system to be analyzed in this paper.

3. The equilibrium and its stability

An economy modeled by Equations (26) through (35) moves in response to gaps between actual and expected values if it has not reached the long-run equilibrium yet. In this section we focus on the steady state. The equilibrium value of each variable is expressed using subscript o. For simplicity, we assume $n = n^*$.

From Equation (13), the equilibrium level of l is determined as

$$l_o = l^d / \bar{V}. \tag{36}$$

Substituting $r = r_o$ into Equation (20), we obtain

$$m_o = h_1 y. \tag{37}$$

From $\hat{l} = 0$,

$$r_o = \rho_o - \pi_o. \tag{38}$$

From $\hat{l} = 0$, $\hat{m} = 0$ and $\hat{p} = \pi_o$,

$$\pi_o = \mu - n. \tag{39}$$

Net export nx is zero in the steady state, therefore, we can derive the equilibrium values of ρ and ω from

$$\begin{cases} \rho_o = y - \delta - \omega_o l^d \\ X_o^p = \omega_o l^d + (1 - s_c)(\rho_o - t_c) + n + \delta + g - y = 0. \end{cases} \tag{40}$$

They are solved as

$$\omega_o = (y-\delta-\rho_o)/l^d,\ \ \rho_o = t_c + \frac{n+g-t_c}{s_c}. \tag{41}$$

The real exchange rate when net export is zero is determined by

$$\eta_o = \frac{l_o(1-\gamma_c^*)(1-s_c^*)(\rho_o^*-t_c^*)}{l_o^*(1-\gamma_c)(1-s_c)(\rho_o-t_c)}. \tag{42}$$

From $\pi_o = \mu - n$, $\pi_o^* = \mu^* - n^*$, $n = n^*$ and $\dot{\varepsilon} = 0$,

$$\varepsilon_o = \mu - \mu^* \tag{43}$$

It is obvious from the above equations that the steady state is independent of any adjustment speed included in the model. Moreover it has been also obvious from the analytical examination in Asada et al. (2003, Ch.10) that the equilibrium can be locally unstable if the adjustment speeds are sufficient large.

4. Typical dynamics of the model

In this section we discuss the dynamical property of the model for two different settings of parameters. The common characteristics of the settings are that monetary policy rules are different between two countries, and that adjustment speeds are high enough to generate persistent growth cycles in both countries. The difference between the settings is whether linkages between countries exist or not.

4.1 Case 1: no linkage

In this subsection we rule out any linkage between countries. The dynamics of home country is modeled by Equations (26) through (29), and that of foreign country is by Equations (32) to (35). Parameters are set as follows:
$s_c = s_c^* = 0.8$, $\delta = \delta^* = 0.1$, $t_c = t_c^* = 0.35$, $g = g^* = 0.35$, $n = n^* = 0.02$, $h_1 = h_1^* = 0.1$, $h_2 = h_2^* = 0.2$, $y = y^* = 1$, $l^d = l^{d*} = 0.5$, $k_w = k_w^* = 0.5$, $k_p = k_p^* = 0.5$, $i = i^* = 0.5$, $\bar{V} = \bar{V}^* = 0.8$, $\alpha_\varepsilon = 0.5$, $\alpha_\pi = \alpha_\pi^* = 0.5$, $\mu = 0.025$, $\mu^* = 0.022$;
$\beta_w = \beta_w^* = 2.0$, $\beta_p = \beta_p^* = 3.0$, $\beta_k = \beta_k^* = 1.0$, $\beta_\pi = 3.9$, $\beta_\pi^* = 3.8$.

The functions $\gamma_c(\eta)$ and $\gamma_c^*(\eta)$ are $\gamma_c(\eta) = \gamma_c^*(\eta) = 1$.

For this setting, the steady state of each country is locally unstable, and limit cycles appear.[6] The limit cycles are graphically illustrated in Figure 1. An economy

[6] The system is numerically integrated by using fourth order Runge-Kutta method.

of home country starting from almost every meaningful point is attracted to the cycle depicted in Figure 1 (top), and moves on it counter-clockwise. Figure 2 shows the time series of growth rates of the nominal wage, the price and output[7] on the limit cycles. It should be noted that the difference of policy rules gives rise to a tiny variation of the periods of limit cycles though both of them are about 9.1.

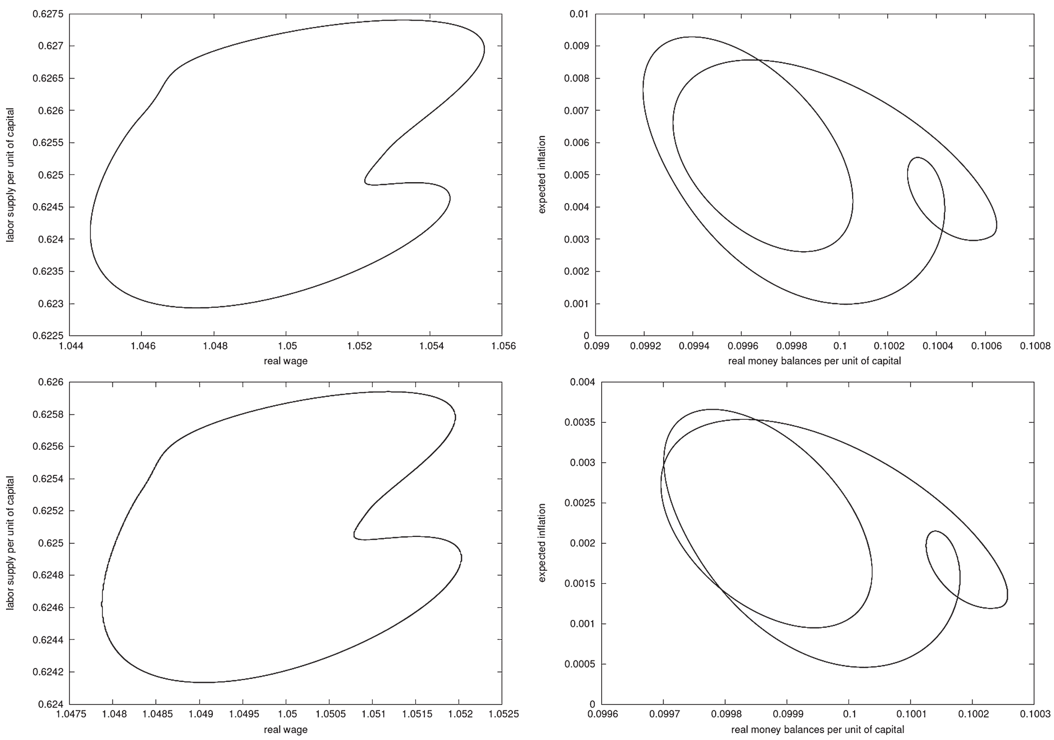

Figure 1: Limit cycles of home country (top) and foreign country (bottom)

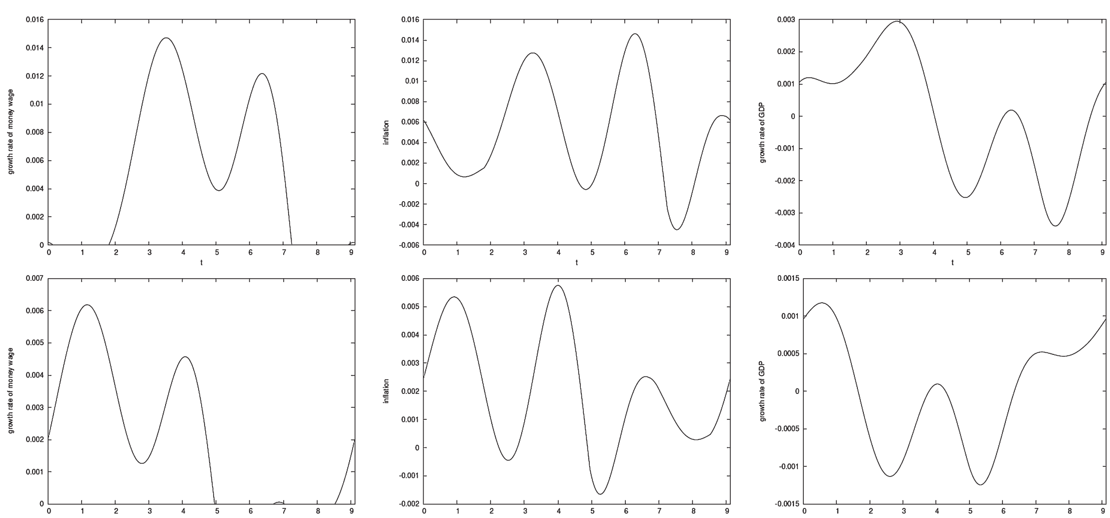

Figure 2: Time series of growth rates in home (top) and foreign (bottom) countries

[7] Strictly speaking, Figure 2 (right) shows the time development of deviation of the actual growth rate of output from the natural growth rate of each country.

4.2 Case 2: linkages through trade and capital flows

Other thins being equal, the functions $\gamma_c(\eta)$ and $\gamma_c^*(\eta)$ are changed as follows:

$$\gamma_c(\eta) = -\max[-\max[0.5+0.2(\eta_o-\eta),0],-1], \tag{44}$$

$$\gamma_c^*(\eta) = -\max[-\max[0.5-0.2(\eta_o-\eta),0],-1]. \tag{45}$$

The piecewise linearity in the above equations confines the propensity to import to an appropriate range. Adjustment parameters concerning trade and capital flows are set as

$\beta = 1,\ \beta_e = 2,\ \beta_\varepsilon = 9.5.$

It should be noted that the adjustment speed of expectation with respect to the nominal exchange rate is much higher than any other speed.

Now we analyze the result of our coupling. Such a generalization of the model often causes the emergence of chaos. There have been many works considering connections of oscillators in the literature of nonlinear macroeconomic dynamics since Lorenz (1987) presented a pioneering work[8] . However, as far as the author knows, there are few models in which exchange rate dynamics is appropriately involved in this context. In regard to this point, the model presented by Asada et al. (2003, Ch.10) is worth analyzing in detail.

Figure 3 shows the chaotic attractor generated by the 10 dimensional dynamical system. The maximum Lyapunov exponent of the attractor is about 0.13. This value means the system exhibits behavior that depends sensitively on the initial conditions. Unlike in Case 1, long-term prediction is impossible in this case. On the other hand, kinks of Phillips curves and the functions of real exchange rate seem to contribute to the viability of economically meaningful fluctuations, and the oscillation in a bounded region generates recurrent time series. Both recurrence and sensitivity on the initial values are characteristics of chaos.

5. Unstable periodic orbits embedded in the attractor

It is known that there are an infinite number of unstable periodic orbits embedded in a chaotic attractor. When similar patterns are observed subsequently in a long time development in a chaotic attractor, it can be considered that the trajectory is

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[8] Lorenz (1987) implied that interaction among three countries can generate chaotic business cycles.

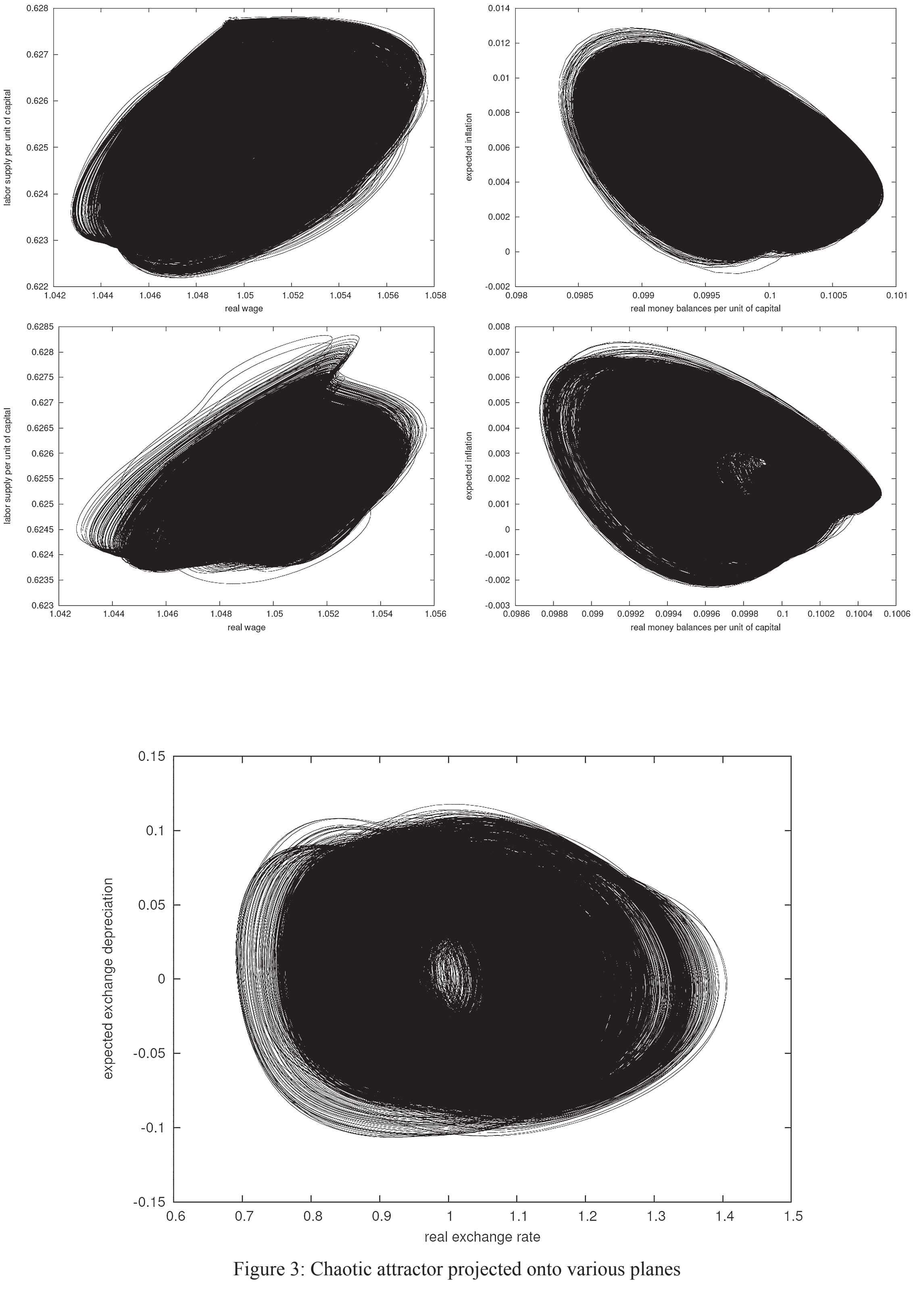

Figure 3: Chaotic attractor projected onto various planes

going along an unstable periodic orbit embedded in the attractor. Let us confirm this conjecture.

Figure 4 (top) shows time series abstracted from the chaotic development in Figure 3. Three patterns of similarity are depicted in the figure. In the chaotic attractor we find an unstable periodic orbit which can be seen as the *original* of those patterns. Figure 4 (bottom) exhibits how an economy starting from a point of the attractor moves along an unstable periodic orbit. This figure implies that chaotic economic dynamics can be characterized by the variety of unstable periodic orbits embedded in the attractor.[9]

Some typical unstable periodic orbits we find are illustrated in Figure 5. Time developments of growth rates of GDP in two countries on these orbits are shown in Figure 6. It is not easy to compare the relationships between the growth rates in home and foreign countries among those periodic solutions. Then, we consider an index to represent the intensity of the relationship between the countries on a trajectory.

We focus on the combination of signs of $\hat{Y}-n$ and $\hat{Y}^*-n^*$, namely, phases of business cycles in home and foreign countries. The home country is considered to be in a good phase if the sign is not negative, otherwise in a bad phase. Hence the relationships between the countries can be described by the relationship between the signs of those values. Let $x=\hat{Y}-n$ and $x^*=\hat{Y}^*-n^*$ be observed at a time point chosen randomly. Note that x and x^* are random variables. If there is no dynamic relation between the countries as in Case 1, then x and x^* should be independent of each other. If there are some relationships between the countries, $\Pr(x \geq 0, x^* \geq 0 \mid x^* \geq 0)$ can be different from $\Pr(x \geq 0)$. The stronger the relation between the countries is, the more the difference between these probabilities may increase. Three differences concerning $\Pr(x \geq 0)$, $\Pr(x^* \geq 0)$, and $\Pr(x^* < 0)$, respectively can be calculated other than that one. We consider the maximum absolute values of these differences as an index of intensity of relationships between phases of the countries and we denote it by ζ.

Table 1 shows the values calculated on unstable periodic orbits and the chaotic orbit. We can see that intensities of relationship between the countries are different among those orbits. That of UPO08 is the highest and that of UPO22 is the lowest and the intensity on the chaotic orbit is between them.[10] It implies that the intensity

[9] Ishiyama and Saiki (2005) discussed in detail the relationships between the variety of typical behavior observed in the chaotic time developments and that of unstable periodic orbits embedded in the chaotic attractor.

[10] The unstable periodic orbits are named based on their periods.

of relationship between countries can change through long time developments, and that it depends on which type of unstable periodic orbits is embedded near the current position.

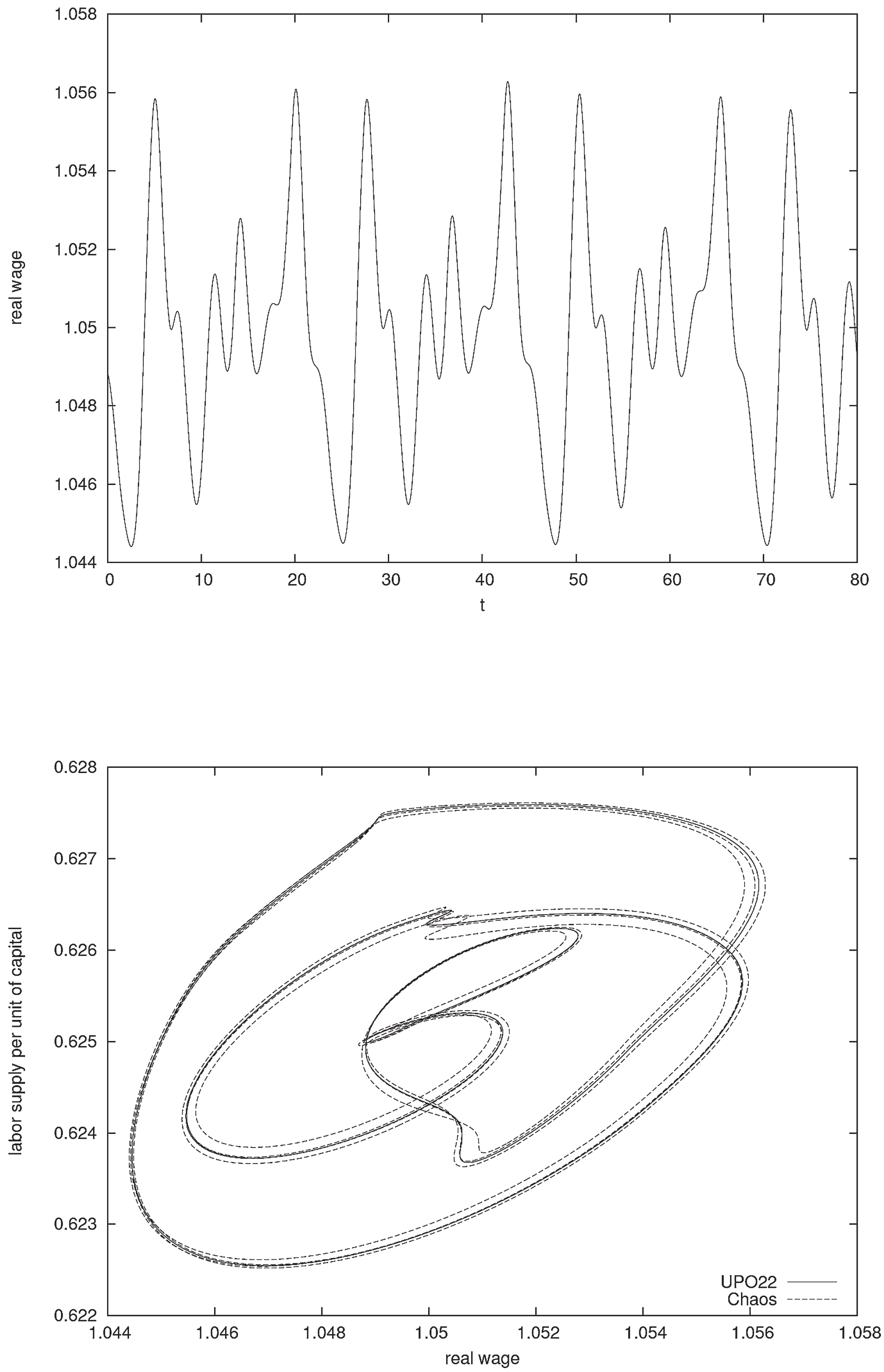

Figure 4: Chaotic time series along an unstable periodic orbit

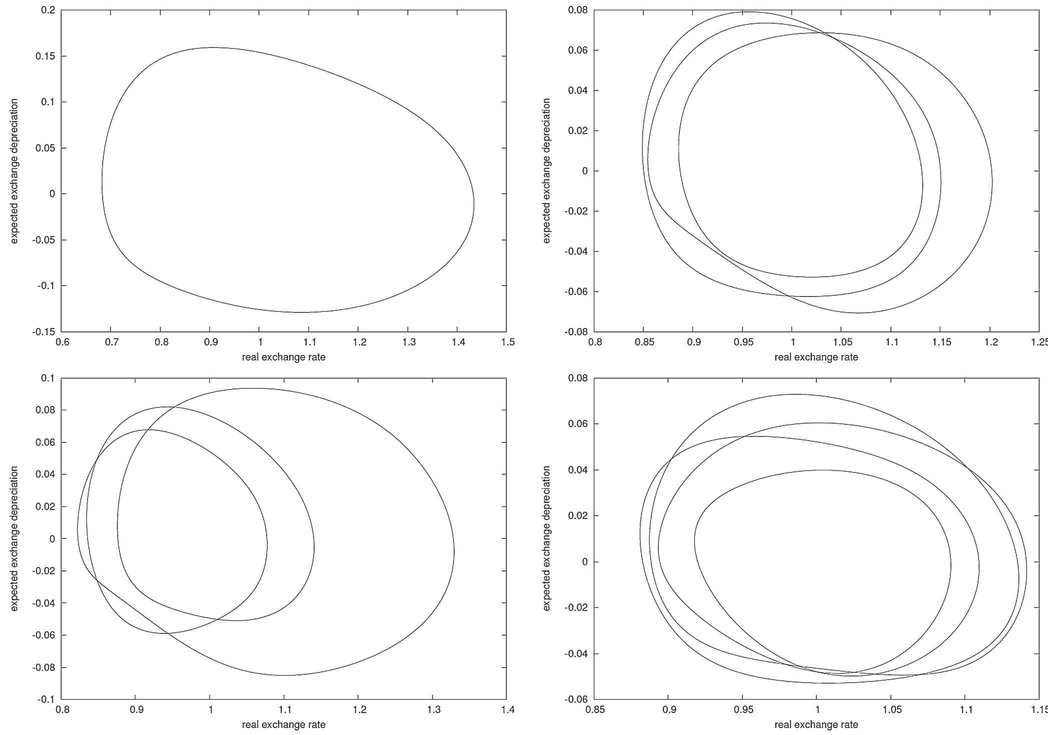

Figure 5: Unstable periodic orbits embedded in the chaotic attractor

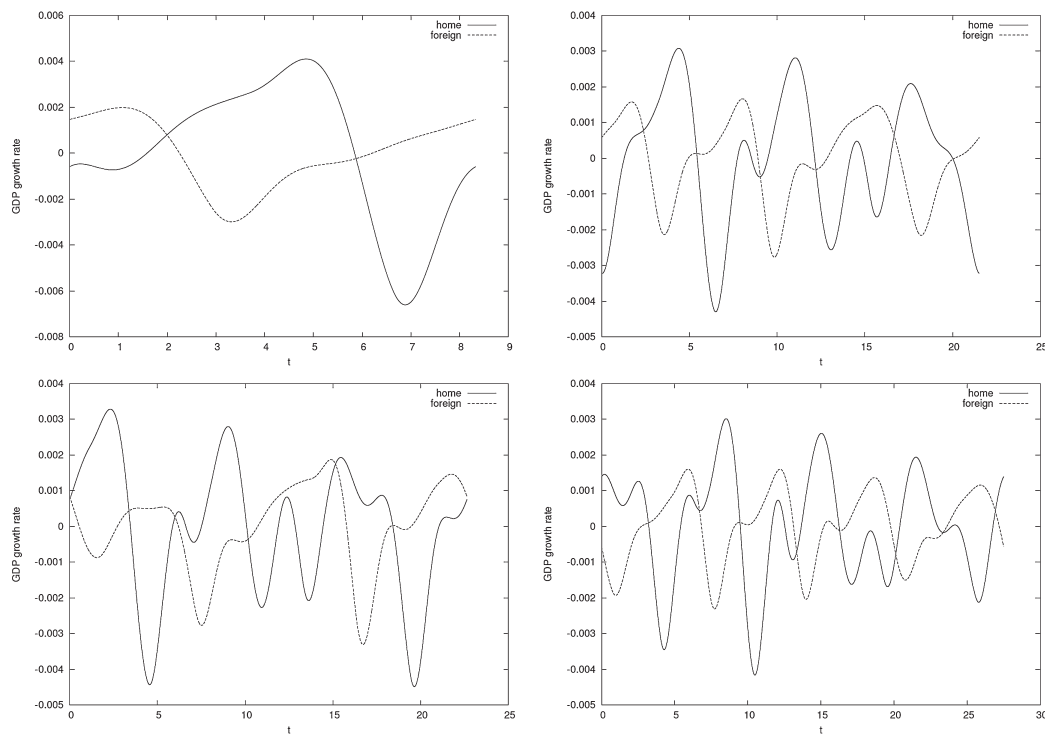

Figure 6: Time series of growth rates of GDP on unstable periodic orbits

Table 1: Intensity of relationships between phases in home and foreign countries

	Period	$P(\hat{Y} > n, \hat{Y}^* > n^*)$	$P(\hat{Y} > n, \hat{Y}^* \leq n^*)$	$P(\hat{Y} \leq n, \hat{Y}^* > n^*)$	$P(\hat{Y} \leq n, \hat{Y}^* \leq n^*)$	ζ
Chaos		0.203	0.350	0.350	0.097	0.23
UPO08	8.33	0.080	0.431	0.446	0.043	0.40
UPO21	21.52	0.187	0.378	0.381	0.064	0.30
UPO22	22.65	0.251	0.338	0.307	0.103	0.19
UPO27	27.15	0.175	0.354	0.389	0.082	0.28
LC		0.291	0.249	0.248	0.212	0.00

6. Conclusion

How will business cycles in each country change when the intensity of linkage between two countries becomes higher? There are two answers opposite to each other. The purpose of this paper is to obtain some implications concerning the problem through analyzing phenomena represented by a nonlinear macroeconomic model. Various macroeconomic models show persistent fluctuations if adjustment speeds of markets are sufficient large. We are interested in the influence of international interaction on such fluctuations. The model we focus on shows a limit cycle as typical behavior of the economy in the case of a closed economy. We exemplify the model can represent chaotic growth cycles when international linkage through trade and capital flows are allowed under the condition that the adjustment speed of expectation with respect to the nominal exchange rate is much higher than other speeds. In order to capture relationships between growth cycles of interacting two countries from complicated time series of real output in the countries, we define an index which can indicate the strength of the relationships. The value of the index calculated along the chaotic solution shows that growth cycles of two countries depend on each other to some extent. The values of the index are also calculated along the unstable periodic solutions numerically detected in the attractor. We confirm some are greater than the value of the chaotic solution, and others are smaller than it. It implies that there exist a period in which growth cycles of the two countries are strongly related and a period in which two economies behave independently in the chaotic trajectory. We conclude the difficulty of the question mentioned above may come from this phenomenon. If it is true, we have to be more careful to conclude the discussion about the problem.

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