Strategically Equivalent Contests

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February 25, 2013

Abstract

We use a Tullock-type contest to show that intuitively and structurally different contests can be strategically equivalent. Strategically equivalent contests generate the same best response functions and, as a result, the same efforts. Two strategically equivalent contests, however, may yield different equilibrium payoffs. We propose a simple two-step procedure to identify strategically equivalent contests. Using this procedure, we identify contests that are strategically equivalent to the original Tullock contest, and provide new examples of strategically equivalent contests. Finally, we discuss possible contest design applications and avenues for future theoretical and empirical research.

JEL Classifications: C72, D72, D74

Keywords: rent-seeking, contest, equivalence, contest design

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An earlier version of this paper was circulated under the title "The Equivalence of Contests." We have benefitted from the helpful comments of Kyung Hwan Baik, Tim Cason, Subir Chakrabarti, Dan Kovenock, Sanghack Lee, John Lopresti, Stephen Martin, Ted Turocy, seminar participants at Indian Statistical Institute Calcutta, IUPUI, Kookmin University, Purdue University, the University of East Anglia and participants at the Midwest Economic Theory Meetings at OSU. The usual disclaimers apply.

1. Introduction

A contest is a game in which players expend costly resources, such as effort, money or time, in order to win a prize. Since the seminal papers of Tullock (1980) and Lazear and Rosen (1981), many different contests have been introduced to the literature. For example, Skaperdas (1992) studies contests where the final payoff depends on the residual resources and the prize. Chung (1996) and Kaplan et al. (2002) examine contests with effort-dependent prizes. Lee and Kang (1998) and Baye et al. (2005) study contests with rank-order spillovers. Although these contests are intuitively and structurally very different, they often share common links.

There are several studies that establish common links between different contests. For example, Che and Gale (2000) provide a link between a rank-order tournament of Lazear and Rosen (1981) and an all-pay auction of Hillman and Riley (1989). Baye et al. (2012) show the connection between the all-pay auction and pricing games (Varian, 1980; Rosenthal, 1980). Hirshleifer and Riley (1992) show how an R&D race between two players which is modeled as a rank-order tournament is equivalent to a rent-seeking contest.[1] Baye and Hoppe (2003) identify conditions under which research tournament models (Fullerton and McAfee, 1999) and patent race models (Dasgupta and Stiglitz, 1980) are strategically equivalent to the rent-seeking contest. These duality results permit one to apply results derived in the rent-seeking contest literature to the innovation, patent race, and rank-order tournament models, and vice versa.

In this paper we show that intuitively and structurally different contests can be strategically and effort equivalent. We consider a two-player Tullock-type contest, where outcome-contingent payoffs are linear functions of prizes, own effort, and the effort of the rival.

[1] Jia (2008) extends the result by proving a more general equivalence between a rank-order tournament and a rent-seeking contest. Fu and Lu (2012) shows that the rent-seeking contest can further include auctions with pre-investment (Tan, 1992). Similarly, Cason et al. (2012) links the rent-seeking contest to a proportional-prize contest. Chowdhury (2009) demonstrates the connection between all-pay auctions (Siegel, 2009) and capacity-constrained price contests (Osborne and Pitchik, 1986; Deneckere and Kovenock, 1996).

Under this structure, we identify strategically equivalent contests that generate the same best response functions and, as a result, the same equilibrium efforts. However, the strategically equivalent contests may yield different equilibrium payoffs.

It is important to emphasize that the aforementioned studies establish links between different families of contests, such as all-pay auctions, rent-seeking contests, and rank-order tournaments. The main result of this paper is conceptually very different from the findings of the previous studies. In particular, we show that even within the same family of Tullock-type contests, different types of contests might produce the same best response functions and the same equilibrium efforts (although not necessarily the same payoffs).

This is an important finding for a number of reasons. First, there exists a substantial literature modeling the rules of the contest as an endogenous choice of a contest designer (Dasgupta and Nti, 1998; Epstein and Nitzan, 2006; Corchón and Dahm, 2011; Polishchuk and Tonis, 2012). A contest designer can choose the parameters of the model to maximize the total rent dissipation (as in the case of rent-seeking), or maximize the equilibrium highest effort (as in R&D races), or minimize the total equilibrium effort (as in electoral races), or simply to enhance public welfare. Our results demonstrate that it is possible for a contest designer to achieve different goals using strategically equivalent contests. For example, the contest designer seeking Pareto improvement may choose a contest that generates the same equilibrium efforts, incurs the same costs, but results in higher expected payoffs for contestants. Finally, certain contests may not be feasible to implement in the field due to regulatory restrictions, or due to the possibility of collusion among contestants. However, such restrictions may not apply to other strategically equivalent contests.

2. The Model

Following Baye et al. (2005, 2012) and Chowdhury and Sheremeta (2011a, 2011b), we consider a two-player contest with two prizes. The players, denoted by i and j, value the winning and the losing prizes as $W > 0$ and $L \in \mathbb{R}$, with $W > L$. Players simultaneously and independently expend efforts $x_i \geq 0$ and $x_j \geq 0$. The probability of player i winning the contest is defined by a lottery contest success function (Tullock, 1980):

$$p_i(x_i, x_j) = \begin{cases} x_i/(x_i + x_j) & \text{if } x_i + x_j \neq 0 \\ 1/2 & \text{if } x_i = x_j = 0 \end{cases} \tag{1}$$

Contingent upon winning or losing, the payoff for player i is a linear function of prizes, own effort, and the effort of the rival:

$$\pi_i(x_i, x_j) = \begin{cases} W + \alpha_1 x_i + \beta_1 x_j & \text{with probability } \quad p_i(x_i, x_j) \\ L + \alpha_2 x_i + \beta_2 x_j & \text{with probability } \ 1 - p_i(x_i, x_j) \end{cases} \tag{2}$$

where α_1, α_2 are cost parameters ($\alpha_1 < 0$, $\alpha_2 \leq 0$), and β_1, β_2 are spillover parameters. We define the contest described by (1) and (2) as $\Gamma(i, j, \Omega)$, where $\Omega = \{W, L, \alpha_1, \alpha_2, \beta_1, \beta_2\}$ is a set of parameters. All parameters in Ω and the contest success function are common knowledge. The players are risk neutral, therefore the expected payoff for player i is

$$E\big(\pi_i(x_i, x_j)\big) = \frac{x_i}{x_i + x_j}\big(W + \alpha_1 x_i + \beta_1 x_j\big) + \frac{x_j}{x_i + x_j}\big(L + \alpha_2 x_i + \beta_2 x_j\big) \tag{3}$$

where $(x_i, x_j) \neq (0,0)$. For $x_i = x_j = 0$, the expected payoff is $E\big(\pi_i(x_i, x_j)\big) = (W + L)/2$.

Player i's best response is derived by maximizing $E\big(\pi_i(x_i, x_j)\big)$ with respect to x_i:

$$x_i^{BRF} = -x_j + \sqrt{\frac{\{(\alpha_1 - \alpha_2) - (\beta_1 - \beta_2)\}x_j^2 - \{W - L\}x_j}{\alpha_1}} \tag{4}$$

Chowdhury and Sheremeta (2011a) show that although the payoff function (3) is not globally concave, the first order condition and the resulting best response function (4) are sufficient for an equilibrium to exist. Moreover, under the appropriate restrictions, i.e., $(\beta_2 - \alpha_1) \geq 0$ and $-(3\alpha_1 + \alpha_2) - (\beta_1 - \beta_2) > 0$, there is a unique symmetric equilibrium defined by:

$$x_i^* = x_j^* = x = \frac{(W-L)}{-(3\alpha_1+\alpha_2)-(\beta_1-\beta_2)}. \tag{5}$$

Given the symmetric equilibrium (5), the equilibrium payoff is

$$E^*(\pi) = \frac{(\beta_2-\alpha_1)(W-L)}{-(3\alpha_1+\alpha_2)-(\beta_1-\beta_2)} + L. \tag{6}$$

The contest $\Gamma(i, j, \Omega)$, defined by (1) and (2), may also generate asymmetric equilibria. Since in the current study we focus only on the symmetric equilibrium, we impose further restriction $(5\alpha_1 - \alpha_2) - (\beta_1 - \beta_2) > 0$, derived by Chowdhury and Sheremeta (2011b), to guarantee the uniqueness of the equilibrium.

3. Equivalent Contests

In this section, we define strategically equivalent contests and show the required parametric restrictions to obtain the equivalence. We start by providing a definition of strategic equivalence.

Definition 1: Contests are *strategically equivalent* if they generate the same best response functions.

This definition of strategic equivalence of contests comes directly from Morris and Ui (2004) who provide a general characterization of best-response equivalent games. It is however different from the definition used in Baye and Hoppe (2003) in which games are strategically equivalent when they generate the same expected payoff functions, and thus the same equilibrium payoffs. Here, we use a less strict definition of strategic equivalence, namely equivalence of the best response functions. It is usually the case in the contest design literature that a contest designer chooses the rules of the contest to induce a specific behavior of contestants (Dasgupta and Nti, 1998; Epstein and Nitzan, 2006). The contest designer is often indifferent towards the resulting payoffs of contestants. Thus, it seems appropriate to have a less restrictive definition of strategic equivalence that mainly relates to strategic behavior of contestants and not their payoffs. Nevertheless, one could use a more restrictive definition of strategic equivalence that also requires the equivalence of payoffs (see Definition 3). Moreover, a number of contests described in this paper are both strategically and payoff equivalent.

To demonstrate strategic equivalence, let us consider two contests $\Gamma^A(i, j, \Omega^A)$ and $\Gamma^B(i, j, \Omega^B)$, where $\Omega^k = \{W^k, L^k, \alpha_1^k, \alpha_2^k, \beta_1^k, \beta_2^k\}$ for $k = A, B$. From equation (4), the sufficient conditions for contests $\Gamma^A(i, j, \Omega^A)$ and $\Gamma^B(i, j, \Omega^B)$ to be strategically equivalent, i.e., to have the same best response functions, are the following:

$$\frac{(\alpha_1^A - \alpha_2^A) - (\beta_1^A - \beta_2^A)}{\alpha_1^A} = \frac{(\alpha_1^B - \alpha_2^B) - (\beta_1^B - \beta_2^B)}{\alpha_1^B} \text{ and } \frac{(W^A - L^A)}{\alpha_1^A} = \frac{(W^B - L^B)}{\alpha_1^B}. \quad (7)$$

Next, we define effort equivalent contests.

Definition 2: Contests are *effort equivalent* if they result in the same equilibrium efforts.

From equation (5), the sufficient condition for contests $\Gamma^A(i, j, \Omega^A)$ and $\Gamma^B(i, j, \Omega^B)$ to be effort equivalent is the following:

$$\frac{(W^A - L^A)}{-(3\alpha_1^A + \alpha_2^A) - (\beta_1^A - \beta_2^A)} = \frac{(W^B - L^B)}{-(3\alpha_1^B + \alpha_2^B) - (\beta_1^B - \beta_2^B)}. \quad (8)$$

Generally, strategic equivalence is a stronger condition than effort equivalence because it requires different contests to generate exactly the same best response functions, and as a consequence the same equilibrium efforts. However, given that in our analysis we impose restrictions to guarantee that only the unique and symmetric equilibrium (5) exists, *strategic equivalence* implies *effort equivalence* and vice versa. It is also important to emphasize that without spillovers, effort equivalence is the same as the revenue equivalence, since revenue of a contest designer is simply the sum of all individual efforts (Baron and Myerson, 1982; Moldovanu and Sela, 2001).[2]

In addition to strategic and effort equivalence, we also define payoff equivalent contests.

Definition 3: Contests are *payoff equivalent* if they generate the same expected payoffs.

From equation (6), the sufficient condition for contests $\Gamma^A(i, j, \Omega^A)$ and $\Gamma^B(i, j, \Omega^B)$ to be payoff equivalent, i.e., to generate the same equilibrium payoffs, is the following:

$$\frac{(\beta_2^A - \alpha_1^A)(W^A - L^A)}{-(3\alpha_1^A + \alpha_2^A) - (\beta_1^A - \beta_2^A)} + L^A = \frac{(\beta_2^B - \alpha_1^B)(W^B - L^B)}{-(3\alpha_1^B + \alpha_2^B) - (\beta_1^B - \beta_2^B)} + L^B. \tag{9}$$

It is easy to verify that strategic equivalence does not automatically imply payoff equivalence. As we show in the next section, depending on the cost and spillover parameters in Ω, one strategically equivalent contest can generate higher payoff than another. Nevertheless, most contests that we discuss are strategically, effort and payoff equivalent.

Finally, to simplify our analysis we assume that all alternative contests have the same winning prize and the same losing prize, i.e., $W^A = W^B = W$ and $L^A = L^B = L$. This assumption is intuitive given that the contest designer usually has specific pre-defined prizes which he can use to design a contest. Given this assumption, strategic and effort equivalence conditions (8) and (9) are simplified to the following condition:

$$\beta_2^A - \beta_1^A - \alpha_2^A = \beta_2^B - \beta_1^B - \alpha_2^B \text{ and } \alpha_1^A = \alpha_1^B. \tag{10}$$

In the rest of the paper, we follow a simple two-step procedure to find strategically equivalent contests to a particular baseline contest. First, we derive the best response function of the baseline contest as in equation (4). Second, from the best response function of the baseline

[2] In contests with spillovers there are different ways to define revenue, and thus effort equivalence may not imply revenue equivalence. For example, revenue can be defined as the sum of individual efforts and both positive and negative spillovers, or as the sum of efforts and only positive spillovers. Such alternative definitions of revenue would require different conditions for revenue equivalence. In this paper, however, we focus only on effort equivalence since eliciting individual efforts is usually the main objective of a contest designer.

contest we derive the restrictions needed, as in (10), for a more general family of contests to generate the same best response functions. This simple procedure is used throughout our analysis. We begin with the original contest of Tullock (1980) as the baseline contest.

3.1. Original Tullock Contest

In the standard rent-seeking contest, introduced by Tullock (1980), there is no losing prize and regardless of the outcome of the contest, both players forgo their efforts. In such a case, the winning prize value $W > 0$, $\alpha_1 = \alpha_2 = -1$, and the other parameters in Ω are zero. The payoff for player i in case of winning or losing is

$$\pi_i(x_i, x_j) = \begin{cases} W - x_i & \text{with probability} \quad p_i(x_i, x_j) \\ -x_i & \text{with probability} \quad 1 - p_i(x_i, x_j) \end{cases} \tag{11}$$

Using our notation, the Tullock contest is defined as $\Gamma(i, j, \{W, 0, -1, -1,0,0\})$. The resulting best response function in such a contest for player i is

$$x_i = -x_j + \sqrt{W x_j}. \tag{12}$$

For a generic contest $\Gamma(i, j, \{W, L, \alpha_1, \alpha_2, \beta_1, \beta_2\})$ to be strategically equivalent to contest $\Gamma(i, j, \{W, 0, -1, -1,0,0\})$, according to condition (10), we need to impose the following restrictions: $\beta_2 - \beta_1 - \alpha_2 = 1$, $\alpha_1 = -1$ and $L = 0$. Such restrictions guarantee that the best response function (4) is exactly the same as the best response function (12). Therefore, by definition these contests are strategically equivalent.

One particularly interesting case arises when we put further restrictions $\beta_1 = -1$ and $\alpha_2 = \beta_2 = 0$. In such a contest, $\Gamma(i, j, \{W, 0, -1,0, -1,0\})$, the new payoff function is:

$$\pi_i(x_i, x_j) = \begin{cases} W - x_i - x_j & \text{with probability} \quad p_i(x_i, x_j) \\ 0 & \text{with probability} \quad 1 - p_i(x_i, x_j) \end{cases} \tag{13}$$

Note that in (13), the winner fully reimburses the loser. This can be interpreted as the 'Marshall system of litigation' (Baye et al., 2005) in which the winner pays his own legal costs and also reimburses all of the legal costs of the loser, whereas the standard Tullock contest can be interpreted as the 'American system of litigation' in which each litigant pays its own legal expenses.[3] It can easily be shown that the unique equilibrium for contests defined by (11) and (13) is the symmetric equilibrium with $x_i^* = x_j^* = W/4$. Moreover, the expected payoff in both contests is exactly the same, $E^*(\pi) = W/4$. Therefore, contests (11) and (13) are strategically, effort and payoff equivalent. This equivalence is surprising, since the two contests are intuitively and structurally very different. However, it has been also shown in an all-pay auction setting under incomplete information (Baye et al., 2005). Therefore, our results provide further evidence that Marshall and American systems of litigation are revenue (in our case, effort) and payoff equivalent.

It is also straightforward to show that the 'input spillover' contest of Chowdhury and Sheremeta (2011a) and Baye et al. (2012), where the effort expended by player j partially affects player i and vice versa, is strategically equivalent to the original Tullock contest. The spillover contest can be defined as $\Gamma(i, j, \{W, 0, -1, -1, \beta, \beta\})$, where $\beta \in (-1,1)$ is the input spillover parameter. This type of contest is motivated by spillover effects in R&D innovation (D'Aspremont and Jacquemin, 1988; Kamien et al., 1992). From strategic equivalence condition (10), one can see that for any value of β, the resulting best response function is exactly same as in (11). Hence, the input spillover contest $\Gamma(i, j, \{W, 0, -1, -1, \beta, \beta\})$ is strategically equivalent to the original Tullock contest $\Gamma(i, j, \{W, 0, -1, -1,0,0\})$. This result suggests that if an R&D competition is modeled as a lottery contest, then the existence of symmetric spillovers may not

[3] Also see Matros and Armanios (2009) and Yates (2011) for further examples of this type of contests.

affect the equilibrium. However, the 'input spillover' contest is not payoff equivalent to the original Tullock contest, since condition (9) is not satisfied. It can be easily shown that a positive (negative) spillover provides a higher (lower) payoff to the players than the Tullock contest.

3.2. Modified Tullock-Type Contests

Researchers often use modified versions of the original Tullock contests in order to address specific questions such as taxes, subsidies, externalities, effort dependent valuations, cost differences, etc. There are instances in the literature where two different Tullock-type contests are strategically equivalent to each other. Here we briefly discuss some of these examples.

Chung (1996) assumes that the value of the winning prize depends on the total effort expenditures in the contest. A simple linear version of the Chung (1996) model would generate the following payoff function:

$$\pi_i(x_i, x_j) = \begin{cases} W + a(x_i + x_j) - x_i & \text{with probability} \quad p_i(x_i, x_j) \\ -x_i & \text{with probability} \quad 1 - p_i(x_i, x_j) \end{cases} \tag{14}$$

Hence, (14) can be described as $\Gamma(i, j, \{W, 0, a-1, -1, a, 0\})$, where $a \in (0,1)$, and the best response function is

$$x_i = -x_j + \sqrt{W x_j/(1-a)} \tag{15}$$

Lee and Kang (1998) study a contest with externalities. In their model the cost of effort decreases with the total effort expenditures. This contest can be captured by

$$\pi_i(x_i, x_j) = \begin{cases} W - x_i + b(x_i + x_j) & \text{with probability} \quad p_i(x_i, x_j) \\ -x_i + b(x_i + x_j) & \text{with probability} \quad 1 - p_i(x_i, x_j) \end{cases} \tag{16}$$

Hence, (16) can be described as $\Gamma(i, j, \{W, 0, b-1, b-1, b, b\})$, where $b \in (0,1)$, and the best response function is

$$x_i = -x_j + \sqrt{W x_j/(1-b)} \tag{17}$$

When $a = b$ the best response functions (15) and (17) and the equilibrium effort expenditures in the two contests are exactly the same. This result indicates that some contests with endogenous prizes, as in Chung (1996), are strategically equivalent to contests with externalities, as in Lee and Kang (1998). Also note that, although both contests are strategically equivalent, they are not payoff equivalent. In particular, the contest defined by (16) results in higher expected payoff than the contest defined by (14), providing a clear Pareto ranking between the two contests. Hence, a benevolent contest designer, such as the government trying to maximize the total social welfare, may opt to choose a contest that elicits the same level of expenditures and, at the same time, results in Pareto improvement for both contestants.

Next, we consider a 'limited liability' contest introduced by Skaperdas and Gan (1995), where the loser's payoff is independent of the efforts expended.[4] The authors motivate this example by stating that contestants may be entrepreneurs who borrow money to spend on research and development and thus are not legally responsible in case of a loss. The loser of such a contest is unable to repay the loan and goes bankrupt. In such a case, $W > 0$, $\alpha_1 = -1$, and the other parameters in Ω are zero. The payoff is:

$$\pi_i(x_i, x_j) = \begin{cases} W - x_i & \text{with probability} \quad p_i(x_i, x_j) \\ 0 & \text{with probability} \quad 1 - p_i(x_i, x_j) \end{cases} \tag{18}$$

The best response function for player i is:

$$x_i = -x_j + \sqrt{x_j^2 + W x_j} \tag{19}$$

For a contest to be strategically equivalent to $\Gamma(i, j, \{W, 0, -1, 0, 0, 0\})$ the required restrictions from (10) are $\beta_2 - \beta_1 - \alpha_2 = 0$, $\alpha_1 = -1$ and $L = 0$. When we impose further

[4] Example of these kinds of contests can also be found in Matros and Armanios (2009).

restrictions $\alpha_2 = -1$, $\beta_2 = -1$ and $\beta_1 = 0$ we obtain a contest with the following payoff function:

$$\pi_i(x_i, x_j) = \begin{cases} W - x_i & \text{with probability} \quad p_i(x_i, x_j) \\ -x_i - x_j & \text{with probability} \quad 1 - p_i(x_i, x_j) \end{cases} \tag{20}$$

This contest can be interpreted as a 'full liability' contest, since the loser has to pay in full the expenditures of both players. Note that although (18) is strategically equivalent to (20), the 'full liability' contest is (by definition) more risky than the 'limited liability' contest. In (18) players do not have to worry about what happens in the case of a loss, since they are not legally responsible. In contrast, the loser in (20) has to pay the expenditures of both players. Therefore, equivalence between (18) and (20) holds only under the assumption of risk neutrality. Moreover, it is easy to verify from (9) that contests (18) and (20) are not payoff equivalent. The equilibrium payoff in the 'full liability' contest is $E^*(\pi) = 0$ and in the 'limited liability' contest it is $E^*(\pi) = W/3$.

Alexeev and Leitzel (1996) study a 'rent-shrinking' contest $\Gamma(i, j, \{W, 0, -1, -1, -1, 0\})$, where the winning prize value decreases by the total effort expenditures. From (10), a strategically equivalent contest would require $\beta_2 - \beta_1 - \alpha_2 = 2$, $\alpha_1 = -1$ and $L = 0$. A 'lazy winner' contest $\Gamma(i, j, \{W, 0, -1, -2, 0, 0\})$ of Chowdhury and Sheremeta (2011a), in which the marginal cost of winning ($\alpha_1 = -1$) is lower than the marginal cost of losing ($\alpha_2 = -2$), definitely satisfies these restrictions. Moreover, the two contests are also payoff equivalent. The equivalence between the 'rent-shrinking' and 'lazy winner' contests enables the designer to achieve the same equilibrium rent dissipation using two alternative contests. Nevertheless, the 'lazy winner' contest is, arguably, easier to implement and it is less susceptible to the collusion problem mentioned in Alexeev and Leitzel (1996).

In many cases a contest designer can use different policy tools to implement a certain contest. Using the same procedure as before it can be shown that under certain restrictions, contests with endogenous valuations (Amegashie, 1999), contests with differential cost structure (Chowdhury and Sheremeta, 2011a), and contests with taxes (Glazer and Konrad, 1999), are strategically equivalent. Specifically, Glazer and Konrad (1999) study a contest $\Gamma(i,j,\{(1-t)w, 0, -1-t, -1, 0, 0\})$, in which a part of the rent seeker's non-negative profit is taxed with tax rate $t \in (0,1)$. Amegashie (1999) studies a contest $\Gamma(i,j,\{W,0,-(1-m),-1,0,0\})$, in which the winner's prize value is a linear function of own effort spent. Chowdhury and Sheremeta (2011a) study the 'lazy winner' contest $\Gamma(i,j,\{W,0,\alpha_1,\alpha_2,0,0\})$, in which the marginal cost of winning is lower than the marginal cost of losing, i.e., $|\alpha_1| < |\alpha_2|$. Using condition (7), when $(1-t)w = W$, $\alpha_1 - \alpha_2 = t = m$, and $\alpha_1 = (t-1) = (m-1)$ then the three contests are strategically and effort equivalent.

The equivalence between these three seemingly unrelated contests conveys an important message. It shows that the designer can either use policy tools, such as taxes, or contests with alternative cost structure to achieve the same objective. Moreover, the three contests do not necessarily generate the same equilibrium payoffs. The equilibrium payoff (under the restriction of strategic equivalence) in Glazer and Konrad (1999) is $E^*(\pi) = (1-t)^2\mathrm{W}/(4-3t)$, in Amegashie (1999) it is $E^*(\pi) = (1-t)W/(4-3t)$, and in Chowdhury and Sheremeta (2011a) it is $E^*(\pi) = (1-t)\mathrm{W}/(2-3t)$. Hence, a contest designer, such as a government trying to maximize the social welfare, can achieve a Pareto improvement by choosing a specific contest structure that generates the highest payoffs for players yet results in the same equilibrium efforts.

4. Discussion

In this paper we use a Tullock-type contest to show that intuitively and structurally different contests can be strategically equivalent. We define strategically equivalent contests as contests that generate the same best response functions. Under the assumption of a unique equilibrium, strategically equivalent contest are also effort equivalent. However, strategically equivalent contests may yield different equilibrium payoffs, and thus may not be payoff equivalent. We describe a simple two-step procedure to identify strategically equivalent contests. Using this procedure, we identify contests that are strategically equivalent to the original Tullock contest, and provide new examples of strategically equivalent contests.

We reestablish some existing results derived under alternative contest success functions and incomplete information, i.e., the equivalence of the American and Marshall systems of litigation. We also introduce new results, such as the equivalence between a standard Tullock contest and an input spillover contest, as well as the equivalence of a number of Tullock-type contests with endogenous valuations, spillovers, and differential cost structures.

Our findings contribute to the contest design literature by demonstrating how different strategically equivalent contests can be used to achieve the same objectives. A contest designer may choose to maximize the total rent dissipation, minimize equilibrium efforts, or simply enhance public welfare. Our results demonstrate that the contest designer can achieve these objectives by imposing appropriate restrictions on contest parameters. For example, we show that the two strategically and effort equivalent contests may yield different equilibrium payoffs. Hence, a contest designer seeking Pareto improvement may choose a contest that generates the same efforts, incurs the same costs, but results in higher expected payoffs for contestants.

It is important to understand the critical conditions required for the equivalence to hold in the field. Following the majority of the rent-seeking contests in the literature, we consider a two-player Tullock-type contest with linear cost and spillover structure under risk neutrality. The strategic equivalence results may not hold if we relax one or more of these assumptions to incorporate behavioral factors that can influence individual decisions but are not modeled in the current setting. For example, it has been shown in laboratory settings that contestants make mistakes (Sheremeta, 2011; Lim et al., 2012), have incorrect judgments (Parco et al., 2005; Amaldoss and Rapoport, 2009), exhibit non-monetary utility of winning (Sheremeta, 2010; Price and Sheremeta, 2011) and are usually risk averse (Millner and Pratt, 1991; Sheremeta and Zhang, 2010).[5] Some of these factors may distort individual behavior in strategically equivalent contests, and thus may break such equivalence. Finally, there are practical applications in which costs are convex (Moldovanu and Sela, 2001) and spillovers influence the payoff function in a non-linearly manner (Kräkel, 2004). A different analysis of equivalence would be required in such cases. Nevertheless, the concept of strategic equivalence and the two-step procedure to obtain strategically equivalent contests would be still relevant for such analyses. Using the two-step procedure one could, for example, find equivalence conditions with more than two players, risk aversion, and non-linear cost/spillover structure. Such analyses as well as the empirical tests of the equivalence in the laboratory are kept for future research.

[5] For an extensive review of the experimental literature on contests see Dechenaux et al. (2012) and for a review of behavioral explanations see Sheremeta (2013).

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