Buletin Teknik Elektro dan Informatika
(Bulletin of Electrical Engineering and Informatics)
Vol. 2, No. 4, December 2013, pp. 252~257
ISSN: 2089-3191

Particle Swarm Optimization in Solving Capacitated Vehicle Routing Problem

M. M. Tavakoli*, Ashkan Sami
CSE Department of Shiraz University, Shiraz, Iran
Corresponding author, e-mail: mtavakoli@cse.shirazu.ac.ir, asami@ieee.org

Abstract

The Capacitated Vehicle Routing Problem (CVRP) is an NP-Hard problem, which means it is impossible to find a polynomial time solution for it. So, researchers try to reach a near optimum solution by using meta-heuristic algorithms. The aim of CVRP is to find optimum route for every vehicle and a sequence of customers, that vehicle serve. This paper proposes a method on how PSO is adjusted for a discrete space problem like CVRP. The process of tweaking solutions is described in detail. At last for evaluation of proposed approach and show the effectiveness of it, the result of running proposed approach over benchmarking data set of capacitated vehicle routing problem is illustrated.

Keywords: *capacitated vehicle routing problem (CVRP), particle swarm optimization (PSO), traveling salesman problem (TSP), meta-heuristic, euclidean distance*

1. Introduction

The Vehicle Routing Problem (VRP) was first proposed by Dantzig and Ramser (1959), and has been widely studied. According to L. Guerra et al. (2007) and S. Masrom et al. (2010), VRP is a combinatorial optimization problem in which a set of routes for a fleet of delivery vehicles based at one or several depots must be determined for a number of customers. The main objective of VRP is to serve customer demands by a minimum cost vehicle routes originating and terminating in a depot. Several variations of the VRP exist in order to adapt to various practical characteristics and constraints such as: Multiple Depot VRP (MDVRP), Split Delivery VRP (SDVRP), Dynamic VRP (DVRP), If a constraint is given on capacity of every vehicle, the problem is known as capacitated vehicle routing problem (CVRP).

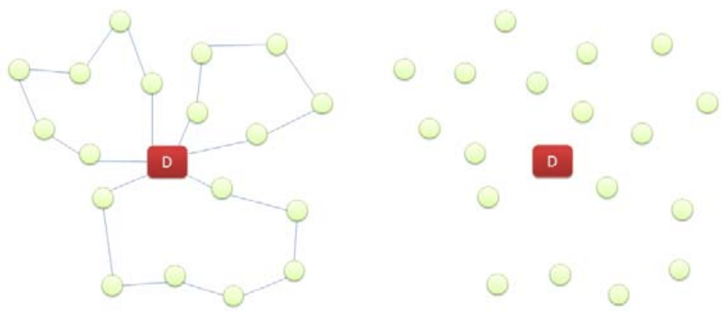

Figure 1. CVRP with One Depot and 18 Customers

1.1. CVRP Model

Capacitated Vehicle Routing Problem as defined by J. F. Cordeau (2002) and J. Lysgaard (2004) is a set of N customers with determined demands which must be served from common depot by fleet of delivery vehicles that has constraint on their capacity. The cost or travel distance of a particular vehicle V_i after completing a tour from depot and serving some

Received July 10, 2013; Revised September 25, 2013; Accepted October 13, 2013

customers in its route, is the summation of Euclidean distance between each pair of nodes that vehicle visit.

The objective of CVRP is to find a collection of simple circuits in the graph of problem (each circuit corresponding to a vehicle route) with a minimum cost such that:

a. Each customer is served exactly once, and by exactly one vehicle
b. Each vehicle route leaves from and returns to depot
c. Sum of the demands of the customers visited by each vehicle route does not exceed given vehicle capacity C.

Suppose that depot is 0 and the customers should be served by available vehicles. The demand of customer C_i is q_i, the capacity of vehicle k is Q_k and the maximum travel distance by vehicle k is D_k. The mathematical model of CVRP by L. Bodin (1983) is described as follows:

If vehicle k travels from customer i to j, $X_{ij}^k = 1$ and otherwise the value is 0.

$$\sum_{k=0}^{K}\sum_{i=0}^{N}\sum_{j=0}^{N} C_{ij}^{k} X_{ij}^{k}$$

Objective function:

$$\sum_{k=0}^{K}\sum_{i=0}^{N} X_{ij}^{k} = 1,\ j = 1,2,...,\ N$$

$$\sum_{k=0}^{K}\sum_{j=0}^{N} X_{ij}^{k} = 1, i = 1,2,...,\ N$$

$$\sum_{i=0}^{N} X_{it}^{k} - \sum_{j=0}^{N} X_{tj}^{k} = 0, k = 1,2,...,\ K; t = 1,2,...,\ N$$

$$\sum_{i=0}^{N}\sum_{j=0}^{N} d_{ij}^{k} X_{ij}^{k} \le D_k, k = 1,2,...,\ K$$

$$\sum_{j=0}^{N} q_j \left(\sum_{i=0}^{N} X_{ij}^{k}\right) \le Q_k, k = 1,2,...,\ K$$

$$\sum_{j=1}^{N} X_{0j}^{k} \le 1, k = 1,2,...,\ K \qquad X_{ij}^{k} \in \{0,1\}; i,\ j = 0,1,...,\ N; k = 1,2,...,\ K$$

$$\sum_{i=1}^{N} X_{i0}^{k} \le 1, k = 1,2,...,\ K$$

The number of customers determined by N, number of vehicles is K, and the travelling cost by vehicle k from customer i to j is shown as C_{ij}^k, and d_{ij}^k is the traveled distance between customer i to j.

The objective function Equation 1 is to minimize the total cost by all vehicles that is the sum of travel distance of vehicles in problem space. Constraints Equation 2, Equation 3 ensure that each customer is served exactly once. Constraint Equation 4 ensures the connectivity of the route. Constraint Equation 5 shows that the total length of each route has a limit. Constraint Equation 6 shows that the total demand of any route must not exceed the capacity of the vehicle. Constraint Equation 7 and Equation 8 ensure that each vehicle is used no more than once. Constraint Equation 9 ensures that the variable only takes the integer 0 or 1.

1.2. Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a global optimization technique. It is originally attributed to Kennedy and Eberhart (1995). A swarm consist of a set of particles that each particle represents a potential solution. Suppose that each solution represented as a point in N-Dimensional space that each point or particle has an initial velocity, particles move through solution space, and after each time step, particles are evaluated according to some fitness criterion. They are accelerated towards particle with best fitness value within their communication group. This property of PSO help particles escape from local optimal solutions. Each particle has a simple memory that remember the position of best solution achieve by itself, this value is called personal best (pbest) and the position of best solution obtained so far by any particle in the neighborhood of that particle, known as global best (gbest). The basic concept of PSO lies in accelerating each particle towards its pbest and the gbest locations, with a random weighted acceleration at each time step.

2. Research Method

Our methodology has described in detail in the following subsections. Section 2-1 provides main assumptions for solving CVRP by PSO. Section 2-2 presents the steps of revised PSO algorithm in detail. Section 2-3 explains solution encoding, and finally section 2-4 investigates heuristics performed on solutions.

2.1. Assumptions

Suppose that both width and height dimensions of the problem space initialize to 100. In order to solve CVRP, the Global neighborhood selected among different neighborhood types have been defined for Particle Swarm Optimization, such as Geographical neighborhood, Social neighborhood, and etc.

The random weighted acceleration values for pbest (W_1) and gbest (W_2) are initializing by generating random numbers multiply by constant values C_1 and C_2 respectively that usually set them in a way which summation of them is 4. Z. Ying et al. (2003) and S. R. Venkatesan (2011) set both C_1 and C_2 to 2, but we empirically set C_1 to 1.5 and C_2 to 2.5.

Weight for pbest: $W_1 = C_1.\text{Rand}()$

Weight for gbest: $W_2 = C_2.\text{Rand}()$

The inertia weight has a well balance mechanism with flexibility to enhance and adapt to both global and local exploration. Large inertia weight facilitates global exploration and small value of it, enhance local exploration. We set the inertia to 0.47.

2.2. Revised PSO Algorithm

The steps of our revised PSO algorithm for solving CVRP are listed below:

(a) Initialize Particles
   1) Generating a set of solutions in a random greedy policy, and assign a particle to each of them.
   2) Evaluate the fitness of each particle
   3) Locate each particle in problem space with random values

(b) Adjust positions
   1) Each particle tries to modify its position using the following information:
      a) the current position,
      b) the current velocity,
      c) the distance between the current position and pbest,
      d) the distance between the current position and gbest
   2) New position components of particle P_i (x-axis, y-axis) computed by using equations below:

$$\text{a) new } X = \text{inertia} * P_i.\text{Velocity} + [(W_1 * (P_i.\text{pbest}.X - P_i.X)) + (W_2 * (P_i.\text{gbest}.X - P_i.X))] + P_i.X$$

$$\text{b) new } Y = \text{inertia} * P_i.\text{Velocity} + [(W_1 * (P_i.\text{pbest}.Y - P_i.Y)) + (W_2 * (P_i.\text{gbest}.Y - P_i.Y))] + P_i.Y$$

(c) Find nearest neighbor of each particle to this new calculated position and tweak solution of it by using TOE, TOI and TSPOE heuristics that completely explained in the next section. If this tweak cause to improve in best fitness obtained so far by algorithm, current particle sets its solution to its nearest neighbor’s tweaked solution and the particle update its location to point (new X, new Y).

(d) Evaluate fitness of each particle by calculating the summation of each vehicle cost.

(e) Check if new pbest or gbest values achieved.
(f) Go to step 2 and repeat this process until the current iteration violate the max iteration constraint.

2.3. Model CVRP and Solution Encoding

The proposed methodology has been written by object oriented programming. In object oriented programming each object in actual world is represented as a class that properties and functionalities of that object are modeled as fields and methods respectively.

Suppose each vehicle (or truck) has two main properties named capacity and route. With this definition each solution is constitute of vehicles (trucks), that the objective of a particular solution computed by summation of each vehicle travel distance, and other attributes shown in Figure 2 are something needed for a particle.

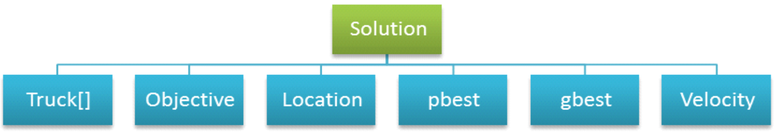

Figure 2. Solution Encoding for CVRP by PSO

In Figure 3 an example of solution by considering encoding is demonstrated for A-n32k5 benchmark. In this benchmark there are 32 customers that should be served by 5 vehicles from one depot. To show each vehicle start its route from depot and return to it, the beginning and end of each vehicle route surrounded by 1. Also the remaining capacity of each vehicle and travel distance of it, are shown in Figure 3.

Truck ID	Route	Capacity	Distance
1	1 - 28 - 25 - 1	56	59.26263
2	1 - 31 - 17 - 8 - 2 - 13 - 1	12	88.50539
3	1 - 21 - 6 - 26 - 11 - 16 - 10 - 23 - 30 - 1	9	236.7426
4	1 - 7 - 18 - 20 - 32 - 22 - 14 - 27 - 1	6	180.9489
5	1 - 15 - 19 - 9 - 12 - 5 - 29 - 24 - 3 - 4 - 1	7	231.9918

Figure 3. An Example of Solution for A-n32k5

2.4. Process of Tweaking Solutions

To improve rudimentary solutions, three heuristics performed on initial solutions. A brief description of them is mentioned below:

(a) Two Optimal Exchange (TOE): first by generating two random numbers, two different vehicles selected, and again by generating two random numbers one customer selected in route of each selected vehicle. If by exchanging these two customers no violation occurs in capacity of vehicles, the result return as new solution, otherwise the process repeated to find a feasible combination.

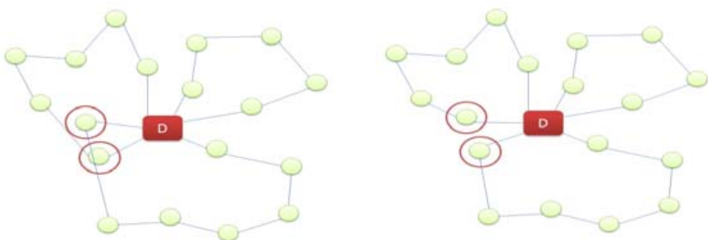

Figure 4. An Example of TOE Heuristic

(b) Two Optimal Insertion (TOI): similar to TOE, first by generating two random numbers two different vehicles are selected and again by generating one random number one customer in route of first vehicle is selected. If no violation occurs in capacity of the second vehicle, the selected customer is removed from first vehicle route and inserted beside of nearest neighbor of that customer in second vehicle route. Like TOE heuristic, this process repeats until a feasible combination found.

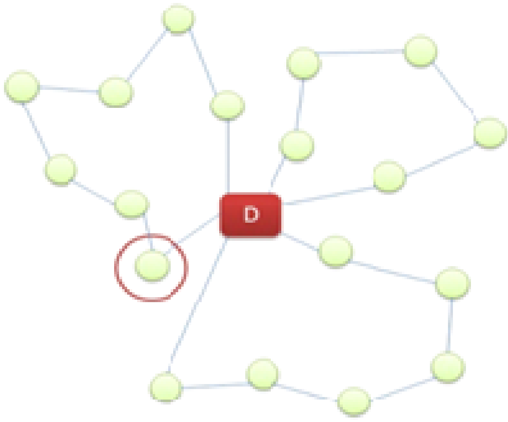

Figure 5. An Example of TOI Heuristic

(c) TSP Optimal Exchange (TSPOE): this heuristic applied to all vehicles in problem instance. This is a contribution in process of tweaking solutions that in each time step, the route of each vehicle considered as a TSP problem and tried to improve quality of solution by exchanging customers in route of each vehicle. In other word, the heuristic tried to find pair of customers iteratively, that exchange of them lead to improvement in quality of solution.

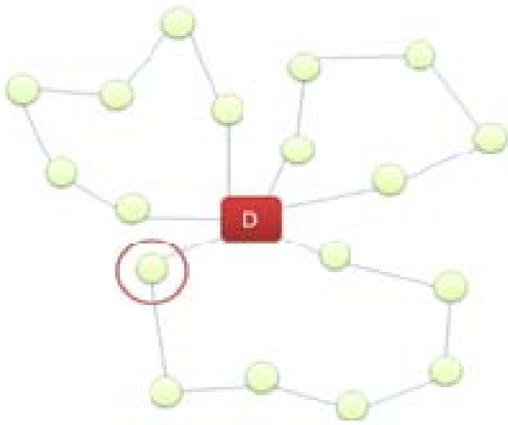

Figure 6. An Example of TSPOE Heuristic

3. Results and Analysis

The proposed algorithm has been written by Microsoft Visual Studio 2008 and C# language of this IDE. In Figure 7 and 8 the average process of running proposed approach over A-n32k5 and A-n80k10 benchmarks demonstrated, for ten runs and less than one minute respectively. Due to employment of strong heuristics in combination of good parameter tuning, the algorithm swiftly improve initial solutions, and achieve near optimum solution in desirable time.

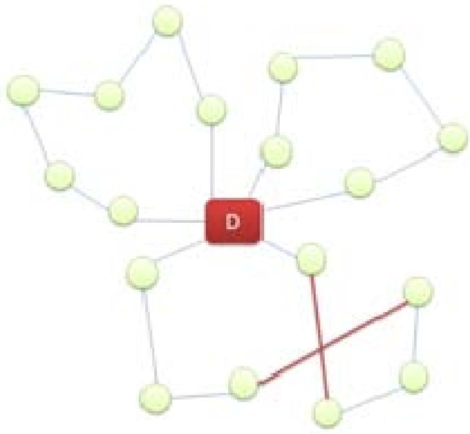

Figure 7. Evolution of Solutions in A-n32k5

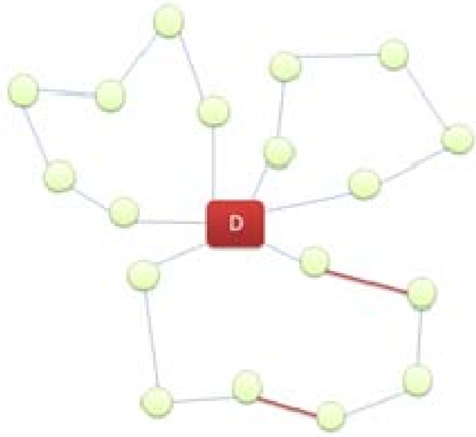

Figure 8. Evolution of solutions in A-n80k10

Table 1 show best results obtained by proposed approach over 7 benchmarks of CVRP. The results have been compared with best known solution (BKS) of each benchmark. The name of benchmark, number of customers and number of vehicles, BKS and proposed approach fitness are given in Table 1 respectively.

Table 1. Best results obtained by proposed approach over 7 benchmarks of CVRP

Benchmark	# Customers	# Vehicles	BKS	Proposed approach
P-n16k8	16	8	450	451.34
P-n20k2	20	2	216	217.42
A-n32k5	32	5	784	787.08
A-n44k6	44	6	937	938.17
A-n61k9	61	9	1034	1050.38
A-n80k10	80	10	1766	1795.09
F-n135k7	135	7	1166	1241.29

4. Conclusion

In this paper a revised PSO algorithm have been presented for solving CVRP. Combination of different heuristics such as TOE, TOI and TSPOE used to improve solutions, according to Table 1, they lead to enhanced solutions and obtain good results in both small and medium problem instances. But in case of large problems like F-n135k7, it seems that the algorithm trapped in local optima and could not find near optimum solution. The future work could be an improvement of proposed approach in a way that performs well on large benchmarks too.

References

[1] Dantzig GB, Ramser JH. The Truck Dispatching Problem Management Science. 1959; 6 (1): 80–91. DOI: 10.1287/mnsc.6.1.80.

[2] Guerra L, Murino T, Romano E. 2007. A heuristic algorithm for the constrained location - routing problem. *International Journal of Systems Applications, Engineering & Development.* 2007; 1(4): 146-154.

[3] Masrom S, Abidin ZZ, Hashimah PN, Rahman AS. Providing flexibility in the Vehicle Route Optimization for the AVL based Transport Monitoring System. *WSEAS Transactions on Computers*. 2010; 9: 526-535.

[4] Cordeau JF, Gendreau M, Laporte G, Potvin JY, Semet F. A guide to vehicle routing heuristics. *J. Operat. Res. Soci.,* 2002; 53: 512-522. DOI: 10.1057/palgrave.jors.2601319

[5] Lysgaard J, Letchford AN, Eglese RW. *A new branch-and-cut algorithm for the capacitated vehicle routing problem*. Mathem. Programm. 2004; 100: 423-445. DOI: 10.1007/s10107-003-0481-8

[6] Bodin L, Golden B, Assad A, Ball M. Routing and scheduling of vehicles and crews: The state of the art. *Computers and Operations Research.* 1983; 10(2): 63-211.

[7] Kennedy J, Eberhart R. *Particle swarm optimization*. Proceeding of IEEE International Conference on Neural Network, (ICNN'95), Perth, Australia. 1995; 1942-1948.

[8] Ying Z, guangjie Z, feihong Yu. *Particle swarm optimization-based approach for optical finite impulse response filter design,* applied optics. 2003; 42(8): 1503-1507.

[9] Venkatesan SR, Logendran D, Chandramohan D. Optimization of capacitated Vehicle Routing Problem using PSO. *International Journal of Engineering Science and Technology (IJEST)*. 2011; 3(10).