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Stability in Time-Delay Systems: Quiet Standing Case Study

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Abstract

The analysis of linear time-delay systems has attractedmuch interest in the literature overlast five decade. Two types of stability conditions, namely delay-independent whichresults guarantee stability for arbitrarilylarge delays and delay-dependent, results take into account the maximumdelay that can be tolerated by the system and, thus, are more useful in applications. Thestability in general for linear time-delay systems, can be checkedexactly only by eigenvalue considerations. When reasonable chosen with intentional delays, case study effects on time-delay of ankle torque on the stability of quiet standing, it can be used to stabilize and improve the close-loop response of these systems.

Keywords: *time-delay, stability, quiet standing, case study*

1. Introduction

The study of time-delay systems has received considerableinterest in the last few decades. Time-delay is the property of a physical system by which the response toan applied force or an action is delayed in its effect and often appears in many practical systems and mathematical formulations such as chemical processes, electrical and control systems, and economical systems [1]. Control systems regularly operate in the presence of delays, primarily due to the time it takes to acquire theinformation needed for decision-making, to createcontrol decisions, and to execute these decisions, asshown in Figure 1.

Time-delay happened such as in traffic-flow, amodelthat is refers to the drivers' delayed reactions, which the reaction delays vary under physicalconditions and stimuli and depend on the drivers' cognitiveand physiological states. These delaysare critical in accounting for human driver's behavior due to drivers needa minimal amount of time to become aware of externalevents which combine sensing, selection, perception, and response to make decision. Delay also occurs when analyzing vehicle dynamics stability through anti-lock braking system, and designing collision-free traffic flowusing adaptive cruise controllers [2].Think about driving a car every time while turning the steering wheel, the tires do not respond for it, that mean, it has a delay between the steering wheel and the tires. These delays may invite collisions, cause traffic jams and stop-and-go waves that effects contributeto casualties on highways and productivity losses due to increased travel times [3].

In most cases, the presence of delays may destructive to the operation of the dynamical system since it isfrequently a source of system instability and make theanalysis and synthesis complicated. A feedbacksystem that is stable without delay may become unstablefor some delays [4]. However, somehow there have beneficial aspects through the time-delay. Main advantages of time-delay are in feedback system that needed the minimumknowledge of the investigated system and no need of a reference signal [5]. In fact, thetime-delayed feedback method generates the reference signal from the delayedtime series of the system under control; yet, judicious introduction of adelay may stabilize an otherwise unstable system [6]. Thepotentially stabilizing and controlling the effect of delay systems is a motivation for exploitingthe ever-present delays in dynamical systems over five decades [7-8]. Foran example, appropriate adjustment of the spindle speed helpsin tuning the delay to avoid chattering in metal machining,while intentionally adding delays to decision-makingallows supply-chain managers to observe consumer trendsto make better purchasing and stocking decisions [9].

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The main objectives and scope of this paper are to study the effects of time-delay for stability and stabilization of the systems in various limitations and opportunities arises that focused on a linear time-invariant (LTI) systems modeled bydelay differential equations (DDEs). Destabilizing and stabilizing effect of single delay systems with feedback law is mentioned in Section 2. Section 3 is dedicated to case study in biology area which is mainly focused on quiet standing analysis. Finally, this paper is concluded in Section 4.

2. Destabilizing and Stabilizing Effect of Delay

Delay differential equations (DDEs) are a large and important class of dynamical systems and often arise in either natural or technological control problems. Most models of systems with delays are obtained basedon inflow-outflow interactions such as conservationlaws involving mass and energy [10]. In these model systems, a controller monitorsthe state of the system, and makes adjustments to the system based on its observations. Sincethese adjustments can never be made instantaneously, a delay arises between the observation andthe control action.There are different kinds of DDEs and the examples been considered here, is a linear discrete-time delay:

$$\frac{d}{dt}x(t) = A_0x(t) + A_1x(t-\tau_1) + \cdots A_ix(t-\tau_i) \qquad (1)$$

where$x(t)$ is the n-dimensional state variable, A_0, A_1, $A_i \in R^{n \times n}$, i=0,…, N, N is a positive integer. In (1), $\tau_i > 0$ is the delay, that is,$\dot{x}(t)$ depends on $x(t)$ at time t as well as at the time instants$t - \tau_i$. The delay is a shift operator that lags an input signalby the constant amount of time τ_i as shown in Figure 2.

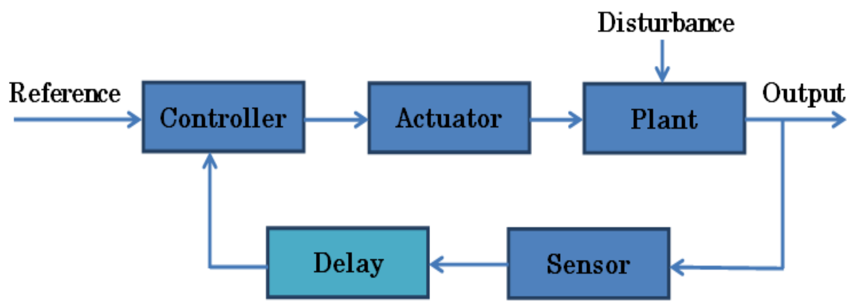

Figure 1. Delay in Feedback System

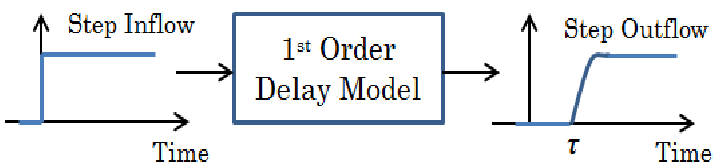

Figure 2. Constant Discrete Delay Model

2.1. Characteristic Equation

The characteristic equation of (1) is given by:

$$f(s;\tau) := \det(-\lambda I + A_0 + A_1e^{-\tau_1\lambda} + \ldots + A_i e^{-\tau_i\lambda}) = 0 \qquad (2)$$

where$I$is the n x n identity matrix, and the exponentialfunctions arise from the Laplace transforms of the delayterms. Due to the presence of the exponential terms, (2)is a quasi-polynomial and thus is a transcendental equation,which possesses an infinite number of roots in thecomplex plane ℂ, called characteristic roots.For a given set of delays, (1) is asymptotically stable ifand only if all of the roots of (2) lie in the open left-halfcomplex plane $\mathbb{C}_-$. Verifying asymptotic stability can bedifficult since (2) has infinitely many characteristic roots [11].

To illustrate how to analyze the stability of a DDE, considerthe plant transfer function of single integrator first-order system, $H(s) = 1/s$ with the controller$C(s) = -ke^{-s\tau}$, where τ is the delay and k is thecontroller gain. The characteristic equation of this system isgiven by $f(s;\tau) := s + ke^{-s\tau}$. If $\tau = 0$, then $f(s;\tau) := 0$ has asingle root at $s = -k$. As we increase τ from zero to 0^+, theroot $s = -k$ moves in $\mathbb{C}$, while at the same time an infinitenumber of roots $s = \tilde{s}_i$, i=1, 2,…, appear in $\mathbb{C}$. Theseroots satisfy two conditions, namely, $\mathcal{R}(\tilde{s}_i) < 0$, and$|\tilde{s}_i| \to \infty$, as $\tau \to 0^+$. That is, for an indefinitely smalldelay, the roots $\tilde{s}_i$are inactive from a stability point ofview. As the delay parameter increases, however, the realparts of these roots may increase, and consequently theseroots can destabilize the closed-loop system.

2.2. Stability Chart

Stability charts are diagrams constructedin the plane of two or more parameters of the system showing the stable and unstabledomains or the numbers of unstable characteristic exponent's or multipliers. When studying the stability of (1), one of the main objectivesis to determine necessary and sufficient conditions forclosed-loop stability in either the delay-parameter space that relies on the τ-decompositiontechnique orthe controller-parameter space using the D-decomposition principle. In the τ-decomposition, the time-lag τ is allowed to vary while other parameters are keptfixed, while in D-decompositionmethod, the time-lag τ is held constantly [12].These decomposition techniques state that boundariesin the parameter space exist to divide the space into regions,where all the values the parameter can attain in each regionmake the system either stable or unstable.A DDE that is exploit information about the delays involved and stable for only some values in the delay parameterspace is called delay-dependent stable. If thestability of a DDE is maintained independently of thedelay which is do not need anyinformation about the delay, then DDE is called delay-independent stable [13]. Multipledisjoint delay regions may also exist, where thesystem may be stable within each region, while becomingunstable outside. These regions, which areknown as stability regions, become stability intervals in asystem with a single delay, that is, when $N = 1$ in (1). Figure 3 shows the stability chart for a closed-loop system with the plant transfer function $e^{-\tau s}b / (s + a)$ and the controller, $C(s) = k$.

2.3. Destabilizing Effects of Delays

Consider the transfer function of a single integrator$H(s)=1/s$ subject to the delayed controller $C(s)= -k^{-s\tau}$with $k > 0$. To determine the stability of the closed-loopsystem, we need to first find the roots $s = j\omega$of the closed-loopcharacteristic equation:

$$s + ke^{-s\tau} \tag{3}$$

for all τ, that is,

$$\cos(\omega\tau) = 0 \tag{4}$$

$$k\sin(\omega\tau) = \omega \tag{5}$$

Due to the periodicity in (4) and (5), there exist infinitelymany delays $\tau_{c,l} = \pi/(2k) + (2\pi l)/\omega_{c,}$, l = 0,1,2,…, all ofwhich yield the crossing frequency $\omega_c = k$, that is, (3) hasroots on the imaginary axis at $s = \pm jk$. By continuity, it followsthat closed-loop stability is guaranteed for all delayssatisfying $\tau \in [0, \tau_c]$, where $\tau_c = \pi/2k$. In this example, thesystem is unstable for $\tau \geq \tau_c$, and thus $\tau_c$is the delay marginof the system.

Let now consider the movement of the rightmost root of(3) as τ changes. As shown in Figure 4 for the controller gain$k = 1$, increasing the delay from zero generates fast-movingcharacteristic roots, which enter from $-\infty$ in $\mathbb{C}$. Note thatthe root located at $-k$ for $\tau = 0$ moves to the left, as thedelay increases. Finally, at the value $\tau_c = \pi/2$, a pair of rootsentering from $-\infty$ crosses the imaginary axis toward $\mathbb{C}_+$.Larger values of k induce smaller delay margins since$\tau_c = \pi/(2k)$.

2.4. Stabilizing Effects of Delays

By considering the second-order feedback open-loop system, $H(s) = 1/(s^2 + \omega_0^2)$with the delayed controller, $C(s) = ke^{-s\tau}$, the closed-loop characteristic equation is given by:

$$s^2 + \omega_0^2 - ke^{-s\tau} = 0 \tag{6}$$

If $\tau = 0$, then the system is unstable for all k. However, thesystem can be made stable either by designing appropriatevalues of k and τ or by using a proportional-derivativecontroller without delay $C(s) = k_p + k_d s$.Let design (k, τ) so that the closed-loop system isstable. As in (3), it can show that two distinct crossingfrequencies exist for each $k > 0$, where $k \in (0, \omega_0^2)$, asgiven by $\omega_{c,1} = \sqrt{w_0^2 - k}$ and $\omega_{c,2} = \sqrt{w_0^2 + k}$, which lead tothe critical delay values $\tau_{c,1,l} = (2l\pi)\sqrt{w_0^2 - k}$ and $\tau_{c,2,l} = (2l\pi)\sqrt{w_0^2 + k}$, for l = 0, 1, 2, ..., respectively. Thesensitivity expression $\mathcal{R}\{[ds/d\tau]\}|_{s=j\omega c} = -2\omega_c^2 / (\omega_0^2 - \omega_c^2)$indicates that the characteristic roots crossing at $\omega_c = \omega_{c,1}$ where the roots move toward $\mathbb{C}_$accommodate the stability, on the other hand,the roots crossing at $\omega_{c,2}$ favor instability.If $\tau = 0$, then the closed-loop system has only a pair ofpoles of the form $s = \pm j\omega_{c,1}$, . As mentioned above, thesepoles provide stability at the delay values $\tau_{c,1,l}$. That is, for sufficientlysmall $\tau = \varepsilon > 0$, the closed-loop system becomesstable since the poles $s = \pm j\omega_{c,1}$, move toward $\mathbb{C}\_$, and noclosed-loop poles are located in $\mathbb{C}_+$ or on the imaginaryaxis. In this case, increasing the delay value has a stabilizingeffect. Considering all critical delays, we conclude that thesystem is stable if and only if, for some nonnegative integer$l$, the delay τ satisfies:

$$\frac{2l\pi}{\sqrt{\omega_0^2 - k}} < \tau < \frac{(2l+1)\pi}{\sqrt{\omega_0^2 + k}} \tag{7}$$

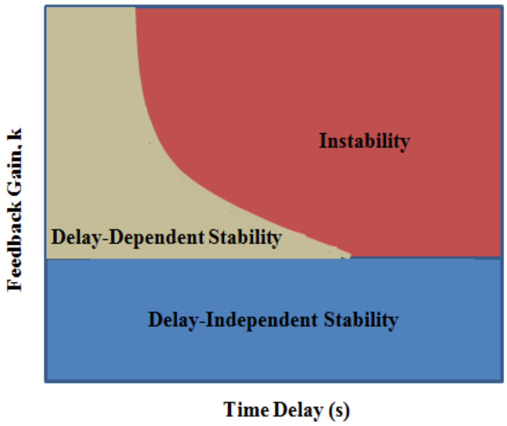

Figure 3.Stability Chart

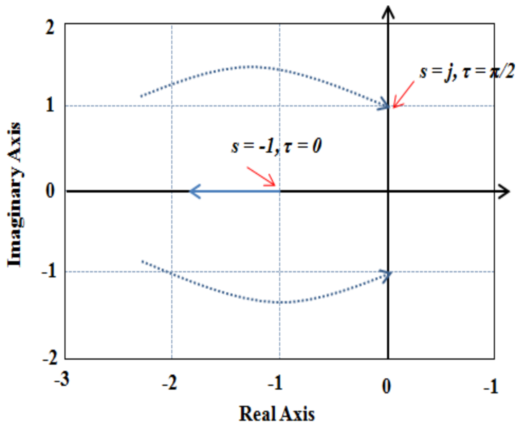

Figure 4. Characteristic roots of the closed-loop system

3. Case Study: Effects of ankle torque on the stability of quiet standing

In Human bipedal stance is inherently unstable because a large body mass is kept in erect posture with its center of mass located high above a relatively small base of support. The mechanism responsible for equilibrium control of quiet stance as shown in Figure 5 involve analyzing muscle activityat the ankles which is maintaining the vertical configuration of the human body. Analysis of quiet standing offers insight on how humans regulate their vertical posture and putslight on how humans walk without falling. The ankle joint torque needed to stabilize the body during quiet stance can be generated actively and passively. Passive torque components are the result of tension or stiffness produced by muscle tonus and by the stiffness of the surrounding tissue, such as ligaments and tendons [14]. On the other hand, active torque component is produced by the central nervous system, which modulates or controls muscle contractions based on the overall body kinematics and dynamics of spontaneous body sway that are influenced by external disturbances [15].

A complete set of dynamic equilibrium equations can be easily derived to establish therelationship between sway movement and the ground reaction forces. Free body diagrams inFigure 6 illustrate the human body as a single segment, single joint inverted pendulum approximation that rotates about the ankle joint [16]. The dynamic equation of the human inverted pendulum model is:

$$I\ddot{\theta} = mgl\sin\theta + T + \varepsilon \tag{8}$$

Figure 6 shows the entire body excluding feet as inverted pendulum rotating about the anklejoint A. m is the mass of body above ankle, $F_h$and $F_v$are horizontal and vertical force actingat ankle joint, I is the moment of inertia of the body, l is the distance of center of mass from the ankle, T is moment acting at ankle joint by muscles, θ is absolute sway angle withrespect to fixed vertical reference, and ϵ is the torque disturbance, which is sufficiently small compared with other torque contributions. The ankle joint torque, T is modeled as:

$$T = k\theta + b\dot{\theta} + f_P(\theta_\tau) + f_D(\dot{\theta}_\tau) \tag{9}$$

where τ is the neural transmission delay, k and b are the passive stiffness and viscosity parameters represent passive feedback torques with no time delay that related to the intrinsic mechanical impedance of the ankle joint, the third and fourth terms represent the active neural feedback torques that are determined as functions of delay-affected tilt angle and angular velocity, respectively.

The delay differential equation (DDE) of inverted pendulum in equations (8) and (9) is numerically integrated by using the forward Euler method where $x(t) = \left[\theta(t), \dot{\theta}(t)\right]$, σ is corresponding amplitude and τ is the feedback delay time. More precisely, the second-order equation of motion is reformulated as the following ordinary DDE:

$$\dot{x}(t) = f(x(t),\ x(t-\tau)) + \sigma\,\varepsilon(t) \tag{10}$$

The model of the neuromusculoskeletal (NMS) torque-generation process for an isometric torque-exertion task can be used for the standing task; since the muscle length change is very small during quiet standing has been concisely modeled as a critically damped, second-order system [17]. The transfer function $H(s)$ is written as:

$$H(s) = \frac{G\omega_n^2}{(s+\omega_n)^2} = \frac{G(1/T_s)^2}{(s+1/T_s)^2} \tag{11}$$

where G is the gain, ω_n is the natural frequency of the second-order system, and T_s is the twitch contraction time represents the time interval from the moment when a stimulus arrives at the muscle to the moment when the generated force reaches its peak.

The dynamic characteristics of equation (11) is determined by the natural frequency, which corresponds to the inverse of the twitch contraction time of the muscle (T_s= $1/\omega_n$). Note that ω_n and T_s equivalently capture the characteristics of the NMS system and the second-order dynamics induce a phase delayas a function of frequency instead of a constant time delay [18]. Since the delay induced by the NMS system is due to the chemical and mechanical muscle and joint dynamics, the T_s is believed to depend not only on the muscle fiber properties involved in the corresponding motor task, but also on the ankle joint and foot condition. Therefore, it was required to identify the NMS system under a condition equivalent to the quiet standing posture.

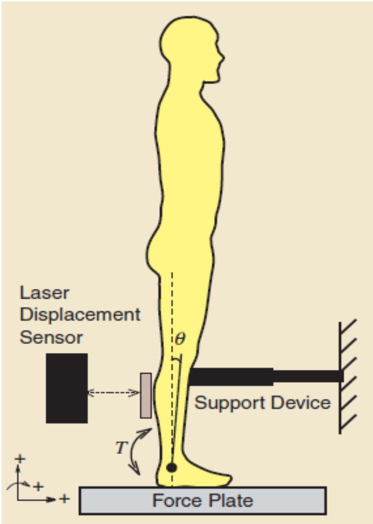

Figure 5. Quiet standing

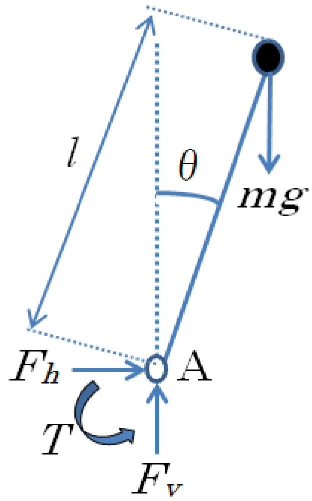

Figure 6. Free body diagram of quiet standing

A block diagram of the closed-loop quiet-standing systemis shown in Figure 7, where the neural controller comprises aproportional-derivative (PD) controller with gains $K_P$and K_D. The main challenge is how to compensate the danger of instability induced by the large neural feedback transmission delay, which is of the order of 200 ms [19]. The standard PD model faces a stringent trade-off that leaves narrow margins for the design of the control parameters: the proportional gain must be large enough for supplementing the insufficient ankle stiffness but not too large for avoiding delay-promoted instability. An active correction mechanism, which is PD controller, emanatesfrom the neural controller and becomes effective after alength of time *t*.Damping of sway patterns requires rather large values of the derivative gain but again the feedback delay sets a stringent upper bound on this parameter.

Following the standard block diagram simplifications in Figure 7, the characteristic equation of quiet standing as:

$$f(s;\tau K_P,K_D) = Q_1(s,K_P,K_D) + e^{-\tau s} Q_2(s,K_P,K_D) = 0 \tag{12}$$

whereQ_1 and Q_2 are polynomials, and τ is the sensory delay ofthe human model. One goal is to find combinations of (K_P, K_D)such that the quiet-standing model (11) is stable for a givendelay τ.

From (1) and (2), by first-order Taylor's series, it can be approximate θ_τ and $\dot{\theta}_\tau$ yield to:

$$(I-D\tau)\ddot{\theta} + (b+D-P\tau)\dot{\theta} + (k+P-mgl)\theta = 0 \tag{13}$$

In other words, the delay tends to decrease the apparent inertia and damping of the inverted pendulum but both must remain positive for stability because the eigenvalues solve the following equation:

$$\lambda^2 + \frac{b+D-P\tau}{I-D\tau}\lambda + \frac{k+P-mgl}{I-D\tau} = 0 \tag{14}$$

As demonstrated in the [20], two additional conditions must be satisfied by the proportional and derivative gains, yielding a set of three conditions to be satisfied by the feedback controller for gaining the asymptotic stability of the upright posture:

$$P > mgl\text{-}k$$
$$D < I/\tau$$
$$D > \tau P - b \qquad (15)$$

In the *P-D* parameter plane as shown in Figure 8, identifies a triangle that limits the set of admissible values for the feedback parameters. As τ decreases, the triangle increases its area and tends to fill the whole first quadrant of the *P-D* plane to the right of the critical value *mgl-k*. On the contrary, as τ increases the triangle decreases its area and vanishes when it reaches a critical value $\tau = [b + \sqrt{b^2 + 4I(mgl - k)}]/[2(mgl - k)]$ which is a function of the physical parameters of the system (*m, l, I, b, k*). A loss of stability of the upright posture occurs when $D > \tau P\text{-}b$ is broken via a Hopf bifurcation, which is a typical critical phenomenon that induces a stable or unstable oscillatory behavior of a dynamical system through instability of an equilibrium state, leading to an unstable oscillation around the upright equilibrium of unstable focus type. Indeed when $D = \tau P\text{-}b$, the real parts of the eigenvalues of the linearized equation (13) vanishes and the upright equilibrium loses its stability.

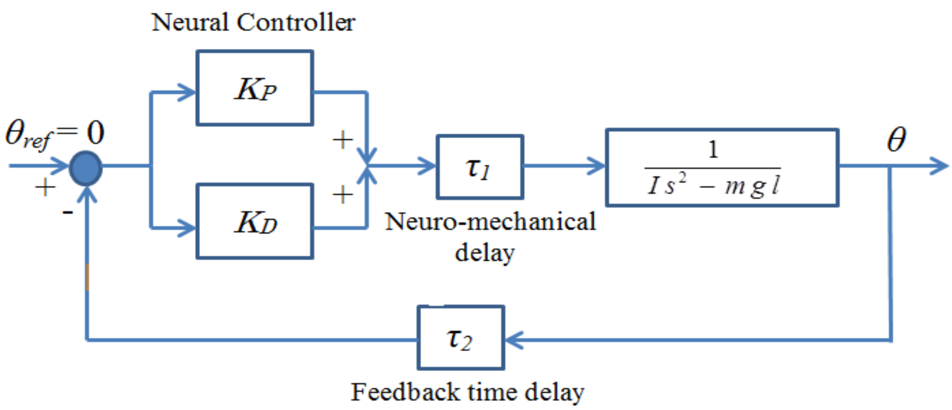

Figure 7. Closed-loop control diagram of quiet standing

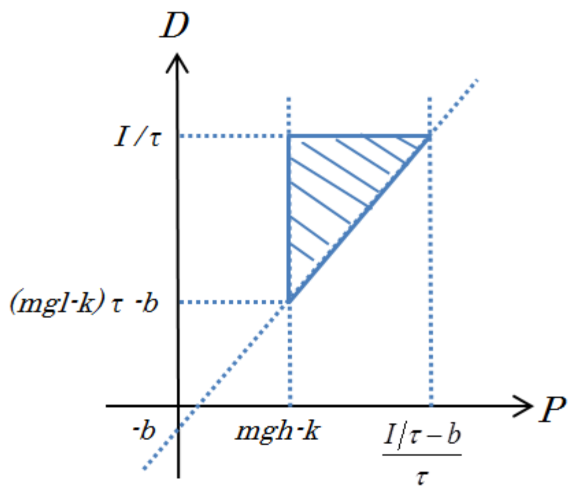

Figure 8. Proportional and derivative plane parameters, region of stability (shaded triangle).

4. Conclusion

In this paper, the effect of delays in dynamical systems modeled by linear time-invariant delay differential equations was expressed. This paper focused on eigenvaluelocations and parametric techniques rather thanLyapunov-based approaches. Example with delay on biological case study was analyzed and discussed. Authors limit the paper to the effects of delays on stability, and believe that delays on controllers for nonlinear systems area deserves further research due to impact of delays continue to grow in many fields.

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