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Published in:
Experimental Thermal and Fluid Science

DOI:
10.1016/j.expthermflusci.2008.06.005

Publication date:
2008

Citation for published version (APA):
Reymond, O., Murray, D. B., & O'Donovan, T. S. (2008). Natural convection heat transfer from two horizontal cylinders. Experimental Thermal and Fluid Science, 32(8), 1702-1709. 10.1016/j.expthermflusci.2008.06.005

Experimental Thermal and Fluid Science 32 (2008) 1702–1709

Contents lists available at ScienceDirect

Experimental Thermal and Fluid Science

journal homepage: www.elsevier.com/locate/etfs

Natural convection heat transfer from two horizontal cylinders

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ARTICLE INFO

Article history:
Received 9 December 2007
Received in revised form 18 June 2008
Accepted 18 June 2008

Keywords:
Natural convection
Cylinders pair
Plume oscillations

ABSTRACT

Natural convection heat transfer from a single horizontal cylinder and a pair of vertically aligned horizontal cylinders is investigated. Surface heat transfer distributions around the circumference of the cylinders are presented for Rayleigh numbers of 2×10^6, 4×10^6 and 6×10^6 and a range of cylinder spacings of 1.5, 2 and 3 diameters. With a cylinder pairing the lower cylinder is unaffected by the presence of the second cylinder; the same is true of the upper cylinder if the lower one is not heated. However, when both cylinders are heated it has been found that a plume rising from the heated lower cylinder interacts with the upper cylinder and significantly affects the surface heat transfer distribution. Spectral analysis of surface heat transfer signals has established the influence of the plume oscillations on the heat transfer. Thus, when the plume from the lower cylinder oscillates out of phase with the flow around the upper cylinder it increases the mixing and results in enhanced heat transfer.

1. Introduction

Natural convection from a single horizontal cylinder has been studied in detail for more than 50 years and more recently the research has been focused on arrays and pairs of cylinders. Research in this area has several applications in engineering including heat exchangers and passive solar energy collectors. Morgan [1], Churchill and Chu [2] and others have determined empirical correlation equations which focus mainly on the area and time-averaged Nusselt number.

More recently, Kuehn and Goldstein [3], Farouk and Güçeri [4] have conducted some numerical analysis of the heat transfer around a single cylinder. Kuehn and Goldstein [3] reported on the angular and radial velocity around a cylinder for Rayleigh numbers of $\leqslant 10^7$. This investigation also presented values of the local and area averaged Nusselt number for a range of Prandtl number fluids, however the study compared numerical solutions to experiments in air only. Saitoh et al. [5] also investigated the natural convection heat transfer from a cylinder in air using finite difference methods. Kuehn and Goldstein [3], Farouk and Güçeri [4] and Cesini et al. [6] have all shown that the heat transfer is at its maximum at the bottom of the cylinder and decreases toward the top of the cylinder. The decrease in heat transfer is attributed to an increase of the thermal boundary layer thickness. The model proposed by Farouk and Güçeri [4] predicted the heat transfer from an isothermal cylinder and also considered a cylinder with a non-uniform wall temperature. It was shown that the plume will shift to the side of the cylinder with the larger temperature gradient.

Most of the research in this area has been limited to the case of a laminar convection flow. This is the simplest but also the most common case in natural convection. Nevertheless the motion of the fluid due to buoyancy can be turbulent. Morgan [1] stated that transition to turbulent flow may occur at a Rayleigh number of 2.0×10^7. Kitamura et al. [7] have investigated the turbulence transition and its influence on the heat transfer around a horizontal cylinder. It was found that the transition occurs for a Rayleigh number of 2.1×10^9, however, even for Rayleigh numbers up to 3.6×10^{13}, only the flow at the top of the cylinder is turbulent. Yang [8] noted that the laminar-turbulent transition varied, depending on whether it was natural convection in air or water. He proposed the Grashoff number as a more appropriate criterion for predicting the transition to turbulent flow and found two values of the Grashoff number experimentally to define the beginning and the end of the transition zone; these were independent of the fluid medium. At $Gr = 5.76 \times 10^8$ the laminar-transition initiated and above $Gr = 4.65 \times 10^9$ the transition to fully turbulent flow occurred.

Cesini et al. [6] investigated the effect of horizontal confinement on natural convection. In this case the aspect ratio was defined as the ratio of the tank width to the diameter of the cylinder. Cesini et al. [6] proposed that for low Rayleigh numbers (1.3×10^3–7.5×10^4) there is an optimum aspect ratio for heat transfer, in the range between 2.1 and 4.3, and that this decreased as Ra increased. For higher Rayleigh numbers Cesini et al. [6] have shown that the effect of the horizontal confinement was small relative to the effects of greater buoyancy forces. The effect of vertical confinement on the natural convection around a heated horizontal

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doi:10.1016/j.expthermflusci.2008.06.005

Nomenclature

D	diameter of cylinder, (m)
G	acceleration due to gravity, (m/s^2)
H	convective heat transfer coefficient, $h = \frac{\ddot{q}}{T_{surf} - T_{bulk}}$, (W/m^{2}K)
H	height of water level above cylinder, (m)
K	thermal conductivity, (W/mK)
Nu	Nusselt number, $Nu = \frac{hD}{k}$, (–)
Nu'	root-mean-square Nusselt number, (–)
Pr	Prandtl number, (–)
$\ddot{q}$	heat flux, (W/m^2)
Ra	Rayleigh number, $Ra = \frac{g\beta(T_{surf} - T_{bulk})D^3}{\nu^2} \times Pr$, (–)
S	separation distance between the centres of two cylinders, (m)
T	temperature, (K)
Greek symbols	
β	thermal diffusivity, (K^{-1})
δ	sensor thickness, (m)
θ	angle about cylinder centre from bottom of cylinder, (°)
ν	kinematic viscosity, (m^2/s)

cylinder was investigated by Atmane et al. [9]. It was found that the heat flux at the top of the cylinder was enhanced by decreasing the confinement ratio (defined as the ratio of the height of water above a cylinder to the diameter of the cylinder). Atmane et al. [9] demonstrated that the enhancement in heat transfer was linked to oscillation of the thermal plume above the cylinder. It was shown that for a confinement ratio larger than 3 the thermal plume is stable and no enhancement of heat transfer occurs.

Both Lieberman and Gebhart [10] and Marsters [11] conducted investigations of natural convection heat transfer from an array of heated cylinders while Yousefi and Ashjaee [12] investigated natural convection from a vertically aligned array of horizontal elliptic cylinders. The effects of spacing and Rayleigh number on the heat transfer were analysed. These investigations found that the upper cylinder behaves in a different way than the lower one. Marsters [11] and Ashjaee and Yousefi [13] have shown that the lower cylinder behaves like a single cylinder and both studies found that the spacing has an effect on the heat transfer of the upper cylinders. For small spacings Marsters [11] has shown that the heat transfer of the upper cylinder could be as much as 50% less than the lower cylinder, while for large spacing the heat transfer could increase by as much as 30%. Lieberman and Gebhart [10] attributed the variation in heat transfer with cylinder spacing to the plume rising from the lower cylinder. They explained that the variations in heat transfer were caused by two opposing effects generated by the plume. The plume temperature is higher than the bulk fluid temperature and this has the effect of reducing the local temperature difference and consequently the surface heat transfer from the upper cylinder. The velocity of the plume however effectively imposes a forced convection flow condition on the upper cylinder which increases the heat transfer.

Sparrow and Niethammer [14] studied free convection heat transfer from an array of 2 vertically separated cylinders in the range of Rayleigh numbers from 2×10^4 to 2×10^5 with a uniform wall temperature boundary condition. Sparrow and Niethammer [14] found that the average Nusselt number increased gradually with increased spacing between the two cylinders. The maximum heat transfer was found to occur when the cylinders were between 7 and 9 diameters apart. Tokura et al. [15] considered free convection heat transfer from vertical arrays of 2, 3 and 5 horizontal cylinders, maintained at a uniform temperature, in the range of Rayleigh numbers from 2.8×10^4 to 2.8×10^5. By measuring the distribution of the heat transfer around the circumference of the cylinder Tokura et al. [15] found that for small spacings the heat transfer at the bottom of the upper cylinder was reduced. However when the space between the cylinders increased, the heat transfer at the bottom of the cylinder was elevated. More recently, Chouikh et al. [16] performed an experimental study of the natural convection flow around 2 heated horizontal cylinders in air. Chouikh et al. [16] reported that the flow field varies with Rayleigh number. It was observed that at the higher Rayleigh numbers considered (10^4 compared to 10^2), the majority of the flow comes from the sides instead of bottom.

The current research investigates the distribution of the surface heat transfer from two horizontal heated cylinders under natural convection conditions in water. This experimental study considers the influence of Rayleigh number and cylinder spacing. The range of Rayleigh numbers from 2×10^6 to 6×10^6 and cylinder spacings of $s/D = 1.5$–3 studied is a common range that is found in many tube bundle heat exchangers. The influence of the plume on the natural convection heat transfer has been established by analysing both time-averaged and time resolved heat transfer measurements at discrete locations on the surface of the cylinder.

2. Experimental rig

This research is conducted on an experimental test rig which was previously used in an investigation by Atmane et al. [9]. As indicated in Fig. 1a, the rig consists of a water tank measuring 600 mm high, 700 mm long and 300 mm wide. Two copper cylinders can be mounted horizontally in the tank, one directly above the other. The cylinders are 30 mm in diameter and span the internal width of the tank (270 mm). The distance between their centres (s), can be varied from 1.5 to 3.0 cylinder diameters. The depth at which the upper cylinder is mounted is 150 mm below the water level; this corresponds to 5 cylinder diameters and so the natural convective flow around the cylinders can be considered to be unconfined vertically, Atmane et al. [9]. The cylinders are considered to be unconfined horizontally also, as the walls of the tank are 5 diameters from the measurement location. Each cylinder is heated by two 500 W cartridge heaters which are inserted in a 10 mm diameter central bore (Fig. 1b). Due to the high thermal conductivity and the thickness of the copper, the heated cylinders approximate a uniform wall temperature boundary condition during testing. For the range of measurements presented the surface temperature has been observed to vary by less than 0.5 °C circumferentially. The bulk water temperature was constant throughout all testing and steady-state was achieved when neither the bulk water nor the surface temperature was observed to vary with time.

As shown in Fig. 1, an RdF Micro-Foil® heat flux sensor and T-type thermocouple are flush mounted on one of the cylinders, hereafter referred to as the instrumented cylinder. The instrumented cylinder can turn about its axis of symmetry so that the circumferential distribution of the surface heat flux can be measured. A thermocouple is also placed in the tank at a depth that is level with the cylinders to measure the bulk water temperature. The RdF Micro-Foil® heat flux sensor consists of a differential thermopile that measures the temperature above and below a known thermal barrier. The heat flux through the sensor is therefore defined by Eq. (1).

$$\ddot{q} = k_s \frac{\Delta T}{\delta} \quad (1)$$

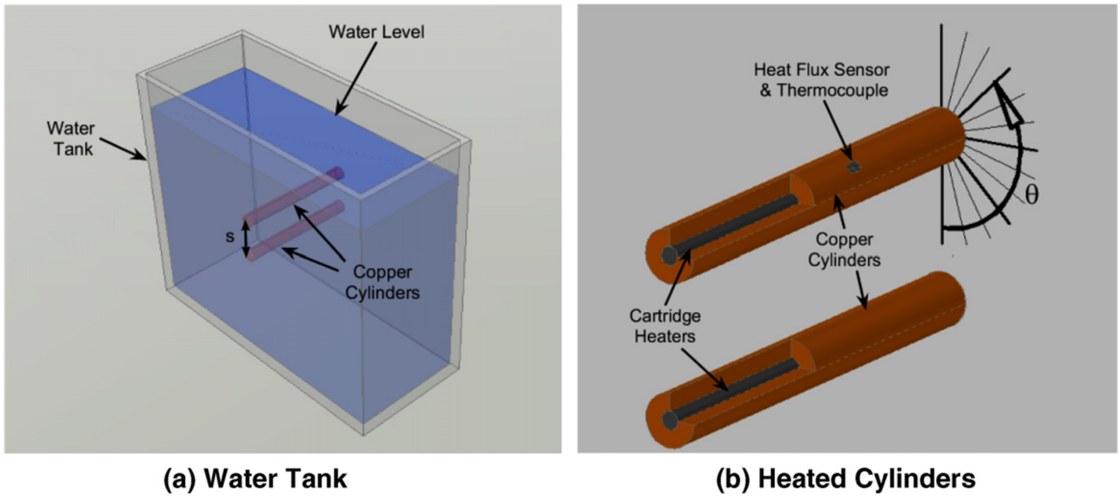

Fig. 1. Experimental test rig.

where k_s is the thermal conductivity of the barrier (kapton) and ΔT is the temperature difference across the thickness (δ) of the barrier. The convective heat transfer coefficient is thus calculated from Eq. (2):

$$h = \frac{\ddot{q}}{T_{\text{surface}} - T_{\text{bulk, water}}} \tag{2}$$

Results are presented in the form or the mean and root-mean-square Nusselt number which are based on the cylinder diameter. The rms Nusselt number (Nu') is the non-dimensional form of the rms convective heat transfer coefficient and is a measure of the unsteadiness (temperature and velocity) of the fluid close to the heated surface.

The emissivity of the sensor surface is approximately 0.5 and radiative heat transfer accounts for less than 1.5% of the measured heat transfer. The emissivity of the copper cylinders is approximately 0.10 and therefore heat transfer due to radiation is considered negligible, accounting for less than 0.3% of the overall heat transfer. The RdF Micro-Foil® heat flux sensor (model 27036-1) has a characteristic 62% response to a step function of 0.02 s. This response time is far higher than the flow and heat transfer fluctuations that have a period in the order of seconds. The output signal from the heat flux sensor was amplified by a factor of 1000 to improve the signal to noise ratio. A National Instrument data acquisition card (DAQ Pad-6015) was used to acquire all temperature and heat flux signals at a frequency of 50 Hz, which again is far greater than the naturally occurring frequencies in the natural convection flow.

The uncertainty associated with each measurement technique is reported with a 95% confidence level in accordance with the ASME Journal of Heat Transfer policy on uncertainty [17]. The uncertainty of the temperature measurements is calculated to be less than 1% and this results in a Rayleigh number uncertainty of 2.7%. The Rayleigh number varied by less than 10% from the mean value reported during experimental testing. Data are presented in the form of the mean and root-mean-square Nusselt number distributions which have an uncertainty of 6% and 90%, respectively. The reason for the very high uncertainty in Nu' is that the typical fluctuation level is low. Data are measured at increments of 10° around the cylinder's circumference with an accuracy of 1°.

3. Results and discussion

Surface heat transfer distributions are presented in this section from a single cylinder firstly and then from cylinders that are vertically aligned. As the surface heat transfer distributions are symmetric about a vertical axis through the cylinder centres, results are presented of the circumferential Nusselt number distribution from 0° (bottom dead centre) to 180° (top dead centre). The effect of varying the Rayleigh number and the spacing between cylinders is investigated. At certain locations around the circumference of the cylinder the temporal variation of the surface heat flux is also examined.

3.1. Single cylinder

Surface heat transfer data are presented in this section in the form of mean and root-mean-square Nusselt number distributions from a single unconfined heated cylinder submerged in water. Fig. 2 presents the variation of the mean heat transfer with angle. These results are consistent with a numerical prediction by Saitoh et al. [5]. The circumferential distribution of the Nusselt number can be related to the thickness of the thermal boundary layer around the cylinder which is thinnest at the bottom, ($\theta = 0°$). Thus the temperature gradient is highest and therefore the surface heat transfer is a maximum at this point. The Nusselt number steadily decreases toward the top of the cylinder as the boundary layer thickness increases. At $\theta \approx 160°$ the surface heat transfer decreases sharply towards 180° (the top of the cylinder). This can be attributed to the presence of a thermal plume which rises above the

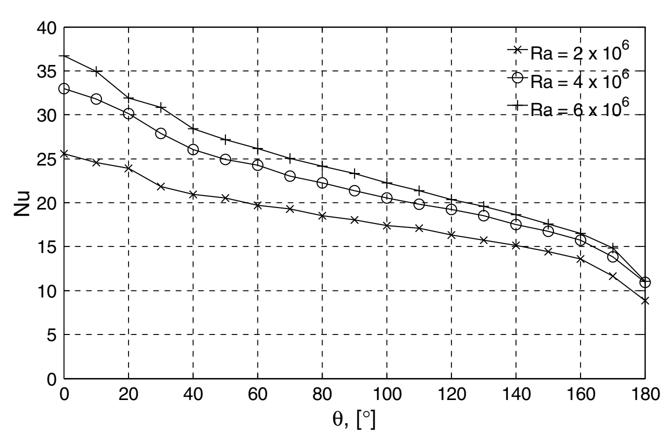

Fig. 2. Effect of Rayleigh number on Nusselt number distributions.

cylinder. The plume effectively insulates the cylinder from the bulk water and results in a lower heat transfer coefficient.

Fig. 2 also illustrates the effect of Rayleigh number on the surface heat transfer distribution. The Rayleigh number was varied by increasing the temperature difference between the cylinder and the bulk water. It can be seen that increasing the Rayleigh number increases the area averaged heat transfer in line with a well established correlation by Morgan [1], Eq. (3):

$$\overline{Nu} = C \cdot Ra^n \tag{3}$$

where $C = 0.48$ and $n = 0.25$ for $10^4 < Ra < 10^7$

Fig. 3 presents the distribution of the root-mean-square Nusselt number for the same range of parameters as in Fig. 2. As the buoyancy driven flow around the cylinder is laminar the heat transfer fluctuations are low, corresponding to around 2% of the mean. As this is less than the uncertainty of the measurement technique, caution should be exercised when drawing conclusions from these data. Nevertheless, some general trends can be seen in the distributions. Overall the magnitude of the fluctuations increase significantly in going from a Rayleigh number of 2×10^6 to 4×10^6 but level off with increasing *Ra* thereafter. At the bottom of the cylinder the fluctuations are generally higher; this is also where the boundary layer is thinnest so small fluctuations in the flow will influence the magnitude of the surface heat transfer fluctuations. With increasing distance around the circumference of the cylinder the magnitude of the heat transfer fluctuations decrease as a steady laminar flow is established. At $\theta \approx 160°$ Nu' increases again as the plume rising above the cylinder is formed and oscillates above the cylinder.

Time-traces of the surface heat transfer and the corresponding spectral analysis at $\theta = 0°$, 70°, 110° and 180° are presented in Fig. 4 for a Rayleigh number of 4×10^6. Clear periodicity at very low frequencies has been found at each location. At $\theta = 180°$ the heat transfer fluctuations occur at a frequency of 0.008 Hz and are attributed to the oscillation of the plume that forms at the top of the cylinder. This oscillation propagates around the circumference of the cylinder and can be seen to occur at each angle. This frequency of oscillation is equivalent to a time period of 125 s and even with long acquisition times of 500 s, the spectral resolution is relatively poor at low frequencies. Nevertheless it can clearly be seen that the frequency of oscillation is constant throughout the range of angles tested with one exception at $\theta = 0°$. In this case, at the bottom of the cylinder, the heat transfer fluctuates at 0.008 Hz and at approximately 0.016 Hz. These frequencies correspond to the natural frequency of the plume and also to twice that frequency. A similar phenomenon has been reported by Scholten and Murray [18] who investigated the heat transfer from a cylinder in cross flow. Scholten and Murray [18] found that the shedding frequency of vortices introduced a periodicity in the flow that influenced the heat transfer at each location around the circumference of the cylinders. The frequency of oscillation at the front stagnation point was shown to be twice that of the shedding frequency and was attributed to the heat flux sensor being insensitive to flow direction. This is comparable to the current natural convection research. As the plume oscillates at a frequency of approximately 0.008 Hz it influences the heat transfer fluctuations at each point of the cylinder's surface and also results in an oscillation frequency double that of the plume frequency at the bottom of the cylinder.

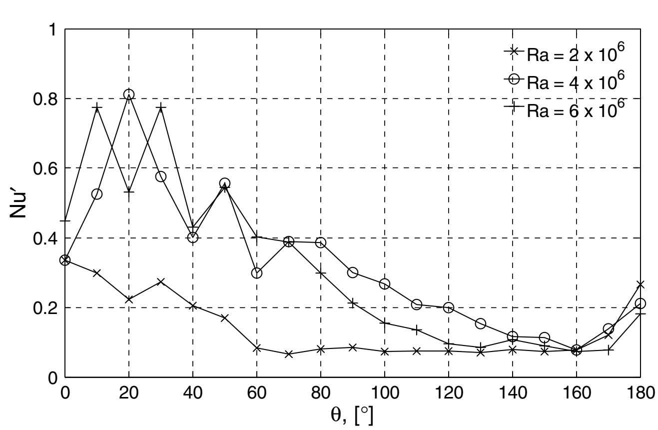

Fig. 3. Effect of Rayleigh number on Nu' distributions.

The magnitudes of the heat transfer fluctuations at different surface locations (presented in Fig. 3) have been discussed previously but are also evident from the power spectra plotted in Fig. 4. The relative magnitude of the oscillations at different frequencies at $\theta = 0°$ however are only evident now from this spectral analysis. It can be seen that heat transfer fluctuations have a similar magnitude at both frequencies. An equivalent analysis has been performed for Rayleigh numbers of 2×10^6 and 6×10^6 however the frequency of oscillation was not found to vary measurably for this range of parameters.

3.2. Two cylinders

The heat transfer from the individual cylinders in a pair of vertically aligned cylinders was measured for a number of different configurations:

- Lower cylinder with upper cylinder heated
- Lower cylinder with upper cylinder unheated
- Upper cylinder with lower cylinder unheated

These data are compared with the heat transfer from a single cylinder in Fig. 5. As can be seen in Fig. 5, it was found that the heat transfer from the lower cylinder was unaffected by the presence of the upper cylinder, whether it was heated or unheated. In fact the two distributions differed by less than 1% from that of the single cylinder, which is well within the margin of error in the measurement technique. This result is arguably at odds with the findings of Atmane et al. [9] who investigated the effect of vertical confinement on the surface heat transfer from a single heated cylinder. Atmane et al. [9] investigated natural convection from a single cylinder at depths of up to 3 diameters, where the water ceiling confines the flow vertically. Atmane et al. [9] have shown that heat transfer at the top of a vertically confined cylinder was enhanced due to the oscillation of the confined plume from one side to the other. This effect was observed for H/D ratios of 0.5 and 1.0, which correspond to the current spacings of $s/D = 1.5$ and 2 diameters if the bottom of the upper cylinder is considered to behave like the water ceiling. Since the upper cylinder has no appreciable effect on the heat transfer from the lower cylinder the lower cylinder can be considered to be unconfined vertically. These findings are consistent with investigations by Marsters [11], Sparrow and Niethammer [14] and Tokura et al. [15].

Results presented in Fig. 5 have also shown that the distribution of the heat transfer from the upper cylinder is unaffected when the lower cylinder is unheated. This result is expected as the unheated cylinder does not induce a natural convective flow and the distance between the two cylinders is far in excess of the thermal boundary layer thickness formed around the heated upper cylinder. Therefore the lower cylinder does not in any way interfere with the boundary layer flow around the upper cylinder. Tokura et al. [15] investigated vertical confinement below a single heated cylinder and have shown that, in the case where a cylinder is close to the bottom of the tank, the heat transfer would be decreased at the bottom of the cylinder. According to Tokura et al. [15] the heat

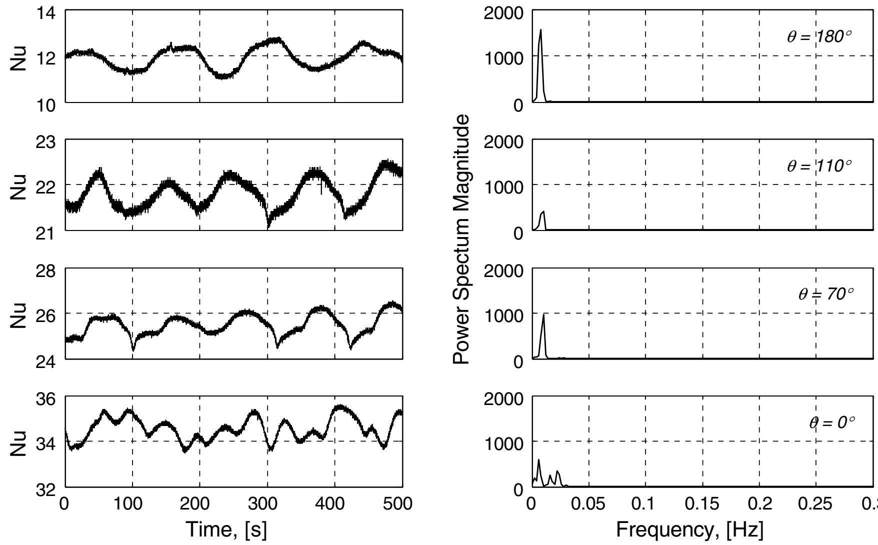

Fig. 4. Nusselt number time-traces & spectral analysis, $Ra = 4.0 \times 10^6$.

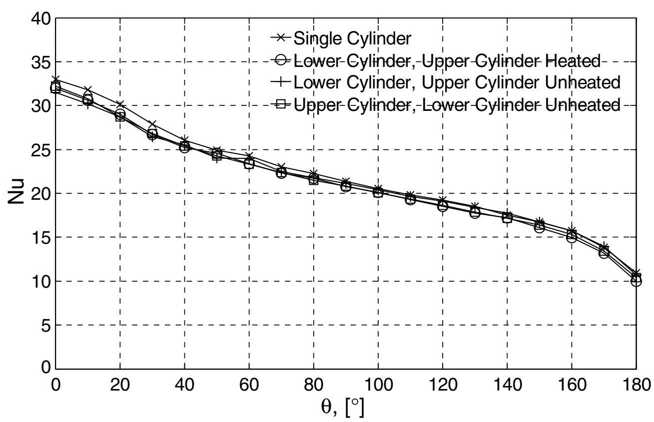

Fig. 5. Nusselt number distributions, $Ra = 4 \times 10^6$, $s/D = 1.5$.

transfer from a heated cylinder is unaffected when it is at a distance greater than 0.1D from the lower surface. This corresponds to a spacing of 1.1D between two cylinders, which is considerably less than the minimum spacing of 1.5D in the current investigation. The distribution of Nu' was also measured for these configurations. It was found that the fluctuations were similar in magnitude to those presented in Fig. 3 and were of the order of 2% of the mean surface heat transfer measurement.

Further data are therefore limited to heat transfer from the upper cylinder when two horizontal cylinders in the same vertical plane are heated. Fig. 6 Nusselt number distribution of the upper cylinder, where the centres of the cylinders are 1.5 diameters apart, to that of a single cylinder for a Rayleigh number of 4.0×10^6. It can be seen from Fig. 6 that the Nusselt number at the bottom ($\theta = 0°$) of the upper cylinder is approximately 30% greater than from the single cylinder; this is consistent with an investigation by Tokura et al. [15]. With increasing circumferential distance, the Nusselt number decreases sharply to a value much lower than that of the single cylinder. The area averaged Nusselt number is calculated to be approximately 8.4% lower than the single cylinder overall. Similar trends were recorded for Rayleigh numbers of 2×10^6 and 6×10^6, with a decrease in area averaged heat transfer of 3.8% and 6.5%, respectively.

Differences between the Nusselt number distribution around the upper cylinder and around the single cylinder are attributed to the interaction of the plume rising from the lower cylinder with the natural convective flow around the upper cylinder. The distribution of the root-mean-square Nusselt number presented in Fig. 7

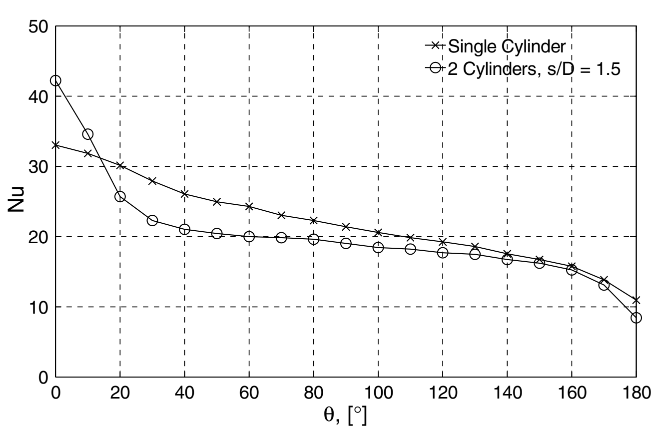

Fig. 6. Comparison of heat transfer from upper cylinder & single cylinder, $Ra = 4 \times 10^6$, $s/D = 1.5$.

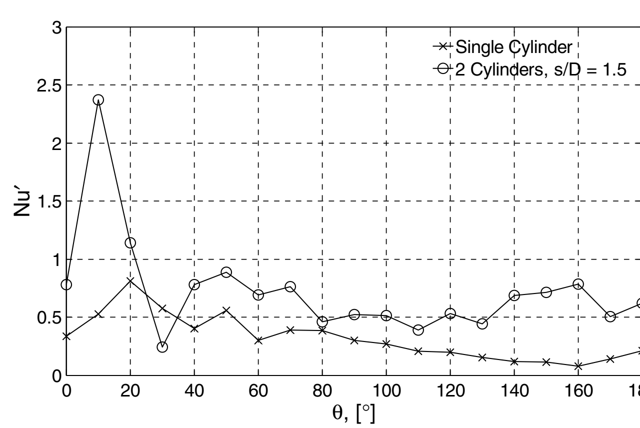

Fig. 7. Comparison of Nu' distribution from upper cylinder & single cylinder, $Ra = 4 \times 10^6$, $s/D = 1.5$.

helps to determine the extent to which this plume affects the flow around the upper cylinder. It has been established earlier that the lower cylinder was not confined by the upper cylinder and therefore the plume rising from the lower cylinder should be reasonably steady. Fig. 7 compares the root-mean-square Nusselt number distribution from the upper cylinder to that from a single cylinder for a Rayleigh number of 4×10^6. While the fluctuations are reasonably small and uniform along the circumference of the single heated cylinder, it can be seen that the largest surface heat transfer fluctuations occur at the bottom of the upper cylinder. This can be directly attributed to the plume rising from the lower cylinder impinging on the upper cylinder. Again, similar trends have been observed for $Ra = 2 \times 10^6$ and 6×10^6.

The rising plume is responsible for two competing effects as described by Lieberman and Gebhart [10], one which enhances and a second which reduces the surface heat transfer from the upper cylinder. Firstly, as the plume from the lower cylinder impinges on the upper cylinder it can be considered as a type of "forced convection" which enhances the heat transfer. In the current vertically aligned configuration of the cylinders, the plume impinges at the bottom dead centre of the upper cylinder and is responsible for the local increase in the surface heat transfer from $\theta = 0^\circ$ to 10° or 20° depending on the Rayleigh number. This can be seen in Fig. 6. The water in the plume, rising from the lower cylinder, is at a higher temperature than the bulk water temperature however. This results in a lower local temperature difference between the upper cylinder and the fluid than is the case for a single cylinder. This effect is amplified by the high heat transfer occurring at the bottom of the upper cylinder, leading to an even lower temperature difference over its sides and top. Despite the increased magnitude of the heat transfer fluctuations along the entire circumference of the upper cylinder, which indicate increased instability in the boundary layer, the heat transfer at greater circumferential distances is reduced. Since the area averaged heat transfer from the upper cylinder is reduced for each Rayleigh number, it is concluded that the increased local fluid velocity does not sufficiently compensate for the adverse effect of the reduced local temperature gradient.

For a Rayleigh number of 2.0×10^6, the Nusselt number distribution around the upper cylinder has been investigated for different spacings ($s/D = 1.5$, 2.0 and 3.0) between the two vertically aligned cylinders. These distributions are compared to the heat transfer distribution from a single cylinder in Fig. 8.

The heat transfer distributions are broadly similar for each spacing; the maximum Nusselt number occurs at the bottom of the cylinder and decreases with distance around the circumference of the cylinder until it is a minimum at the top. The relative magnitude of the surface heat transfer, however, is strongly dependent on the spacing between the cylinders. For each of the spacings tested, the heat transfer is enhanced at the bottom of the upper cylinder when compared to a single cylinder. This can be attributed to the "forced convection" at this location due to the impingement of the plume rising from the lower cylinder. Fig. 8 shows that the enhancement at this location increases from $s/D = 1.5$ to 2 and then falls again for $s/D = 3$. It can be seen also from Fig. 8 that the region over which the heat transfer enhancement exists increases with increasing distance between the two cylinders. This can be explained by considering the plume that rises from the lower cylinder. As the plume rises the local fluid velocity decreases due to viscous dissipation and the spreading of the plume over a wider region.

The effect of the plume spreading can be clearly seen in the distributions of Nu' which are plotted for a similar range of parameters in Fig. 9. For $s/D = 1.5$ a region of high heat transfer fluctuations extends from 0° to beyond 20°. This region is larger for $s/D = 2$ where it extends beyond $\theta = 40^\circ$. This region of high heat transfer fluctuations is directly attributable to the width of the plume and is evidence of the plume spreading with increased distance between the two cylinders. Beyond this region of high heat transfer fluctuations, the magnitude drops significantly and remains low and uniform over the rest of the cylinder surface as a more steady boundary layer flow develops.

Investigations by Sparrow and Niethammer [14] and Tokura et al. [15] were conducted for similar geometries but differ from the current study as the working fluid was air and only time-averaged results for a lower range of Rayleigh numbers ($10^4 \leqslant Ra \leqslant 10^5$) are presented. Nevertheless, some general trends make for comparison with the current research. Sparrow and Niethammer [14] and Tokura et al. [15] have reported that there is an optimum spacing for enhanced heat transfer from the upper cylinder. This is consistent with findings presented in Fig. 8. According to Sparrow and Niethammer [14] the optimum spacing is Rayleigh number dependent.

The spacing of 2 diameters appears to be the optimum spacing for enhanced heat transfer at the bottom of the cylinder. This location corresponds to where the spreading of the plume has been sufficient to optimise the effect of forced convection by spreading the plume over a large area without reducing the arrival velocity of the plume significantly. As the distance between the two cylinders increases, the plume velocity decreases further and the local heat transfer coefficient at the bottom of the upper cylinder decreases. The increased spreading of the plume with increased s/D also explains why the heat transfer enhancement is sustained over a greater area. In the case of $s/D = 1.5$ and 2.0 the heat transfer

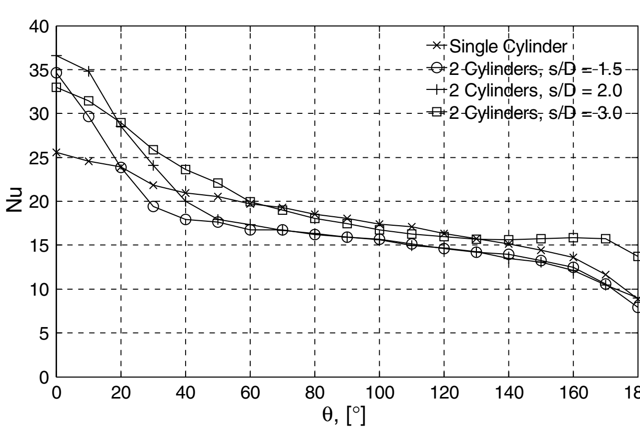

Fig. 8. Effect of spacing on Nusselt number distribution from upper cylinder, $Ra = 2 \times 10^6$.

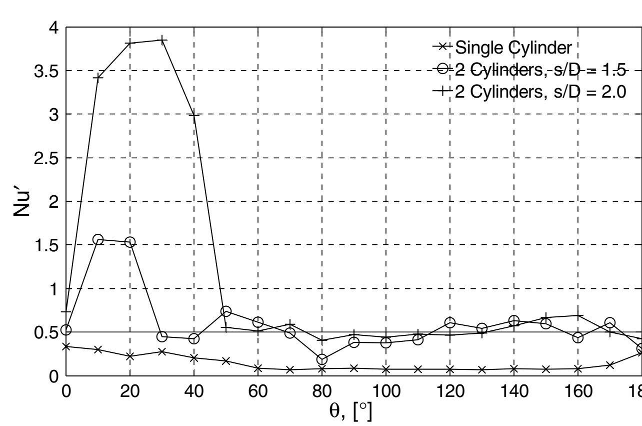

Fig. 9. Effect of spacing on Nu' distribution from upper cylinder, $Ra = 2 \times 10^6$.

enhancement lasts up to an angle of $\theta = 20^\circ$ and 40° respectively before it drops significantly below that of a single cylinder (Fig. 8). The local Nusselt number from the upper cylinder remains lower than that of a single cylinder until $\theta \approx 160^\circ$ beyond which all three spacings exhibit similar values of Nusselt number.

While the increase of the local Nusselt number above that of a single cylinder is more modest for $s/D = 3.0$, the region of enhanced heat transfer extends up to 60° and at no point along the circumference of the cylinder does the Nusselt number fall below that of the single cylinder. At an angle of $\theta \approx 160^\circ$ the heat transfer distribution diverges from that of a single cylinder once again, remaining high as the heat transfer from the single cylinder falls sharply towards the top of the cylinder. Therefore, while $s/D = 2$ may be the optimum spacing for enhanced heat transfer over the bottom of the upper cylinder, the overall heat transfer can be enhanced by increasing the spacing to $s/D = 3$. Further investigation may reveal an optimum spacing for the area averaged heat transfer from the upper cylinder.

The time varying heat transfer from the upper cylinder in a pair of vertically aligned heated cylinders was also investigated. Figs. 10 and 11 present heat transfer time-traces and spectral analysis of the signal for cylinder spacings of 1.5 and 2 diameters respectively and a single Rayleigh number of 6×10^6. There are many similarities between the surface heat transfer signals from the upper cylinder and the single cylinder presented in Fig. 4. At the bottom of the cylinder ($\theta = 0^\circ$) the surface heat transfer fluctuates at frequencies which are multiples of the plume frequency; the lower frequency is equivalent to the plume oscillation frequency and the dominant frequency is twice this naturally occurring frequency. In some cases a third frequency peak occurs at a harmonic of the natural frequency. At the bottom of a single cylinder the magnitudes of the two frequency peaks are similar as shown in Fig. 4 but this is not the case for a cylinder pairing.

The differences between the surface heat transfer signals from a single cylinder and from an upper cylinder can be attributed to the interaction of the plume rising from the lower cylinder with the upper cylinder. At $\theta = 0^\circ$ on the upper cylinder it is obvious from both the time-traces and the spectral analysis that the surface heat transfer fluctuates at a higher frequency than it does for the single cylinder. For both $s/D = 1.5$ and 2 (Figs. 10 and 11, respectively) the higher frequency of 0.016 Hz is dominant and the lower frequency of 0.008 Hz is only a subharmonic of this dominant frequency. This can be understood by considering the oscillation of the plume rising from the lower cylinder and its interaction with the flow around the upper cylinder. The plume rising from the lower cylinder oscillates at the same frequency (0.008 Hz) as the plume from a single cylinder or the upper cylinder. When the plume from the lower cylinder interacts with the upper cylinder it is likely to oscillate out of phase with the flow fluctuations at the bottom of the upper cylinder. The resultant frequency at the bottom of the upper cylinder therefore is twice as high as the single cylinder plume frequency. At $\theta = 70^\circ$ for both $s/D = 1.5$ and 2, the higher frequency is reduced significantly but crucially still exists (see inset in Fig. 10). This cannot be attributed to effects from both sides of the cylinder and can only be attributed to the plume rising out of phase with that from the lower cylinder.

It is thought that at lower spacings, outside the scope of this investigation, the proximity of the cylinders could result in the arriving plume oscillating in phase with the existing flow oscillations at the bottom of the upper cylinder. At larger spacings the upper cylinder flow will not oscillate in phase with the lower and therefore surface heat transfer fluctuations from the upper cylinder will occur at the higher frequency. The apparent phase lag is proportional to the spacing between the cylinders (s/D) and explains why the magnitude of surface heat transfer fluctuations from the under side of the upper cylinder are also spacing dependent. It can be seen from Fig. 9 that the surface heat transfer fluctuations at the under side of the upper cylinder increase with increasing s/D and this can be attributed to the larger phase difference between the effectively opposing flow streams. When the two natural convective flows (rising plume from the lower cylinder and thermal boundary layer around the upper cylinder) meet at the bottom of the upper cylinder the phase lag will determine the amount of turbulence generated. As the two flows move further out of phase with greater s/D the flows may directly oppose each other. This results in a more turbulent flow on the lower surface and can explain why the overall surface heat transfer increases in this region with increasing s/D. It is expected that at even greater s/D the two flows would eventually reach 180° out of phase with each other and therefore move together. This would reduce the mixing in the flow close to the surface and hence reduce the mean heat transfer once more. An optimum spacing between the two cylinders exists where the surface heat transfer at the bottom of the upper cylinder will be maximised. The phase lag between the two natural convection flow streams will be a contributing factor along with the effects of plume spreading and viscous dissipation of the plume from the lower cylinder. Future work will concern the measurement of the

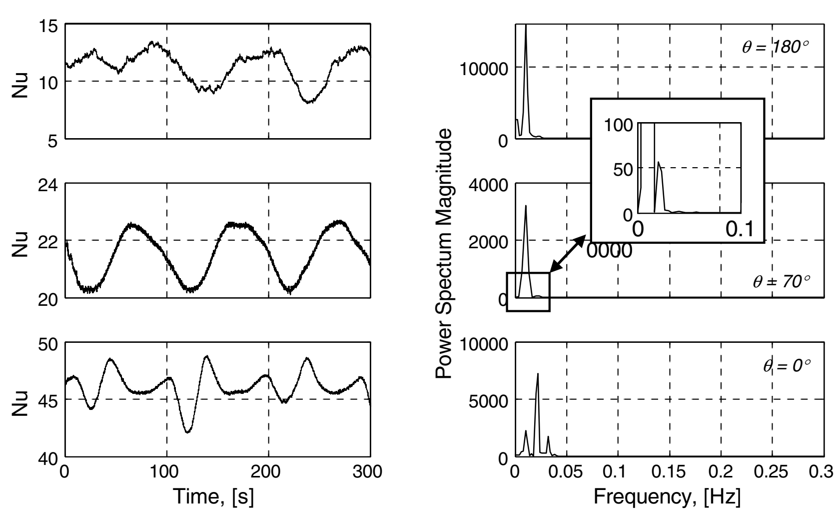

Fig. 10. Nusselt number time-traces & spectral analysis (upper cylinder), $s/D = 1.5$, $Ra = 6.0 \times 10^6$.

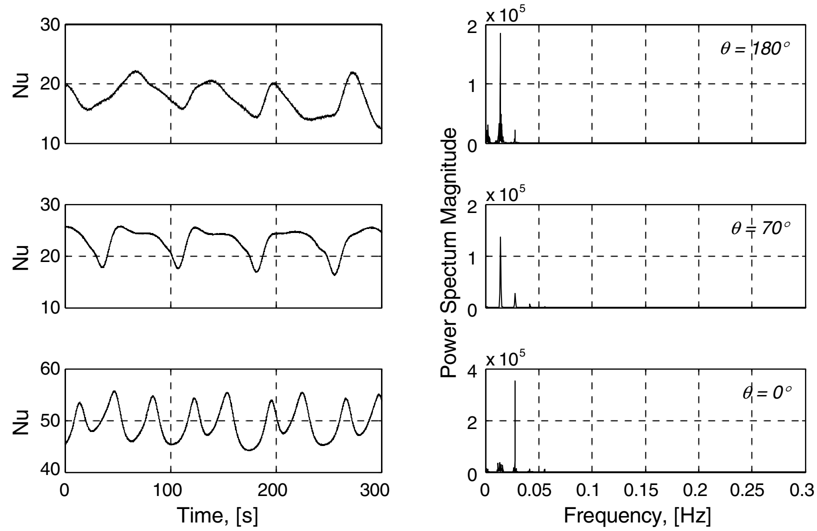

Fig. 11. Nusselt number time-traces & spectral analysis (upper cylinder), $s/D = 2$, $Ra = 6.0 \times 10^6$.

velocity flow field around the two cylinders using a Particle Image Velocimetry Technique.

4. Conclusions

Surface heat transfer measurements from both a single horizontal cylinder and a pair of vertically aligned horizontal cylinders have been presented. The distribution of the mean Nusselt number around the circumference of a single cylinder has been shown to be a maximum at the bottom of the cylinder ($\theta = 0°$) and to decrease towards the top ($\theta = 180°$) as the boundary layer develops. The root-means-square Nusselt number (Nu') distributions have also been presented and are indicative of the unsteadiness in the flow around the cylinder, which influences the overall heat transfer. For the single cylinder, the magnitude of the unsteadiness is low overall but some general trends can be seen. At the bottom of the cylinder the fluctuation is a maximum; it decreases around the sides of the cylinder and increases sharply again at the top of the cylinder. These regions of high Nu' have been attributed, by spectral analysis of the surface heat transfer signals, to plume oscillation which enhances the heat transfer overall.

For a pair of cylinders and the range of spacings investigated, the heat transfer from the lower cylinder is unaffected by the presence of an upper cylinder. Similarly the heat transfer from the upper cylinder is unaffected by an unheated lower cylinder. Therefore, the investigation concentrated on the effect of cylinder spacing on the heat transfer from the upper cylinder with both cylinders heated. It was found that the plume rising from the lower cylinder has a large effect on the surface heat transfer from the upper cylinder. The combined effect of "forced convection" and lower local temperature difference have been discussed. Analysis of the time varying heat transfer from the surface of the upper cylinder has given new insight into the convective heat transfer mechanism. It has been shown that the plume rising from the lower cylinder interacts with the natural convective flow around the upper cylinder and enhances the turbulent mixing and as a consequence the mean surface heat transfer. It has also been shown that this enhancement is a function of the spacing between the two cylinders. Further investigation of the phase difference between the plume rising from the lower and upper cylinders is required to further our understanding of the convective heat transfer mechanism. It is also intended that future work will consider a wider range of cylinder spacings to ascertain the optimal set-up for high rates of area averaged heat transfer.

References

[1] V.T. Morgan, The overall convective heat transfer from a smooth cylinder, Advances in Heat Transfer 111 (1975) 99–264.
[2] S.W. Churchill, H.H.S. Chu, Correlating equations for laminar and turbulent free convection from a horizontal cylinder, International Journal of Heat and Mass Transfer 18 (9) (1975) 1049–1053.
[3] T.H. Kuehn, R.J. Goldstein, Numerical solution to the Navier-Stokes equations for laminar natural convection about a horizontal isothermal circular cylinder, International Journal of Heat and Mass Transfer 23 (7) (1980) 971–979.
[4] B. Farouk, S.I. Güçeri, Natural convection from a horizontal cylinder – laminar regime, Journal of Heat Transfer 103 (3) (1981) 522–527.
[5] T. Saitoh, T. Sajiki, K. Maruhara, Bench mark solutions to natural convection heat transfer problem around a horizontal circular cylinder, International Journal of Heat and Mass Transfer 36 (5) (1993) 1251–1259.
[6] G. Cesini et al., Natural convection from a horizontal cylinder in a rectangular cavity, International Journal of Heat and Mass Transfer 42 (10) (1999) 1801–1811.
[7] K. Kitamura, F. Kami-iwa, T. Misumi, Heat transfer and fluid flow of natural convection around large horizontal cylinders, International Journal of Heat and Mass Transfer 42 (22) (1999) 4093–4106.
[8] S.-M. Yang, Improvement of the basic correlating equations and transition criteria of natural convection heat transfer, Heat Transfer – Asian Research 30 (4) (2001) 293–300.
[9] M.A. Atmane, V.S.S. Chan, D.B. Murray, Natural convection around a horizontal heated cylinder: the effects of vertical confinement, International Journal of Heat and Mass Transfer 46 (19) (2003) 3661–3672.
[10] J. Lieberman, B. Gebhart, Interactions in natural convection from an array of heated elements, experimental, International Journal of Heat and Mass Transfer 12 (11) (1969) 1385–1396.
[11] G.F. Marsters, Arrays of heated horizontal cylinders in natural convection, International Journal of Heat and Mass Transfer 15 (5) (1972) 921–933.
[12] T. Yousefi, M. Ashjaee, Experimental study of natural convection heat transfer from vertical array of isothermal horizontal elliptic cylinders, Experimental Thermal and Fluid Science 32 (2007) 240–248.
[13] M. Ashjaee, T. Yousefi, Experimental study of free convection heat transfer from horizontal isothermal cylinders arranged in vertical and inclined arrays, Heat Transfer Engineering 28 (5) (2007) 460–471.
[14] E.M. Sparrow, J.E. Niethammer, Effect of vertical separation distance and cylinder-to-cylinder temperature imbalance on natural convection for a pair of horizontal cylinders, Journal of Heat Transfer 103 (4) (1981) 638–644.
[15] I. Tokura, S. Saito, K. Kishinami, K. Muramoto, An Experimental Study of Free Convection Heat Transfer from a Horizontal Cylinder in a Vertical Array set in Free Space between Parallel Walls, Journal of Heat Transfer 104 (1983) 102–107.
[16] R. Chouikh et al., Experimental study of the natural convection flow around an array of heated horizontal cylinders, Renewable Energy 21 (1) (2000) 65–78.
[17] J.H. Kim, T.W. Simon, R. Viskanta, Journal of heat transfer policy on reporting uncertainties in experimental measurements and results, ASME Journal of Heat Transfer 115 (1993) 5–6.
[18] J.W. Scholten, D.B. Murray, Unsteady heat transfer and velocity of a cylinder in cross flow – 1. Low freestream turbulence, International Journal of Heat and Mass Transfer 41 (1998) 1139–1148.