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APPLICATIONS OF TOPOLOGY OPTIMISATION IN STRUCTURAL ENGINEERING

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ABSTRACT

This study introduces applications of structural topology optimisation to buildings and civil engineering structures. Topology optimisation problems utilize the firmest mathematical basis, to account for improved weight-to-stiffness ratio and perceived aesthetic appeal of specific structural forms, enabling the solid isotropic material with penalization (SIMP) technique. Structural topology optimisation is a technique for finding the optimum number, location and shape of "openings" within a given continua subject to a series of loads and boundary conditions. Aerospace and automotive engineers routinely employ topology optimisation and have reported significant structural performance gains as a result. Recently designers of buildings and structures have also started investigating the use of topology optimisation, for the design of efficient and aesthetically pleasing developments. This paper examines two examples of where topology optimisation may be a useful design tool in civil/structural engineering in order to overcome the frontiers between civil engineers and engineers from other disciplines. The first example presents the optimised structural design of a geometrically complex high-rise structure and the optimal design of its architectural building shape. The second one focuses on the optimisation and design of a perforated steel I-section beam, since such structural members are widely used nowadays in the vast majority of steel buildings and structures while they provide numerous of advances. Conclusions are drawn regarding the potential benefits to the more widespread implementation of topology optimisation within the civil/structural engineering industry.

1 INTRODUCTION

Structural optimisation is concerned with maximizing the utility of a fixed quantity of resources to fulfill a given objective. Three categories of structural optimisation exist; shape, size and topology. Structural topology optimisation is the most general of the three categories yielding information on the number, location, size and shape of "openings" within a continuum. The first solutions to a topology optimisation problem (fig.1) were presented by Michell [1]. Modern topology optimisation techniques can be applied to generalised problems through the use of the Finite Element (FE) method, as a relatively recent innovation. Aerospace, automotive and mechanical engineers have successfully utilised topology optimisation in order to achieve weight savings in structures. Enthusiasm for topology optimisation in the field of civil/structural engineering, where weight savings are seen as less critical due to the one off nature of building structures, is generally accepted as being more muted [2]. However, in the era of sustainable and resilient infrastructures, where the concept of redundancy plays a significant role, we should reconsider optimising every single structure to the best of its efficiency. Indeed the one off nature of every civil-structural engineering project necessitates the use of rigorous optimisation techniques to drive efficiencies on the increasingly complex projects of today.

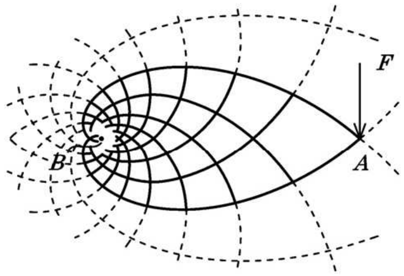

Figure 1 - One of the first proposed solutions to a structural topology optimisation problem

Topology optimisation has found several novel applications in the field of civil engineering, most notably; a novel technique for geotechnical analysis [3] and reinforcement layout optimisation in concrete structures [4]. The main focus of this review study is applications of topology optimisation to the design of large scale structures and structural engineering components.

This paper briefly details the two most popular topology optimisation techniques currently available; presents the theoretical background and practical implementation of the most commonly used Solid Isotropic Material with Penalisation (SIMP) technique; and reveals previous applications of topology optimisation in both structural engineering and architecture. Moreover, the implementation of topology optimisation within the field of structural engineering, and potential opportunities beyond the present frontiers, are examined through various examples. A description of studies conducted by the authors using the topology optimisation technique for: (i) the design of a high-rise structure, and (ii) the development of a novel steel I-section with atypical web openings configurations, is also presented.

2 TOPOLOGY OPTIMISATION IN STRUCTURAL ENGINEERING AND ARCHITECTURE

2.1 Background

During the 20th century architects and engineers have used innovative and novel methods to develop optimum forms of structures and sculptures. Of particular note would be the works from Antonio Gaudi, Félix Candela, Frei Otto, Pier Luigi Nervi, Heinz Isler, Richard Buckminster Fuller and Robert le Ricolais [5,6,7]. All these individuals shared the “key theme” of attempting to create structurally efficient and functional forms that are architecturally pleasing at the same time.

Whilst the techniques employed by these innovators generated efficient and aesthetic forms, they shared a common limitation. All of the techniques employed required that the number of holes within the structure had to be known apriori to the structural form finding exercise, which usually involves the use of a physical analogue model. Topology optimisation is not restricted by this limitation and it can effectively “carve” the optimum structure form from a block of material defined by the designer. In addition, the increased freedom of being able to optimize the number of openings within a structure offers an exciting new chapter in the study of improved structural forms.

Topology optimisation has been used randomly in the structural design process and has no a clear defined role. It has been described as “an intellectual sparring partner” [8] during the early conceptual design phase. In some cases the results of a topology optimisation study have been directly translated into the geometry of the final structure in a process called Computational Morphogenesis [9,10]. The results of many topology optimisation exercises often show a strong resemblance to structures that are found in nature [11,12] and are usually structurally efficient as well as aesthetically pleasing.

2.2 Topology optimisation in structural engineering applications

Significant work on the design of bracing systems for high-rise structures using topology optimisation has been presented [13]. Engineers Skidmore, Owings and Merrill (SOM) have utilised the theoretical work on high-rise bracing design, in order to develop conceptual designs for high-rise buildings that are both aesthetically pleasing and structurally efficient [14].

Topology optimisation was used to derive the optimal number, location and shape of holes in the exterior reinforced concrete walls of an office building near to the Takatsuki JR Station in Japan [15]. The walls were modelled as simple rectangular plates and optimised for vertical and horizontal loading combinations. The result (fig.2a) was found to be both aesthetically pleasing and structurally sound. It should be noted that the architecture of the entire building was totally governed by structural considerations arising from the results of the topology optimisation study.

Topology optimisation has also been used for purely architectural purposes. The architectural aspiration of the Doha Education Centre's roof canopy support was to mimic the form of a Sidra tree [16]. Topology optimisation studies were performed in order to define the geometry of the canopy support structure. It was found that the resulting form has strong resemblances to a tree trunk indeed (fig.2b).

Another support structure, for a doubly curved roof canopy, was designed using topology optimisation and constructed from reinforced concrete as is shown in fig.2c [17]. Computer Numerical Controlled (CNC) milling technology was employed to create the formwork for such a geometrically complex topology form. In general, it is worth noting that advanced manufacturing practices are required in cases where topology optimised designs are to be implemented on a larger scale.

Figure 2 - a) An office building in Japan with topology optimised Walls; b) A topology optimised support structure; c) Topology optimised support structure for a Canopy in Doha

3 TOPOLOGY OPTIMISATION TECHNIQUES

3.1 Theory

The term "topology" is derived from the Greek word "topos" meaning position/place. The application of topology optimisation extends to the number of holes, their location, their shape and the connectivity of the structural domain [8]. Shape and sizing optimisation are more limited than topology optimisation in the respect that the designer must specify the topology of the proposed structure which is then fixed throughout the optimisation process. The general form of the topology optimisation problem is to determine the optimum distribution of material within a designable domain to fulfil a given objective (fig.3).

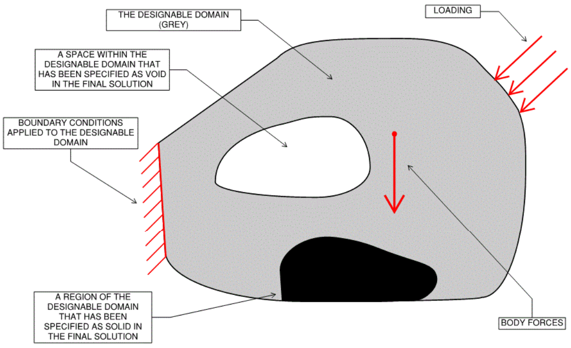

Figure 3 - Basic terms used in topology optimisation

An in depth review of all the topology optimisation techniques suggested to date is beyond the scope of this paper. Currently the two most popular ones are the Solid Isotropic Material with Penalisation (SIMP) technique and Evolutionary Structural Optimisation (ESO) technique. Both of them involve the discretisation of the designable domain into the finite elements, and utilize the FE analysis technique to determine the response of the structure to be optimised. Whilst ESO has received significant research interest [18,19], it is generally criticised for being a heuristic technique that does not guarantee convergence to an optimum solution [20,21]. The SIMP technique is generally accepted in the literature as being the most prevalent tool for the topology optimisation of linear elastic structures [20,21]. Solution methods for nonlinear, dynamic and buckling responses are currently under investigation and are not yet at a stage of maturity to enable their application in routine design. The only commercially available technique for solving nonlinear topology optimisation problems currently is the Equivalent Static Loads Method (ESLM) [22]. The lack of techniques for solving problems involving more complex behaviours is a major barrier to the more widespread implementation of topology optimisation in civil and structural engineering.

3.2 SIMP technique

The basis of the SIMP technique is to determine the optimum structural shape by varying the density of the material within the designable domain [23]. The designable domain is generally discretised while the FE analysis technique is employed to determine the behaviour of the structure to be analysed. Conceptually the use of a discretised design space may be considered thus; “one may consider the design domain as a black and white television screen divided into a lot of small pixels (finite-elements) and by turning the material on and off in each pixel, one can produce a picture of the optimal structure” [24]. Computationally the SIMP technique involves the FE analysis of the design space followed by an optimisation of the density of each finite element within the mesh. The structure, with the altered element densities, is then re-analysed and the optimisation performed again. The procedure continues until the convergence.

It is desirable to develop the so-called “0-1” design, where the final distribution of material within the design space, is comprised entirely of either solid material or voids (no material). The solution of the “0-1” problem has been attempted; however, it is generally the case that the application of such techniques is computationally prohibitive due to the number of finite elements necessary to model the design space. The SIMP technique addresses this problem by defining the material within each of the finite elements, as a continuous design variable. By converting the design variable from discrete to continuous it is possible to use more computationally efficient mathematical programming methods for the solution of the original problem [8].

Intermediate density material, which neither takes the value of solid nor void, is generally not desirable since it is not possible to correspond such intermediate densities to real world structures. In order to avoid the presence of intermediate densities, within the final design, a penalisation is used to disproportionately decrease the benefit derived by the presence of intermediate density material. Penalisation of intermediate densities is achieved within SIMP, by relating the stiffness of the material to the density, thus:

$$E = \rho^P$$

Two basic approaches to topology optimisation using the SIMP technique exist as follow:

• *Minimum Compliance Design*; the minimisation of a specific performance measure subject to a constraint on the available resources. Usually the compliance of the structure will be defined as the optimisation objective with a constraint on the available material. This constraint is generally defined in terms of a fraction of the material in the designable domain prior to the optimisation.

• *Minimum Weight Design*; the minimisation of the mass of the structure with constraints on specific performance measures. The specific performance measures will usually be defined as stress, displacement, buckling load factor or any combination thereof.

The minimum compliance approach has proven effective at identifying conceptual structural design but it has been criticised by some practicing engineers for not enabling any specific performance targets such as the stress or the displacement to be included in the optimisation process [25]. Whilst the minimum weight design would seem to be an obvious solution to this problem, the topology optimisation problems containing specific performance constraints such as the maximum stress, the buckling load or the displacement are significantly more complex to solve [26,27].

4. TOPOLOGY OPTIMISATION IN HIGH-RISE STRUCTURAL DESIGN

4.1 Background

Requirements for high-rise structures as solutions to overcrowding in modern cities and as landmarks pose a significant challenge for structural engineers. This challenge is elegantly described by the "premium for height effect" [28] whereby the material required to construct taller buildings is disproportionately greater than for low-rise construction due to the increased bracing requirements. An even more significant structural challenge in the 21st century is the increasing tendency for architectural aspirations in high-rise construction to tend towards "aerodynamic", "twisted" and "free" forms [29]. The geometric complexity of "twisted" and "free" form structures often causes engineering intuition to fail when attempting to determine the optimum structural layout. An overview of an investigation into the use of topology optimisation for the design of a geometrically complex high-rise structure, conducted by the authors, is presented.

4.2 Optimised complex high-rise structures

In order to convince the civil engineering community to use the topology optimisation technique in the design of a geometrically complex high-rise structure a proposal for a tower with a "freeform" architectural intent was sought. The Bionic Tower is an architectural proposal for a high-rise tower in Abu Dhabi as it is shown in fig.4a. The project reached the feasibility stage in 2007 but was never progressed. An investigation was conducted herein to determine how topology optimisation could have been used at the conceptual structural design phase.

A braced outrigger structural arrangement was selected for the Bionic Tower (fig.4b) whereby the structural core is stabilised by a series of structural elements on the perimeter of the building. The core is connected to the external bracing elements by a truss at the pinnacle of the tower. The braced outrigger was selected on the basis that it fulfills the architectural intent of an externally visible structure and provides a viable structural solution for a tower of this height. The core is connected to the perimeter columns by a series of horizontal trusses. Lateral loading was applied to the tower and topology optimisation studies were performed on the entire exterior surface as well as on the trusses connecting the core to the perimeter surface.

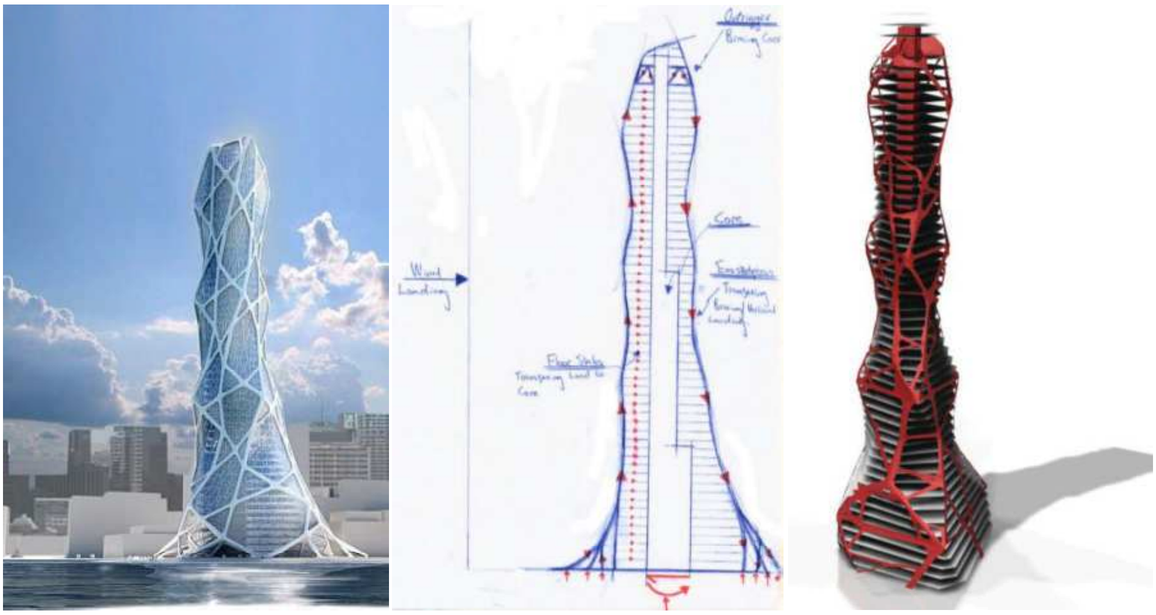

Figure 4 - a) Architectural aspiration of the Bionic Tower; b) Braced outrigger structural concept adopted for the bionic tower; c) Results of a topology optimisation study considering the exterior of the Bionic Tower as the designable domain

Despite the highly irregular shape of the tower, it was found that a series of discrete structural load paths could be identified from the results of the topology optimisation (fig.4c). An inspection of the trusses connecting the core to the perimeter surface (fig.5) showed completely rational truss layouts with strong similarities to typical optimal truss layout solutions found in the literature. Furthermore, the aesthetics of its structural layout were compatible with the "freeform" architectural initial intent of the architect and the client.

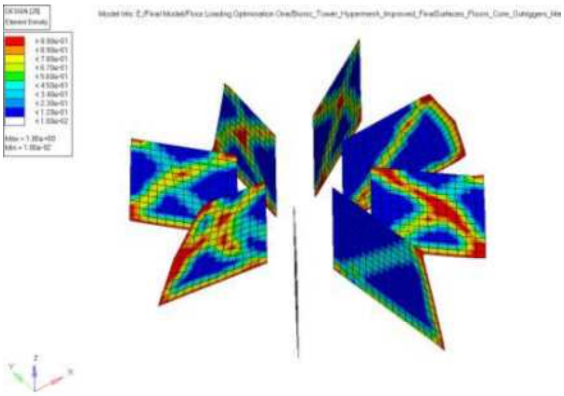

Figure 5 - Rational truss structures suggested for the outriggers in the results of the topology optimisation study

It is worth noting that the topology optimisation technique has been applied to the structural design of irregular and twisted high-rise structures previously; however the example presented is the first of its kind, where the topology optimisation has been applied to a completely "freeform" geometry. The results exemplify how topology optimisation is a useful design tool for designing structures for complex forms, where intuition may fail.

5 TOPOLOGY OPTIMISATION IN STRUCTURAL DESIGN OF MEMBERS

5.1 Background

The judicious placement of holes in the webs of steel beams has been employed to design lighter and stiffer beams for over 100 years. The original concept of creating a beam with web openings can be attributed to Geoffrey Murray Boyd [30], who patented what is now known as the castellated beam.

Castellated beams are formed by the expansion of a parent I-section to form a deeper stiffer section with web openings (fig.6a). Cellular beams, which contain circular openings, are currently the most widely used perforated beams due to their beneficial weight-to-stiffness ratio, and the ability to pass services (eg. hydraulic pipes, electric wires, etc.) through large holes, while the stresses are distributed evenly in the vicinity of the circular holes. An alternative to the castellation process of fabrication is the plate assembly. Plate assembly involves the fabrication of the I-section from a series of three flat steel plates (fig.6b). Plate assembly has the advantage of increased flexibility in terms of the position and the shape of the openings.

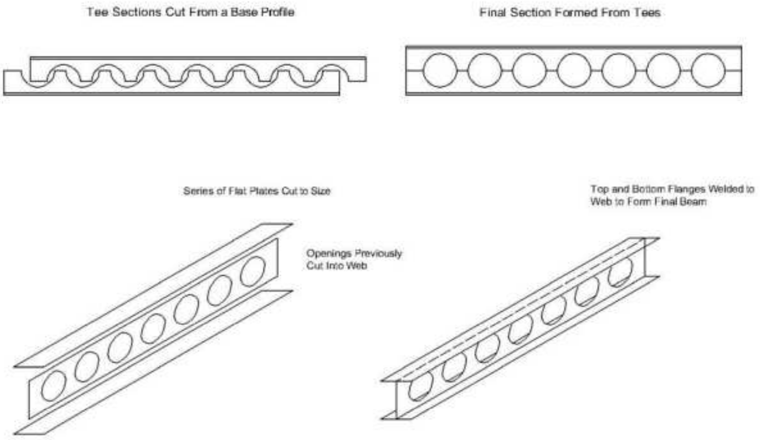

Figure 6 - a) Castellation technique; b) Plate assembly technique for fabricating perforated beams

The constant desire for improvement and mature level of understanding of the structural action of perforated steel sections has recently led to novel opening shapes, such as ellipses, being investigated [31]. These novel opening shapes were proposed as they promote an efficient and economic fabrication, improved structural performance and aesthetic qualities when compared to the standard opening types.

5.2 Optimised perforated steel beams

A comprehensive investigation was conducted with the use of topology optimisation techniques for the optimal design of the web openings in structural steel beams used in Civil Engineering applications [32]. The use of the continuum structural topology optimisation approach for the design of an I-section beam web has not previously been presented in the literature. The SIMP technique was implemented in this study. Various constraints and objectives were investigated.

The study was conducted on a standard 305x165x40 Universal Beam (UB). The section was selected on the basis that it has been widely used in prior to both experimental and numerical studies [33] and represents a typical 5m span section in building construction. The beam was subjected to uniformly distributed loading along the top compression steel flange to simulate the load from a steel-concrete composite (SCC) or reinforced concrete deck with the partial shear strength (i.e. lateral stability was not provided).

The topology optimisation was performed on the beam web only with the objective of maximizing the stiffness of the beam subject to a constraint on the area of the beam web that must be massless. In perforated beams like this, the web plays a very important role in providing the vertical shear capacity, forming the so called Vierendeel mechanism as well as providing resistance to the out-of-plane web-post (steel part between two consecutive openings) buckling failure mechanism. Both these local failure modes are directly associated to perforated beams, hence the study of the web only. On the other hand, steel flanges are providing the global bending capacity and hence they are not considered in the current investigation. Initially, it was specified that a minimum of 60% of the beam web should be open (massless). The topology optimisation results (fig.7a) suggested a truss like structure for the entire length of the beam, with a large opening in the centre where maximum moments but low shear forces exist. The overall design appeared to follow the lines of the principle stresses within the beam web and the openings took a rhomboidal shape. In order to rationalize the results of the topology optimisation, a complementary study was conducted where the results were constrained so as to be symmetrical about the longitudinal axis of the beam web. The symmetry constrained study resulted in a similar design with rhomboidal openings, but it was better balanced along the length of the beam (fig.7b).

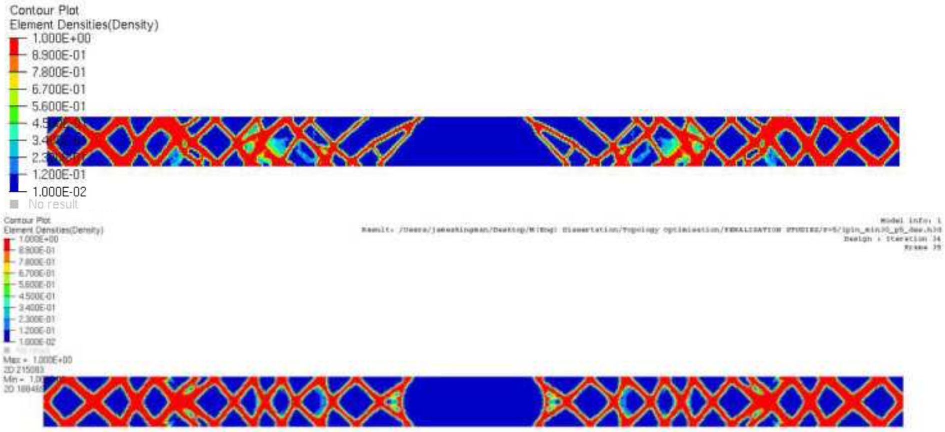

Figure 7 - a) Results of a topology optimisation study on a beam web; b) Results of a topology optimisation study on a beam web incorporating a symmetry constraint

The results of the topology optimisation study were post-processed in order to generate the finalised geometry of the optimised beam web (fig.8a). In order to further investigate the structural performance of the beam web in comparison to a typical beam with circular web openings, a nonlinear FE analysis was employed. The size of the circular web openings was determined based on the maximum size generally used widely in industry, equal to 0.75 times the depth of the web. A total of 17 web openings were placed along the length of the beam, in order to make the weight of the cellular beam as similar as possible to the optimised one, whilst retaining the same flange dimensions. It was desirable to compare a cellular beam of a similar mass in order to be able to draw valuable conclusions regarding the structural efficiency of the topology optimised design.

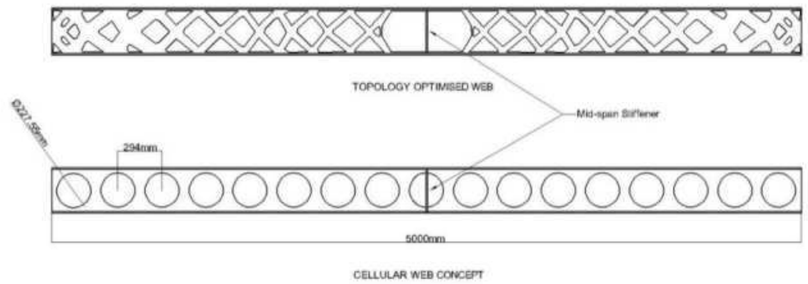

Figure 8 - a) Geometry of topology optimised beam web; b) Geometry of cellular beam web

Cellular beams often exhibit complex failure behaviour which may include localised buckling modes (eg. Vierendeel mechanism, web-post buckling, buckling due to local compression, etc.) or yielding and redistribution of stresses in the vicinity of the openings. Previous FE studies [34,35] were used to verify the FE model and provide accurate assessment of the effectiveness of the topology optimised beam web.

The basis of the FEA method employed is a three-step process whereby an initial pre-stress is applied to the FE model and a linear static analysis performed. The results of the linear static analysis are then used in an eigenvalue analysis of the FE model to determine the first buckling frequency and its associated mode shape. Imperfections are applied to the FE mesh, using the mode shape taken from the eigenvalue analysis, with a magnitude of the web thickness divided by 200. A geometric and materially nonlinear FE analysis is then performed to determine the load response of the beam. It is worth to note that the geometric complexity of the topology optimised beam web design necessitated the refinement of the FE mesh adding to the time required to complete the analysis process. The analyses were performed using the commercial FE package ANSYS v.14.

The results of the FE analysis suggest that the beam with an optimised web has a higher yield load and a greater stiffness in the linear range compare to the cellular beam (fig.9). Since both of the beams are formed from the same amount of structural material it can be concluded that the use of material in the topology optimised design is more efficient; therefore the proof-of-concept was achieved. The results also demonstrate that at the yield load level the stresses in the web of the cellular beam increase towards the support. Oppositely, it was found that the stresses in the optimised web, particularly close to the critical area of the supports, were uniform despite the localised stress concentrations at the corners of openings.

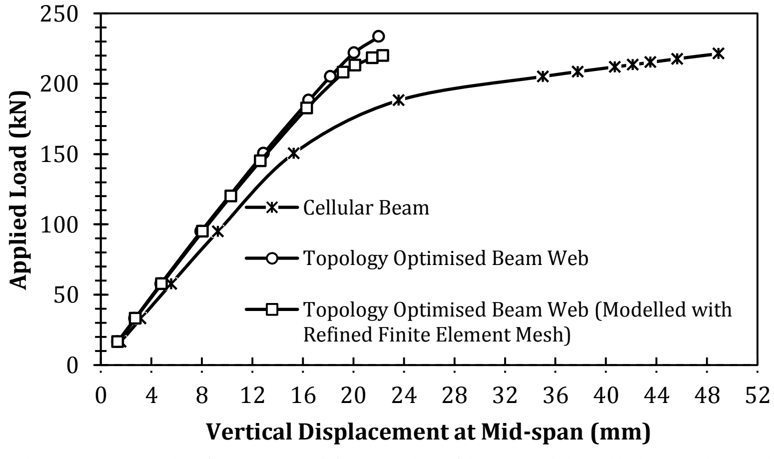

Figure 9 - Load displacement results for comparitive study of beam with cellular and topology optimised webs

5.3 Novel web opening architecture

In light of the results detailed above and Kingman et al. [32], it is concluded that the topology optimisation is a useful tool identifying alternative improved beam configurations and improving the in depth understanding of their structural behaviour. However, when it was applied to a full length section, the resulting design is generally complex and somewhat difficult to justify and be used in most practical applications. Consequently, a localised study approach was later established in order to identify optimum web opening shapes and locations. In the local study a short beam section was modelled while shear forces and bending moments applied directly to the section and the topology optimisation was then performed.

Further, a parametric investigation on a large number of cross-sections indicated that only the depth of the section alters the optimal topology of the web openings (fig.10). It can be concluded that for beams of depth between 270mm and 700mm, the optimum web opening topology is the same. Based on these results a web opening configuration is suggested that offers advantageous structural performance.

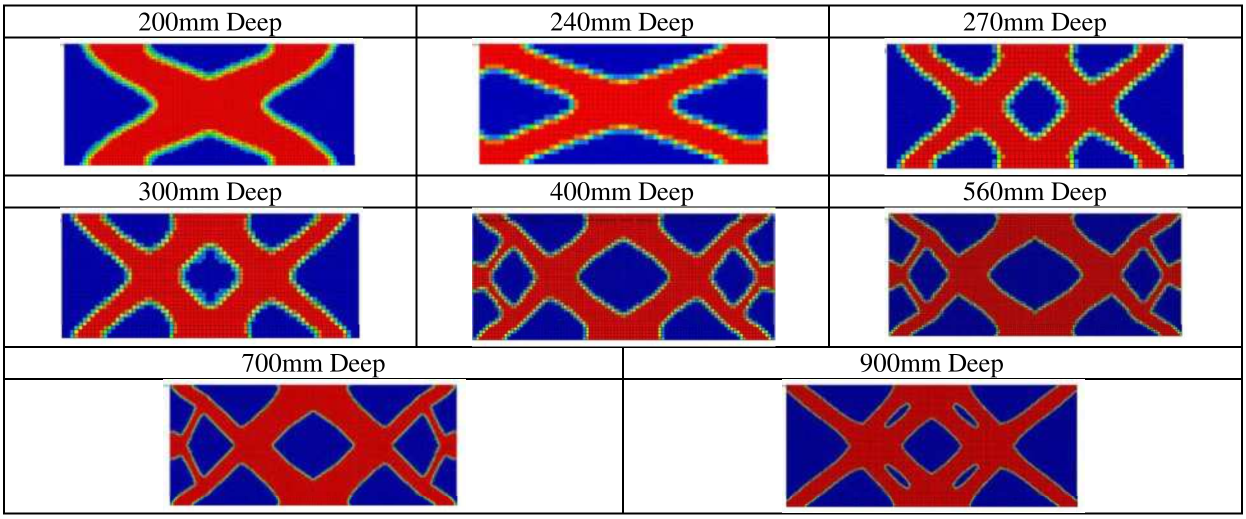
200mm Deep
240mm Deep
270mm Deep
300mm Deep
400mm Deep
560mm Deep
700mm Deep
900mm Deep

Figure 10 - Results of topology optimisation studies on localised beam sections of varying depths

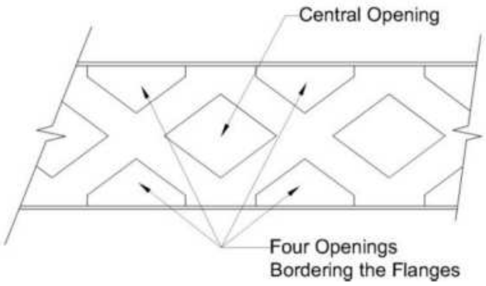

Figure 11 - Suggested web opening configuration based on the results of the local topology optimisation study

Therefore, a topology optimised beam web tends to offer improved structural performance compared to the typical perforated beams. The major disadvantage of directly applying continuum topology optimisation algorithms to the design of a long-span beam is the geometric complexity of the resulting design and the associated difficulty to determine analytically the load capacity of the section.

A local approach was implemented in order to identify a more generalised opening type. Based on the results of this study a novel opening architecture has been suggested (fig.11). It is anticipated that this new configuration is possible to be fabricated using the plate assembly technique, while no cost implies, compared to any other opening shapes. Further study is, however, required to examine various failure mechanisms that might have been introduced due to the complexity of these web openings as well as derive an analytical and/or empirical method to determine the load carrying capacities.

6 CONCLUSIONS

Topology optimisation offers significant opportunities in civil/structural design and architecture. It has been suggested as a tool that can lead to greater collaboration between engineers and architects during the conceptual design process. A limited number of examples of topology optimisation being used in structural engineering and architecture can be found in the literature and have been presented in this paper. The work of the authors on the topology optimisation of a high-rise structure and the topology optimisation of perforated steel beams is presented in more detail. In both cases it was found that topology optimisation is a useful design tool which promotes efficient designs.

At present, the major barriers to the widespread implementation of topology optimisation methods are: (i) the complex geometry of the optimised designs and (ii) the difficulty in solving problems involving nonlinear behaviour (such as buckling) and dynamics. The increasing use of advanced manufacturing techniques such as

CNC machining and 3D-printing may offer a solution to the complex geometry often arising during topology optimisation studies. Methods for solving topology optimisation problems involving nonlinear behaviour as well as dynamics are currently under investigation with a promising area of research being the Equivalent Static Load (ESL) method.

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REFERENCES

[1] Michell, A.G.M. (1904). The Limits of Economy of Material in Frame Structures. "*Philosophical Magazine*". 8(6):589-597.

[2] Pucker, T. and Grabe, J. (2011). "Structural Optimization in Geotechnical Engineering: Basics and Application". *ACTA Geotechnica.* 6(1):41-49.

[3] Smith, C. and Gilbert, M. (2007). "Application of Discontinuity Layout Optimization to Plane Plasticity Problems". *Proc. R. Soc.* 463:2461-2484.

[4] Kim, H. (2002). "Topology Optimization for Reinforced Concrete Design". *Proc. 5th World Congress on Computational Mechanics.* Vienna, Austria.

[5] Otto, F. and Rasch,B. (1996). "Finding Form: Towards An Architecture of the Minimal". Stuttgart: Edition Axel Menges.

[6] Abruzzese, D. and Tursi, A. (2003). "Form Finding Research: Development Between Empirical and Numerical Methods". *System-Based Vision for Strategic and Creative Design.* 569-578.

[7] Isler, H. (1961). "New Shapes for Shells". *Bulletin of the International Association.* 8:123-130.

[8] Bendsøe, M.P. and Sigmund, O. (2003). *Topology Optimisation: Theory, Methods, and Applications*. 2nd ed. Berlin: Springer Verlag.

[9] Ohmori, H. 2008. "Computational Morphogenesis: It's Current State and Possibility for the Future". *Proc. 6th Int. Conf. on Computation of Shell and Spatial Structures*. 28-31 May 2008. New York, USA.

[10] Rudavski, S. 2009. "Towards Morphogenesis in Architecture". *International Journal of Architectural Computing*. 7(3)345-374.

[11] Xie, Y.M., Zuo, Z.H., Huang, X., Tang, J.W., Zhao, B. and Felicetti, P. (2011). "Architecture and Urban Design Through Evolutionary Structural Optimisation Algorithms". *International Symposium on Algorithmic Design for Architecture and Urban Design*. Tokyo, Algode.

[12] Frattari, L., Vadori, R., Aria, R.D. and Leoni, G. (2009). "Topology Optimisation in Architecture: May it be a Design Tool?". *8th World Congress on Structural and Multidisciplinary Optimisation*. Lisbon, Portugal.

[13] Beghini, L.L. (2013) "Building Science Through Topology Optimisation". Ph.D. Illinois Institute of Technology.

[14] Stromberg, L.L., Beghini, A., Baker, W.F., and Paulino, G.H. (2012). "Topology Optimisation for Braced Frames: Combining Continuum and Beam/Column Elements". *Engineering Structures*. 37: 106-124.

[15] Ohmori, H., Futai, H., Ijima, T., Muto A. and Hasagawa, H. (2005). "Application of Morphogenesis to Structural Design*". Proc. of Frontiers of Computational Science Symposium*. Nagoya, Japan.

[16] Burry, J. and Burry, M. (2010). *The New Mathematics of Architecture*. London: Thames & Hudson Ltd.

[17] Dombernowsky, P. and Søndergaard, A. (2009). Three-dimensional Topology Optimisation in Architectural and Structural Design of Concrete Structures. *Proc. of the Int. Assoc. of Shell and Spatial Structures Symposium.* Valencia, Spain.

[18] Xie, Y.M. and Steven, G.P. 1993. A evolutionary procedure for structural optimisation. [pdf] *Computers and Structures.* 49(5):885-896.

[19] Querin, O.M., Steven, G.P. and Xie, Y.M. 1998. Evolutionary Structural Optimisation (ESO) Using a Bi-Directional Algorithm. Engineering Computations. 15(8):1031-1048.

[20] Rozvany, G.I.N. (2008). A Critical Review of Established Methods of Structural and Topology Optimisation. *Structural and Multidisciplinary Optimisation.* 37(3):217-237.

[21] Eschenauer, H.A and Olhoff, N. (2001). "Topology Optimisation of Continuum Structures: A Review". *Applied Mechanics Review.* 54(4)331-390.

[22] Park, K.J. and Park, G.J. (2005). "Optimisation of Structures with Nonlinear Behaviour using Equivalent Loads". *6th World Congress of Structural and Multidisciplinary Optimisation.* Rio de Janeiro, Brazil.

[23] Bendsøe, M.P. and Kikuchi, N. (1988). "Generating Optimal Topologies in Structural Design using a Homogenization Method". *Computer Methods in Applied Mechanics and Engineering.* 71(2):197-224.

[24] Sigmund, O. (2000). "Topology Optimisation: A Tool for the Tailoring of Structures and Materials". *Philosophical Transactions of the Royal Society.* 358:211-217.

[25] París, J., Casteleiro, M., Navarrina, F. and Colominas, I. (2007). "Topology Optimisation of Structures with Local and Global Stress Constraints". *International Conference on Computational & Experimental Engineering and Sciences.* 2(1): 13-20.

[26] Zhou, M. (1996). "Difficulties in Truss Topology Optimisation with Stress and Local Buckling Constraints". *Structural Optimisation.* 10: 71-81.

[27] Rozvany, G.I.N. (1996). "Difficulties in Truss Topology Optimisation with stress, Local Buckling and System Buckling Constraints". *Structural Optimisation.* 11: 134-136.

[28] Khan, F.R. (1969). "Recent Structural Systems in Steel for High-Rise Buildings". *Proc. of the British Constructional Steel Association on Steel in Architecture.* British Constructional Steel Association: London.

[29] Ali, M.M. and Moon, K.S. (2007). "Structural Developments in Tall Buildings: Current Trends and Future Prospects". *Architectural Science Review*. 50(3)205-223.

[30] Knowles, P.R. (1991). Castellated Beams. *Proc. of the Institution of Civil Engineers.* 90:521-536.

[31] Tsavdaridis, K.T. and D'Mello, C. (2012). "Optimisation of Novel Elliptically-Based Web Opening Shapes of Perforated Steel". *Journal of Constructional Steel Research.*

[32] Kingman, J.J., Tsavdaridis, K.D. and Toropov, V.V. (2013). "The Application of Topology Optimisation to the Design of Steel I-Section Beam Web Openings". *Proc. of the 14th Int. Conf. on Civil, Structural and Environmental Engineering Computing*. Cagliari, Italy.

[33] Tsavdaridis, K. and D'Mello, C. (2012). "Vierendeel Bending Study of Perforated Steel Beams with Various Novel Web Opening Shapes through Nonlinear Finite-Element Analyses". Journal of Structural Engineering. 138(10):1214–1230.

[34] Tsavdaridis, K.D. and D'Mello, C., (2011). "Web Buckling Study of the Behaviour and Strength of Perforated Steel Beams with Different Novel Web Opening Shapes". *The Journal of Constructional Steel Research*, 67(10)1605-1620.

[35] Tsavdaridis, K.D. and D'Mello, C. (2011). "Finite Element Investigation on Web-Post Buckling of Perforated Steel Beams with Various Web Opening Shapes Subjected Under Different Shear-Moment Interaction". 6th European Conference *on Steel and Composite Structures (EUROSTEEL)*, Budapest, Hungary, Vol. C, 1851-1856.