A Stability Approach to Mean-Variance Optimization

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Abstract

I jointly treat two critical issues in the application of mean-variance portfolios, i.e., estimation risk and portfolio instability. I find that theory-based portfolio strategies known to outperform naive diversification ($1/N$) in the absence of transaction costs, heavily underperform it under transaction costs. This is because they are highly unstable over time. I propose a generic method to stabilize any given portfolio strategy while maintaining or improving its efficiency. My empirical analysis confirms that the new method leads to stable and efficient portfolios that offer equal or lower turnover than $1/N$ and larger Sharpe ratio, even under high transaction costs.

Keywords: Portfolio Choice; Stability; Estimation Risk; Transaction Costs

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1 Introduction

Mean-variance analysis of Markowitz (1952) is an important portfolio choice model in academia and investment practice. However, practical applications suffer from two critical issues. First, the parameters that define mean-variance efficient portfolios are unknown and need to be estimated in finite samples. Potential estimation errors add risk to the portfolio selection process, coined as estimation risk in the literature, and negatively affect out-of-sample portfolio performance (e.g., see Michaud, 1989, and Best and Grauer, 1991). Second, estimated portfolio weights tend to be very unstable over time. This instability translates into high transaction costs and further decreases portfolio returns. The recent work of Kirby and Ostdiek (2012) highlights the relation between these two issues: portfolio instability tends to increase with estimation risk. In this context, the present paper treats estimation risk and portfolio instability in a joint manner.

A vast literature has developed around proposing portfolio strategies that are less sensitive to estimation risk (Brandt, 2009). This literature has been recently challenged by the influential work of DeMiguel, Garlappi and Uppal (2009) who find that most sample-based strategies perform worse out-of-sample than the equally-weighted portfolio, also known as $1/N$. In response to this finding, DeMiguel, Garlappi, Nogales and Uppal (2009), Tu and Zhou (2011), Kirby and Ostdiek (2012) and Kourtis, Dotsis and Markellos (2012) develop more efficient strategies that offer significantly higher risk-adjusted returns than $1/N$.

I find that $1/N$ still outperforms most existing sample-based strategies when transaction costs are present. The reason is that most sample-based strategies are very sensitive to small changes in the underlying sample. As new observations enter the sample and portfolio is rebalanced, portfolio composition tends to change dramatically. As a result, portfolio turnover and the associated transaction costs are magnified. An empirical exercise similar to that of

DeMiguel, Garlappi and Uppal (2009) shows that the strategies in DeMiguel, Garlappi, Nogales and Uppal (2009), Tu and Zhou (2011) and Kourtis, Dotsis and Markellos (2012) can even produce negative average returns net of transaction costs. Such results question the practical value of mean-variance optimization in the presence of transaction costs.

My findings motivate the main objective of this paper: to develop sample-based strategies that are both efficient and stable. The traditional approach to promote stability is to explicitly incorporate proportional transaction costs in the mean-variance framework.[1] However, the nonlinear form of proportional transaction costs does not generally allow a closed-form solution to the portfolio problem. As a result, many of the recent advances in the estimation risk literature cannot accommodate proportional transaction costs, since they require an analytical representation of the optimal weights (for instance, the methods in Kan and Zhou, 2007; Tu and Zhou, 2011; Kourtis, Dotsis and Markellos, 2012). Also, a computational algorithm is required for the derivation of the optimal portfolio, but such algorithms tend to be inefficient when the number of assets is large.

To resolve these issues, I propose a new stability approach to mean-variance optimization. In particular, I directly impose an instability penalty to the mean-variance objective. The penalty controls the deviation of the portfolio weights in each period from the weights before rebalancing. According to the new portfolio objective, the investor optimizes a trade-off between efficiency and stability. The novelty is that stability is measured in terms of the deviation from the weights before rebalancing using the Σ_t-norm, instead of the 1-norm that is traditionally

[1] Woodside-Oriakhi, Lucas and Beasley (2013) review the recent literature that accounts for transaction costs in the mean-variance framework. Alternatively, several studies investigate the problem of portfolio choice under proportional transaction costs in a continuous-time setting (see Cvitanić, 2001, for a review).

applied in the literature, where Σ_t is the covariance matrix of asset returns. Because of this differentiation, the stability approach developed here offers several attractive features.

First, it leads to an intuitive analytical solution of the portfolio optimization problem: the stable portfolio weights are a linear combination of a portfolio from the efficient frontier and the weights before trading. Since there is no constraint on how one can estimate the first portfolio, the stability approach can be easily applied to stabilize any sample-based strategy improving its performance under transaction costs. For example, it can be directly used with the strategies from DeMiguel, Garlappi, Nogales and Uppal (2009), Tu and Zhou (2011), Kirby and Ostdiek (2012), and Kourtis, Dotsis and Markellos (2012). I show that the turnover of the stabilized strategies is only a fraction of the turnover of their original counterparts.

Second, the stability approach may improve the performance of a sample-based strategy even in the absence of transaction costs. To understand this, I provide two interpretations of the instability penalty. I show that it is equivalent to imposing a norm constraint in the original mean-variance problem. Jagannathan and Ma (2003) and DeMiguel, Garlappi, Nogales and Uppal (2009) show that such portfolio constraints can increase risk-adjusted returns in the presence of estimation risk. I further find that the instability penalty promotes investment in a momentum strategy. The latter performs well when the serial correlation of asset returns is positive. In this case, the stabilized strategy offers higher returns than its original counterpart.

Third, the investor can control the importance of stability in the portfolio choice process through a stability parameter. I study two selection criteria for this parameter. The first chooses the parameter that matches the turnover of the portfolio to that of $1/N$. This is because the latter portfolio is known to be relatively stable over time. Given that $1/N$ is the benchmark in this study, it is reasonable to ask if there can be sample-based strategies that offer the same stability levels and higher risk-adjusted returns. The second criterion aims to further increase both risk-adjusted returns and stability by taking into account the positive autocorrelation of portfolio returns reported by Campbell, Lo and MacKinlay (1997).[2] By construction, it leads to portfolios with a lower turnover than $1/N$.

The instability penalty in this paper is inspired by the model of quadratic transaction costs studied by Gârleanu and Pedersen (2013). Gârleanu and Pedersen use this special form of transaction costs based on the same incentive: to allow analytical solutions to the portfolio choice problem. However, there are several important differences between the work of Gârleanu and Pedersen and the present paper. First, this paper also deals with estimation risk. Second, this paper considers transaction costs that have a proportional form following the standard convention in the literature. The instability penalty serves as a mean to reduce portfolio turnover rather than to represent transaction costs. Third, the instability penalty in this paper involves a parameter that the investor can adjust to achieve a desired level of portfolio stability. Fourth, my analysis is carried out in asset returns rather than in price changes considered by Gârleanu and Pedersen.

To evaluate the stability approach, I apply it to ten sample-based strategies from the literature. These include three portfolios from the sample-based mean-variance frontier, a short-sale constrained mean-variance portfolio, a portfolio based on the shrinkage estimator of Ledoit and Wolf (2004) as well as five portfolios from the studies of DeMiguel, Garlappi, Nogales and Uppal (2009), Tu and Zhou (2011), Kirby and Ostdiek (2012), and Kourtis, Dotsis and Markellos (2012). In this manner, I derive stable versions of all sample-based strategies. I compare their

[2]DeMiguel, Garlappi, Nogales and Uppal (2009) and Kourtis, Dotsis and Markellos (2012) use a similar criterion for calibrating their strategies.

out-of-sample performance to that of $1/N$ and their original counterparts in 5 data sets of real asset returns following the approach of DeMiguel, Garlappi and Uppal (2009).

The results of the empirical analysis confirm that the stability approach proposed in this work leads to both efficient and stable portfolios. In all data sets, most of the stabilized sample-based strategies outperform $1/N$ under transaction costs, with regards to both Sharpe ratio and turnover. For example, in a data set of 10 industry portfolios the original strategies by DeMiguel, Garlappi, Nogales and Uppal (2009) and Kourtis, Dotsis and Markellos (2012) lead to Sharpe ratios of -0.076 and -0.070 under transaction costs of 100 basis points (bp) while the Sharpe ratio of $1/N$ is 0.522. I find that the stable versions of these portfolios offer a net Sharpe ratio of 0.600 and 0.645, respectively, outperforming $1/N$. At the same time, the stable strategies generate a turnover equal to or lower than $1/N$ by construction. In most cases considered, the stability approach improves Sharpe ratios even in the absence of transaction costs, with the improvement mainly coming from an increase in portfolio return. The conclusions from my empirical analysis are robust to alternative assumptions about the level of transaction costs, the size of the sample available and the frequency of rebalancing.

2 The mean-variance framework

I consider a mean-variance investor in a market of N risky assets and a risk-free asset. The returns on the risky assets in period t are denoted by R_t while R_t^f stands for the risk-free return. In each period, the investor chooses the portfolio weights w_t that minimize the conditional variance of the portfolio excess returns

$$\min_{w_t} w_t' \Sigma_t w_t, \tag{1}$$

subject to the constraints

$$w_t{}'\mu_t = \mu_t^0 \tag{2}$$

$$w_t{}'1_N = 1. \tag{3}$$

In the above, μ_t and Σ_t are respectively the conditional mean and covariance matrix of the excess returns $r_{t+1} = R_{t+1} - R_{t+1}^f$. 1_N is an N-dimensional vector of 1's. The first constraint sets the level of return (μ_t^0) that the investor requires. Solving (1), (2) (3) for different values of μ_t^0 generates the minimum variance frontier. The second constraint ensures that the investor only holds risky assets. I impose this constraint to perform comparisons between different portfolio strategies without the need of accounting for a risk aversion parameter. My analysis though can be easily modified to allow investment in the risk-free asset.[3]

The above investment setting has two attractive features. First, it allows time-varying moments for the asset returns as well as time-varying target return. As such, it can accommodate changes in the set of investment opportunities while the investor can adjust the amount of risk to take in each period. Second, it enables the analytical expression of the optimal portfolios. Following Kirby and Ostdiek (2012), I express the solution of the problem (1), (2), (3) as a linear combination of two special portfolios. The first is the global minimum variance portfolio, i.e., the portfolio of minimum risk. Its weights are defined as:

$$w_t^{MIN} = \frac{\Sigma_t^{-1}1_N}{1_N{}'\Sigma_t^{-1}1_N}. \tag{4}$$

The second portfolio is the tangency portfolio defined as the point where the Capital Market Line is tangent to the efficient frontier. Its weights are given by

[3]The conclusions in this work are valid in the case that investing in the riskless asset is allowed. The corresponding results are available upon request.

$$w_t^{TP} = \frac{\Sigma_t^{-1}\mu_t}{1_N{}'\Sigma_t^{-1}\mu_t}. \tag{5}$$

Applying first order conditions to the mean-variance problem gives the optimal portfolio w_t^{MV} as a linear combination of the global minimum variance portfolio and the tangency portfolio

$$w_t^{MV} = \left(1 - x_t\right) w_t^{MIN} + x_t w_t^{TP}, \tag{6}$$

where $x_t = \dfrac{\mu_t^0 - \mu_t^{MIN}}{\mu_t^{TP} - \mu_t^{MIN}}$. $\mu_t^{MIN} = \mu_t{}' w_t^{MIN}$ and $\mu_t^{TP} = \mu_t{}' w_t^{TP}$ are the expected excess returns on the global minimum variance portfolio and on the tangency portfolio, respectively.

If short sales are not allowed, the constraint $w_t \geq 0$ should be added in the optimization problem to ensure the positivity of the weights. The short-sale constrained optimization problem cannot be solved analytically, but there are fast quadratic optimization algorithms available that can be used to obtain the solution.

3 Estimation risk and portfolio instability

The framework in the previous section does not incorporate estimation risk and transaction costs, both inevitable in practice. This section shows that these two issues combined can largely deteriorate portfolio performance.

The optimal mean-variance portfolio is a function of the first two moments of the excess returns, μ_t and Σ_t. In practice, μ_t and Σ_t are unknown and the investor has to estimate them. The estimation is usually performed using a sample from the history of the asset excess returns

$\{r_{t-T+1},....,r_t\}$, where T is the sample size.[4] For example, the traditional practice employs the Maximum Likelihood (ML) estimators:

$$\hat{\mu}_t = \frac{1}{T}\sum_{j=t-T+1}^{t} r_j \tag{7}$$

$$\hat{\Sigma}_t = \frac{1}{T}\sum_{j=t-T+1}^{t} (r_j - \hat{\mu}_t)\left(r_j - \hat{\mu}_t\right)' \tag{8}$$

and applies them to estimate the portfolio weights in (6). Using estimates instead of the true values of the parameters introduces estimation risk in the portfolio choice process and tends to produce low or even negative risk-adjusted portfolio returns.

Kirby and Ostdiek (2012) show that the effects of estimation risk can be more prominent in the presence of transaction costs, because estimation errors lead to unstable portfolios over time. To understand why estimated portfolio weights tend to be unstable, consider a sample-based portfolio strategy p that at each time t uses the available sample to produce the portfolio weights $\hat{w}_t^p$. Let also $\tilde{w}_t^p$ be the portfolio of the investor before rebalancing to $\hat{w}_t^p$. The instability of the strategy can be measured using the portfolio turnover metric:

$$\tau_t^p = \left\|\hat{w}_t^p - \tilde{w}_t^p\right\|_1 = \sum_{i=1}^{N}\left|\hat{w}_{i,t}^p - \tilde{w}_{i,t}^p\right|, \tag{9}$$

where $\|\cdot\|_1$ is the 1-norm. Since the weight for an asset i before rebalancing is given by

$$\tilde{w}_{i,t}^p = \frac{\hat{w}_{i,t-1}^p\left(1+R_{i,t}\right)}{1+\sum_{i=1}^{N}\hat{w}_{i,t-1}^p R_{i,t}}, \tag{10}$$

the turnover for asset i is

[4]For exposition purposes, I assume that T does not change with t, but my analysis also holds when T varies with time.

$$\left|\hat{w}_{i,t}^{p}-\tilde{w}_{i,t}^{p}\right|=\left|\left(\hat{w}_{i,t}^{p}-\hat{w}_{i,t-1}^{p}\right)-\frac{\hat{w}_{i,t-1}^{p}\left(R_{i,t}-R_{t}^{p}\right)}{\left(1+R_{t}^{p}\right)}\right| \tag{11}$$

where $R_t^p=\sum_{i=1}^{N}\hat{w}_{i,t-1}^{p}R_{i,t}$ is the portfolio return from strategy p at time t.

The above equation implies that the turnover for an asset for a sample-based strategy may be larger by $\left|\hat{w}_{i,t}^{p}-\hat{w}_{i,t-1}^{p}\right|$ compared to a constant-weights strategy, such as $1/N$. Given that the samples used for the estimation of $\hat{w}_{i,t}^{p}$ and $\hat{w}_{i,t-1}^{p}$ differ by just one observation, the magnitude of $\left|\hat{w}_{i,t}^{p}-\hat{w}_{i,t-1}^{p}\right|$ depends on the sensitivity of the strategy in small sample changes. In particular, $\left|\hat{w}_{i,t}^{p}-\hat{w}_{i,t-1}^{p}\right|$ can be large for strategies that are very sensitive to estimation risk or have a large dependence on the most recent observation. Such strategies are expected to produce a large turnover.[5]

Turnover determines the magnitude of transaction costs. Given transaction costs that are proportional to the trade, the return of the strategy net of transaction costs is given by $\tilde{R}_t^p=\left(1+R_t^p\right)\left(1-\kappa\tau_t^p\right)-1$, where κ stands for the proportional transaction cost.[6] As a result, sample-based strategies that are very sensitive to sample changes may be subject to a large decline in portfolio returns when transaction costs are present. I explore this argument by studying the out-of-sample efficiency and stability of several popular sample strategies from the

[5]Other factors that have a significant contribution to the portfolio turnover are the risk of the portfolio and of the individual assets, as Kourtis (2014) shows.

[6]While several alternative different models of transaction costs have been considered in the literature such as fixed or quadratic, I adopt the most common form of proportional transaction costs used by Magill and Constantinides (1976), Davis and Norman (1990), and Balduzzi and Lynch (1999), among many others. This form directly corresponds to brokerage fees and bid-ask spreads.

literature. In the following, I present each considered strategy and summarize the findings from previous studies with regards to its performance. Table 1 reports all strategies employed in this work.

3.1 Naive diversification ($1/N$)

Naive diversification stands as the benchmark strategy in this paper, similarly to DeMiguel, Garlappi and Uppal (2009). This simple strategy assigns the same weight to each asset. Despite the lack of a theoretical foundation, naive diversification is not subject to estimation risk which explains its favorable performance compared to several strategies in the literature, reported by DeMiguel, Garlappi and Uppal (2009). Other attractive features of this strategy include the positivity of the weights and the low turnover due to the constant weights.

3.2 Sample tangency portfolio (TP)

The sample tangency portfolio results from replacing μ_t and Σ_t with $\hat{\mu}_t$ and $\hat{\Sigma}_t$ in (5). This portfolio results in the worst performance in DeMiguel, Garlappi and Uppal (2009), associated with negative Sharpe-ratios in several cases and with very high levels of turnover. Kirby and Ostdiek (2012) attribute this performance to the form of the denominator in the tangency portfolio weights formula. In particular, $1_N{}'\hat{\Sigma}_t^{-1}\hat{\mu}_t$ can take values close to zero. As a result, the sample tangency portfolio tends to have extreme weights leading to poor out-of-sample performance.

3.3 Sample global minimum variance portfolio (MIN)

The means μ_t are generally considered more challenging to estimate than variances-covariances (see, Merton, 1980; Chopra and Ziemba, 1993). Motivated by this, several authors propose the use of the sample global minimum variance portfolio computed using $\hat{\Sigma}_t$. In particular, Jagannathan and Ma (2003) and DeMiguel, Garlappi and Uppal (2009) find that the global minimum variance portfolio outperforms the sample tangency portfolio in several data sets of real returns. Kourtis, Dotsis and Markellos (2012) analytically confirm these findings. The sample global minimum variance portfolio is more stable than the tangency portfolio over time for two reasons. First, it is less sensitive to estimation risk, since it does not involve expected returns. Second, the denominator does not attain values close to zero because $\hat{\Sigma}_t$ is positive semidefinite.

3.4 Sample mean-variance portfolio (MV, MVC)

The sample-based mean-variance portfolio is the sample counterpart of (6), estimated using $\hat{\mu}_t$ and $\hat{\Sigma}_t$. Besides defining the target return, the choice of μ_t^0 in practice controls the estimation risk and the stability of the sample mean-variance strategy. In particular, as μ_t^0 decreases towards $\hat{\mu}_t^{MIN}$, then $\hat{x}_t$ decreases towards 0 and the portfolio weights $\hat{w}_t^{MV}$ become less sensitive to estimation errors in $\hat{\mu}_t$. This in turn improves stability. In this study, following Kirby and Ostdiek, I choose μ_t^0 to be equal to the maximum sample mean return between $1/N$ and the global minimum variance portfolio. As shown by Kirby and Ostdiek, this setting produces more efficient portfolios than the tangency portfolio and $1/N$. I further consider the short-sale constrained version of this portfolio denoted by MVC.

3.5 Ledoit-Wolf minimum variance portfolio (LW)

All sample-based portfolios discussed above employ the ML estimators of μ_t and Σ_t. Ledoit and Wolf (2004) develop an improved estimator of the covariance matrix based on the shrinkage methodology of James and Stein (1961). The estimator is a convex combination of $\hat{\Sigma}_t$ and the identity matrix *I*. I use this estimator to compute the global minimum variance portfolio and denote the resulting portfolio with LW.

3.6 2-norm-constrained portfolio (NC)

Portfolio constraints such as the no short-sales constraint are known to improve performance in the presence of estimation risk (Jagannathan and Ma, 2003). In this context, DeMiguel, Garlappi, Nogales and Uppal (2009) augment (1), (3) with the constraint $\| w_t \| \leq \delta_t$, where $\| \cdot \|$ can be the 1-norm, 2-norm or a matrix norm. They use two criteria to select the parameter δ_t. The first involves a cross-validation method for computing the parameter that minimizes out-of-sample portfolio variance. The second chooses the value of δ_t that maximizes the last period's portfolio return on the basis of the positive autocorrelation in monthly portfolio returns reported by Campbell, Lo and MacKinlay (1997). DeMiguel, Garlappi, Nogales and Uppal find that the resulting norm-constrained minimum variance portfolios can outperform $1/N$ out-of-sample in terms of risk and risk-adjusted returns. In my analysis, I only include the 2-norm constrained portfolio (NC) that maximizes last period's return on the basis of the empirical results in DeMiguel, Garlappi, Nogales and Uppal.

3.7 Three-fund strategy of Tu and Zhou (2011) (3F)

Tu and Zhou (2011) propose a strategy that diversifies among the tangency portfolio, the global minimum variance portfolio, $1/N$ and the risk-free. Their strategy produces higher risk-adjusted returns than $1/N$. To make the performance of this strategy independent of the risk-free asset and comparable to the remaining strategies in this work, I divide the portfolio weights by their sum in order to end up with a portfolio of risky assets only.

3.8 Volatility timing strategy (VT)

Kirby and Ostdiek (2012) propose several strategies that mimic some of the attractive features of $1/N$, while they take into account the information contained in past asset returns. Among these strategies, I use the global minimum variance portfolio that comes for using the sample diagonal covariance matrix, i.e., a matrix with the asset variances in the main diagonal and zeros elsewhere. This volatility timing strategy is particularly attractive since it offers a very low turnover and positive weights.

3.9 Portfolios based on shrinking the inverse covariance matrix (ICV, ICR)

The two remaining strategies I include in my analysis are derived by Kourtis, Dotsis and Markellos (2012) who apply the shrinkage methodology of James and Stein (1961) directly to the inverse covariance matrix and use the resulting estimator to compute the global minimum variance portfolio. Kourtis, Dotsis and Markellos propose the use of a linear combination of the identity matrix and the inverse covariance matrix from a 1-factor model as a shrinkage target. They further calibrate their portfolio strategies in two ways, similar to DeMiguel, Garlappi, Nogales and Uppal (2009); the first uses the cross-validation technique to reduce out-of-sample variance (ICV) and the second maximizes last period's return (ICR). They find that the first approach offers low levels of risk and turnover while the second results in higher Sharpe Ratio and turnover compared to $1/N$ and other strategies from the literature.

3.10 Empirical analysis of portfolio efficiency and stability

3.10.1 Methodology

I now study the stability of the sample-based strategies presented above along with their out-of-sample efficiency in the presence of transaction costs. For this purpose, I employ 5 data sets of real monthly excess returns that include 3 data sets that have been used in several previous studies. These are the 3 Fama-French factors (3FF), the 10 industry portfolios (10Ind) and the 25 size and book-to-market portfolios (25SBM). The fourth data set (8Int) consists of US-dollar denominated returns on 8 international market portfolios (Australia, Canada, France, Germany, Japan, Italy, UK, USA). Finally, I consider a data set of returns on 50 stocks that are constituents of S&P500 for the period 01/1981-09/2012.[7] As a risk-free rate, I adopt the 1-month T-bill rate. All data sets are obtained from Kenneth French's website, except for the stocks data set which is obtained from Thomson-Reuters. My selection of data sets allows the evaluation of portfolio performance for different numbers and types of assets at both US and international level. Table 2 includes a list of these data sets along with their respective time periods.

In each data set, I study how the strategies perform using the rolling window approach of DeMiguel, Garlappi and Uppal (2009). In particular, for each month $t \geq T$, I use the returns for the months $t-T+1,...,$ t to compute the portfolio weights $\hat{w}_t^p$ for each strategy p. In this manner, I obtain a time series of monthly excess returns $\{ r_{t+1}^p = r_{t+1}' \hat{w}_t^p \}$ for all strategies and data sets. To assess the out-of-sample efficiency of a particular strategy p, I compute the associated mean $\hat{\mu}^p$ and variance $(\hat{\sigma}^p)^2$ of each strategy returns as well as the Sharpe ratio given by:

[7]While it is customary to use randomized samples of stocks to avoid survivorship bias, it is more appropriate for this work to keep the set of stocks fixed. Changing the stocks periodically would correspond to large transaction costs which would not reflect the stability of the considered strategies.

$$\hat{\theta}^p = \frac{\hat{\mu}^p}{\hat{\sigma}^p}. \quad (12)$$

The latter is my main measure of out-of-sample portfolio efficiency in this work, following the literature. To study the statistical significance of the difference between the Sharpe ratio of each sample-based strategy p and $1/N$, I test the hypothesis "$H_0 : \hat{SR}_p - \hat{SR}_{1/N} = 0$". For this test, I follow the circular block bootstrap approach discussed by Ledoit and Wolf (2008) with 10,000 trials and with an average block size of 10 to obtain the respective p-value for the difference. I further evaluate the stability of each strategy by computing the average turnover $\hat{\tau}^p = \frac{1}{M-T-1}\sum_{t=T+1}^{M-1}\tau_t^p$, where M is the total number of observations in the data set. Finally, I study how the strategies perform in the presence of transaction costs by estimating the Sharpe ratio of the portfolio returns net of proportional transaction costs.

I present in Panel A of Tables 3-7 the mean, variance, Sharpe ratio in annual terms, and the average monthly turnover for each portfolio strategy in absence of transaction costs, as well as the Sharpe ratio in the presence of transaction costs ($\tilde{\theta}^p$) of 100 bp ($\kappa = 0.01$).[8] One, two and three '*' next to the Sharpe ratios reflect the statistical significance of the difference from the Sharpe ratio of $1/N$ at 10%, 5% and 1%, respectively. I compute all performance measures for a

[8]I choose this relatively high value for the proportional transaction cost in most of this work on the basis of three reasons. First, this value is close to the actual average trading cost that an investor would encounter if she invested in the US equity market directly in the period spanned by the data sets considered. This is based on the estimates of Stoll and Whaley (1983), Bhardwaj and Brooks (1992), and French (2008). Second, even though transaction costs in the US market have declined over time, they can still be very high in an international diversification setting or in other countries according to the findings of Domowitz, Glen and Madhavan (2001). Third, a high transaction cost allows testing the value of the stability approach for the small investor. To ensure that this value does not affect the validity of my conclusions, I further consider the more typical value of 50 bp, also assumed in Balduzzi and Lynch (1999), DeMiguel, Garlappi and Uppal (2009) and Kirby and Ostdiek (2012). My conclusions remain unchanged.

sample size T of 120 months. Since the computation of the portfolio weights of the 3-fund strategy from Tu and Zhou (2011) requires the input of a risk aversion parameter, I have set 3 as that parameter.[9]

3.10.2 Discussion of efficiency under no transaction costs

When there are no transaction costs, the results are generally in line with the literature. With regards to portfolio risk, the strategies based on the shrinkage estimator of the covariance matrix or its inverse (LW and ICV, respectively) offer lower variance than the remaining strategies, including MIN. The tangency portfolio is the worst performer in terms of Sharpe ratio. It offers a higher Sharpe ratio than $1/N$ only for the 3FF and 25SBM data sets while it even results in a negative average return of -0.025 for the 50SP set. The Sharpe ratio for the sample mean-variance portfolio MV is higher than that of $1/N$ and of the global minimum variance portfolio in all data sets, except for the set of 50 stocks. In the latter set, $1/N$ achieves a Sharpe ratio of 0.703 compared to a Sharpe ratio of 0.232 for the MV portfolio justifying the need for more efficient treatment of estimation risk.

The five recent strategies that aim to outperform $1/N$ (NC, 3F, VT, ICV, ICR) are indeed more efficient, in most cases. In the 10Ind and 25SBM portfolios all five sample-based strategies lead to higher risk-adjusted returns. In the 3FF set, the norm-constrained portfolio and the 3-fund rule of Tu and Zhou (2012) offer the highest Sharpe ratios, namely, 0.961 and 0.946. However, all sample-based strategies are challenged in the sets of international portfolios and 50 S&P stocks. In the 8Int data set, none outperforms $1/N$ statistically significantly. In fact, all lead to

[9]I have also considered alternative values for the risk aversion parameter that defines TZ without important changes in the results.

lower Sharpe ratio except for the volatility timing strategy and the ICV portfolio. In the 50SP data set, only NC, VT and ICR achieve a higher Sharpe ratio than $1/N$.

3.10.3. Discussion of stability and efficiency under transaction costs

Examining the turnover for the strategies reported in Panel A of Tables 3-7 leads to four observations. First, all sample-based strategies apart from the volatility-timing portfolio produce significantly higher turnover than $1/N$. The tangency portfolio is the most unstable strategy producing up to 2000 times the turnover of $1/N$.[10] Setting the target mean equal to the return of $1/N$ (as for MV) and adding a short-sale constraint (as for MVC) helps improve stability. For example, in the 25SBM data set (Table 6) the turnover for the tangency portfolio (TP), the mean-variance portfolio (MV), and its short-sale constrained counterpart (MVC), is 38.19, 0.862, 0.501, respectively. However, $1/N$ is still significantly more stable with a turnover of 0.017.

Second, the turnover for most strategies is higher in the larger data sets (25SBM and 50SP). For example, the turnover of the sample mean-variance portfolio MV is 0.036 in the 3FF data set, 0.165 in the 8Int set, 0.216 in the 10Ind set and 0.862 in the 25SP set. To understand this finding, note that when the number of assets is large, the estimation risk is higher which increases the turnover.

Third, the strategies LW, VT, ICV that focus on reducing the risk of the sample global minimum variance portfolio (MIN) also reduce its turnover. This is because these strategies use more stable estimators of the covariance matrix or its inverse than the ML estimators.

[10]The turnover for the TP strategy takes very high values in some of the datasets (e.g., 25SBM). The reason is that the denominator in the formula of the TP weights ($1_N' \hat{\Sigma}_t^{-1} \hat{\mu}_t$) takes values close to 0 in some cases, resulting in very high absolute weights that, in turn, magnify the turnover. This problem of the TP strategy is also reported by Kirby and Ostdiek (2012, pp. 442). In the same fashion, 3F attains a very high turnover in the 8Int dataset.

Indicatively, in the set of 10 industry portfolios (Table 5), the turnover for MIN is 0.157 while the turnover for LW, VT and ICV is 0.107, 0.025 and 0.114, respectively.

Fourth, the strategies NC, 3F and ICR from the recent literature that are known to outperform $1/N$ in terms of Sharpe ratio are highly unstable over time. In fact, they generate higher turnover than all remaining strategies apart from the tangency portfolio. For instance, in the 25SBM data set, NC, 3F and ICR respectively have a turnover of 3.794, 1.372 and 3.738, while the turnover for all remaining strategies, except TP, is less than 0.863.

Why are strategies that are known to effectively treat estimation risk so unstable? The reason is that strategies such as NC, 3F and ICR aim to improve out-of-sample performance by using the available sample more efficiently. Therefore, they tend to be very sensitive to sample changes. Especially, NC and ICR are primarily determined by last month's returns. As a result, when a new month of returns enters the sample, the portfolio associated with those strategies may have a significantly different composition from the previous period. This difference corresponds to a large increase in the turnover.

A high turnover can heavily affect portfolio efficiency in practice as the results in the last column of Panel A in Tables 3-7 indicate. This column reports the Sharpe ratio for each strategy adjusted for proportional transaction costs of 100 bp. The results confirm that sample-based strategies that generate high levels of turnover can be very inefficient in the presence of transaction costs. As expected, the tangency portfolio is again the most inefficient strategy leading to negative average return in all data sets besides 3FF.

More importantly, even strategies that are very effective in dealing with estimation risk also lead to negative returns under transaction costs. The Sharpe ratio for 3F is negative in 2 out of 5 data sets. The two strategies that make use of the autocorrelation in portfolio returns (NC, ICR) generate negative average return in 4 out of 5 data sets. The sample-based strategies perform worse in the data sets of 8 international portfolios (Table 4) and 50 stocks (Table 7). In the international data set, NC, 3F and ICR lead to a Sharpe ratio of -0.472, -0.527 and -0.642, respectively, while the Sharpe ratio for $1/N$ is 0.327. In the stocks data set, the sample-based strategies perform even worse due to the large number of assets and the magnitude of estimation risk. In this set, all strategies apart from VT produce a lower Sharpe ratio than $1/N$. In fact, six out of ten sample strategies lead to negative average return out-of-sample. I observe that only the volatility timing strategy outperforms $1/\mathrm{N}$ and this result is not statistically significant.

Overall, the results in Panel A of Tables 3-7 indicate that most of the sample-based strategies that are known to effectively deal with estimation risk tend to be highly unstable over time. This instability can even lead to negative returns in the presence of transaction costs. In this setting, $1/N$ is superior than most theory-based strategies in both turnover and risk-adjusted returns. These findings highlight the need for both stable and efficient sample-based portfolios. The next section provides a method for constructing such portfolios.

4 A stability approach to mean-variance optimization

The natural approach to promote stability is to penalize turnover in the portfolio optimization problem. This can be achieved by modifying the constraint (2) to accommodate proportional transaction costs:

$$w_t{}'\mu_t - \kappa \left\| w_t - \tilde{w}_t \right\|_1 = \mu_t^0 \qquad (13)$$

The resulting portfolio choice problem involves a nonlinear constraint. As a result, the derivation of an analytic representation of the optimal weights is no longer feasible and their calculation may be hard for a large number of assets. It is also uncertain to what extent the constraint (13) improves the stability and efficiency of the mean-variance portfolio under transaction costs. In this context, I study the out-of-sample performance of the sample-based mean-variance portfolio (and its short-sale constrained counterpart) that use (13) to account for transaction costs. Following the same procedure as in the previous section, the target return is set equal to the maximum sample mean return between the $1/N$ and the MIN portfolio. In Table 8, I present the turnover and Sharpe ratio net of transaction costs of 100 bp for both portfolios for the 5 data sets considered. I include the corresponding statistics for the original portfolios and $1/N$, taken from Table 3-7, for comparison purposes. I find that the penalty (13) reduces portfolio turnover to the levels of $1/N$. As a result, the net Sharpe ratio increases. However, $1/\mathrm{N}$ still outperforms the sample-based mean-variance portfolio in 2 out of 5 sets and the short-sale constrained portfolio in 3 out of 5 sets.

The results in Table 8 indicate the need for more effective treatment of estimation risk under transaction costs. However, the constraint (13) cannot be efficiently applied to many of the recent advances in the estimation risk literature for two reasons. This is because the analytical form of the portfolio weights, required, for example, by the strategies of Tu and Zhou (2011) and Kourtis, Dotsis and Markellos (2012), is not feasible here due to the nonlinearity of (13). Second, the constraint (13) involves a term that depends on the unknown means and a term that is independent of any unknown parameter. Because of this, DeMiguel, Nogales and Uppal (2013) show that potential errors in the estimation of μ_t may lead to suboptimal portfolios. Therefore, when estimation risk is high, (13) can be inefficient in improving portfolio performance under transaction costs. The results in Table 8 for the MV strategy in the 50SP data set confirm this argument.

To resolve these issues, I propose an alternative method to produce efficient portfolios under transaction costs. I explicitly introduce an instability penalty in the mean-variance objective function as:

$$\min_{w_t} w_t '\Sigma_t w_t + c_t \left(w_t - \tilde{w}_t\right)'\Sigma_t \left(w_t - \tilde{w}_t\right). \qquad (14)$$

According to (14), the investor aims to minimize both the portfolio variance and the deviation of the portfolio weights from the weights before trading ($\tilde{w}_t$) at the same time. In this manner, the new investor's objective represents a trade-off between efficiency and stability, controlled by the stability parameter c_t. To better expose this trade-off, I rewrite (14) as:

$$\min_{w_t} \left\| w_t - w_t^{MIN} \right\|_{\Sigma_t}^2 + c_t \left\| w_t - \tilde{w}_t \right\|_{\Sigma_t}^2 , \qquad (15)$$

where $\|\cdot\|_{\Sigma_t}$ is the Σ_t -norm defined by $\|x\|_{\Sigma_t} = \sqrt{x'\Sigma_t x}$.[11] Then, the higher the stability parameter c_t, the smaller the deviation of the optimal weights is from $\tilde{w}_t$ and the larger the deviation is from the global minimum variance portfolio.

The main difference between the instability penalty proposed here and the constraint is that the deviation from $\tilde{w}_t$ is measured by the Σ_t -norm instead of the 1-norm. In this manner, the instability penalty offers two comparative advantages. First, all terms in the objective function are now subject to estimation error in Σ_t. As a result, there is not an interaction between error-free terms and terms subject to errors. Second, the optimal portfolio weights can be expressed in closed form for any number of assets, as the next proposition shows:

[11]This norm is well defined, because Σ_t is positive semidefinite.

Proposition 1. *Let* $\tilde{\mu}_t = \tilde{w}_t' \mu_t$ *be the expected excess return of the portfolio of the investor before rebalancing. The solution to the portfolio choice problem defined by* (14) *and the constraints* $w_t ' \mu = \mu_t^0$ *and* $\mathrm{w}_t' 1_{\mathrm{N}} = 1$ *can be expressed as*

$$\bar{w}_t^{MV} = \frac{1}{1+c_t}\omega_t^{MV} + \frac{c_t}{1+c_t}\tilde{w}_t \tag{16}$$

where ω_t^{MV} *is the solution of the original mean-variance problem* (1)-(3) *when the target return is* $\dot{\mu}_t^0 = \mu_t^0 + c_t\left(\mu_t^0 - \tilde{\mu}_t\right)$.

Proof. The first order conditions for this optimization problem are:

$$2\left(1+c_t\right)\Sigma_t w_t - \delta_1\mu_t - \delta_2 1_N - 2c_t\Sigma_t \tilde{w}_t = 0, \tag{17}$$

where δ_1, δ_2 are respectively the Lagrange multiplies for the two constraints. Solving for w_t gives:

$$w_t = \frac{\delta_1}{2\left(1+c_t\right)}\Sigma_t^{-1}\mu_t + \frac{\delta_2}{2\left(1+c_t\right)}\Sigma_t^{-1}1_N + \frac{c_t}{\left(1+c_t\right)}\tilde{w}_t \tag{18}$$

Substituting the above to the constraints (2), (3) results in a linear system of two equations:

$$\delta_1\frac{\mu_t '\Sigma_t^{-1}\mu_t}{2\left(1+c_t\right)} + \delta_2\frac{\mu_t '\Sigma_t^{-1}1_N}{2\left(1+c_t\right)} + \frac{c_t\tilde{\mu}_t}{2\left(1+c_t\right)} = \mu_t^0 \tag{19}$$

$$\delta_1\frac{\mu_t '\Sigma_t^{-1}1_N}{2} + \delta_2\frac{1_N '\Sigma_t^{-1}1_N}{2} = 1 \tag{20}$$

Solving the system for δ_1 and δ_2 and substituting in (18) gives (16).

The optimal portfolio is a linear combination of a portfolio in the efficient frontier and the portfolio of the investor before trading. This result illustrates the impact of the instability penalty on portfolio rebalancing. At each time t, instead of rebalancing the whole investment to w_t^{MV}, the investor will move only $\frac{1}{1+c_t}$ of her investment to the efficient portfolio ω_t^{MV} and leave the remaining amount intact. This practice can help treat both estimation risk and instability.

4.1 Treating estimation risk

The main advantage of the stability approach developed here is that it leads to a simple closed-form expression of the portfolio weights. In this manner, it can be effectively combined with the recent developments in the portfolio choice literature to treat estimation risk under transaction costs. This is because the estimation of the mean-variance weights ω_t^{MV} in (16) can be performed using any portfolio strategy. Then, for a sample-based strategy p with weights $\hat{w}_t^p$, there is a stable counterpart given by

$$\bar{w}_t^p = \frac{1}{1+c_t^p}\hat{w}_t^p + \frac{c_t^p}{1+c_t^p}\tilde{w}_t^p \qquad (21)$$

If p is known to be efficient out-of-sample, the stable analogue of p may also lead to good performance. I illustrate this in Section 5 by studying the out-of-sample performance of the stable versions of the sample-based strategies considered in this work.

The imposition of the instability penalty may improve the efficiency of sample-based strategies even in the absence of transaction costs. To better understand this argument, I provide two interpretations of the instability penalty. First, note that c_t^p can be thought of as the Lagrange multiplier of the constraint $\left\| w-\tilde{w} \right\|_{\Sigma_t}^2 < \delta$, for some constant δ when this constraint is added to the original mean-variance problem (1)-(3). In this sense, applying the instability penalty is equivalent to imposing a norm constraint. While this constraint has not been considered in the literature, Jagannathan and Ma (2003) and DeMiguel, Garlappi, Nogales and Uppal (2009) show that portfolio constraints generally improve portfolio performance under estimation risk. Therefore, strategies which are subject to significant estimation errors, such as the tangency portfolio may benefit from the instability penalty, irrespective of the presence of transaction costs.

Second, the stable portfolio $\bar{w}_t^p$ results from diversifying the original sample-based strategy p with a buy-and-hold strategy defined by $\tilde{w}_t^p$. On one hand, typical sample-based strategies assume rebalancing in each period and they tend to be contrarian exploiting potential stock reversals. On the other hand, the buy-and-hold strategy $\tilde{w}_t^p$ is a momentum strategy offering high returns when the asset returns are positively autocorrelated between consecutive periods. Therefore, the stable strategy diversifies between a contrarian and a momentum strategy. As a result, it improves the performance of the original strategy when serial correlations in the market are positive.

4.2 Treating portfolio instability

It is straightforward to verify that the turnover of the stable version of a strategy p given in (21) is

$$\bar{\tau}_t^p = \frac{1}{1+c_t^p}\tau_t^p. \qquad (22)$$

Hence, the turnover of the stable portfolio is decreasing with c_t^p being less than the turnover of the original portfolio for $c_t^p \geq 0$. The larger c_t^p is, the more stable the portfolio is and the less the proportional transaction costs ($\kappa\hat{\tau}_t^p$) are. In the special case that c_t^p goes to infinity, the investor will tend to not trade at all and transaction costs will approach 0.

The selection of the parameter c_t^p is critical for the application of the stability approach. In this work, I study two alternative selection criteria. First, in each period t, I set the value of c_t^p that matches the turnover of the stable strategy to that of $1/N$. This is because $1/N$ is known to offer very low levels of turnover, compared to sample-based strategies, while it is the benchmark in this paper.[12] The stability parameter is then given by

$$\hat{c}_t^p = \frac{\tau_t^p - \tau_t^{1/N}}{\tau_t^{1/N}} \qquad (23)$$

which leads to $\bar{\tau}_t^p = \tau_t^{1/N}$.

The natural incentive for using the instability penalty is to reduce turnover and the associated transaction costs. This also motivates the first selection criterion for the stability parameter. However, it is interesting to examine whether c_t^p can be chosen in such a way so that it also improves risk-adjusted returns. In this context, I propose a second selection criterion for c_t^p that aims to further increase the returns of a given portfolio strategy while keeping the level of turnover and the associated transaction costs lower than $1/N$. This criterion uses the reported positive autocorrelation of monthly portfolio returns similarly to the works of DeMiguel, Garlappi, Nogales and Uppal (2009) and Kourtis, Dotsis and Markellos (2012). In particular, I choose the value of c_t^p that lies in the interval $\left[\hat{c}_t^p, +\inf\right)$, so that the resulting stable portfolio maximizes the return of the previous month. This is because when portfolio returns are positively autocorrelated, the resulting stable portfolio is expected to generate a high return for the current month as well.

[12] In a similar context, the good performance of $1/N$ has motivated Tu and Zhou (2011) to combine it with other portfolio strategies to improve out-of-sample performance.

The form of the stable portfolio weights $\bar{w}_t^p$ implies that in each period, the investor will rebalance to the stable portfolio for $c_t^p = \hat{c}_t^p$ if $R'_t \, \hat{w}_t^p > R'_t \, \tilde{w}_t^p$ or otherwise she will not trade. Setting the lower bound for c_t^p equal to $\hat{c}_t^p$ ensures that the average turnover will not overcome the turnover of $1/N$, keeping transaction costs low. In the case that the investor does not trade, the turnover will be zero, further reducing the average turnover and transaction costs over time.

5 Performance of the stable portfolios

I evaluate the stability approach by applying it to the sample-based strategies presented in Section 3. Panels B and C of Tables 3-7 contain the mean, variance, and turnover of the resulting stable strategies as well as their Sharpe ratio in both the absence and presence of transaction costs. For Panel B, the stability parameter is selected to match the turnover of the sample-based strategy to that of $1/N$ while in Panel C the stability parameter maximizes last month’s portfolio return. For direct comparison with Panel A, all performance metrics are derived following exactly the same procedure as in Section 3 and under the same assumptions.

5.1 Performance in the 3FF data set

This data set contains the returns for only 3 assets leaving less room for estimation risk and portfolio instability. As a result, transaction costs have a smaller impact except for the norm-constrained portfolio and the ICR strategy from Kourtis, Dotsis and Markellos (2012), which generate a relatively high turnover. According to the last column of Panel A these two strategies produce an annual Sharpe ratio of 0.626 and 0.270 net of transaction costs of 100 bp and they are outperformed by $1/\mathrm{N}$ which has a net Sharpe ratio of 0.798. It is interesting to see whether the instability penalty can improve the efficiency of these strategies under transaction costs.

The last column of Panels B and C shows that the stability approach leads to an increase in the net Sharpe ratio and stability for all sample-based strategies. The improvement is more noticeable for NC and ICR. As shown in Panel B, the transaction costs-adjusted Sharpe ratio for the stable counterparts of NC and ICR is 0.840 and 0.846, respectively, higher than that of $1/N$. At the same time their turnover decreases to match that of $1/N$ according to the first selection criterion for the stability parameter. Most of the remaining stabilized sample-based strategies also outperform $1/N$, with the 3F strategy offering the highest Sharpe ratio (0.944). The results in Panel C indicate that when the stability parameter is selected, according to the second criterion, portfolio performance is further improved to the point that all sample-based strategies, apart from the volatility timing strategy, outperform $1/N$. I also observe a large decrease in the turnover which is less than 0.016 for all sample-based strategies, while $1/N$ has a turnover of 0.023.

Another observation made is that the instability penalty appears to improve the Sharpe ratio of most sample-based strategies, even in the absence of transaction costs. This finding is consistent with the discussion in Section 4. The most significant improvement is observed for the ICR strategy where the Sharpe ratio increases from 0.784 to 0.918, exceeding that of $1/N$. I find that the increase in the Sharpe ratio mainly comes from an increase in the mean return rather than a decrease in the variance.

5.2 Performance in the 10Ind and 25SBM data sets

Because the findings are similar, I discuss the results for both data sets (Tables 5 and 6). The Sharpe ratios under no transaction costs confirm the findings of previous studies, since all sample strategies apart from the tangency portfolio outperform $1/N$. The largest Sharpe ratio is observed for the ICR strategy in the 10Ind set (0.908) and for the 3F strategy in the 25SBM set

(1.183). However, once I account for transaction costs of 100 bp these strategies result to negative risk-adjusted returns. In this setting, $1/N$ offers higher Sharpe ratio than most sample-based strategies.

In panels B and C of Tables 5 and 6, I report the Sharpe ratios for the stable counterparts of all strategies. In the absence of transaction costs, the results are mixed with some strategies performing slightly better and others somewhat worse than their original analogues. However, under transaction costs of 100 bp, all stable strategies, but TP, offer higher Sharpe ratios than $1/N$. The most significant improvement is observed for TP, NC, and ICR that are very unstable. For example, in the 25SBM data set the Sharpe ratio of ICR increases from -1.538 to 0.585 when the turnover is equal to that of $1/\mathrm{N}$ and increases to 0.589 when the stability parameter maximizes last month's portfolio return. In most cases, the two selection criteria for the stability parameter lead to similar levels of Sharpe ratio.

The potential of the stability approach in improving portfolio performance is best illustrated by examining the results for the tangency portfolio in the 10Ind data set. As discussed in Section 3, this portfolio is very sensitive to estimation errors and very unstable. This is clear by the results in Panel A; the Sharpe ratio is comparatively the lowest (0.289) and the turnover is 50 times higher than that of $1/N$. As a result, the tangency portfolio generates a negative Sharpe ratio net of transaction costs of -0.295. However, imposing the instability penalty largely improves both the Sharpe ratio and the turnover. Notably, under the second criterion for selecting the stability parameter, the tangency portfolio is more stable and efficient than $1/N$, offering a Sharpe ratio of 0.524 net of transaction costs and a turnover of 0.014.

5.3 Performance in the 8Int and 50SP data sets

The data set of 8 international portfolios (Table 4) and the data set of 50 stocks (Table 7) are the most interesting environment for testing the stability approach. According to the results in the first set, none of the sample strategies significantly outperforms $1/N$, while most of the strategies are very unstable. The highest turnover comes from the TP, 3F, NC and ICR portfolios and leads to negative average returns. Only the VT strategy outperforms $1/N$ in the presence of transaction costs. In the 50SP set, the sample-based strategies perform even worse due to the large number of assets that increases the magnitude of estimation risk. Apart from VT and NC, all strategies perform worse than $1/N$ in terms of both Sharpe ratio and turnover. Due to instability, most strategies result in negative Sharpe ratio under transaction costs of 100 bp. Therefore, it is interesting to examine to what extent the stability approach improves performance in such a challenging setting.

As Panels B and C of Tables 4 and 7 indicate, the stability approach manages to drastically increase the Sharpe ratio of the sample-based strategies while reducing their turnover. For the 8Int data set, I find that the Sharpe ratio for all stable portfolios except from TP and 3F is higher than that of $1/N$. This holds in both the presence and the absence of transaction costs. The Sharpe ratio of TP and 3F still gains a large increase from 0.071 and 0.147 to 0.304 and 0.324, respectively. The highest Sharpe ratios here are offered from the stable analogues of the portfolios based on shrinking the inverse covariance matrix (ICV, ICR). Again, the improvement in the Sharpe ratios can be attributed to an increase in average return. Also, as in the 3FF data set, the second criterion for the stability parameter is more successful in improving the Sharpe ratio than the first criterion.

Similar conclusions can be drawn by studying the results for the 50 stocks data set (Table 7). The stability approach improves all strategies in terms of Sharpe ratio and turnover, even under no transaction costs. The improvement is more impressive when the stability parameter is selected according to the second criterion. In this case, the Sharpe ratio under transaction costs for NC, 3F, and ICR increases from -0.718, -0.035 and -0.707 to 0.704, 0.675 and 0.767, respectively. The corresponding turnover decreases from 1.630, 0.577 and 1.680 to less than 0.048. These three sample-based strategies outperform $1/N$ which has a Sharpe ratio of 0.664 and a turnover of 0.050.

5.4 Summary of the results and robustness checks

The results in this section lead to five conclusions. First, the instability penalty improves the Sharpe ratio and shrinks the turnover of all strategies under transaction costs in all data sets. The improvement is more prominent for the strategies that tend to produce higher rates of turnover. Second, in most cases, Sharpe ratios also improve under no transaction costs. This finding indicates the effectiveness of the stability approach in dealing with estimation risk, as well. Third, the stability approach increases Sharpe ratios mainly by increasing out-of-sample portfolio returns while maintaining similar levels of risk. Fourth, calibrating the stable portfolio strategies on the basis of last month's return is more effective than the first criterion for choosing the stability parameter. Fifth, the stable counterparts of most sample-based strategies outperform $1/N$ under transaction costs in all data sets, in contrast to the original strategies.

In this work, I assume that (1) the proportional transaction cost is 100 bp; (2) the sample size is 120 months; and (3) rebalancing is performed at the end of each month. To study the sensitivity of my conclusions to these assumptions, I also consider proportional transaction costs equal to 50 bp per trade, a sample size to 60 months and daily rebalancing instead of monthly. I find that my conclusions about the performance of the stability approach are not significantly sensitive to the assumptions (1)-(3). As expected, the merits of the stability approach are more prominent under a small sample size and daily rebalancing where portfolio instability is higher. These results are available from the author upon request.

6 Conclusions

In this paper, I jointly deal with two critical obstacles in the application of mean-variance portfolios, i.e., estimation risk and instability. I find that most existing sample-based strategies underperform $1/N$ in the presence of transaction costs. This is because they are very unstable over time leading to high portfolio turnover and transaction costs. Motivated by this finding, I propose a new method to promote stability in mean-variance optimization: I augment the standard mean-variance objective with a new instability penalty that controls the deviation from the portfolio before rebalancing. Compared to traditional approaches in the literature, the main advantage of this method is that it leads to intuitive analytical representations of the portfolio weights for any number of assets. Therefore, it can be easily applied to stabilize any sample-based strategy.

In my empirical analysis I apply the stability method to several sample-based strategies from the literature and evaluate their performance in five data sets of real asset returns. I find that the stabilized portfolio strategies outperform $1/N$ under most scenarios, even in the presence of high transaction costs. At the same time, they offer a turnover which is equal or less than the turnover of $1/N$. Overall, my analysis confirms that the new stability approach enables the effective application of sample-based mean-variance portfolios under both estimation risk and transaction costs.

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Table 1

List of portfolio strategies

This table reports the portfolio strategies considered in the paper.

Abbreviation	Description
1/*N*	Equally-weighted portfolio
TP	Sample tangency portfolio
MIN	Sample global minimum variance portfolio
MV	Sample mean-variance portfolio with a required return equal to the maximum return between 1/*N* and MIN
MVC	Short-sale constrained mean variance portfolio with a required return equal to the maximum return between 1/*N* and MIN
LW	Minimum variance portfolio that results from shrinking the covariance matrix to the identity matrix (Ledoit and Wolf, 2004)
NC	2-norm constrained minimum variance portfolio that exploits positive autocorrelation in portfolio returns (DeMiguel, Garlappi, Nogales and Uppal, 2009)
3F	The linear combination of MEAN, GMV and 1/*N* portfolios proposed in Tu and Zhou (2011)
VT	Minimum variance portfolio that results from using a diagonal covariance matrix (Kirby and Ostdiek, 2012)
ICV	Minimum variance portfolio that results from shrinking the ML estimator of the inverse covariance matrix to a linear combination of the identity and the inverse covariance matrix from a 1-factor model. The estimation minimizes out-of-sample variance (Kourtis, Dotsis and Markellos, 2012)
ICR	Minimum variance portfolio that results from shrinking the ML estimator of the inverse covariance matrix to a linear combination of the identity and the inverse covariance matrix from a 1-factor model. The estimation maximizes last month's return (Kourtis, Dotsis and Markellos, 2012)

Table 2

List of data sets

This table lists the data sets of monthly excess returns that I use in my empirical analysis. It also reports the time period each data set spans. All data are obtained from Kenneth French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html), apart from the stocks data set which is obtained from Thomson-Reuters. I use the 30-day T-bill rate, also taken from Kenneth French's website, for the derivation of excess returns.

Abbreviation	Time period	Description
3FF	07/1969-09/2012	3 Fama-French factors
8Int	01/1981-12/2011	8 international market portfolios (Australia, Canada, France, Germany, Japan, Italy, UK and USA)
10Ind	07/1969-09/2012	10 industry portfolios
25SBM	07/1969-09/2012	25 size and book-to-market portfolios
50SP	01/1981-12/2012	50 S&P 500 stocks

Table 3

Results for the 3 Fama-French factors data set

This table summarizes the out-of-sample performance of the strategies described in Table 1 in the 3 Fama-French factors data set. Panel A reports the average $\hat{\mu}^p$, variance $(\hat{\sigma}^p)^2$ and Sharpe ratio $\hat{\theta}^p$ of a strategy's excess returns in annual terms as well as the average monthly turnover $\hat{\tau}^p$. It also reports the annualized Sharpe ratio in the presence of proportional transaction costs of 100 basis points ($\tilde{\theta}^p$). The returns for the strategies are derived using a rolling window approach with a sample size of 120 months. The first 120 observations in the data set are held out for the initial estimation of the portfolio weights. Panels B and C report the same metrics as in Panel A for the stable counterparts of the strategies of Table 1 derived using the approach developed in this work. For Panel B, stability is enforced by setting the turnover of the strategy equal to the turnover of $1/N$, as described in the text. For Panel C, stability is enforced by maximizing last month's portfolio return (see text for more details).

	No transaction costs			Transaction Costs	
Strategy	$\hat{\mu}^p$	$(\hat{\sigma}^p)^2$	$\hat{\theta}^p$	$\hat{\tau}^p$	$\tilde{\theta}^p$
1/*N*	0.0547	0.0042	0.8405	0.0229	0.7983
Panel A: Original strategies					
TP	0.0651	0.0051	0.9143	0.0509	0.8289
MIN	0.0415	0.0032	0.7336*	0.0243	0.6833*
MV	0.0518	0.0033	0.8966*	0.0358	0.8232
MVC	0.0518	0.0033	0.8966*	0.0358	0.8232
LW	0.0424	0.0032	0.7473*	0.0237	0.6983*
NC	0.0562	0.0034	0.9613**	0.1609	0.6263**
3F	0.0608	0.0041	0.9455*	0.0360	0.8795*
VT	0.0348	0.0034	0.5956***	0.0231	0.5492***
ICV	0.0404	0.0032	0.7150*	0.0252	0.6627*
ICR	0.0473	0.0036	0.7840	0.2553	0.2704***
Panel B: Stable strategies (criterion 1*)*					
TP	0.0678	0.0053	0.9283	0.0229	0.8910
MIN	0.0419	0.0032	0.7380*	0.0229	0.6899*
MV	0.0538	0.0034	0.9280*	0.0229	0.8809*
MVC	0.0527	0.0033	0.9148	0.0229	0.8674
LW	0.0432	0.0033	0.7543*	0.0229	0.7066*
NC	0.0547	0.0038	0.8841	0.0229	0.8401
3F	0.0650	0.0043	0.9850**	0.0229	0.9438**
VT	0.0352	0.0035	0.5985***	0.0229	0.5520***
ICV	0.0405	0.0032	0.7146*	0.0229	0.6664*
ICR	0.0554	0.0039	0.8891	0.0229	0.8456
Panel C: Stable strategies (criterion 2*)*					
TP	0.0744	0.0057	0.9810	0.0091	0.9666
MIN	0.0498	0.0035	0.8398	0.0059	0.8278
MV	0.0611	0.0038	0.9939**	0.0113	0.9720**
MVC	0.0584	0.0038	0.9542*	0.0073	0.9399*
LW	0.0503	0.0035	0.8525	0.0059	0.8404
NC	0.0569	0.0040	0.9008*	0.0142	0.8737*
3F	0.0698	0.0048	1.0101***	0.0079	0.9963***
VT	0.0438	0.0037	0.7188**	0.0070	0.7050**
ICV	0.0486	0.0035	0.8223	0.0061	0.8099
ICR	0.0586	0.0041	0.9178*	0.0158	0.8881*

***, **, * indicate that the difference from the Sharpe ratio of $1/N$ is statistically significant at the 0.01, 0.05 and 0.10 level, respectively.

Table 4

Results for the 8 international market portfolios data set

This table summarizes the out-of-sample performance of the strategies described in Table 1 in the 8 international market portfolios data set. Panel A reports the average $\hat{\mu}^p$, variance $(\hat{\sigma}^p)^2$ and Sharpe ratio $\hat{\theta}^p$ of a strategy's excess returns in annual terms as well as the average monthly turnover $\hat{\tau}^p$. It also reports the annualized Sharpe ratio in the presence of proportional transaction costs of 100 basis points ($\tilde{\theta}^p$). The returns for the strategies are derived using a rolling window approach with a sample size of 120 months. The first 120 observations in the data set are held out for the initial estimation of the portfolio weights. Panels B and C report the same metrics as in Panel A for the stable counterparts of the strategies of Table 1 derived using the approach developed in this work. For Panel B, stability is enforced by setting the turnover of the strategy equal to the turnover of $1/N$, as described in the text. For Panel C, stability is enforced by maximizing last month's portfolio return (see text for more details).

	No transaction costs			Transaction Costs	
Strategy	$\hat{\mu}^p$	$(\hat{\sigma}^p)^2$	$\hat{\theta}^p$	$\hat{\tau}^p$	$\tilde{\theta}^p$
1/*N*	0.0577	0.0282	0.3438	0.0241	0.3265
Panel A: Original strategies					
TP	0.0956	1.8203	0.0708*	8.3554	-0.6725*
MIN	0.0432	0.0203	0.3031	0.1190	0.2026*
MV	0.0553	0.0248	0.3512	0.1650	0.2250*
MVC	0.0569	0.0256	0.3556	0.0939	0.2850
LW	0.0453	0.0204	0.3171	0.0833	0.2469
NC	0.0438	0.0231	0.2881	0.9667	-0.4722***
3F	0.5332	13.2279	0.1466**	20.4056	-0.5265***
VT	0.0596	0.0265	0.3660	0.0242	0.3482
ICV	0.0500	0.0205	0.3486	0.0992	0.2652
ICR	0.0372	0.0230	0.2453	1.1273	-0.6416***
Panel B: Stable strategies (criterion 1)					
TP	0.2236	0.5427	0.3035	0.0241	0.2995
MIN	0.0528	0.0225	0.3525	0.0241	0.3332
MV	0.0570	0.0237	0.3704	0.0241	0.3516
MVC	0.0595	0.0264	0.3660	0.0241	0.3482
LW	0.0509	0.0221	0.3427	0.0241	0.3232
NC	0.0599	0.0265	0.3681	0.0241	0.3503
3F	0.0544	0.0283	0.3237	0.0241	0.3065
VT	0.0598	0.0266	0.3670	0.0241	0.3493
ICV	0.0570	0.0227	0.3784	0.0241	0.3592
ICR	0.0635	0.0262	0.3924*	0.0241	0.3745*
Panel C: Stable strategies (criterion 2)					
TP	0.1056	0.2580	0.2080	0.0157	0.2043
MIN	0.0606	0.0251	0.3825	0.0094	0.3754
MV	0.0646	0.0258	0.4017*	0.0122	0.3926*
MVC	0.0601	0.0262	0.3709	0.0124	0.3617
LW	0.0602	0.0253	0.3785	0.0090	0.3717
NC	0.0602	0.0265	0.3700	0.0203	0.3551
3F	0.0549	0.0280	0.3277	0.0102	0.3204
VT	0.0602	0.0269	0.3670	0.0060	0.3627
ICV	0.0650	0.0258	0.4050*	0.0098	0.3977*
ICR	0.0635	0.0263	0.3916*	0.0227	0.3748*

***, **, * indicate that the difference from the Sharpe ratio of $1/N$ is statistically significant at the 0.01, 0.05 and 0.10 level, respectively.

Table 5

Results for the 10 industry portfolios data set

This table summarizes the out-of-sample performance of the strategies described in Table 1 in the 10 industry portfolios data set. Panel A reports the average $\hat{\mu}^p$, variance $(\hat{\sigma}^p)^2$ and Sharpe ratio $\hat{\theta}^p$ of a strategy's excess returns in annual terms as well as the average monthly turnover $\hat{\tau}^p$. It also reports the annualized Sharpe ratio in the presence of proportional transaction costs of 100 basis points ($\tilde{\theta}^p$). The returns for the strategies are derived using a rolling window approach with a sample size of 120 months. The first 120 observations in the data set are held out for the initial estimation of the portfolio weights. Panels B and C report the same metrics as in Panel A for the stable counterparts of the strategies of Table 1 derived using the approach developed in this work. For Panel B, stability is enforced by setting the turnover of the strategy equal to the turnover of $1/N$, as described in the text. For Panel C, stability is enforced by maximizing last month's portfolio return (see text for more details).

	No transaction costs			Transaction Costs	
Strategy	$\hat{\mu}^p$	$(\hat{\sigma}^p)^2$	$\hat{\theta}^p$	$\hat{\tau}^p$	$\tilde{\theta}^p$
1/*N*	0.0804	0.0220	0.5418	0.0240	0.5223
Panel A: Original strategies					
TP	0.0774	0.0718	0.2889	1.3037	-0.2954*
MIN	0.0865	0.0148	0.7107*	0.1570	0.5557
MV	0.0848	0.0159	0.6734*	0.2160	0.4665
MVC	0.0766	0.0156	0.6130	0.1143	0.5032
LW	0.0849	0.0143	0.7114*	0.1066	0.6044
NC	0.1053	0.0164	0.8221***	0.9483	-0.0755***
3F	0.0817	0.0170	0.6257	0.1797	0.4613
VT	0.0818	0.0189	0.5945*	0.0249	0.5727*
ICV	0.0868	0.0146	0.7191*	0.1142	0.6058*
ICR	0.1188	0.0171	0.9082***	1.0547	-0.0698***
Panel B: Stable strategies (criterion 1)					
TP	0.1476	0.1201	0.4260	0.0240	0.4178
MIN	0.0897	0.0153	0.7251*	0.0240	0.7017*
MV	0.0896	0.0173	0.6806**	0.0240	0.6586**
MVC	0.0781	0.0163	0.6115	0.0240	0.5889
LW	0.0844	0.0146	0.6997*	0.0240	0.6757*
NC	0.0820	0.0178	0.6150*	0.0240	0.5933*
3F	0.0798	0.0172	0.6083	0.0240	0.5863
VT	0.0819	0.0190	0.5946*	0.0240	0.5736*
ICV	0.0857	0.0148	0.7044**	0.0240	0.6806**
ICR	0.0863	0.0167	0.6678*	0.0240	0.6454*
Panel C: Stable strategies (criterion 2)					
TP	0.1231	0.0537	0.5314	0.0141	0.5241
MIN	0.0901	0.0168	0.6956*	0.0112	0.6853*
MV	0.0896	0.0185	0.6580*	0.0120	0.6474*
MVC	0.0789	0.0177	0.5936	0.0124	0.5824
LW	0.0866	0.0158	0.6885*	0.0116	0.6774*
NC	0.0823	0.0177	0.6181*	0.0205	0.5996*
3F	0.0795	0.0179	0.5940	0.0127	0.5826
VT	0.0817	0.0191	0.5906*	0.0058	0.5855*
ICV	0.0877	0.0158	0.6964*	0.0115	0.6854*
ICR	0.0854	0.0169	0.6561*	0.0228	0.6351*

***, **, * indicate that the difference from the Sharpe ratio of $1/N$ is statistically significant at the 0.01, 0.05 and 0.10 level, respectively.

Table 6

Results for the 25 size and book-to-market portfolios dataset

This table summarizes the out-of-sample performance of the strategies described in Table 1 in the 25 size and book-to-market portfolios data set. Panel A reports the average $\hat{\mu}^p$, variance $(\hat{\sigma}^p)^2$ and Sharpe ratio $\hat{\theta}^p$ of a strategy's excess returns in annual terms as well as the average monthly turnover $\hat{\tau}^p$. It also reports the annualized Sharpe ratio in the presence of proportional transaction costs of 100 basis points ($\tilde{\theta}^p$). The returns for the strategies are derived using a rolling window approach with a sample size of 120 months. The first 120 observations in the data set are held out for the initial estimation of the portfolio weights. Panels B and C report the same metrics as in Panel A for the stable counterparts of the strategies of Table 1 derived using the approach developed in this work. For Panel B, stability is enforced by setting the turnover of the strategy equal to the turnover of $1/N$, as described in the text. For Panel C, stability is enforced by maximizing last month's portfolio return (see text for more details).

	No transaction costs			Transaction Costs	
Strategy	$\hat{\mu}^p$	$(\hat{\sigma}^p)^2$	$\hat{\theta}^p$	$\hat{\tau}^p$	$\tilde{\theta}^p$
1/*N*	0.0893	0.0304	0.5123	0.0173	0.5004
Panel A: Original strategies					
TP	0.4869	0.5918	0.6329	38.1853	-5.3242**
MIN	0.1448	0.0177	1.0882***	0.8140	0.3462*
MV	0.1428	0.0177	1.0722***	0.8623	0.2870**
MVC	0.1282	0.0344	0.6919*	0.5008	0.3595*
LW	0.1212	0.0159	0.9617***	0.3197	0.6552**
NC	0.1416	0.0197	1.0098***	3.7935	-1.8537***
3F	0.2010	0.0289	1.1826***	1.3717	0.1968**
VT	0.0915	0.0279	0.5481*	0.0182	0.5351*
ICV	0.1267	0.0169	0.9754***	0.5659	0.4488
ICR	0.1589	0.0250	1.0059***	3.7375	-1.5378***
Panel B: Stable strategies (criterion 1)					
TP	0.2561	0.3141	0.4570	0.0173	0.4533
MIN	0.1158	0.0208	0.8025**	0.0173	0.7881**
MV	0.1175	0.0219	0.7950**	0.0173	0.7809**
MVC	0.0845	0.0255	0.5291	0.0173	0.5160
LW	0.1094	0.0201	0.7707**	0.0173	0.7560**
NC	0.0978	0.0271	0.5941	0.0173	0.5814
3F	0.1121	0.0288	0.6607*	0.0173	0.6484*
VT	0.0914	0.0279	0.5473*	0.0173	0.5348*
ICV	0.1117	0.0202	0.7865**	0.0173	0.7718**
ICR	0.0958	0.0257	0.5982	0.0173	0.5853
Panel C: Stable strategies (criterion 2)					
TP	0.2526	0.3162	0.4491	0.0117	0.4466
MIN	0.1106	0.0211	0.7610**	0.0094	0.7533**
MV	0.1120	0.0219	0.7573**	0.0095	0.7496**
MVC	0.0847	0.0248	0.5376	0.0102	0.5299
LW	0.1037	0.0203	0.7274*	0.0101	0.7189*
NC	0.0984	0.0270	0.5986*	0.0157	0.5872*
3F	0.1071	0.0290	0.6286*	0.0115	0.6205*
VT	0.0919	0.0279	0.5502*	0.0043	0.5471*
ICV	0.1061	0.0201	0.7486**	0.0093	0.7407**
ICR	0.0960	0.0255	0.6009*	0.0166	0.5885*

***, **, * indicate that the difference from the Sharpe ratio of $1/N$ is statistically significant at the 0.01, 0.05 and 0.10 level, respectively.

Table 7

Results for the 50 S&P 500 stocks dataset

This table summarizes the out-of-sample performance of the strategies described in Table 1 in the 50 S&P 500 stocks data set. Panel A reports the average $\hat{\mu}^p$, variance $(\hat{\sigma}^p)^2$ and Sharpe ratio $\hat{\theta}^p$ of a strategy's excess returns in annual terms as well as the average monthly turnover $\hat{\tau}^p$. It also reports the annualized Sharpe ratio in the presence of proportional transaction costs of 100 basis points ($\tilde{\theta}^p$). The returns for the strategies are derived using a rolling window approach with a sample size of 120 months. The first 120 observations in the data set are held out for the initial estimation of the portfolio weights. Panels B and C report the same metrics as in Panel A for the stable counterparts of the strategies of Table 1 derived using the approach developed in this work. For Panel B, stability is enforced by setting the turnover of the strategy equal to the turnover of $1/N$, as described in the text. For Panel C, stability is enforced by maximizing last month's portfolio return (see text for more details).

	No transaction costs			Transaction Costs	
Strategy	$\hat{\mu}^p$	$(\hat{\sigma}^p)^2$	$\hat{\theta}^p$	$\hat{\tau}^p$	$\tilde{\theta}^p$
1/*N*	0.1074	0.0233	0.7031	0.0501	0.6638
Panel A: Original strategies					
TP	-0.0253	0.1658	-0.0621	2.7254	-0.8265**
MIN	0.0352	0.0223	0.2353***	0.5227	-0.1862***
MV	0.0346	0.0222	0.2323***	0.5326	-0.1991***
MVC	0.0726	0.0135	0.6245	0.1284	0.4919*
LW	0.0661	0.0135	0.5685*	0.2133	0.3475**
NC	0.0992	0.0174	0.7518	1.6295	-0.7181***
3F	0.0641	0.0220	0.4319	0.5765	-0.0352***
VT	0.0970	0.0178	0.7270	0.0479	0.6841
ICV	0.0698	0.0134	0.6039	0.2033	0.3921**
ICR	0.1072	0.0178	0.8031*	1.6804	-0.7076***
Panel B: Stable strategies (criterion 1)					
TP	0.5950	12.7863	0.1664	0.0501	0.1647
MIN	0.0764	0.0328	0.4215*	0.0501	0.3883*
MV	0.0842	0.0479	0.3849***	0.0501	0.3574***
MVC	0.0748	0.0139	0.6342	0.0501	0.5831
LW	0.0737	0.0154	0.5943*	0.0501	0.5457*
NC	0.0934	0.0162	0.7342	0.0501	0.6868
3F	0.0923	0.0184	0.6802	0.0501	0.6358
VT	0.0972	0.0179	0.7260	0.0501	0.6810
ICV	0.0751	0.0138	0.6381	0.0501	0.5868
ICR	0.0929	0.0135	0.7982*	0.0501	0.7463*
Panel C: Stable strategies (criterion 2)					
TP	0.1822	5.2951	0.0792	0.0293	0.0777
MIN	0.1018	0.0606	0.4138*	0.0237	0.4022*
MV	0.1169	0.0909	0.3879***	0.0268	0.3772***
MVC	0.0752	0.0146	0.6232	0.0216	0.6018
LW	0.0765	0.0204	0.5354*	0.0226	0.5164*
NC	0.0938	0.0161	0.7405	0.0385	0.7040
3F	0.0953	0.0186	0.6982	0.0265	0.6749
VT	0.0967	0.0172	0.7371*	0.0081	0.7297*
ICV	0.0775	0.0150	0.6337	0.0196	0.6145
ICR	0.1001	0.0152	0.8132*	0.0472	0.7672*

***, **, * indicate that the difference from the Sharpe ratio of $1/N$ is statistically significant at the 0.01, 0.05 and 0.10 level, respectively.

Table 8

Performance of mean-variance portfolios that directly account for proportional transaction costs

This table reports the out-of-sample Sharpe ratio in annual terms (Panel A) and the average monthly turnover (Panel B) for the equally-weighted portfolio (1/*N*), the sample mean-variance portfolio (MV) and its short-sale constrained counterpart (MVC) under proportional transaction costs of 100 basis points. The statistics for the sample-based portfolios are reported for the case that the portfolios do not account for transaction costs (original) and when they directly account for proportional transaction costs (tcs), as described in Section 4. All performance measures are derived using a rolling window approach for a sample size of 120 months. The required level of return net of transaction costs for the mean-variance portfolios is set equal to the maximum return between 1/*N* and the global minimum variance portfolio.

Strategy	3FF	8Int	10Ind	25SBM	50SP
Panel A: Sharpe Ratio					
1/*N*	0.7983	0.3265	0.5223	0.5004	0.6638
MV (original)	0.8232	0.2250*	0.4665	0.2870	-0.1991***
MVC (original)	0.8232	0.2850	0.5032	0.3592*	0.4919*
MV (adjusted for tcs)	0.8658	0.3020	0.7065*	0.9132***	0.2052**
MVC (adjusted for tcs)	0.9096*	0.3241	0.6120*	0.4739	0.6016
Panel B: Turnover					
1/*N*	0.0229	0.0241	0.0240	0.0173	0.0501
MV (original)	0.0358	0.1650	0.2160	0.8623	0.5326
MVC (original)	0.0358	0.0939	0.1143	0.5008	0.1891
MV (adjusted for tcs)	0.0100	0.0236	0.0329	0.0382	0.1423
MVC (adjusted for tcs)	0.0083	0.0156	0.0165	0.0239	0.0651

***, **, * indicate that the difference from the Sharpe ratio of $1/N$ is statistically significant at the 0.01, 0.05 and 0.10 level, respectively.