MATHEMATICAL MODELLING OF EMBODIED CARBON EMISSIONS OF BUILDING PROJECTS

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It is increasingly recognised that if the emphasis remains on reducing operating carbon emissions (OC) of buildings, embodied carbon emissions (EC) will eventually attain a significant proportion of buildings' lifetime carbon emissions (LC). Emphasis on minimising EC is equally desirable if LC is to be reduced. A first step to minimising EC is quantification, in order to know what quantities to minimise. However, several prevalent approaches of quantifying EC pose challenges in promoting potential alternative actions to reduce EC. In many cases, besides the limitations associated with the boundaries usually adopted, it is difficult (if not impossible), to attribute the respective sources of energy (e.g. diesel, coal, biomass etc.) to the resulting EC. This paper presents a mathematical model for computing EC of building projects and in contrast to previous studies, a concept of disaggregation is adopted in order to identify EC with the respective energy sources. The approach enables the specific sources of energy to bear on the quantification of EC, in a manner that allows differentiation of the contribution of the different sources of energy. The model is presented in a series of mathematical equations. The major benefit associated with the nature of the developed model is that, even without recourse to material substitution (e.g. timber for concrete), it is possible to achieve emission reductions from the same material by optimising the parameters (e.g. energy used in manufacturing and transportation) associated with its EC.

Keywords: building projects, embodied carbon emissions, mathematical model.

INTRODUCTION

The building sector has earned a reputation of being both energy and carbon intensive – it consumes up to 40% of the global final energy and releases 50% of the annual global emissions (WBCSD 2012; UNEP 2009). Meanwhile, national and international climate-change regulatory regimes (e.g. UK Climate Change Act 2008; Kyoto Protocol 1998) set ambitious targets to progressively reduce carbon emissions to the smallest possible count. Such ambitions do not exclude buildings, given the reputation of the sector. The total lifetime carbon emissions (LC) of a building arise from embodied carbon (EC) (e.g. emissions from material manufacture and transportation) and operating carbon (OC) (e.g. emissions from lighting and heating). Focussing only on reducing OC, as the case has hitherto been, has a knock-on effect on EC. Several studies (e.g. Iddon and Firth 2013; Sartori and Hestnes 2007) report that reducing OC increases the relative contribution of EC to LC. Even though it is widely acknowledged that OC takes the larger proportion of LC, with the current trend, it may not be the case in the near future – OC will approach 100% of LC. Avoiding this likelihood necessitates simultaneous efforts of reducing EC too.

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Initiatives of reducing EC should begin with quantifying it in a disaggregated but not aggregated approach. In disaggregated approaches, the different energy sources (e.g. diesel, coal, biomass etc.) that contribute to EC can be readily accounted for, unlike aggregated approaches. The major shortcoming of aggregated approaches is that they assume emissions from the different energy sources to be homogeneous. Such assumptions present shortcomings similar to those in economics, when inflation is interpreted based on a specific 'basket of goods', yet goods in that basket may widely differ (e.g. in quality, preference, and price changes), making the sole inflation figure rather non-representative for different goods. For instance, the study (Huberman and Pearlmutter 2008) used a carbon emission factor of 100kgCO_2 per unit energy for all the different energy sources that were involved in calculating EC. Such an approach and several similar ones (see Kua and Wong 2012; Broun and Menzies 2011; Dimoudi and Tompa 2008) stifle potential efforts to minimise EC. Without articulating what each energy source contributes to emissions means that it would be difficult, if not impossible, to trade off for better options (i.e. opting for energy sources with lower emissions). Relating to the inflation analogy again, the figure for inflation may not provide enough information for someone to identify goods that might be cheaper. Meanwhile, disaggregated approaches are not easily achievable especially in processes (e.g. steel manufacturing) where it is difficult to distinguish the proportions of various sources of energy used (see Hammond and Jones 2011). Even so, the benefits associated with disaggregation make the temptation to disaggregate EC irresistible. Although some studies (Gustavsson *et al*. 2010; Dias and Pooliyadda 2004) attempted to disaggregate EC, they still leave a lot to be desired – the boundaries they adopted did not take full account of components (i.e. materials, plant, and workforce) that contribute to EC of a building project.

From extant literature reviewed, aggregation is promoted in various ways, commonest of which include: use of ball-pack average carbon emission factors for varying materials (see Aye *et al*. 2012; Huberman and Pearlmutter 2008); use of generic country average emission factors (see González and García Navarro 2006; Cole 1998); and use of emission factors with undisclosed energy sources (see Broun and Menzies 2011; Dimoudi and Tompa 2008; Asif *et al*. 2007). EC results possess significant levels of uncertainty due to variation of energy mixes, among other reasons (Hammond and Jones 2010). Aggregation certainly compounds such uncertainties. In this paper, we present a mathematical model that can facilitate disaggregation in the quantification of EC of building projects.

METHODOLOGY

This work was about developing a mathematical model and therefore, the methodology adopted followed standard mathematical modelling principles. Mathematical modelling *"…mimic[s] reality by using the language of mathematics"* (Bender 1978: 1). Several texts on mathematical modelling (e.g. Meerschaert 2007; Edwards and Hamson 2001; Hangos and Cameron 2001; Murthy *et al*. 1990; Burghes and Wood 1980) suggest that it generally involves: formulating the problem, stating assumptions, mathematical formulations (e.g. equations), solving the mathematical equations and interpreting the results, verifying that the mathematical model is correct and finally, using the mathematical model/solution to address the problem. However, rarely are all these stages executed, or even executed in a perfect sequence. It is usual for a mathematical modelling process to involve rounds of iterations, often excluding some steps that are not of interest or are out of scope (Burghes and Wood 1980). Since the major aim of this paper was to present a mathematical model, the scope was limited to problem formulation, assumptions, mathematical formulations, and verification.

Problem formulation

Problem formulation necessitates a thorough understanding of the world associated with the problem (Berry and Houston 1995; Murthy *et al*. 1990). As elaborated in the introductory part of this paper, the problem to address was elicited from the extant literature. The major prevalent problem was aggregation of EC results and this work set out to address this problem by developing a mathematical model that can accord disaggregation. The task was to develop a model to compute EC of buildings in a way that enables the energy sources to bear on the quantification, in a manner that allows differentiation of the contribution of the different energy sources.

Assumptions

Relaxing assumptions drifts the model away from the reality of the problem, whereas stringent assumptions present difficult solutions (and analysis) but drift the model closer to the reality of the problem (Burghes and Wood 1980). A balance between strictness and relaxation of assumptions is necessary. In deriving assumptions, Bender (Bender 1978: 2-3) suggested that a model should delineate the world into three parts: the part to be neglected, the part potentially affecting the model but not included, and the part the model studies. Too many considerations (i.e. number of variables) can complicate the model, whereas neglecting the 'correct' ones can invalidate conclusions drawn from the model (ibid). The assumption stage is therefore concerned with delineating the appropriate variables of the model. The biggest proportion of a building's EC occurs prior to commissioning the building i.e. during the pre-construction and construction phases. Upon review of literature, it was concluded that the appropriate model's input variables were:

- emissions from construction materials, including process emissions (e.g. resulting from chemical reactions like calcination of lime during cement manufacture) and material transportation emissions (see Chang *et al*. 2012; Monahan and Powell 2011; Asif *et al*. 2007; Nässén *et al*. 2007);
- emissions from plant (i.e. equipment, appliances, machinery and the like) used during construction; this includes emissions from transportation of plant and emissions from onsite-use (see Hughes *et al*. 2011; Kofoworola and Gheewala 2009; Guggemos and Horvath 2006); and
- emissions from workforce, limited to emissions associated with the mode (or energy used) for commuting to and from the construction site (see Gustavsson *et al*. 2010; Cole 1998).

Mathematical formulations

Caution should be exercised when choosing the appropriate mathematical formulations to define relationships between variables (Edwards and Hamson 2001). Meerschaert referred to the 'formulation stage' as *"selecting the modelling approach"* and noted that *"... success at this step requires experience, skill, and familiarity with the relevant [mathematics] literature"* (Meerschaert 2007: 8). In order to formulate a model, it is imperative to understand the various alternative kinds of formulations (Murthy *et al*. 1990) in order to choose a model that is appropriate for the problem in question.

Type of mathematical model used
The taxonomy of mathematical models is delineated by various attributes. Quantitative models respond to questions of inquiry prescribing quantification (e.g. how much?, how many?), whereas qualitative models are broadly concerned with studying a system and its properties, without necessarily reducing anything to numbers (Saaty and Alexander 1981). A quantitative model was appropriate in this case since modelling dealt with numbers (e.g. quantity of emissions). Unlike dynamic models which are suited for studying systems that entail processes evolving over time (e.g. spread of a disease), static models are time independent (Meerschaert 2007; Murthy *et al*. 1990). The proposed model considered static systems whereby emissions are computed at a specific instance in time. This was appropriate due to the great uncertainty usually associated with anticipating change in policy and technology related to emission reductions. Since in deterministic systems the values of the variables are predictable with certainty and rather not random as the case is for stochastic or probability systems (Edwards and Hamson 2001; Murthy *et al*. 1990), a deterministic approach was adopted for the modelling exercise. Furthermore, various types of equations can be used in mathematical modelling: differential, integral, algebraic, and difference (Meerschaert 2007; Edwards and Hamson 2001; Murthy *et al*. 1990). In Murthy *et al*. (1990), it is indicated that static-algebraic formulations are suitable for modelling deterministic systems. Of the 54 equations in the 25 models (related to embodied energy, greenhouse gases, waste and time-cost parameters of building-projects) of previous studies that were reviewed in Abanda *et al*. (2013), 40 equations were ‘static-algebraic’. Thus algebraic equations were considered appropriate for deriving the model. Consequently, the derived mathematical model was a quantitative-deterministic-static-algebraic type of model.

The analysis technique
Life cycle assessment (LCA) is a commonplace technique of analysing environmental profiles of buildings. The life cycle of a building consists of its construction, use, maintenance, demolition and related waste handling (Gustavsson and Joelsson 2010), all of which have impacts on the environment. Research suggests that as the interest to reduce such impacts developed (Van Ooteghem and Xu 2012), the need for better methods to understand and therefore quantify the impacts (e.g. energy use, emissions, water use) in a lifecycle perspective increased, which saw LCA emerge (Sartori and Hestnes 2007). Combined with energy, LCA evolved into lifecycle energy analysis (LCEA). LCEA of buildings is the LCA analysis that uses energy as the measure for gauging the environmental impacts of buildings (Huberman and Pearlmutter 2008). The LCEA method is deemed appropriate for buildings and its intentions are not to substitute LCA but rather, enable assessment of energy efficiency (Fay *et al*. 2000). In the procedure, LCEA accounts for all energy intakes throughout the building’s life time and upon understanding the amount of energy, the associated carbon emissions can be deduced and the environmental impacts of the building can also be conceptualised (Ramesh *et al*. 2010). For the developed model, it subscribed to the partial LCEA approach of cradle to construction site as per modules A1 to A5 (BS EN 15978:2011) and relevant LCA standards (see ISO 14040: 2006; ISO 14044: 2006).

Modelling techniques adopted
Commonly referenced are three primary modelling techniques used in LCEA: process analysis (PA), input-output analysis (IOA), and hybrid analysis (HA). In Alcorn and Baird (1996: 319), PA is referred to as one entailing *“... systematic examination of the direct and indirect energy inputs to a process”*. In other words, PA deals with tracing

all the energy inputs of products that are dependent on the process (Mortimer 1991). Meanwhile, the IOA method credits its roots from macro-economics, as it was initially developed in economic research problems and later adopted for energy analysis (Hammond and Jones 2008; Bullard *et al*. 1978; Roberts 1978). IOA traces energy flows by analysing monetary flows to and from economic sectors, through mapping the financial output of each sector with the corresponding energy used (Alcorn and Baird 1996). HA, as the name suggests, is an amalgam of PA and IOA. Since HA combines data from PA and IOA in various ways (Crawford *et al*. 2006), hybrid-variants can be realised (e.g. PA-based and IOA-based hybrids), depending on dominance of a method in the approach adopted. As such, each of these three – PA, IOA and HA – methods has its own merits and demerits.

Several studies (e.g. Murray *et al*. 2010; Hammond and Jones 2008; Crawford *et al*. 2006; Lenzen and Dey 2000; Alcorn and Baird 1996; Mortimer 1991) discuss the merits and demerits associated with PA, IOA and HA, based on which a judgement can be made on the appropriate technique to adopt. PA is suitable for assessing direct but not indirect impacts, while the reverse applies for both IOA and HA. For indirect impacts, PA is criticised for the subjectivity involved in deciding the truncation point (Lenzen and Dey 2000). The unavoidable use of sector averages in IOA implies that the method poses challenges in evaluating a specific individual product (Murray *et al*. 2010). Thus IOA is usually associated with aggregated results (Bourgault *et al*. 2012). PA is suitable for a specific process or product and can also take into account technological advancements in the system under study (Gustavsson *et al*. 2010). Although PA does not give ‘complete’ results, by 50% sometimes (Lenzen and Dey 2000), accuracies of up to 90% can be registered (Hammond and Jones 2010; Murray *et al*. 2010). Most models based on static-algebraic formulations – to which the derived model in this work subscribes – are usually based on PA (see Abanda *et al*. 2013). Since the interest of this work was centred on disaggregation using algebraic equations, PA techniques were adopted.

Verification

Verification involves *“determining whether the model is behaving correctly”* (Hangos and Cameron 2001: 29) i.e. does the model give the correct or expected output? Although verification is often presented last in sequence, in reality, it is usually done concurrently with other stages (i.e. formulation stage and solution stage). In this work, verification was done concurrently with the formulation of equations. Meanwhile, in modelling, *“mathematical modelling of a physical world makes sense only if the models are dimensionally correct”* (Berry and Houston 1995: 121) or rather, dimensionally homogeneous (Bender 1978). Therefore, as a tool, dimension analysis can be used verify that the developed model’s formulations are correct. The fundamental dimensions of physical quantities are Mass (M), Length (L) and Time (T) (Berry and Houston 1995; Murthy *et al*. 1990; Bender 1978), from which all other dimensions of quantities can be derived. If all the terms which constitute an equation have the same dimensions, then it can be claimed that the equation is dimensionally homogeneous (Bender 1978: 35). Consequently, as a verification measure, derived equations were rigorously checked for dimension homogeneity.

RESULTS AND DISCUSSION

EC of a building project equals to the sum of emissions from materials, emissions from plant, and emissions from workforce (see Hughes *et al*. 2011; ICE 2010). The model was thus composed of a series of equations related to emissions from materials, plant, and workforce. In each equation, a dimensionless disaggregation factor was introduced. This factor is defined as the proportion of energy used (e.g. for manufacturing, transportation), derived from a specific energy source j. Multiplying the disaggregation factor with the carbon emission factor of that energy source enables the outputs of the model to be presented in a disaggregated manner.

Emissions from construction materials

Emissions from manufacturing and transporting n construction materials, using e different sources of energy are given by Equations (1) and (2) below, respectively. Three options A, B, and C, were considered in Equation (2). Option A is applicable where the weight of materials is significant and known, and the distance of transportation can be estimated. Option B is applicable where the weight of materials is insignificant (whether known or unknown) and the quantity of energy used is known. Option C is suitable where weight of materials is insignificant (whether known or unknown) and the distance of transportation can be estimated:

$$EC_{m1} = \sum_i^n \rho_i \left(\sum_j^e V_{ij}\, C_j^a \theta_j^a + S_i \right) \tag{1}$$

$$EC_{m2} = \begin{bmatrix} \sum_i^n \rho_i \left(\sum_j^e W_{ij}\, X_j^a C_j^a \alpha_j^a \right); If\ option\ A\ conditions\ apply \\ \sum_i^n \sum_j^e W_{ij}^a\, C_j^a\, \alpha_j^a; If\ option\ B\ conditions\ apply \\ \sum_i^n X_j^a \left(\sum_j^e C_j^b\, \alpha_j^a \right); If\ option\ C\ conditions\ apply \end{bmatrix} \tag{2}$$

where: EC_{m1} is the total emissions from manufacturing materials (in $kgCO_2$); ρ_i is the quantity of material type i (in kg); V_{ij} is the quantity of energy j to manufacture a unit of material i (in kWh/kg); C_j^a is the carbon emission factor (in $kgCO_2/kWh$) per unit energy j used; θ_j^a is a disaggregation factor in manufacturing material i; S_i is a constant for process emissions per unit of material i (in $kgCO_2/kg$); EC_{m2} is the total emissions from transporting materials (in $kgCO_2/kg$); W_{ij} is the quantity of energy j to transport a unit of material i per unit distance (in kWh/kgkm); X_j^a is the transport distance for material i (in. km); α_j^a is a disaggregation factor in transporting materials; C_j^b is the carbon emission factor per unit distance (in $kgCO_2/km$) with respect to the corresponding transportation energy j; W_{ij}^a is the quantity of energy j to transport material i (in kWh).

Emissions from plant

Emissions from operation and transportation of p plant, using e different sources of energy are given by Equation (3) and (4) respectively:

$$EC_{q1} = \sum_q^p \varphi_q \left(\sum_j^e U_{qj}\, C_j^b \theta_j^b \right) \tag{3}$$

$$EC_{q2} = \sum_q^p \varphi_q^b \left(\sum_j^e Y_{qj}\, X_j^b C_j^b \alpha_j^b \right) \tag{4}$$

where: EC_{q1} is the total emissions from operating plant (in $kgCO_2$); φ_q is the number of plant type q; U_{qj} the quantity of energy j used for operating plant q (in kWh); C_j^b is the carbon emission factor (in $kgCO_2/kWh$) per unit energy j used; θ_j^b is a disaggregation factor in operating the equipment; EC_{q2} are the total emissions from transporting plant; φ_q^b is the weight of plant q (in kg); Y_{qj} is the quantity of energy j to transport a given weight of plant q per unit distance (in. kWh/kgkm); X_j^b is the transport distance for plant q (in km); α_j^b is a disaggregation factor in transporting the plant. Options mentioned in Equation (2) about material transportation can equally apply to transportation of plant in Equation (4).

Emissions from workforce

Emissions from transporting workforce for duration r, using e different sources of energy were given by Equation (5) considering two options A and B. Option A is applicable where the duration of using the workforce and the quantity of energy used per unit duration are known. Option B is applicable where the duration of using the workforce, the quantity of workforce, the distance travelled, and the modes of transport used are all known.

$$EC_l = \begin{bmatrix} \sum_f^r \beta_f \left(\sum_j^e Z_{fj}\, C_j^c \alpha_j^c\right);\ If\ option\ A\ conditions\ apply \\ \sum_f^r \beta_f\, L_f X_j^c \left(\sum_j^e C_j^d\, \alpha_j^d\right);\ If\ option\ B\ conditions\ apply \end{bmatrix} \tag{5}$$

where: EC_l is the total emissions from transporting workforce (in $kgCO_2$); β_f is the duration f workforce is used (in days); Z_{fj} is the quantity of energy j to transport workforce per duration (in kWh/day); C_j^c is the carbon emission factor of the transport energy used (in $kgCO_2$/kWh); α_j^c is a disaggregation factor for transporting workforce; L_f is the number of people in the workforce required; X_j^c is the distance travelled by a person per duration (in km/day); C_j^d is the carbon emission factor per person per unit distance depending on the mode (e.g. bus, train, cycle) of transport used (in $kgCO_2$/personkm); α_j^d is a disaggregation factor for the mode used in transportation.

Conditions (constraints) subjected to the model

The direct and indirect emissions (defined as per Defra/DECC 2013) were to fulfil Equation (6), whereas the disaggregation factors for all the different sources of energy e were to sum to unity, as expressed by Equations (7) and (8):

$$C_j^{a,b,c,d} = D_j + I_j \tag{6}$$

$$\sum_j^e \theta_j^{a,b} = 1\,;\ 0 \le \theta_j^{a,b} \le 1 \tag{7}$$

$$\sum_j^e \alpha_j^{a,b,c,d} = 1\,;\ 0 \le \alpha_j^{a,b,c,d} \le 1 \tag{8}$$

where: D_j and I_j are the direct and indirect emissions resulting from energy source j, respectively.

The final model

The final derived consolidated model for the total embodied carbon emissions (EC_T) of a building project is given by Equation (9) below.

$$EC_T = (EC_{m1} + EC_{m2}) + \left(EC_{q1} + EC_{q2}\right) + EC_l \tag{9}$$

Model verification

All derived equations were checked for dimensional homogeneity and they satisfied this condition. An example of Equation (1) is illustrated below:

$$EC_{m1} = \sum_i^n \rho_i \left(\sum_j^e V_{ij}\, C_j^a \theta_j^a + S_i\right)$$

from inspection, the above equation can be broken down into three terms which are: EC_{m1}, $\rho_i V_{ij} C_j^a \theta_j^a$ and $\rho_i S_i$, whose dimensions can be deduced as follows: EC_{m1} is measured in $kgCO_2$ (i.e. mass) and thus $[EC_{m1}] = M$; ρ_i is measured in kg and thus $[\rho_i] = M$; V_{ij} is measured in kwh/kg and thus $[V_{ij}] = (ML^2T^{-2})/M$; C_j^a is measured in $kgCO_2$/kWh and thus$[C_j^a] = M/(ML^2T^{-2})$; θ_j^a is a dimensionless constant and thus $[\theta_j^a] = 1$; S_i is measured in $kgCO_2$/kg and thus $[S_i] = M/M$. Substituting the deduced dimensions into the three terms of the equation shows that $[EC_{m1}] = M$,

$[\rho_i V_{ij} C_j^a \theta_j^a] = (M \times (ML^2T^{-2})/M \times M/(ML^2T^{-2}) \times 1) = M$, and $[\rho_i S_i] = (M \times M/M) = M$. Therefore, Equation (1) is dimensionally consistent.

IMPLICATIONS

Considering a building project, if attention is drawn to materials, as they are a major source of EC, many studies have hitherto concentrated their efforts on discussions involving material-type comparisons (i.e. what is the 'greenest' amongst steel, timber, and concrete?). This work contends that it is equally important to highlight 'green from what energy source?' On a suitable energy-mix palette, it is equally possible to achieve emission reductions by varying the disaggregation factors related to that material, without recourse to material substitution. For instance, in Equation (1) and (2), the disaggregation factors θ_j^a and α_j^a can be varied until a desired level of emissions from materials is attained. This may for instance imply reconsidering where the construction materials are sourced from. In Equation (5), a construction practice can vary α_j^d, which is related to the proportion of the different modes or energy sources used for transporting workforce, in order to arrive at a desired level of emissions. Demonstration of how the model can contribute to several of such 'real-world utilities' falls in the last phase of mathematical modelling – using the model to address a real-world problem. The present work sets the foundation to embark on this phase that is beyond the scope of this paper.

CONCLUSIONS

It has been argued that the prevailing approaches of computing EC do aggregate results and this stifles plausible alternatives to reducing EC. Using mathematical modelling, this paper has presented a mathematical model for computing EC of building projects. The model considers all plausible components of a building project that cause emissions. More importantly and contrary to most previous efforts, the model can present disaggregated outputs. Although a disaggregated approach may not be easy to apply in some cases, it is worth the effort. The approach enables the specific sources of energy to bear on the quantification, in a manner that allows differentiation of the contribution of the different sources of energy to the resulting EC. In that way, it is possible to achieve emission reductions by varying the disaggregation factors, which are the proportions of energy sources used. This opens up more alternatives of reducing EC, thereby promoting sustainable construction.

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