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*Published in:*  
Journal of Modern Optics

*DOI:*  
[10.1080/09500340008235134](https://doi.org/10.1080/09500340008235134)

*Publication date:*  
2000

[Link to publication in Heriot-Watt Research Gateway](#)

*Citation for published version (APA):*  
Maniscalco, S., Messina, A., & Napoli, A. (2000). Preparation of macroscopically distinguishable superpositions of circular or linear oscillatory states of a bidimensionally trapped ion. *Journal of Modern Optics*, 47(12), 2113-2126. [10.1080/09500340008235134](https://doi.org/10.1080/09500340008235134)



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Publisher: Taylor & Francis

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## Journal of Modern Optics

Publication details, including instructions for authors and subscription information:

<http://www.tandfonline.com/loi/tmop20>

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Version of record first published: 03 Jul 2009

To cite this article: S. Maniscalco, A. Messina & A. Napoli (2000): Preparation of macroscopically distinguishable superpositions of circular or linear oscillatory states of a bidimensionally trapped ion, *Journal of Modern Optics*, 47:12, 2113-2126

To link to this article: <http://dx.doi.org/10.1080/09500340008235134>

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## Preparation of macroscopically distinguishable superpositions of circular or linear oscillatory states of a bidimensionally trapped ion

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(Received 16 November 1999)

**Abstract.** A simple scheme for the generation of two different classes of bidimensional vibrational Schrödinger cat-like states of an isotropically trapped ion is presented. We show that by appropriately adjusting an easily controllable parameter having a clear physical meaning, the states prepared by our procedure are quantum superpositions of either vibrational axial angular momentum eigenstates or Fock states along two orthogonal directions.

### 1. Introduction

Quantum mechanics works extremely well in describing microscopic systems. A huge number of experiments have confirmed, without any doubt, the linear character of the theory, which allows, at a microscopic level, the existence of superpositions of states possessing well defined and distinct properties. However, the superposition principle gives rise to serious conceptual difficulties when extended to macroscopic systems. In fact, as brilliantly pointed out in 1935 by Schrödinger [1], coherent superpositions of macroscopic states possessing mutually exclusive properties ( $|dead\ cat\rangle + |live\ cat\rangle$ ,  $|on\ TV\rangle + |off\ TV\rangle$ ,  $|open\ window\rangle + |closed\ window\rangle$ , and so on), are never observed in our daily experience.

Many efforts have been made to overcome this embarrassing situation, the most part of which is aimed at identifying the elusive border between the microscopic world, described by quantum laws, and the macroscopic world, described by classical laws. Quite recently there have been fascinating theories developed according to which the coupling of the system to the environment is responsible for decoherence phenomena converting quantum superpositions into statistical mixtures [2]. The decoherence times were shown to be strongly dependent on the system's size, thus denying the possibility of detecting quantum

features of macroscopic systems except than for extremely short times. It is then interesting to study the existence and the practical observability of superpositions of distinguishable states in mesoscopic systems, since they are somehow in between macroscopic and microscopic systems.

During the last few years, the growing development of techniques of laser cooling and trapping of ions, has made possible the realization of important experiments for testing fundamental aspects of quantum mechanics [3–6]. Trapped and cooled ions are systems very well isolated from the external environment. This fact allows the experimental realization of Schrödinger cat-like atomic states and the detection of quantum interference effects between the two states composing the superposition [6].

It is of relevance to emphasize that the ability of generating and detecting coherent superpositions of macroscopically distinguishable states in such systems provides a good occasion to study the decoherence phenomena and thus the nebulous quantum-classical boundary.

In this paper we present a simple scheme for generating quantum superpositions of two macroscopically distinguishable states of the vibrational motion of a bidimensionally trapped two-level ion.

The states generated in accordance with our proposal, which is based on the wave packet reduction approach, belong to two different classes. The first class contains quantum superpositions of axial ( $z$  component) angular momentum eigenstates pertaining to the maximum and minimum eigenvalue respectively. The states belonging to the second class are coherent combinations of number states describing linear oscillatory motions of the ion along two orthogonal directions. It is worth noting that, in the context of our scheme, it is possible to guide the vibrational state of the trapped ion toward one class or the other, simply by controlling an adjustable parameter related to the initial condition imposed on the confined system.

## 2. The system

Consider the quantized motion of a two-level ion of mass  $m$  confined in a 2D isotropic harmonic potential characterised by the trap frequency  $\nu$ . The annihilation operators of vibrational quanta in the  $X$  and  $Y$  directions respectively, are defined as:

$$\hat{a}_x = \frac{1}{2^{1/2}} \left( \left( \frac{m\nu}{\hbar} \right)^{1/2} \hat{X} + i \frac{1}{(m\nu\hbar)^{1/2}} \hat{P}_x \right), \quad (1)$$

$$\hat{a}_y = \frac{1}{2^{1/2}} \left( \left( \frac{m\nu}{\hbar} \right)^{1/2} \hat{Y} + i \frac{1}{(m\nu\hbar)^{1/2}} \hat{P}_y \right). \quad (2)$$

Accordingly the position and momentum operators are given by

$$\hat{X} = \left( \frac{\hbar}{2\nu m} \right)^{1/2} (\hat{a}_x^\dagger + \hat{a}_x), \quad \hat{Y} = \left( \frac{\hbar}{2\nu m} \right)^{1/2} (\hat{a}_y^\dagger + \hat{a}_y), \quad (3)$$

$$\hat{P}_x = i \left( \frac{\hbar\nu m}{2} \right)^{1/2} (\hat{a}_x^\dagger - \hat{a}_x), \quad \hat{P}_y = i \left( \frac{\hbar\nu m}{2} \right)^{1/2} (\hat{a}_y^\dagger - \hat{a}_y). \quad (4)$$

Let us denote with  $|n_x, n_y\rangle = |n_x\rangle|n_y\rangle$  the simultaneous eigenstates of the number operators  $\hat{n}_x = \hat{a}_x^\dagger \hat{a}_x$  and  $\hat{n}_y = \hat{a}_y^\dagger \hat{a}_y$ , such that

$$\hat{n}_x |n_x, n_y\rangle = n_x |n_x, n_y\rangle, \quad (5)$$

$$\hat{n}_y |n_x, n_y\rangle = n_y |n_x, n_y\rangle. \quad (6)$$

In what follows we will call the eigenvalues  $n_x$  and  $n_y$  *linear quanta* along the  $X$  and  $Y$  directions respectively. It is important to underline that, since the potential energy is invariant under rotation about  $z$ , we could just as well have chosen another system of orthogonal axes  $\bar{X}$  and  $\bar{Y}$  instead of  $X$  and  $Y$ . Therefore, in order to take better advantage of the symmetry of the problem under scrutiny, it is convenient to introduce the component  $\hat{L}_z$  of the angular orbital momentum, defined as:

$$\hat{L}_z = \hat{X}\hat{P}_y - \hat{Y}\hat{P}_x = i\hbar(\hat{a}_x\hat{a}_y^\dagger - \hat{a}_y\hat{a}_x^\dagger). \quad (7)$$

Introducing the two bosonic operators

$$\hat{a}_l = \frac{1}{2^{1/2}}(\hat{a}_x + i\hat{a}_y), \quad (8)$$

$$\hat{a}_r = \frac{1}{2^{1/2}}(\hat{a}_x - i\hat{a}_y), \quad (9)$$

the angular momentum  $\hat{L}_z$  can be recast in the form:

$$L_z = \hbar(\hat{a}_r^\dagger \hat{a}_r - \hat{a}_l^\dagger \hat{a}_l). \quad (10)$$

Following [7] it is reasonable to call the excitations associated with the number operators  $\hat{n}_r = \hat{a}_r^\dagger \hat{a}_r$  and  $\hat{n}_l = \hat{a}_l^\dagger \hat{a}_l$ , *right and left circular quanta* respectively. For the scope of this paper it is convenient, at this point, to observe that the simultaneous eigenstates  $|n_l, n_r\rangle$  of the operators  $\hat{n}_l$  and  $\hat{n}_r$ ,

$$|n_l, n_r\rangle = \frac{1}{(n_l!n_r!)^{1/2}}(a_l^\dagger)^{n_l}(a_r^\dagger)^{n_r}|n_x = 0, n_y = 0\rangle, \quad (11)$$

are eigenstates of  $\hat{L}_z$  belonging to the eigenvalue  $\hbar(n_r - n_l)$ . Inserting equations (8) and (9) into (11), under the conditions  $n_l = N, n_r = 0$  or  $n_l = 0, n_r = N$ , we find that the eigenstates of  $\hat{L}_z$ , corresponding to its minimum ( $-\hbar N$ ) and maximum ( $+\hbar N$ ) eigenvalues respectively, can be written down as follows:

$$|n_l = N, n_r = 0\rangle = \sum_{k=0}^N \frac{1}{2^{N/2}} \binom{N}{k}^{1/2} (-i)^k |n_x = N - k, n_y = k\rangle, \quad (12)$$

$$|n_l = 0, n_r = N\rangle = \sum_{k=0}^N \frac{1}{2^{N/2}} \binom{N}{k}^{1/2} (i)^k |n_x = N - k, n_y = k\rangle. \quad (13)$$

It has been shown that irradiating the trapped ion with two laser beams applied along the two orthogonal directions,  $\bar{X}$  and  $\bar{Y}$ , with an angle of  $\pi/4$  relative to the  $X$  and  $Y$  axes respectively and both tuned to the second red vibrational sideband, the physical system under scrutiny can be described, in the rotating wave approximation, by the following Hamiltonian model [8]:

$$\hat{H}_{\text{RWA}} = \hbar\nu(\hat{a}_x^\dagger\hat{a}_x + \hat{a}_y^\dagger\hat{a}_y) + \frac{\hbar\omega_0}{2}\hat{\sigma}_z + [d\varepsilon^{(-)}(\hat{x}, \hat{y}, t)\hat{\sigma}_- + h.c.]. \quad (14)$$

In equation (14),  $d$  is the appropriate dipole matrix element,  $\omega_0$  is the electronic transition frequency and  $\hat{\sigma}_z = |+\rangle\langle+| - |-\rangle\langle-|$ ,  $\hat{\sigma}_+ = |+\rangle\langle-|$ ,  $\hat{\sigma}_- = |-\rangle\langle+|$  describe the internal degrees of freedom,  $|+\rangle$  and  $|-\rangle$  being the ionic excited and ground states respectively. The negative frequency part of the classical driving field is given by:

$$\varepsilon^{(-)}(\hat{x}, \hat{y}, t) = E_1 \exp[i(\omega_L t - k_1 \hat{x} + \varphi_1)] + E_2 \exp[i(\omega_L t - k_2 \hat{y} + \varphi_2)] \quad (15)$$

where  $\omega_L = \omega_0 - 2\nu$  is the laser's frequency and  $\hat{x}$  and  $\hat{y}$  are the position operators along the axes  $\bar{X}$  and  $\bar{Y}$ . Finally we indicate with  $E_1$ ,  $\varphi_1$ ,  $k_1$  and  $E_2$ ,  $\varphi_2$ ,  $k_2$  amplitudes, phases and wave vectors of the two lasers respectively. In what follows we assume that

$$E_1 = E_2 = E_0,$$

$$\varphi_1 = \varphi_2 + \pi,$$

$$k_1 = k_2 = k.$$

In a frame rotating around  $z$  at the laser frequency  $\omega_L$ , Hamiltonian (14) has to be transformed as follows

$$\hat{H}_L = U^\dagger \hat{H}_{\text{RWA}} U + i\hbar \left( \frac{\partial}{\partial t} U \right) U, \quad (16)$$

where

$$U = \exp\left(-i\omega_L \frac{\hat{\sigma}_z}{2} t\right). \quad (17)$$

$\hat{H}_L$  can be explicitly cast in the form

$$\begin{aligned} \hat{H}_L &= \hbar\nu(\hat{a}_x^\dagger\hat{a}_x + \hat{a}_y^\dagger\hat{a}_y) + \frac{\hbar\Delta}{2}\hat{\sigma}_z + \hbar\Omega\{[\exp[-i\eta(\hat{a}_x + \hat{a}_x^\dagger)] - \exp[-i\eta(\hat{a}_y + \hat{a}_y^\dagger)]\}\hat{\sigma}_- + h.c. \} \\ &\equiv \hat{H}_0 + \hat{H}_{\text{int}} \end{aligned} \quad (18)$$

In equation (18),  $\Delta = \omega_0 - \omega_L = 2\nu$  is the detuning of the laser beams frequency  $\omega_L$  from the transition frequency  $\omega_0$ ,  $\Omega = dE_0/\hbar$ ,  $\eta = (k^2\hbar/2\nu m)^{1/2}$  is the Lamb-Dicke parameter and  $\hat{a}_x^\dagger(\hat{a}_x)$  and  $\hat{a}_y^\dagger(\hat{a}_y)$  are the annihilation (creation) operators along the directions  $\bar{X}$  and  $\bar{Y}$  respectively. The operators  $\hat{a}_x$  and  $\hat{a}_y$  are of course related to  $\hat{a}_x$  and  $\hat{a}_y$  by the following relations

$$\hat{a}_x = \frac{1}{2^{1/2}}(\hat{a}_x + \hat{a}_y), \quad \hat{a}_y = \frac{1}{2^{1/2}}(-\hat{a}_x + \hat{a}_y). \quad (19)$$

If the Lamb-Dicke limit is satisfied,  $\eta \ll 1$ , it can be shown [8] that, in the weak coupling regime  $\nu \gg \Omega$  and in the interaction picture, the Hamiltonian model describing the system under scrutiny assumes the form

$$\hat{H} = g[(\hat{a}_x\hat{a}_y)\hat{\sigma}_+ + (\hat{a}_x^\dagger\hat{a}_y^\dagger)\hat{\sigma}_-] \quad (20)$$

where  $g = 2\Omega(\eta^2/2)\hbar \exp(-\eta^2/2)$  is the effective coupling constant.

### 3. The dynamics

We presume to prepare the ion in the initial state  $|\Psi(0)\rangle = |n_x = N, n_y = 0\rangle|-\rangle$ . This state can be created following one of the several procedures for the generation of Fock states which have been proposed in the context of trapped ions [4, 5, 9]. It is relevant to note that, starting from the initial 3D zero point energy vibrational state, the Boulder group has succeeded in generating one-dimensional Fock states  $|n_x = N\rangle$  up to  $N = 16$  [4, 10].

If at  $t = 0$  we turn on the two laser fields realizing the Hamiltonian model given by equation (20), the momentum exchange between the classical laser beams and the ion entangles its internal and external degrees of freedom. As a consequence, the probability  $P_-(t)$  of finding the ion in its electronic ground state, at a subsequent instant of time  $t$ , is in general less than 1. In particular, we have demonstrated that, at a generic time  $t$ ,  $P_-(t)$  can be exactly written as:

$$\begin{aligned} P_-(t) &= \sum_{k=0}^N |P_k|^2 \cos^2(f_k t) \\ &= \frac{1}{2} \left[ 1 + \sum_{k=0}^N |P_k|^2 \cos(2f_k t) \right], \end{aligned} \quad (21)$$

where  $f_k = 2g[k(N - k)]^{1/2}$  are the so-called Rabi frequencies and

$$P_k = \frac{1}{2^{N/2}} \binom{N}{k}^{1/2}.$$

From equation (21) it is reasonable to seek instants of time at which the internal and external degrees of freedom disentangle or, stated another way,  $P_-(t)$  comes back to its initial value 1. These revivals of  $P_-(t)$  are of course related to the rephasing of the oscillating terms appearing in equation (21). In order to evaluate these special instants of time, we observe that  $|P_k|^2$  is a binomial distribution sharply peaked around its mean value  $\langle k \rangle = N/2$ , with a variance equal to  $N^{1/2}/2$ . If  $N \gg 1$ , it is reasonable to assume that only the terms satisfying the inequality

$$\frac{N}{2} - \frac{N^{1/2}}{2} \leq k \leq \frac{N}{2} + \frac{N^{1/2}}{2} \quad (22)$$

effectively contribute to the sum appearing in equation (21). Indicating with  $\bar{t}$  the instant of time at which  $P_-(t)$  has its first revival, it is easy to convince oneself that, at this time, the necessary condition

$$2(f_k - f_{k+1})\bar{t} = 2m_k\pi, \quad m_k = 0, \pm 1, \dots \quad (23)$$

must be satisfied. Taking into account the considerations above, we have proved that equation (23) can be linearized as follows:

$$\begin{aligned} 2(f_k - f_{k+1})\bar{t} &= 4g\bar{t} \left[ \frac{2k - N + 1}{N} + O\left(\frac{(2k - N + 1)^4}{N^4}\right) \right] \\ &\simeq \frac{2k - N + 1}{N} 4g\bar{t}. \end{aligned} \quad (24)$$

In particular for  $k = \bar{k}$  with

$$\bar{k} = \begin{cases} \frac{N}{2}, & \text{if } N \text{ is even,} \\ \frac{N+1}{2}, & \text{if } N \text{ is odd,} \end{cases} \tag{25}$$

we get

$$2(f_{\bar{k}} - f_{\bar{k}+1})\bar{t} = \begin{cases} \frac{4g\bar{t}}{N}, & \text{if } N \text{ is even,} \\ \frac{8g\bar{t}}{N}, & \text{if } N \text{ is odd,} \end{cases} \tag{26}$$

so that, in view of equation (24), the following relation between  $(f_k - f_{k+1})$  and  $(f_{\bar{k}} - f_{\bar{k}+1})$  may be written down

$$2(f_k - f_{k+1})\bar{t} \equiv \Delta_k 2\bar{t}[f_{\bar{k}} - f_{\bar{k}+1}]. \tag{27}$$

In this equation

$$\Delta_k = \frac{2k - N + 1}{2 - \delta_{2\lfloor \frac{N}{2} \rfloor, N}} \tag{28}$$

is an integer whatever the value of  $k$  is and the symbol  $[x]$  denotes the integer part of the real number  $x$ .

On the other hand the linearized frequency  $f_{\bar{k}}$  may be cast as follows

$$f_{\bar{k}} = \begin{cases} gN, & \text{if } N \text{ is even,} \\ gN + gO(1/N) \simeq gN, & \text{if } N \text{ is odd.} \end{cases} \tag{29}$$

From equation (27) it is possible to deduce the following key relation

$$\cos(2f_{k+1}\bar{t}) = \cos[2f_{\bar{k}}\bar{t} - 2\Delta_k\bar{t}(f_{\bar{k}} - f_{\bar{k}+1})], \tag{30}$$

where  $k = \bar{k}, \bar{k} \pm 1, \dots$ . In view of equations (21) and (30), time  $\bar{t}$  of the first revival can be found solving the system

$$2(f_{\bar{k}} - f_{\bar{k}+1})\bar{t} = 2\pi, \tag{31}$$

$$2f_{\bar{k}}\bar{t} = 2n\pi, \tag{32}$$

where  $n$  is an unknown integer to be determined simultaneously with  $\bar{t}$ . As suggested by equations (26) and (31),  $\bar{t}$  depends on the parity of  $N$ . We have in fact proved that, if  $N \gg 1$  is even,  $\bar{t} = \pi N/2g \equiv t_e$  and  $n = N^2/2$ , whereas if  $N \gg 1$  is odd  $\bar{t} = t_e/2 - \pi/4gN \equiv t_o$  and  $n = (N^2 - 1)/2$ . Thus, in the case of odd  $N$ , the first revival of  $P_-(t)$  occurs at a time ( $t_o$ ) which turns out to be almost one half of the instant of time ( $t_e$ ) at which  $P_-(t) = 1$  if  $N$  is even. For this reason and in view of the scheme we are going to propose, in what follows we concentrate our attention on time  $t_o$ . It is possible to persuade oneself, by direct substitution into equation (31), that, if  $N$  is even, at the instant  $t = t_e/2 \simeq t_o$

$$2(f_{\bar{k}} - f_{\bar{k}+1})\frac{t_e}{2} = \pi. \tag{33}$$



As a consequence, looking at equation (30), we immediately deduce that

$$\cos\left(2f_k \frac{t_e}{2}\right) \propto (-1)^k \quad (34)$$

so that, in view of equation (21) we get  $P_-(t_e/2) \simeq \frac{1}{2}$  if  $N$  is even. Summing up, at  $t = t_e/2 \simeq t_0$ , the ionic internal and external degrees of freedom are disentangled or maximally entangled in correspondence to  $N$  odd or even respectively.

These results bring to the light a peculiar non-classical property of our system, namely a sensitivity to the granularity of the initial total number of vibrational quanta  $N$ . The physical origin of this intrinsically quantum behaviour directly stems from the specific two-boson coupling mechanism envisaged in this paper.

In order to reach the goal of this paper we now analyse the state  $|\Psi(t)\rangle$  of the system under scrutiny at  $t = t_0$ . By applying the time evolution operator to the initial state  $|\Psi(0)\rangle$ , it is not difficult to show that, at a generic time  $t$ ,  $|\Psi(t)\rangle$  can be written as

$$\begin{aligned} |\Psi(t)\rangle &= |c(t)\rangle|-\rangle - i|s(t)\rangle|+\rangle \\ &= \sum_{k=0}^N P_k \cos(f_k t) |N-k, k\rangle|-\rangle - i \sum_{k=1}^{N-1} P_k \sin(f_k t) |N-k-1, k-1\rangle|+\rangle. \end{aligned} \quad (35)$$

Let us focus our attention on the case  $N \gg 1$  odd. The conditions of full revival of  $P_-(t)$  under which the instant  $t_0$  has been found, may be expressed, in connection with the form of  $|\Psi(t)\rangle$ , saying that the state of the system at  $t = t_0$  has the factorized form

$$|\Psi(t_0)\rangle \simeq \sum_k P_k \cos(f_k t_0) |N-k, k\rangle|-\rangle \equiv |c(t_0)\rangle|-\rangle. \quad (36)$$

Let us moreover observe that

$$\cos(f_k t_0) = \cos\left(\frac{\pi(N^2-1)}{4}\right) = \begin{cases} +1, & \text{if } \frac{N+1}{2} \text{ is even,} \\ -1, & \text{if } \frac{N+1}{2} \text{ is odd,} \end{cases} \quad (37)$$

and that

$$(f_k - f_{k+1})t_0 = \Delta_k \pi \quad (38)$$

as a consequence of equations (27) and (31). In light of these considerations, and remembering that  $\Delta_k \in \mathbb{Z}$ , it is possible to verify that  $\cos(f_k t)$  can assume only the values  $+1$  and  $-1$ . Moreover in detail we have proved that its  $k$ -dependence may be expressed as

$$\cos(f_k t_0) \simeq (-1)^{\bar{k}} \cdot i^{(k-\bar{k})} \frac{(1-i(-1)^{k-\bar{k}})}{i-1}. \quad (39)$$

In view of equation (39) the vibrational state of the trapped ion at  $t = t_0$ , within an irrelevant overall phase factor, assumes the form:

$$|c(t_0)\rangle \simeq \frac{1}{2^{1/2}} \left( \sum_k P_k i^k |N-k, k\rangle - i(-1)^{(N+1)/2} \sum_k P_k (-i)^k |N-k, k\rangle \right). \quad (40)$$

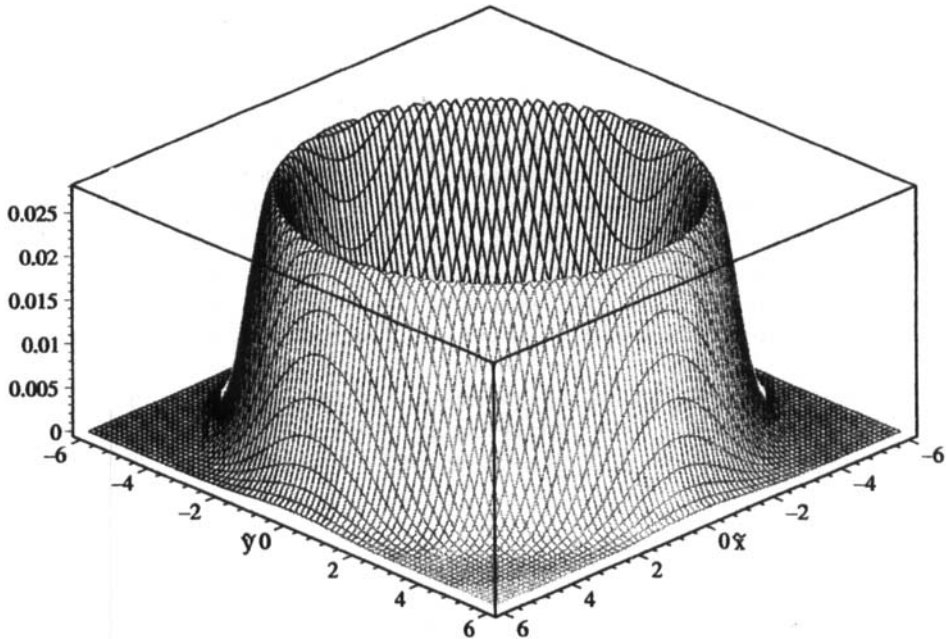


Figure 1. Spatial distribution of the states  $|n_l = N, n_r = 0\rangle$  and  $|n_l = 0, n_r = N\rangle$ , for  $N = 21$ , against  $\bar{x} = x/\beta$  and  $\bar{y} = y/\beta$  with  $\beta = (m\nu/\hbar)^{1/2}$ .

Taking into consideration equations (12) and (13), equation (40) can be written as:

$$|c(t_0)\rangle \simeq \frac{1}{2^{1/2}} (|n_l = N, n_r = 0\rangle - i(-1)^{(N+1)/2} |n_l = 0, n_r = N\rangle). \quad (41)$$

Equation (41) expresses one of the main results of this paper. We can in fact say that, if at  $t = t_0 \simeq t_c/2$  we turn off the external laser beams, the vibrational state of the trapped ion coincides with a superposition of two macroscopically distinguishable states. The two contributing terms in the quantum superposition (41) are, in fact, eigenstates of the orbital angular momentum  $\hat{L}_z$  corresponding to the maximum  $(+\hbar N)$  and minimum  $(-\hbar N)$  eigenvalue respectively, with  $N \gg 1$ . The spatial distribution of the two states  $|n_l = N, n_r = 0\rangle$  and  $|n_l = 0, n_r = N\rangle$  appearing in equation (41), are coincident and their relative plot is shown in figure 1. In figure 2 we report the spatial distribution of the vibrational state (41) generated with the help of our scheme. This plot clearly shows interference effects which represent a signature of the quantum nature of the superposition (41). Note that the spatial interference fringes are sensitive to vibrational decoherence [11] and can, thus, be used to study the decoherence induced transition of the quantum superpositions (41) into the corresponding statistical mixture whose spatial distribution is shown in figure 3.

Consider now the case  $N \gg 1$  even. The circumstance that  $P_-(t_c/2) \simeq \frac{1}{2}$ , as previously seen, suggests that, in contrast with the conclusions relative to the case  $N$  odd, the state of the system exhibits maximum entanglement between its fermionic and bosonic degrees of freedom. We have found that, also in this case, the vibrational state  $|c(t_c/2)\rangle$ , appearing in equation (36), possesses interesting

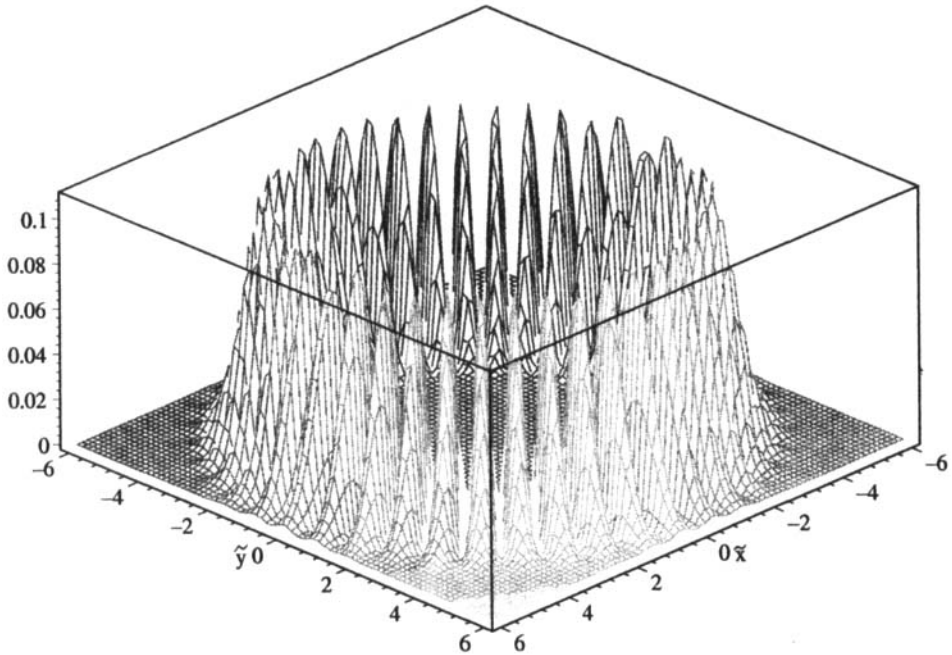


Figure 2. Spatial distribution of the superposition  $2^{-1/2}(|n_l = N, n_r = 0\rangle + i|n_l = 0, n_r = N\rangle)$ , for  $N = 21$ , against  $\tilde{x} = x/\beta$  and  $\tilde{y} = y/\beta$  with  $\beta = (m\nu/\hbar)^{1/2}$ .

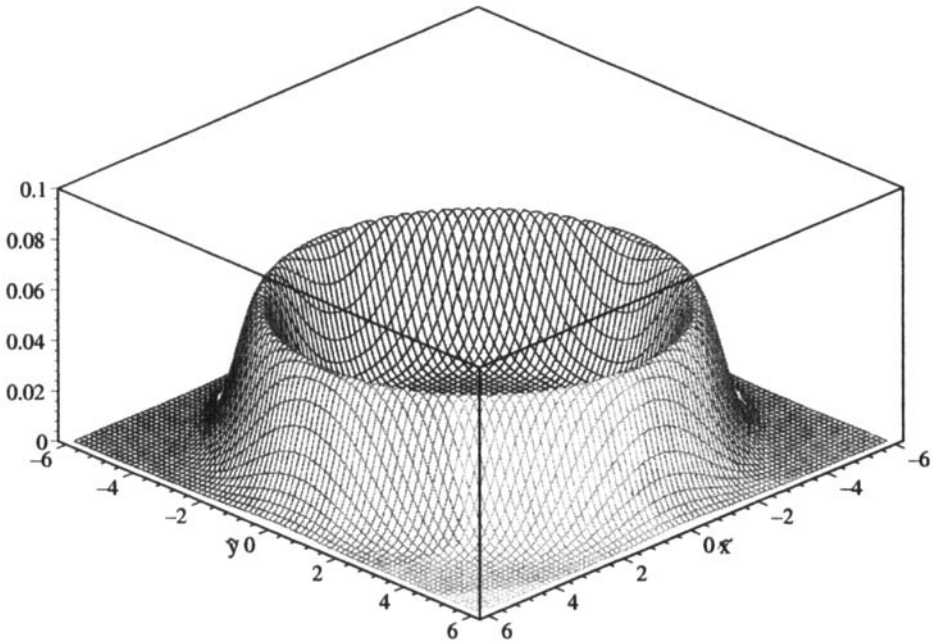


Figure 3. Spatial distribution of the statistical mixture of the two states  $|n_l = N, n_r = 0\rangle$  and  $|n_l = 0, n_r = N\rangle$ , for  $N = 21$ , against  $\tilde{x} = x/\beta$  and  $\tilde{y} = y/\beta$  with  $\beta = (m\nu/\hbar)^{1/2}$ .

properties. In fact, following an analysis similar to the one adopted for  $N$  odd, it is not difficult to establish that

$$\cos\left(f_k \frac{t_e}{2}\right) = \cos\left(\frac{\pi N^2}{4}\right) = \begin{cases} 1, & \text{if } \frac{N}{2} \text{ is even,} \\ -1, & \text{if } \frac{N}{2} \text{ is odd,} \end{cases} \quad (42)$$

and

$$(f_k - f_{k+1}) \frac{t_e}{2} = A_k \frac{\pi}{2}. \quad (43)$$

We wish to underline that the integer  $A_k$ , given by equation (28), is an odd integer whatever  $k$  is, if  $N$  is even. This property allows us to write  $\cos[f_k(t_e/2)]$  in the form

$$\cos\left(f_k \frac{t_e}{2}\right) \simeq \frac{1}{2}[(-1)^k + (-1)^{N/2}] \quad (44)$$

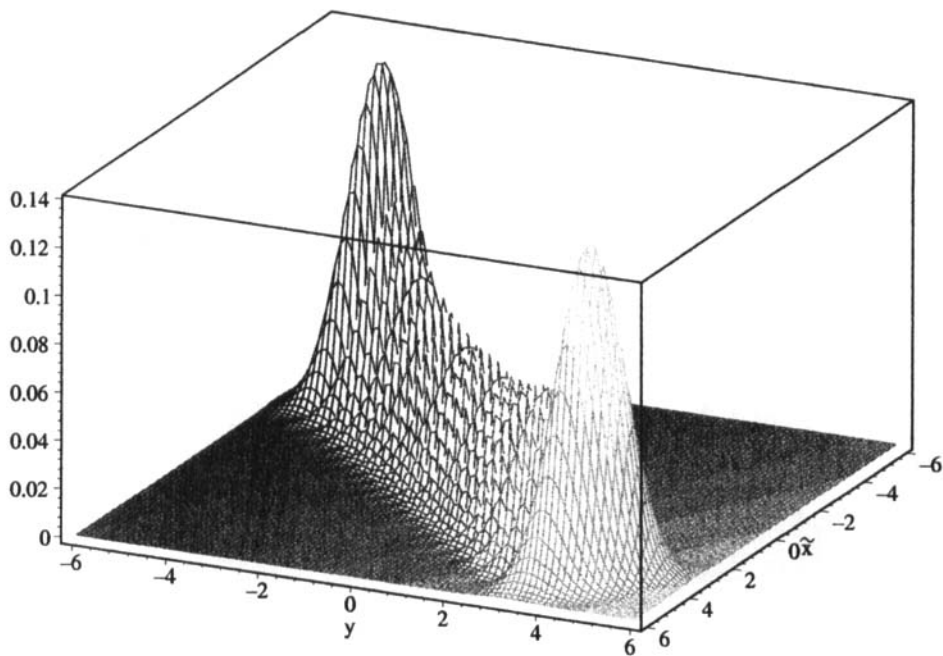
so that, substituting equation (44) into equation (36)

$$\begin{aligned} |c\left(\frac{t_e}{2}\right)\rangle &\simeq \frac{1}{2} \left( \sum_k P_k (-1)^k |N-k, k\rangle + (-1)^{N/2} \sum_k P_k |N-k, k\rangle \right) \\ &\equiv \frac{1}{2} \left[ \exp\left(-i \frac{\pi}{4} \hat{L}_z\right) |n_x = N, n_y = 0\rangle + (-1)^{N/2} \exp\left(i \frac{\pi}{4} \hat{L}_z\right) |n_x = N, n_y = 0\rangle \right] \\ &\equiv \frac{1}{2} (|n_{\bar{x}} = 0, n_{\bar{y}} = N\rangle + (-1)^{N/2} |n_{\bar{x}} = N, n_{\bar{y}} = 0\rangle). \end{aligned} \quad (45)$$

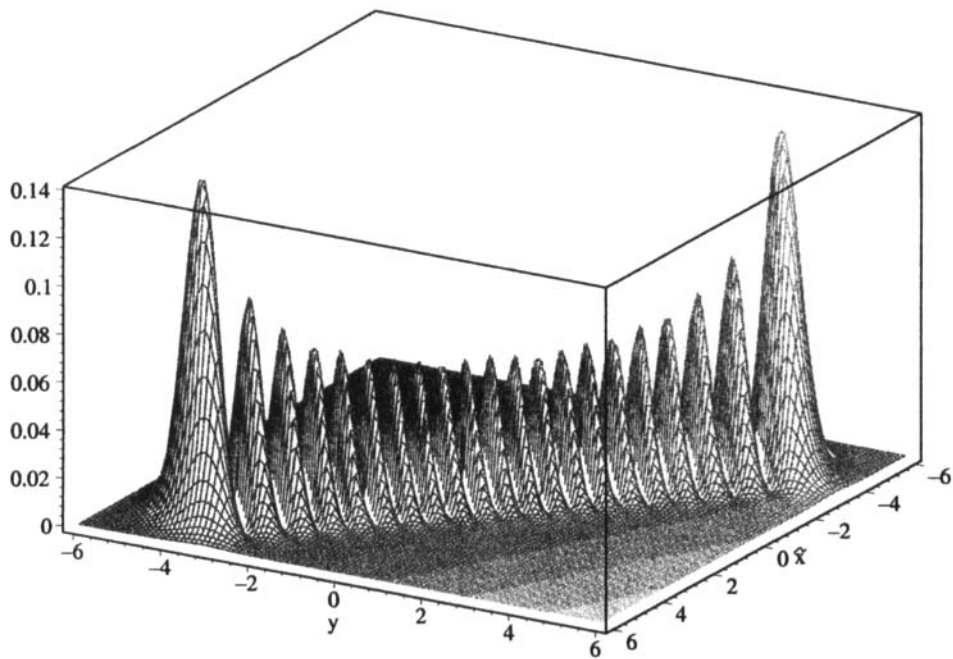
Equation (45) shows that  $|c(t_e/2)\rangle$  is a superposition of two states describing ionic oscillations along the two orthogonal directions  $\bar{X}$  and  $\bar{Y}$  respectively. In this sense we may claim that, if at  $t = t_e/2$  we turn off the laser field realizing Hamiltonian model (20) and we measure, by means of quantum jumps techniques [4, 5, 12], the electronic state of the ion as  $|-\rangle$ , its vibrational state is projected onto the quantum superposition of two macroscopically distinguishable states given, apart from an appropriate renormalization factor, by equation (45).

In [13] Bollinger *et al.* propose a different method for the generation of the vibrational state  $2^{-1/2}(|n_{\bar{x}} = N, n_{\bar{y}} = 0\rangle + |n_{\bar{x}} = 0, n_{\bar{y}} = N\rangle)$  and illustrate why this state is particularly interesting to consider in connection with the Mach-Zender boson interferometer. In fact, as shown in [13] this state yields exactly the Heisenberg uncertainty limit for an interferometer for any  $N$ . We wish to underline that our generation procedure makes possible the preparation not only of the quantum superposition  $2^{-1/2}(|n_{\bar{x}} = N, n_{\bar{y}} = 0\rangle + |n_{\bar{x}} = 0, n_{\bar{y}} = N\rangle)$  but also of the orthogonal superposition  $2^{-1/2}(|n_{\bar{x}} = N, n_{\bar{y}} = 0\rangle - |n_{\bar{x}} = 0, n_{\bar{y}} = N\rangle)$  characterized by very different quantum properties [14].

The spatial distributions of the two superimposed states  $|n_{\bar{x}} = N, n_{\bar{y}} = 0\rangle$  and  $|n_{\bar{x}} = 0, n_{\bar{y}} = N\rangle$ , are displayed in figures 4(a) and (b) respectively. Also in this case, there appear significant interference effects in the spatial distribution of the superpositions  $2^{-1/2}(|n_{\bar{x}} = N, n_{\bar{y}} = 0\rangle \pm |n_{\bar{x}} = 0, n_{\bar{y}} = N\rangle)$ , as illustrated in figure 5. For the sake of completeness, figure 6 illustrates the spatial distribution of the corresponding statistical mixture.



(a)



(b)

Figure 4. Spatial distribution (a) of the state  $|n_{\bar{x}} = N, n_{\bar{y}} = 0\rangle$  and (b) of the state  $|n_{\bar{x}} = 0, n_{\bar{y}} = N\rangle$ , for  $N = 20$ , against  $\tilde{x} = x/\beta$  and  $\tilde{y} = y/\beta$  with  $\beta = (m\nu/\hbar)^{1/2}$ .

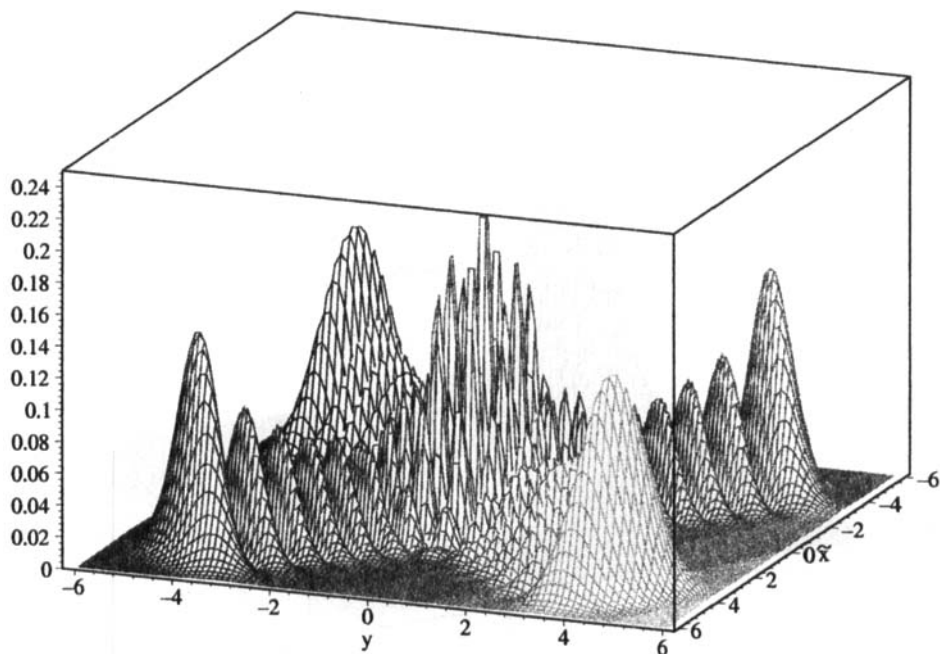


Figure 5. Spatial distribution of the superposition  $2^{-1/2}(|n_{\tilde{x}} = N, n_{\tilde{y}} = 0\rangle - |n_{\tilde{x}} = 0, n_{\tilde{y}} = N\rangle)$ , for  $N = 20$ , against  $\tilde{x} = x/\beta$  and  $\tilde{y} = y/\beta$  with  $\beta = (m\nu/\hbar)^{1/2}$ .

#### 4. Conclusions

Taking advantage from the peculiar coupling mechanism between the internal and external degrees of freedom of an ion trapped in an isotropic bidimensional harmonic potential we have proved the possibility of realizing two different classes of bimodal Schrödinger cat states. In particular, we have shown that varying the initial total number of vibrational quanta  $N$  of just one unit, the system evolves towards states characterized by very different quantum properties. Moreover, we have demonstrated the existence of instants of time at which a conditional measurement of the electronic state of the ion, generates superpositions of vibrational axial angular momentum eigenstates or Fock states along orthogonal directions, depending on the parity of  $N$ . A relevant aspect of our proposal is that, in both cases, the state generation is accomplished in a single step. This amounts to saying that only simultaneous laser pulses are required.

Unfortunately, the bigger  $N$  is the faster are the decoherence processes, due to coupling with the external environment, which destroy the quantum coherences of the superpositions generated. This circumstance, together with the fact that the generation times  $t_c/2$  and  $t_o$  are proportional to  $N$ , gives rise to serious difficulties for the experimental realization of the proposed scheme. However, very recently, there have been proposed quantum state protection methods [15] which basically consist in realizing a state-dependent artificial reservoir competing with the natural reservoir. Such methods are of relevance in connection with our proposal since it has been shown [15] that superpositions of the form (45) can be protected in this way.

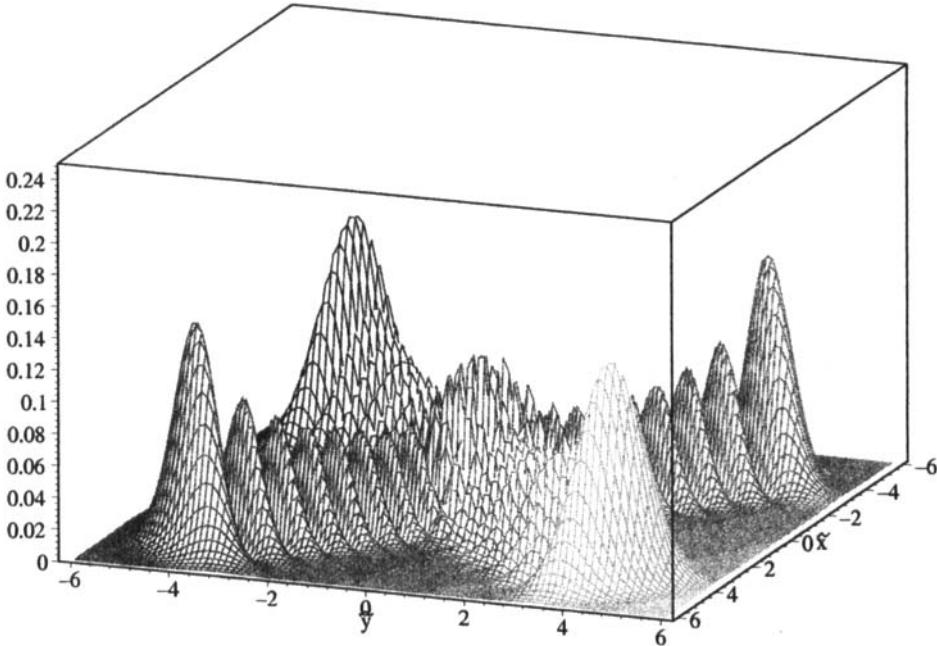


Figure 6. Spatial distribution of the statistical mixture of the states  $|n_{\tilde{x}} = N, n_{\tilde{y}} = 0\rangle$  and  $|n_{\tilde{x}} = 0, n_{\tilde{y}} = N\rangle$ , for  $N = 20$ , against  $\tilde{x} = x/\beta$  and  $\tilde{y} = y/\beta$  with  $\beta = (m\nu/\hbar)^{1/2}$ .

Concluding, we wish to emphasize that, in the context of the scheme illustrated in this paper, the total number of excitations  $N$  present in the initial state of the centre of mass motion, plays the role of an adjustable parameter allowing the generation of very different bimodal vibrational Schrödinger cat-like states.

### Acknowledgments

The financial support from CRRNSM-Regione Sicilia is greatly acknowledged. This work was also supported by MURST 40% co-financial support in the framework of the research project ‘Amplificazione e Rivelazione di Radiazione Quantistica’.

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