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Adaptive control of nonlinear dynamical systems using a model reference approach

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1. Introduction

Control of systems which exhibit nonlinear dynamical behavior, particularly chaos, has been an active area of interest in recent years following the work of Ott, Grebogi, & Yorke (1990). In addition to this approach, nonlinear systems which exhibit chaotic dynamics have also been controlled using a more standard control engineering approach including techniques such as feedback linearization and adaptive control (Di Benardo, 1996). Of these approaches, adaptive control is often most useful in a practical engineering environment, as it requires only a limited knowledge of the plant structure and parameters. When a reference model is used, adaptive controllers can also be applied to systems where the details of the plant cannot be fully known *a priori* or vary with time. Using these type of algorithms without knowledge of plant parameters, such that we assume zero initial conditions for the controller gains, has become known as the "minimal control synthesis" approach (Stoten, 1993). Basing adaptive control schemes on a reference model enables the system to be controlled to behave like the model itself. This type of approach is applied to a wide range of systems, but is based primarily on linear control theory, as the required model is usually a linear one. In this type of system, the effect of nonlinearities and/or disturbances is compensated for by the adaptive nature of the controller.

In this paper we review linear reference model adaptive control, in particular the "minimal control synthesis" approach. We consider how linear adaptive controllers are designed using both hyperstability and Lyapunov stability analysis. We then discuss using model reference adaptive controllers to control systems which exhibit nonlinear dynamical behaviour.

We consider the example of a Duffing oscillator being controlled to follow a linear reference model. For this system we show that if the

nonlinearity is "small" then standard linear model reference control can be applied. A second example, which is often found in synchronization applications, is when the nonlinearities in the plant and reference model are identical. Again we show that linear model reference adaptive control is sufficient to control the system.

Finally we consider controlling more general nonlinear systems. For these systems we discuss how adaptive feedback linearization control can be used to control scalar systems. As an example we use two systems with chaotic behaviour; the Lorenz and Chua systems. The Lorenz system is used as a reference model and a single coordinate from the Chua system is controlled to follow one of the Lorenz system coordinates. Some examples of using feedback linearization to control mechanical systems are given by Goodwine & Stépán (2000) who consider control of towed wheels, Lin & Lin (2001) who consider spacecraft control and Menson & Ohlmeyer (2001) who consider missile guidance and autopilot systems.

2. Theoretical formulation for general linear systems

We begin by considering the general *linear* state space formulation for model reference adaptive control. The state equation can be written in the form

$$\dot{x}(t) = Ax(t) + Bu(t) + d(t), \tag{1}$$

where x is the state variable vector, u the control signal vector, A represents the linear dynamics of the plant, B the linear dynamics of the controller and $d(t)$ is a disturbance term.

In general we consider a controller which uses both the state variables and the reference signal, factored respectively by the adaptive gains $K(t)$ and $K_r(t)$, such that

$$u(t) = K(t)x(t) + K_r(t)r(t), \tag{2}$$

where $r(t)$ is the reference (demand) signal, $K(t)$ is the feedback adaptive gain and $K_r(t)$ the feed forward adaptive gain. This type of assumption on the structure of the control signal requires that the demand signal r has sufficient persistence of excitation (Sastry & Bodson, 1989).

Substituting equation (2) into equation (1) gives

$$\dot{x} = Ax + B(Kx + K_r r) = (A + BK)x + BK_r r + d(t). \tag{3}$$

The plant is controlled to follow the output from a reference model with known dynamics

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t), \tag{4}$$

2

where x_m is the state of the reference model and A_m and B_m are linear reference equivalents of A and B.

We need to monitor the performance, or effectiveness of the controller, and this is done using the error signal $x_e = x_m - x$. The objective of the control algorithm is $x_e \to 0$ as $t \to \infty$. We can reformulate the dynamics of the system as the error dynamics

$$\dot{x}_e = A_m x_e + (A_m - A - BK)x + (B_m - BK_r)r - d(t). \qquad (5)$$

Comparing equations (4) and (3), we can see that for exact matching between the plant and the reference model, the following relations hold

$$\begin{aligned} A + BK^* &= A_m \\ BK_r^* &= B_m \end{aligned} \qquad (6)$$

where $()^*$ denotes a constant value.

Assuming all matrix (pseudo) inverses exist we can obtain expressions (generally known as the Erzberger conditions (Khalil, 1992) for the gain values as

$$\begin{aligned} K^* &= B^{-1}(A_m - A) \\ K_r^* &= B^{-1}B_m \end{aligned} . \qquad (7)$$

However, one of the main aims of model reference adaptive control is to operate without explicit knowledge of the plant parameter values, which in this case are the matrices A and B. In fact one of the most significant advantages of this type of controller is that it can operate without explicit values for the matrices A and B. As a result we cannot usually solve equation (7), but we can use it to express the matrix terms given in equation (5), as

$$\begin{aligned} B(K^* - K) &= (A_m - A - BK) \\ B(K_r^* - K_r) &= (B_m - BK_r) \end{aligned} . \qquad (8)$$

Now we define functions which represent the variation of the adaptive gains $K(t)$ and $K_r(t)$ from the exact matching solution values K^* and K_r^*. Let

$$\begin{aligned} \phi(t) &= (K^* - K(t)) \\ \psi(t) &= (K_r^* - K_r(t)) \end{aligned} , \qquad (9)$$

then using equation (6) and (9) we can rewrite equation (5) as

$$\dot{x}_e = A_m x_e + B(\phi x + \psi r) - d(t) \qquad (10)$$

(Banks, 1986, Khalil, 1992). This can be further abbreviated by defining $\Phi(t) = \{\phi, \psi\}^T$ and $w(t) = \{x, r\}^T$, such that

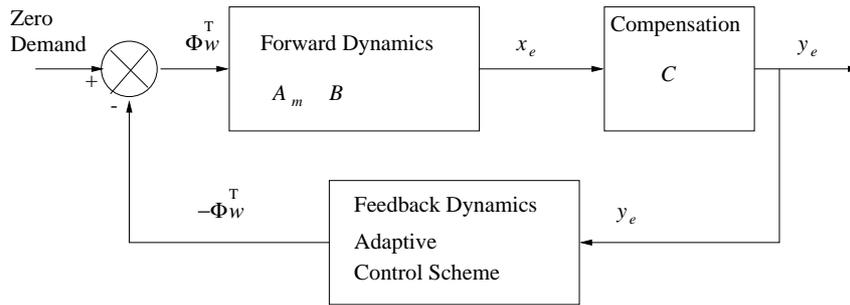$$\dot{x}_e = A_m x_e + B\Phi^T w - d(t) \qquad (11)$$

3

Figure 1. Schematic representation of error dynamics as a feedback system.

Thus the error system can be reduced to a single ordinary differential equation, with distinct linear and nonlinear parts. Note by denoting $k = \{K, K_r\}^T$ and $k^* = \{K^*, K_r^*\}^T$, we can write

$$\Phi = (k^* - k). \tag{12}$$

In this formulation Φ can be thought of as a gain or parameter error: i.e. the difference between the exact matching conditions and the current adaptive gain values.

3. Controller design

In general, for linear systems, we wish to design an adaptive controller for as wide a class of systems $\{A, B\}$ as possible. Of primary importance is the overall stability of the system. Thus controller design is usually approached with some stability criteria in mind. For adaptive systems there are two commonly used stability criteria; Lyapunov stability and hyperstability. We will first consider hyperstability, often referred to as the Popov Criteria (see Banks (1986) for a derivation). In both stability analyses we exclude the disturbance term $d(t)$ which is assumed to be small and bounded, and therefore to be a robustness not a stability issue (Sastry & Bodson, 1989).

A key feature of the hyperstability criteria is that the system can be divided into "blocks", each of which can be proved stable independently. This technique can greatly simplify the stability problem for a wide class of control systems (Landau, 1979). In the case of the model reference system, we spilt the system into a feed forward block and a feedback block (e.g. see Stoten & Benchoubane (1990)). This is illustrated schematically in figure 1.

In figure 1, the schematic feedback system has a zero demand signal corresponding to the desired equilibrium solution of the error dynamics;

$x_e = 0$. The feed forward dynamic block is represented by equation (11) and gives an output error signal x_e. In the general case we assume that the error signal is compensated via the constant linear matrix C such that $y_e = Cx_e$. C can be chosen to ensure the stability of the feed forward block (Landau, 1979). This compensated error is used in the adaptive control scheme which is represented as a feedback block in figure 1. The control scheme gives an output of $-\Phi(t)w(t)$ which is added to the demand in the feedback loop.

Having split the system into forward and feedback blocks, we use hyperstability to show the stability of each block. The forward dynamics are linear, and we can show that $\{A_m, B, C\}$ form a hyperstable block using the Lyapunov equation

$$A_m^T P + P A_m = -Q, \tag{13}$$

where $C = B^T P$ and Q is positive definite. In other words the transfer function $C(sI-A)^{-1}B$ must be strictly positive real (SPR). This is now referred to as the Kalman-Yacoubovich-Popov Lemma, and a proof can be found in Sastry 1999.

For the feedback block, we use the integral inequality

$$\int_{t_0}^{t} y_e(t)(-\Phi(t)^T w(t))\mathrm{d}t \geq -\gamma^2, \tag{14}$$

where γ is a finite constant. By differentiating, equation (14) can be expressed as

$$-y_e \Phi^T w(t) \geq 0. \tag{15}$$

from which we can see that the solution $\Phi^T = -C_1 y_e w^T$, $C_1 > 0$ satisfies the inequality. This is effectively a simple proportional controller, however we need a controller which maintains a gain value as $y_e \to 0$. Adding an integral term will allow the adapted value to remain when $y_e = 0$. Thus we adopt an adaptive gain of the form

$$k(t) = C_1 y_e w + C_2 \int_{t_0}^{t} y_e w \mathrm{d}t, \tag{16}$$

where C_1 and C_2 are arbitrary positive constants. By denoting $k(t) = k_1 + k_2$ as the sum of the proportional and integral parts, we note that $k_1 = (C_1/C_2)\dot{k}_2$ and that $\Phi = (k^* - (k_1 + k_2))$ and $-k_1 = -C_1 y_e w$. Now we partition Φ such that $\Phi = \Phi_\alpha + \Phi_\beta$, where $\Phi_\alpha = k^* - k_2$ and $\Phi_\beta = -k_1$. From this we note that $\Phi_\beta = -k_1 = (C_1/C_2)\dot{\Phi}_\alpha$ (k^* is a constant so it goes to zero when differentiated). Then substituting these relations into equation (14) gives firstly

$$-\int_{t_0}^{t} y_e(t)(\Phi_\alpha + \Phi_\beta)w(t)\mathrm{d}t \geq -\gamma^2, \tag{17}$$

then, substituting $-y_e w = -k_1/C_1 = \Phi_\beta/C_1 = \dot{\Phi}_\alpha/C_2$ we can write

$$\frac{1}{C_2} \int_{t_0}^{t} \Phi_\alpha \dot{\Phi}_\alpha \mathrm{d}t + \frac{1}{C_1} \int_{t_0}^{t} \Phi_\beta \Phi_\beta \mathrm{d}t \geq -\gamma^2. \qquad (18)$$

The second integral term is now always greater than zero providing $C_1 > 0$. For the first integral term we have that,

$$\frac{1}{C_2} \int_{t_0}^{t} \Phi_\alpha \dot{\Phi}_\alpha \geq -\frac{1}{2C_2} \Phi_\alpha(t_0)^T \Phi_\alpha(t_0) \geq -\gamma^2, \qquad (19)$$

using the relation

$$\int_{t_0}^{t} f(t) \dot{f}(t) \mathrm{d}t = \frac{1}{2} (f^2(t) - f^2(t_0)) \geq -\frac{1}{2} f(t_0)^2, \qquad (20)$$

($f(t)$ scalar). Thus the hyperstability condition is satisfied (Popov, 1973).

For general model reference adaptive control, the gains are defined setting $C_1 = \beta$ and $C_2 = \alpha$ such that

$$k = \alpha \int_{0}^{t} y_e w(t) \mathrm{d}t + \beta y_e w(t). \qquad (21)$$

Equation (21) gives an expression which computes the adaptive gain values, $k(t)$, from time $t = 0$ to an arbitrary time t. However, α and β need to be selected in advance, and clearly have a significant influence on the rate of adaption. As yet there is no theoretical approach to choosing values for α and β.

3.1. LYAPUNOV STABILITY ANALYSIS

An alternative stability criteria is Lyapunov stability, see for example Sastry (1999). Here we consider the analysis of a single input single output system with error dynamics

$$\dot{x}_e = -a_m x_e + b \Phi^T w \qquad (22)$$

where $x_e, a_m > 0$ and b are scalar. As with the hyperstability proof, we define $\Phi = \Phi_\alpha + \Phi_\beta$. To prove the stability of the system, we select a trial Lyapunov function of

$$V = \frac{x_e^2}{2} + \frac{b}{2\alpha C} \Phi_\alpha^T \Phi_\alpha \qquad (23)$$

where C in this example is positive scalar. Equation (23) is of quadratic form, and as a result can be used to prove the global stability of the system. This is because if $b/(\alpha C) > 0$, then V is a positive real function

(Sastry, 1999). Note also we are only using Φ_α *not* Φ to define the Lyapunov function.

Essentially, we must show that the derivative of the Lyapunov function $\dot{V}$ is negative definite for all initial values and for all time. The derivative of V is given by

$$\dot{V} = x_e \dot{x}_e + \frac{b}{\alpha C} \Phi_\alpha^T \dot{\Phi}_\alpha. \tag{24}$$

Substituting for $\dot{x}_e$ using equation (22) gives

$$\dot{V} = -a_m x_e^2 + x_e b (\Phi_\alpha + \Phi_\beta)^T w + \frac{b}{\alpha C} \Phi_\alpha^T \dot{\Phi}_\alpha. \tag{25}$$

The key step is now to substitute for $\dot{\Phi}_\alpha$, which from the previous section, equation (18), can be expressed as $\dot{\Phi}_\alpha = -\alpha y_e w$, which gives

$$\dot{V} = -a_m x_e^2 - x_e b \Phi_\beta^T w. \tag{26}$$

Finally substituting for Φ_β gives

$$\dot{V} = -a_m x_e^2 - \beta C x_e^2 w^T w, \tag{27}$$

which for $a_m, \beta, C > 0$ is always negative definite. A matrix version of this type of analysis can be found for example in Landau (1979) or Sastry (1999). In addition, an interesting variation of this type of analysis for non-quadratic Lyapunov functions is described by Rao (1998). A comparison between hyperstability and Lyapunov stability analysis is given in Narendra & Valavani (1980).

4. Nonlinear Systems

We now consider the case when both the plant and reference model have a nonlinear structure (without a disturbance) of the form

$$\begin{aligned} \dot{x}(t) &= f_1(x,t) + g_1(u,t), \\ \dot{x}_m(t) &= f_2(x_m,t) + g_2(r,t), \end{aligned} \tag{28}$$

where f_1, f_2, g_1 and g_2 are nonlinear functions. In this work we will consider the case where these functions are nonlinear but have conditions on their structure. For example in the case where g_1 and g_2 have the standard linear structure $g_1 = Bu$ and $g_2 = B_m r(t)$, and f_1 and f_2 can be written in affine form: $f_1 = Ax + h_1(t)$ and $f_1 = A_m x_m + h_2(t)$, then we can write the error dynamics as equation (11) with $d(t) = h_1 - h_2$. In this case the nonlinearities h_1 and h_2 are expressed as a disturbance to

7

the linear system. This approach can be applied to systems where the term $d(t)$ remains small and bounded such that the linear stability analyses can be applied. For example we can use this approach to control a range of nonlinear systems to behave in a linear way providing the nonlinearity remains small.

Example 1: Nonlinearity small and bounded
In this case $h_2 = 0$, and $d(t) = h_1$. So, if we consider the case where a linear reference model is used, and the plant is a Duffing type oscillator

$$
\begin{aligned}
\ddot{x} + \delta\dot{x} + \sigma x - \epsilon x^3 &= u(t) \\
\ddot{x}_m + \delta\dot{x}_m + \sigma x_m &= q\cos(\omega t)
\end{aligned} \quad . \tag{29}
$$

then for small ϵ the Duffing oscillator can be controlled to track the output of the reference model. A numerical example of this using MCS control is shown in figure 2, where $\delta = 0.1$ and $\sigma = q = \omega = 1$. In figure 2 (a) we show the linear case, $\epsilon = 0$ where the plant tracks the model output after approximately 15 seconds. In figure 2 (b) we show the case when $\epsilon = 0.1$, where again the plant tracks the model output after approximately 15 seconds. However in this case the effect of the nonlinearity means that the control signal is significantly larger in amplitude after tracking occurs than in the linear case. For this particular example linear control works up to approximately $\epsilon = 0.5$ after which the tracking deteriorates significantly. However, by decreasing the numerical integration timestep and increasing controller gain values it is possible to simulate stable control for larger ϵ values.

4.1. A GENERAL FORMULATION FOR NONLINEAR SYSTEMS

For general nonlinear functions we can write the error dynamics as

$$
\dot{x}_e(t) = \dot{x}_m(t) - \dot{x}(t) = f_2(x_m, t) - f_1(x, t) + g_2(r, t) - g_1(u, t). \tag{30}
$$

This can then be expressed as

$$
\dot{x}_e(t) = \Delta f(t) + \Delta g(u, t) . \tag{31}
$$

where $\Delta f(t) = f_2 - f_1$ and $\Delta g(t) = g_2 - g_1$ representing the difference in plant and controller dynamics. For plant and reference models with identical nonlinearities, $\Delta f(t)$ will have a particular form such that the control becomes linear in the steady state as demonstrated by the following example.

Example 2: Identical nonlinearities in the system
In the case where the reference model and plant have identical non-linearities, such as in many synchronization problems, linear adaptive

8
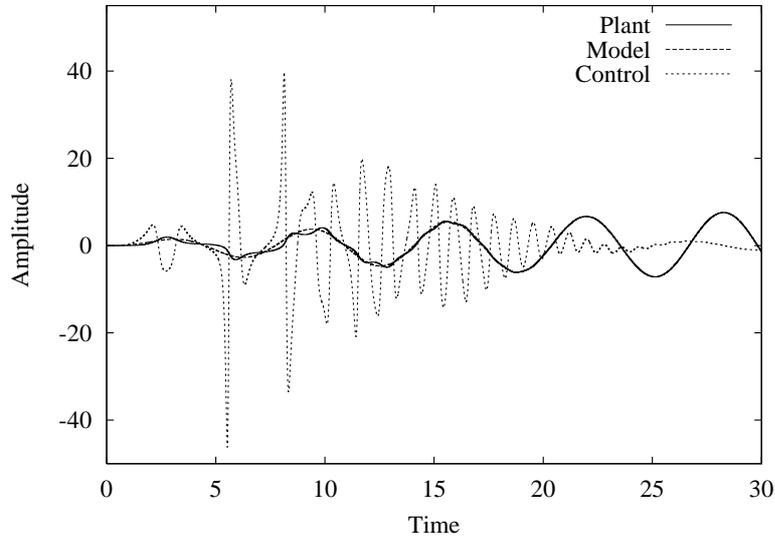
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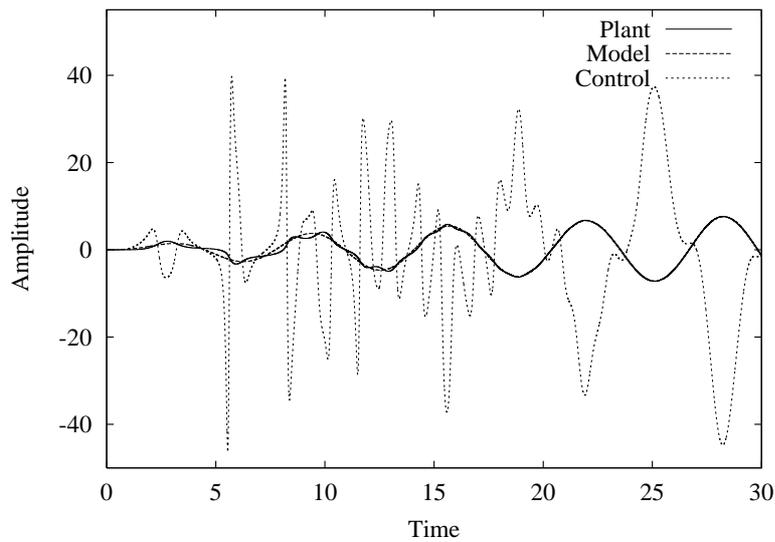
(b)

Figure 2. Numerical simulation of Duffing system being controlled to the output of a linear reference model using MCS. The reference signal is a sine wave with amplitude and frequency equal to 1. The controller parameters are $\alpha = 0.1$, $\beta = 0.01$ and settling time $t_s = 0.5$. (a) response of the system without nonlinearity $\epsilon = 0$, (b) response of the system with $\epsilon = 0.1$.

control can often be used successfully (Stoten & Di Bernardo, 1996). For example the problem discussed in Stoten & Di Bernardo (1996) is to synchronize the motion of two Duffing oscillators

$$\ddot{x} + \delta\dot{x} - \sigma x + \epsilon x^3 = q\cos(\omega t) + u(t)$$
$$\ddot{x}_m + \delta_m\dot{x}_m - \sigma_m x_m + \epsilon_m x_m^3 = q_m\cos(\omega t) \quad . \tag{32}$$

In this case $\sigma_m = \sigma = 1$, $\delta_m = \delta = 0.4$ and $\epsilon_m = \epsilon = 1$, $\omega = 1.8$, $q = 1.8$ and $q_m = 0.62$. As a result the error dynamics can be written as

$$\begin{bmatrix} \dot{x}_{e1} \\ \dot{x}_{e2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \sigma_m & -\delta_m \end{bmatrix} \begin{bmatrix} x_{e1} \\ x_{e2} \end{bmatrix} + \begin{bmatrix} 0 \\ -(x_m^3 - x^3) + (q_m - q)\cos(\omega t) + u(t) \end{bmatrix} \tag{33}$$

Here we have selected the parameter values such that the reference model has periodic behaviour when the initial conditions at the start of the simulation are $x_m(0) = 0.348751$ and $\dot{x}_m(0) = 0.194153$. The plant has zero initial conditions, and a forcing amplitude which results in chaotic motion. In figure 3 (a) the controller is disabled, $u(t) = 0$, and the model and plant motions are quite different. In figure 3 (b) the controller is enabled and the two systems quickly synchronize.

We also note from equation (33) that as the two system synchronize $x \to x_m$ then the $(x_m^3 - x^3)$ term tends to zero and the system becomes effectively linear in the steady state. Thus for this type of system, the linear adaptive controller only has to cope with nonlinear dynamics during the transient (or adaptive) stage.

4.2. ADAPTIVE FEEDBACK LINEARIZATION CONTROL

Dealing with generalised nonlinear adaptive control systems such as equation (31) is as yet an unsolved problem and an active area of research interest (Khalil, 1992), (Krstić, *et al*, 1995) (Sastry, 1999). A key approach to these types of systems is that of linearization (Sastry, 1999). In this work, we will consider the (scalar) case when $g_2 = 0$ such that we can write

$$\dot{x}_e(t) = -\lambda x_e + L - g_1(u, t), \tag{34}$$

where $L = \Delta f + \lambda x_e$, and $\lambda > 0$. This formulation is possible with a wide variety of both linear and nonlinear systems (Di Benardo, 1996). It is clear from equation (34) that $(L - g_1(u, t)) \to 0$, and $\lambda > 0$ will stabilize the required equilibrium, $x_e = 0$. Therefore if the explicit structure of f_1 and f_2 is known, L is the feedback linearization controller for the system. e.g. $g_1 = L$ and $\lambda > 0$ gives $x_e \to 0$ as $t \to \infty$.

For systems where the structure of f_1 and f_2 is known but actual parameter values are not, then we consider the class of systems where L can be expressed as $L = k^*\xi$, where k^* represents a set
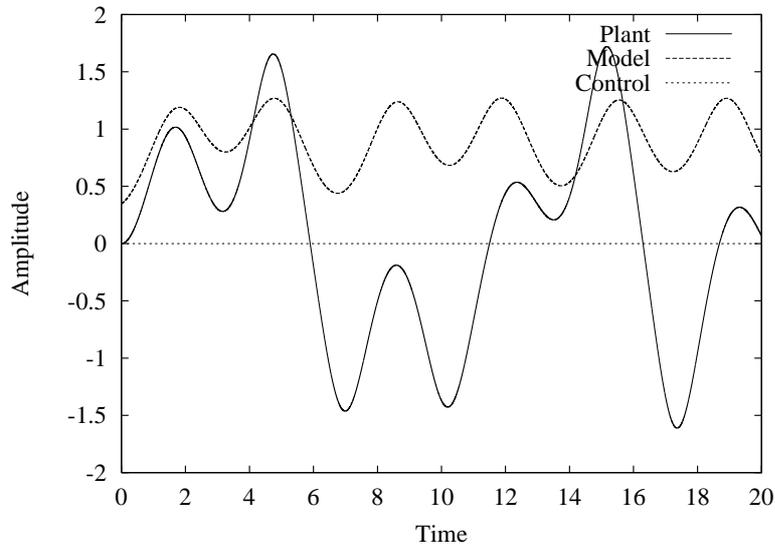
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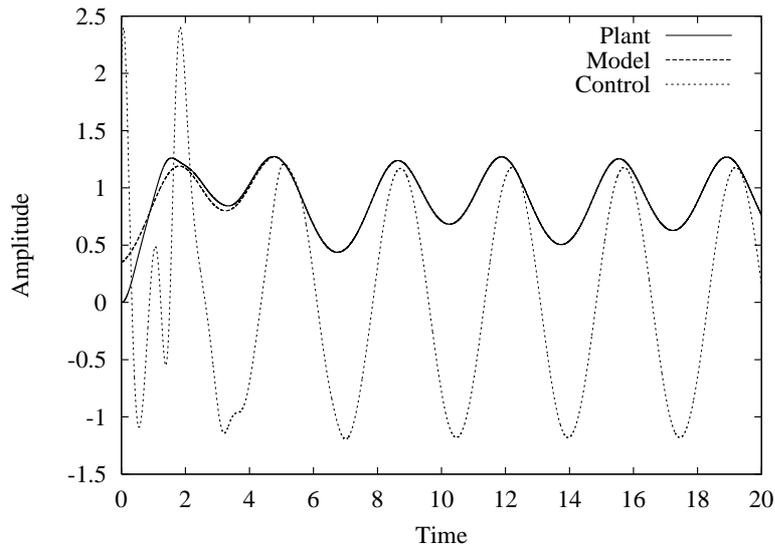
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Figure 3. Numerical simulation of two Duffing systems being synchronized using MCS control. The reference signal is a sine wave with amplitude and frequency equal to 1. The controller parameters are $\alpha = 0.1$, $\beta = 0.01$ and settling time $t_s = 0.5$. (a) response of the system without control where the model has a periodic motion and the plant is chaotic, (b) response of the system with adaptive control applied.

11

of constant parameters, and ξ represents the vector of system variables. For such systems we can use an adaptive controller of the form $g(u,t) = u = k(t)\xi$, where $k(t)$ is the adaptive gain. This approach will give an adaptive feedback linearization controller for the system. Using these definitions equation (34) can be expressed as

$$\dot{x}_e(t) = -\lambda x_e + \phi(t)\xi(t), \tag{35}$$

where $\phi(t) = k^* - k(t)$ is the parameter error. We then need to find an expression for $k(t)$ which stabilises the system such that $\phi(t) \to 0$ as $t \to \infty$. This we can achieve by choosing a trial Lyapunov function of the form

$$V(t) = \frac{x_e^2}{2} + \frac{\phi^T \phi}{2\rho}, \tag{36}$$

where ρ is the controller gain. Then the derivative of V with respect to time is

$$\dot{V}(t) = x_e(-\lambda x_e + \phi(t)\xi(t)) + \frac{1}{\rho}\phi\dot{\phi}, \tag{37}$$

such that choosing $\dot{\phi} = -\rho x_e \xi$, results in $\dot{V} = -\lambda x_e^2$ which implies that the controller is Lyapunov stable. Thus $\dot{\phi} = -\dot{k} = -\rho x_e \xi$, such that the adaptive gain becomes

$$k = \rho \int_{t=t_0}^{t} x_e \xi \mathrm{d}t. \tag{38}$$

Thus $k(t) \to k^*$ as $\phi \to 0$ and $x_e \to 0$. We note also that providing $\phi \to 0$, the final adaptive gain values correspond to the unknown set of system parameters k^*. In general $k(t) \to k^*$ providing the adaptive controller has a persistently exciting signal.

4.2.1. *Example 3: Non-identical nonlinear systems*
In this example we use a Chua system defined as

$$\begin{aligned}
\dot{x}_1 &= \alpha_1(x_2 - x_1) + \alpha_2 x_1 - \alpha_3(|x_1 + 1| - |x_1 - 1|) \\
\dot{x}_2 &= x_1 - x_2 + x_3 \\
\dot{x}_3 &= -\delta x_2
\end{aligned} \tag{39}$$

and a Lorenz system

$$\begin{aligned}
\dot{y}_1 &= -\sigma(y_1 - y_2) \\
\dot{y}_2 &= Ry_1 - y_2 - y_1 y_2 \\
\dot{y}_3 &= y_1 y_2 - b y_3
\end{aligned} \tag{40}$$

We select parameters such that both systems are chaotic: $\alpha_1 = 10$, $\alpha_2 = 0.68$, $\alpha_3 = 0.59$, $\sigma = 10$, $R = 28$ and $b = 8/3$. Initial conditions

for the system were selected as $x_1(0) = 1.1$, $x_2(0) = 1.0$, $x_3(0) = 7.0$, $y_1(0) = -1.1$, $y_2(0) = -1.0$ and $y_1(0) = -5.0$.

As our analysis is limited to control of scalar variables, we will consider the case when we wish to control x_3 from the Chua system to follow y_1 from the Lorenz system. To develop a controller we need to express L, the feedback linearization controller, as a product of an unknown parameter vector, k^* and a system variable vector, ξ. Thus $x_e = y_1 - x_3$ and by taking $\lambda = \sigma$ we obtain

$$L = \sigma(y_2 - x_3) + \delta x_2 = \{\sigma, \delta\} \left\{ \begin{array}{c} y_2 - x_3 \\ x_2 \end{array} \right\} \tag{41}$$

so that the system variable vector is $\xi = \{(y_2 - x_3), x_2\}^T$ and the parameter vector is $k^* = \{\sigma, \delta\}$, and as before the adaptive feedback linearization controller is $u = k(t)\xi$. In this example the control signal, $u(t)$, is added to the expression for $\dot{x}_3$

The response of the system is shown in figure 4, where the controller was initiated at time $t = 10$. In this figure x_3 is referred to as the plant and y_1 the model. The control parameters for this example were $\rho = 50$ and $t_s = 4.0$. The control tracking after $t = 10$ is excellent, but the control effort, as shown in figure 4 (b), is much larger than the linear adaptive control examples.

5. Conclusions

In this paper we have considered the application of model reference adaptive control techniques to nonlinear systems. In particular we have considered using the minimal control synthesis approach to model reference adaptive control. This approach has been outlined in detail including a discussion on stability of such systems.

We have discussed how nonlinear systems with either small or identical nonlinearity can often be controlled using a standard linear adaptive control, and a numerical example of each of these cases has been shown. For non-identical nonlinear systems we can take only a more limited approach, and we discussed how an adaptive feedback linearization controller could be developed. As an example we demonstrated how scalar variables from two different chaotic systems can be synchronized using this adaptive control approach.

In general, many of the results obtained from control of chaos and synchronization techniques can also be achieved using adaptive control. The significant advantage of using adaptive controllers is that they have a proven track record in application to physical systems.
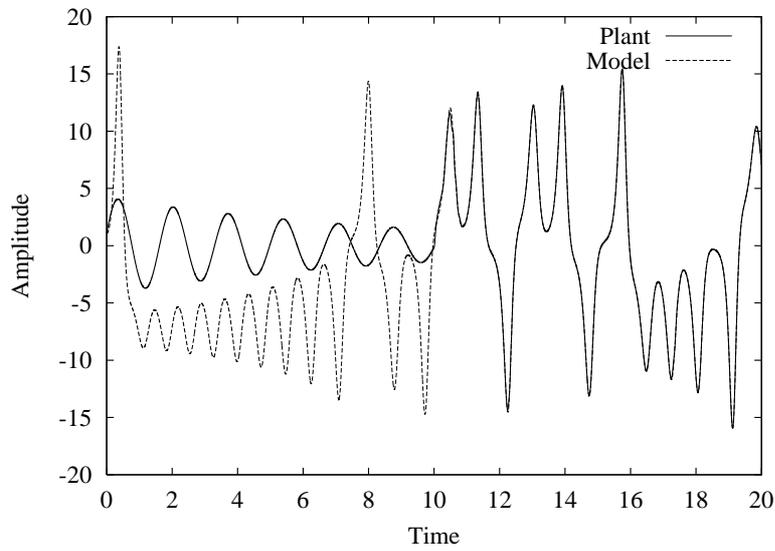
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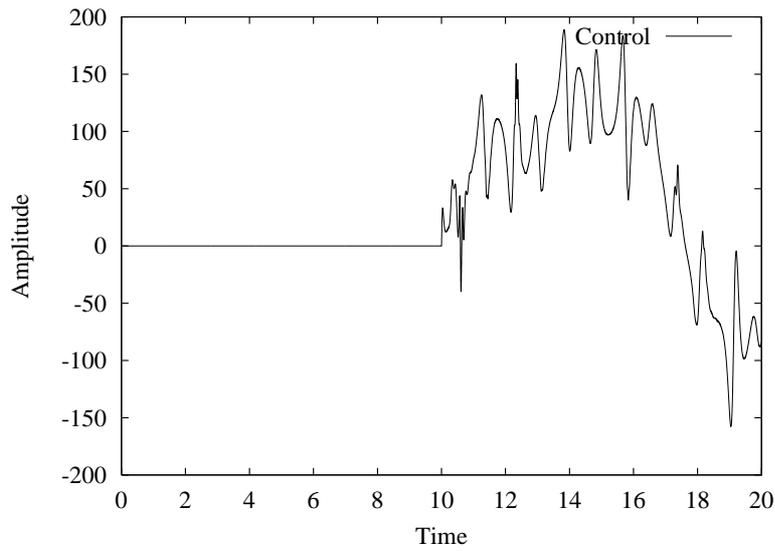
(b)

Figure 4. Numerical simulation of coupling a Chua and Lorenz system with adaptive feedback linearization control. (a) response of the system without control up until time $t = 10$ when the controller is activated, (b) control signal for the system.

Acknowledgements

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