

Mathematic & Mathematics Education: Searching for Common Ground, edited by M. Fried and T. Dreyfus, New York, Springer, 2014, 402 pp., £90, ISBN: 978-94-007-7472-8

Hyman Bass (2005) writes in the Bulletin of the American Mathematical Society that the very close connection between mathematics and mathematics education is characterised by the fact that:

In no other science is there such an old tradition of scientists' commitment to education questions. (Bass, 2005, p. 417)

A quick search for “mathematics education” in the archives of the Notices of the American Mathematical Society (a journal aimed at mathematicians and widely read also outside the US where it is published) confirms this statement, returning 25,800 hits. The book *Mathematics & Mathematics Education: Searching for Common Ground*, to which Bass is one of the contributors, discusses some of the benefits, problems and characteristics of this close connection and of the ensuing collaborations between mathematicians and mathematics educators. But in what perspectives should this discussion be grounded?

Tony Becher famously talked of *academic tribes and territories* in his 1989 study of academic disciplines and their organization and in an early section of this book Steve Lerman reminds us that mathematics and mathematics education are two subjects with two very different ways of constructing knowledge: the first relies on deductive inference, the second on the analysis of empirical data. In Becher's words (1989) the first is a “hard-pure” discipline and the second a “soft-applied” one. Lerman also suggests that there is a need for a careful sociological analysis of both disciplines in order to understand their distinctions, commonalities and interdependence.

While this book is not, itself, that sociological analysis, it is an attempt to address some of these issues from the perspectives of the various chapter authors and respondents who have roles within and across both of the *academic tribes* of mathematicians and mathematics educators.

This book was written as a result of the symposium held at Ben Gurion University in May 2012 on the occasion of Ted Eisenberg's retirement. Eisenberg's background as both a mathematician and a distinguished researcher in mathematics education framed the symposium. Indeed after the *Preface*, the book starts with a *Dialogue on a Dialogue* between Presmeg, Eisenberg and Fried. This dialogue revisits an exchange that appeared in 2009 in the pages of ZDM about the nature of mathematics education and its relation to mathematics (Eisenberg and Fried, 2009; Presmeg, 2009a, 2009b). It exposes two contrasting views on what the relations between these two disciplines should be and especially on the nature of mathematics education. Eisenberg, and to some extent Fried, make a very strong argument for the advanced training in mathematics of researchers engaged in mathematics education at all levels and their right to be in mathematics departments. They stress how the exchange between mathematics and mathematics educators, facilitate by inhabiting the same academic department, can be and must be two ways: the mathematics educator can contribute to the mathematics knowledge of the mathematician and the mathematician can contribute to the research in mathematics education. A "win-win" situation as Eisenberg describes it (p. 40). Presmeg on the other hand elaborates how mathematics education has become a discipline in its own right, still concerned with the teaching and learning of mathematics, but importing ideas and research methods for a variety of other disciplines. Although she maintains the importance of advanced mathematics content knowledge for mathematics educators, Presmeg also recognises the importance of training in other disciplines such as for example sociology, psychology and philosophy. These two views are

clearly very different, and the difference of these views will emerge again in several sections of this book.

The structure of the book varies across sections. The general structure intended for the main part of the book is a sequence of sections in which one author provides a position paper on an area of mathematics education, followed by a chapter in which another author collates the reflections of a number of other scholars on that area. The list of areas is admirably wide including, for example, history of mathematics, policy, visualisation, argumentation and proof. There are also two chapters in the main section of the book that do not follow this structure, Chapter 5: *Mutual expectations*, and Chapter 8: *Problem solving*. In addition to these main chapters and the Dialogue, there is an Appendix focussed directly on Eisenberg's career that reflects the affection with which he is held and the gratitude felt for his contribution to mathematics education.

Before I discuss my reading of this book I think it is necessary to explain my own background – as my discussion will be inevitably shaped by my own professional history. I *fell* into mathematics education as the result of a chance encounter in a corridor with a colleague in the Education Department at the University of East Anglia. At the time I was working in the Mathematics Department after having completed my doctorate in pure mathematics. After this chance encounter I started *collaborating* with the colleague in the Education Department on a project investigating the learning of Analysis at university level. Participants were students on a module I was teaching. This led me, through a series of different collaborations, through coincidence and convenience to join the Education Department and my work straddles research in mathematics education and teaching in both education and mathematics. That background gives me an insight into some of the questions of identity which permeate this book: who is a mathematician, who is a mathematics educator, who is a mathematics education researcher? In

my current job I teach mostly in a Mathematics Department and I work with mathematicians on curriculum design while my research is in an Education Department. When I talk about my research to colleagues in the Mathematics Department I am often asked: “And what does this mean for my teaching”? It has been my experience that mathematics education researchers rarely find a language suitable to discuss the applied side of their research with the colleagues who teach mathematics, and that mathematicians do not often grasp the kind of questions that mathematics education can reasonably answer. So my experience of working in both communities and, in some way, of being at their interface has made me sensitive to issues of communication and language.

It is always difficult to review edited books with so many contributors but to summarise what I took away from this book, on the one hand it is commendable and important that efforts are made to bring together mathematicians and mathematics educators and to engage them in reflection on collaboration. Every mathematician and/or mathematics educator knows that working together can be a difficult experience (the difficulties have been discussed in several forums, see for example Ralston, 2004) and that lack of communication is always detrimental both to mathematics and to mathematics education. They would also recognise that mathematics and mathematics education are necessary to each other, as it is pointed out throughout the book, as neither could exist in isolation. It is also important to look at this collaboration closely, to present examples where this collaboration has been very fruitful and to analyse, where possible, the characteristics that have made projects successful.

On the other hand I found that the book lacks coherence both at macro level and micro level. At the macro level, while the choice of the overarching theme is clearly justified, it is not clear why the themes of some of the chapters were chosen (why history of mathematics and not for

example early number formation?). Although an attempt to link the themes of the chapters to Eisenberg's work is made in the concluding note by Thompson, this link is not convincing. At the micro level, the chapter format of a position paper and a series of coordinated reactions is not always successfully realised, as I elaborate below.

Chapter 11 on visualisation in mathematics and mathematics education is a successful realisation of the format intended by the editors. As a non-specialist in this part of mathematics education, I have found it informative and a pleasure to read, especially the position paper by Clemens. This is a detailed reflection on what is meant by visualisation, its relation to problem solving and the verbal-visual divide. I found particularly helpful the concrete examples of tasks from research studies included in the paper. For example the description of one of the tests used by Wattanawaha in his work (1977) is effective at illustrating the four components of his characterisation of spatial tasks: Dimensionality (Does the test require 1D, 2D or 3D thinking?); Internalization (Does the solution require a visual image or not?); Presentation (Does the answer require a visual image or not?); Thought process (Is this specified in the task or not?). The task included in the chapter consisted in applying a given rotation to a 3D object with labelled corners shaped as a capital H so that the solver would have to work out in which position two of the corners landed after the rotation. As a reader I welcomed the opportunity to engage with the test as this helped me clarify the classification proposed, and my reasoning within it. The short reply sections are also relatively coherent and helpful for the reader to construct an image of the main debates in this part of mathematics education. Other chapters realise the original format less well, where the position paper is less clear and the replies appear to have little connection to what the position paper asks us to consider, nor to the issues of collaboration and common ground. One such example is the chapter about mathematical reasoning, justification and proof (Chapter 13).

Tall's position paper illustrates mathematics learners' progress from informal justifications and argumentations to the highest level of formal proof, from school to more advanced mathematics studies. In order to do so, Tall discusses several important constructs which have emerged from studies on proof such as met-befores and crystalline concepts. The position paper is interesting and coherent, although it offers a very particular way of looking at proof and argumentation. The accompanying reactions chapter is interesting in itself, but rather than reflecting on Tall's chapter, the different 'reflections' each offer their own particular view. They tackle separate aspects of argumentation and proof and most fail to consider both the content of the position paper and the overarching theme of collaboration. As a result the section appears mostly a collection of authors' own theories and studies on argumentation and proof with little connection between them or to the theme of the book.

One section, however, explicitly examines the issue of collaboration between the two *tribes* and comprises a collection of personal experiences and examples of such collaboration. The main chapter in this section helps indicate what can happen when commonality and compatibility of aims and objectives are achieved. It reviews the long-standing research collaboration between Hyman Bass and Deborah Ball, which, within this book, may be the clearest example of what happens when one finds common ground and I would add a common language. The collaboration between Ball and Bass spans over more than 15 years and, as the authors explain, is funded on mutual respect and it is grounded in mathematics as a discipline of study, instruction as practice and mathematics education research as the study (amongst other things) of such practice. Interestingly, the authors also mention the difference in the respective disciplines of what counts as an evidence, they acknowledge such difference and describe the way in which they have transformed this difference into strength of their collaboration. The other examples of

collaboration are particularly valuable in terms of what has been successful in different contexts: for example, Artigue reflects on her collaborations: from personal interactions with mathematicians, through working with mathematicians on national groups and commissions, to collaborating on teaching programmes and dissemination activities. In his contribution, Törner even includes an insightful set of “dos and don’ts” which come from his experience of collaboration. For me, the most immediate of these is:

Collaboration must first be grounded in communication. (Törner, 2014, p. 321)

Communication however can only be achieved by finding the right language, and this issue of language is of course intimately related to that of readership. It is not always clear for whom this book was written. If it aims to address both communities of mathematicians and of mathematics educators (researchers), then there are language issues here that may hinder clear communication. Some of the sections are difficult to read for a non-specialist: in particular, a mathematician who is not well versed in the discourse of mathematics education as embodied in the language of its research papers and theories may be frustrated. Indeed the book is written predominantly in the language of mathematics education and my experience suggests that mathematicians who do care about their students’ learning but who may not wish to learn the language and theories of a whole new field would not be willing to engage with some parts of this book. Of course, those well versed in mathematics education language will be able to read about parts of mathematics education which are outside their own specialism. This was true for me in, for example, the section on visualisation. The two *tribes* may not have yet found their common language in this book.

Finally I would like to notice that the war metaphor has pervaded too much in mathematics education, the US “math wars” being probably the most unseemly, though the *tribes* involved in this *war* were not simply mathematicians and mathematics educators but included politicians and policy makers. This book, while flawed in some respects, at its best does show that different communities such as mathematicians and mathematics education researchers can work together, can understand each others’ position and thus can be considerably less *tribal*.

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