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Comparison between the bivariate Weibull probability approach and linear regression for assessment of the long-term wind energy resource using MCP

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Highlights

- Measure-correlate-predict approach based on bivariate Weibull probability tested at 22 sites
- Deviations from ideal bivariate Weibull behaviour investigated using observed and artificial data
- Error metrics calculated using 120 test periods over an 11 year data record
- Performance compared to existing regression methods using variable onsite measurement periods

Keywords: measure-correlate-predict, wind resource assessment, bivariate Weibull distribution

Abstract

A detailed investigation of a measure-correlate-predict (MCP) approach based on the bivariate Weibull (BW) probability distribution of wind speeds at pairs of correlated sites has been conducted. Since wind speeds are typically assumed to follow Weibull distributions, this approach has a stronger theoretical basis than widely used regression MCP techniques. Building on previous work that applied the technique to artificially generated wind data, we have used long-term (11 year) wind observations at 22 pairs of correlated UK sites. Additionally, 22 artificial wind data sets were generated from ideal BW distributions modelled on the observed data at the 22 site pairs. Comparison of the fitting efficiency revealed that significantly longer data periods were required to accurately extract the BW distribution parameters from the observed data, compared to artificial wind data, due to seasonal variations. The overall performance of the BW approach was compared to standard regression MCP techniques for the prediction of the 10 year wind resource using both observed and artificially generated wind data at the 22 site pairs for multiple short-term measurement periods of 1-12 months. Prediction errors were quantified by comparing the predicted and observed values of mean wind speed, mean wind power density, Weibull shape factor and standard deviation of wind speeds at each site. Using the artificial wind data, the BW approach outperformed the regression approaches for all measurement periods. When applied to the real wind speed observations however, the performance of the BW approach was

comparable to the regression approaches when using a full 12 month measurement period and generally worse than the regression approaches for shorter data periods. This suggests that real wind observations at correlated sites may differ from ideal BW distributions and hence regression approaches, which require less fitting parameters, may be more appropriate, particularly when using short measurement periods.

1 Introduction

The installed capacity of wind energy systems has seen rapid growth over the last decade [1] as governments, businesses and individuals seek to reduce their carbon emissions in response to growing concern over climate change. In the UK, where a legally binding commitment exists to reduce CO_2 equivalent emissions by 80% in 2050 compared to 1990 levels, wind power is considered a key part of the Government's strategy to decarbonise electricity supplies [2]. To maximise the UK's favourable wind potential, wind energy systems on a range of scales should be utilised.

Vital to the successful deployment of wind power systems on any scale is an accurate assessment of the available wind energy resource. Since wind flows are stochastic in nature, the wind resource must be characterised using long-term averages which describe the available power at the proposed (target) site. For large-scale installations, this typically involves onsite measurements of wind speed and direction covering 1-3 years [3], in addition to long-term correlation with a nearby reference site to account for inter-annual variations. The correlation is achieved using one of a family of approaches known collectively as measure-correlate-predict (MCP). A typical MCP approach involves using regression or other techniques to relate wind speed measurements at a target site with concurrent measurements at a nearby reference site [4], or with appropriate atmospheric data from reanalysis projects [5]. Long-term historical reference data is then used with the established relationship to predict the long-term wind resource at the target site.

For small-scale installations, a long-term measurement campaign may not be practical or financially viable and developers may rely on wind maps, empirical correction factors [6] or boundary layer scaling approaches [7, 8]. MCP applied to very short-term measurement periods may also be a viable approach [9] providing the performance of the techniques as a function of the measurement period has been investigated.

The literature related to MCP is extensive, encompassing industry reports, commercial software, and conference and academic papers dating back to the 1940s [10]. Here we mention only the major classes of MCP techniques, a more detailed review can be found in [10]. Early MCP approaches [11, 12] involved simple scaling of the short-term mean wind speed using long-term reference site measurements, thus providing only limited information regarding the long-term wind resource. Later studies [4, 13-15] used linear regression of the scalar wind speeds at the target and reference sites to predict a long-term time series based on short-term measurements, from which parameters related to the wind speed distribution could be estimated. More complex regression models, including two-dimensional [16], vector [14] and non-linear [17] have also been investigated. Mortimer [18] proposed binning wind data according to the reference site wind speed and direction and construction of a matrix containing ratios of the short-term reference and target site wind speeds. The ratios were used along with a matrix of standard deviations to predict the long-term target site wind speeds. A matrix approach was also proposed by Woods and Watson [19] where wind data was binned according to reference and target site wind direction. Further processing was undertaken to account for the directional wind veer that may occur in complex terrain. Learning based techniques such as artificial neural networks (ANNs), which represent learned patterns between input and output data by weighted interconnections, are increasingly being applied to MCP [20-24]. Given training data with known reference and target site wind speeds, the patterns can be learnt and applied to unseen data to make predictions at the target site. MCP approaches based on the joint probability distribution function (pdf) between reference and target site wind speeds have also been proposed [25, 26], although such approaches have received relatively little attention considering their attractive theoretical properties. Despite the variety of proposed approaches, MCP implementation in commercial software packages [27-29] is often restricted to top-down linear regression or scaling approaches, presumably due to their simplicity and empirical success.

This study is concerned with an MCP approach based on the joint pdf between the reference and target site wind speeds. The motivation for this approach is that whilst simple linear regression techniques are based on the assumption of a bivariate Gaussian distribution between two variables [26, 30], univariate Weibull distributions are typically used in wind resource assessment [31]. Hence there is a stronger theoretical justification for describing the correlation between target and reference site wind speeds using a bivariate Weibull (BW) distribution. Such an approach provides a direct mathematical basis for modelling the distribution of wind speeds at the target site given a specific input wind speed at the reference site. The modelled distributions are known as conditional distributions since they are conditional on the input reference site wind speed. This approach contrasts with regression techniques which treat the conditional distributions as scatter or residual errors about a true mean value. Recently, Perea *et al*. [26] used artificially generated wind speed data to investigate the utility of an MCP approach based on BW probability distributions. Their results indicated that the approach performed better than several established MCP techniques. However, a vital question is whether such a promising approach can be successfully applied to real wind speed observations which will likely deviate from idealised BW distributions and which may contain terms dependent on season and wind angle.

In this work, the BW approach is applied to wind speed observations at 22 pairs of UK sites located in a variety of terrains, in addition to artificially generated wind data drawn from ideal BW distributions. A sliding window technique is applied to data records covering 11 years, using short-term measurement periods of 1-12 months, to predict the long-term (10 year) wind resource at each site. The accuracy of the wind resource predictions is assessed through a variety of error metrics and the results compared to widely used regression MCP approaches. The aims of this work are: (I) To investigate the practical challenges of applying the BW approach to real wind data compared to artificial data drawn from ideal BW distributions, (II) To compare the performance of the BW approach with widely used linear MCP techniques using real wind data from a number of sites.

2 Methodology

MCP approaches are generally concerned with predicting a long-term historical time-series of wind speeds (and possibly directions) using short-term concurrent wind measurements at a correlated reference/target site pair. The short-term measurements are used to model the relationship between the two sites, while long-term historical reference data are used as model inputs to predict the long-term target site wind speeds.

Using simple linear regression, any input reference site wind speed has a corresponding single-valued output prediction at the target site. Repeating this process for the full historical time-series at the reference site produces an estimated long-term historical time series at the target site that is assumed to be a suitable predictor of the future wind resource. The BW probability approach involves a similar process but with the following distinctions. Firstly, the BW approach seeks to directly model the underlying distribution of target site wind speeds rather than the historical time-series. Secondly, rather than the restriction that a specific reference site wind speed corresponds to a specific target site wind speed, the BW approach predicts a distribution of target site wind speeds for every reference site wind speed in the form of a conditional probability distribution. Since wind power is proportional to the cube of the wind speed, these characteristics are important in achieving accurate wind resource predictions. The BW approach will now be described in more detail.

2.1 A bivariate probability approach to MCP

Given two correlated random variables, their relationship may be described by a bivariate pdf. The height of the pdf surface at a point describes the probability of observing a particular combination of variable pairs. The distribution can be thought of as being composed of a series of one-dimensional, conditional probability distributions or vertical slices through the two-dimensional probability surface. Each slice describes the probability of observing particular values of one variable given a fixed value of the second. In addition, the conditional probability slices can be integrated across one of the variables to yield the marginal, or complete, distribution of the other variable.

For wind speeds observed at a correlated reference/target site pair, the conditional and marginal probability densities have a direct physical interpretation. The conditional probability density is given by [26]:

$$f(u_t|u_r = u_r') = \frac{f(u_r', u_t)}{f(u_r')}$$

Equation 1

where u_r and u_t represent wind speed observations at the reference and target sites respectively and u_r' is a specific value of u_r, $f(u_r, u_t)$ is the bivariate pdf and $f(u_r)$ represents the univariate pdf at the reference site.

The marginal pdf at the target site, $f(u_t)$, is obtained by integrating the product of the conditional pdf in Equation 1 and the marginal pdf at the reference site, $f(u_r)$, over all reference site wind speeds using [26]:

$$f(u_t) = \int f(u_t|u_r = u_r')\, f(u_r)\mathrm{d}u_r$$

Equation 2

The marginal pdf of wind speeds at the target site $f(u_t)$, represents the key descriptive quantity of the target site wind resource.

Implementation of an MCP approach based on an underlying bivariate pdf requires a prediction of the long-term marginal pdf of wind speeds at the target site, $f_{long}(u_t)$, based on a short-term measurement period. Combining Equation 1 and Equation 2:

$$f_{long}(u_t) = \int \frac{f(u_r, u_t)}{f_{short}(u_r)} f_{long}(u_r) \mathrm{d}u_r$$

Equation 3

where the subscripts 'short' and 'long' refer to the short-term training period and long-term prediction period respectively.

In line with previous work [26], it is assumed that the short-term measurement period is sufficient to determine the form of the underlying bivariate pdf, $f(u_r, u_t)$ using some fitting procedure and that this function does not change with time. To obtain $f_{long}(u_t)$ from a short-term measurement campaign also requires an estimate of the long-term reference site wind speed distribution $f_{long}(u_r)$. This is obtained by fitting a univariate Weibull distribution to the long-term wind speed observations at the reference site. In practice, the wind speed observations are discrete rather than continuous and the integral in Equation 3 is replaced with a summation at discrete intervals.

2.2 Application of the bivariate Weibull probability approach

While a number of BW constructions are possible [32], the present application requires a formulation that yields two-parameter, univariate, Weibull marginals and whose likelihood function is analytically tractable. Here the BW previously employed by Johnson *et al*. [33] in relation to strength properties of lumbar, which was later applied to artificial wind data by Perea *et al*. [26] is used. The BW pdf contains five parameters and is described by [33]:

$$f(u_r, u_t) = \frac{k_r}{c_r}\left(\frac{u_r}{c_r}\right)^{\left(\frac{k_r}{d}\right)-1} \frac{k_t}{c_t}\left(\frac{u_t}{c_t}\right)^{\left(\frac{k_t}{d}\right)-1} \left\{\left(\frac{u_r}{c_r}\right)^{\frac{k_r}{d}} + \left(\frac{u_t}{c_t}\right)^{\frac{k_t}{d}}\right\}^{d-2}$$

$$\mathrm{x} \left\{\left[\left(\frac{u_r}{c_r}\right)^{\frac{k_r}{d}} + \left(\frac{u_t}{c_t}\right)^{\frac{k_t}{d}}\right]^{d} + \frac{1}{d} - 1\right\} \exp\left\{-\left[\left(\frac{u_r}{c_r}\right)^{\frac{k_r}{d}} + \left(\frac{u_t}{c_t}\right)^{\frac{k_t}{d}}\right]^{d}\right\}$$

Equation 4

where k and c are the Weibull shape and scale factors respectively, $0 < d \leq 1$ describes the degree of association between wind speed observations at the two sites and the subscripts r and t refer to the reference and target sites. The magnitude of d is inversely related to the degree of correlation between the two sites [32].

Johnson *et al*. [33] showed that the log-likelihood ($\ln L$) function for this distribution is tractable and may be used to fit the BW to concurrent observations of the two correlated variables using the method of maximum likelihood (MML). The $\ln L$ is given by:

$$\ln L = n\ln\left(\frac{k_r}{c_r}\right) + n\ln\left(\frac{k_t}{c_t}\right) + \left[\left(\frac{k_r}{d} - 1\right)\sum_{i=1}^{n}\ln\left(\frac{u_{r,i}}{c_r}\right)\right]$$
$$+\left[\left(\frac{k_t}{d} - 1\right)\sum_{i=1}^{n}\ln\left(\frac{u_{t,i}}{c_t}\right)\right] + \left\{(d-2)\sum_{i=1}^{n}\ln\left[\left(\frac{u_{r,i}}{c_r}\right)^{\frac{k_r}{d}} + \left(\frac{u_{t,i}}{c_t}\right)^{\frac{k_t}{d}}\right]\right\}$$
$$+\sum_{i=1}^{n}\ln\left\{\left[\left(\frac{u_{r,i}}{c_r}\right)^{\frac{k_r}{d}} + \left(\frac{u_{t,i}}{c_t}\right)^{\frac{k_t}{d}}\right]^{d} + \frac{1}{d} - 1\right\} - \sum_{i=1}^{n}\left[\left(\frac{u_{r,i}}{c_r}\right)^{\frac{k_r}{d}} + \left(\frac{u_{t,i}}{c_t}\right)^{\frac{k_t}{d}}\right]^{d}$$

Equation 5

where n is the total number of observations, $u_{r,i}$ and $u_{t,i}$ represent the i^{th} concurrent wind speed observation at the reference and target sites respectively and $\ln$ is the natural logarithm.

Here, short-term wind speed observations at the reference and target sites were used to obtain the fitted BW pdf by minimising the negative $\ln L$ (equivalent to maximising $\ln L$) using a multidimensional, non-linear Nelder-Mead search implemented in MATLAB [34]. Using the method of Johnson *et al*. [33], the minimisation was implemented as follows: (I) starting estimates of k_r, k_t, c_r and c_t were obtained through fitting univariate Weibull distributions to the short-term wind speed observations at the target and reference sites and these were used with an initial value of d = 0.5 to minimise $\ln L$ with respect to d only, (II) these starting parameters were used for a second minimisation search with respect to all five parameters to obtain the final fitted BW distribution, $f(u_r, u_t)$. The predicted long-term target site wind speed distribution $f_{long}(u_t)$, was then obtained using Equation 3.

A second approach was also implemented for comparison. Final estimates of k_r, k_t, c_r and c_t were extracted through univariate Weibull fits to the short-term reference and target site wind observations.

The association parameter d was then obtained using the relation between d and the covariance of u_r and u_t proposed in [32]:

$$cov(u_r, u_t) = c_r c_t \left[\Gamma\left(\frac{d}{k_r}+1\right)\Gamma\left(\frac{d}{k_t}+1\right)\Gamma\left(\frac{1}{k_r}+\frac{1}{k_t}+1\right)\right.$$
$$\left.-\Gamma\left(\frac{1}{k_r}+1\right)\Gamma\left(\frac{1}{k_t}+1\right)\Gamma\left(\frac{d}{k_r}+\frac{d}{k_t}+1\right)\right] \div \Gamma\left(\frac{d}{k_r}+\frac{d}{k_t}+1\right)$$

Equation 6

where Γ is the gamma function.

Equation 6 was solved numerically to obtain an estimate for d with the restriction $0 < d \leq 1$. This approach allows all five parameters to be obtained without fitting the full two-dimensional distribution. This modified technique is referred to as BW2 in the following discussion. As with the BW approach, $f_{long}(u_t)$ was obtained using Equation 3.

To determine the statistical parameters that describe the predicted wind resource, 10^6 random wind speed samples were drawn from the predicted $f_{long}(u_t)$. These were used to calculate the error metrics described in Section 2.6. Since the angular dependent upwind roughness can affect the scaling between the reference and target site wind speeds [8], the BW approach was implemented using wind data binned into 90^0 angular sectors with respect to the reference site wind direction, except when investigating the convergence efficiency (Section 3.1) where no binning was applied. This sector width was chosen based on the performance of the BW approach for sector widths of 30^0-360^0. For training periods where there were less than 80 observations within an angular bin, the fitted BW parameters behaved erratically and hence the data from the full range of angles was used.

2.3 Generation of artificial wind speed data

In addition to the long-term observed wind data at multiple sites, which is crucial to investigating the performance of the BW approaches, samples of artificial data drawn from known BW distributions were also used. The purpose of using additional artificial data was (I) to validate the proposed theoretical framework for BW-based MCP (II) to investigate differences in the fitting efficiency of the BW distribution using real and idealised data, and thereby infer how observed data differs from idealised BW distributions and (III) to investigate to what extent conclusions based on artificial data may be extrapolated to observed data.

Samples of artificial wind data drawn from specified BW distributions were constructed using an approach reported by Lu and Bhattacharyya [32] and others [33, 35]. The artificial data was used to mimic the results of a short-term measurement campaign at two correlated sites with an ideal BW distribution, thus providing a first step to validating the methodology.

Correlated, artificial random variables representing n pairs of concurrent wind speeds at two sites are here denoted as ($\mathbf{x} = [x_{1,} \, x_{2,} \dots x_n]$, $\mathbf{y} = [y_{1,} \, y_{2,} \dots y_n]$) and written in terms of the independent random variables ($\mathbf{v} = [v_{1,} \, v_{2,} \dots v_n]$, $\mathbf{w} = [w_{1,} \, w_{2,} \dots w_n]$) for the i^{th} pair using the expressions [32]:

$$x_i = v_i^{d/k_x} w_i^{1/k_x} c_x$$

Equation 7

$$y_i = (1 - v_i)^{d/k_y} w_i^{1/k_y} c_y$$

Equation 8

where k, c and d are the BW distribution parameters defined previously, $\mathbf{v}$ is a random variable distributed uniformly in the interval [0,1] and $\mathbf{w}$ has an exponential and gamma mixture pdf given by [32]:

$$f(w) = (1 - d + wd) \, exp(-w) \, , \quad w > 0$$

Equation 9

Using the method of Johnson *et al*. [33], the following procedure was used to generate random samples from the BW distribution. First, five random variables ($\mathbf{s}_{1,}\mathbf{s}_2, \mathbf{s}_3, \mathbf{s}_4, \mathbf{s}_5$) were generated in the interval [0,1] along with the assignments $\mathbf{v} = \mathbf{s}_1$ and:

$$\mathbf{w} = \begin{cases} -\ln(\mathbf{s}_2) - \ln(\mathbf{s}_3), & \text{if } \mathbf{s}_5 \leq d \\ -\ln(\mathbf{s}_4)\,, & \text{if } \mathbf{s}_5 > d \end{cases}$$

Equation 10

After defining the variables ($\mathbf{v}, \mathbf{w}$), artificial wind speed samples ($\mathbf{x}, \mathbf{y}$) were generated with the desired distribution parameters using Equation 7 and Equation 8. Artificial data sets representing 11 years of hourly wind speed entries were generated for each of the 22 site pairs considered in this study using distribution parameters extracted from BW fits to the observed long-term data records. These were used for comparing the performance of the BW approach using artificial versus real wind data. Since the artificial data was generated using distribution parameters extracted from observations at each of the monitoring sites, they represent idealised BW versions of the observed data.

2.4 Baseline MCP approaches

To assess the utility of the BW approach, its success was compared with two widely used linear MCP techniques, linear regression (LR) and the variance ratio method (VR). While more sophisticated MCP approaches exist, linear methods are widely used both in the wind industry [10] and as a baseline for testing new approaches [15, 22, 36]. Hence, as a minimum requirement, the performance of the BW approach should first be tested against these techniques.

In line with previous studies [4, 19], the LR and VR techniques were applied to wind data binned in 30^0 angular sectors with respect to the reference site wind direction, resulting in 12 separate regressions for each reference/target site pair. For training periods with less than 20 entries in a particular angular bin, the regression parameters for the bin were obtained using data from the full range of angles.

2.4.1 Linear regression

For LR, the target and reference site wind speeds are related by:

$$u_t = \alpha + \beta u_r + \varepsilon$$

Equation 11

where α and β are regression coefficients obtained using a least squares fit and ε represents the residual errors.

Previous work [9] demonstrated that the success of the LR technique can be significantly improved by accounting for the residual errors. Hence, here ε is modelled using random samples from a zero-mean Gaussian distribution of the form:

$$\varepsilon \sim \mathcal{N}(0, \sigma_{res}^2)$$

Equation 12

where σ_{res} is the standard deviation of the residuals estimated during the short-term training period, given by [37]:

$$\sigma_{res} = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}(u_{t,i} - \hat{u}_{t,i})^2}$$

Equation 13

and $u_{t,i}$ and $\hat{u}_{t,i}$ are the i^{th} observed and predicted target site wind speeds respectively and n is the total number of observations.

2.4.2 Variance ratio method

The variance ratio method is an approach derived from linear regression that attempts to account for the fact that, where no account is taken of the ε term, the variance of the target site wind speeds is underestimated by a factor $1\ /r$, where r is the linear correlation coefficient. The method is discussed in detail elsewhere [15] and so here we simply present the descriptive equation. The predicted target site wind speeds are given by:

$$\hat{u}_t = \left[\bar{u}_t - \left[\frac{\sigma_t}{\sigma_r}\right]\bar{u}_r\right] + \left[\frac{\sigma_t}{\sigma_r}\right]u_r$$

Equation 14

where $\bar{u}$ represents the mean wind speed and σ represents the standard deviation of wind speeds about the sample mean as estimated from the short-term measurement period. The subscripts r and t refer to the reference and target sites respectively.

The baseline MCP approaches described above were applied previously to the meteorological monitoring sites used in this study for a fixed short-term measurement period of three months [9]. They are included here to serve as a comparison for assessing the success of the BW approaches using multiple measurement periods.

2.5 Meteorological Measurements

The MCP approaches were implemented using long-term wind data from monitoring sites across the UK obtained from the UK Met Office anemometer network [38]. For all sites, the data consisted of hourly averaged wind speed and direction with a resolution of 10^o and 0.51 ms^{-1} (0.51 ms^{-1} = 1 knot), and covered the same 11 year period of August 2001 – July 2012. The MCP approaches were applied to 22 target sites designated as *urban*, *sub-urban*, *rural* or *coastal* using satellite images. A range of terrains were used to calculate average statistics that can be generalised to a range of site types. In addition to the target sites, 15 nearby meteorological stations were selected as reference sites for the implementation of the MCP algorithms. Wherever possible, reference sites were located in open rural terrain, or in coastal areas when paired with coastal target sites. Standard Met Office observational practice requires siting anemometers at 10 m above ground level. Sites where the anemometer height is known to differ from this are noted in Table 1. The approximate locations of the monitoring sites are shown in Figure 1 and further details are in Table 1. The wind speed frequency distributions for all sites were deemed to be adequately described by univariate Weibull distributions. The average and maximum differences in estimated wind power density calculated from observed data and the fitted Weibull distributions was 2.2% and 5.7% respectively.

To obtain robust error statistics, multiple test periods were used by implementing a sliding window approach [9] across the entire 11 year data record as follows: (I) A 12 month training window was shifted in steps of one month across the entire data record using a total of 120 steps. At each step, data not covered by the window had a combined length of 10 years and was designated as the test data such that the training and test data did not overlap. (II) Within the training window, the training length was varied between 1 and 12 months representing a range of short-term onsite measurement periods. For each training period the MCP approaches were applied to predict the 10 year wind resource at the target sites over the test period. (III) The predictions were repeated for each window position resulting in 120 predictions for each training data length. These predictions were then compared with the observed target site wind data during the test periods in order to calculate error statistics.

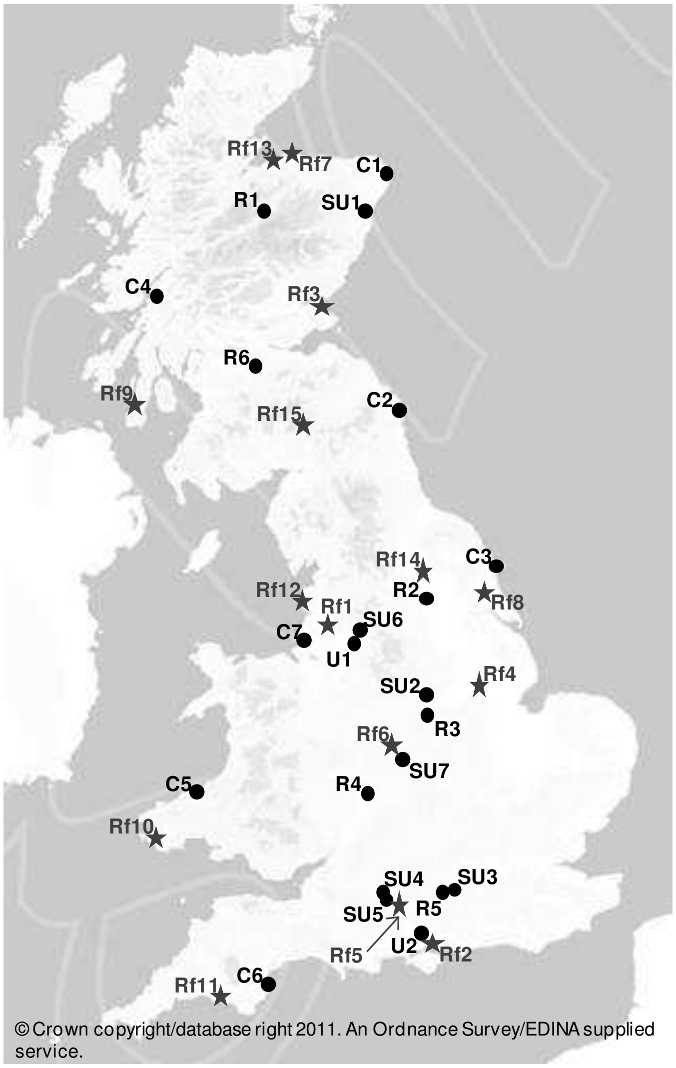

Figure 1: Approximate locations of the UK monitoring sites used in this study. Target sites (black circles) are designated as Urban, Sub-Urban, Rural or Coastal. Reference sites (grey stars) are designated as Rf.

Reference sites			Target sites					
Site	OS grid	Elev (m)	Site	OS grid	Elev (m)	$\bar{u}_{tar}/\bar{u}_{ref}$	d (km)	r
Rf1	SD6614	440	U1*	SJ8396	33	0.49	25	0.79
Rf2	SU5501	9	U2**	SU4210	26	0.72	16	0.87
Rf3	NO4620	10	SU1	NJ8712	65	0.94	101	0.55
Rf4	TF0049	63	SU2	SK5045	117	0.67	49	0.82
Rf5	SU3039	90	SU3	SU8554	65	0.92	58	0.85
Rf5	SU3039	90	SU4	SU1344	132	0.90	17	0.88
Rf5	SU3039	90	SU5	SU1740	126	1.13	13	0.92
Rf1	SD6614	440	SU6	SD8812	110	0.35	22	0.73
Rf6	SP2186	96	SU7	SP3180	119	0.87	12	0.81
Rf7	NJ2169	7	C1	NK1345	15	1.06	96	0.51
Rf3	NO4620	10	C2	NU2514	23	1.06	133	0.66
Rf8	TA0243	7	C3	TA1967	15	1.20	30	0.68
Rf9	NR6622	10	C4	NM8834	3	0.64	113	0.70
Rf10	SM8905	44	C5	SN2452	133	1.27	59	0.79
Rf11	SX4952	50	C6	SX9456	58	1.17	46	0.67
Rf12	SD3131	10	C7	SD3000	9	1.10	31	0.88
Rf13	NJ0662	5	R1	NH8914	228	0.59	51	0.53
Rf14	SE4961	14	R2	SE5238	8	1.13	24	0.88
Rf4	TF0049	63	R3	SK5026	43	0.67	55	0.79
Rf6	SP2186	96	R4	SO9749	35	1.03	44	0.85
Rf5	SU3039	90	R5	SU7349	118	1.14	45	0.86
Rf15	NT2302	236	R6	NS8264	277	1.68	74	0.73

Table 1: Summary of the UK monitoring sites used in this study. Reference sites are designated as Rf, target sites are designated as Urban, Sub-Urban, Rural or Coastal. The ordnance survey grid references (OS grid), elevations above sea level (Elev), ratio of wind speeds at the target and reference sites (u_{tar}/u_{ref}), separation distances (d) and linear correlation coefficients (r) are also shown. Anemometer heights known to differ from 10 m above ground level: *h = 20.6 m, **h = 22.5 m.

2.6 Error metrics

To assess the accuracy of the MCP approaches, the error metrics of mean absolute error (MAE), mean bias error (MBE) and absolute percentage error (%Error) were used to compare predicted statistical parameters with those observed at the target sites. For an arbitrary parameter of interest z, and a collection of N sites, these metrics are defined as:

$$MAE = \sum_j \left| z_{obs,j} - z_{pred,j} \right| / N$$

Equation 15

$$MBE = \sum_j (z_{pred,j} - z_{obs,j}) / N$$

Equation 16

$$\%Error = 100 \sum_j \frac{\left|z_{obs,j} - z_{pred,j}\right|}{z_{obs,j}} / N$$

Equation 17

where the subscripts refer to the observed and predicted values of the parameter at the j^{th} site.

These metrics were applied to the predicted mean wind speed in addition to three further parameters of particular importance in characterising the wind resource, as defined below.

The mean Betz power density in the wind given by [39]:

$$\bar{p}_d = (16/27) 0.5 \rho \overline{u^3}$$

Equation 18

where (16/27) is the Betz limit, ρ = 1.225 kgm^{-3} is the air density and $\overline{u^3}$ is the mean of the cubed wind speeds.

The univariate Weibull shape factor k, where the univariate Weibull pdf is defined by:

$$f(u) = k \frac{u^{k-1}}{c^k} \exp\left[-\left(\frac{u}{c}\right)^k\right]$$

Equation 19

where u and c represent the wind speed and univariate Weibull scale factor respectively.

The standard deviation of wind speeds defined as:

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (u_i - \bar{u})^2}$$

Equation 20

where n is the total number of observations, u_i is the i^{th} wind speed observation and $\bar{u}$ is the mean wind speed.

Since the sliding window approach results in 120 predictions for each target site and training length, the error metrics for each training length were calculated as the average across all window positions and target sites.

3 Results and Discussion

3.1 Convergence efficiency of the bivariate Weibull parameters using artificial verses observed wind data

To investigate the efficiency with which the fitted BW parameters converged with respect to the sample length when using observed versus artificial wind data, four reference/target site pairs (one from each terrain type) were chosen, along with their associated artificially generated wind data, for detailed investigation. Since similar trends were observed for each site pair, the results of a single site pair Rf4/R3 located in open, flat terrain, are presented here.

The five parameters associated with the fitted BW pdf for the two sites were first determined using MML as described in Section 2.2 using the full 11 year data record. The extracted parameters were k_r = 2.04, c_r = 6.01, k_t = 1.96, c_t = 3.98 and d = 0.48. These parameters were used as inputs to create samples of artificial data from the specified BW distribution as described in Section 2.3. To compare the fitting efficiency for the artificial and observed wind data, MML was used to extract the five BW parameters using progressively increasing sample sizes of observed or artificial data. A step size of 24 data points was used, representing one day of hourly averaged wind speeds.

The artificial data was sampled randomly from the specified distribution, hence for each sample of a particular size, the fitted BW parameters will vary until the sample size is large enough for the parameters to converge. For observed wind data, a real wind measurement campaign was replicated by choosing samples of consecutive wind data thus introducing additional complexity due to seasonal variations. The variability in the extracted parameters was investigated using a Monte Carlo approach, whereby for each sample size the fitting procedure was repeated using 200 trials. For the artificial data, the 200 trials were generated randomly from the required distribution. For the observed wind data, the 200 trials were consecutive observations with random starting points throughout the 11 year data record, thus replicating measurement campaigns initiated at different times. The Monte Carlo approach was used to extract the predicted mean and standard deviation for each distribution parameter and sample size.

Figure 2 shows the results of this procedure for the BW target site parameters of k_t, c_t and d. The standard deviation across the 200 trials for each sample size is related to the precision of the fits. A large standard deviation indicates that the fitted parameter is dependent on the exact locations of the samples; hence increased fitting efficiency is associated with a faster reduction in standard deviation with sample size. For all three parameters, Figure 2 shows that the fitting efficiency is considerably greater when using artificial wind data compared to observed wind data. In the case of the observed data, seasonal variations in the wind speeds and directions are likely to impact on the form of the BW distribution leading to the large variations across different trials. Hence significantly longer data samples may be required to accurately extract the distribution parameters when using observed wind data compared to artificial data.

The mean values from the Monte Carlo averaging are also of interest since they represent the accuracy of the fits. Figure 2 shows that for the artificial samples, the mean parameter values reach the true distribution values with a sample size of just a few days. For the observed wind data however, there is a large over estimation in the mean value of k_t when using small samples. An increased value of k_t indicates a narrower wind speed distribution, likely due to 'clumping' of wind speeds in a relatively narrow range related to seasonal weather patterns. Similarly, the observed wind data results in an over estimation of the mean fitted value of d when using small samples indicative of poor correlation between the two sites. In contrast, the mean value of c_t, related to the target site mean wind speed, remains close to the true distribution value even for small samples of observed data. This is not surprising since c_t is directly related to the mean wind speed which can be accurately determined from many snapshots of concurrent wind speed observations taken across multiple years (the Monte Carlo approach). For observed sample lengths of around 40 days, the mean fitted parameters are relatively close to the true distribution values. However, the large standard deviation indicates that the extracted parameters lack precision, with large variations possible depending on the measurement season. Similar trends were observed in the fitted parameters of k_r and c_r.

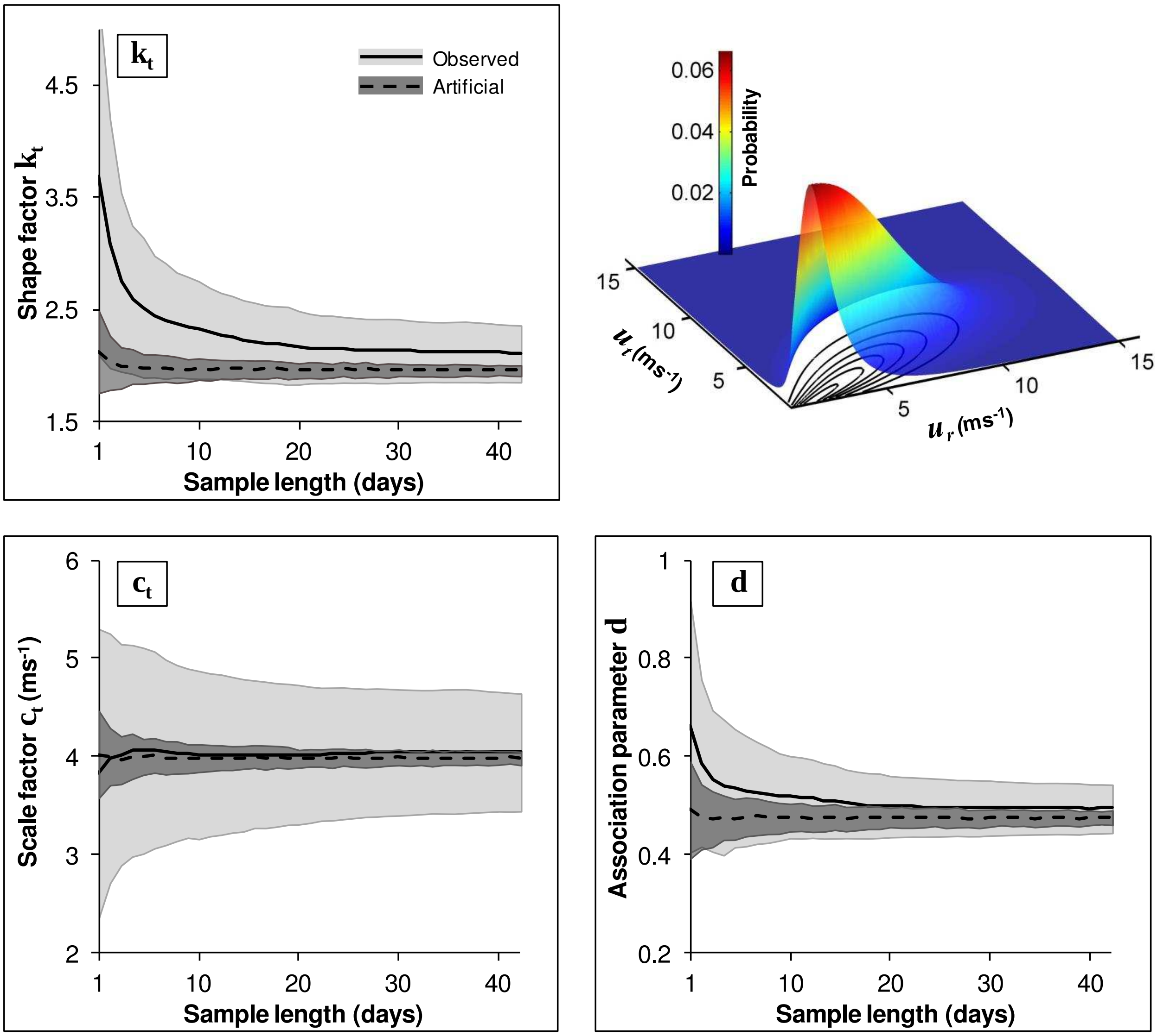

Figure 2: Variation in the fitted BW parameters of k_t, c_t and d using artificial (dotted line, dark shading) and consecutively sampled observed (solid line, light shading) wind data from a single reference/target site pair. The lines indicate a mean value averaged across 200 trials, the shading represents +/- one standard deviation from the mean. The inset shows the full BW probability surface.

To investigate if these results were related to seasonal effects, the Monte Carlo procedure was repeated using random, rather than consecutively sampled wind speed observations. Using this approach, concurrent pairs of wind speed observations at the reference and target sites were drawn at random throughout the 11 year data record. This random sampling procedure removes the effect of seasonal weather patterns and mirrors more closely the random sampling of artificial wind data.

Figure 3 shows the results of this procedure for the BW parameters of k_t and c_t. The mean and standard deviation of k_t and c_t follow almost identical trends using the artificial and observed wind data with rapid convergence of both the Monte Carlo mean value and the standard deviation. Similar trends were observed for the remaining three BW parameters, indicating that it is the restriction of consecutive sampling, and most likely the associated seasonal weather patterns, which result in the loss of fitting efficiency when using observed rather than artificial wind data.

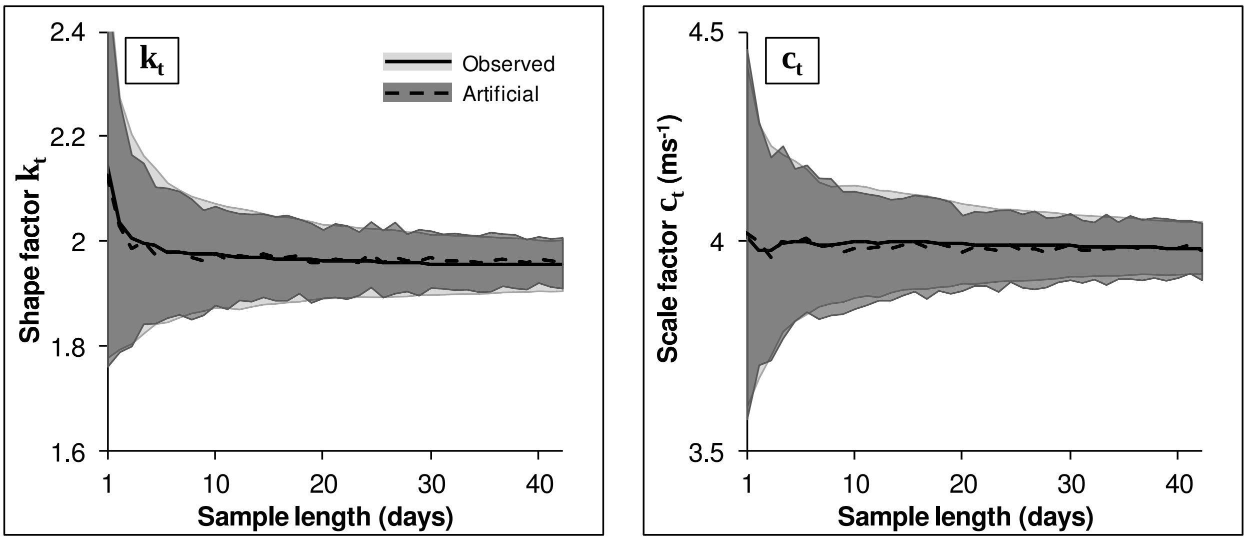

Figure 3: Variation in the fitted BW parameters of k_t and c_t, using artificial (dotted line, dark shading) and randomly sampled observed (solid line, light shading) wind data from a single reference/target site pair. The lines indicate a mean value averaged across 200 trials, the shading represents +/- one standard deviation from the mean.

These results highlight some important factors related to the implementation of the BW approach to observed wind data. Firstly, the convergence time is likely to be significantly longer than in the case of artificial data as highlighted by Figure 2. This could result in relatively large errors in the estimated parameters when using short data periods. Secondly, assuming these results can be generalised, the values of the parameters k and d may be overestimated on average, when using short data periods. Note that when conducting a measurement campaign, consecutive sampling of wind speeds is the most likely approach due to the time and expense of installing a meteorological mast. However, with the improvement in portable measurement devices and where multiple sites are to be investigated, a non-consecutive sampling approach which captures seasonal variability [40] may be a viable alternative. A final observation is noteworthy regarding the two methods outlined in Section 2.2 for extracting the distribution parameters. For the four sites considered, the extracted values of k_t, c_t, k_r and c_r were almost identical (within ~1.5%) using both the BW and BW2 approaches. However, BW2 resulted in consistently lower estimates of d (by ~10% - 40%) compared to BW. This suggests that estimates of d based on the covariance are associated with a higher predicted correlation between the reference and target site wind speeds. Interestingly, when applied to the artificial wind data this difference almost vanished indicating that the effect may be due to deviations of the real wind data from idealised bivariate Weibull distributions.

3.2 Comparison between the bivariate Weibull and baseline measure-correlate-predict approaches

To compare the success of BW and BW2 with the existing MCP methods of LR and VR, each approach was applied to observed and artificially generated wind data for the 22 site pairs to predict the 10 year wind resource. The error metrics were calculated as described in Section 2.6. Figure 5 shows the %Error metrics for $\bar{u}$ and $\bar{p}_d$ using the artificially generated data for all 22 site pairs and training lengths of 1-12 months. The BW approaches clearly perform better than the regression approaches for all training lengths in line with previous work [26]. Equivalent trends were also observed for σ and k.

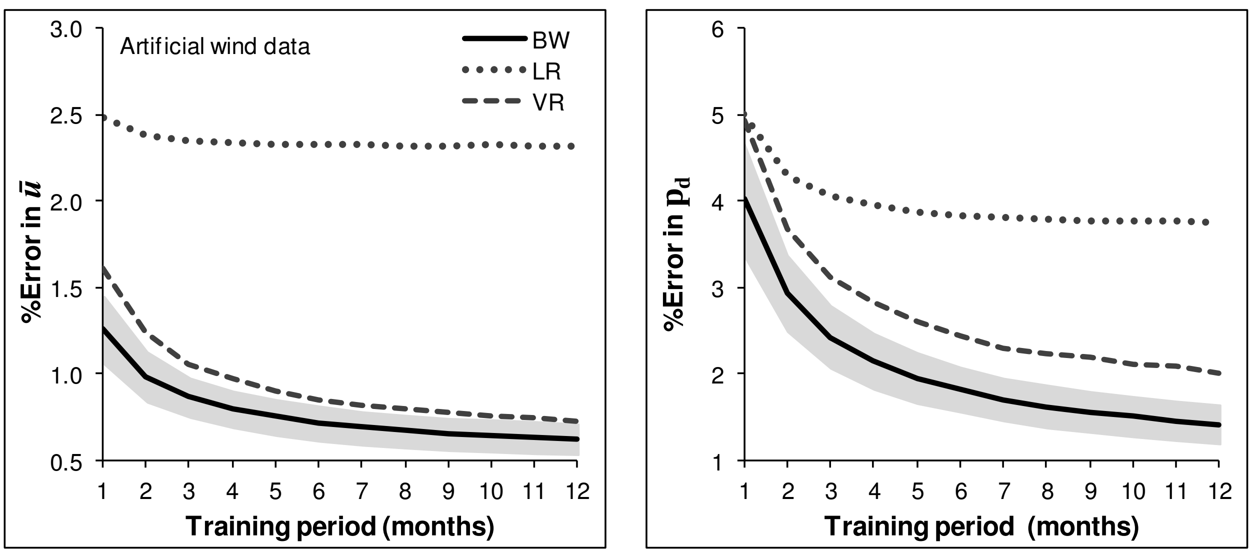

Figure 4: %Error metric as a function of training period for the wind resource parameters of $\bar{u}$ and $\bar{p}_d$ using artificially generated wind data. Lines show the mean value averaged across 22 site pairs. The shaded region represents +/- one standard deviation for the BW approach as calculated across the 120 test periods.

Figure 5 shows the equivalent %Error metrics for $\bar{u}$, $\bar{p}_d$, σ and k using observed wind data for all 22 site pairs. Note that applying the sliding window approach to observed wind data ensures that the average error metrics are independent of the season or year in which the short-term measurements were taken, while the standard deviation of the percentage errors (shading Figure 5) indicates the magnitude of the intra- and inter-annual variations.

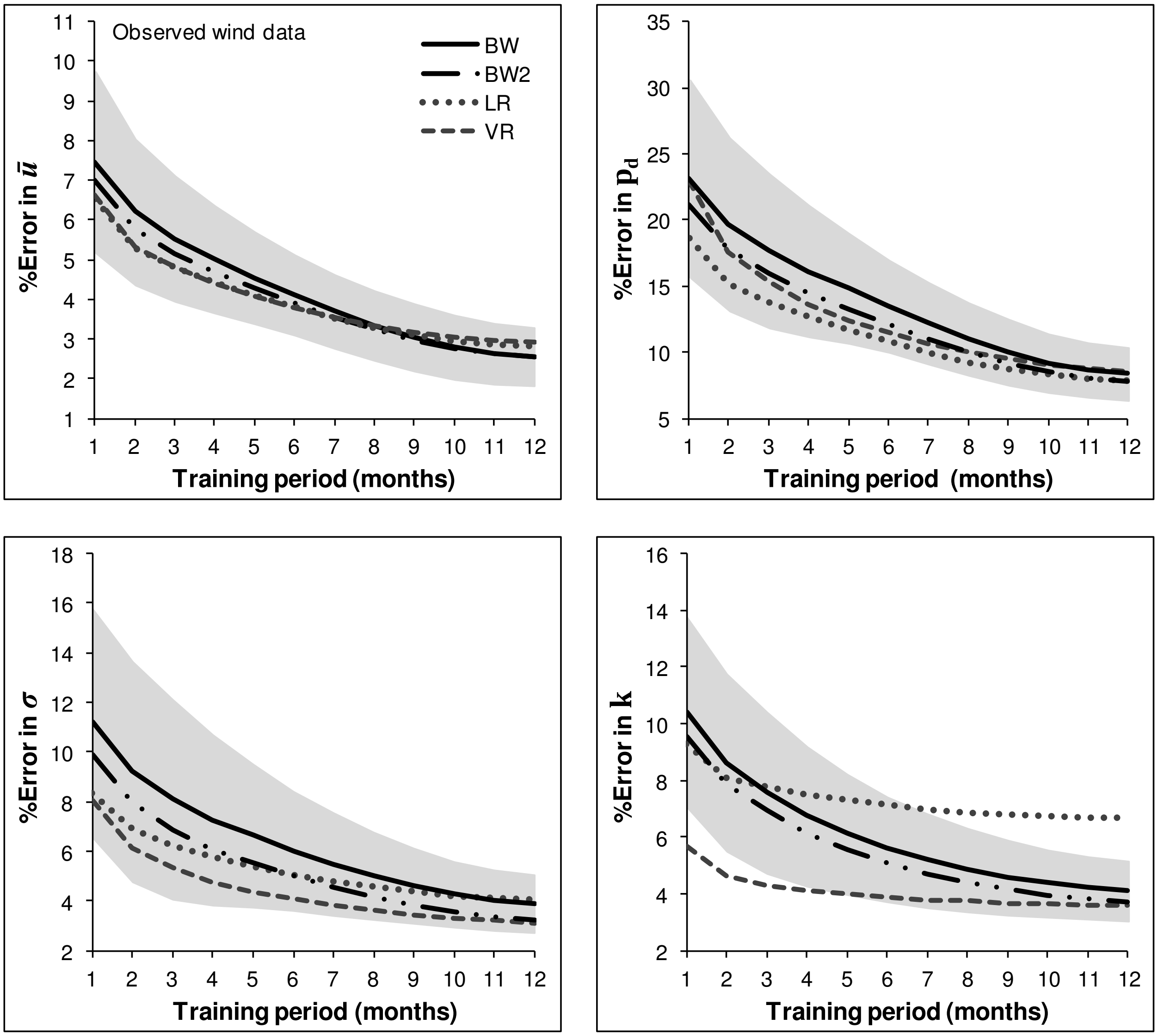

Figure 5: %Error metrics as a function of training period for the wind resource parameters of $\bar{u}$, $\bar{p}_d$, k and σ using observed wind data. Lines show the mean value averaged across 22 site pairs. The shaded region represents +/- one standard deviation in for the BW approach as calculated across the 120 different starting months.

Clearly, the error metrics behave quite differently when the MCP approaches are applied to observed wind data. Generally, for short training periods, one or more of the regression approaches results in lower %Error than either BW or BW2. Using a full 12 month training period, the BW2 approach performs as well as the best regression approach in terms of the %Error in $\bar{p}_d$, σ and k and slightly better than the best regression method in terms of $\bar{u}$. It is of interest that for training periods less than 8 months, the relatively simple LR method consistently performs as well or better than the other approaches in predicting $\bar{u}$ and $\bar{p}_d$, while for longer training periods all the MCP approaches tend to converge. For the parameters σ and k which describe the form of the wind speed distribution, the VR approach performs better than the other approaches at short training periods converging with BW2 at longer training periods. For all four parameters, the %Error metric is notably lower for the BW2 approach compared to BW. Since, as discussed previously, the BW2 approach only differs in the estimation of the d parameter, this suggests that the reference/target site covariance provides a more suitable indicator for this parameter compared to MML. These results indicate that when using real wind data, the MCP approaches of BW and BW2 may not consistently produce more accurate predictions compared to regression approaches despite their stronger theoretical basis. This is in contrast to results obtained when using artificial wind data (Figure 4Figure 5) and could be due to deviations of the observed wind data from idealised BW distributions. It should be noted that the LR approach implemented here includes a Gaussian model of the scatter term ε about the predicted wind speeds, which has been shown to increase the accuracy of predictions [9]. Without this term, the LR method would be considerably less competitive with the BW and BW2 approaches.

Figure 6 shows the MBE metrics, which describe the tendency to overestimate or underestimate a parameter, based on the observed data. For a full 12 month training period, the BW approach results in the lowest bias in $\bar{u}$. However, in terms of $\bar{p}_d$, BW2 performs best closely followed by LR, BW and VR. Note that while BW2 and LR slightly overestimate $\bar{u}$, these approaches also underestimate the width of the wind speed distribution, as indicated by the MBE in σ and k, and these two effects may offset each other resulting in a low net negative bias in $\bar{p}_d$. VR exhibits a very small bias in σ and k and hence the positive bias in $\bar{p}_d$ is a more direct reflection of the positive bias in $\bar{u}$ using this approach. As suggested in Section 3.1, both BW and BW2 tend to overestimate k, especially for short training periods. The behaviour of the MBE across these parameters reveals that the errors in $\bar{p}_d$ are due to a relatively complicated combination of factors, including possible cancellation of errors. Despite these complications, the MBE is generally small across all MCP approaches for training periods of 12 months, with greater differences at shorter training periods.

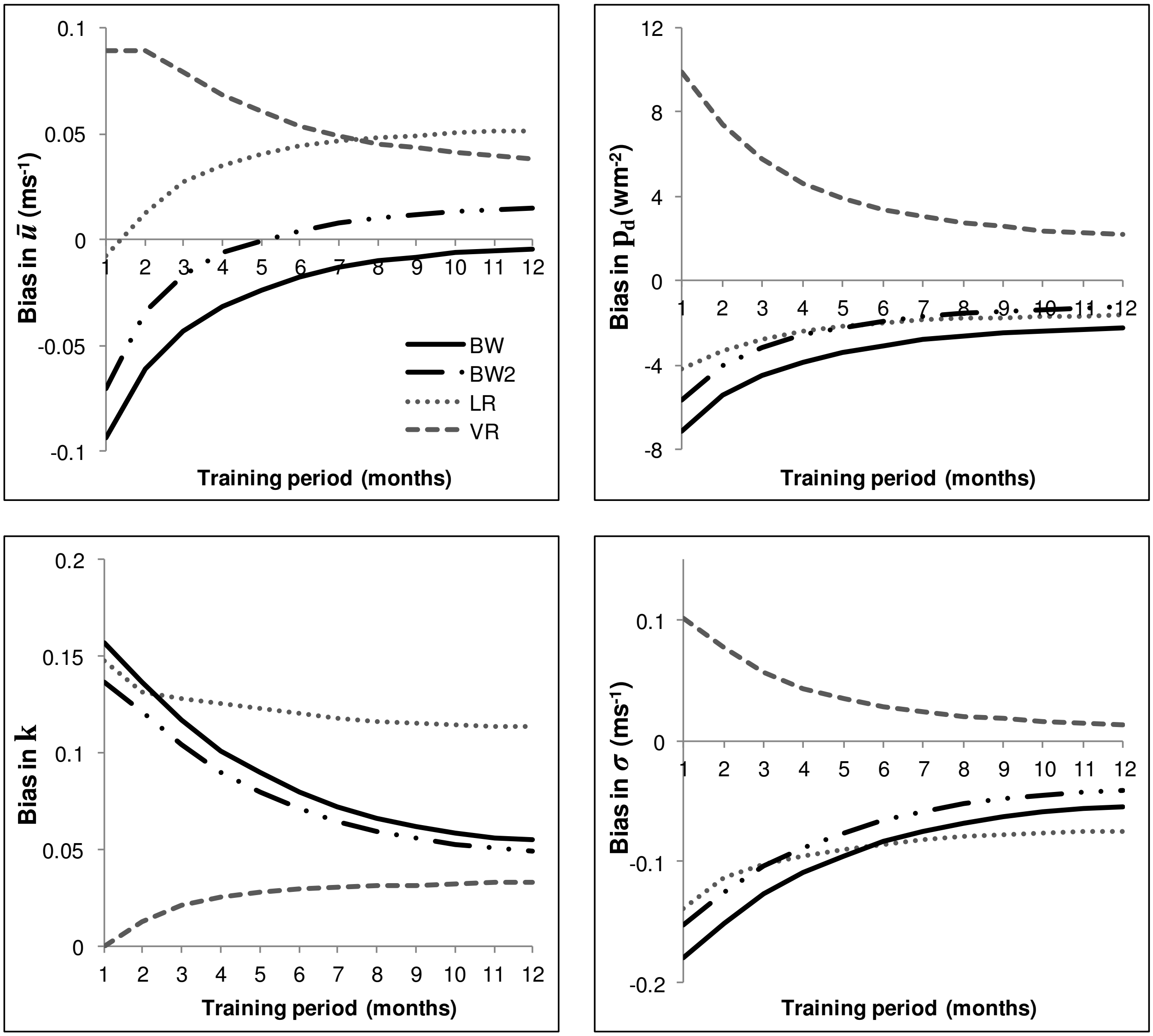

Figure 6: MBE metrics as a function of training period for the wind resource parameters of $\bar{u}$, $\bar{p}_d$, σ and k using observed wind data. Lines show the mean value averaged across 22 site pairs.

Table 2 summarises the metrics of %Error, MAE and MBE for training periods of 3 and 12 months using the observed data. At 12 months, the performance of all four MCP approaches is very similar with BW2 performing very slightly better on average than the remaining approaches. For a shorter training period of 3 months there are clearer differences with the regression techniques of LR and VR generally resulting in smaller errors than the BW approaches. This is likely because the BW approaches require a greater number of fitting parameters and thus requiring longer training periods. Overall, errors are approximately halved by increasing the training period from 3 to 12 months.

3 M	Method	$\bar{u}$	$\bar{p}_d$	σ	k
%Error	BW	5.5	18	8.1	7.6
	BW2	5.5	17	7.7	7.3
	LR	4.8	14	6.2	7.8
	VR	4.8	15	5.3	4.3
		$\bar{u}$ (ms^{-1})	$\bar{p}_d$ (wm^{-2})	σ (ms^{-1})	k
MAE	BW	0.25	15	0.19	0.15
	BW2	0.23	13	0.17	0.14
	LR	0.21	11	0.16	0.14
	VR	0.21	11	0.13	<0.1
MBE	BW	<0.1	-4.5	-0.13	0.12
	BW2	<0.1	-3.7	-0.11	0.11
	LR	<0.1	-2.8	-0.10	0.13
	VR	<0.1	5.7	<0.1	<0.1

12 M	Method	$\bar{u}$	$\bar{p}_d$	σ	k
%Error	BW	2.6	8.4	3.9	4.1
	BW2	2.6	7.8	3.2	3.7
	LR	2.8	7.9	4.0	6.7
	VR	2.9	8.5	3.1	3.6
		$\bar{u}$ (ms^{-1})	$\bar{p}_d$ (wm^{-2})	σ (ms^{-1})	k
MAE	BW	0.11	6.1	<0.1	<0.1
	BW2	0.11	5.7	<0.1	<0.1
	LR	0.12	5.8	0.10	0.12
	VR	0.12	6.1	<0.1	<0.1
MBE	BW	<0.1	-2.3	<0.1	<0.1
	BW2	<0.1	-1.2	<0.1	<0.1
	LR	<0.1	-1.6	<0.1	0.11
	VR	<0.1	2.2	<0.1	<0.1

Table 2: Error metrics for the wind resource parameters of $\bar{u}$, $\bar{p}_d$, σ and k using training periods of 3 months (left) and 12 months (right) averaged across 22 target sites and 120 starting months.

4 Conclusion

An MCP approach based on modelling of the underlying BW probability distribution of reference and target site wind speeds has been implemented at 22 pairs of UK sites using multiple test periods over an 11 year data record. Building on previous work that applied the technique to artificial wind data, we have carried out a detailed comparison between the performance of the approach using observed and artificially generated data. The results indicate that due to seasonal effects, the data period required for convergence of the extracted BW parameters is likely to be significantly longer when using observed compared to artificially generated wind data and that the Weibull shape factor k and association parameter d may be overestimated on average when using short measurement periods. In addition, estimating d from the covariance of the target/reference site wind speeds was found to result in improved performance across all error metrics compared to estimations based on MML.

The performance of the BW approach was compared quantitatively with two established regression MCP methods using observed wind data at the 22 site pairs as well as artificial wind data generated from ideal BW distributions modelled on the same sites. In line with a previous study [26], the BW approach outperformed the regression approaches for all measurement periods when applied to idealised wind data. However, when applied to observed wind data, the regression approaches generally performed better than the BW approaches for short training periods, while all approaches performed similarly for training periods of 12 months. The results suggest that the improved performance of the

BW approach when using artificial wind data may not always be transferable to real wind observations since they may not precisely follow idealised BW distributions.

Future work should investigate whether certain sites may respond better to the BW approaches than others and to what extent this may be predicted from short-term observations.

5 Acknowledgements

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6 References

[1] Global Wind Energy Council. Global wind statistics 2012, Available from: www.gwec.net, 2013.
[2] Department of Energy and Climate Change (DECC). UK Renewable Energy Roadmap, URN 11D/698, 2011.
[3] AWS Scientific Inc. & National Renewable Energy Laboratory (U.S.). Wind Resource Assessment Handbook - fundamentals for conducting a successful monitoring program, 1997.
[4] A. Derrick, Development of the measure-correlate-predict strategy for site assessment. Proceedings of the BWEA, 1992, 259-265.
[5] M.C. Bower. The use of NCEP/NCAR Reanalysis data in MCP, European Wind Energy Conference and Exhibition, Athens, Greece, 2006.
[6] Department of Energy and Climate Change (DECC). Microgeneration installation standard: MIS 3003, Issue 3.1a, 2008.
[7] UK Meteorological Office. Small-scale wind energy - Technical Report, 2008.
[8] S.M. Weekes, A.S. Tomlin, Evaluation of a semi-empirical model for predicting the wind energy resource relevant to small-scale wind turbines. Renew. Energy, 2013;50:280-288.
[9] S.M. Weekes, A.S. Tomlin, Data efficient measure-correlate-predict approaches to wind resource assessment for small-scale wind energy. Renew. Energy, 2014;63:162-171.
[10] J.A. Carta, S. Velázquez, P. Cabrera, A review of measure-correlate-predict (MCP) methods used to estimate long-term wind characteristics at a target site. Renewable and Sustainable Energy Reviews, 2013;27:362-400.
[11] P. Putnam, Power from the wind, D. Van Nostrand Company, New York, 1948.
[12] C.G. Justus, K. Mani, A.S. Mikhail, Interannual and month-to-month variations of wind speed. J. Appl. Meteorol., 1979;18:913-920.
[13] T.A. Nygaard, Estimating Expected Energy Capture at Potential Wind Turbine Sites in Norway. Journal of Wind Engineering and Industrial Aerodynamics, 1992;39:385-393.
[14] C. Achberger, M. Ekstrom, L. Barring, Estimation of local near-surface wind conditions - a comparison of WASP and regression based techniques. Meteorological Applications, 2002;9:211-221.
[15] A.L. Rogers, J.W. Rogers, J.F. Manwell, Comparison of the performance of four measure-correlate-predict algorithms. Journal of Wind Engineering and Industrial Aerodynamics, 2005;93:243-264.
[16] M. Nielsen, L. Landberg, N.G. Mortensen, R.J. Barthelmie, A. Joensen, Application of measure-correlate-predict approach for wind resource measurement. Proceedings of the EWEA, 2001.

[17] V. Riedel, M. Strack, H. Waldl. Robust approximation of functional relationships between meteorological data: Alternative measure-correlate-predict algorithms, European Wind Energy Conference. Wind Energy for the New Millennium, 2001, pp. 806-809.
[18] Mortimer A., A new correlation/prediction method for potential wind farm sites. Proceedings of the BWEA, 1994, 1994.
[19] J.C. Woods, S.J. Watson, A new matrix method of predicting long-term wind roses with MCP. Journal of Wind Engineering and Industrial Aerodynamics, 1997;66:85-94.
[20] D.A. Bechrakis, J.P. Deane, E.J. McKeogh, Wind resource assessment of an area using short term data correlated to a long term data set. Sol. Energy, 2004;76:725-732.
[21] M. Monfared, H. Rastegar, H.M. Kojabadi, A new strategy for wind speed forecasting using artificial intelligent methods. Renew. Energy, 2009;34:845-848.
[22] S. Velazquez, J.A. Carta, J.M. Matias, Comparison between ANNs and linear MCP algorithms in the long-term estimation of the cost per kW h produced by a wind turbine at a candidate site: A case study in the Canary Islands. Applied Energy, 2011;88:3869-3881.
[23] S. Velázquez, J.A. Carta, J.M. Matías, Influence of the input layer signals of ANNs on wind power estimation for a target site: A case study. Renewable and Sustainable Energy Reviews, 2011;15:1556-1566.
[24] B. Saavedra-Moreno, S. Salcedo-Sanz, L. Carro-Calvo, J. Gascón-Moreno, S. Jiménez-Fernández, L. Prieto, Very fast training neural-computation techniques for real measure-correlate-predict wind operations in wind farms. Journal of Wind Engineering and Industrial Aerodynamics, 2013;116:49-60.
[25] J.A. Carta, S. Velazquez, A new probabilistic method to estimate the long-term wind speed characteristics at a potential wind energy conversion site. Energy, 2011;36:2671-2685.
[26] A. Romo Perea, J. Amezcua, O. Probst, Validation of three new measure-correlate-predict models for the long-term prospection of the wind resource. Journal of Renewable and Sustainable Energy, 2011;3.
[27] EMD International WindPRO. Available from: www.emd.dk. [accessed April 2013].
[28] Schlez W, GH WindFarmer theory manual - Garrad Hassan and Partners Ltd. 2009.
[29] ReSoft. WindFarm: wind farm anlysis, design and optimisation. Available from: www.resoft.co.uk. [accessed April 2013].
[30] P. Clive, Non-linearity in MCP with Weibull Distributed Wind Speeds. Wind Engineering, 2008;32:319-323.
[31] C.G. Justus, W.R. Hargraves, A. Yalcin, Nationwide assessment of potential output from wind-powered generators. J. Appl. Meteorol., 1976;15:673-678.
[32] J.C. Lu, G.K. Bhattacharyya, Some new constructions of bivariate Weibull models Ann. Inst. Stat. Math., 1990;42:543-559.
[33] R.A. Johnson, J.W. Evans, D.W. Green, Some bivariate distributions for modeling the strength properties of lumber. United States Department of Agricutlure, Forest Service, Research Paper FOL-RP-575, 1999.
[34] MATLAB 7.8, The MathWorks Inc., Natick, MA,. 2000.
[35] L. Lee, Multivariate distributions having Weibull properties. Journal of Multivariate Analysis, 1979;9:267-277.
[36] J.H. Bass, M. Rebbeck, L. Landberg, M. Cabre, A. Hunter. An improved measure-correlate-predict algorithm for the prediction of the long term wind climate in regions of complex environment, Joule Project JOR3-CT98-0295, 2000.
[37] S.L.R. Ellison, V.J. Barwick, T.J.D. Farrant, Practical statistics for the analytical scientist, a bench guide, RSC Publishing, 2009.
[38] UK Meteorological Office. Met Office Integrated Data Archive System (MIDAS) Land and Marine Surface Stations Data (1853 - current). NCAS British Atmospheric Data Centre 2012.

[39] P. Ramirez, J.A. Carta, Influence of the data sampling interval in the estimation of the parameters of the Weibull wind speed probability density distribution: a case study. Energy Conv. Manag., 2005;46:2419-2438.
[40] M.A. Lackner, A.L. Rogers, J.F. Manwell, The round robin site assessment method: A new approach to wind energy site assessment. Renew. Energy, 2008;33:2019-2026.