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Implementation of Inventory System by P (R, T) Model with Differenced Time of Known Priced Increase at PT Inti Vulkatama

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ABSTRACT

PT IntiVulkatama is a manufacturing company forthe tire retreading. The demand of the product is probabilistic causing the PT IntiVulkatama often experience shortages of raw material inventory (backorder) that could produce the company to lose the trust of its customers. Currently, PT IntiVulkatama address this problem using a forecasting method by looking at the demand of the past. PT IntiVulkatama must have a better inventory system that can minimize the expected total cost especially when facing the condition of differenced time of known priced increase for some of the raw materials.

It is suggested that a good method for solving the problem is fixed order interval system. With this system, PT IntiVulkatama can perform a joint order with a certain time interval and the order size is adjusted to the difference between the maximum inventory and the amount of stock that is available when the order is made. In addition, PT IntiVulkatama should able to determine the size of special order when one or some of raw materials experiencing the differenced time of known priced increase. Study has derived a model that can be used when the company experienced the priced increase for two raw materials.

Based on data, PT IntiVulkatama should make a joint order for its six raw materials with a time interval of 1.12 weekwith each order size reaches the maximum of the inventory. The result of the price increase problem is a method of the differenced time of known priced increase with the savings value greater than zero. The study results in a recommendation that not to perform a special order due to the saving is less than the total cost.

CHAPTER I. Introduction

One of the problems that often occur in the company is the inventory of raw materials. According Tersine (1994), an inventory is the material waiting to be sold, used or transformed. The inventory is also needed by the company to anticipate the fluctuationof demands.

It is used to that companies operate the vehicles in carrying out its activities. One types of the vehicles that the most widely used bycompanies for the delivery the product is the truck, and one of the truck components that the most important is the truck tire. There are some companies producing truck tires that compete to get the customers. In order to satisfy customers, companies must able to provide the product or services that fit tothe customer needs or requirements.

PT IntiVulkatama is a manufacturing company in the tire retreading. PT IntiVulkatama processes the used slick tires into new tires by adding yeast (a specific tires model made by rubber) in accordance with the requirement of consumers. There are six types of tires that are often requested by consumers; the type of tire 700-14, 700-16, 750-15, 750-16, 900-20 and 1000-20. The tires 700-16 and 750-16 are processed by using the rubber raw materials ofhot process, while the type 750-15, 700-14, 900-20 and 1000-20 are processed by using the rubber raw material of the cold process. The number of new tire demand is probabilistic causing PT IntiVulkatama often experiences shortages for the raw materials (stockout).

PT IntiVulkatama sometimes experiences thatthe purchase price of raw materials, supplied by a supplier, has increases in the future, and the time for the priced increase is also known. Thissituation may occur for one or more of raw material. Specifically, for the priced increase for more than one raw materials, it was assumed that the time for the priced increase are similar.In the real situation, this assumption is violated.

Based on the problem identification explained above, the followings are the problem statements of the study:

1. How do the order time interval T and the optimal order size of the rubber raw material by using P (R; T) model to minimize the expected total cost?

2. How do the inventory policies that must be performedby PT IntiVulkatama when there is an increase in the price of the rubber raw material supplied by the supplierwith different period?

The following are the assumptions used in this study:

1. Lead time is 4 days.

2. The cost of stock out is determined by the backorder cost

3. The increase in the price of raw material is known with certainly.

4. The raw materials sent to PT IntiVulkatama are assumed in the good condition, so the cost of the damage is not considered in the study.

6. Neither the increasing or decreasing are in the area of warehouses for study

7. There are only two prices of raw materials that are increase with known differenced period.

Chapter II. LITERATURE REVIEW

There are two types of inventory systems, the fixed order size and fixed order interval systems (Tersine, 1994). Both the inventory system can be used to determine the number of orders and when the right time to place an order.Tersine also describes the policies that can be doneif an inventory system experiences the priced increase for the raw material suppliedby the supplier. The model only considers one raw material.

It was known that the inventory activities always creates cost to the company. Those cost are the purchasing cost, holding cost, ordering cost, and stockout cost. Usually, the purchasing cost will not be considered in the calculation especially when the value of periodic interval T is interested. Hadley derived an equation of total cost $K_{jointorder}$by considering more than one material supplied by the suppliers.

$$K_{Joint\ order} = \frac{L+(n-1)a}{T} + \sum_{i=1}^{n}\left[IC_i\left(R_i - \mu_i - \frac{\lambda_i T_i}{2}\right) + \sum_{i=1}^{n}\pi_i E(R,T)_i\right] \ldots\ldots\ldots\ldots\ldots\ldots 1.$$

where :

L : the ordering cost per order

a : the cost due to the additional of product in calculation

IC_i : the inventory cost for the ith product.

μ_τ : the lead time demand

T_i : The single order time period for i^{th} material

λ_i : the average demand per period for the i^{th} product.

π_i : the backordering cost for the ith product.

$E(R,T)_i$: the expectation number of backorder for the i^{th} product.

If there is a priced increase, Tersine derived the formula to determine a special order size and the optimal saving for the inventory;

$$\hat{Q}_1^* = \frac{k_1\lambda_1}{IC_1} + \frac{(P_1+k_1)Q_{1a}^*}{P_1} - q_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.$$

Where :

$\hat{Q}_1^*$: The special order sizefor the first product

k_1 : the price increase

P_1 : The regular price of the first raw material before the price increase

Q_{1a}^* : The EOQ after the price increase

q_1 : The inventory position when the special order size is performed

The following g* is the company's saving due to the special order size for the first raw material;

$$g_1^* = L\left[\frac{P_1}{P_1+k_1}\left(\frac{\hat{Q}_1^*}{Q_{1a}^*}\right)^2 - 1\right] = L\left[\left(\frac{Q_1^*}{Q^*}\right)^2 - 1\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.$$

$$\hat{Q}_{1a}^{*} = \sqrt{\frac{2L\lambda_1}{(P_1 + k_1)I}}$$

For fulfilling the demand, the company is still using a method of demand forecasting by analyzing the past demand. It is suggested that the company uses a right method to address the stockout problem. The P (R,T) method is recommended to use, because this method can also be applied effectively for multi productsof the single supplier.

The previous study [Aritonang, 2012] has considered thecondition for the priced increase for more than one raw materials, it was assumed that the time for the priced increase are similar. In the real situation, this assumption is not fulfilled, where the time for priced increase for more than one raw material has different in orderingtime.

Chapter III. RESEARCH METHODOLOGY

The research uses the following figureas the model of thestudy;

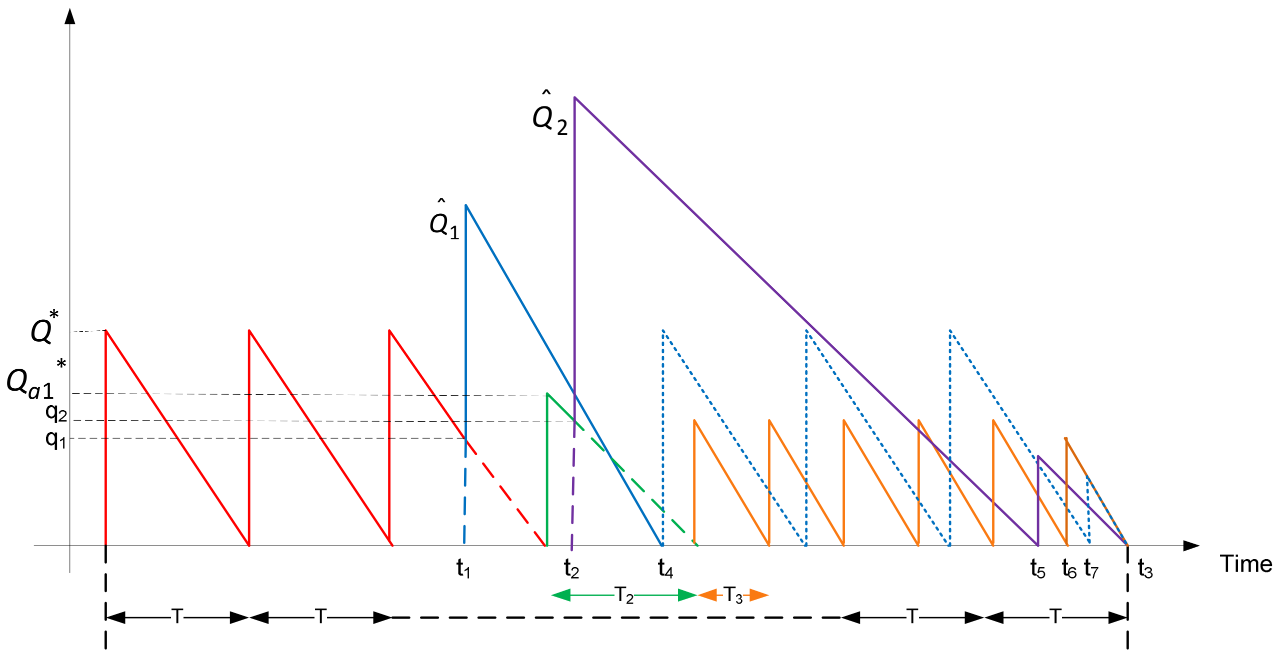

Figure 1. The model of the study

The model can be explained as follows;

I. When there is no priced increase, all the raw materials are ordered using the P(R,T) model. It means that all the raw materials will be ordered at every time period T. The value of T can be found by minimizing the following total cost K $_{\text{joint order}}$ per year given by the equation 1.

II. At time t_1, one of the raw material experiences the priced increase. The special order size will be performed using the equation 2.

The special order will be used until to $\frac{\hat{Q}_1^*}{\lambda_1}$ time unit or until to the point t_4, then $t_4 - t_1 = \frac{\hat{Q}_1^*}{\lambda_1}$

The rest of five raw materials will ordered jointly by using T_2.

III. At time t_2, a second of the raw material also experiences the priced increase. In order to make a decision whether to perform a special order or not, the following calculation should be considered;

1. The special order and the company's saving for the second material are given by the equation 2 and 3. This special order will finish in $\frac{\hat{Q}_2^*}{\lambda_2}$ time units or until to the point t_5, then;

   $t_5 - t_2 = \frac{\hat{Q}_2^*}{\lambda_2}$

   The company's saving due to the special order size for the second raw material is follow;

$$g_2^* = L\left[\frac{P_2}{P_2 + k_2}\left(\frac{\hat{Q}_2^*}{Q_{2a}^*}\right)^2 - 1\right] = L\left[\left(\frac{Q_2^*}{Q^*}\right)^2 - 1\right]$$

2. The total saving for doing a special order for both the first and the second raw material is;

   $$g_{total} = g_1^* + g_2^*$$

   The rest of four raw materials will be ordered jointly by using T_3.

3. After all the raw materials that experiecing the special order have been used up (in this case, the first and the second raw material), all the six raw materials will be ordered jointly at time t_3. Then t_3 is the time where all the products will be jointly ordered in the future. The following is the formula to calculetes t_3.

   If $\frac{\hat{Q}_1^*}{\lambda_1} > \frac{\hat{Q}_2^*}{\lambda_2}$, then ;

   $\mathrm{t}_3 = t_1 + \left[\mathrm{Roundup}\left(\frac{t_4 - t_1}{T}\right)\right] x\, T$ . . . . 4.

   Otherwise :

   $\mathrm{t}_3 = t_2 + \left[\mathrm{Roundup}\left(\frac{t_5 - t_2}{T}\right)\right] x\, T$ . . . . 5.

   As an example, it is assumed that the equation 5 is chosen.

4. After the first raw material has been used until to point t_4, a multi-single order will be carried out. The total multi-single order m could not be greater than t_3. The following formula is used to determine the number of multi-single order m;

   $\mathrm{m} = \mathrm{Roundown}\left[\frac{t_3 - t_4}{T_{single\ order\ for\ raw\ material\ 1}}\right]$ . . . . 6.

   The total cost TC_1 due to this multi-single order is;

   TC_1 = m x (the purchasing cost the holding cost + the ordering cost + the stockout cost) . . . . 7.

   To reach the point t_3, an order is also performed for material 1 with the following order size;

   $(\mathrm{t}_3 - (\mathrm{t}_4 + mT_{single\ order\ for\ material\ 1}))\lambda_1$

   It also creates cost TC_2.Then total cost due to this condition for the raw material 1 is $TC_1 + TC_2$.

5. Each of the rest four raw materials will also do an order to reach the time t_3, and this also creates cost to the company. This order is called the adjustment order. For the ith raw material, His the cost due to of this condition;

$$\mathrm{H} = L + (n-1)a + \sum_{i=1}^{4}(0.5\left(t_3 - \left(t_2 + nT_3 + {}^{q_2}/_{\lambda_2}\right)\right)\lambda_i IC_i x\ adj\ period + \sum_{i=1}^{4} E(R,T)_i)\pi_i \ldots\ldots 8.$$

Where;

$$\mathrm{n} = \mathrm{Roundown}\left[\frac{t_3 - t_{2-}{}^{q_2}/_{\lambda_2}}{T_3}\right] \ldots\ldots 9.$$

IV. A special Order for the second raw material is recommended if the following condition is fulfilled;

$$\boldsymbol{g_1^* + g_2^* > TC_1 + TC_2 + H} \quad \ldots\ldots 10$$

CHAPTER IV. RESEARCH SCHEDULLE

The research was performed since February 2014 to october 2014. The schedule of the research is shown below;

	2014											
	Jan	Feb	Mar	Apr	Mei	Jun	Jul	Agt	Sep	Okt	Nov	Des
Research for previous inventory models associated with the priced increase												
Derivation of a formula to determine the special order size by considering the different time of priced increase												
Writing articles for national journals accredited												
Evaluation of models to consider the capacity of the warehouse												
Evaluation of models to consider the ordering constraints												
Evaluation for all models and test models												
Writing articles for national journals accredited												
Writing reports, and lecture module and software												

CHAPTER V- RESULT AND DISCUSSION

In this chapter, a hypothetical case of inventorysystem is developed to find out how the procedure in Chapter 3 works. This hypothetical case is found from the rubber inventory case at PT IntiVulkatama (Kinley&Matrisia, 2013). The detail can be described as below.

CASE DESCRIPTION

There are 6 kind of rethreaded tires produced by PT IntiVulkatama. Each tire needs different kind of rubber. According to historical data of rubber demand, it can be estimated that each rubber has normally distributed demand with parameter value as shown in Table 1. The joint ordering cyclefor all raw material of rubbers is 0,8 week.

Table 1. Weekly Rubber Demand Characteristic

Rubber for Tire Type	Mean (pack/week)	St Dev (pack/week)
1000-20	39.61	13.67
900-20	8.96	3.55
750-15	4.01	1.21
750-16	17.88	6.90
700-16	8.92	3.96
700-14	2.34	1.04

A scenario is developed where there will be 2 times of priced increase event. These two events are the priced increase on 1000-20 rubber type and 900-20 rubber type. Both events occur on the 2.5thweek and 3.5th week, respectively. The amount of priced increase for both 1000-20 and 900-20 are 8% and 10%. As explained in Chapter 3, in the beginning, the P(R,T) inventory system will be applied for all the raw materials. First, the optimal solution to P(R,T) inventory system has to be found because the optimal order interval in P(R,T) system will be the input to run the procedure in Chapter 3. The final solution to this hypothetical case will be whether PT IntiVulkatama has to do the special order or not. There are several possible solutions, i.e. doing the special order for both products, just one of the products, or no special order at all.

A. Rubber Selling Price

The data of rubber selling prices are shown on the table 2.

Table 2. Rubber Selling Price

No	Rubber	Price(Rp)
1	1000-20	2,560,250
2	900-20	2,327,500
3	750-15	1,745,625
4	750-16	2,189,700
5	700-16	1,946,400
6	700-14	1,396,500

B. Ordering Cost

The ordering cost is calculated based on the administrative, telecommunication, loading-unloading, and inspection activities.It includes the communication cost, stationary cost, and proportional salary of workers who do loading-unloading, accounting, and inspection processes. Based on these two kindsof costs, the total cost for individual order will be Rp 118,681.9. If the company does a joint order, total order cost will be increased by 20% (equal to Rp 6,835.93) for every product included. For example, if the company does the joint order for three types of rubbers, then the total ordering cost will be 118,681.9 + (2 x 6,835.93, which equals to Rp 132,353.7.

C. Inventory Cost

The capital cost and warehouse cost will be considered when the inventory cost is calculated. The capital cost is defined as the profit company can get by depositing its money in a bank rather than put it on the inventory of raw material. Based on this definition, an interest rate of a certain bank's money investment product is used to calculate the cost. The referred net interest rate is 5.2% per year. The capital cost for every type of rubber will be calculated by multiplying the rubber's selling price by 5.2% interest rate. The second aspect of the inventory cost is warehouse cost. This cost will consider the amount of electricity used for the warehouse and also the proportional warehouse worker's salary. From these two type of cost components, the total inventory cost for each product are shownas follow;

Table 3. Material Rubber Inventory Cost

No	Rubber	InventoryCost (Rp)
1	1000-20	139,142.32
2	900-20	127,679.48
3	750-15	95,654.24
4	750-16	121,196.15
5	700-16	107,868.00
6	700-14	81,021.91

Itshould be considered the increase in total inventory cost when there is the priced increase event. This table below shows the new inventory cost (for two rubber material) when the rubber selling price increases.

Table 4. Rubber Inventory Cost after Price Increase

No	Rubber	Inv. Cost (Rp)
1	1000-20	149,792.96
2	900-20	139,782.48

D. Stockout Cost

There are two possible kindsof stock-outs, i.e. lost sale and back-order. For PT IntiVulkatama, the backorder condition is what actually happens. According to this information, the company will suffer from the backorder cost if it can't fulfill customer demand. The amount of backorder cost suffered by the company can be calculated from the tires selling price. The backorder cost is defined as the amount of money that the company could get by depositing his revenue in the bank. Failure in fulfilling demand makes the company lost its chance to get this certain amount of money. The interest rate of 5.2% is used again for this cost calculation. These backorder costsare presented on Table 5.

Table 5. Rubber Backorder Cost

No	Rubber	Backorder Cost (Rp)
1	1000-20	247,000.00
2	900-20	213,200.00
3	750-15	143,000.00
4	750-16	148,200.00
5	700-16	138,840.00
6	700-14	85,800.00

The priced increase of rubber will also affect the tire price. Assuming that it will be proportionally increased by the increment on the rubber price. The following are the new backorder cost for rubber 1000-20 and 900-20.

Table 6. Rubber Backorder Cost after Price Increase

No	Rubber	Back Order Cost (Rp)
1	1000-20	266.760
2	900-20	234.520

CALCULATION

Fromexplanation about the rubber inventory system, the procedure in Chapter 3 can be performed by the following steps;

1. Finding the optimal joint order interval of six types of rubbers
   This step is done by minimizing the value of Equation 1.

$$\boldsymbol{min}\ K_{Joint\ order} = \frac{118.681{,}9 + (6-1)6.835{,}93}{T} + \sum_{i=1}^{6}\left[IC_i\left(R_i - \mu_i - \frac{\lambda_i T}{2}\right) + \pi_i E(R,T)_i\right]$$

   a. IC_i has been calculated and is shown in the explanation part of inventory cost components (Table 3 or Table 4).
   b. R_i is the maximum inventory number for the i^{th} rubber. It can be calculated by considering the probability of backorder, the average demand, andthe standard deviation of demand during the interval order and lead time.
   c. μ_i is average number of demand for the i^{th}rubberduring the lead time.

d. λ_i is weekly average number of demand for the i^{th}product(Table 1 on the column "Mean")
e. π_i is the backordering cost for the ith product as shown in Table 5 or Table 6.
f. $E(R,T)_i$ is expected number of backorder for the i^{th} product.

Completing this unconstrained optimization problem, it is found that the optimal interval of joint order (T) of the six rubber materials is 1.12 week with parameters value as follow;

Table 7. The parameters of six Products

Rubber	R (pack)	μ (pack)	E(R,T) in pack
1000-20	119	31.69	3.54
900-20	29	7.16	0.536
750-15	12	3.21	0
750-16	55	14.30	2.40
700-16	29	7.13	1.40
700-14	8	1.87	0.002

2. Calculating the saving cost of 1000-20 rubber type when a special order is performed. Equation 2 & 3 are used to calculate the saving cost of 1000-20 in special order condition. The data needed for solving the Equation 2 and 3 are listed on table 8 below.

Table 8. Data Used for Equation 2&3

Type of Data	Amount
k	Rp 204,820
lambda (λ)	39.61 packs/week (2060 packs/year)
IC	Rp 139,142.32 / pack/ year
P	Rp 2,560,250 / pack
L	Rp 118,681.9 / order
IC increased price	Rp149,792.96/ pack/ year

In order to get $\hat{Q}_a^*$, it can be used the EOQ formula with the priced increase as follow;

$$\hat{Q}_a^* = \sqrt{\frac{2\ x\ 118{,}681.9\ x\ 2060}{149{,}792.96}} = 57.04\, packs$$

The average inventory when special order arrives (q) is estimated from the average ofstock which is 0.5 of maximum inventory(R). This maximum inventory is calculated by considering the backorder probability, standard deviation, and average demand during the order period. The maximum inventory(R) for 1000-20 with joint order interval for 1,12 week is 119 packs, so that the qwill be 59,5 packs. The special order size of 1000-20 rubber is

$$\hat{Q}_1^* = \frac{204{,}820\ x\ 2060}{139{,}142.32} + \frac{(2{,}560{,}250 + 204{,}820)\ x\ 57.04}{2{,}560{,}250} - 59.5 = 3035\, packs$$

Q^* is a fixed order size of 1000-20 when the price has not increased yet. Using the EOQ formula, the Q^* will be 59.28 packs. Completing all the calculations above, the saving for 1000-20 rubber material is

$$g_1^* = 118{,}681.9\left[\left(\frac{3035}{59.28}\right)^2 - 1\right] = Rp\ 310{,}970{,}917.2$$

The saving is a positive so that it is decided to run the special order for 1000-20.

3. Calculating optimal joint order interval for the rest of five rubber products.
   Equation 1 is used to calculate the optimal joint order interval for the rest of 5 rubber material products. The result of completing Equation 1 is the optimal joint order interval ($T_{joint\ 5}$) of 1.56 weeks with the R and E(R,T) as follow;

Table 9. Detail of 5 Products Joint Order

Rubber	R (pack)	E(R,T) in pack
900-20	33	0.943
750-15	14	0
750-16	64	2.514
700-16	34	1.14
700-14	9	0.203

4. Calculating the end usage period of 1000-20 special order.
   The special order of 1000-20added by its inventory (q) when the special order arrives will be consumed as long as $(\hat{Q}_1^* + q\ )/\lambda_1$which is (3035 + 59.5)/39.61 = 78.1144 weeks and it is similar to($t_{price\ increase}$ + special order usage period)week orequal to 80,6135 weeks.

5. Calculatingt$_{joint}$which is the time point when the joint order of six rubber types will be performed again.
The previous joint order interval (T) is used as the basic to calculating t_{joint}. The logic presented by Equation 4 or 5 can be used to get the t_{joint}.

$$t_{joint} = t_{price\ increase} + \left[\text{Round up}\left(\frac{t_{end\ of\ spec\ order} - t_{price\ increase}}{T}\right)\right] x\ T$$

$$= 2.5 + \left[\text{Round up}\left(\frac{80.6135 - 2.5}{1.12}\right)\right] x\ 1.12 = 80.9$$

6. Finding the optimal individual order interval for 1000-20.
   Equation 1 is used to find the optimal 1000-20 order interval by considering the new price of 1000-20. This new price will affect inventory cost and backorder cost.

$$\boldsymbol{min}\ \ K_{individual\ order\ 1000-20} = \frac{118.681{,}9}{T} + 149.792{,}96\left(R - \mu - \frac{T}{2}\right) + 266.760\ E(R,T)_1$$

Completing this unconstrained optimization problem, it is found that the optimal order interval is 1,25 week with R, μ, and E(R,T) are 125 packs, 31,69 packs, and 3,57 packs respectively.

7. Calculating how many times the individual order of 1000-20 can be performed during the end of special order usage tot$_{joint}$.

   Rounddown [($t_{joint} - t_{end\ of\ special\ order}$) / $T_{ind\ order\ 1000\text{-}20}$] = (80.9 – 80.6135) / 1.25 = 0 times

   It is 0 times, so that the individual order with 1.25 weeks order interval can't be performed. The order process will just be carried out in the amount necessary for fulfilling demand as long as ($t_{joint} - t_{end\ of\ special\ order}$)or equal to 0.2865 weeks. Based on this condition, the order size of 1000-20 rubber will be

$$(80.9 - 80.6135)\ x\ 39.61\frac{packs}{week} = 11.348\ packs$$

8. Calculating how many times the 5 rubbers joint order can be performed untilt$_{joint}$.
   Frequency of 5 rubbers joint order can be calculated as below

   Rounddown [($t_{joint} - t_{priced\ increase} - q_{1000\text{-}20}/\lambda_{1000\text{-}20}$) / $T_{joint\ 5}$]= (80.9 – 2.5 – 59.5/39.61) / 1.56 = 49 times
   This 49 times joint order will end at the following week;

   $t_{priced\ increase} + q_{1000\text{-}20}/\lambda_{1000\text{-}20}$ + frequency x $T_{joint\ 5}$ = 2,5 + 1,50 + 49 x 1,56 = 80,44.

   The t_{joint} will be at week of 80.9. During the rest (80.9 – 80.44) or equal to 0,46 weeks, every rubber will be ordered in a necessary amount to fulfill the demand during this period. This order can be called the adjustment order. The order size for every rubbermaterial which is done just to reach the t_{joint} is shown on the table below;

   Table 10. Adjustment Order Size

Rubber	Mean (λ) packs/week	Order Size (0.46 x λ)
900-20	8.96	4.1
750-15	4.01	1.8
750-16	17.88	8.2
700-16	8.92	4.1
700-14	2.34	1.07

9. Calculating the new optimal policy of joint order interval for six rubbers.
   Equation 1 is used to calculate this optimal joint order interval, but it will include the new price of 1000-20. This calculating process results in anew optimal joint order interval of 1.03 weeks.

Note: the resulting inventory policies from step 5 till 9 are just temporary policies and will be executed only if there is no another special order carried out.

10. The effect for another priced increase for another product.

In this case, there is another priced increase for another material rubber, which is the type of900-20. First of all, it should be identified whetheranother priced increase occurs in special order period/multi-single order cycle period or not. It is known that the time for 900-20 priced increase is in 3.5 week, which is in the special order usage period of 1000-20. The decision in doing a special order for the second product (the type of 900-20) will depend on the result of comparing the total saving cost of doing the special order for both products to the total cost of multi-order cycle. If the saving amount is greater than multi-single order cost, then the special order for the second product will be performed (equation 10). The calculating processes in order to get solution to this situation are started from the step 11.

11. Calculating the saving cost of 900-20 rubber type when the priced increases

This calculating procedure is just the same as one in step 2.

Data needed to get the amount of saving are presented on table below

Table 11. Data Input for Calculating Saving Cost

Type of Data	**Amount**
k	Rp 232,750
lambda (λ)	8.961 packs/week (466 packs/year)
IC	Rp 127.679.48 / pack/ year
P	Rp 2,327,500 / pack
L	Rp 118,681.9 / order
IC increased price	Rp 139,782.48/ pack/ year

The EOQ formula is used to calculatedQ_a^* by considering the price increase as follow;

$$\hat{Q}_a^* = \sqrt{\frac{2 \ x \ 118{,}681.9 \ x \ 466}{139{,}782.48}} = 28.06 \ packs$$

The inventory position (q) when a special order arrives is estimated by the average of inventory, which is 0.5 of maximum inventory(R). The maximum inventory(R) for 900-20 with the joint order interval of 1.56 week (step 3) is 33 packs, so that the q equals to 16.5 packs. The special order size of 900-20 rubber is

$$\hat{Q}_2^* = \frac{232{,}750 \ x \ 466}{127{,}679.48} + \frac{(2{,}327{,}500 + 232{,}750) \ x \ 28{,}06}{2{,}327{,}500} - 16.5 = 864 \ packs$$

Q* will be the order size of 900-20 when the price has not increased. Using the EOQ formula, the Q* will be 29.43 packs. Completing all the calculations above, it is found that the saving for 900-20 rubber is

$$g_2^* = 118{,}681.9 \left[\left(\frac{864}{29.43} \right)^2 - 1 \right] = Rp \ 102{,}170{,}911.2$$

The saving is a positive number, but it is too early to decide whether to perform the special order or not. Two products saving cost (1000-20 and 900-20) have to be totaled and then compared to the total cost resulted from individual orders ofthe multi-single orderand the rest of 4 products in the joint order until to t_3. From the calculation, it is found that the total saving from 1000-20 and 900-20 isRp 413,141,828.4

12. Calculating the optimal joint order interval period for the rest of 4 products.
Thecalculation is performed by using equation 2. The optimal solution for the joint order interval is 1.71weekswith R and E(R,T) are shown in the following table;

Table 12. Detail of 4 Products Joint Order

Rubber	R (pack)	E(R,T) in pack
900-20	33	0.943
750-15	14	0
750-16	64	2.514
700-16	34	1.14
700-14	9	0.203

13. Calculating the end usage period of 900-20 special order.
The special order of 900-20and its inventory (q)will be used for$(\hat{Q}_2^* + q\ )/\lambda_2$ weeks, which is (864 + 16.5)/8.96 = 98.25 weeks. Due to the price is increased started on the 3.5 weeks, so the product will finish at ($t_{price\ increase}$ + special order usage period) week, or equal to 101,75 week.

14. Calculating the new time of joint ordert_{joint}.
The joint interval (T) in step 1, when there is no price increase, is the basic for calculating t_{joint}. The$t_{joint}$is started from the priced increase time of 900-20.

$$\mathrm{t_{joint}} = t_{price\ increase} + \left[\text{Round up}\left(\frac{t_{end\ of\ spec\ order} - t_{price\ increase}}{T}\right)\right] x\ T$$

$$\mathrm{t_{joint}} = 3.5 + \left[\text{Round up}\left(\frac{101{,}75 - 3{,}5}{1{,}12}\right)\right] x\ 1{,}12 = 102.06$$

Thist_{joint}will be compared to the previous $t_{joint}$of 80.9. Set the final $t_{joint}$to the maximum value. The final value of $t_{joint}$is 102.06. This final $t_{joint}$is used to set to t_3 (equation 4 and 5).

15. Calculating the frequency of 1000-20 multi-single order.
The number of times (m_1)of1000-20 multi-single order order can be calculated by using equation 2;

$$m_1 = \text{Roundown}\left[\frac{t_3 - t_4}{T_{single\ order\ for\ raw\ material\ 1}}\right]$$

or

$$m_1 = \text{Roundown}\left[\frac{t_3 - t_{end\ of\ special\ order\ of\ 1000-20}}{T_{single\ order\ for\ 1000-20}}\right]$$

m_1= (102.06 – 80.6135) / 1.25 = 17 times

The t_3 is determinedafter considering the price increase on 900-20. The end time of this 17 times multi-single order is at the week of 101.86, which is 80.6135 + (17 x 1.25). The rest of 0.1965 (calculated by 102.06 -101.86) weeks until it reaches the final t_3 is the period whenthe 1000-20 will be ordered by 0.1965 x $\lambda_{1000\text{-}20}$. This 1000-20 order size is 7.78 packs.

16. Calculating the frequency of 900-20 individual order

The number of times (m_2) of 900-20 multi-single order can be calculated as follow;

$$m_2 = \text{Roundown}\left[\frac{t_3 - t_{end\ of\ special\ order\ of\ 900-20}}{T_{single\ order\ for\ 900-20}}\right] = 0$$

So, there is no multi-single order of 900-20will be performed. This 900-20 order will only be performed to fulfill the demand between the endof period of special order to t_3, which is 0.31 weeks. The number of 900-20 that should be ordered equals to 0.31 x $\lambda_{900\text{-}20}$,which is equal to 2,77 packs.

17. Calculating the order size for each of the rest 4 products to reach the t_3.

First of all, the end of 4 joint-order period should be calculated. The joint order frequency since the time of 900-20 price increases can be estimated using Equation 9;

$$\text{n} = \text{Round down}\left[\frac{102.06 - 3.5 - {}^{16.5}/_{8.96}}{1.71}\right] = 56 \text{ times}$$

The end time of 4 rubbers joint order is onthe week (3.5 + 16.5 / 8.96 + 56 x 1.71) = 101.10. Then the rest period until it reaches t_3 is 102,06 – 101,1 = 0,958 weeks. From this information, the order size for each rubber can be calculated and the results are shown on the table 13;

Table 13. Adjustment Order Size for 4 Joint Product

Rubber	Mean (λ) packs/week	Order Size (0,958 x λ) packs
750-15	4.01	3.85
750-16	17.88	17.13
700-16	8.92	8.54
700-14	2.34	2.24

18. Calculating the total cost of multi-single order and adjustment period

This cost component consists of the multi-singleorder cycle cost of and product adjustment order cost to reach the t_3.

a. Multiple order cycle

This multi-singleorder cycleexists on the 1000-20 rubber for 17 times. Total cost of this 17-time order can be calculated using Equation 7.

TC = m x (purchasing cost + holding cost+ order cost+ stock-out cost)

Inventory cost

The Inventory cost during the optimal individual order interval for 1000-20 can be calculated by the following equation;

$$IC_{1000-20} x \frac{T_{1000-20}}{52}\left(R_{1000-20} - \mu_{1000-20} - \frac{\lambda T_{1000-20}}{2}\right)$$

The values of $IC_{1000-20}$, $R_{1000-120}$ and $\mu_{1000-20}$ are Rp 149,792,96.125 packs, and 31.69 packs, respectively, while $\lambda_{1000-20}$ and $T_{1000-20}$ are 39,62 packs / week and 1,25 weeks. The equation results in the inventory cost for 1000-20 of Rp 246,827.4.

The Ordering cost

The ordering cost for a single order is Rp 118,681.9

Backorder cost

The stock-out cost or backorder cost for 1000-20 can be estimated by using the following equation;

$$\pi_{1000-20} E(R_{1000-20}, T_{1000-20})_i \, x \frac{T_{1000-20}}{52},$$

where $\pi_{1000-20}$ is backorder cost per unit product and $E(R_i, T_{1000-20})$ is the average number of backorder per year,andthe values equal to Rp 266.760 and 3,57 packs, respectively. The equation will result in backorder cost by Rp 22,898.89.

Purchasing Cost

This cost is incurred by purchasing a certain amount of rubber to fulfill demand during the multi-single cycle periods. The number of rubbers purchased during this period is estimated using the multi-single cycle demand, which is 17 x 1.25 x 39.61 = 841.7125 packs. This number of products will cost the company by the following value;

841.7125 x (2,560,250 x 1.08) = Rp 2,327,393.982

According to those three cost components, the total cost for multi-singleorder cycle is

TC_1 = 17 x (Rp 246,827.4 + Rp 118,681.9 + Rp 22,898.89) + 2,327,393.982

TC_1= Rp2,333,996.922

b. Adjustment Order Cost for 1000-20 and 900-20

Adjustment order cost consists of the one-time order cost and inventory cost. Assuming that the backorder cost is very small compared to other costs, then this cost would be neglected in the calculation.

- Adjustment order cost for 1000-20

TC = order cost + (adjustment order size/2 x inventory cost/pack/year x adjustment period) + purchasing cost

TC = 118.681.9 + ((7.78/2) x 149,792.96 x 0.1965/52) + 7.78 x (2,560,250 x 1.08)

TC = Rp 21,633,133.41

- Adjustment order cost for 900-20

With the same calculation, it is found that;

TC = 118,681.9 + ((2.77/2) x 139,782.48 x 0.31/52)) + 2.77 x (2,327.500 x 1.1)

TC = Rp 7,211,728.54

c. Adjustment Order Cost for the rest of 4 products

According to Equation 9, the total order cost for these 4 products is

$L + (n-1)a$ = 118,681.9 + (4-1) 6,835.93 = Rp 139,189.69

The total inventory cost for these 4 products is

$$\sum_{i=1}^{4} 0.5\ (adj.period)\lambda_i IC_i x\ adj\ period$$

The adjustment period is 0,958 week. The calculation of each inventory cost during the adjustment period is shown on the table below;

Table 14. Inventory Cost during Adjustment Period

Rubber	λ	IC	Inventory Cost during Adjustment Period
750-15	4.019	95,654.24	176,420.15
750-16	17.885	121,196.2	994,648.08
700-16	8.923	107.868	441,680.56
700-14	2.346	81,021.91	87,228.81
		Total	1,699,977.61

The following table is the purchasing cost of these four products;

Table 15. Purchasing Cost of the 4 Joint Products

Order Size	Current Price	Total Price
3.85	1.745.625	6,721,008.866
17.13	2.189.700	37,517,947.55
8.54	1.946.400	16,638,282.66
2.24	1.396.500	3,138,589.062
Total		64,015,828.14

Then the total cost TC_2 is Rp2,428,696,779

19. Comparing the total saving to the total cost of multiple order cycle and the adjustment order cost

    From step 11 we get the total saving cost is equal to Rp 413,141,828.4

    From step 18 we can get the total cost for multiple order cycle and adjustment order cost equals to Rp 2,428,696,779

    The total saving cost is less than the total cost of multiple order cycle and adjustment order cost, so that the solution is not performing the special order**.**

20. Calculating the final six products joint order

The final step is to give the recommendation of six products joint order interval that will be started on t_3 week. Equation 2 is used to find the solution. The optimal interval for this new six products joint order is 1.03 weeks with the following R and the E(R,T) values.

Table 17. Detail of The New 6 Products Joint Order

Rubber	R (packs)	E(R,T)
1000-20	115	3,43
900-20	28	0,557
750-15	11	0,379
750-16	53	2,49
700-16	28	1,426
700-14	8	0

CHAPTER VI.CONCLUSIONS AND RECOMMENDATIONS

1. If there is no increase in the price of raw rubber materials, the company can use the method of P(R,T) to make an order to obtain the minimum cost in the inventory management. It is found that the joint order interval for the entire raw material rubber is 1:12 weeks.
2. It has been derived a procedure if there is an increase in raw material prices on the two different time. The decision to place an order on a special rubber raw materials that increase depending on whether the saving obtained is greater than the costs incurred in the implementation of the policy. The case is taken as an example decided not to make special reservations on the second rubber raw materials that experience the priced increase.

Study also provides the following suggestions;

1. It is recommend that companies implement methods resulting in inventory management activities , using procedures that have been derived above, so the company can minimize the total cost of inventory.
2. It is recommended that the company is also considering the capacity of the warehouse in using these procedures. This is due to the amount of special ordering very large.

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