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Nature of Spatiotemporal Light Bullets in Bulk Kerr Media

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We present a detailed experimental investigation which uncovers the nature of light bullets generated from self-focusing in a bulk dielectric medium with Kerr nonlinearity in the anomalous group velocity dispersion regime. By high dynamic range measurements of three-dimensional intensity profiles, we demonstrate that the light bullets consist of a sharply localized high-intensity core, which carries the self-compressed pulse and contains approximately 25% of the total energy, and a ring-shaped spatiotemporal periphery. Subdiffractive propagation along with dispersive broadening of the light bullets in free space after they exit the nonlinear medium indicate a strong space-time coupling within the bullet. This finding is confirmed by measurements of a spatiotemporal energy density flux that exhibits the same features as a stationary, polychromatic Bessel beam, thus highlighting the nature of the light bullets.

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Propagation-invariant electromagnetic wave packets—light bullets—have been long sought in many areas of modern optics and attract a great deal of interest in fundamental and applied research [1]. The generation of three-dimensional light bullets, which propagate in the medium without natural dispersive broadening and diffractive spreading, is a nontrivial task from the analytical and numerical points of view, and even more complicated to achieve in real experimental settings [2–4].

The pursuit of three-dimensional light bullets considers two essentially different physical concepts. The first is based on the generation of spatiotemporal solitons, which could be regarded as ideal light bullets with rapidly decaying tails, constituting a high degree of energy localization. The formation of the spatiotemporal soliton relies on simultaneous balancing of diffraction and dispersion by nonlinear effects, such as self-focusing and self-phase modulation [5]. In the first approximation, these conditions could be met in media with Kerr nonlinearity in the anomalous group velocity dispersion (GVD) range [5]. However, the proposed light bullet is a three-dimensional extension of the universal Townes profile, therefore possessing similar properties: it forms only at the nonlinear focus [6] and is modulationally unstable [7]. Therefore, for achieving a spatiotemporally invariant propagation, linear and nonlinear optical properties of the medium must be suitably tailored—see, e.g., Refs. [8–12]—thus raising difficult technological challenges. So far, an experimental demonstration of three-dimensional light bullets was demonstrated in coupled waveguide arrays with specifically designed optical properties [13–16]. In such an environment, vortex light bullets were also recently reported [17].

The second concept for achieving light bullets is based on the precise tailoring of the input wave packet so as to match the material properties, i.e., defeating the natural diffractive spreading and dispersive broadening in the linear propagation regime. Linear light bullets are nonsolitary, weakly localized wave packets, whose stationary propagation is achieved due to the Bessel-like profile of the beam, whose spectral components are distributed over certain propagation cones, so as to continuously refill the axial part, which contains an ultrashort pulse. Moreover, this approach is equally effective in media with normal, as well as anomalous, GVD. To this end, nonsolitary spatiotemporal linear light bullets have been experimentally demonstrated in the form of the *X* waves [18], Airy bullets [19,20], and ultrashort-pulsed Bessel-like beams [21]. However, practical realizations of the linear light bullets require precise control of propagation angles and phases of the spectral components, and therefore intricate experimental techniques.

Another route for achieving nonsolitary, weakly localized light bullets is based on the self-reshaping of the entire wave packet by means of nonlinear effects, producing the nonlinear analogs of the *X* waves [22,23] and Airy bullets [24]. In particular, spontaneous formation of the nonlinear *X* waves was demonstrated by means of femtosecond filamentation in transparent dielectrics, where the input Gaussian-shaped wave packet self-adjusts its spatiotemporal shape via nonlinear effects into a specific spatiotemporal *X* shape, which maintains its stationarity even in the presence of the nonlinear losses [25–27]. However, pulse splitting, which occurs during the filamentation of intense femtosecond pulses in the normal GVD regime [28], prevents the formation of a single *X* wave.

Conversely, studies of filamentation in the conditions of anomalous GVD predicted the generation of isolated spatially and temporally compressed pulses [29–34]. Recent investigations have uncovered a filamentation regime, in which quasistable three-dimensional nonspreading pulses are generated [35]. However, the interpretation of these light bullets has, to date, relied on fundamentally different "solitonic" and "nonsolitonic" (i.e., conical) concepts, which coexist as long as they are being employed to explain different (spatial, temporal, spectral) features separately, as based on a very simple experimental characterization, where the details of fundamental importance are not captured or overlooked. In this Letter, we provide a comprehensive experimental characterization (high dynamic range measurements of spatiotemporal intensity profiles, angularly resolved spectra, energy fluxes and free-space propagation), which uncovers the physical nature of the light bullets in bulk media with anomalous GVD. We explicitly demonstrate that these light bullets are polychromatic Bessel-like wave packets which bear the basic properties of the nonlinear O waves [36].

The experiment was performed using 90-fs Gaussian pulses with a center wavelength of 1.8 μm from an optical parametric amplifier. Its output beam (an idler wave, in the present case) was suitably attenuated, spatially filtered, and focused by an $f = +100$-mm lens into a 45-μm (FWHM) spot size which was located on the front face of the sapphire sample. The input pulse energy of 3.1 μJ (corresponding to $3.4P_{\mathrm{cr}}$, where $P_{\mathrm{cr}} = 10$ MW is the critical power for self-focusing in sapphire) was set so as to induce a light filament, which formed after 4 mm of propagation, as verified by the supercontinuum emission in the visible spectral range. The spatiotemporal intensity distribution at the output of the sapphire sample was measured by a three-dimensional imaging technique, based on recording the spatially resolved cross-correlation function [23,37,38]. More specifically, the output beam was imaged onto a 20-μm-thick beta-barium borate crystal and gated by means of broadband sum-frequency generation with a short, 25-fs pulse with a central wavelength of 720 nm from a noncollinear optical parametric amplifier. The cross-correlation signal with a center wavelength of 515 nm was then imaged onto the CCD camera with a 14-bit dynamic range (Grasshopper 2, Point Grey). By changing the time delay of the gating pulse in an 8-fs step, we acquired a sequence of cross-correlation images, which afterward were merged together to reproduce the entire spatiotemporal intensity distribution of the light bullet with 25-fs temporal and 5-μm spatial resolution.

The evolution of the spatiotemporal intensity distribution over a propagation distance z was captured by placing sapphire samples of different lengths in such a way that the output face of the sample was always kept at the same fixed position, while moving the focusing lens accordingly, to ensure the location of the input focal plane at the front face of the sample. Figure 1 presents the spatiotemporal intensity profiles as measured at various propagation distances in sapphire, showing how the input Gaussian wave packet transforms into a spatially and temporally compressed three-dimensional light bullet. Over the first few mm of propagation, the input Gaussian beam shrinks due to self-focusing and the input Gaussian pulse experiences self-compression due to the interplay between self-phase modulation and anomalous GVD. After the nonlinear focus ($z = 4.2$ mm), the wave packet transforms into a spatially and temporally compressed three-dimensional light bullet, which has a FWHM diameter of 15 μm and a pulse duration of 40 fs, as evaluated from the deconvolution of the on-axis cross-correlation function. High dynamic range measurements reveal that the light bullet consists of a sharply localized high-intensity core, which carries the self-compressed pulse and a low-intensity, ring-shaped spatiotemporal periphery and propagates without an apparent change of its spatiotemporal shape. Almost identical spatiotemporal shapes were also measured for the light bullets with other input wavelengths (2.2 μm) and in other nonlinear media, such as fused silica (not shown). Propagation dynamics of the light bullet at 1.8 μm are summarized in Fig. 2, where full circles show the beam width and pulse duration versus propagation distance, demonstrating that the high-intensity core maintains its localization over more than 10 mm of propagation. For a comparison, diffraction and dispersion lengths for a Gaussian wave packet of equivalent dimensions are 0.5 and 7.5 mm, respectively.

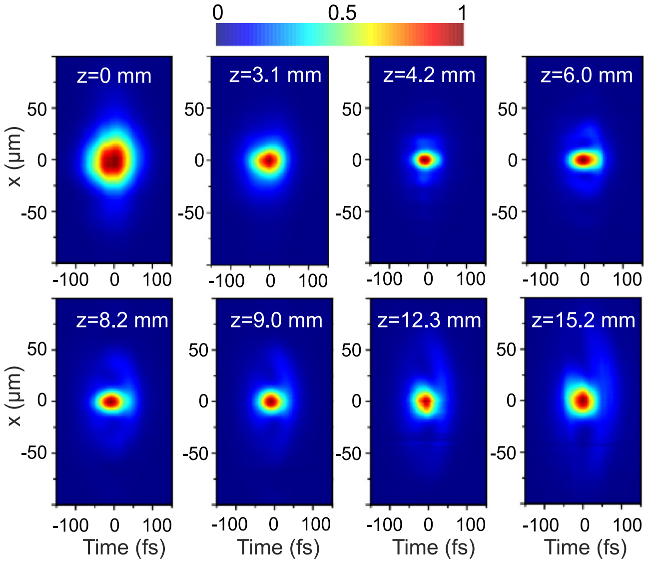

FIG. 1 (color online). Spatiotemporal intensity distributions of the self-focusing Gaussian wave packet, which transforms into a light bullet, as measured at various propagation lengths z in sapphire.

In a further experiment, where the light bullet after 6 mm of propagation in sapphire was thereafter allowed to

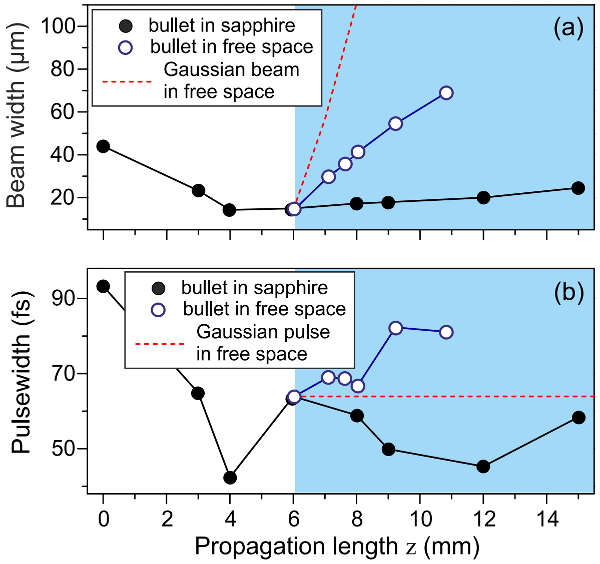

FIG. 2 (color online). Evolution of (a) beam FWHM diameter and (b) pulse width versus propagation length. Full circles show the formation and propagation dynamics of the light bullet in sapphire, as summarized from Fig. 1. Open circles show propagation of the light bullet in free space (air), which starts at $z = 6$ mm. Dashed curves indicate the expected free-space propagation of a strongly localized Gaussian wave packet.

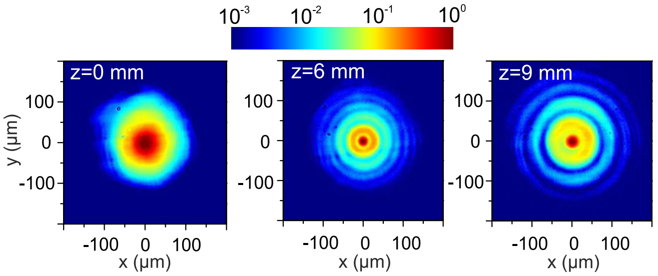

FIG. 3 (color online). Spatial profiles of the input beam ($z = 0$ mm) and the light bullet at $z = 6$ mm and $z = 9$ mm.

propagate in free space (air), we recorded interesting and very important propagation features. We observe a gradual increase of both spatial and, more importantly, temporal dimensions of the central core, as the propagation distance in free space increases, as shown by open circles in Fig. 2. These data are compared with the calculated linear evolution of a strongly localized Gaussian wave packet with identical spatial and temporal dimensions (shown by the dashed curves), which represent the expected spreading of a solitonlike object in free space. The distinctive differences in free-space propagation between the present light bullet and a solitonlike object are immediately clear: the diffraction spreading of the bullet is almost 5 times less than that of a Gaussian-shaped beam, and its temporal width increases by a factor of 1.3, just after 3 mm of propagation, in the absence of dispersion, while for a solitonlike object, it is expected to remain constant. These results demonstrate that the light bullets, as they exit the nonlinear medium, exhibit subdiffractive and dispersive propagation in free space that is incompatible with the behavior of highly localized spatiotemporal solitonlike objects. The linear and nonlinear propagation features of the light bullets arise from a dramatic spatiotemporal reshaping of the input Gaussian wave packet and resulting strong space-time coupling, which is a distinctive property of conically shaped wave packets [39].

In support of this claim, we studied the relevant characteristics of the light bullet in more detail. Figure 3 illustrates the spatial profiles of the input Gaussian beam and the light bullet, as obtained by the time integration of spatiotemporal profiles presented in Fig. 1. The spatial profiles are presented in a logarithmic intensity scale and reveal an intense central core, which contains approximately 25% of the total energy and is surrounded by a low-intensity concentric ring-shaped periphery, thus resembling a distinct Bessel-like intensity distribution, which emerges from the interplay of self-focusing, nonlinear absorption and diffraction [40]. A Bessel-like intensity distribution of the light bullet explains the spatial features of the central core observed in the experiment: its robustness during the nonlinear propagation in sapphire, as well as the subdiffractive propagation in free space, which are achieved via energy refilling from the beam periphery [40,41].

Figure 4 highlights the characteristic properties of the input wave packet and the light bullet. The top row compares the near-field (x, t) spatiotemporal intensity profiles in the logarithmic intensity scale, so as to better visualize the entire spatiotemporal structure of the light bullet. The middle row presents the corresponding angularly resolved (θ, λ) spectra, as measured by scanning the far field with a 200-μm fiber tip of a fiber spectrometer (AvaSpec-NIR256-2.5, Avantes). The angularly resolved spectra suggest the occurrence of an elliptical, or O-shaped, pattern of conical emission that is expected from the Kerr-driven spatiotemporal instability gain profile in the case of anomalous GVD [42,43]. The rings observed in the near-field profile (Fig. 3) are due to the interference effects inherent to Bessel beams, giving rise to the nontrivial conical emission observed in the far field. The near- and far-field intensity profiles are combined together to obtain the full spatiotemporal phase profile of the light bullets by means of an iterative Gerchberg-Saxton retrieval algorithm; see Refs. [44,45] for details. The gradient of the retrieved phase profile is used to explicitly visualize the transverse energy density flux in the full spatial and temporal coordinates, as shown as normalized in the bottom row of Fig. 4, along with an overlayed contour plot of the retrieved intensity profiles. Here, blue and red colors indicate downward and upward fluxes with respect to the vertical axis, respectively.

The transverse energy density flux of the light bullet indicates a radially symmetric pulse-front tilt resulting from

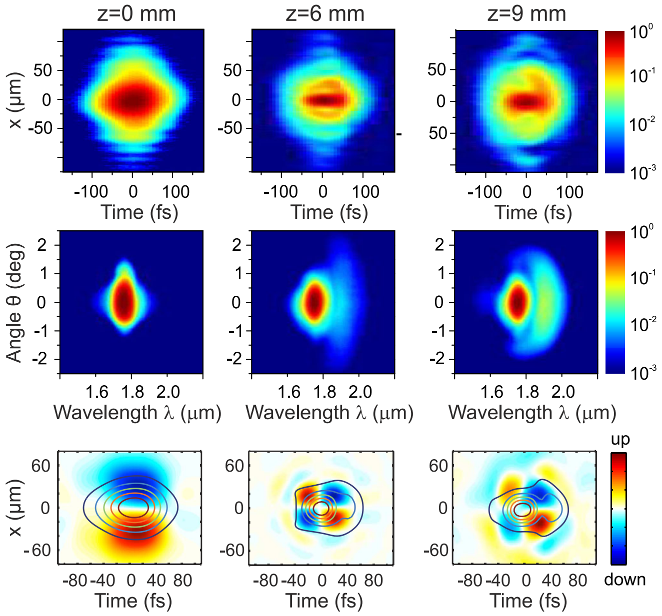

FIG. 4 (color online). Top row: Spatiotemporal intensity profiles of the input wave packet and the light bullet. Middle row: Angularly resolved spectra. Bottom row: Retrieved transverse energy fluxes.

strong space-time coupling, and the spatiotemporal distribution of the currents is almost identical to that of a stationary, polychromatic Bessel beam with a subluminally propagating envelope peak [46], as schematically depicted in Fig. 5. The established subluminal propagation of the envelope peak is very much in line with the results of numerical simulations in fused silica [35,47], where the position of the peak is shown to continuously shift toward positive times with propagation. The radially symmetric pulse-front tilt, which is owed to angular dispersion [48], explains the observed features of the temporal behavior of the light bullet in the nonlinear and free-space propagation regimes, as illustrated Fig. 2(b). The propagation angles of the spectral components comprising the light bullet compensate for material dispersion in a dispersive medium, whereas such angular distribution causes the dispersive spreading of the pulse, as the bullet exits the dispersive medium and propagates in free space. In addition, the presence of the pulse-front tilt may readily explain the reported pulse width dependence on the aperture size in the autocorrelation measurements [34]. We underline that a solitonlike object would be expected to exhibit a flat phase and hence no transverse energy density flow in the spatiotemporal domain. Conversely, the conical flux unveiled here is inherent to polychromatic Bessel-like pulses and hence serves as an unambiguous demonstration of the nature of the light bullets generated by self-focusing in a bulk dielectric medium with anomalous GVD. More precisely, the entirety of established properties of the bullet —a quasistationary *O*-shaped spatiotemporal intensity profile and characteristic angularly resolved spectrum, a Bessel-like spatial intensity distribution, and a transverse energy flux along with nonlinear and free-space propagation features—closely resemble those of the nonlinear *O* waves featuring weak losses and a subluminally propagating envelope peak [36].

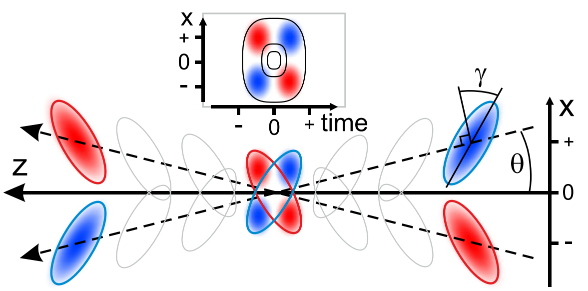

FIG. 5 (color online). Schematic representation of the transverse energy flux in an ultrashort-pulsed (polychromatic) Bessel beam with a subluminally propagating envelope peak. θ and γ are arbitrary cone and pulse-font tilt angles, respectively.

In conclusion, we uncovered the nature of three-dimensional light bullets generated from the self-focusing of intense femtosecond pulses in bulk dielectric media with anomalous GVD. The self-focusing dynamics of 100-fs, 1.8-μm pulses in sapphire was experimentally captured in detail in full four-dimensional space by means of a three-dimensional imaging technique. We demonstrate that the emerging light bullets consist of a sharply localized high-intensity core, which carries the self-compressed pulse and a weak, delocalized low-intensity periphery, comprising a Bessel-like beam. We explicitly demonstrate that the seemingly weak periphery, as viewed in the linear intensity scale, is nonetheless an important integral part of the overall wave packet, as it continuously balances energy losses in the central core and prevents it from spreading during its linear and nonlinear propagation. We disclose that spatiotemporal reshaping of the input Gaussian wave packet results in the development of a very distinct spatiotemporal flow of the energy that is not compatible with a spatiotemporal soliton but rather finds a natural explanation in terms of a polychromatic Bessel beam, which could be qualified as a nonlinear *O* wave which has weak losses and a subluminally propagating envelope peak [36]. As a consequence of this, the light bullets exhibit a rather remarkable behavior as they exit the sample and propagate in free space, i.e., in the absence of any nonlinear or dispersive effects: the bullets disperse temporally, yet continue to propagate with strongly suppressed diffraction. We expect that these important features are characteristic of an entire family of spatiotemporal light bullets, which are generated by femtosecond filamentation in bulk dielectric media with anomalous GVD and should be carefully accounted for building the basis for diverse future applications that may require a simple setup for creating intense, temporally compressed, and subdiffractive wave packets [49].

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