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Billings, S.A., Stansby, P.K., Swain, A.K. et al. (1 more author) (1996) Accurate Prediction of Nonlinear Wave Forces: Part II(Responding Cylinder). Research Report. ACSE Research Report 625 . Department of Automatic Control and Systems Engineering

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Research Report No: 625

June,1996

Accurate Prediction of Nonlinear Wave Forces : Part-II (Responding Cylinder)

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Abstract

In part-I of this paper, continuous time models have been fitted to wave forces acting on fixed cylinders and a new equation structure called the Dynamic Morison model has been proposed as an alternative to the traditional Morison equation. In this, the second part of the paper,continuous time nonlinear differential equation models are fitted to wave forces from responding cylinders and corresponding equation structures are proposed as an alternative to the Morison equation for the prediction of wave forces for the moving cylinder case. It is shown that the new models explain the wave force mechanisms and perform considerably better than the Morison equation.

1 INTRODUCTION

Prediction of wave forces acting on fixed structures composed of members of circular cross section of relatively small diameter is generally based on the Morison equation given by

$$\begin{aligned} F(t) &= 0.25\pi\rho D^2 C_m \dot{u} + 0.5\rho D C_d u|u| \\ &= C_m A_i \dot{u} + C_d A_d u|u| \end{aligned} \tag{1}$$

where 'F(t)' is the force per unit axial length, 'u(t)' is the instantaneous flow velocity, 'ρ' is the water density and 'D' is the diameter. C_m and C_d are dimensionless inertia and drag coefficients respectively. $A_i = 0.25\rho\pi D^2$ and $A_d = 0.5\rho D$.

When the cylinder is free to respond dynamically, the physics of the flow becomes increasingly complex and it is often difficult to determine an accurate representation of the wave force mechanics. Forces acting on moving structures of circular cross section are traditionally computed by modifying the Morison equation for fixed cylinders as reviewed by Chakrabarti (1987). The expression for the force is generally given in terms of relative motion as

$$\begin{aligned} F(t) &= C_m A_i(\dot{u} - \ddot{x}) + A_i\ddot{x} + C_d A_d(u - \dot{x})|(u - \dot{x})| \\ &= C_m A_i \dot{u}_r + A_i\ddot{x} + C_d A_d u_r|u_r| \end{aligned} \tag{2}$$

where $u_r = (u - \dot{x})$ denotes the relative velocity. Eqn.(2) is known as the relative velocity model of the wave force.

The equation is widely used to calculate wave forces acting on a variety of offshore structures including jacket platforms, articulated columns,risers,tension leg platforms etc. Flow around a cylinder has been simulated numerically by Slaouti and Stansby (1992) for laminar flow conditions (where simulations are accurate) with the cylinder allowed to respond with two degrees of freedom. It has been observed that eqn.(2) fails to accurately represent the force time history and can perform poorly in predicting the dynamic response.

In part-I, the limitations of the Morison equation for predicting wave forces on fixed cylinders were investigated and a new Dynamic Morison equation was proposed. The second part the paper makes similar investigations to develop a corresponding new equation for the accurate prediction of wave forces on moving cylinders.

The organisation of the paper proceeds as follows. Section-2 describes the behaviour of the Morison equation for a responding cylinder in the frequency domain. Section-3 fits continuous time differential equation models between the inline force and the relative velocity. In section-

4 the performance of Morison equation models fitted to the data sets are shown. Based on the knowledge of the model structure from section-3 and hydrodynamic reasoning a new equation structure is proposed in section-5.

2 Frequency Domain Characteristics of the Morison Equation For Moving Cylinders

The mechanics of wave forces on fixed cylinders is a complex phenomenon which becomes increasingly complex when the cylinder responds dynamically. In this section the characteristics of the Morison equation based on relative motion (Relative Motion Morison Equation) given by

$$\begin{aligned} F(t) &= C_m A_i(\dot{u} - \ddot{x}) + A_i\ddot{x} + C_d A_d(u - \dot{x})|(u - \dot{x})| \\ &= K_m \dot{u}_r + A_i\ddot{x} + K_d u_r|u_r| \end{aligned} \tag{3}$$

where $u_r = (u - \dot{x})$ is the relative velocity, $K_d = 0.5\rho\pi C_d$ and $K_m = A_i C_m$ will be analysed in the frequency domain based on a Volterra series expansion. The direct Volterra series expansion of eqn(3) is not possible because of the presence of the drag term $u_r|u_r|$, and hence an approximated form of eqn(3) will be considered. It is possible to approximate the drag term $u_r|u_r|$ (Bendat and Piersol,1986) by

$$u_r|u_r| = a_1 u_r + a_3 u_r^3 + a_5 u_r^5 + \ldots.. \tag{4}$$

Eqn(3) can be expressed as

$$\begin{aligned} F(t) &= K_m \dot{u}_r + A_i\ddot{x} + K_d u_r|u_r| \\ &= K_m \dot{u}_r + A_i\ddot{x} + K_d((a_1 u_r + a_3 u_r^3 + a_5 u_r^5 + ..) \\ &= K_m \dot{u}_r + A_i\ddot{x} + K_{d1} u_r + K_{d3} u_r^3 + K_{d5} u_r^5 \end{aligned} \tag{5}$$

where $K_{d1} = K_d a_1$, $K_{d3} = K_{d1} a_3$ and so on.

Assuming that the drag term can be approximated by retaining up to the third order nonlinear term, the approximate Morison equation for a moving cylinder can be expressed as

$$F(t) = A_i\ddot{x} + K_m \dot{u}_r + K_{d1} u_r + K_{d3} u_r^3 \tag{6}$$

The linear transfer function between the output F(t) and the relative velocity u_r from eqn.(6) is given by

$$H_1(j\omega_1) = K_{d1} + j\omega_1 K_m \quad (7)$$

The transfer function suffer from high frequency instability as in the fixed cylinder case since as the frequency $\omega_1 \to \infty$, the magnitude $|H_1(j\omega_1)| \to \infty$ implying that when the cylinder is subjected to a high frequency input wave of very small amplitude, this will induce an extremely high force.

3 Discrete Time Modelling

The wave conditions for the experimental data sets used for modelling are identical to the Salford fixed cylinder data (Baker,1994) as described in Table-1 of Part-I of the paper. The flow parameters and the ratio of maximum cylinder velocity to the maximum input velocity are tabulated in Table-1.

Table-1: Flow Conditions for Responding Cylinder

Tests on Responding Cylinder	Flow Parameters (KC,β, Re)	$\frac{max(\dot{x})}{max(u)}$
Data Set-1	KC=4.74, β=667.2 Re=3.1638E+03	0.2737
Data Set-2	KC=4.95, β=653.11 Re=3.2332E+03	0.3273
Data Set-3	KC=4.46, β=649.51 Re=2.9021E+03	0.3662

From the table it is evident that the cylinder velocity is small in relation to the flow velocity. As a first step in modelling the data, the possibility of relating the inline force (actual output y) with the relative velocity (u-$\dot{x}$) which is compatible with our analysis procedure, will be explored. That is the cylinder acceleration term is omitted. To achieve this, the input-output and displacement data were decimated by a factor of 2 (effective sampling frequency of 25Hz) using the decimate function available in MATLAB (1992). This function filters the data with an eighth order Chebyshev Filter that is basically a lowpass filter having a cutoff frequency

$$f_{cutoff} = \frac{0.8 * f_s}{2R} \quad (8)$$

where f_s is the sampling frequency of the original data and $R = decimationfactor$. The spectral content of the present data is not lost due to this decimation. The above sampling was chosen using the procedures described in section-3 of Part-I of the paper. Discrete NARMAX models were fitted between y and $(u - \dot{x})$ with an initial model specification of $n_u = 3$, $n_y = 3$, $n_e = 10$, $N_l = 3$. The terms that were selected together with the associated parameter estimates, error reduction ratios and standard deviation of the parameters are tabulated in Table-2.1,2.2 and 2.3.

Table-2.1 : Results of the Orthogonal Estimator applied to Data Set-1

terms	estimates	ERR	$\sigma_{est.}$
y(k- 1)	0.17310E+01	0.97756E+00	0.10574E+00
y(k- 2)	-0.87252E+00	0.20326E-01	0.18252E+00
y(k- 3)	0.67490E-01	0.19022E-03	0.85476E-01
u_r(k- 3)	-0.18432E+01	0.41944E-04	0.19645E+00
u_r(k- 1)	0.19862E+01	0.29097E-03	0.21608E+00
u_r(k- 3)u_r(k- 3)u_r(k- 3)	0.11265E+02	0.61020E-04	0.19157E+01

0.478 e(k-1)+0.213e(k-5)-0.190e(k-2)+0.107e(k-7)-0.0747e(k-4)
+0.0692e(k-3)-0.0345e(k-8)-0.022e(k-10)-0.0188e(k-9)-0.028e(k-6)

Table-2.2 : Results of the Orthogonal Estimator applied to Data Set-2

terms	estimates	ERR	$\sigma_{est.}$
y(k- 1)	0.13461E+01	0.97745E+00	0.10913E+00
y(k- 2)	-0.21205E+00	0.21425E-01	0.18535E+00
y(k- 3)	-0.23536E+00	0.11446E-03	0.85756E-01
u_r(k- 1)u_r(k- 1)u_r(k- 1)	0.96770E+01	0.95923E-05	0.13079E+01
u_r(k- 3)	-0.19558E+01	0.72028E-05	0.43870E+00
u_r(k- 1)	0.23944E+01	0.18436E-03	0.29759E+00
u_r(k- 2)	-0.16772E+00	0.17910E-04	0.64868E+00

+0.944e(k-1)+0.268e(k-3)+0.301e(k-2)-0.0793e(k-6)+0.0657e(k-9)
-0.0735e(k-5)-0.0682e(k-4)+0.026e(k-10)+0.065e(k-7)-0.0394e(k-8)

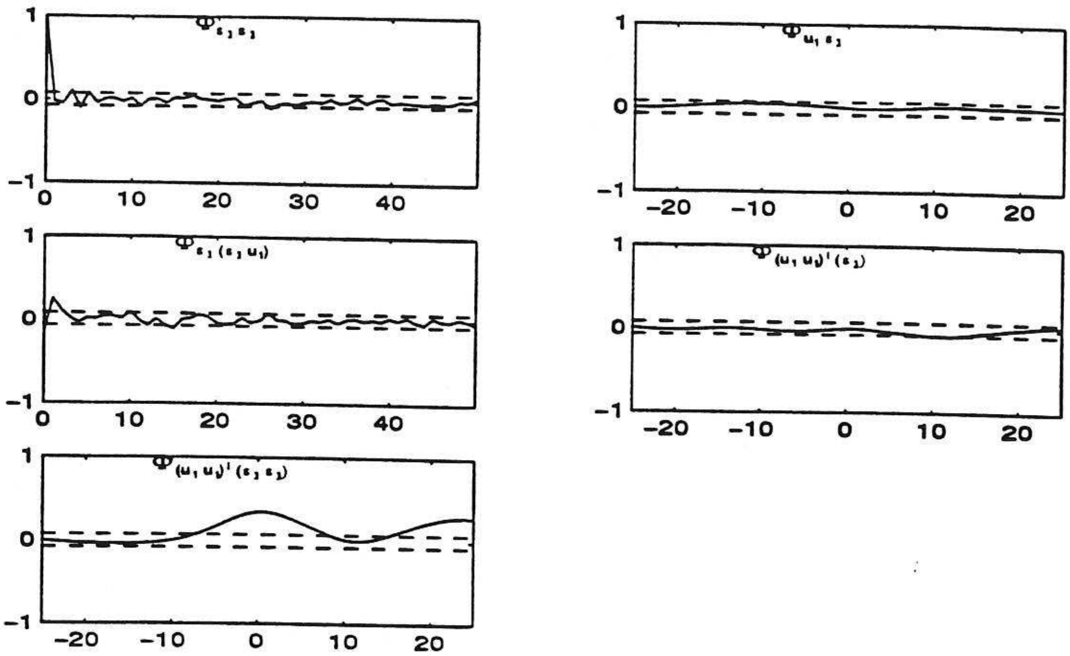

Figure 1: Correlation tests of the NARMAX model of moving cylinder based on relative velocity for data set-1

Table-2.3 : Results of the Orthogonal Estimator applied to Data Set-3

terms	estimates	ERR	$\sigma_{est.}$
y(k- 1)	0.17159E+01	0.97650E+00	0.24047E-01
y(k- 2)	-0.77429E+00	0.22530E-01	0.22424E-01
u_r(k- 2)u_r(k- 2)u_r(k- 2)	0.89534E+01	0.15349E-04	0.13035E+01
u_r(k- 1)	0.29072E+01	0.49061E-05	0.23855E+00
u_r(k- 2)	-0.28354E+01	0.14124E-03	0.23668E+00

+0.348e(k-3)+0.249e(k-1)+0.189e(k-5)-0.160e(k-4)-0.130e(k-8)
-0.0826e(k-2)-0.0625e(k-6)-0.0632e(k-7)-0.0605e(k-9)-0.0262e(k-10)

The correlation plots and model predicted output over the estimation and test sets for data set-1 are shown in Figure-1, 2 and 3.

Corresponding plots for the models fitted to the other data sets were quite satisfactory but are not given here to save space. The normalised mean squared errors based on one step ahead predictions and model predicted outputs are shown in Table-3.

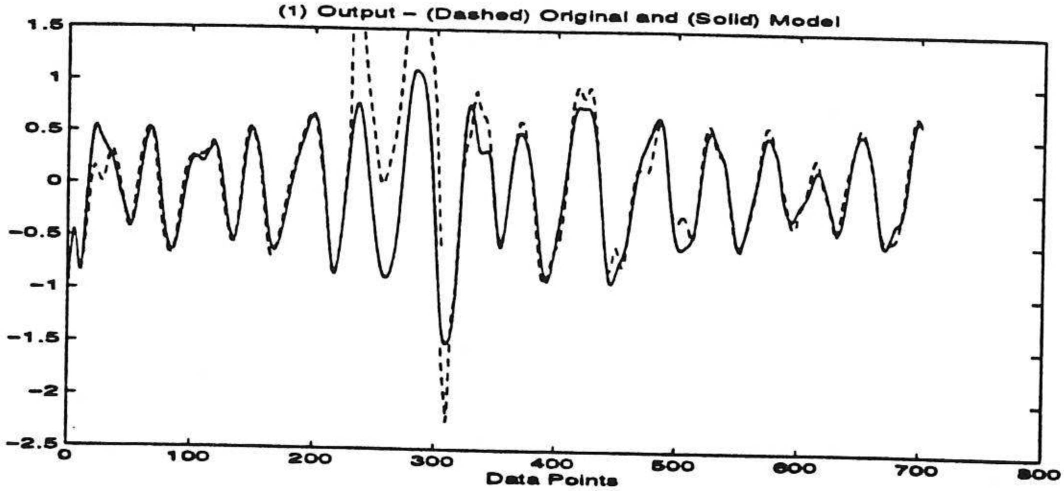

Figure 2: Model Predicted Output of the NARMAX model (relative velocity) for data set-1 :Estimation Set

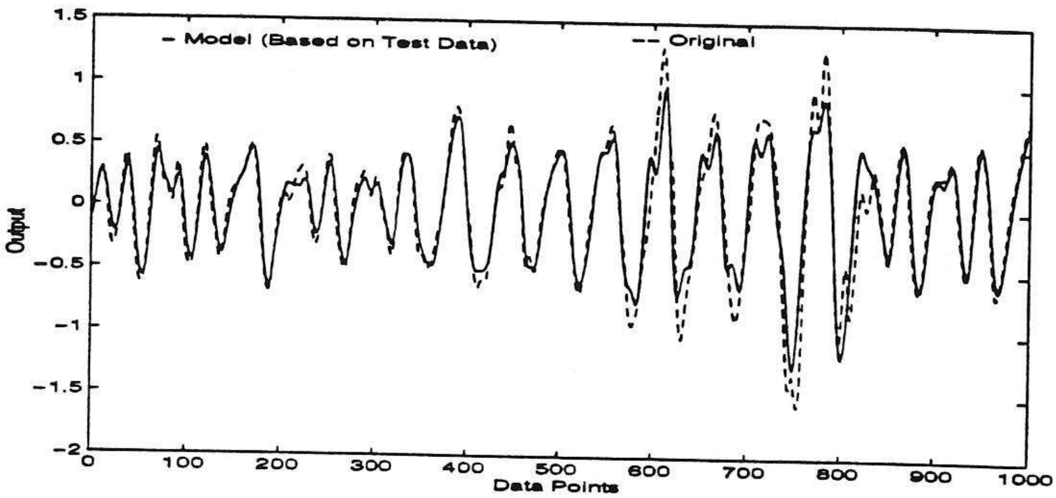

Figure 3: Model Predicted Output of the NARMAX model (relative velocity) for data set-1 over the Test Set

Table-3: Normalised Mean Square Errors (NMSE) for the NARMAX models

Model	NMSE of one-step ahead output	NMSE of Model Predicted Output
Data Set-1	0.00129	0.1193
Data Set-2	0.000658	0.1228
Data Set-3	0.000649	0.0974

The results suggest that the estimated NARMAX model is unbiased and the predictive performance of the model is excellent. Fig-3 shows that the model for data set-1 generalises well over the rest of the data points.

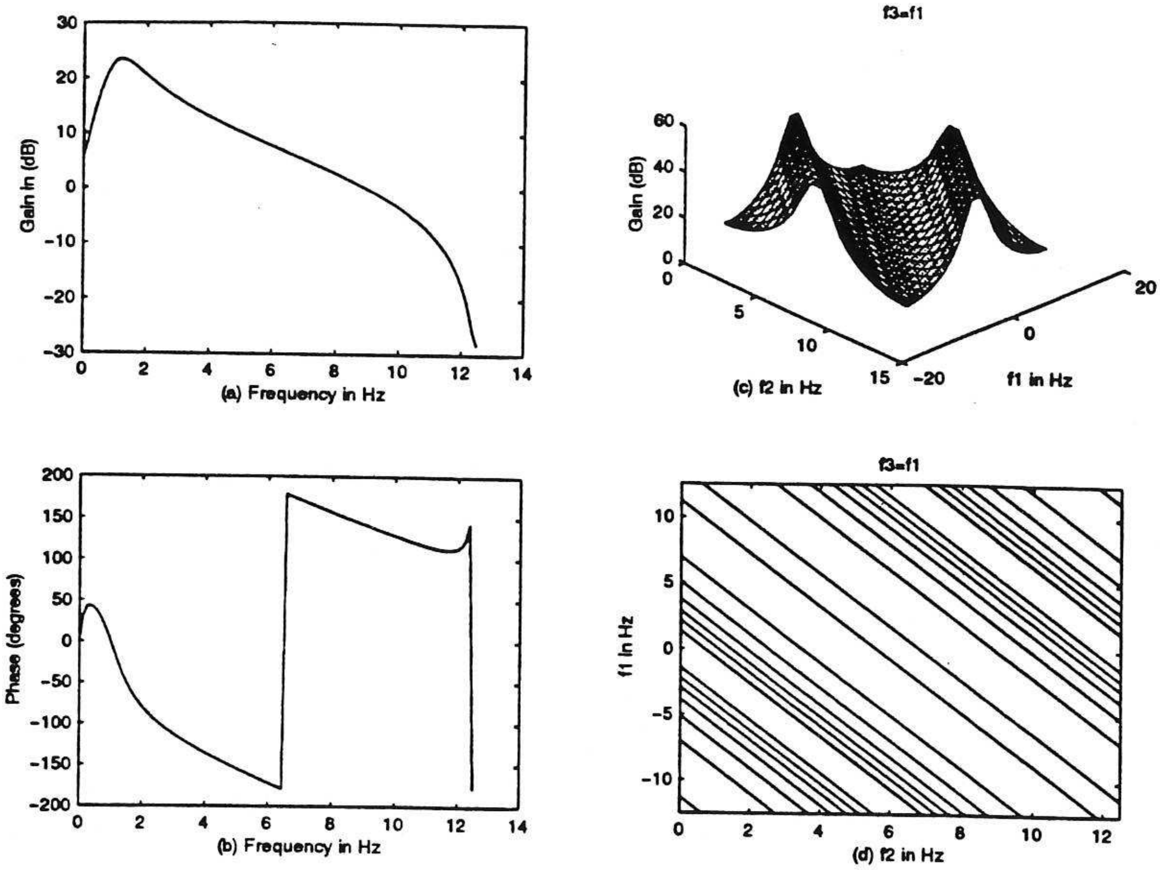

Figure 4: (a). Gain ($H_1(f_1)$ (b).Phase, (c) Gain $H_3(f_1, f_2, f_3)$ (d) Gain Contour for the NARMAX model(relative velocity) of Data-Set-1

3.1 Frequency Domain Analysis

Plots of the linear and third order frequency response functions between the inline force(output) and relative velocity(input) of the estimated discrete NARMAX model for data set-1 (Table-2.1) are shown in Fig-4a,b,c and d.

The plot of the magnitude of $H_1(f_1)$ shows a peak value of 23.53db at 1.2626Hz which corresponds to the linear resonant frequency and then decreases monotonically. This is in contrast to the linear frequency response of the standard form of the Morison model which exhibits a response which increases with frequency. Since the NARMAX model of the system does not contain any second order nonlinear terms, $H_2(f_1, f_2)$ is absent. The peak magnitude of $H_3(.)$ is found to be 44.46db compared with a maximum of 23.53db for $H_1(.)$ and thus shows that the system possesses dominant nonlinear characteristics which corresponds to the ridges in Fig-4d when $f_1 + f_2 + f_3 = 0.85$Hz. These can not be explained by the Morison's equation which has an $H_3(.)$ which is constant for all frequencies. The information from all the frequency response functions of the estimated models are shown in Table-4 and these show acceptable consistency despite the variability of the experimental conditions across the data sets.

Table-4: Characteristics of the Frequency Response Functions of Models of Table-2

Model	Resonant Frequency (Linear) in Hz	Max.Linear Gain (dB)	Ridge Equation	Max. Nonlinear Gain(dB)
Data Set-1	1.2626	23.53	$f_1+f_2+f_3=0.85$	44.46
Data Set-2	1.20	22.311	$f_1+f_2+f_3=0.875$	40.68
Data Set-3	1.05	22.1995	$f_1+f_2+f_3=0.75$	44.9945

3.2 Estimation of Nonlinear Continuous Time Models

Nonlinear continuous time models were reconstructed from the GFRF of the estimated NARMAX models. In order to reconstruct the linear part of the continuous time system, 100 equally spaced frequency response data were generated in the frequency range 0-5Hz and the weighting parameter'λ' was chosen to be 4.0 For the reconstruction of third order nonlinear part 64 equally spaced frequency response data were generated in the frequency range 0-0.15Hz. It was found that with the inclusion of a 'u_r^3' term the sum of the error reduction ratios values equaled 99.99% suggesting that the u_r^3 term is adequate to capture the nonlinear dynamics of the system. The results of the reconstruction for all the data sets are summarised in Table-5.

Table-5 : Summary of Results of Reconstruction

Reconstructed	Model	C_m	C_d	NMSE
Data Set-1	$0.02369\,\ddot{y}+0.1686\dot{y}+y=$ $2.0406\,\dot{u}_r+1.9518\,u_r+151.430\,u_r^3$	1.7985	1.8956	0.0784
Data Set-2	$0.02267\,\ddot{y}+0.15288\dot{y}+y=$ $1.6824\,\dot{u}_r+2.684\,u_r+95.162\,u_r^3$	1.4828	1.9138	0.0509
Data Set-3	$0.0276\,\ddot{y}+0.1671\dot{y}+y=$ $1.9572\,\dot{u}_r+1.24\,u_r+153.10\,u_r^3$	1.7250	1.5687	0.0970

4 Morison Model Vs Reconstructed Model

The Morison model fitted to data set-1 using traditional least squares is given by

$$y(t)=2.2652\dot{u}_r+1.1346\ddot{x}+47.4843u_r|u_r| \qquad (9)$$

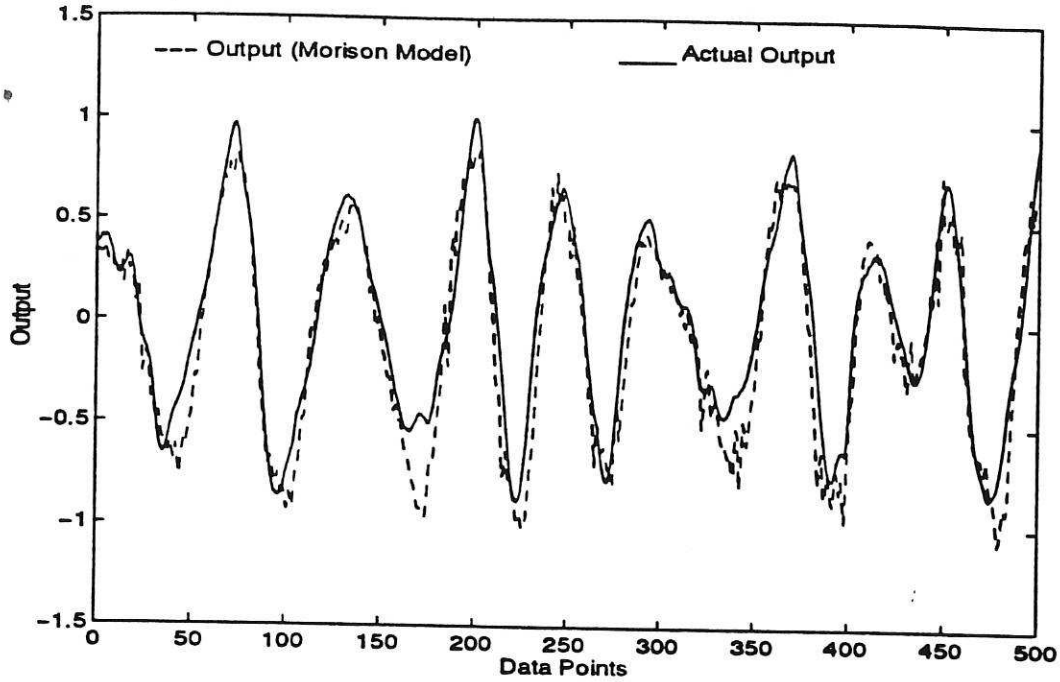

Figure 5: Morison Fit to Data Set-1

The approximated Morison model is given by

$$y(t) = 2.2639\dot{u}_r + 1.1346\ddot{x} + 3.7450u_r + 109.504u_r^3 \tag{10}$$

A comparison of the output of the Morison model with the original data is shown in Fig-5.

From the plot it is apparent that the model fits reasonably well. But it has been discussed in section-2 that the linear transfer function between the force and relative velocity will increase as the frequency increases showing high frequency instability. It should be emphasised that the monotonic increase in the linear transfer function computed from the Morison equation is inherent in the structure and not just the coefficients. Consequently any method of estimating the parameters in the Morison model can never lead to a transfer function which falls off for increasing frequency ; a feature exhibited by almost all the real data. The results of fitting Morison models to all the data sets are summarised in Table-6.

Table-6 : Morison Models For Responding Cylinder

	Model	C_m	C_d	NMSE
Data Set-1	$y = 2.2652\dot{u}_r + 1.1346\ \ddot{x} + 47.4843u_r\lvert u_r\rvert$	1.9965	2.4992	0.1553
Data Set-2	$y = 2.2853\dot{u}_r + 1.1346\ \ddot{x} + 37.1043u_r\lvert u_r\rvert$	2.0142	1.9528	0.1450
Data Set-3	$y = 2.4099\dot{u}_r + 1.1346\ \ddot{x} + 35.2771u_r\lvert u_r\rvert$	2.124	1.8567	0.1049

5 Proposed Equation Structure

A successful model should exhibit the following properties :

- fit to the input-output data
- represent the underlying dynamics of the system
- be structurally stable
- contain just a few parameters which can be physically interpreted

In section-4 models based on the Morison equation were estimated for different data sets and were shown to fit the data quite well but fail to emulate the underlying dynamics. The Morison equation therefore only satisfies the first and last criteria above.

Nonlinear continuous time models have been fitted between the output (force) and relative velocity in section-3. The estimation procedure makes use of the orthogonal least squares algorithm of Billings et al(1988) and the weighted complex orthogonal estimator of Swain and Billings,(1995) to correctly determine the structure of the model and to estimate the unknown parameters. The structure of the models estimated based on the relative velocity are of the form

$$\alpha_2\ddot{y} + \alpha_1\dot{y} + y = \beta_m\dot{u}_r + \beta_{d1}u_r + \beta_{d3}u_r^3 \tag{11}$$

The model has been shown to predict the wave force accurately and appears to capture the underlying physics. It has been shown that this model does not suffer from high frequency instability and is structurally sufficient to represent all the dynamic features of the data. The last two terms in the models are assumed to be associated with the drag term.

Models based on eqn(11) do not however include the effect of cylinder acceleration. Eqn.(11) may be simply extended to include cylinder acceleration which is likely to become more significant as the maximum cylinder velocity increases in relation to the maximum fluid velocity by a model structure of the form

$$\alpha_2\ddot{y} + \alpha_1\dot{y} + y = A_i\ddot{x} + \beta_m\dot{u}_r + \beta_d u_r|u_r| \tag{12}$$

Inclusion of higher order derivative terms in the original Morison equation (eqn(2)) again avoids the feature of high frequency instability in $H_1(.)$ and also allows the model to be structurally sufficient to capture the history effects. Models based on eqn(12) were therefore fitted and the results are summarised in Table-7.

Table-7 : Summary of Results of Proposed Model

	Model	C_m	C_d	NMSE
Data Set-1	$0.0162\,\ddot{y}+0.1124\dot{y}+y=$ $1.1346\,\ddot{x}+1.8459\,\dot{u}_r+49.7482\,u_r\|u_r$	1.6269	2.6183	0.0453
Data Set-2	$0.0181\,\ddot{y}+0.1272\dot{y}+y=$ $1.1346\,\ddot{x}+1.8174\,\dot{u}_r+47.1716u_r\|u_r\|$	1.6018	2.4827	0.0539
Data Set-3	$0.0268\,\ddot{y}+0.1321\dot{y}+y=$ $1.1346\,\ddot{x}+1.9840\,\dot{u}_r+40.20\,u_r\|u_r\|$	1.7486	2.1161	0.0324

The Normalised Mean Square Error (NMSE) for the Dynamic Morison Equation without history terms for models of data set-1,2 and 3 are 0.1284,0.1405 and 0.1111 respectively. As for the fixed cylinder, the Dynamic Morison equation(12) has two additional terms which may be non-dimensionalised with reference to a time scale which we again choose to be associated with the mid point of the rectangular wave spectrum. Eqn(12) is thus given by

$$\gamma_2 T_w^2 \ddot{y} + \gamma_1 T_w \dot{y} + y = A_i \ddot{x} + \beta_m \dot{u}_r + \beta_d u_r |u_r| \qquad (13)$$

This is again rewritten as

$$\gamma_0(\gamma_1 T_w)^2 \ddot{y} + \gamma_1 T_w \dot{y} + y = A_i \ddot{x} + \beta_m \dot{u}_r + \beta_d u_r |u_r| \qquad (14)$$

and values of γ_0, γ_1, C_m and C_d together with the test conditions are given in Table-8. The corresponding results for the fixed cylinder are shown in Table-9 (Table-17 of Part-I of the paper).

Table-8 : Summary of Results for Responding Cylinder

Model	T_w	γ_0	γ_1	C_m	C_d	KC	Re
Data Set-1	2.164	1.282	0.05195	1.627	2.618	4.74	3.16×10^3
Data Set-2	2.2109	1.188	0.0575	1.602	2.483	4.95	3.23×10^3
Data Set-3	2.2232	1.537	0.0594	1.749	2.116	4.46	2.90×10^3

Table-9 : Summary of Results for Fixed Cylinder

Model	T_w	γ_0	γ_1	C_m	C_d	KC	Re
Data Set-1	2.164	0.90	0.103	1.89	1.82	4.74	3.16×10^3
Data Set-2	2.2109	0.83	0.0997	1.84	1.75	4.95	3.23×10^3
Data Set-3	2.2232	1.26	0.0869	1.67	1.74	4.46	2.90×10^3

The γ_0 values are slightly larger for the responding cylinder while the γ_1 values are roughly halved suggesting the history effects to be less. This is consistent with cylinder motion reducing the flow velocity relative to the cylinder and thus creating a less strong wake and weaker vortex shedding. The C_m values are changed slightly by cylinder response while the C_d values are increased by up to 40%. This should be associated with a stronger wake which is inconsistent with the previous argument. Clearly the influence of cylinder response on wake formation and vortex shedding is complex and while the Dynamic Morison equation with relative motion terms captures the underlying dynamic structure, the effect of the relative values of the non-dimensional coefficients do not have a simple physical interpretation. Prediction of cylinder response is however of great practical importance and progress would be made by further analysis of data, preferably at high Reynolds numbers associated with full scale flows.

6 Conclusions

A nonlinear continuous time model has been estimated for the wave forces on cylinders free to respond dynamically. The model is the same as that for a fixed cylinder with the inclusion of relative velocity and the cylinder acceleration terms. The Dynamic Morison equation based on relative motion captures the underlying dynamics while the simple Morison equation based on relative velocity does not represent the true dynamics. The data analysed are only for a small range of wave flume tests but the non-dimensional coefficients show some consistency with fixed cylinder values for identical wave conditions apart from the drag coefficient which can be markedly increased due to cylinder response. Analysis of more data sets, preferably at high Reynolds number, is needed to establish the Relative Motion Dynamic Morison equation as a predictive tool for practical application.

Acknowledgments

AKS gratefully acknowledges the financial support provided by the Commonwealth Scholarship Commission of the United Kingdom and is thankful to Board of Governors, Regional Engineering College, Rourkela, India for granting study leave. SAB gratefully acknowledges that part of this work was supported by EPSRC.

References

[1] Baker,M (1994), "Wave loading on a small diameter flexibly mounted cylinder in random waves", Ph.D dissertation, University of Salford.

[2] Bendat,J.S and Piersol,A.G.(1986),"Decomposition of Wave Forces into Linear and Nonlinear Components", J. Sound and Vibration, Vol-106(3), pp.391-408.

[3] Billings,S.A.,Korenberg,M.J. and Chen,S (1988),"Identification of nonlinear output affine systems using an orthogonal least square algorithm",Int.J. System Science,Vol.19,pp.1559-1568

[4] Chakrabarti,S.K.(1987), "Hydrodynamics of Offshore Structures" ,Southampton: Computational Mechanics Publications, Springer Verlag.

[5] Leontaritis,I.J. and Billings,S.A. (1985), "Input-Output parametric models for Nonlinear Systems,Part-I-Deterministic Nonlinear Systems, Part-II-Stochastic Nonlinear Systems,Int.J.Control,vol.41, pp.303-344.

[6] MATLAB (1992), "MATLAB High Performance Numeric Computation and Visualization Software : Reference Guide", The MATH WORKS Inc.

[7] Palm,G. and Poggio,T. (1977) "The Volterra representation and the Wiener expansion: validity and pitfall ", SIAM J. on Applied Mathematics, 33, pt-2

[8] Slaouti,A. and Stansby,P.K.(1992), "Response of a Circular Cylinder in Regular and Random Oscillatory Flow at KC=10", Proc. BOSS-92, pp.308-321, London.

[9] Swain,A.K. and Billings,S.A.(1995), "Weighted Complex Orthogonal Estimator for Identifying Linear and Nonlinear Continuous Time Models from Generalised Frequency Response Functions", Submitted for publication

[10] Swain,A.K., Billings,S.A., Stansby,P.K. and Baker,M. (1996), "Accurate Prediction of Nonlinear Wave Forces : Part-I (Fixed Cylinder)"