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Nonunitary generation of nonclassical states of a bidimensional harmonic oscillator

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A scheme for generating quantum superpositions of macroscopically distinguishable states of the vibrational motion of a bidimensionally trapped ion is reported. We show that these states possess highly nonclassical properties controllable by an adjustable parameter simply related to the initial condition of the confined system

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Over the last few years there have been developed sophisticated techniques of laser cooling and trapping of atoms, opening a new research field for testing fundamental features of atomic physics and quantum optics [1–4]. It is possible, in fact, to demonstrate that an ion confined in an electromagnetic trap is describable as a particle in a harmonic potential in the sense that its center of mass (c.m.) can be quantized as a harmonic oscillator [5,6]. Appropriately driving the trapped ion by classical laser beams, its internal and external degrees of freedom can be coupled. Thus, simply by controlling the configuration of the driving lasers, it becomes possible to manipulate the external motion of the confined ion. It is, moreover, of particular relevance that, if the Lamb-Dicke limit is satisfied and the driving field is tuned to one of the vibrational sidebands of the atomic transition, then the quantum dynamics of such systems may be deduced from generalized nonlinear Jaynes-Cummings models wherein the quantized radiation field is, obviously, replaced by the quantized c.m. motion of the ion [7]. This prominent feature directly leads to the possibility of testing the rich dynamics predicted by these models using trapped ions instead of cavities. One of the advantages of exploiting such systems is related to the circumstance that typical dissipative effects strongly limiting the performance of experiments in cavities, in the optical as well as in the microwave regime, can be significantly suppressed for the ion motion, thanks to the extremely weak coupling between the vibrational modes and the external environment. It is thus not surprising that ions confined by electromagnetic fields are eligible systems for producing, for example, specific nonclassical (vibrational) states. Several schemes for the generation of Fock states, coherent states, and squeezed states have, in fact, been reported and realized in this context [8–13]. Quite recently, Monroe *et al.* [14] have proposed an experimental scheme for generating and detecting a Schrödinger-cat-like state of a trapped ion providing insight into the fuzzy boundary between the classical and quantum worlds. The state generated by exploiting the procedure of Monroe *et al.* is given by a superposition of two coherent state wave packets of a single trapped ion. Over the last few years, some interesting methods for creating generalized coherent states of the bidimensional vibrational c.m. motion have been reported [15–18]. In this paper we present an original scheme aimed at generating quantum superpositions of bosonic SU(2) macroscopically distinguishable coherent states [19,20] by exploiting the wave packet reduction method. In particular, we show that these states possess nonclassical properties controllable by an adjustable parameter simply related to the initial condition imposed on the confined system.

Consider a two-level ion of mass M confined in a bidimensional isotropic harmonic potential characterized by a trap frequency ν. The operators $\hat{a}$ ($\hat{a}^\dagger$) and $\hat{b}$ ($\hat{b}^\dagger$) defined as

$$\hat{a}=\frac{1}{\sqrt{2}}\left(\sqrt{\frac{M\nu}{\hbar}}\hat{X}+i\frac{1}{\sqrt{M\nu\hbar}}\hat{P}_x\right), \tag{1}$$

$$\hat{b}=\frac{1}{\sqrt{2}}\left(\sqrt{\frac{M\nu}{\hbar}}\hat{Y}+i\frac{1}{\sqrt{M\nu\hbar}}\hat{P}_y\right) \tag{2}$$

are the annihilation (creation) operators of vibrational quanta in the X and Y directions, respectively. It has been shown [17] that by irradiating the trapped ion with an appropriate configuration of laser beams the physical system under scrutiny can be studied, in the Lamb-Dicke limit and in the interaction picture, by the following Hamiltonian model:

$$\hat{H}=g[(\hat{a}\hat{b})\hat{\sigma}_+ +(\hat{a}^\dagger\hat{b}^\dagger)\hat{\sigma}_-], \tag{3}$$

where $\hat{\sigma}_z=|+\rangle\langle+|-|-\rangle\langle-|$, $\hat{\sigma}_+=|+\rangle\langle-|$, $\hat{\sigma}_-=|-\rangle\langle+|$ describe the internal degrees of freedom, $|+\rangle$ and $|-\rangle$ being the ionic excited and ground states respectively. Let us denote by $|n_a,n_b\rangle=|n_a\rangle|n_b\rangle$ the simultaneous eigenstates of $\hat{a}^\dagger\hat{a}$ and $\hat{b}^\dagger\hat{b}$ such that $\hat{a}^\dagger\hat{a}|n_a,n_b\rangle=n_a|n_a,n_b\rangle$ and $\hat{b}^\dagger\hat{b}|n_a,n_b\rangle=n_b|n_a,n_b\rangle$.

We suppose that the initial state of the ion has the form

$$\begin{aligned}|\Psi(0)\rangle&=\left|\tau=1,j_0=\frac{N}{2}\right\rangle|-\rangle\\ &\equiv\frac{1}{2^{N/2}}\sum_{k=0}^{N}\binom{N}{k}^{1/2}|N-k,k\rangle|-\rangle\\ &\equiv\sum_{k=0}^{N}P_k|N-k,k\rangle|-\rangle.\end{aligned} \tag{4}$$

The vibrational state $|\tau=1,j_0=N/2\rangle$ belongs to the class of the so called SU(2) coherent states defined as

$$|\tau,j\rangle=\frac{1}{(1+|\tau|^2)^j}\sum_{k=0}^{2j}\binom{2j}{k}^{1/2}\tau^k|2j-k,k\rangle, \tag{5}$$

where $\tau\in C$ and $2j\in N$. The states $|N-k,k\rangle$ appearing in Eq. (4) are eigenstates of the operator $(\hat{a}^\dagger\hat{a}+\hat{b}^\dagger\hat{b})$ all pertaining to the eigenvalue $N=2j_0$ representing the initial total number of vibrational quanta. Very recently, number states of the ion motion along the X direction of an electromagnetic trap have been experimentally realized by Meekhof *et al.* [11]. Of course, the applicability of this method is by no means restricted to oscillations along the X axis only. As pointed out by Gou and Knight [18], the generation of the initial state $|\tau=1,j_0\rangle$ of a bidimensionally confined ion amounts to realizing a Fock state of the ion motion along the direction with an angle $\pi/4$ relative to the X axis.

If we turn on, at $t=0$, the laser fields realizing the Hamiltonian model given by Eq. (3), then at any subsequent instant of time t the state of the system in the Schrödinger picture, apart from an overall phase factor, can be written as

$$|\Psi(t)\rangle=|\varphi_-(t)\rangle|-\rangle+|\varphi_+(t)\rangle|+\rangle \tag{6}$$

with

$$|\varphi_-(t)\rangle=\sum_{k=0}^{N}P_k\cos(f_kt)|N-k,k\rangle \tag{7}$$

and

$$|\varphi_+(t)\rangle=-i\sum_{k=1}^{N-1}P_k\sin(f_kt)|N-k-1,k-1\rangle, \tag{8}$$

where

$$f_k=2g\sqrt{(N-k)k} \tag{9}$$

are the Rabi frequences. Equation (6) shows that, starting from the factorized state $|\Psi(0)\rangle$, the Hamiltonian model (3) leads to entanglement between the external and internal degrees of freedom of the trapped ion, giving rise to far-reaching interesting dynamical consequences. In order to appreciate the meaning of this assertion, we focus our attention on the time evolution of the vibrational entropy defined as follows

$$S_v(t)=-\mathrm{Tr}[\rho_v(t)\ln\rho_v(t)], \tag{10}$$

ρ_v being the reduced density operator describing the external motion of the ion. A straightforward calculation gives

$$S_v(t)=-\ln\{c(t)^{c(t)}[1-c(t)]^{1-c(t)}\}, \tag{11}$$

where

$$c(t)=\sum_{k=0}^{N}|P_k|^2\cos^2(f_kt)=\frac{1}{2}\left(1+\sum_{k=0}^{N}|P_k|^2\cos(2f_kt)\right). \tag{12}$$

Exploiting an original analytical method based on analysis of the Rabi frequencies relative to our system, we now demonstrate that there exist N-dependent instants of time at which the internal and external degrees of freedom of the trapped ion are disentangled $[c(t)=1$ or $c(t)=0]$ or maximally entangled $[c(t)=1/2]$.

Consider first of all the case of disentanglement and, in particular, the case in which $c(t)$ comes back to its initial value 1. These revivals of $c(t)$ are of course related to the rephasing of the oscillating terms appearing in Eq. (12). In order to evaluate these special instants of time, we observe that $|P_k|^2$ is a binomial distribution sharply peaked around its mean value $\langle k\rangle=N/2$, with a variance equal to $\sqrt{N}/2$. If $N\gg1$, it is reasonable to assume that only the terms satisfying the inequality

$$\frac{N}{2}-\frac{\sqrt{N}}{2}\leqslant k\leqslant\frac{N}{2}+\frac{\sqrt{N}}{2} \tag{13}$$

effectively contribute to the sum appearing in Eq. (12). Indicate with $\bar{t}$ the instant of time at which $c(t)$ has its first revival. It is easy to convince oneself that, at this time, the necessary condition

$$2(f_k-f_{k+1})\bar{t}=2m_k\pi,\quad m_k=0,\pm1,\ldots, \tag{14}$$

must be satisfied. Taking into account the considerations above, we have proved that Eq. (14) can be linearized as follows:

$$2(f_k-f_{k+1})\bar{t}=4g\bar{t}\left[\frac{2k-N+1}{N}+O\left(\frac{(2k-N+1)^4}{N^4}\right)\right]$$
$$\simeq\frac{2k-N+1}{N}4g\bar{t}. \tag{15}$$

In particular, for $k=\bar{k}$ with

$$\bar{k}=\begin{cases}\dfrac{N}{2} & \text{if } N \text{ is even,}\\[2mm] \dfrac{N+1}{2} & \text{if } N \text{ is odd,}\end{cases} \tag{16}$$

we get

$$2(f_{\bar{k}}-f_{\bar{k}+1})\bar{t}=\begin{cases}\dfrac{4g\bar{t}}{N} & \text{if } N \text{ is even}\\[2mm] \dfrac{8g\bar{t}}{N} & \text{if } N \text{ is odd,}\end{cases} \tag{17}$$

so that, in view of Eq. (15), the following relation between (f_k-f_{k+1}) and $(f_{\bar{k}}-f_{\bar{k}+1})$ may be written down:

$$2(f_k-f_{k+1})\bar{t}\equiv\Delta_k2\bar{t}[f_{\bar{k}}-f_{\bar{k}+1}]. \tag{18}$$

In this equation,

$$\Delta_k=\frac{2k-N+1}{2-\delta_{2[N/2],N}} \tag{19}$$

is an integer whatever the value of the natural k, and the symbol $[x]$ means the integer part of the real number x.

On the other hand, the linearized frequency $f_{\bar{k}}$, defined by Eq. (9) with $k=\bar{k}$, may be cast as follows:

$$f_{\bar{k}}=\begin{cases} gN & \text{if } N \text{ is even,} \\ gN+gO(1/N)\simeq gN & \text{if } N \text{ is odd.} \end{cases} \tag{20}$$

From Eq. (18) it is possible to deduce the key relation

$$\cos(2f_{k+1}\bar{t})=\cos[2f_k\bar{t}-2\Delta_k\bar{t}(f_{\bar{k}}-f_{\bar{k}+1})], \tag{21}$$

where $k=\bar{k},\bar{k}\pm1,\ldots$. In view of Eqs. (12) and (21) and recalling that Δ_k is integer, the time $\bar{t}$ of the first revival can be found by solving the system

$$2(f_{\bar{k}}-f_{\bar{k}+1})\bar{t}=2\pi, \tag{22}$$

$$2f_{\bar{k}}\bar{t}=2n\pi, \tag{23}$$

where n is an unknown integer to be determined simultaneously with $\bar{t}$. As suggested by Eqs. (17) and (22), $\bar{t}$ depends on the parity of N. We have in fact proved that, if $N \gg 1$ is even, $\bar{t}=\pi N/2g\equiv t_e$ and $n=N^2/2$ whereas if $N\gg1$ is odd it is not difficult to verify that $\bar{t}=t_e/2-\pi/4gN\equiv t_o$, with $n=(N^2-1)/4$ up to $O(1/N)$. Thus, in the case of odd N, the first revival of $c(t)$ occurs at a time (t_o) which turns out to be almost one-half of the instant of time (t_e) at which $c(t)=1$ if N is even. In view of the scheme we are going to propose, in what follows we concentrate our attention on the time t_o. It is possible to persuade oneself, by direct substitution into Eq. (22), that, if N is even, at the instant $t=t_e/2 \simeq t_o$,

$$2(f_{\bar{k}}-f_{\bar{k}+1})\frac{t_e}{2}=\pi \tag{24}$$

and then

$$2f_{k+1}\frac{t_e}{2}=2f_k\frac{t_e}{2}-\Delta_k\pi \tag{25}$$

in view of Eq. (18). Taking into account that by definition (19) Δ_k is odd when N is even, we immediately deduce that

$$\cos\left(2f_k\frac{t_e}{2}\right)\propto(-1)^k \tag{26}$$

so that, from Eq. (12) we definitively get $c(t_e/2)\simeq\frac{1}{2}$ if N is even.

Summing up, we have proved analytically that if $N\gg1$ is odd, at the instant $t_o=t_e/2-\pi/4gN\simeq t_e/2$, $c(t_o)=1$, and thus $S_v(t_o)$ reaches its absolute minimum, indicating that the internal and external degrees of freedom manifest a marked tendency to disentangle from each other. On the contrary, at $t_e/2=\pi N/4g$, if $N\gg1$ is even, $c(t_e/2)=1/2$, and thus the vibrational entropy $S_v(t_e/2)$ reaches its maximum value. This means that the vibrational and electronic degrees of freedom are, in this case, maximally entangled at this instant

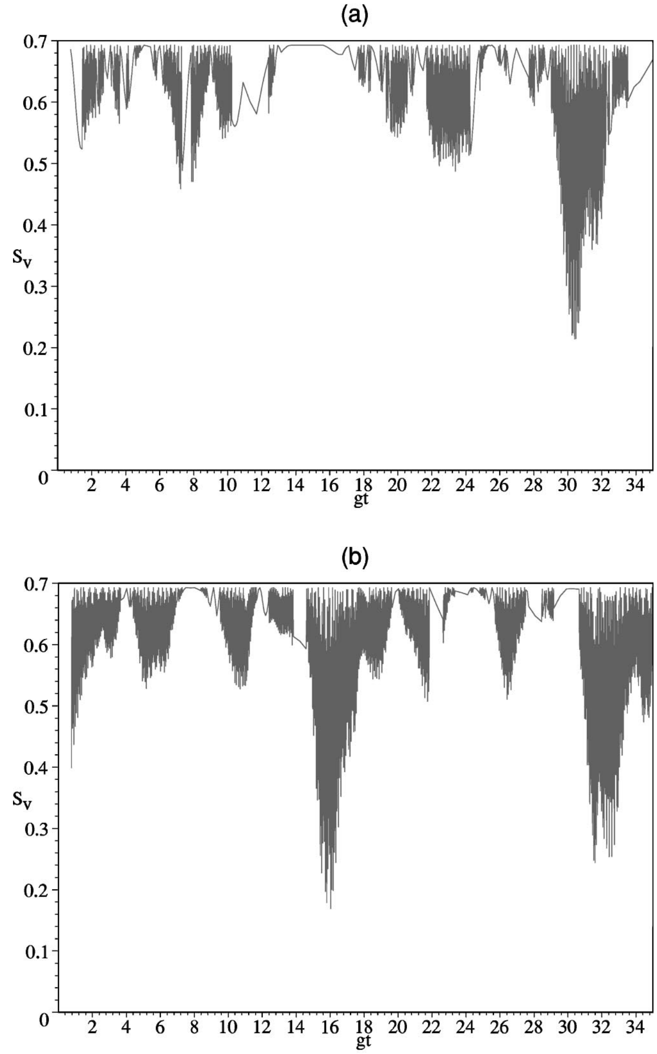

FIG. 1. Time evolution of the vibrational entropy $S_v(gt)$ for (a) $N=20$ and (b) $N=21$.

of time. Figure 1 displays the time evolution of S_v, as given by Eq. (11), corresponding to $N=20$ and $N=21$. These figures illustrate our prediction and, in particular, the existence of an N-dependent instant of time $t_o\simeq t_e/2$ at which the system under scrutiny exhibits different quantum behaviors dependent on the parity of N.

These results bring to light a peculiar nonclassical property of our system, namely, a sensitivity to the granularity of the initial total number of vibrational quanta, N. The physical origin of this intrinsically quantum behavior stems directly from the specific two-boson coupling mechanism envisaged in this paper.

In order to bring to light the link between the quantum dynamics followed by our system and the occurrence of such a nonclassical feature, it appears highly interesting to construct a detailed representation of $|\varphi_-(t)\rangle$ and $|\varphi_+(t)\rangle$ at $t=t_e/2$ and $t=t_o$. To this end we introduce and study the time evolution of the SU(2) Q function defined as

$$Q^{(j)}(\tau)=\langle\tau,j|\rho_v|\tau,j\rangle. \tag{27}$$

$Q^{(j)}(\tau)$ is a j-dependent quasiprobability distribution function defined over the phase space parametrized by $\tau \in C$ [21]. For what follows it is of relevance to underline that the total excitation number operator $\hat{N} = \hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b} + \hat{\sigma}_z + 1$ is a constant of motion and that the initial state of our system, $|\psi(0)\rangle$, is an eigenstate of $\hat{N}$ corresponding to the eigenvalue $n = 2j_0$ with $j_0 = N/2$. Observing that $|\tau, j\rangle$ is orthogonal to $|n_a, n_b\rangle$ when $2j \neq n_a + n_b$, then it is easy to convince oneself that $|\varphi_-(t)\rangle$ $[|\varphi_+(t)\rangle]$ can be expressed as a quantum superposition of different SU(2) coherent states $|\tau, j = j_0 = N/2\rangle$ $[|\tau, j = j_0 - 1 = (N-2)/2\rangle]$ obtained by varying τ. This circumstance directly leads to us considering the Q functions $Q^{(j=j_0)}(\tau)$ or $Q^{(j=j_0-1)}(\tau)$ only. In particular, we fix our attention here on the quasiprobability function $Q^{(j)}(\tau)$ with $j = j_0 = N/2$. Figures 2(a) and 2(b) [3(a) and 3(b)] display, for $N = 20$ ($N = 21$), $Q^{(j=j_0)}(\tau)$ at $t = 0$ and $t = t_e/2$ ($t = t_o$), respectively. A careful analysis of these figures suggests that detection at these N-dependent instants of time of the electronic state of the trapped ion in its ground state $|-\rangle$ projects the c.m. motion into a superposition of two macroscopically distinguishable SU(2) coherent states. In addition, we find that such a superposition exhibits a high sensitivity to the parity of the total number of vibrational quanta present at $t = 0$, in accordance with our conclusions previously deduced on the basis of the properties of S_v.

In fact, Figs. 2(b) and 3(b) strongly suggest that, after the measurement act, the two components of the vibrational state are $|\tau = 1, j = N/2\rangle$ and $|\tau = -1, j = N/2\rangle$ if N is even, or $|\tau = i, j = N/2\rangle$ and $|\tau = -i, j = N/2\rangle$ if N is odd. It is of relevance to emphasize that measurement of the internal state of a trapped ion, as required in this paper, is currently performed using the quantum jump technique [11,14]. A theoretical description of this method may be found in a paper of Poyatos *et al.* [22]. Of course it is very difficult to guess the exact superposition of the two vibrational SU(2) coherent states into which the c.m. motion is projected after measurement of the electronic state from this kind of analysis of the $Q^{(j)}(\tau)$ plots. For this reason, in order to know the exact form of the vibrational state generated by the procedure proposed in this paper, let us consider the following two classes of normalized quantum superpositions:

$$|\Psi_{N_{even}}\rangle = \frac{1}{\sqrt{2}}\left(\left| \tau = 1, j = \frac{N}{2} \right\rangle + e^{i\varphi} \left| \tau = -1, j = \frac{N}{2} \right\rangle \right) \tag{28}$$

and

$$|\Psi_{N_{odd}}\rangle = \frac{1}{\sqrt{2}}\left(\left| \tau = i, j = \frac{N}{2} \right\rangle + e^{i\varphi} \left| \tau = -i, j = \frac{N}{2} \right\rangle \right). \tag{29}$$

Assuming that N is even (odd), the modulus of the scalar product $f(\varphi, t)$ $[g(\varphi, t)]$ between the state $|\Psi_{N_{even}}\rangle$ ($|\Psi_{N_{odd}}\rangle$) and the normalized state

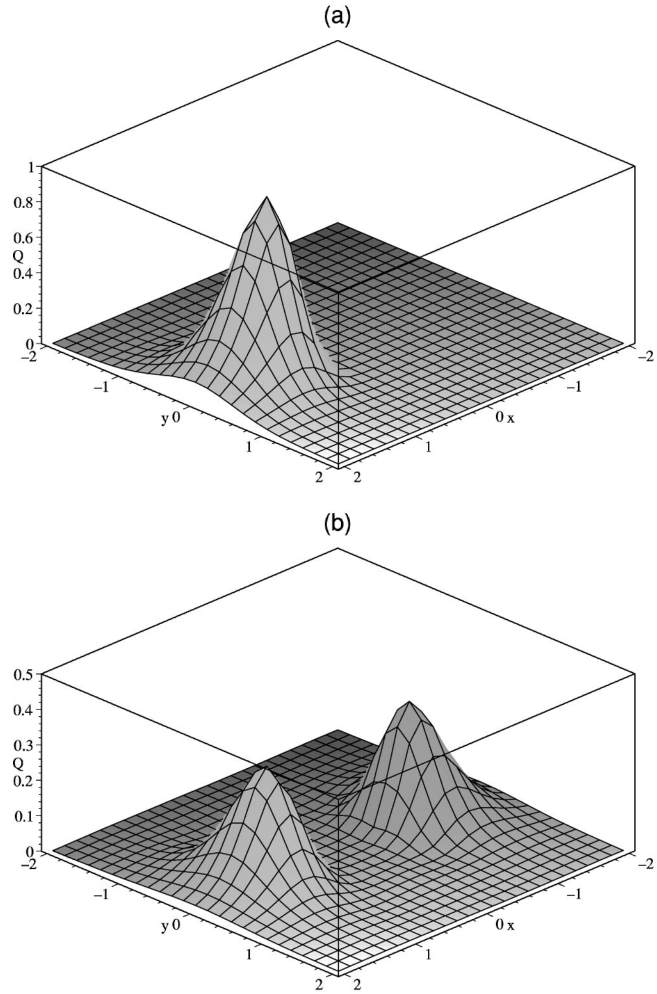

FIG. 2. Plot of $Q^{j=j_0}$ ($x = \mathrm{Re}[\tau], y = \mathrm{Im}[\tau]$) for $N = 20$, corresponding to (a) $t = 0$, (b) $t = t_e$.

$$|\psi(t)\rangle = \frac{1}{\sqrt{c(t)}} |\varphi_-(t)\rangle \tag{30}$$

obtained by detecting, at the instant t, the internal state of the ion as $|-\rangle$, can be written as

$$f(\varphi, t) = |\langle \psi(t) | \Psi_{N_{even}} \rangle| = \frac{1}{\sqrt{2c(t)}} \left| \sum_{k=0}^{N} P_k^2 \cos(f_k t)[1 + e^{i\varphi}(-1)^k] \right|, \tag{31}$$

$$g(\varphi, t) = |\langle \psi(t) | \Psi_{N_{odd}} \rangle| = \frac{1}{\sqrt{2c(t)}} \left| \sum_{k=0}^{N} P_k^2 \cos(f_k t)[i^k + e^{i\varphi}(-i)^k] \right|. \tag{32}$$

Let us focus our attention on the case $N \gg 1$ odd. Using Eqs. (20), (18), and (22), it is simple to deduce that

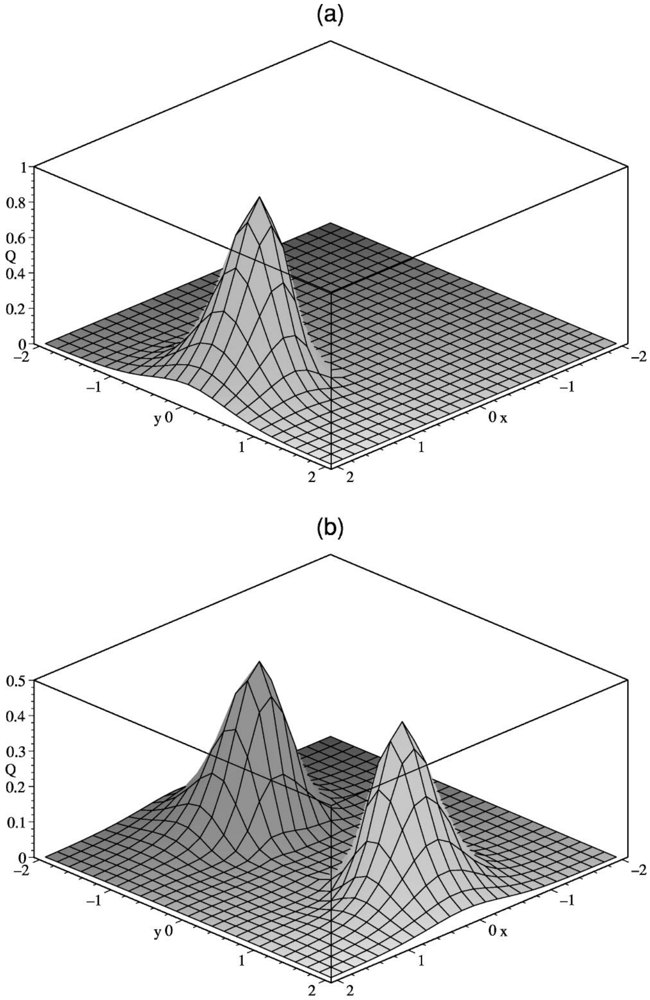

FIG. 3. Plot of $Q^{j=j_0}$ $(x=\mathrm{Re}[\tau], y=\mathrm{Im}[\tau])$ for $N=21$, corresponding to (a) $t=0$, (b) $t=t_o$.

$$\cos(f_{\bar{k}}t_o)=\cos\left(\frac{\pi(N^2-1)}{4}\right)=(-1)^{(N+1)/2} \tag{33}$$

and that

$$(f_k-f_{k+1})t_o=\Delta_k\pi. \tag{34}$$

In view of these considerations, and remembering that $\Delta_k \in Z$, it is possible to verify that $\cos(f_k t_o)$ can assume only the values $+1$ and -1. In more in detail, its k dependence may be expressed as

$$\cos(f_k t_o)\simeq(-1)^{\bar{k}} i^{(k-\bar{k})}\frac{[1-i(-1)^{(k-\bar{k})}]}{i-1} \tag{35}$$

with $\bar{k}=(N+1)/2$. Remembering that, at $t=t_o$, $c(t_o)=1$, and substituting Eq. (35) into Eq. (32) we obtain

$$g(\varphi,t_o)=\tfrac{1}{2}|e^{i\varphi}-i(-1)^{(N+1)/2}|. \tag{36}$$

Looking at Eq. (36), we may conclude that if N is odd, then $g(\varphi=\pi/2,t=t_o)\simeq 1$ or $g(\varphi=3\pi/2,t=t_o)\simeq 1$ when $(N+1)/2$ is odd or even, respectively.

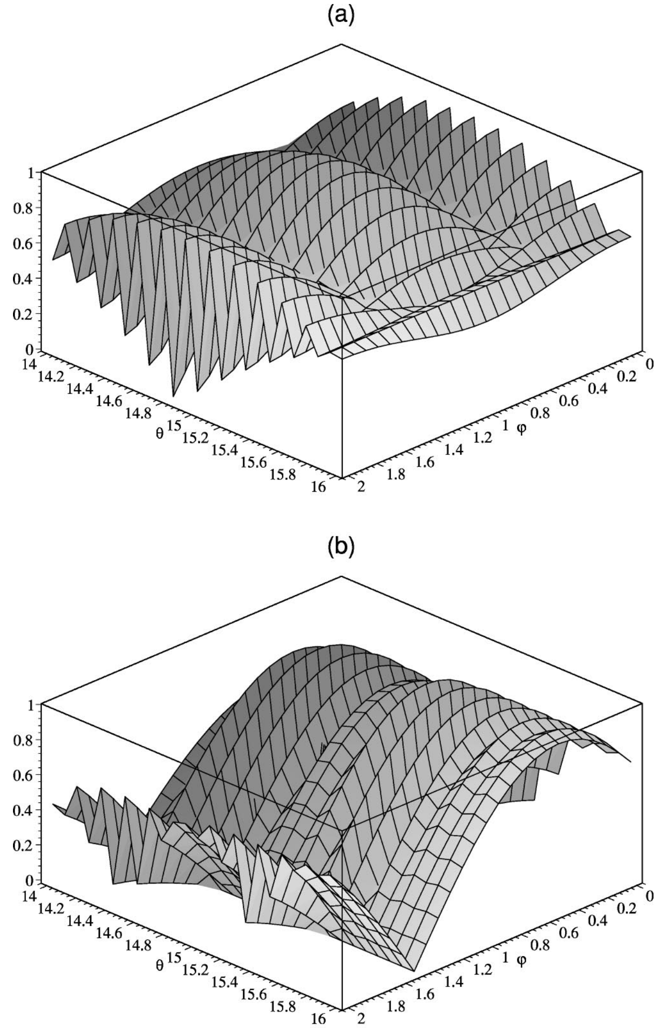

FIG. 4. (a) Plot of the function $f(\varphi,\theta)=|\langle\psi_{N_{even}}|\psi\rangle|$ for $N=20$, with $\theta=gt$. (b) Plot of the function $g(\varphi,\theta)=|\langle\psi_{N_{odd}}|\psi\rangle|$ for $N=21$, with $\theta=gt$.

Consider now the case $N\gg 1$ even. Following an analysis similar to the one adopted for N odd, it is not difficult to establish that

$$\cos\left(f_{\bar{k}}\frac{t_e}{2}\right)=\cos\left(\frac{\pi N^2}{4}\right)=(-1)^{N/2} \tag{37}$$

and

$$(f_k-f_{k+1})\frac{t_e}{2}=\Delta_k\frac{\pi}{2}. \tag{38}$$

We recall that the integer Δ_k given by Eq. (19) is an odd integer whatever k is, if N is even. This property allows us to write $\cos(f_k t_e/2)$ in the form

$$\cos\left(f_k\frac{t_e}{2}\right)\simeq\tfrac{1}{2}[(-1)^k+(-1)^{N/2}] \tag{39}$$

so that, substituting Eq. (39) into Eq. (31) and remembering that $c(t_e/2)=1/2$, we get

$$f(\varphi,t_e/2)=\tfrac{1}{2}|e^{i\varphi}+(-1)^{N/2}|. \tag{40}$$

In this case Eq. (40) tells us that $f(\varphi_e=2\pi,t_e/2)\simeq 1$ or $f(\varphi=\pi,t_e/2)\simeq 1$, when $N/2$ is even or odd, respectively.

In Fig. 4 we report the functions $f(\varphi,t)=|\langle\Psi_{N_{even}}|\psi\rangle|$ for $N=20$ and $g(\varphi,t)=|\langle\Psi_{N_{odd}}|\psi\rangle|$ for $N=21$, in a range of values of t centered around $t_o\simeq t_e/2$.

These results allow us to conclude that the properties of the state generated following the procedure envisaged in this article strongly depend on the parity of the total initial number N of vibrational quanta. In more detail, we get

$$\left|\psi\left(\frac{t_e}{2}\right)\right\rangle=\frac{1}{\sqrt{2}}\left(\left|\tau=1,j=\frac{N}{2}\right\rangle+(-1)^{N/2}\left|\tau=-1,j=\frac{N}{2}\right\rangle\right) \tag{41}$$

if N is even, whereas we obtain

$$|\psi(t_o)\rangle=\frac{1}{\sqrt{2}}\left(\left|\tau=i,j=\frac{N}{2}\right\rangle-i(-1)^{(N+1)/2}\left|\tau=-i,j=\frac{N}{2}\right\rangle\right) \tag{42}$$

if N is odd. In words this means that if N is even the state $|\psi(t_e/2)\rangle$, defined by Eq. (41), has the form of an even (odd) SU(2) coherent state [20], if $N/2$ is even (odd). On the other hand, if N is odd, the two states $|\psi(t_o)\rangle$ obtained by measuring at $t=t_o$ the internal state of the ion as $|-\rangle$ may be called SU(2) Yurke-Stoler-like coherent states, with a difference of $\pi/2$ $(3\pi/2)$ in the relative quantum phase, when $(N+1)/2$ is even (odd).

Summarizing, in the context of our conditional scheme, based on a single measurement act, the total number of excitations N present in the initial state of the ion c.m. motion behaves as an adjustable parameter, allowing the realization of vibrational states possessing very different nonclassical bosonic number distributions. It is worth emphasizing that, whatever the parity of N, the states discussed in this paper are quantum superpositions of two distinguishable SU(2) coherent states of a bidimensional isotropic harmonic oscillator.

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