Interactive Open-Pit Design Using Parameterization Techniques

by

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The candidate confirms that the work submitted is his own and that appropriate credit has been given where reference has been made to the work of others

To my wife Hadda, son Abdellatif, daughter Meriam, my mother, my brother, and all my family for their encouragement and support throughout all these years which have helped me to finish this research

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Abstract

The standard approach to open pit design is to optimize the pit shape using the criterion of maximum total profit on the basis of a revenue block model of the orebody. There are some difficult problems inherent in such an approach. For example, scheduling and production rates will have a significant effect on the shape of the pit; if the bulk of the rich (and thus high revenue earning) ore is at the bottom of the pit and will not be mined until near the end of the life of the mine, then the time value of money may make the simple revenue block approach meaningless. In addition, optimality is a function of economic parameters which may change significantly over time.

The aim of the parametric approach is to express the solution (i.e., the optimal pit shape) as a function of one or more parameters such as costs, prices or cut-off grade. Matheron's parametric approach is to use a grade block model together with the techniques of functional analysis without making any economic assumptions. This leads to a set of technically optimal nested pits which can be used for mine scheduling.

Whittle uses the traditional revenue block approach with the Lerchs-Grossmann algorithm and finds a set of optimal pits which are functions of the price/cost ratio.

The aim of this project is to combine the two approaches mentioned above and provide a complete parametric solution in terms of technical and economic parameters. The project includes the development of an interactive computer program for the parametric design and scheduling of open pits.

The author reviews the literature on optimal open pit design and scheduling and then provides an overview of the parameterization method. In this research project the parameterization method has been extended to allow for the selection of an economically optimum pit. Scheduling is then discussed in detail and a new method that combines linear programming and user-activated simulation is introduced.

All software developed during this project is described in detail in the final Chapter.

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CHAPTER ONE

Optimal open pit design and optimal production scheduling: Literature review and survey of previous work

1. Optimal open pit design

1.1 Introduction

An open pit mine is characterised physically by a large hole in the ground. The determination of the shape and location of this hole with respect to the mineralization are the essential features of the optimality problem. Once the optimal shape and location of the pit are determined the mine must be equipped with the necessary operating plant and labour. This development requires the investment of a large amount of capital which must be repaid as quickly as possible. Many studies (eg, Dowd, 1976, Lane, 1964) have shown that the combination of the time value of money and the need to pay back the initial investment as quickly as possible require operations to begin with a relatively high cut-off grade which then declines over the life of the mine. Estimation of block grades and consequent cut-off grades is thus essential for optimal mine planning. These estimations are based on the available information gathered during the exploration and feasibility stages by a drilling and sampling programme. From this information a model of the orebody is constructed in the form of estimated block grades which are then used as the basis on which the optimum open pit is designed and the optimal production schedule for mine design is determined.

The objective of any optimal open pit design algorithm is to determine the final pit limits of an orebody and its associated grade and tonnage, which will maximize some specified economic and/or technical criteria whilst satisfying practical operational requirements. Since the advent and widespread use of computers, open pit design has been implemented by the application of different methods and various algorithms, all with a common objective:

to maximize the overall mining profit within the designed pit limit

One of the most important economic aspects of open pit mining is the cut-off grade. There are many different types of cut-off grade each defined and used for a different purpose. In the most general sense a cut-off grade is any grade:

- that is used for distinguishing between two different courses of action (e.g. to mine or not to mine; to process or not to process; to separate marginal ore from run-of-mine ore)

or

- that is used to classify material (e.g. into ore and waste; into graded fractions)

For the purposes of this work the term *cut-off grade* is used in its general economic sense to distinguish between ore and waste. The cut-off grade is a very important factor in mine planning as it affects the overall reserves of ore and the amount of waste and overburden to be removed.

Cut-off grades are known as technico-economic parameters and are a complex function of grade distributions and variables such as mining costs, processing costs and metal prices. They define what is mined and what is milled from a mine's output. To be optimal a cut-off grade must be such that it maximizes the realisable total net discounted value of the orebody. If the cut-off grade is too high it will reduce the mineral recovered and possibly the life of the mine; if it is too low the cut-off will reduce the average grade (and hence profit) below acceptable levels.

It is important to differentiate between planning and the operational cut-off grades. There are a number of techniques available for optimising cut-off grades. Roman (1973) introduced dynamic programming as a means of optimising production rates and Dowd (1976) extended this work to include cut-off grades. Lane (1964) is generally regarded to have provided landmarks in the understanding and general communication of cut-off grade theory and its application. He introduced (Lane, 1964) three stages in determining the optimal cut-off grade: extraction, processing and marketing. Costs were determined for each stage as well as the effect on the Present Value of each stage of varying cut-off grade. Blackwell (1970) revised Lane's method, again with three stages as the pit, the concentrator and the market constraints. With the inclusion of an additional time cost to be borne by the operation, he was able, by the use of a computer-based algorithm, to determine the maximum Net Present Value, which resulted again in a declining cut-off grade.

Perhaps one of the best definitions of cut-off grade is that of Taylor (1972). Taylor firstly defines the breakeven grade as that grade from which the recoverable revenue exactly balances the costs of mining, treatment and marketing, and the cut-off grade is any grade that, for any specifically reason is used to separate two courses, e.g. to mine or to leave, to mill or to dump according to their appropriate conditions. Such a definition allows the forecasting of future marketing conditions in terms of probability.

The essential difference between planning and operational cut-off grades is the time scale to which they relate. Planning cut-off grades are long term and generally required before production starts to define geographically and quantitatively the potential ore limits. Operating cut-off grades are those required during production to define on a shorter term basis those parcels of ore that may contribute to unmined ore reserves or to streams of broken ore.

The use of a planning cut-off grade constructed on breakeven principles certainly appears justifiable for the determination of pit limits, whereas the operating cut-off grade appears justified for production scheduling. In the first specification it is assumed that when a cut-off grade is applied to the block then the whole block is either above or below the cut-off grade, i.e. the selective mining unit is the whole block. The second specification allows for selective mining on a scale smaller then the planning block.

Almost all methods are based on orebody block models which are either a :

- **Revenue block model** obtained by dividing the deposit into blocks and assigning a revenue value to each block according to its estimated grade and tonnage above a specified cut-off grade

or a

- **Block grade model** which is usually in the form of average grade above a specified cut-off grade.

There are a number of other methods which are essentially based on geological models and ignore the block concept but, in general, these are only applicable in the simplest of orebodies such as those described in section 1.3.1 of this Chapter. More general applications of geological models are not considered here because the models are not suited to pit design.

Although many methods have been proposed over the past 30 years very few enjoy any significant use today. The major reason for this is that most methods cannot be guaranteed to yield a true optimum.

The initial section of this thesis examines the general understanding behind the concept of optimal pit determination followed by some of the techniques which are applicable in mining operations.

1.2 Optimization criteria

The first step in any optimization problem is to define the optimization criteria. For pit design there can be any number of criteria: technical, geological, economic or a combination of all three. The most commonly used criteria are economic such as maximum profit, maximum extraction of metal, maximum net present value, optimal mine life. Of these, the most widely accepted are variants of maximum profit. However, an orebody can be mined at a range of cut-off grades each of which (at least over practical values) will yield similar amounts of metal for different tonnages of ore mined. Because of the time value of money it will always be more profitable to mine at a higher cut-off grade in the early years and then at a declining cut-off grade over the later years. Thus the optimizing criterion should be maximum net present value rather than maximum total profit. There are, however, very difficult problems with implementing net present value as an optimizing criterion which will be discussed in Chapter 2.

For manual methods of pit design it is generally very difficult, if not impossible, to use criteria such as maximum net present value or even maximum total profit as an optimizing criterion. The implicit optimizing criterion in most manual methods is, for a given cut-off grade, either maximum extraction of metal or minimum extraction of waste.

Whatever the method the optimizing criteria may be overridden by other considerations such as technical constraints (e.g., safe pit slopes), environmental and planning requirements or government policies, the latter often imposed in the form of taxation schemes. Although these factors are important they are not explicitly considered in the remainder of this thesis. The purpose here is to examine and derive methods that optimize economic criteria. However, many technical constraints (such as pit slopes) will be included inherently in the methods or the formulation of the models.

1.3 Review of methods for open pit optimization

In this review of the literature attention is restricted to those optimization techniques in common use. In particular, attention is focused on those techniques used frequently in commercial and research software for the determination of the optimum open pit limits; these methods include the Lerchs-Grossmann algorithm, the various moving cone algorithms, dynamic programming, the corrected form of the Korobov algorithm and the parameterization technique. In addition, traditional pit design, in its original form and in its later, computerised form will be discussed.

1.3.1 Traditional pit design

For simple, well-defined mineralizations there is often no need to use sophisticated computer algorithms to design optimal pits: the true optimum can be found by the application of well-known, elementary mathematical techniques. An example of such simple cases (taken from Dowd, 1994a) is in the mining of dipping, stratigraphically defined structures of uniform grade as shown in figure 1. As the pit is deepened more and more waste must be removed. Here the pit shape can be defined as a function of the net value of mining ore and waste down to a given depth. Once the pit slopes are defined the objective is to determine the depth which gives the maximum profit. Simple calculus can be used to determine the optimal depth and thus optimal pit shape. To illustrate this consider the simple case shown in figure 1.

Assume that the ore has constant width w and a strike length of ℓ. Table 1 shows the derivation of the optimal mining depth. Similar, but more complex, formulas can be derived for more realistically shaped and oriented stratigraphic deposits or sequences.

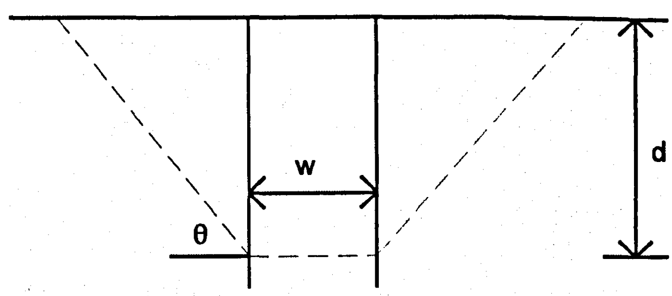


Figure 1

In the more general case, where the grades vary in three dimensions and the ore is not confined to simple stratigraphic boundaries, such exact elementary approaches are not possible and a more complex algorithmic solution is required. A few years ago, even in cases such as this, the determination of optimum pit limits was done by hand using traditional, logical pit design methods based largely on cross-sectional interpretations of the mineralization and on grade contour maps. In the manual version of this method it is not possible to use a total systems approach, i.e. to take into account simultaneously all relevant geological, technical and economic factors. However, the method can be largely computerized and a systems approach then becomes possible; the only role of the computer is in speeding up manual calculations.

	S		selling price per tonne of ore
	C_o		cost of mining one tonne of ore
	C_w		cost of mining one tonne of waste
	C_t		cost of processing one tonne of ore
	g_o		specific gravity of ore
	g_w		specific gravity of waste
	d		depth of mining
	ℓ		strike length of orebody
	w		width of orebody
	θ		wall slope of pit
	r		processing recovery
Tonnage of ore mined	T_o	=	$d \times w \times \ell \times g_o$
Tonnage of waste	T_w	=	$2 \times \frac{1}{2} \times d \times d/\tan\theta \times \ell \times g_w$
Profit		=	$S \times r \times T_o - C_o \times T_o - C_t \times T_o - C_w \times T_w$
		=	$d \times w \times \ell \times g_o \times (rS - C_o - C_t) - d^2 \times C_w \times g_w \times \ell/\tan\theta$

Differentiating profit with respect to depth and setting to zero gives optimum mining depth:

$$d_{opt} = \frac{w \times \tan\theta \times g_o \times (rS - C_o - C_t)}{2 \times g_w \times C_w}$$

Table 1 : Derivation of optimum mining depth for case illustrated in figure 1 (from Dowd, 1994a)

In the manual approach it is not possible to work completely in three dimensions and a two-dimensional (or, at best, "two-and-a-half dimensional") approach is used based on vertical and/or horizontal sections. These vertical and horizontal sections (or, in fact, sections of any orientation) are only approximations to the three-dimensional shape of the pit; the pit shape on any section is designed independently of the shape on the other sections and the results are then modified to create a continuous (smoothed) pit surface.

This "two-and-a-half dimensional" approach was also applied in the early computerised attempts at optimal open pit design. The best example of this is Johnson's dynamic programming algorithm (Johnson, 1971), which is described in section 1.3.4. Whilst it is easy to find examples for which such an approach fails to find the optimal solution it was, nevertheless, a useful, approximate technique at a time when computers were very much slower and less powerful than they are today.

1.3.2 Lerchs-Grossmann algorithm

The first rigorously optimal method for the general case was proposed by Lerchs and Grossmann (1965). This method overcomes the limitations of traditional pit design and can be proved always to yield the optimal solution. The Lerchs-Grossmann algorithm is based on Graph Theory.

1.3.2.1 Graph theory

The graph theory approach developed by Lerchs and Grossmann (1965), for the determination of the optimum pit limit is based on the construction of a maximum closure of a graph. The Lerchs-Grossmann algorithm converts the three-dimensional grid of blocks in the orebody model into a directed graph. Each block in the grid is represented by a vertex which is assigned a mass equal to the net revenue value of the corresponding block. The vertices are connected by arcs in such a way that the connections leading from a particular vertex to the surface define the set of vertices (blocks) which must be removed if that vertex (block) is to be mined. A simple two-dimensional example is shown in figure 2.

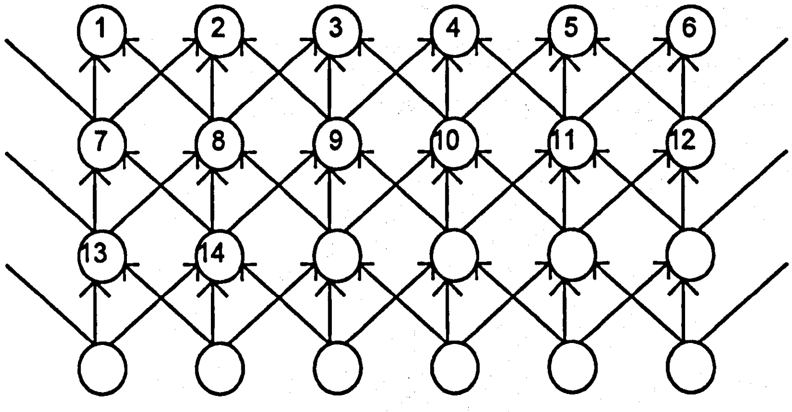


Figure 2 : Directed graph representing a 2-D deposit model
nodes represent blocks and arcs define mining constraints

Vertices connected by an arc pointing away from a vertex are termed successors of that vertex, i.e. the vertex y is a successor of the vertex x if there exists an arc directed from x to y. The set of all successors of x is denoted Γx. For example, in figure 2, $\Gamma x_9 = \{x_2, x_3, x_4\}$. A closure of a directed graph, which consists of a set of vertices X, is a set of vertices $Y \subset X$ such that if $x \in Y$ then $\Gamma x \in Y$. For example, in figure 2, $Y = \{x_1, x_2, x_3, x_4, x_5, x_8, x_9, x_{10}\}$ is a closure of the directed graph. The value of a closure is the sum of the masses (revenue values) of the vertices in the closure. Each closure defines a possible pit; the closure with the maximum value defines the optimal pit.

This method is the only method which can be proved rigorously, mathematically always to lead to the correct optimal solution. However, a number of recently published new methods have also made similar claims but they remain to be independently verified.

Most of the stated disadvantages of the Lerchs-Grossmann are perceived rather than real. The most commonly stated disadvantages are:

- Complexity of the method
- Computing time
- Difficulty of incorporating variable pit slopes

Complexity

In principle, it is desirable that users of a technique understand the mechanisms being used otherwise it becomes a "black box" which may generate results that cannot be properly assessed and questioned. However, it might also be said that once a technique has been proved and implemented in a validated software package it is no longer necessary for the user to have a detailed knowledge of the workings of the algorithm. After all, no user of a proprietary **CAD** package demands to understand the algorithms that it employs before he or she agrees to use it. Thus complexity of the algorithm cannot really be accepted as a disadvantage. The important things are for the user to be aware of any limitations in the algorithm and/or in the software implementation.

Computing time

There is no doubt that the Lerchs-Grossmann algorithm requires significantly more computing time than most of its (non-optimal) competitors. Increased computing time is the price to be paid for a truly optimal solution. However, computing speed is rapidly being increased and a **PC** can now solve optimal open pit problems that only a few years ago could only be attempted on large mainframes. Computing time is fast becoming irrelevant.

Variable pit slopes

This has been a problem in the past because of the difficulty of defining the joining arcs in the network (see figure 2) in any general and flexible sense. However, there are now a number of published solutions to this problem; see, for example, Dowd and Onur (1993).

1.3.2.2 Maximal flow techniques

It can be easily verified that finding the maximal closure of a graph, on which the Lerchs-Grossmann algorithm method is based, is essentially the same as finding the maximal flow through a network. Maximal flow techniques have been applied with some success, but they do not seem to have been adopted to any great extent largely because they share the same perceived disadvantages as the Lerchs-Grossmann method.

Of the various known alternative algorithms which have been employed to overcome the perceived disadvantages and limitations of the Lerchs-Grossmann algorithm the most well-known are the moving cone algorithm, dynamic programming, the corrected form of the Korobov algorithm and the parameterization technique.

1.3.3 Moving cones

The main alternatives, in current use, to the Lerchs-Grossmann algorithm are the various versions of the floating or moving cone algorithm in which the extraction volume for each block is defined by a cone that is centred on that block. The moving cone is the simplest method for determining the optimal pit shape and is the most widely used of the heuristic algorithms. Each block is assigned a cone which is defined by the pit wall slopes in all directions around the block. This cone is called a removal cone as it defines all blocks which must be mined in order to mine the block on which the cone is positioned. The optimum pit is a combination of sets of removal cones of blocks as shown in figure 3.

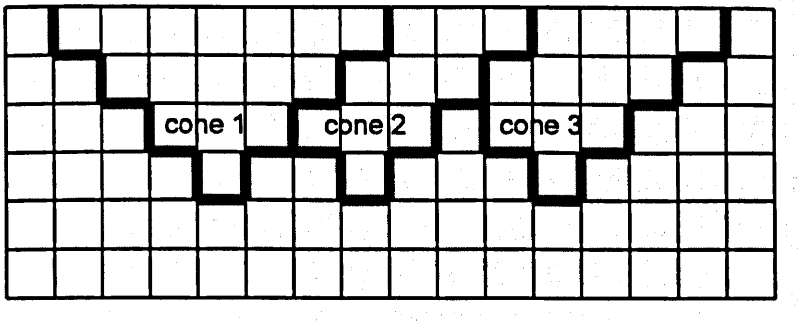


Figure 3 : Pit outline formed by the combined removal cones of three blocks

There is a serious problem with moving cone methods as illustrated in figures 4 and 5 where, for example, the value of the removal cone for the first positive block is equal to -1 and so the block is rejected as shown in figure 4.

Similarly, the value of the removal cone for the next positive block is also -1. Treated separately, neither of the positive valued blocks would be removed and the optimal solution is not to mine this simple, two-dimensional orebody. However, if the blocks are removed together, as in figure 5, the value of their combined removal cones is +4 and the blocks can be profitably mined.

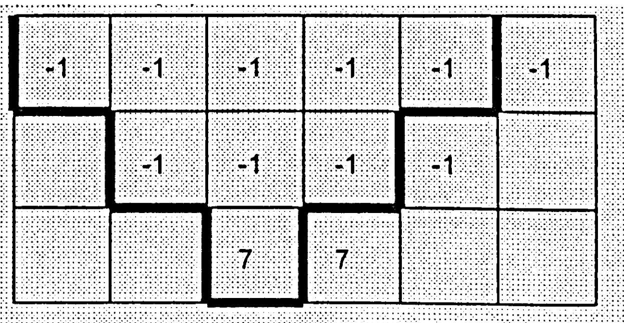


Figure 4 : Removal cone for first positive block

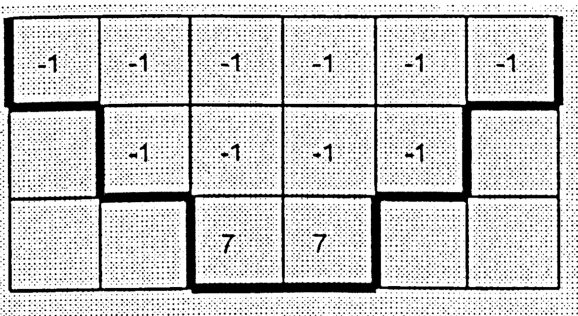


Figure 5 : Removal cone for combined positive blocks

It is thus not sufficient to consider the removal cone of each block independently of all other removal cones which intersect it. Various techniques have been proposed to overcome this problem but the numerous forms of the moving cone method remain heuristic algorithms for which rigorous proofs of optimization are not possible and for which a counter example of non-optimisation can usually be found.

1.3.4 Dynamic programming algorithm

Dynamic programming was advanced as an early alternative to the Lerchs-Grossmann algorithm.

Dynamic programming is the name given to a technique used to find optimal sequences of decisions for problems which can be described as sequential decision processes. Problems to which the technique can be applied must be such that they can be divided into a sequence of smaller problems for each of which an optimal solution can be found. Problems which can be solved by dynamic programming are characterized by :

✏a system	which defines the problem to be optimized.
✏stages	which are sub-problems into which the overall problem can be divided; they usually correspond to periods of time.
✏state	the condition of the system at any given stage.
✏optimal policy	sequence of decisions which optimizes a criterion function.
✏transfer function	an expression which defines the manner in which the state of the system at one stage is related to the state of the system at the preceding stage, ie the manner in which the next state will be determined by the current state and decision.
✏recursive relationship	is a mathematical expression which defines the optimal solution at each stage.

Dynamic programming is based on the application of a simple property of multi-stage decision processes. This property has been formulated by Bellman (1957) in his principle of optimality as:

An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

In other words, an optimal set of decisions has the property that if a particular decision is optimal, all subsequent decisions that depend on that decision must also be optimal.

Unlike techniques such as linear programming there is no standard mathematical formulation of dynamic programming and so particular equations (transfer functions and recursive relationships) must be developed to fit each individual situation. As a theory, dynamic programming was first formulated by Bellman (1957). It has been applied successfully to a number of mining problems (David and Dowd, 1976, Dowd, 1976, Dowd, 1980, Dowd and Elvan, 1987, Onur and Dowd, 1993).

The dynamic programming formulation of the optimal open pit problem is relatively simple. The decisions are all of the possible combinations of blocks which satisfy the mining constraints and the problem is to choose the sequence of decisions which maximises the net present value. This is a particularly attractive approach as it will solve the problem on the basis of the ideal criterion. In principle, dynamic programming will yield the correct optimal solution. However, in practice, the large numbers of blocks (and hence combinations of decisions) in orebody block models result in too many alternative decisions and, as a consequence, computing time and storage are prohibitive, even allowing for recent advances in PC technology.

In an attempt to overcome these problems the method has been implemented by subdividing the orebody block model into a sequence of two-dimensional vertical slices of blocks and applying dynamic programming to these two-dimensional arrays (Johnson, 1971). This leads to a series of correct solutions for each of the two-dimensional slices, i.e. considering each slice as a separate, independent "orebody". The sequence of two-dimensional optima are then combined, by another dynamic programming technique, into a quasi-optimal solution for the total three-dimensional block model. This approach may yield a solution which is not significantly different to the true optimum. However, there is no way of knowing how close any solution is to the true optimum and, in practice, the difference may be highly significant. It is very easy to devise simple examples for which the two-dimensional, approximate dynamic programming method will yield solutions significantly different to the true optimum.

Notwithstanding the time and storage problems associated with the full implementation of the dynamic programming method it still attracts interest as a possible method for optimal open pit design. To be practical and efficient an accurate method must be found to eliminate all sub-optimal decision sequences as soon as they arise. Such an approach would be a fruitful avenue for future research.

1.3.5 The Korobov algorithm

This method is originally due to Korobov (1974) and is reported in David, Dowd and Korobov (1974), Dowd and Onur (1993). It is a cone based algorithm which uses the idea of allocating values from positive blocks against the negative or zero blocks contained in the extraction cones of the positive blocks. A flowchart for the algorithm is given in Korobov (1974).

An extraction cone is assigned to every positive block in the orebody model and the positive block values within each cone are allocated against the negative block values within the cone until no negative block remains or until the values of the positive block have all been allocated. If, when this allocation is completed, the positive block on which the extraction cone is based remains positive, then this extraction cone is accepted as a member of the optimum solution set. When a non-empty extraction cone is added to the solution set, the algorithm starts again from the beginning with original block values restored to the blocks not yet extracted from the block model. If an extraction cone is empty the positive block is added to the solution and the algorithm proceeds on the current or next level.

The method can best be explained by means of a simple, two-dimensional example based on that given in Dowd and Onur (1993). For the sake of simplicity slope angles are assumed to be 45° in all directions and the blocks are squares. The initial block values are shown in figure 6. There are two numbers in each block: the upper one designates the block number and the lower one is the value of the block.

	1	2	3	4	5	6	7	8	9	10	11
1	1 +1	2 -1	3 -1	4 -1	5 +1	6 +1	7 -2	8 -2	9 +1	10 +1	11 +1
2		12 +2	13 -1	14 -1	15 -1	16 -1	17 +1	18 +1	19 +1	20 +1	
3			21 -2	22 -1	23 +2	24 +2	25 -1	26 -1	27 -1		
4				28 -1	29 -1	30 +4	31 +4	32 +4			

Figure 6 : example of orebody block model
upper number is block identifier; lower number is block value

Step 1: The procedure starts with the first (uppermost) level and by convention, works from left to right. All blocks with positive values (blocks 1,5,6,9,10,11) are removed from level 1 and added to the solution set. Blocks from level 2 are then added as shown in step 1 of figure 7. The value of the extracted blocks is 6 units.

Step 2: On level 2 there are five positive blocks (12,17,18,19,20). The extraction cone of block 12 comprises blocks 1, 2 and 3 with block 1 already removed. A value of +1 is allocated from block 12 to block 2 leaving block 2 with a value of 0 and block 12 with a value of +1. The remaining +1 value of block 12 is then allocated against block 3 leaving block 3 with a value of 0 and block 12 with a value of 0. In the same manner cones are established for blocks 17, 18, 19 and 20 and the positive values of these blocks are allocated against the negative block values within their extraction cones. The result of these allocations is shown in step 2 figure 7. As block 20 remains positive and its extraction cone is empty it is added to the solution set which now has a value of 7. As none of the remaining blocks on level 2 are positive the next step is to add level 3 as shown in step 3 figure 7.

Step 3: For level 3, the two units of block 23 are allocated against blocks 4 and 14 leaving all three blocks with values of 0. **Step 4:** The two units of block 24 are allocated to blocks 15 and 16 leaving the values of all three blocks with values of zero as shown in step 4 in figure 7. There are no positive blocks left on level 3 after allocating the positive values of blocks 23 and 24 and so no further blocks can be added to the solution set at this stage. **Step 5:** Add level 4 to the other levels as shown in step 5 of figure 7.

Step 6: Now consider block 30. Of the blocks within the extraction cone of block 30 (3, 4, 7, 8, 14, 15, 16, 17, 18, 23, 24, 25) only blocks 8 and 25 have negative values and two units from block 30 are allocated against these two blocks as shown in step 6 figure 7. After allocation block 30 remains positive (+2) and so this block and all blocks within its extraction cone are added to the solution set. The net value of this extraction cone, given that blocks 5, 6 and 9 have already been extracted, is zero; the net pit value remains at 7. As the extraction cone for block 30 is non-empty the algorithm starts again at the beginning with the original values restored to all non removed blocks.

Step 1 $S = 1 + 1 + 1 + 1 + 1 + 1 = 6$

	1	2	3	4	5	6	7	8	9	10	11
1		2 -1	3 -1	4 -1			7 -2	8 -2			
2		12 +2	13 -1	14 -1	15 -1	16 -1	17 +1	18 +1	19 +1	20 +1	

Step 2 $S = 6 + 1 = 7$

1	2 0	3 0	4 -1			7 0	8 -1			
2	12 0	13 -1	14 -1	15 -1	16 -1	17 0	18 0	19 0	20 +1	

Step 3

1	2 0	3 0	4 -1			7 0	8 -1	
2	12 0	13 -1	14 -1	15 -1	16 -1	17 0	18 0	19 0
3		21 -2	22 -1	23 +2	24 +2	25 -1	26 -1	27 -1

Step 4 $S = 7$

1	2 0	3 0	4 0			7 0	8 -1	
2	12 0	13 -1	14 0	15 0	16 0	17 0	18 0	19 0
3		21 -2	22 -1	23 0	24 0	25 -1	26 -1	27 -1

Figure 7 : steps in Korobov algorithm applied to example in figure 6 (continued......)

Step 5

1	2 0	3 0	4 0			7 0	8 -1	
2	12 0	13 -1	14 0	15 0	16 0	17 0	18 0	19 0
3		21 -2	22 -1	23 0	24 0	25 -1	26 -1	27 -1
4			28 -1	29 -1	30 +4	31 +4	32 +4	

Step 6 S = 7 + (0) = 7

1	2 0	3 0	4 0			7 0	8 0	
2	12 0	13 -1	14 0	15 0	16 0	17 0	18 0	19 0
3		21 -2	22 -1	23 0	24 0	25 0	26 -1	27 -1
4			28 -1	29 -1	30 +2	31 +4	32 +4	

Step 7 S = 7 + 1 = 8

1	2 -1							
2	12 +2	13 -1						19 +1

Step 8 S = 8 + 1 = 9

1	2 0	
2	12 +1	13 -1

Figure 7 (........continued........)

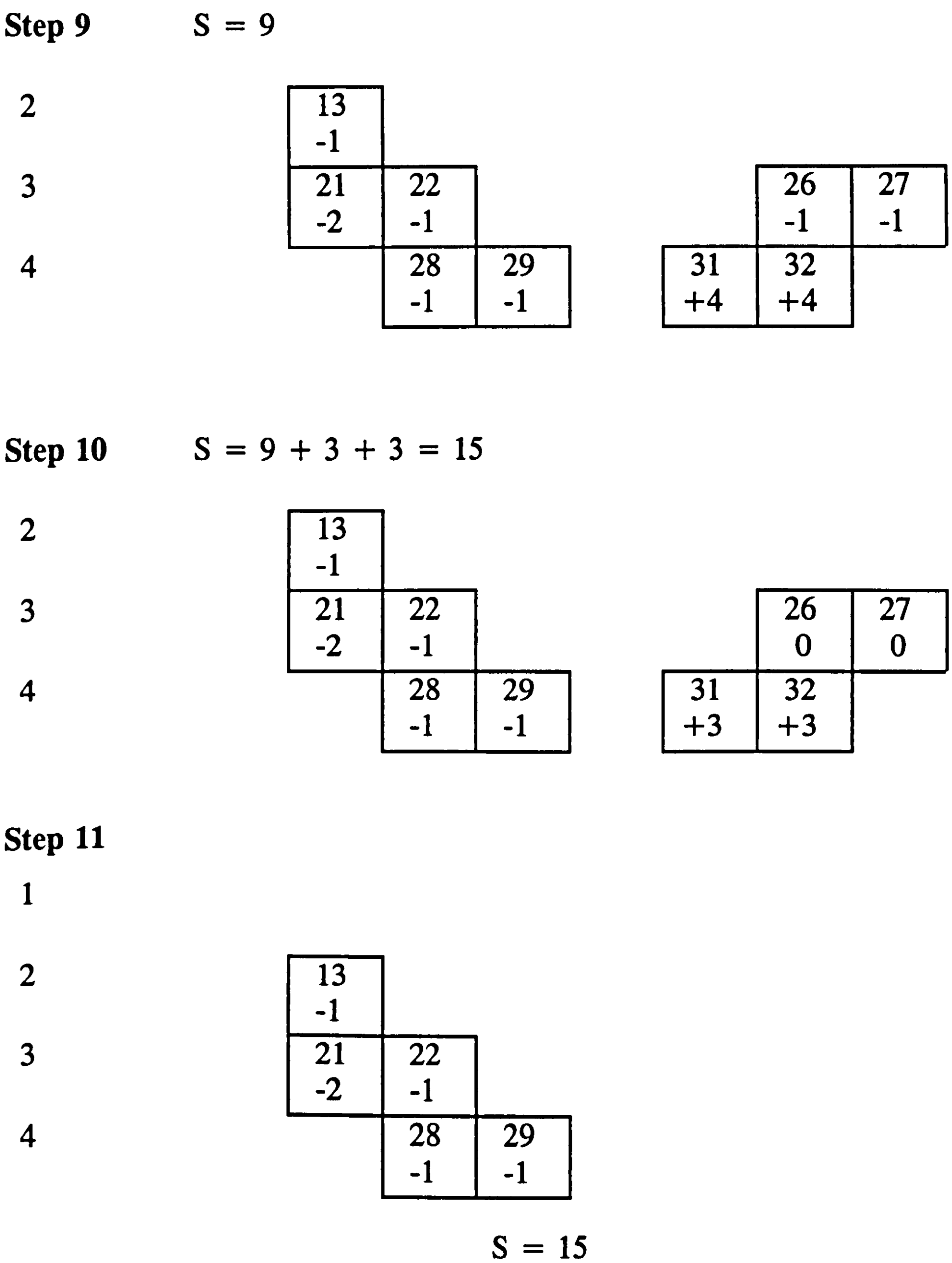


Figure 7 (.....continued)

Add level 1; there are no positive blocks and thus none can be removed. **Step 7:** Add level 2 as shown in step 7 in figure 7. The extraction cone of block 19 is empty and this block is added to the solution giving a net pit value of 8. **Step 8:** Block 2 is the only block within the extraction cone of positive block 12 and one unit from the latter is allocated against the former. Block 12 and its extraction cone are removed giving a net pit value of 9 as shown in step 8 in figure 7.

Step 9: Add level 4 as shown in step 9 in figure 7. **Step 10:** Block 26 is allocated one unit from block 31 and block 27 is allocated one unit from block 32 as shown in step 10 in figure 7. **Step 11:** As blocks 31 and 32 remain positive after allocation they and the blocks in their extraction cones are added to the solution set giving the final pit shape shown in step 11 of figure 7. The net pit value is 15 and the only unmined blocks remaining in the block model are 13, 21, 22, 28 and 29.

Soon after the method had been introduced it was realised that the algorithm did not reach the optimum solution in all cases. For some types of block models the optimum solution is missed by the algorithm as shown by the example in figure 8.

	1	2	3	4	5	6
1	1 -1	2 -1	3 -1	4 -1	5 -1	6 -1
2		7 -1	8 -1	9 -1	10 -1	
3			11 3	12 7		

Figure 8: example in which Korobov algorithm will not yield optimal solution

The first positive block encountered is on level 3 (block 11), the three units of which are allocated against blocks 1, 2 and 3 (figure 9a). The extraction cone of block 12 now contains six negative valued blocks each of which becomes zero after allocation of values from block 12 (figure 9b). After allocation, block 12 has a value of +1 and it, together with its extraction cone, is added to the solution set (figure 9c).

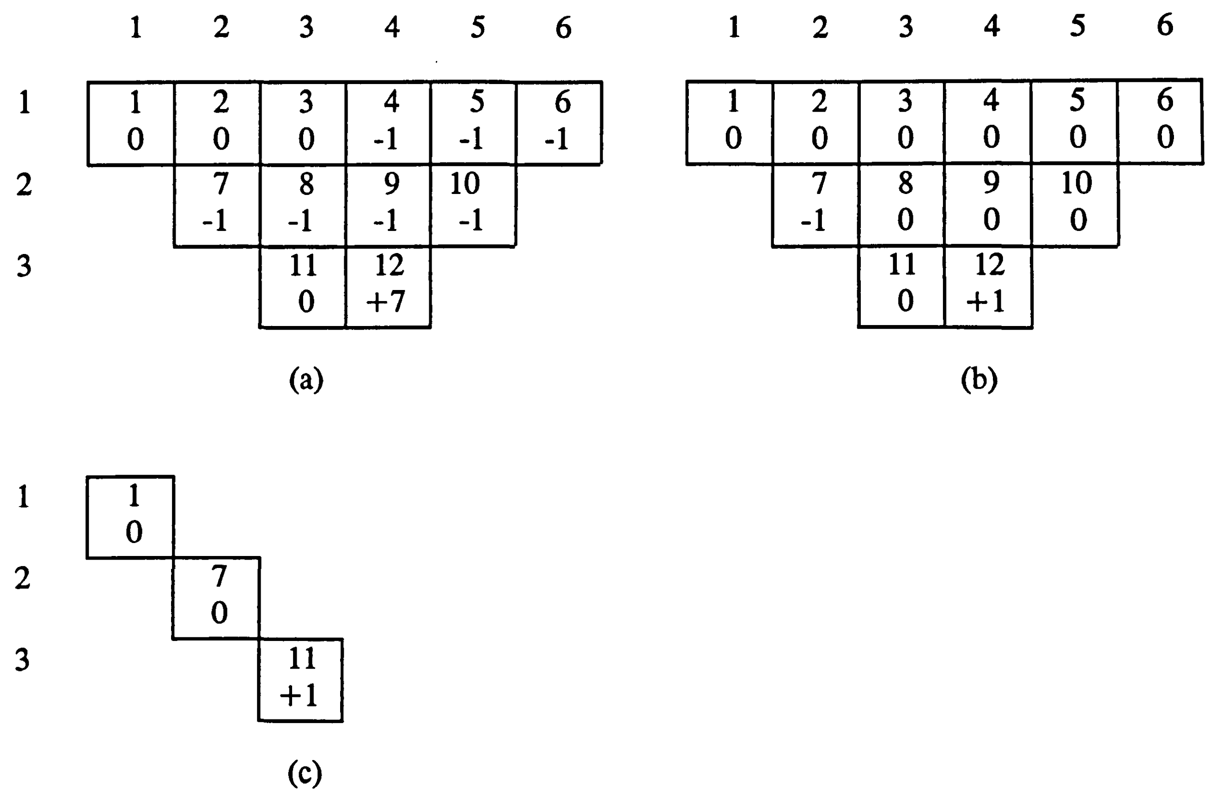


Figure 9 : steps in the Korobov algorithm applied to the example in figure 8

The extraction cone of block 12 has a value of -1 which is the net pit value at this stage. The algorithm now starts again from the beginning with the original values restored to the non-removed blocks. There are two negative valued blocks (1 and 7) in the extraction cone for block 11 and after allocation (figure 9c) block 11 remains positive. Block 11 and the blocks within its extraction cone, with a net value of +1, are added to the solution set. The solution yielded by the algorithm is thus to mine all blocks at a net profit of zero.

The error is caused by blocks which are common to both extraction cones. Blocks 2, 3, 4, 5, 8 and 9 are common blocks. Blocks 1 and 7 are only members of extraction cone 1 and are not in common with extraction cone 2. If the allocation procedure began with these non-common blocks the error would not occur. Thus if two or more cones have blocks in common, allocation must first be made against the non-common blocks; allocation against common blocks is done only after the values of all non-common negative blocks have been reduced to zero.

1.3.6 Corrected form of the Korobov algorithm

The fault in the Korobov algorithm was rectified by Dowd and Onur (1993). The correction to the Korobov algorithm is based on the following logic:

> *If two or more cones have blocks in common, then blocks not in common must be paid for first; common blocks are only paid for after all blocks not in common have been paid for.*

The number of cone searches is significantly reduced by means of paths which define links between zero valued blocks in an extraction cone and any negative block in an intersecting cone. A flowchart for the algorithm is given in Dowd and Onur (1993). The corrected form of the Korobov algorithm can be demonstrated by means of the simple, 3-D example taken from Dowd and Onur and shown in figure 10.

Suppose that there are two levels and the slope angle is 45°. The block dimensions are the same for all directions. In this example, extraction cones for blocks on level 2 are defined by taking the 9 blocks on level 1 over a positive block on level 2. The extraction cone for positive block (i,j,2) on level 2 consists of blocks (m, n, 1) where m=i-1,i+1 and n=j-1,j+1.

As there is no positive block on level 1 the search moves to level 2 and starts from block (2,2,2) (as this is the first cone of the model it will be called cone 1). The member blocks for cone 1 are blocks (m, n, 1) where m=1,3 and n=1,3. The original allocation is shown in figure 11 in which each block has two numbers. The upper number designates the cone from which this block has been allocated a value; the lower number is the net block value after allocation.

	1	2	3	4	5
1	-1	-1	-1	-1	-1
2	-1	-1	-1	-1	-1
3	-1	-1	-1	-1	-1
4	-1	-1	-1	-1	-1
5	-1	-1	-1	-1	-1

2	2	2	2
3	4	4	4
4	2	2	2

Figure 10 : three-dimensional block model (level 1 above; level 2 below)

Within cone 9 there are 8 blocks allocated values by cones other than cone 9. Blocks (3,3,1), (3,4,1), (4,3,1) have been allocated values by cone 5, blocks (3,5,1), (4,4,1) and (4,5,1) have been allocated values by cone 6 and blocks (5,3,1) and (5,4,1) have been allocated values by cone 8. Consider cone 5 first. Take out all the blocks of cone 5 which are in common with cone 9 and determine whether any of the remaining blocks in cone 5 are negative. There are no such blocks but there are some blocks allocated values by cones 2, 3 and 4. Consider cone 3 and determine whether there are any negative blocks in cone 3 which are not in common with the intersection of cones 5 and 9. There are two such blocks: (1,4,1) and (1,5,1). A path has now been established from the positive valued extraction cone

9 via cones 5 and 3 to a negative block. The path, in terms of blocks is: (5,5,1), (3,3,1), (2,3,1), (1,4,1). This path defines the re-allocation: the value (+1) previously allocated to block (2,3,1) by cone 3, is re-allocated to block (1,4,1); the value (+1) which was previously allocated to block (3,3,1) by cone 5 is re-allocated to block (2,3,1); block (3,3,1) is allocated a value of +1 from cone 9 (ie, block (4,4,2)). There are no positive values in the block model and the algorithm stops. The final form of the result is shown in figure 12.

	1	2	3	4	5
1	1 0	2 0	3 0	-1	-1
2	1 0	2 0	3 0	5 0	6 0
3	4 0	4 0	5 0	5 0	6 0
4	4 0	4 0	5 0	6 0	6 0
5	7 0	7 0	8 0	8 0	9 0

2	1 0	2 0	3 0
3	4 0	5 0	6 0
4	7 0	8 0	9 +1

Figure 11 : step 1 in the corrected Korobov algorithm applied to the example in figure 10

	1	2	3	4	5
1	1 0	2 0	3 0	3 0	 -1
2	1 0	2 0	3 0	5 0	6 0
3	4 0	4 0	5 0	5 0	6 0
4	4 0	4 0	5 0	6 0	6 0
5	7 0	7 0	8 0	8 0	9 0

2	1 0	2 0	3 0
3	4 0	5 0	6 0
4	7 0	8 0	9 0

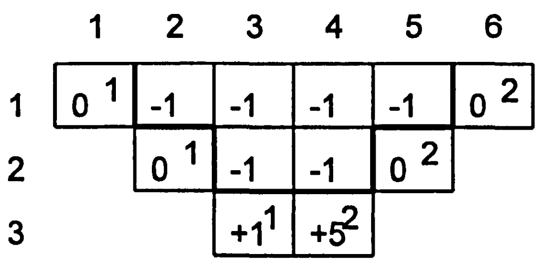
Figure 12 : step 2 in the corrected Korobov algorithm applied to the example in figure 10

Note that an alternative path to a negative block could have been defined via cone 2 - (5,5,1), (3,3,1), (2,2,1), (4,4,1). When alternative paths are available it is irrelevant which is chosen : the algorithm will always lead to the same solution.

The corrected Korobov algorithm has also been applied to the same example in figure 8, where the original Korobov algorithm misses the optimum solution. The basis of the extraction cones for the positive blocks (3,3) and (3,4) are in level 3. In a similar way after allocation each block of the two-dimensional example is represented by two numbers. The upper number designates the cone from which this block has been allocated a value; the lower number is the net block value after allocation.

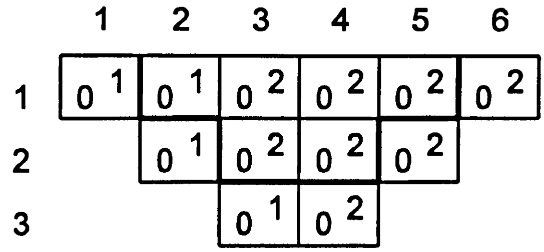
Blocks (1,2), (1,3), (1,4), (1,5), (2,3) and (2,4) are common blocks for both cones 1 and 2. Bocks (1,1) and (2,2) are only members of extraction cone 1 and are not common to cone 2. The same is true for blocks (1,6) and (2,5) which are only members of extraction cone 2 and are not common to cone 1.

The non-common blocks are paid first, starting with the extraction cone 1, two units are allocated for blocks (1,1) and (2,2) leaving block (3,3) with a value of +1. Two units are allocated against blocks (1,6) and (2,5), leaving block (3,4) of the extraction cone 2 with a value of +5. These allocations are shown in the following figure(a).



Figure(a)

The algorithm re-starts again by paying the common blocks of the two cones. One unit is allocated against block (1,2) leaving block (3,3) with a value of zero. Five units are allocated against blocks (1,3), (1,4), (1,5), (2,3) and (2,4), leaving block (3,4) with a value of zero. After this allocation is completed, the positive blocks on which the extraction cones 1 and 2 were based are zero as shown in the following figure(b). The algorithm stops.



Figure(b)

The solution is thus not to mine at a zero profit, as neither extraction cone is part of the optimum solution. In contrast, in the original Korobov algorithm, both extraction cones were part of the solution.

1.3.7 The parameterization technique

Some methods such as the 'pillar method' have been shown to produce non rigorously optimal solutions that have been abstracted to uses for which they were proposed.

An alternative approach to pit optimization is to parameterize the pit design as a function of a number of variables. This algorithm, which uses grade values instead of a revenue block model, is based on techniques of functional analysis. The parameterization method was developed and implemented at the Paris School of Mines at Fontainebleau, France. The aim of this method is to transform a parametric optimization problem with severe geometric constraints into a simple one with no constraints; it does not take any economic parameters into account.

This technique could also be applied for the determination of mining sequences for the optimization of the recoverable reserves of any particular pit, where in economic terms the mining sequence is more important during the early stages and plays a major role in capital investment.

The technique, described briefly above, is not rigorous, and has some weaknesses on the economic side in comparison with other algorithms. It forms the fundamental basis behind Chapter 2 where it is discussed in detail.

2. Production schedule optimization

2.1 Introduction

Production scheduling is of vital importance in pit design and mine planning. Production scheduling is the development of a sequence of depletion schedules leading from the initial state of the deposit to the ultimate pit limits. Production scheduling can be either long range or short range depending on the duration of the scheduling period. Short range scheduling is the development of a depletion sequence on a daily, weekly or monthly basis; long range scheduling is mainly concerned with yearly plans and includes ore reserves, stripping ratios and capital investments.

The main objective of short and long range mine planning (scheduling) in an open pit operation is to maximize the profits realized within every mining period and throughout the life of the mine.

Although in practice production scheduling for both surface and underground mining operations is similar in nature, a large range of techniques is applied in solving such planning problems. The techniques consist of both rigid operational research (**OR**) methods and practical procedures which are heuristically based.

It is widely expected that new and improved **OR** and mathematical techniques will lead to better and more efficient methods of solving production scheduling problems. The combination of these methods with the geostatistical simulation of orebodies should lead to realistic scheduling packages which take adequate account of uncertainties, errors in variables and risk.

In spite of their potential, very few **OR** techniques have been applied to the solution of production scheduling problems in the mining industry and attention has been focussed on the simplest of such techniques such as Linear Programming. Although such techniques have been under-used this does not imply that they are not applicable. Furthermore, the lack of use is attributed to a combination of many causes, some of which are no longer relevant today.

Based on what has already been achieved, (operational research and computer techniques) goal programming can be effectively applied to solve the problem of open pit production planning optimization (Zhang, Cheng and Su (1993)). Goal programming is applied to overcome the limitations of single objective linear programming applications.

A common approach to the solution of the problem of optimal open pit production scheduling involves combining two or more operational research techniques. In the work described in this thesis, Simulation and Linear Programming have been combined to provide the basis of a method for the solution of the problem.

The success of production scheduling methods will undoubtedly continue to increase and spread to areas of mining where these methods have not yet been applied. Success depends on the ability to formulate good operational research and computer models.

2.2 Review of previous techniques for the optimization of production schedules

The following sections give short resumés of some the operational research techniques that are applicable to production scheduling problems in mining operations.

2.2.1 Simulation

Simulation can be described as the use of a model to experiment with any given system. It is one of the most powerful and versatile of the operational research techniques available for assessing complicated, non-analytical problems. In production scheduling problems, simulation is often used to help choose the correct number of haulers assigned to an excavator, to evaluate different sizes of equipment, or to assess the output of a given operating subsystem. However, it does not guarantee the optimality of the solution and needs considerable computing time.

2.2.2 Linear programming

Linear programming is the most frequently applied operational research technique in the solution of production scheduling problems in both surface and underground mining. The most frequent applications have been in surface mining where the size of the operation and the difficulty of meeting grade and resource constraints combine to create a problem ideally suited to the technique. The Linear Program can be solved by a general procedure known as the simplex method. A major restriction still facing the implementation of the technique is the number of constraints which must be kept relatively small. In some cases the complexity of multilevel open pit mining, especially the precedence constraints, can lead to very large linear programming models that can be computationally expensive, or in some cases impossible, to solve.

2.2.3 Integer programming

In recent years, the use of integer programming methods has become more popular. However, the applications of integer programming to production scheduling problems seem to be oriented towards truck and shovel assignment problems. Integer Programming is a less frequently used technique in optimal open pit scheduling because of the complexity of the solution algorithms. A special group of integer programming models is the 0-1 integer programming model, where each variable is allowed to take a value of only 0 or 1. Such a model would be ideal for a production scheduling problem since it permits the assignment of a 0-1 variable to each block of the block model. For example a block can have a 0 value if it is not mined within a mining period or a value of 1 if it is designated to be mined in that period. However, the solution time of a 0-1 integer programming model tends to increase exponentially with the number of variables. Although the solution algorithms improve with time, there is little hope of even being able to solve really large problems, such as the optimal open pit production scheduling problem.

2.2.4 Dynamic programming

Dynamic programming (DP) is another operational research technique that has been used in solving open pit scheduling problems, e.g. Onur and Dowd (1993). It was first applied to the open pit scheduling problem by Roman (1974). Wright (1989) has also applied dynamic programming to the open pit scheduling problem.

The following formulation is taken from Onur and Dowd (1993). The system is the orebody, the stages are mining periods and the state at any stage is the set of blocks remaining in the orebody.

Let a be the ore/waste ratio,

b be the allowable limits of the grade,

c be the minimum operating room for equipment,

d	be the working slope angle,
e	be the maximum movement distance of the shovels,
r	be the discount rate,
g	be the production rate.

Further, let :

$f_n(p)$	be the total discounted profit after n decisions placing the system in state p when an optimal decision policy has been followed.
S	be the set of all possible decisions which satisfy all the scheduling constraints.
$R_n(p,m(a,b,c,d,e,r,g))$	be the immediate profit obtained by taking the decision m, which is a function of a, b, c, d, e, r, g, and thus placing the system in state p.
$T(n\text{-}1,p,m(a,b,c,d,e,r,g))$	be the state of the system at step n-1 as a result of taking decision m(a,b,c,d,e,r,g) at step n. i.e. the transfer function (noting that the method uses reverse chronology).

The principle of optimality is then expressed in the following recursive relationship:

$$f_n(p) = \max_{(m(a,b,c,d,e,r,g) \in S)} \{R_n(p,m(a,b,c,d,e,r,g)+f_{n-1}(T(n,p,m(a,b,c,d,e,r,g))\}$$

The dynamic programming formulation of the scheduling problem can best be explained with a simple example taken from Onur and Dowd (1993). In this example some assumptions have been made for the sake of simplicity but in a real case all relevant characteristics of the mine and the orebody must be applied to the formulation.

In this example the assumptions are :

1) the working slope angle is 45°
2) a total of three positive blocks (which represent ore) and between three and five negative blocks (which represent waste) can be mined in the same stage to represent a specific stripping ratio,
3) the discount rate is 10%,
4) minimum access space is one block,
5) there are no limitations on the number of shovels or on where they can work.

The orebody to be scheduled is shown in figure 13.

	1	2	3	4	5	6	7	8	9
1	0.5 -1	0.5 -1	0.5 -1	0.5 -1	0.5 -1	0.5 -1	1.0 +2	1.5 +3	1.5 +3
2		1.0 +2	1.0 +2	1.0 +2	0.5 -1	0.5 -1	0.5 -1	0.5 -1	
3			0.5 -1	1.5 +3	0.5 -1	0.5 -1	1.5 +3		
4				1.5 +3	2.0 +4	2.5 +5			

Figure 13 : orebody to be scheduled
upper number is grade; lower number is revenue x 10^2

To keep the example simple, only the four possible schedules shown in figures 14, 15, 16 and 17 will be considered here; table 2 displays the discounted value of each decision in each stage.

	1	2	3	4	5	6	7	8	9
1	4	3	2	2	2	1	1	1	1
2		4	3	2	2	2	1	1	
3			4	3	3	3	2		
4				4	4	3			

Figure 14 : solution no. 1 to scheduling problem in figure 13
number in block is the period in which block is mined

	1	2	3	4	5	6	7	8	9
1	3	2	2	2	2	1	1	1	1
2		3	2	2	2	2	1	1	
3			3	3	3	4	4		
4				3	4	4			

Figure 15 : solution no. 2 to scheduling problem in figure 13

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	2	2	3	3
2		1	1	1	2	2	3	4	
3			2	2	3	4	4		
4				3	4	4			

Figure 16 : solution no. 3 to scheduling problem in figure 13

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	2	2	2	3
2		1	1	1	2	2	3	4	
3			3	3	3	4	4		
4				3	4	4			

Figure 17 : solution no. 4 to scheduling problem in figure 13

Year	Schedule 1	2	3	4	Discount factor	Discounted Schedule 1	2	3	4
1	500	500	100	100	0.91	455	455	91	91
2	0	-200	100	200	0.83	0	-166	83	166
3	700	500	700	600	0.75	525	375	525	450
4	700	1100	1000	1000	0.68	476	748	680	680
Total	2200	2200	2200	2200		1450	1412	1379	1387

Table 2:
discounted cash flows for the four mining sequences in figures 14-17

There are two options satisfying all requirements for the first stage (period). These are the groups of blocks {(1,6), (1,7), (1,8), (1,9), (2,7), (2,8)} and {(1,1), (1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4)}. The other stages shown in the figures are the stages resulting from the blocks chosen in the first stage.

From the four possibilities given, the optimal policy is given by the sequence beginning in stage 1 with the schedule with a value of 500; the states resulting from this decision are $f_2(0)$, $f_3(700)$, $f_4(700)$.

This path gives the optimum schedule as far as NPV is concerned and was obtained after all other paths had been considered. Discounted cash flow for each possible schedule is shown in figure 18.

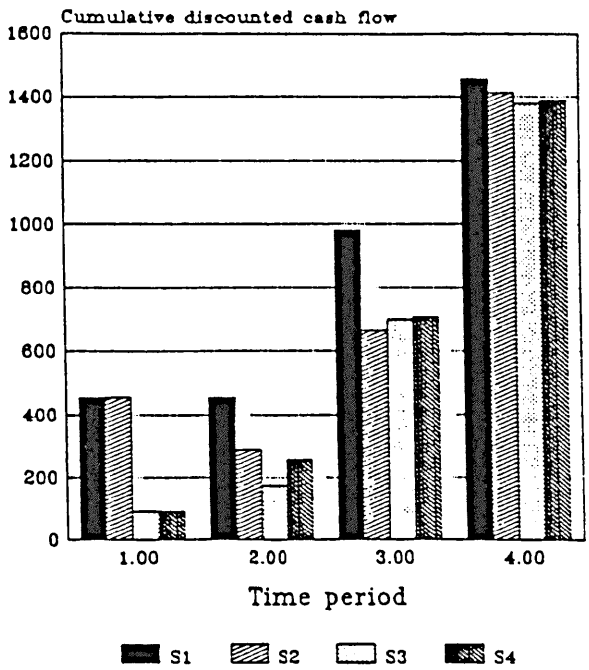


Figure 18 : discounted cash flows for mining alternatives given in figures 14-17

The tree representation of the example is as shown in figure 19.

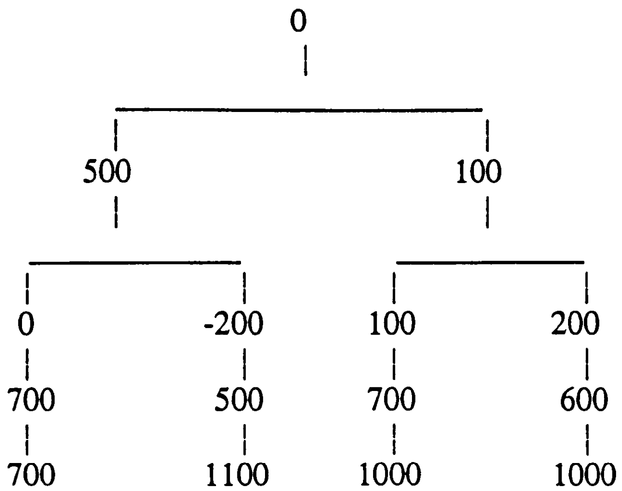


Figure 19 : tree representation of npv of mining sequences from figures 14-17

The main problem with the dynamic programming approach is the limitation in terms of the total number of variables and constraints that can be taken into account. Every dynamic programming model suffers from the 'dimensionality curse'. Only a limited number of mining periods and possible states (production rates) can be examined each time.

2.2.5 Graph and network theory

Graph and network theory have also been applied to production scheduling problems. A graph is set of junction points ordinarily called nodes which may be connected together by lines called branches. A simple graph is illustrated in figure 20.

The graph shown is called a connected graph because each node is connected to every other node by one or more of the branches provided, ignoring the direction arrows. If we take a graph and consider a situation where the branches are associated with some sort of flow, then the mathematical structure is called a network. If all the flows are given a particular sense of direction, then the network is said to be oriented with a flow in the specified direction, e.g. from left to right as shown in figure 21.

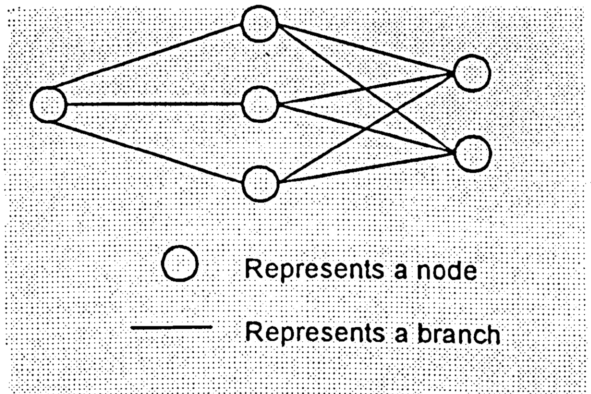


Figure 20 : Illustration of a simple graph.

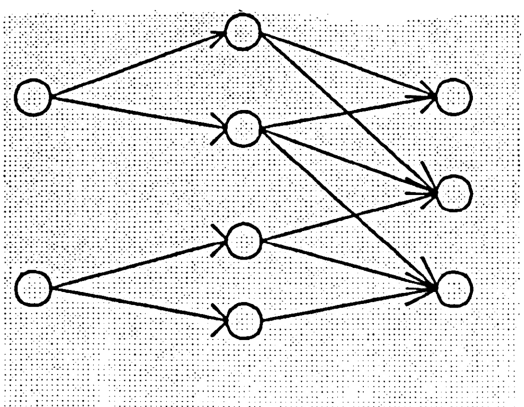


Figure 21 : illustration of a flow network.

Many problems can be expressed in a network format. The most common of these are the critical path method (CPM) and the project evaluation and review technique (PERT). These network problems are used in project control but are used only occasionally in typical production scheduling applications.

The area of maximizing the flow through a network is another important use of network theory. This problem is characterized by a network containing branches with a given capacity. The objective is to maximize the total flow quantity through the network.

The area of graph and network theory has found some application in production scheduling. However, the methods applied have differed considerably, and no one method has been extensively utilized. The most useful of the methods applied to practical problems from the area of graph and network theory are **CPM** and **PERT**.

2.2.6 Heuristic methods

Heuristic methods are procedures which are not mathematically proven or centred but which are based upon practical or logical operating procedures.

Heuristic models are common in the solution of mining problems but seldom appear in the operational research literature. The reason is simply that these methods are often quite subjective and apply only to one particular operation. However, a few of these models have appeared in the mining literature.

3. The objectives of the research programme

In many cases the application of a single operational research approach imposes limitations, especially for the rigorous optimization methods which usually require strict constraints and a single optimum solution. The limitation of methodology has been an obstacle to the general solution of the optimal scheduling problem. In recent years there has been a tendency to combine two or more different operational research methods to solve complicated mining problems.

The optimal open pit design method adopted in this work is parameterization.

This method is an application of technical parameterization and the subsequent optimization of the results. As such, it does not include mine scheduling and takes no account of the effects of roadways and other facilities.

The objective of this research was to develop methods, and associated software, for the determination of optimum mining sequences and the optimal open pit shape.

The research programme was divided into two main parts :

✏1 *optimum pit limit*
✏2 *optimum mining schedule*

The first part is done independently of the second using the parameterization technique, which gives a set of nested pits, which includes the optimum pit. This part includes the selection of the optimum pit from the nested set of pits for a specified set of economic conditions.

The second part deals with the mining schedule of the material within the optimum open pit limits obtained in the first part, using a combination of physical and economical parameters. A new approach, 'Simulation-linear programming' has been developed for the optimal scheduling of the mining blocks within the optimal open pit.

The simulation part of the model handles the extraction of ore and waste blocks once they are scheduled by the linear programming module. The development of sets of blocks is controlled by the geometrical (mining access) constraints. These sets of blocks are then submitted to the linear programming part of the model where the movements of ore and waste are optimized. The two parts of the model operate separately to overcome the difficulties of having a large number of constraints within a single technique.

CHAPTER TWO

The determination of optimal open pit limits by parameterization

1. Introduction

1.1 The concept of parameterization

The basic idea of parameterization is to provide a solution to a problem in a parametric form, i.e. as a function of one or more parameters. For example, the formula:

$$v = u + at$$

is a parametric solution to the problem of determining, after **t** seconds, the velocity (**v** m/s) of an object that is moving with an initial velocity of **u** m/s and accelerates at a rate of **a** m/s^2. Rather than providing a solution for a specific instance the formula provides a general solution in terms of all relevant parameters. By substituting values of the parameters (**u**, **a** and **t**) all possible solutions can be found rather than finding individual solutions from first principles.

For open pit optimization the objective is to generate a family of pits as a function of one or more parameters. For example, to generate the pits corresponding to all possible cut-off grades. The optimal open pit problem may be parameterized as a function of a single variable or as a function of several variables. However, only the single variable parameterization has been successfully implemented. Two-variable parameterization has met with limited success but it is doubtful whether general multivariable parameterization of the open pit problem will ever be successfully implemented because of:

- the complexity of the problem
- the computing power and time that would be required
- the doubt that any more than a single variable (cut-off grade) parameterization is really needed

1.2 Need for parameterization

The design of an open pit is based on an orebody model which, in turn, is based on estimated grades and tonnages. Estimated grades and tonnages are subject to significant errors which depend on the amount and location of information (drilling and other forms of sampling) which are available for estimation. Each orebody model is only one possible representation of the orebody constructed from the information available. The reserves, final pit design, location of roadways and operating requirements are only as good as the model of the orebody. All standard algorithms for open pit design are based on a revenue block model which is constructed from the orebody model. Each block in the revenue block model is assigned a net revenue value calculated from the estimated recoverable grade and tonnage in the block and from predicted costs, prices, cut-off grades, mining dilution and mill recovery. The "optimal" open pit derived from this model will not be the same as the true optimal open pit based on perfect information. The difference between the true optimal pit and the estimated optimal pit may be significant and has important implications for feasibility studies, cash flows, risk analysis and all subsequent mine design and planning.

The two major problems associated with the traditional approaches to optimal open pit design are:

- the inability to optimize on the basis of maximum net present value
- the need to solve the problem separately for each change in the value of a variable (e.g., cut-off grade, price, costs)

The first of these problems has been discussed, amongst others, by Dowd (1994b) and Dowd and Onur (1993) and can be stated as :

The net present value of a block cannot be determined until it is known when the block will be mined. However, it is not known when the block will be mined until the pit is designed and a mining sequence is established. But the pit cannot be designed on the basis of maximizing net present value until each block is assigned its net present value.

This is an intractable problem for the traditional approaches based on moving cones, graph theory (Lerchs-Grossmann), flows through a network and related techniques.

The second problem is largely operational. A single pit design based on a fixed set of costs, prices and cut-off grades can often provide a misleading picture of the possible working pit and of the minable reserves. It is always advisable to test the sensitivity of the pit design to changes in all of the variables used to calculate the revenue block model. In addition, it is also advisable to test the pit design to sensitivity to grade and tonnage estimation errors (cf. Dowd, 1994b). These types of analyses could result in the need to generate several dozen pit designs each of which could take significant computing time. The problem is that optimal open pit design algorithms do not express the solution **parametrically**, i.e. as a function of the parameters that were used to calculate the block model or of other design parameters such as pit wall slopes. A parametric solution to optimal open pit design might also lead to a method of solving the problem of optimizing on the basis of maximum net present value.

The parameterization method of pit design was developed and implemented in the early 1980s at the Ecole Nationale Supérieure des Mines de Paris at Fontainebleau, France. This algorithm uses the grade values of blocks instead of a revenue block model and is based on the techniques of functional analysis. The aim of this method is to transform a parametric optimization problem with severe geometric constraints into a simple one with no constraints. The method does not take into account any economic parameters and this is the fundamental difference in its approach. It is also the cause of most of the controversy which the method has attracted.

Parameterization could also be applied to the determination of mining sequences for the optimal extraction of the recoverable reserves of any particular pit. In economic terms the mining sequence is more important during the early stages and plays a major role in capital investment. This aspect is discussed in Chapter 3.

The variables used in the technique are the total tonnage, selected tonnage, the quantity of metal and the method of exploitation. These variables depend on a number of parameters including the raw data and the manner in which they are used to generate a block grade model; the cutoff grade used to define ore and waste; and the operational objectives which may have a significant influence on initial pit designs and hence the evaluation of the economic potential of the mineral deposit.

The method is not rigorous, and has some weaknesses on the economic side when compared to other economic or revenue based algorithms. However, the aim of this study is to stress the advantages of the theoretical and practical aspects of technical parameterization, as applied to an estimated block grade model of an orebody, and to present the additional work done by the author during the course of this research project in:

- improving the software implementation of the method
- implementing ways of graphically displaying and interrogating the parametric solutions generated by the method
- determining economically optimum pit limits from a parametric family of technically optimal pits

- using the results generated by a parameterization program as the basis for optimal scheduling of the mining operation

Standard computer programs which implement the parameterization algorithm only apply technical parameterization and optimize the results; they do not include any economic optimization and do not determine mining sequences, mine scheduling, roadway design or other factors.

The main areas undertaken in this research project were :

1. The development of a computer program to implement the parameterization method and include the economic optimization for a final operating pit design. Such parameterization is based on metal content, though it is acknowledged that other parameters need to be considered if the work is continued.

2. Development of an algorithm for the determination of mining sequences which will optimize the net present value of recovered ore. This is a critical element in optimization because in economic terms the mining sequence is more important during the early stages of mining and plays a much greater role in investment decisions than does the shape of the ultimate open pit.

3. Development of a computer program to determine the optimal mining schedule and integration of this program with the parameterization to determine the optimal mine schedule.

2. The parameterization model

This section is based largely on Moks (1983b) and Dowd (1992). The basic idea of the parameterization technique is to reduce the number of parameters to those which are of primary interest and then, as far as possible, separate the technical parameters from the economic ones. It is then easy to deal with the determination of the optimal pit limits for an orebody using only the technical parameters and leaving the economic ones to be considered post-optimization. The original version of the technique has been presented in a number of papers, principally Matheron (1975a, 1975b, 1975c), François-Bongarçon (1978,1980), François-Bongarçon and Guibal (1980, 1981, 1982).

The parameterization algorithm developed by Matheron (1975a,b,c) works with estimated block grades and is used to find a family of technical pits which maximise the quantity of metal for a given total tonnage and selected (above a cut-off grade) tonnage, without assuming any values for the economic parameters. Amongst this family of technically optimum pits there is at least one which satisfies the criteria for an economic optimum. However, the objective is to find a total family of technically optimum pits corresponding to every possible value of total tonnage and selected tonnage. For this purpose a fundamental hypothesis is made :

The technical factors which influence the definition of a particular pit are :

- (i) Mineralization
- (ii) Mining method
- (iii) Total tonnage (T_t)
- (iv) Selected tonnage (T_s)
- (v) Planning cut-off grade
- (vi) Metal quantity
- (vii) Pit geometry

These interdependent factors do not constitute an exhaustive list but they are the most significant. If the problem is considered as a comparison of the possible options for a particular deposit (i.e. a block grade file), then the mineralization and basic mining method can be assumed fixed. Furthermore, the mining method is mainly represented by geometric constraints; in particular the limiting pit slope. This slope is defined for the deposit on rock and soil mechanics criteria.

The planning cut-off grade is used directly for the selection of ore and waste at the planning stage and, for a given total tonnage, defines the selected tonnage of ore and quantity of metal within the selected tonnage.

The total tonnage of the pit is perhaps the most fundamental factor in defining that pit. Once this factor is assigned a value then the shape and position of the pit is also defined provided that the following fundamental hypothesis is made.

2.1 Fundamental hypothesis

The fundamental hypothesis in parameterization is that, for a given set of conditions and technical factors, the pit plan of particular interest is that which maximises the quantity of metal.

This hypothesis is generally true, because most revenue functions increase with metal quantity Q. This allows a pit to be defined for a particular deposit by a minimum number of technical parameters:

- metal quantity Q
- total tonnage T_t
- selected tonnage T_s

For given T_t and T_s the optimal pit (B) is then :

$$B_i = (Q_{max_i}, T_{t_i}, T_{s_i}) \qquad (2\text{-}1)$$

Provided that Q_{max} can be found for given values of T_t and T_s, this will also define the position and shape of the pit B_i subject to the geometric access constraints.

2.2 Defining the orebody and the pits

The resource model E consists of a finite set of blocks $x \in E$. Each block has a total tonnage, $T(x) > 0$, and a tonnage, $Q(x) \geq 0$, of the valuable constituent(s). The grade of a block is then $q(x) = Q(x)/T(x)$.

The extraction cone of a block x is denoted $\Gamma(x)$ and is such that:

$$y \in \Gamma(x) \Leftrightarrow \Gamma(y) \subset \Gamma(x) \qquad (2\text{-}2)$$

A feasible pit B consists of the union or intersection of extraction cones; in particular:

$$B = \cup \{\Gamma(x) : x \in B\} \qquad (2\text{-}3)$$

3. Single selection parameterization

The formulation given above is very awkward, since for a given T_t and T_s there exists an extremely large number of feasible pits and the direct calculation of these would be prohibitive in terms of time and money. To avoid searching a vast number of alternatives the problem is reformulated in terms of convex analysis techniques.

3.1 Convex analysis

Each particular pit B_i can be viewed as a point in the three-dimensional space of (Q, T_t, T_s). As there is a finite number of blocks in any deposit model, the number of blocks in any pit, even without geometric constraints, is also finite. Thus there is a finite number of possible pits or points in this space. These points form a cloud within a sub-domain of this space. Their limits are determined by the following constraints:

$$\begin{aligned} &Q,\ T_s,\ T_t \quad \text{all positive} \\ &T_t < \text{deposit total tonnage } T_D \\ &T_s < T_t < T_D \\ &Q < T_s < T_D \end{aligned} \qquad (2\text{-}4)$$

This sub-domain is shown in figure 22.

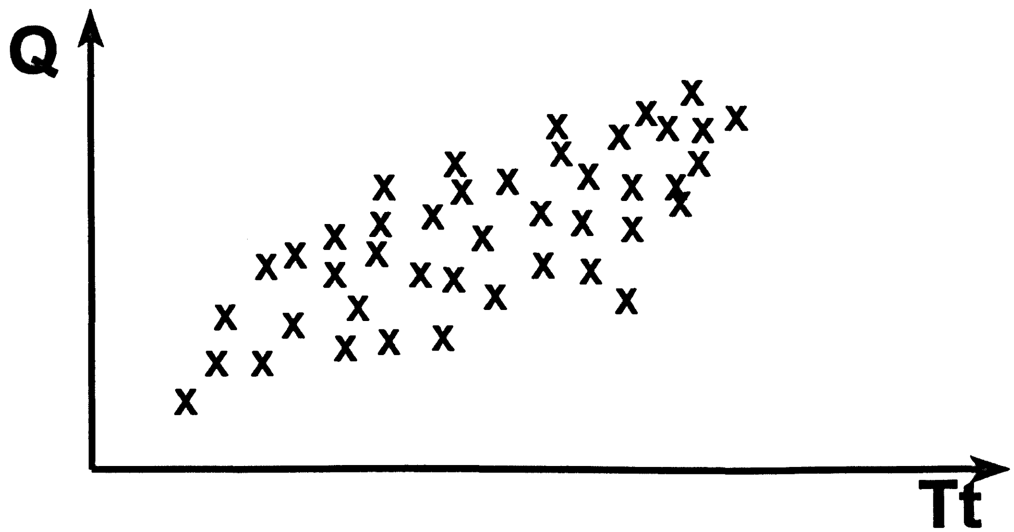


Figure 22:
Sub-domain of feasible pits

According to the fundamental hypothesis not all the points in the space are of interest, only those which maximise the surface of the cloud of points relative to the Q axis. A further reduction in the number of these points is made by considering only the convex hull of the cloud surface as shown in figure 23.

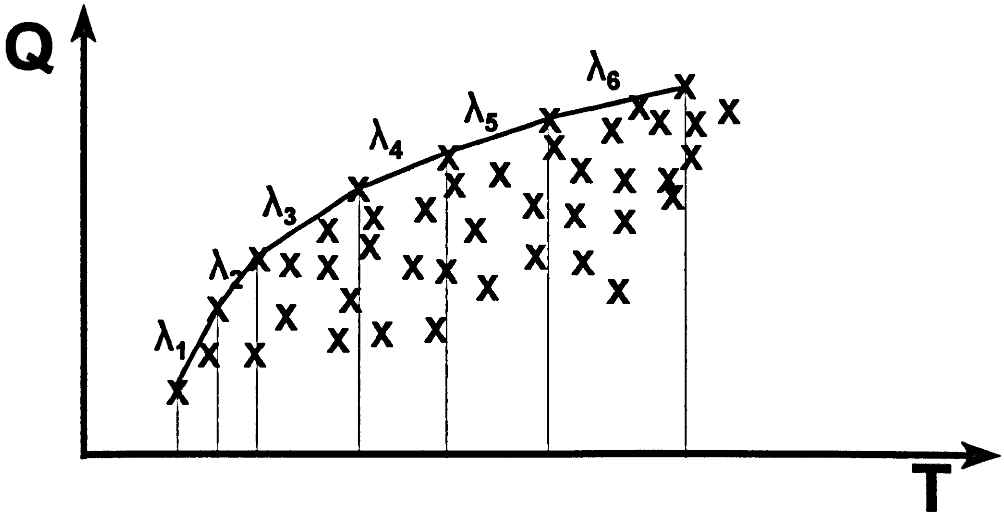


Figure 23

The pits that constitute the convex hull are shown by vertical lines; the remainder are not considered further. A point lying on the hull which does not correspond to a change of gradient (λ) of the hull is also omitted. Extension to a third dimension requires a convex hull defined by planes.

The points eliminated during this reformulation are assumed to be surrounded by points representing similar or larger quantities of metal; this approximation is known as convex analysis, Matheron (1975a, 1975b, 1975c), François-Bongarçon (1978). However, it is assumed that the critical points of the convex hull occur in sufficient density as to include all the possible conditions for which the optimal pit might exist.

4. Double selection parameterization

Instead of taking this direct approach, Matheron (1975) replaced the problem by its dual equivalent. The objective of this re-formulation is to find the family of pits which satisfies the following expression:

$$B_i = \max (Q - \lambda T_t - \theta T_s) \qquad (2\text{-}5)$$

where λ and θ are cutoff parameters for two levels of selection (which tonnage T_t to extract and how much, T_s, of that tonnage to select as ore) and they correspond to different gradients of the planes on the convex hull of the points in the space (Q, T_t, T_s). These parameters, however, can be applied on a local scale to individual blocks, as will be seen later.

The new formulation of the problem defines a pit in terms of λ and θ and the problem now is to find the family of pits which maximizes expression (2-5) for all values of λ and θ.

The re-formulation can also be found in Dagdalen and François-Bongarçon (1982), Coleou (1989), François-Bongarçon and Guibal (1981), all of which can be consulted for a more detailed presentation.

The examination of the dual problem was considered in the simplest possible situation of free selection of blocks in which there are no geometric or other access constraints.

4.1 Free selection

A simple way to examine the problem as expressed in (2-5) is to consider the case where no geometric or other constraints of access exist. In this unrealistic case there can only be one level of selection and therefore only one selection parameter θ; and the problem becomes:

$$B_i = \max (Q - \theta T) \qquad (2\text{-}6)$$

In this case, the maximum is always reached by arranging the blocks in decreasing order of grade and, for any particular extraction tonnage, taking the blocks of highest grade until that tonnage is reached. Thus in this case, Q is always a convex function of T (see figure 24) and defines a cut-off grade in the selection.

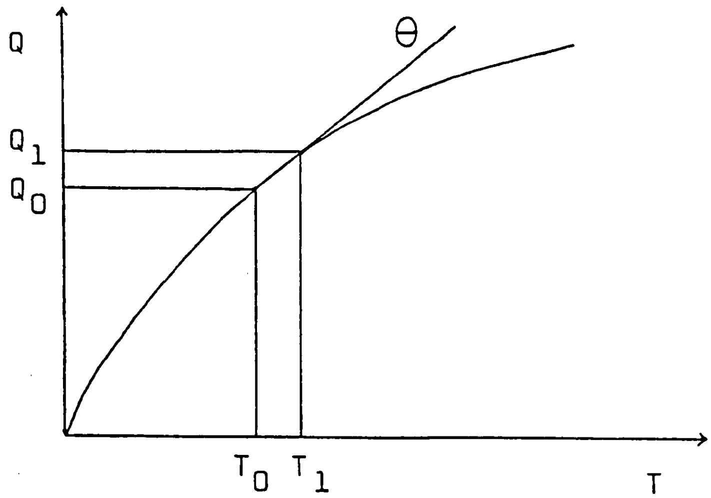


Figure 24

At a given T_0 the corresponding tonnage Q_0 of metal is obtained. If the tonnage is increased to T_1, then the total tonnage will be increased by $T_1 - T_0 = \Delta T$. This block ΔT will increase the quantity of metal by $Q_1 - Q_0 = \Delta Q$. The average grade of this marginal block is, of course, $\Delta Q/\Delta T = \theta$. Thus θ acts as a cut-off grade in the selection, so the maximum in (2-6) can be reached for a given θ by taking all blocks of grade greater than or equal to θ.

The double selection with constraints (2-5) is approached in a very similar way. One selection parameter θ is fixed $\theta=\theta_0$ and then the quantity $(Q-\theta_0 T_s)/T_t$ is projected onto a particular functional space which is characterised by its automatic satisfaction of those constraints. A free selection can then take place on the projected value Λ by the cut-off parameter λ. The approach to the final solution of the parameterization problem relies on the definition of the functional space on which the projection of the quantity $(Q - \theta_0 T_s)/T_t$ is made.

4.2 The functional space

To extract a block x, all of the blocks in its extraction cone $\Gamma(x)$ must also be taken. This is a free selection only if all of the blocks $y' \in \Gamma(x)$ are of grade higher than or equal to that of x and all blocks $y \notin \Gamma(x)$ outside this cone of grade below that of x.

Thus the ordered relationship provided by the extraction cone can be used to characterise this particular functional space F.

A function f is in this space if, for every block y in the extraction cone of x, $f(y) \geq f(x)$ for each block x. Such a function is said to be Γ-increasing, i.e.:

$$f \in F \Leftrightarrow \{f(y) \geq f(x), \ \forall y \in \Gamma(x)\} \quad \forall x \qquad (2\text{-}7)$$

In practice, (2-7) is replaced by an approximation which decomposes the space into a series of linear identities. The cone, of circular or elliptical base, is replaced by one of polygonal base, usually of six sides in conformance with Matheron's method (1975a, 1975b) (see figure 25).

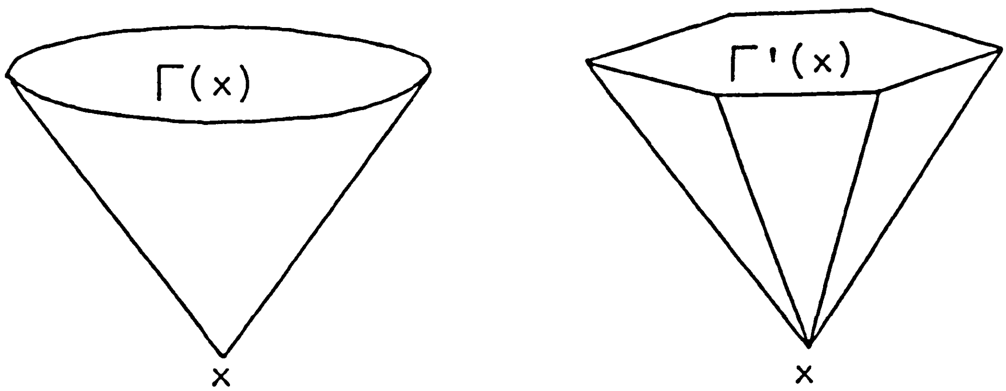


Figure 25: Approximation of the extraction cone

In most cases the six-sided polygonal base is sufficient to describe the cone and the six linear forms can then be divided into two groups of three independant shapes (ϕ_1, ϕ_2, ϕ_3) and (ϕ_4 , ϕ_5 , ϕ_6) which are chosen in such a way that each of the two groups corresponds to a triangle circumscribed by the base curve (horizontal directrix) of the cone as illustrated in figure (a).

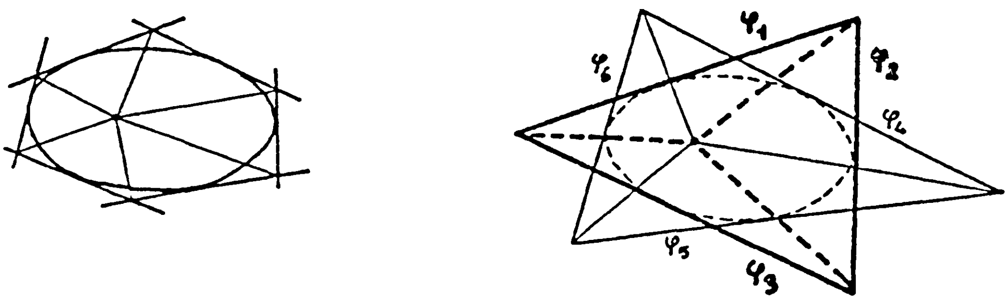


Figure (a): Illustration of the cone co-ordinates transformation

Each of the sides k of the approximate extraction cone $\Gamma'(x)$ can be defined by equal values of a normal axis ϕ_k. In the same way, a block y is within the extraction cone of x if and only if:

$$y \in \Gamma'(x) \Leftrightarrow \phi_k(y) \geq \phi_k(x), \qquad \forall\ k = 1,6 \qquad (2\text{-}8)$$

A simple linear transformation of a block's coordinates from the Cartesian to the pyramidal ϕ_k can be achieved: $y = (Y_1, Y_2, \ldots, Y_6)$, $x = (X_1, X_2, \ldots, X_6)$ and y is in the extraction cone of x if and only if $(Y_1 \geq X_1, Y_2 \geq X_2, \ldots, Y_6 \geq X_6)$. It is now easier to characterise the functional space F.

$$f \in F \Leftrightarrow f(y) \geq f(x), \quad \forall (Y_1 \geq X_1, Y_2 \geq X_2, \ldots, Y_6 \geq X_6) \qquad (2\text{-}9)$$

4.3 The projection

The projection of $q = (Q - \theta_0 T_s)/T_t$ onto the functional space F is denoted by:

$$\Lambda = \Pi_F \bar{q} \qquad (2\text{-}10)$$

Even with the simplification in the characterisation of this space, Λ has to be approached iteratively. This is done by decomposing the space F of 6 coordinates into two spaces F_1 and F_2 of 3 coordinates each :

$$\Lambda_1 = \Pi_{F_1} q \; ; \qquad \Lambda_2 = \Pi_{F_2} q \qquad (2\text{-}11)$$

and then approximating Λ by projecting q onto the space of $\Lambda_1 \Lambda_2$.

$$\Lambda = \Pi_{\Lambda_1 \Lambda_2} q \qquad (2\text{-}12)$$

This two-dimensional projection is very quick and easy to compute and allows the further approximation of approaching Λ_1 and Λ_2 by a series of similarly two-dimensional projections. Matheron (1975a, 1975b) developed two methods for this approximation, the first is the spiral method and the second is the triangle method. The latter method is more powerful in its approach but can be blocked at a false solution, which can then be unblocked by the former method. Details of the two methods are given in section 5.

5. Practical techniques

This section again draws heavily from the works of Matheron (1975a, 1975b, 1975c) and François-Bongarçon (1978). The functional space F has been defined and the existence of the projection Λ of grade onto this space proved. However, as this projection cannot be obtained directly, the space is decomposed and the projection approached iteratively. This method is not rigorous because there is no guarantee that the limiting projection found iteratively is the required projection.

5.1 Γ-increasing space

The extraction cone, Γ(x), can be represented by an infinite family I of linear identities on R^3 such that:

$$y \in \Gamma(x) \Leftrightarrow \varphi_i(y) \geq \varphi_i(x) \qquad (2\text{-}13)$$

By modifying the cone slightly, I can be assumed finite, say 6, which amounts to approximating the base of the cone by a polygon: Γ is now defined by 6 linear identities:

$$y \in \Gamma(x) \Leftrightarrow \varphi_i(y) \geq \varphi_i(x) \qquad i = 1,2,....,6 \qquad (2\text{-}14)$$

With Γ defined by (2-14), a function f on E is said to be Γ-increasing if and only if:

$$k_i \geq k_i', \quad I = 1, 6 \Rightarrow f(k_1, k_2, ..., k_6) \geq f(k_1', k_2', ..., k_6') \qquad (2\text{-}15)$$

The space of Γ-increasing functions F is the family of functions of the form: $f(\varphi_1(x), \varphi_2(x), ..., \varphi_6(x))$ where f is increasing on R^6 in relation to the six variables simultaneously:

$$(k_i \geq k_i', \ \forall I = 1, 6) \Rightarrow f(k_1, k_2, ..., k_6) \geq f(k_1', k_2', ..., k_6') \quad (2\text{-}16)$$

5.2 Spiral theorem

This theorem and its proof are found in Matheron (1975a); the proof will not be given here. The functions can be regarded as vectors whose dimension is equal to the number of blocks in E.

The problem is to calculate $\Lambda' = \Pi_{F(\varphi_1,\varphi_2, ..., \varphi_p)}\, q$ knowing any function Λ_1 of $F(\varphi_1,\varphi_2,....,\varphi_p)$. The recurrence relationship:

$$\begin{aligned} Y_n &= \Pi_{(Y_{n-1},\, \varphi_i)}\, q \qquad n \geq 1; \;\; i = n \text{ modulo } p \\ \textit{with:} \quad Y_1 &= \Pi_{F(\Lambda_1,\, \varphi_1)}\, q \end{aligned} \quad (2\text{-}17)$$

defines a set of functions $\{Y_n\}$ in $F(\varphi_1, \varphi_2, ..., \varphi_p)$ which converge in a finite number of iterations towards a limit Λ_0. Moreover: $|Y_{n+1}| \geq |Y_n| \;\; \forall n \geq 1$ with equality only if $Y_{n+1} = Y_n$.

5.3 Triangle theorem

This theorem and its proof are found in Matheron (1975b).

The problem is to calculate $\Lambda'' = \Pi_{F(\varphi_1,\varphi_2,\varphi_3)}\, q$ knowing three independent functions X_0, Y_0, Z_0 of $F(\varphi_1, \varphi_2, \varphi_3)$. The triple recurrence relationship:

$$X_n = \Pi_{F(Y_{n-1},\, Z_{n-1})}\, q \qquad n \geq 1$$

$$Y_n = \Pi_{F(Z_{n-1},\, X_{n-1})}\, q \qquad n \geq 1 \qquad (2\text{-}18)$$

$$Z_n = \Pi_{F(X_{n-1},\, Y_{n-1})}\, q \qquad n \geq 1$$

defines three sets of functions $\{X_n\}$, $\{Y_n\}$, $\{Z_n\}$ of $F(\varphi_1, \varphi_2, \varphi_3)$ which converge in a finite number of iterations towards the same limit $\Lambda_0^{//}$. Moreover, $|X_{n+2}| \geq |X_n|$ $\forall$ $n \geq 0$ with equality only if $X_{n+2} = X_n$. The same is true for $\{Y_n\}$ and $\{Z_n\}$.

This algorithm is an iterative projection algorithm which uses dynamic programming for the maximization process. It provides a complete λ-parameterization (maximising the expression Q - θT for each value of λ).

The projections of Λ_1 and Λ_2 are obtained by iterative processes where every step has the form:

$$Z_n = \Pi_{F(X_{n-1}\, Y_{n-1})}\, q \qquad (2\text{-}19)$$

and the elementary projection is of the form :

$$Z = \Pi_{F(X,\, Y)}\, q \qquad (2\text{-}20)$$

The problem is now a projection in a two-dimensional matrix (X,Y).

The matrix is constructed by assigning values of the parameter λ (cut-off grade) to each block using a modified two-dimensional Lerchs-Grossmann algorithm; the critical values of λ are determined in such a way as to avoid missing pits.

5.4 Computation

The six linear identities $\varphi_i(x)$, i = 1,2,..,6 are easily calculated for each block x from its Cartesian coordinates and the pit slopes in each of the six directions.

The triangle method is then used to find the projection Λ_0'' of grade onto the Γ-increasing space of $F(\varphi_1, \varphi_2, \varphi_3)$. The coordinates $\varphi_1(x)$, $\varphi_2(x)$, $\varphi_3(x)$ themselves can be used as initial functions. The blocked solution Λ_0'' can then be used as an initial function for the spiral method to obtain Λ_0'. This amounts to finding a family of pits which satisfy the constraints of an extraction cone of triangular base. The same is done with axes $\varphi_4(x)$, $\varphi_5(x)$, $\varphi_6(x)$ to obtain the projection Λ_0''' and the final projection is found from:

$$\Lambda_0 = \Pi_{F(\Lambda_0', \Lambda_0''')} q \qquad (2\text{-}21)$$

This computation requires the ability to achieve the two-dimensional projections.

5.5 The two-dimensional projection

At Fontainebleau, this projection is done by assigning values of λ and then using a modified (Francois-Bongarçon, 1978) version of the two-dimensional Lerchs-Grossmann (1965) algorithm. This algorithm is extremely rapid and efficient. A search is then made for the critical values of λ by first calculating λ_{min} and λ_{max} and then progressively subdividing this interval.

At Leeds, Moks (1983) and Dowd (1992) approached the problem differently. They search for all of the critical values of λ directly by an iterative method similar to the moving cone method. This approach has been used in the current research project and has been incorporated into the software.

The formulation of the two-dimensional parameterization is analogous to the development of the three-dimensional parameterization outlined in section 3.

If the grades of blocks x ∈ E are projected onto the two-dimensional space E_2 defined by functions X and Y, then the cloud of points in this plane corresponds to the blocks x ∈ E. The grades are unchanged.

6. Using simple examples to explain the concept of parameterization

The projection of the function Λ can be illustrated by some simple examples in one and two dimensions where the function Λ can be easily determined by sight. In the grade matrices suppose that each block has a weight of one tonne.

Example 1

Γ-increasing : to remove a block, all blocks above it must be removed first. This is illustrated by the example in figure 26.

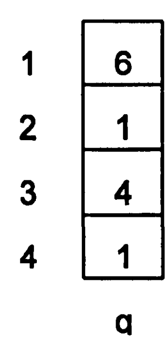


Figure 26

Start by considering the high grade values; the block of grade 6 must be the first that is removed. To remove blocks 1 and 2, the value of λ decreases because the average grade of the two blocks is 3.5. The third block will increase the values of pits 1 and 2, therefore the following pit will contain blocks 1, 2, and 3 (average grade 3.666). Finally, for λ less than or equal to 1 all four blocks will be taken.

As the function Λ will take a constant value for all blocks (average grade) this will give the arrangement shown in figure 27.

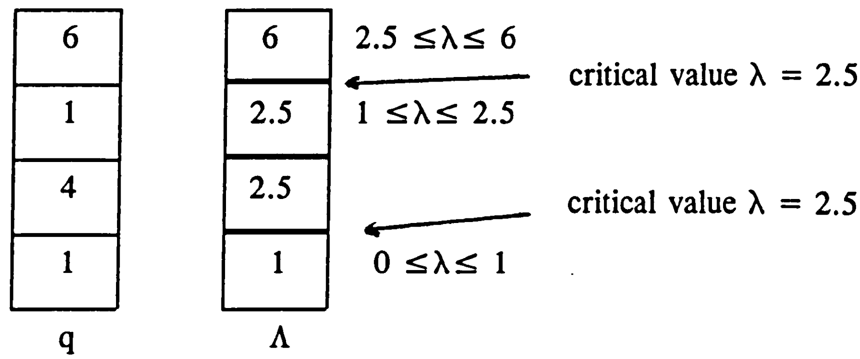


Figure 27

Example 2

Blocks as in figure 28 and same analysis as in example 1. If the high value of λ is chosen then the third block will be removed first. Because of the Γ-increasing constraint this block can only be removed if blocks 1 and 2 have already been removed. This gives an average grade of 5.0 for each block. For λ less than or equal to 1.0 all blocks will be chosen as shown in figure 28.

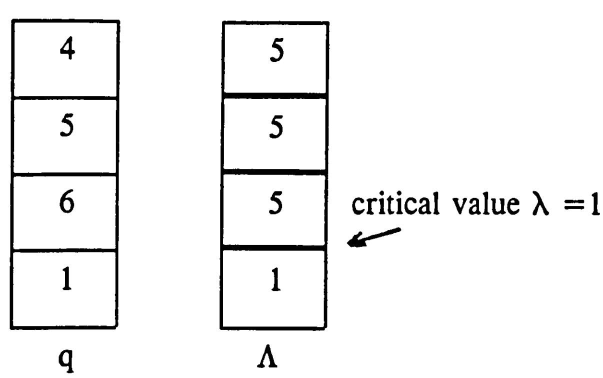
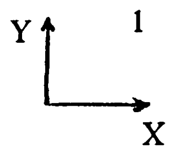


Figure 28

Example 3

This is a two-dimensional example with oriented X and Y axes. In this example, a particular block can only be removed by the removal of the blocks of coordinates greater than or equal to those of that block. In figure 29 the marked area is the extraction cone of block (X=4, Y=3) , which has a grade value of 4. Following the same reasoning as above the results shown in figure 29 are obtained. NB. It is convention to commence this exercise from the top right hand corner.

4	3	6	5	5	6
3	3	3	3	4	5
2	3	2	3	2	4
1	5	4	3	3	1
	1	2	3	4	5



q

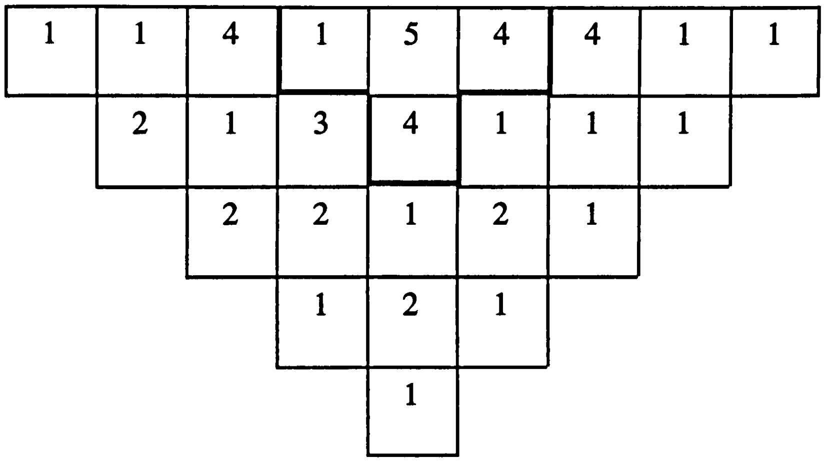
3	5.33	5.33	5.33	6
3	3	3	4	5
2.88	2.88	2.88	2.88	4
2.88	2.88	2.88	2.88	2.88

Λ

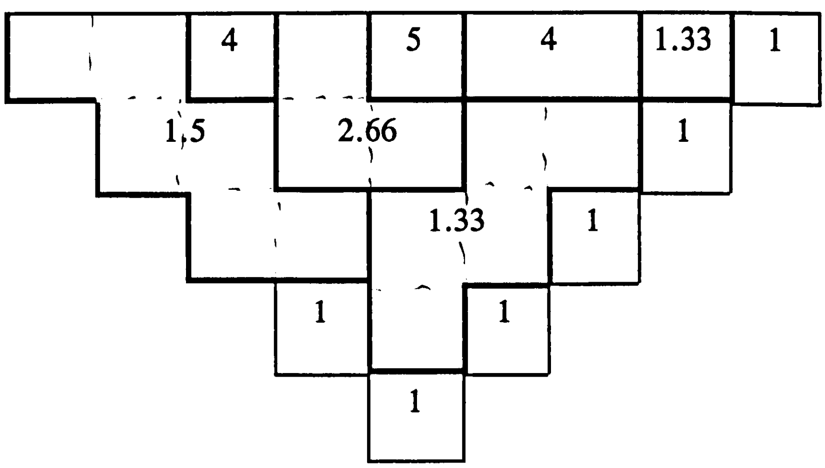
Figure 29

Example 4

Consider the two-dimensional example with natural extraction cone shown in figure 30. If the extraction cone is defined by single block steps, then Λ, the projection of the grade onto a Γ-increasing space, can be found directly by searching for the pit (cone or set of cones) with the highest average grade. This average grade, Λ, is then assigned to each block in the pit and the search then starts again for the pit with the highest average grade amongst the remaining blocks. This process is repeated until a solution is found. As can be seen the pits obtained are those with cone values greater than or equal to the value of each block.



q



Λ

Figure 30

Example 5

A second two-dimensional example is shown in figure 31 together with the extraction cone used in the example. This time Λ, the projection of the grade onto a Γ-increasing space, can be found directly by searching for the pit (cone or set of cones) with the highest average grade where valid pits are defined with reference to the extraction cone. This average grade, Λ, is then assigned to each block in the pit and the search then starts again for the pit with the highest average grade amongst the remaining blocks. In this example, the pits of highest average grade have a value of 0.60; there are two consisting of one block each, one consisting of four blocks and one of nine blocks. The largest pit is always chosen. This process is repeated until the solution shown in figure 31 is found.

0.1	0.1	0.3	0.4	0.3	0.2	0.4	0.6	0.4	0.6	0.3	0.2	0.3	0.2
	0.1	0.2	0.4	0.3	0.4	0.6	0.8	0.8	0.6	0.4	0.2	0.1	
		0.1	0.2	0.4	0.3	0.5	0.8	0.9	0.4	0.2	0.2		
			0.2	0.2	0.3	0.3	0.5	0.9	0.5	0.3			
				0.3	0.5	0.3	0.6	0.8	0.4				
					0.1	0.2	0.4	0.6					

Figure 31(a): two-dimensional example; block grades

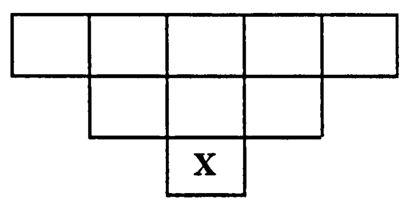


Figure 31(b): extraction cone Γ(x) for two-dimensional example

0.16	0.24	0.37	0.40	0.43	0.53	0.60	0.60	0.60	0.60	0.60	0.48	0.40	0.30
	0.16	0.24	0.37	0.38	0.43	0.53	0.60	0.60	0.60	0.48	0.40	0.30	
		0.16	0.24	0.37	0.38	0.43	0.53	0.60	0.48	0.40	0.30		
			0.16	0.24	0.33	0.38	0.43	0.48	0.40	0.30			
				0.16	0.24	0.33	0.38	0.40	0.30				
					0.10	0.20	0.33	0.30					

Figure 31(c): Λ values for the example in figure 31(a)

The Λ values are seen to be Γ-increasing, i.e. for any given block, the Λ values of all blocks within its extraction cone are greater than or equal to the Λ value for that block. Thus for any given cut-off value λ, the set of blocks defined by $\Lambda \geq \lambda$ constitute a feasible pit. Moreover, every technically optimal pit is completely defined by a parameter λ.

7. Implementation of parameterization

The parameterization algorithm is implemented by using the triangle and spiral methods. The triangle method is applied to determine the projection of Λ_1 onto the Γ-increasing space $F(\varphi_1, \varphi_2, \varphi_3)$ using the grade and volume of the selected blocks. The blocked solution can then be used as an initial function for the spiral method to find the optimal solution of pits which satisfy the constraints of an extraction cone of triangular base. The same is true for Λ_2 but this time for the Γ-increasing space $F(\varphi_4, \varphi_5, \varphi_6)$

The projections of Λ_1 and Λ_2 are obtained by iterative processes which yield two-dimensional matrices of functions (x,y). The projections are done by assigning grade and tonnage values to each block and then using the two-dimensional Lerchs-Grossmann algorithm.

The orebody block model is divided into parallel sections and the modified Lerchs-Grossmann method is applied to each section to determine the optimal shape of the two-dimensional pit for each level on this section. Thus for each section there will be an optimum pit limit by level together with the corresponding optimal value. These values are then accumulated into columns, which determine a two-dimensional numerical matrix for the pit.

When the two-dimensional matrix is found for all sections representing the pit then a parameterization method is issued to parameterize the two-dimensional matrix. In this study the search of the matrix is based on the two-dimensional parameterization algorithm written by Moks (1983) and modified by Dowd (personal communication). The algorithm is the reverse of the original one. Instead of the search for the right hand quadrant, Moks saw in his study that the larger pits corresponding to lower values of grade are of interest and he thus found it desirable to reverse the solution procedure by defining the lower left hand quadrant, and searching and removing the pits satisfying the extraction cone of minimum grade.

The two-dimensional matrix is searched for the panel with the minimum grade for its left hand quadrant. The minimum grade of each quadrant is compared with that of the previous one; if it is less than the previous one, the two quadrants are accumulated together and removed. If not, then all the panels are set back to their original values and the procedure recommences for the next quadrant. The results of each procedure are stored and compared with the previous one. The final results are returned to the main program.

During the use of the triangle and spiral methods it sometimes happens, after a certain number of iterations, that the norm gives only slight changes around a few pits. In such cases efficiency is improved by stopping the convergence procedure.

The subroutine stack is used in the subroutine projection to control the norm value for each projection. It is then checked against the following value until the last iteration of the subroutine (six iterations with six norm values for the triangle method and only three for the spiral method). If the norm stops increasing or shows only a few variations around the pits then the solution is blocked.

To ensure convergence of the projection after a finite number of iterations its progress should be systematically checked. At the end of each projection a subroutine test is called to check whether the norm values are similar. If so, convergence is deemed to have been reached and the procedure is halted; if not, the procedure continues. The maximum number of iterations is set by the user.

When the pits are found, they are returned to the main program and a number of characteristic values of each individual pit are calculated. These characteristic values are:

- the average grade
- the selected tonnage
- the total tonnage
- the quantity of metal
- the stripping ratio and
- the ore-total tonnage ratio.

8. Economic optimization

8.1 Selection of the final pit

Matheron (1975) approaches the economic parameterization very theoretically and derives the parameterizing function which characterises the optimum for maximum revenue. The isovalue curves of this function might give the optimum pit design. The aim of this method was to consider the variability of an optimum pit when a given economic parameter λ changes. For each block the profit value will appear as a function $W(\lambda)$ of this parameter (monotonic function). This can also be seen in the paper by François-Bongarçon and Marechal (1976).

In this work the final pit limits have been estimated by methods which are based upon two factors :

(1) the orebody block model, on which all calculations are based, consists of values estimated from sparse data and it is thus subject to uncertainty and error

(2) the cost constraints are applied through what is called a parameterizing function which is based on both geometrical constraints and varying economic values.

The objective of the ultimate pit limit design is to determine the projected final pit limits of an orebody and its associated projected grade and tonnage, which will maximize some pre-specified economic criteria while satisfying practical operational requirements.

The parameterization approach enables us to present various alternative pit designs to the user so that the best plan, as defined by a specific combination of parameters (mining and processing costs etc...) can be selected. This is not possible with the standard block revenue model approach as applied by such methods as the original Lerchs-Grossmann algorithm (though it can be handled by the Whittle (1988) 4-D proprietary software).

The objective is to select the pit that has the maximum net worth attainable within an acceptable risk range and within any given constraints. For our purposes, this plan is the optimal plan.

8.1.1 Profit matrix

The determination of optimal ultimate pit limits, by nearly all of the computerised techniques, requires the transformation of the block grade matrix into a revenue block matrix which can be optimised (scheduled) according to a pre-determined economic criterion subject to geometric constraints of access.

Block profit comes from the evaluation of a function of many variables such as grade of ore, mining costs, transportation costs, price of mineral, etc. The net value for each block is the actual cost or profit realised by mining and processing a block. Blocks with positive profit have a final value which covers all costs (mining, production, transport, etc.) whereas negative profit blocks do not.

8.1.2 The general costing equation

The selection of the optimum pit amongst the set of pits produced by the parameterization method is done by using a simple profit function of the form:

$$A - (B + C)$$

where: A is the revenue from the sale of the finished metal,
B is the total processing cost, and
C is the mining cost.

Clearly A, B and C are all functions and it is assumed that :

(i) all variables affecting profit are included in the functions A, B or C, and

(ii) blocks of all grades are acceptable for processing. However, very low grades may give no finished metal, so A = 0.

Clearly :

(i) If A < B then profit < - C. In this case the block would be of very low grade, regarded as waste and not processed. This gives A = B = 0 and profit = - C.

(ii) If A > (B + C) then a positive profit is realized from mining and processing a block regarded as ore.

(iii) If A > B and A < (B + C) then by mining and processing the ore a loss is incurred but the loss is less than mining with no subsequent processing. In practice this is marginal ore and is regarded here as ore.

The cut-off grade is used as a constraint. If the block grade g_i is greater than or equal to the cut-off grade then it is treated as an ore block, if it is less than the cut-off grade then it is treated as a waste block. Normally, ore blocks have positive net profit values and waste blocks are negative. The plant cut-off grade is that for which A = B.

In practice, of course, this representation is too simplistic and it is necessary to allow for taxation, overheads, capital investment and other financial features.

8.1.3 The optimum pit

Ideally, mining is done in such a manner as to maximize profit throughout the mine life while maintaining operational continuity. To illustrate the use of the above formulation the simplest revenue formula is usually of the type:

$$Pr(i) = A\ V\ q(i) - B\ V - C\ V \qquad \text{if ore } q(i) \geq COG$$

$$Pr(i) = -\ C\ V \qquad \text{if waste } q(i) < COG$$

Where :

Pr(i)	is the profit value of block i
q(i)	is the grade of block i
COG	is the cut-off grade
V	is the tonnage of block i
A	is the sale price of metal per tonne
B	is the processing cost per tonne
C	is the mining cost per tonne

For this revenue formula, the implicit assumptions are :

1. The economic values assigned are constant throughout the mine and throughout the mine life.

2 The profit motive (maximizing revenue) is in use, but without time discounting.

3. The mining costs are the same for ore and waste.

4. The price is net of transport and marketing costs.

By applying the profit function to each block, a block profit matrix is generated. Pit profit is obtained by summing the profit of each block that has to be mined within that pit limit to produce the required profit.

Such a definition (selection) of an optimum pit is taken to be the configuration of blocks whose pit profit is a maximum.

9. Application of parameterization

9.1 Computer program

The application of the parameterization method and the interpretation of the results in terms of conventional open pit characteristics require additional work. This extra work involves the construction of plan, metal inventories and ore-waste ratios for each pit defined by a critical cutoff grade. These calculations have been incorporated into a computer program.

The program which has been used in this study is based on an original coding by Moks (1983), updated and amended by Dowd, and written in FORTRAN 77. The original versions have been extended in the current project to include the choice of the final pit and the integration with a scheduling algorithm for more detailed mine planning. The performance of the software has been improved to enable it to run faster and more efficiently and in a more interactive manner. The original parameterization method is retained in the new developments.

9.2 Input and Output

The user provides to the interactive part of the program :

- the cutoff grade,
- the block dimensions,
- the specific gravity of the ore,
- the pit wall slope angle

- the estimated set of data (block grades) stored in a computer file in 3-D form, (including X, Y and Z co-ordinates).

The program then produces a set of nested pit designs (characteristics and plans), based on the specified input data, at an average of approximately 30 seconds of computer run time on a Sun workstation. The output is provided in two forms. The first is a binary file created for efficient storage and data transfer to other programs (not discussed here). The second file contains full details of the complete characteristics and plans and a summary of each individual pit. The plans of the nested pits are given as grade distributions or level heights, both of which can be used to provide pit contours. The characteristics are those parameters which define the pits in economic terms such as cash flows.

The characteristics associated with each pit design, such as metal content, total tonnage, average grade and stripping ratio, are easily calculated and interpreted. But the most difficult is the representation and the location of the shape in the form of an open pit design. However, the design of pits is always determined by boundaries rather than by internal grade distributions and the contour plan is the better tool for these representations.

9.3 Interpretation and presentation: display of output

A program was written to process the numerical data into graphical output and produce graphical representations of the characteristics (metal content, total tonnage, average grade, stripping ratio, etc...) of any pit plan. The program was written for a Sun workstation using graphics routines from the UNIRAS package (1989).

UNIRAS is a software system consisting of a set of subroutines and functions (UNIRAS Reference and User Manuals) used by an applications program to generate images and pictures on display input/output devices. The graphics display program developed in this research project was written in FORTRAN 77, and is designed to provide users with the choice of generating the following output:

1- Selection of the final pit
2- Two-Dimensional contouring of mining levels
3- Three-Dimensional representation of the optimum pit plan
4- Cross-sectional representation of pit elevations
5- Three-dimensional views of blocks mined

The graphics display programs will be presented and described more comprehensively, and examples will be given to illustrate their use, in Chapter 6 of this thesis.

10. Conclusions

The parameterization method allows the calculation of many optimal pits at the same time, corresponding to different values of the parameter λ . These pits are obtained without considering economic parameters, and also construct the relationship between cutoff grade and tonnage, or the quantity of metal and total tonnage of exploitation. In one computer run, it gives all possible pits of the orebody for a given parameter λ. The user can decide the size of the blocks to be optimised without creating a problem about the presentation of the results.

The objective of parameterization is to find a complete family of technically optimal pits corresponding to every possible value of the total tonnage and the selected tonnage. The crucial assumption for the success of the method is that this family contains the pit that is optimal for any specified sets of economic parameters. The only drawback is the limitation on the slope angle of the cone.

However, practical applications of parameterization are not well documented and some questions remain about the general applicability of the method. It is still not clear whether there are significant practical cases in which the following may prevent a true optimum being reached:

1 Approximating the surface found by Q_{maxi} (at any T) by the convex hull Q - λT is likely to miss some pit designs.

2. Approximating the extraction cone by one of polygonal base.

3. The grouping of blocks by their polygonal coordinates into discrete ordered subsets.

4. The approach to the projection by decomposition and iteration.

5. If used, the imposition of a minimum significant tonnage between pits.

However, parameterization appears to be at least as good as most algorithms and better than most in respect of the computing time and, more importantly, the block model it uses. Even with the introduction of technical parameterization as the most recent development the problem is not yet solved to everybody's satisfaction.

CHAPTER THREE

Determination and optimization of mining sequences

Page

1. Introduction

A major problem in mine planning and design is the determination of the mining sequence which will optimize some specified criterion. The criterion of most interest, especially during feasibility studies, is maximum net present value. Ideally, the maximum net present value should also be the criterion for the optimal open pit design. However, as noted in Chapter 2, this involves an intractable circular argument. The optimal open pit design problem with net present value as the optimizing criterion could also be viewed as a scheduling problem:

> *Schedule the mining of the blocks, subject to mining access constraints, in such a way that the maximum net present value is achieved.*

The solution to this problem must also yield the pit shape which has maximum net present value. However, whilst the problem can be formulated in this manner and solution algorithms are available (e.g., dynamic programming), the number of blocks comprising most practical orebody models means that algorithms are prevented from reaching a solution because of prohibitive storage requirements or equally prohibitive computing times or a combination of the two.

1.1 Scheduling as a general problem

The problem of scheduling arises in many other applications outside the mining industry and there is an extensive literature on the subject in many of these non-mining applications.

In general, scheduling problems can be subdivided into two categories : deterministic scheduling and stochastic scheduling.

The deterministic scheduling approach is to formulate the problem in such a way that there is only one (deterministic) solution. However, in many applications the variables have uncertain values and solutions can, or at least should, be expressed in terms of likelihood or probability. In essence the deterministic approach assumes that the values of all variables are fixed and known whereas the stochastic scheduling approach recognizes and includes the uncertainty on each variable.

A realistic approach to mine scheduling should formulate the problem as a stochastic scheduling problem. However, in this thesis a deterministic approach has been adopted mainly because this is seen as an essential first step towards a stochastic formulation. For example, the linear programming approach described in the next Chapter could readily be adapted to stochastic linear programming and this is an area for further work and development of the methods described here.

Deterministic scheduling problems can be subdivided into two further categories (Garey and Johnson, 1979) :

P-problems for which there exist optimal solution algorithms of polynomial complexity.

NP-hard problems which can only be solved by algorithms of non-polynomial complexity.

There are four major methods applied in the solution of NP-hard problems :

Relaxation

In these approaches the original problem is replaced by a similar one in which restrictions on one or more parameters have been weakened or relaxed and/or some additional constraints have eliminated possibilities and reduced the solution space.

Approximate optimization

Either a heuristic approach is taken or an existing optimizing algorithm is assisted by the addition of some heuristic aspects. An example of this approach in mine design is the modified moving cone method of open pit optimization.

Optimization by enumeration

This approach enumerates all possibilities and then seeks the optimal sequence. As such it leads to the true optimal solution. An example of this approach is dynamic programming as applied, for example, to the mine scheduling problem. Other examples include branch and bound methods and iterative methods.

Artificial intelligence approaches

These approaches include expert systems, genetic algorithms and neural networks. Neural network techniques, in particular, offer the possibility of reaching optimal or near optimal solutions to the mine scheduling problem.

1.2 Parameterization as a means of scheduling

The parameterization method could be used as a means of scheduling. Each pit in the nested family generated by the application of the parameterization algorithm represents a set of blocks of a given average grade. On the assumption that net profit is directly proportional to grade, sets of blocks arranged in descending order of grade correspond to sets of blocks that maximize net present value (provided that the sets are not restricted by size). Consider the two-dimensional example in figure 31 of Chapter 2. Each grade increment could be regarded as a pushback as shown in figure 32 in which blocks comprising the first few grade increments are shown by different shadings.

0.16	0.24	0.37	0.40	0.43	0.53	0.60	0.60	0.60	0.60	0.60	0.48	0.40	0.30
	0.16	0.24	0.37	0.38	0.43	0.53	0.60	0.60	0.60	0.48	0.40	0.30	
		0.16	0.24	0.37	0.38	0.43	0.53	0.60	0.48	0.40	0.30		
			0.16	0.24	0.33	0.38	0.43	0.48	0.40	0.30			
				0.16	0.24	0.33	0.38	0.40	0.30				
					0.10	0.20	0.33	0.30					

Figure 32:
Parameterization results from the example given in figure 31 used as production schedule that maximizes net present value

The difficulty of course is that the grade increments do not necessarily coincide with production increments or multiples of such increments and it may not be possible to satisfy production capacities by blocks belonging to these increments. Nevertheless, this approach offers the possibility of a good practical approximation.

1.3 Simplifying the problem

In practice the optimal open pit scheduling problem can be significantly reduced in size simply because it is impractical to schedule a mining operation for more than relatively short time periods (usually 1 to 2 years maximum and occasionally even less than 1 year). The reasons for this are :

- Financial variables, especially metal prices, cannot be predicted with any accuracy beyond this period. Significant changes in metal prices will have a significant effect on the revenue block model which in turn will alter the optimal mining sequence.

- Grade and tonnage values assigned to blocks are only estimates not true values. These estimates are based on relatively sparse data and are subject to error. In general, the error decreases as more data become available and are used in the estimation. In addition, the estimated grades and tonnages may change significantly as more data become available. Inevitably, in the initial stages, the most accessible blocks will be better estimated than those at greater depth. As mining progresses, more data will become available and the revenue block model may change significantly.

- Many other variables and factors may change throughout the life of the mine. Such variables include geotechnical properties and conditions which may affect wall slopes (and hence the mining access constraints).

Given these restrictions on the time period over which scheduling can realistically be applied a sensible approach is :

1 Design an optimal open pit on the basis of any of the common optimizing criteria (maximum total undiscounted revenue or maximum metal recovery). This pit defines an outer shell within which all scheduling (and therefore mining) will be done. This pit also defines total minable reserves.

2 Schedule blocks within the pit shell over specified scheduling periods updating the schedule over time and/or as the values of significant variables (e.g., financial, geological, geotechnical) change.

In certain circumstances, developments during any scheduling period may require the re-definition of the optimum pit. This feature has not been addressed in this thesis and remains an important issue to be tackled if the work continues.

1.4 Pit scheduling

Unlike the problem of determining optimal open pit limits, there are very few references available which address the subject of pit sequencing in a practical way. Those papers which do cover the subject recognize the fact that orebodies are normally mined in stages or sequences. The principles behind open pit mine scheduling were first clearly stated by Lerchs and Grossmann (1965) :

> *The same optimal open pit limits could be reached by a multitude of mining sequences each of which will produce a different cash flow. Therefore there exists an optimum mining sequence which can be defined as that sequence which maximizes the present worth of the deposit. The possible sequences from which to choose from are of course limited by various constraints imposed by pit wall stability, the ore requirements of the mill and good mining practice. The optimum pit limits, therefore, cannot be defined without first determining an optimum mining sequence.*

The basis of the most commonly accepted methodology and its underlying philosophy are discussed in a number of papers and textbooks, most notably, Lerchs and Grossmann (1965), Roman (1973), Dowd (1976), Wilke and Reimer (1979), Wilke, Mueller and Wright (1984), Huang (1993), Fytas, Hadjigeorgiou and Collins (1993).

A new approach, which the author has called the *'combination of linear programming and simulated, user-activated waste stripping'* method, has been developed as part of this PhD research project. The method consists of three parts: ***waste stripping***, ***linear programming*** and ***simulation*** modules.

These references are used to introduce the general concepts of the method that forms the fundamental basis of the present and the following Chapter.

The algorithms have been developed for complete three-dimensional implementation although they could equally well be applied to two-dimensional cases (e.g., strips of mineable material). The algorithm has been used for a three-dimensional case study and the results have been validated; part of this case study is given in Chapter 6.

1.5 Characteristics of mining sequences

As mentioned earlier the optimization procedure is required to select the material to be mined in each sequence of a mining period, to maximize the output at the early stages, that is to quickly reach the rich ore of the technically optimum pit. This will maximize the chosen financial objective over the life of the mine, satisfying a carefully selected set of operational constraints. However, mining could proceed along unlimited alternate paths each of which would generate a unique cash flow pattern. The path with the highest discounted cash flow should be selected as the best mining sequence.

The following simple example, taken from Onur and Dowd (1993), illustrates the general concepts raised in the previous paragraph.

1.5.1 Illustration

The simple example shown in figure 33 illustrates the importance of net present value in the determination of mining sequences. In this example the same section of an orebody is depleted in the same time period with a discount rate of 10% by using two different planning sequences. The results are shown in table 1.

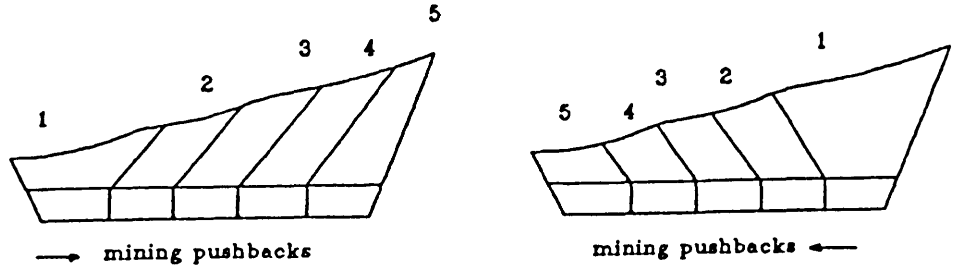


Figure 33 : two possible mining sequences

It can be seen from table 1 that the best way of mining the orebody is to mine the most profitable parts during the early stages of the mining operation. The same results are presented graphically in figure 34.

Time period	Alternative 1 Profit	Alternative 2 Profit	Discount Rate	Discounted Alternative 1	Discounted Alternative 2
1	100	60	0.909	90.9	54.4
2	90	70	0.826	74.3	57.8
3	80	80	0.751	60.1	60.1
4	70	90	0.683	47.8	61.5
5	60	100	0.621	37.3	62.1
Total	400	400		310.4	296.0

Table 1 : Discounted cash flows from two mining sequences

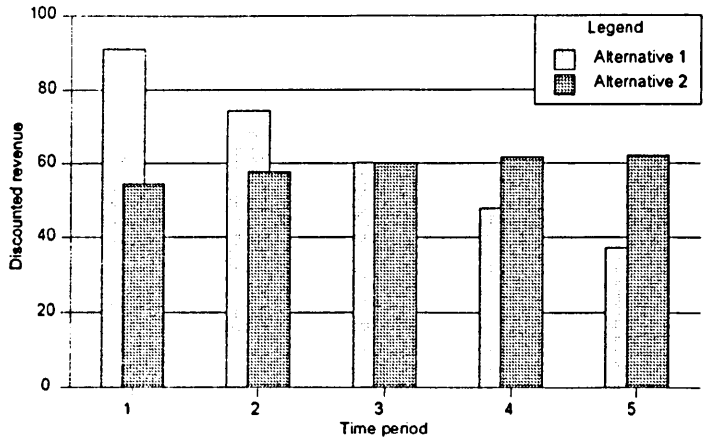


Figure 34: graphical form of results in table 1

1.5.2 General characteristics

Long range production scheduling is mainly concerned with such criteria as ore reserves, stripping ratios and major investment usually on a year by year basis. Short range scheduling, on the other hand, is the development of a sequence of depletion schedules on a daily, weekly or monthly basis, which complies with restrictions imposed by the long range plans, plant capacities, inventories, equipment availability and the existing mining operation.

Regardless of the type of ore or mineral product mined, there are certain basic data that are required in any production scheduling problem. These are :

1) the tonnage and grade of each block
2) specific gravity of ore and waste
3) the revenue value of each block
4) deposit block model
5) present pit/quarry layout or overall optimum limits of the pit/quarry

to be scheduled,

6) mine life or production rate (depending on whether short term or long term scheduling is used)

7) the maximum and minimum allowable grade to be mined or to be fed to the processing plant in any time period

8) maximum and minimum allowable production rate of waste and ore

9) working slope angle in the pit

10) minimum pit bottom dimensions

11) discount rate

12) preproduction rate and period, (if required)

Because of the large number of blocks in most orebody block models it is not practical to optimize a schedule which simultaneously includes the entire model covering all periods of the mine life and all types of mining constraints. For this reason the scheduling problem has been divided into a set of sequential sub-problems. The two major divisions are for time and mining constraints. These divisions can be summarized as :

Time

The scheduling process proceeds one mine period at a time, and

Constraints

Within each time period the constraints are treated in two major classes.

The simulation

This part handles the physical constraints and forms the main subject of the present Chapter.

The Linear Programming technique

This part handles the production constraints and parameters and is the main subject of Chapter 4.

The blocks to be mined in each mining period are selected subject to the given mining access constraints. The constraints on the possible mining sequences include both physical constraints on the mining operation and economical constraints. Only the physical constraints are dealt with here.

The following is a list of the operating constraints used in this application and which the computer program attempts to satisfy :

1. Mining access constraints. These constraints define the access to each block within the orebody block model. In their simplest form these constraints prevent the mining of a block which is directly under another unmined block and prevent the mining of a block for which any adjacent block has an unmined block above it; in this version mining must be done in a step-wise manner.

2. The one-for-one restriction requires that for every unit volume which is mined from a block that is restricted by a number of other blocks, at least an equal volume must be removed from each of the restricting blocks.

3. Maximum working slope angle.

4. Minimum radius of a mining area. This constraint is used to impose access for mining equipment to mining blocks and may include such specifications as minimum truck turning circle and minimum allowable access for loaders.

1.6 Mining sequence constraints

Determination of an allowable mining sequence is primarily a problem of geometry. In an actual mining operation the material is removed in such a way that the safe wall slope is never exceeded in any direction. The practice of benching gives the open pit mine a step-like structure. These conditions justify the representation of the outline of a mining plan or the volume of the mineral to be removed, consistent with the wall slope constraints and mining practices, by a series of frustums.

Given the block size, two methods are used to describe the allowable mining sequence relative to the block concept; one is to use a cone to define the set of allowable or required blocks, Erickson and Pana (1966) and the other is to use a pattern or set of blocks which closely approximates a thin frustum to define the allowable set Johnson (1968). The second method will be used in the mathematical formulation given in this Chapter.

1.6.1 Cone generation

A cone (or pit) is generated by the removal of whole blocks from the block matrix. As a pit is mined, the slope of the sides must not exceed the angle of failure and a maximum pit slope must be defined and observed. The pit slope considered here is the average slope generated by a step pattern (as whole blocks are removed). Thus, considering different step patterns amounts to considering different slopes.

Many other authors have considered cone generation, for example, Boyce (1969), Hartman, et al (1966), Johnson (1968), Lerchs and Grossmann (1965). The method adopted here is that of forming a cone with a single block as a base. Two methods are illustrated, both using cubic blocks. Due to the symmetry and uniform grid considerations, this requires that alternative benches must have the closest five blocks in the case illustrated in figure 35a, and the closest nine blocks in the case illustrated in figure 35b, mined in the bench above before the bench can be opened.

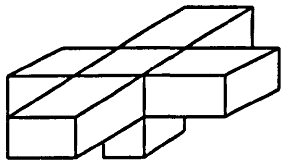


fig. 35a - 5:1

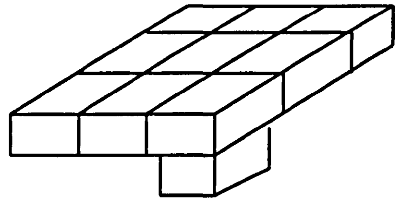


fig. 35b - 9:1

Figure 35

The 9:1 configuration method produces a cone with slopes ranging from 35° to 45°. The 5:1 configuration method produces a cone with slopes ranging from 45° to 55°.

By changing the dimensions of the blocks, i.e. height and width, the slope produced by the various methods changes and it should be possible in most instances to arrive at block dimensions and a cone generation procedure which will produce a close enough approximation to the required pit slope. Block dimensions must, however, be kept to reasonable size.

For convenience, the block height is generally the bench height of the pit but the horizontal dimensions are often completely arbitrary. If the size of the block is large, however, model coarseness can cause the orebody to be completely misrepresented in terms of recoverable ore.

In attempts to achieve greater precision there is a tendency to reduce the size of the block, but this is usually restricted to some extent by the drill hole spacing and possibly by the interpolation (estimation) procedure used. For a given area, the block size affects the size of the block matrix directly, and hence the storage size and the amount of computing time required by the program. In other words, accuracy requires small blocks whereas computing economy is achieved with large ones; however, the over-riding determinant of block size may be the drilling or sampling grid.

2. Optimization of mining sequences using linear programming

2.1 Introduction

In recent years several people have worked on the optimal open pit design problem as a means of finding the optimum mining schedule. The optimization criteria vary with application but are usually based on Net Present Value (NPV) or Maximum Total Return.

The optimum schedule depends on both physical and economic parameters which can be considered either together or separately. Considering both parameters at the same time is computationally time consuming and, for large problems, as the number of states increases, the solution may be impossible to attain.

The problem can be split into two parts. One part uses Linear Programming (LP) to find the optimum path, whilst the second part checks the feasibility of the first solution in terms of mining constraints and improves the scheduling sequence. The effect of this check is that the final LP solution will always be confined to blocks which become accessible during the period under consideration.

The idea of free-ore develops the above consideration further. By submitting for LP selection only those blocks which are immediately available for mining, the constraints on precedence and accessibility of blocks will be eliminated. Such an approach cannot lead to a rigorous overall optimum in the mathematical sense. However, the mining constraints, which play the major role in the definition of the open pit mine design, are chosen only to constrain the practicability of the solution.

In contrast, the use of combinations of different mining constraints in one algorithm throughout the scheduling will make the long-term project much more flexible especially for quick decision making in the case of unforeseen circumstances, such as unexpected sudden changes in ore (waste), or equipment break-down etc.

Scheduling is mainly a combination of the physical restrictions and the economic parameters as discussed above, which usually produce optimum results in terms of material extracted or in monetary return. Different people have different ideas on how to achieve this goal.

From the above analysis the scheduling model considered here is divided into three parts:

1 *Waste stripping module.*
This deals only with waste and overburden to be mined to expose ore blocks.

2 *Linear programming module.*
This deals mainly with the ore blocks to be mined .

3 *Simulation module*
This deals with the re-adjustment of sets of blocks submitted to the LP module.

The approach used to solve the problem is a combination of Linear Programming and simulated, user-activated waste stripping with the overall objective of maximizing the profit or the Net Present Value (total discounted profit).

The pit limits part is done independently of the second (scheduling) using all combinations which satisfy the mining constraints to give the maximum number of combinations of blocks that can be mined during a given period of time. This part has already been described in Chapter 2.

This approach is the main subject of the next Chapter and deals with the mining schedule of the material within the optimum open pit limit obtained by the methods described in Chapter 2.

2.2 Linear programming

During recent years it has become increasingly apparent that operational research and the use of computers can greatly improve mine planning and increase the possibility of attaining the management's ultimate goal of total maximum profit. The operational research technique which has been applied to more production scheduling problems than any other is Linear Programming (LP).

LP is a mathematical method for determining the optimum allocation of limited resources to products or activities e.g. the determination of an optimum production mix or the number of items produced to maximize profit or minimize costs. The objective is either to maximize the benefits while using limited resources or to minimize costs while meeting certain requirements.

All linear programming problems have three common characteristics :

1 - A linear objective function: maximize, minimize or equalize z where:

$$z=c_1x_1+c_2x_2+...+c_nx_n$$

This is a mathematical statement of what management wishes to achieve. This could be a statement concerning maximizing profit or minimizing cost.

2 - A set of linear constraints :

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1$$

.

.

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq b_m$$

The constraints are the physical limitations on the objective function caused by factors such as budgets, labour, raw materials etc. Linearity means that the constraints are linear and have fixed coefficients.

3 - A set of non-negative constraints :

$$x_i \geq 0 \qquad with \quad i=1...n$$

This limits the solution to positive (or zero) values of the variables.

Where :	z	=	objective function
	a	=	coefficient matrix
	b	=	vector of constraint right-hand-side (RHS)
	x	=	vector of variables

In all linear programming problems there is a set of possible answers all of which satisfy the constraints. These answers constitute the **feasible** region. LP then finds a point in the feasible region which optimises the objective function.

LP problems are usually solved by a technique known as the Simplex Method, first developed in the 1940's. This method works by taking a sequence of square sub-matrices of "a" and solving for "x", in such a way that successive solutions always improve, until a point in the algorithm is reached where improvement is no longer possible. The method is used in this work and is further explained in sub-section 2.2.1.

2.2.1 Simplex method for solving mining scheduling problems

We first need to specify the objective function in standard form, and convert all the inequalities to equations. Having specified the problem we convert the inequalities to equations by adding non-negative slack variables. The slack variables represent any unused capacity in the constraint . Each constraint will have its own slack variable.

For example : $3x_1 + 4x_2 \leq 50$ becomes $3x_1 + 4x_2 + x_3 = 50$ where : $x_3 \geq 0$

The slack variable represents any unused capacity in the constraint and in this case $0 \leq x_3 \leq 50$. Where $x_3 = 50$ represents the case of zero production and $x_3=0$ represents the case of full utilisation of the resources and zero unused capacity. Each constraint will have its own slack variable.

The Simplex method is a step by step arithmetic method of solving LP problems whereby one moves from a position of zero contribution until no further contribution can be made. Each step produces a feasible solution and an answer which is better than the previous one. The method is better explained in the following example.

Maximize the following linear objective function :

$$z = 8x_1 + 5x_2 + 10x_3$$

Subject to the linear constraints :

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &\leq 400 \\ x_1 + x_3 &\leq 150 \\ 2x_1 + 4x_3 &\leq 200 \\ x_2 &\leq 50 \end{aligned}$$

And the non-negativity constraints :

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

Solving the above example first we add the slack variables,

$$\begin{aligned} 2x_1 + 3x_2 + x_3 + x_4 &= 400 \\ x_1 + x_3 + x_5 &= 150 \\ 2x_1 + 4x_3 + x_6 &= 200 \\ x_2 + x_7 &= 50 \end{aligned}$$

with $x_i \geq 0$ and $i = 1 \ldots 7$

Writing the objective function in standard form: $z - 8x_1 - 5x_2 - 10x_3 = 0$

and then the simplex tableau takes the form:

	Basic	Entering Variables X_1	X_2	X_3	X_4	X_5	X_6	X_7	Solution
Obj. Function	z	-8	-5	-10	0	0	0	0	0
Constraints	X_4	2	3	1	1	0	0	0	400
&	X_5	1	0	1	0	1	0	0	150
	X_6	2	0	4	0	0	1	0	200
Slack Var.	X_7	0	1	0	0	0	0	1	50

Table 2 : step 1 in the simplex algorithm

The basic column identifies the current basic (or basic variables) whose values are given in the solution column. This implicitly assumes that these variables not present have the value zero.

The solution above gives the current solution :

$$x_1 = x_2 = x_3 = 0,\ x_4 = 400,\ x_5 = 150,\ x_6 = 200, \text{ and } x_7 = 50$$

If z = 0 (i.e. zero production) then the solution is feasible. For a problem with n variables (including slack variables) and m constraints. n - m variables must be zero and the remaining m variables form the basis. In the above example n = 7, m = 4, =====> 7 - 4 = 3 zero variables and 4 solution variables.

If we examine the current zero variables (x_1, x_2 and x_3) all have negative coefficients ($-8x_1-5x_2-10x_3$) which is equivalent to positive coefficients in the original objective function ($8x_1+5x_2+10x_3$). Since we are maximizing, the value of z can be increased by increasing x_1, x_2 or x_3 above the zero value. However, we always select the variable with the most negative objective coefficient because such a solution is most likely to lead to the optimum solution rapidly (i.e. coefficient of x_3).

The optimality condition is, in the case of maximization, i.e. (-8-5-10) ==> (0, 0, 0). If all non-basic variables have non-negative values in the z equation in the current tableau, the current solution is optimal. Otherwise the non-basic variable with the most negative coefficient is selected as the variable entering the basis.

The feasibility condition determines which variable leaves the basis, this variable is the one which will be first to reach zero when the entering variable reaches its maximum value. It is identified by dividing the solution value by the corresponding positive values in the column of the variables entering the basis. The row with the smallest value identifies the variable leaving the basis.

The algorithm for the above method proceeds as follows:

Step 1. Identify the largest negative value in the z-row (i.e. -10)

Step 2. Divide the solution quantity by the positive values in this column. (i.e. 400/1, 150/1, 200/4)

Step 3. Select the row which has the element with the smallest positive value (200/4 = 50) and identify the value that appears in both the row and the column. This element is the PIVOT element (i.e. 4).

Step 4. Divide all the elements in the row by the pivot and replace the basic variable by the entering variable. In this case x_6 is replaced by x_3.

The new pivot equation is equal to the old pivot equation divided by the pivot element (i.e. new pivot equation).

The new pivot equation is :

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Solution
X_3	½	0	1	0	0	¼	0	50

Step 5. Using row operations (Gauss-Jordan method) make all other elements in this column zero using the PIVOT row.

Iteration	Basic	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Solution
1	z	-3	-5	0	0	0	2½	0	500
X_3 enters	X_4	1½	3	0	1	0	-¼	0	350
	X_5	½	0	0	0	1	-¼	0	100
X_6 leaves	X_3	½	0	1	0	0	¼	0	50
	X_7	0	1	0	0	0	0	1	50

Table 3 : step 5 in the simplex algorithm

This procedure is repeated (Step1 to Step 5) using the next highest coefficient in the objective function, until all the values in the z row are greater than or equal to zero. In the above table 2, the new z line is equal to the old line -(-10)x, new x_3 line.

Iteration	Basic	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Solution
2	z	-3	0	0	0	0	2½	5	750
X_2 enters	X_4	1½	0	0	1	0	-¼	-3	200
	X_5	½	0	0	0	1	-¼	0	100
	X_3	½	0	1	0	0	¼	0	50
X_7 leaves	X_2	0	1	0	0	0	0	1	50

Table 4 : step 5 iteration 2 in the simplex algorithm

Iteration	Basic	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Solution
3	z	0	0	6	0	0	4	5	1050
X_1 enters	X_4	0	0	-3	1	0	-1	-3	50
	X_5	0	0	-1	0	1	-½	0	50
X_3 leaves	X_1	1	0	2	0	0	½	0	100
	X_2	0	1	0	0	0	0	1	50

Table 5 : step 5 iteration 3 in the simplex algorithm

As all quantities in the z-row are greater than or equal to zero we have reached the optimum.

i.e. $x_1 = 100$, $x_2 = 50$, $x_3 = 0$, $x_4 = 50$, $x_5 = 50$, $x_6 = 0$ and $x_7 = 0$, with a maximum profit of : $z = 1050$.

In minimisation problems the entering variable must have the largest positive coefficients in the z equation. The solution is reached when all the values in the z row are less than or equal to zero and the non-basic variables have non-positive coefficients.

2.2.2 Other LP algorithms for solving mining scheduling problems

Other LP algorithms known as Interior-Point methods come from non-linear programming approaches proposed in 1958 and further developed in the late 80's. These methods can be faster for many large-scale problems. Such methods are characterized by constructing a sequence of trial solutions that go through the interior of the solution space, in contrast to the Simplex Method which stays on the boundary and examines only the corners (vertices).

Integer LP models are ones where the answers must not take fractional values. Integer models may be ones where only some of the variables are to be integer and others may be real-valued termed Mixed Integer Linear Programming (MILP), or Mixed Integer Programming (MIP); or they may be ones where all the variables must be integer termed Integer Linear Programming (ILP). The class of ILP is often further subdivided into problems where the only legal values are Binary (0,1), and general integer problems.

Although various algorithms for MIP have been studied, most if not all available general purpose large-scale MIP codes use a method called 'Branch and Bound' to try to find an optimal solution. Branch and Bound solves MIP by solving a sequence of related LP models. Good codes for MIP distinguish themselves more by solving shorter sequences of LP's, than by solving the individual LP's faster. Even more so than with regular LP, a costly commercial code may prove its value if the MIP model is difficult.

There are certain models whose LP solution always turns out to be integer, assuming the input data is integer to start with. The theory of unimodular matrices is fundamental here (unimodular is: if every square sub-matrix has a determinant equal to 0, +1, or -1). Such problems are best solved by specialized routines that take major shortcuts in the Simplex Method, and as a result are relatively quick-running compared to ordinary LP.

Nowadays, with good commercial software, models with a few thousand constraints and several thousand variables can be tackled on a 386 PC. Workstations can often handle models with variables in the tens of thousands, or even more, and mainframes can go larger.

The choice of code can make more difference than the choice of computer hardware. It is hard to be specific about model sizes and speed, a priori, due to the wide variation in things like model structure and variation in factorizing the basis matrices; just because a given code has solved a model of a certain dimension, it may not be able to solve all models of the same size, or in the same amount of time.

For the application of LP to the Mining Scheduling Problem, a code, written in "C" language, called lp_solve was supplied to the author by Proll (1995). Lp_solve can solve general LP problems or mixed integer LPs. The code uses a Simplex Algorithm and sparse matrix techniques, for pure LP problems. If one or more of the variables is declared integer, then the simplex algorithm is iterated with a Branch and Bound algorithm, until the desired optimal solution is found. Using the present code to solve the LP problem an input file has to be created in suitable format for lp_solve.

The input is a set of algebraic expressions and integer declarations in the following order :

- **Objective function,** is a linear combination of variables, ending with a semi-colon, optionally preceded by (max: or min:) to indicate whether it is maximization or minimization is sought.

- **Constraint,** is an optional constraint name followed by a colon plus a linear combination of variables and constants, followed by a relational operator, followed again by a linear combination of variables and constants, ending with semi-colon. The relational operator can be any of the following: less, or less or equal to, equal, greater or greater or equal to.

- **Declaration,** is of the form: 'int' followed by variable and ending with semi-colon, commas are allowed between variables.

The following is an example of input of general form:

The problem : minimize $x_1 + x_2$ (or maximize $-(x_1 + x_2)$)

subject to:

$$x_1 \geq 1$$
$$x_2 \geq 1$$
$$x_1 + x_2 \geq 2$$

with x_1 integer

The lp_solve input file is:

max : $-x_1$ + $-x_2$;

(or min: x_1 + x_2;)

$x_1 \geq 1$;

$x_2 \geq 1$;

$x_1 + x_2 \geq 2$;

int x_1;

The results from Lp_solve are the values of the decision variables, that represent the status of the blocks, together with the profit gained from mining that set of blocks. The values of the decision variables are 0 (for blocks not mined) and 1 (for mined blocks) in any given time period examined. More details are given in Chapter 6.

The software is unlimited in size but the computer platform on which it is implemented sets upper limits to the size of the problems that can be practically solved.

CHAPTER FOUR

Optimal production scheduling in open pit operations

1. Introduction

The production scheduling problem can be stated simply as the determination of the sequence in which blocks must be mined, subject to mining, economic and geotechnical constraints, so as to optimize a specified objective function.

In this application the objective is to maximize the net present value of the total mined product. The author uses a combination of linear programming and simulated, user-activated waste stripping to obtain the optimum schedule.

2. Linear programming

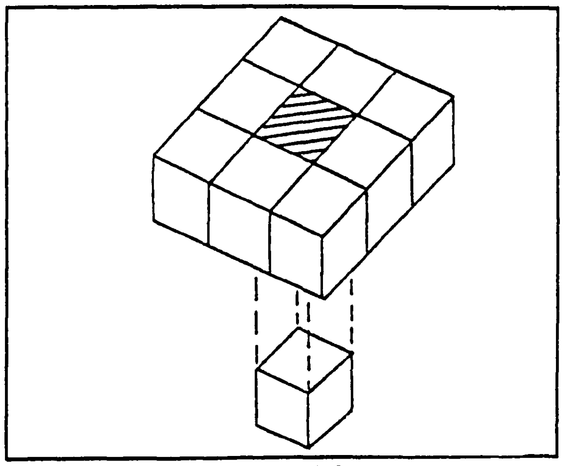


Figure 36:
illustration of mining constraints

To illustrate the linear programming formulation assume that the blocks in the block model are regular cubes and that the pit slope constraints are 45° in all directions. It is further assumed that the pit slope constraints also define the mining access constraints as illustrated in figure 36 in which the lower block can only be mined in a given period if the nine blocks on the level above have been mined in previous periods and/or are mined in the given period. This simplistic example does not in any way constrict the generality of the formulation which follows.

Let $b_{ijk}(t)$ be a binary valued variable which takes the value 1 if block (i,j,k) is mined in period t and takes the value 0 otherwise. The indices i,j,k are block counters in the east-west (x), north-south (y) and vertical (z) directions respectively with k increasing with depth.

Let V_{ijk} be the net return (in monetary value) obtained from mining, processing and selling block (i,j,k) and let r be the discount rate for the mining project.

A linear programming formulation of the optimal sequencing problem is:

$$\textit{Maximize:} \quad \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=1}^{N_z} \sum_{t=1}^{T} b_{ijk}(t)\, V_{ijk}\, (1 + r)^{-t}$$

subject to:

$$\sum_{t=1}^{t'} b_{\ell m\, k-1}(t) \; - \; b_{ijk}(t') \;\; \geq \;\; 0$$

for:

$\ell = i-1,\, i,\, i+1$
$m = j-1,\, j,\, j+1$
$\forall\; i, j, k, t'$

where N_x, N_y, N_z are, respectively, the number of blocks in the block model in the x, y and z directions and T is the total (maximum) number of time periods considered. The constraints define the access to block (i,j,k) in time period t. These constraints can readily be adapted to describe any other slope and access constraints. It is also possible to include additional technical and operational constraints but the absence of these constraints does not detract from the general nature of the formulation given here.

The difficulty with this formulation is that the number of variables and (especially) the number of constraints are prohibitive for any realistic problem. A similar problem is encountered with the dynamic programming formulation of the problem: the number of possible decision sequences rapidly exceeds any practical storage and computing facilities. It is possible to reduce significantly the computations involved in the linear programming formulation by taking advantage of the properties of the unimodular matrix formed by the constraints and by using various algorithmic approaches that are well documented in the linear and integer programming literature. However, the linear program, as formulated above, is still too large to be solved for all but the smallest of block models.

Some authors (Wilke and Reimer (1979), Wilke et. al. (1984) and Huang (1993)) have attempted to separate the access constraints from the linear program but it is intuitively obvious (and easy to demonstrate) that such approaches cannot possibly lead to optimal solutions in any problem that involves sequential decisions. In all but the simplest cases they do not even lead to near optimal solutions and thus the use of such simplifications is a dangerous approach to optimal scheduling.

3. Practical solutions: reducing the numbers of constraints and variables

It is possible to adapt the linear programming formulation by recognising some subtle differences between optimal open pit design and the optimal sequencing of mining blocks and by recognising that the solution yielded by any general formulation will not necessarily be a practical mining solution:

(1) An initial pit will always be designed well in advance of any need to schedule the mining operation: scheduling can be seen as the order in which blocks are removed so as to reach an ultimate pit shape.

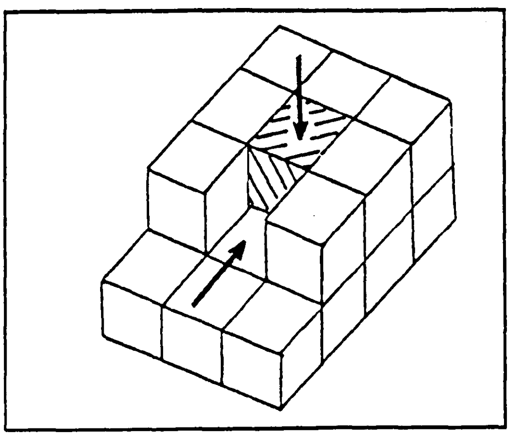


Figure 37: simplified block access for scheduling

(2) Mining generally proceeds in fronts

and mining access to a particular block is not as stringent as that required for pit design. For example, mining is usually done by "push backs" on benches and, in its simplest form, access could be defined in terms of one block above and one block in front as shown in figure 37.

(3) Blocks that are already accessible do not require an access constraint.

(4) For technical and economic reasons blocks must be mined (more or less) contiguously in groups. In general it is not feasible, or at least not good mining practice, to mine isolated blocks in different parts of the orebody. Similarly, mining will not be done on a large number of widely separated levels in any given period or even over short to medium sequences of consecutive periods; the number of levels opened and the distance between them will usually be restricted.

(5) Scheduling for any practical purposes is very rarely considered for more than relatively short periods of time (3 to 5 years maximum)

4. The model

The approach adopted here is similar in structure to that of Huang (1993). There are three components in the model:

(1) A user-activated waste stripping module.

(2) A linear programming module that determines an optimal mining sequence from sub-sets of blocks submitted from the overall orebody block model.

(3) A mining simulator which removes mined blocks from the block model and adjusts the subsets of blocks for submission to the linear programming module.

It is assumed that an overall pit has already been designed (in this work by means of the parameterization algorithm). This is not an essential requirement but it does simplify the procedure. It is further assumed that a minimum amount of ore must always be exposed in any given time period. This is a fairly widespread and sensible operational requirement. Before production begins, for example, a certain amount of overburden and/or waste must be stripped. The final assumptions are for access to blocks:

(1) access to any block in the uppermost layer of the orebody block model is determined solely by removal of the waste block immediately above it.

(2) for all other layers in the orebody block model an accessible block (i,j,k) is one for which the block immediately above (i,j,k-1) has been removed and at least one contiguous block on the upper level has been removed: (i-1,j,k-1) or (i,j-1,k-1) or (i,j+1,k-1) or (i+1,j,k-1).

5. The waste stripping module

A minimum tonnage of ore must be exposed at the start of each period. This minimum tonnage is specified by the user and is determined by the program in terms of numbers of accessible ore blocks.

The minimum exposed ore tonnage constraint is a common production requirement and, in this application, is a valuable means of reducing the number of constraints for the linear programming module.

All ore blocks that are not accessible are assigned a priority code depending on:

(1) The number of faces of the block (if any) that are already exposed.

(2) The value of the block.

In descending order of priority each ore block is then examined to determine whether access is prevented by a waste block. If so, the waste block is scheduled for removal. Stripping continues until the minimum tonnage of ore is exposed. Apart from any pre-production stripping, all waste stripping incurred by this module is apportioned equally over the periods considered by the linear programming module and capacity constraints are adjusted accordingly.

At any stage the user can intervene to identify specific blocks to be stripped. This option can be used to override part or all of the automatic stripping operation.

When the minimum amount of ore tonnage is exposed control passes to the linear programming module.

6. The linear programming module

This module schedules a subset of blocks submitted from the orebody block model. Membership of a subset is defined below.

Let: $b_{ijk}(t)$ be a binary valued variable such that:

$$b_{ijk}(t) = \begin{cases} 1 & \text{if block (i,j,k) is mined in period t or earlier} \\ 0 & \text{otherwise} \end{cases}$$

X_{ijk} be the tonnage of ore in block (i,j,k)

Y_{ijk} be the tonnage of waste in block (i,j,k)

V_{ijk} be the net revenue obtained from block (i,j,k)

r be the discount rate for the project

X_{min} be the minimum ore production requirement per period

X_{max} be the ore production capacity per period

C be the total (ore and waste) capacity per period

N_x be the number of blocks in the block model for the x direction

N_y be the number of blocks in the block model for the y direction

N_z be the number of blocks in the block model for the z direction

$N_z(1)$, $N_z(2)$ be the mininmum and maximum vertical interval (number of levels) of blocks currently considered

t_p be the number of time periods considered for scheduling

The linear program is:

$$\text{Maximize:} \quad \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=N_z(1)}^{N_z(2)} \sum_{t=1}^{t_p} b_{ijk}(t)\; V_{ijk}\; (1 + r)^{-t}$$

subject to:

$$\sum_{t=1}^{t'} b_{ijk-1}(t) \; - \; b_{ijk}(t') \;\geq\; 0 \qquad (1)$$

$$\sum_{t=1}^{t'} \{b_{i-1jk}(t) + b_{ij-1k}(t) + b_{ij+1k}(t) + b_{i+1jk}(t)\} \; - \; b_{ijk}(t') \;\geq\; 0 \qquad (2)$$

with, for (1) *and* (2):

$$i = 1, N_x$$
$$j = 1, N_y$$
$$k = N_z(1), N_z(2)$$
$$t' = 1, t_p$$

$$X_{\min} \;\leq\; \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=N_z(1)}^{N_z(2)} b_{ijk}(t)\, X_{ijk} \;\leq\; X_{\max} \qquad \text{for } t = 1, t_p$$

$$\sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=N_z(1)}^{N_z(2)} b_{ijk}(t)\, (X_{ijk} + Y_{ijk}) \;\leq\; C \qquad \text{for } t = 1, t_p$$

The access constraints for block (i,j,k) in period t are that the block immediately above it (constraint 1) and at least one contiguous block on the same level (constraint 2) must have been mined prior to t or be mined in t. The constraints for any block which already satisfies these conditions is removed from the linear program. This results in a significant reduction in the number of access constraints from that required in the general optimal open pit formulation.

The current formulation is limited to a horizontal array of 2500 (e.g., 50 x 50) blocks, a maximum of three levels open in any one time period and an upper limit of three time periods in any linear program. These limitations generate a maximum of 22,500 variables, 22,500 type (1) access constraints (each containing a maximum of 4 variables), 22,500 type (2) access constraints (each containing a maximum of 13 variables) and 10 production capacity constraints. The number of constraints can (and generally must) be reduced significantly by eliminating those for blocks that are already accessible, by using the stripping module to make more blocks accessible and, once mining has begun, by limiting the lateral and horizontal distances between production areas within and between production periods. This reduction in the number of constraints is an essential step in the operation of the program. If the linear program is too large to be solved the user is invited to reduce the number of constraints, via the *waste stripping* and *mining simulation* modules, by undertaking additional stripping and/or reducing the extent of working areas. If the lateral extent of working areas is reduced the block indices (N_x and N_y) in the linear program must be adjusted.

7. The mining simulation module

This module:

(1) removes blocks from the block model for one scheduling period

(2) adds additional blocks to the set to be considered for scheduling

(3) applies equipment moving constraints to remove blocks from the subset to be considered for future scheduling

(4) returns control either to the stripping module or to the linear programming module

The linear programming module operates on a subset of blocks from the orebody block model. In a simple, three-dimensional rectangular array of blocks this subset would initially correspond to the uppermost three layers of the array. The solution of the linear program for this subset yields an optimal schedule for three periods. The blocks corresponding to the first period are removed from the orebody block model and recorded as the scheduled production for the first period. Any blocks within three levels immediately below a mined block are now added to the subset. If the minimum ore tonnage is exposed control passes back to the linear programming module where the subset is scheduled for periods 2, 3 and 4. If the minimum tonnage is not exposed control passes back to the stripping module before entering the linear programming module. (NOTE that the linear programming module may inherently schedule waste to be mined during any production period).

The program continues in this manner, scheduling three periods at a time, selecting only those blocks scheduled for the first of these periods, adjusting the subset and then scheduling for a further three periods.

Once the very first production period is scheduled an additional constraint is imposed on membership of the subset of blocks offered to the linear programming module. Blocks can only enter the subset if they are within user-specified horizontal and vertical distances from the blocks scheduled in previous periods. This condition is critical in reducing the numbers of variables and constraints in the linear program to yield a formulation that can be solved.

8. Solving the linear program

The major problem in this formulation is in obtaining a solution to the linear program. Although the formulation is in fact an integer program the nature of the access constraints is such that they form a unimodular matrix. The capacity constraints can be manipulated so as to conform to the same matrix structure. The advantage of this is that the program can be solved as a continuous linear program without recourse to the more computationally demanding integer programming.

If the numbers of variables and/or constraints in the linear program are too large control is returned to the mining simulation module where the user is invited to reduce the horizontal and vertical distance requirements for membership of the subset of blocks.

It is also possible to use approximate methods of solution such as that reported in Dowd (1989). At present, however, the software requires a workstation for implementation. Future developments will focus on the possibility of using various forms of decomposition to speed up the solution and to reduce storage requirements.

8.1 Additional constraints

It is possible to include additional operational constraints. For example, it may be desirable for blending or similar purposes to maintain the average grade of production in each time period between upper and lower limits:

$$\sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=N_z(1)}^{N_z(2)} b_{ijk}(t)\, g_{ijk}\, X_{ijk} \geq G_\ell \qquad \textit{for } t=1,..,t_p$$

$$\sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=N_z(1)}^{N_z(2)} b_{ijk}(t)\, g_{ijk}\, X_{ijk} \leq G_u \qquad \textit{for } t=1,..,t_p$$

where G_ℓ and G_u are the lower and upper limits on grade values (these may also be specified as functions of time or geographical locations)

g_{ijk} is the grade value of block (i,j,k).

The inclusion of such additional constraints will however substantially increase computing requirements.

9. Conclusion

The formulation of the scheduling problem given here allows the determination of an optimal mine schedule using the criterion of maximum net present value. It is believed that the algorithms used yield a true practical optimum or a solution which is very close to the true optimum. At present the algorithm described is limited to a workstation implementation but it is believed that the use of decomposition methods and approximate solution algorithms for the linear programming component will ultimately yield a PC version.

CHAPTER FIVE

Software description

1. Introduction

The methods described in the previous Chapters have been coded into a set of programs that have been tested on trial data and on a real data set from a gold deposit. This Chapter describes the main programs in the software package, presents the different types of data input and output, the major paramters and describes the execution of the software.

The programs developed during this research project are:

- Optimal open pit design by parameterization
- Final pit selection
- Mine scheduling
- Graphics display of the results

All programs are contained in an integrated computer package. The package is described, section by section, in this Chapter.

2. Programming platform

The software consists of a main program, four main **FORTRAN** subroutines, and five **UNIRAS** routines. The problem is solved entirely in core.

The main program calls the major executable subroutines and graphics routines.

2.1 Fortran subroutines

The following are the main Fortran subroutines of the software:

2.1.1 Subroutine master

This subroutine finds the optimal open pit limits using the parameterization technique. The result is a set of technically optimum nested pits, one of which will be the economic optimum for a given set of economic parameters.

The parameterization technique can produce a set of nested pit designs for a grade block model (characteristics and plans), in about 30 seconds, or less, of computer run time on a Sun workstation.

2.1.2 Subroutine select

The determination of optimal ultimate pit limits requires the transformation of the block grade matrix into a revenue block matrix which can be optimised (scheduled).

Block profit comes from the evaluation of a function of many variables such as grade of ore, mining costs, transportation costs, price of mineral, etc. The net value for each block is the actual cost or profit realised by mining and processing that block. Blocks with positive profit have a final value which exceeds the total of all costs (mining, production, transport, etc.) whereas negative profit blocks do not.

2.1.3 Subroutine Mstrip

The subroutine strips waste blocks so as to expose the minimum ore tonnage at the start of each mining period. The minimum ore tonnage is specified by the user and is determined by the program in terms of numbers of accessible ore blocks.

2.1.4 Subroutine Mlinprog

This is the linear programming subroutine and it is used in conjunction with subroutine **Mstrip** to schedule the mining of a subset of ore blocks submitted from the orebody block model.

2.1.5 Subroutine Msimul

This is the simulation subroutine and it is used in conjunction with subroutine **Mlinprog** to remove and adjust the subset of blocks being processed by **Mlinprog.** This subroutine checks whether the minimum ore tonnage is exposed and returns control either to **Mstrip** or to **Mlinprog** depending on whether or not more waste must be stripped.

2.2 Graphics routines

As explained in section 2.5 Chapter 6, the **Graphics routines** have been incorporated into the software to help to produce better representations of the numerical data associated with each optimal pit shape. The routines are designed to produce contour plans, three-dimensional views, and cross-sectional views.

3. Major parameters of the software

Most of the input data are read in the individual modules of the software. Some of the major parameters for the subroutines are described below:

mtng	-	minimum tonnage required (in stripping module)
ibtng	-	total tonnage in each block i (block weight)
zc	-	cut-off grade
ippr	-	production period
tp	-	time period
nac	-	number of access constraints

ncc - number of capacity constraints
mco - mining cost of ore
mcw - mining cost of waste
mpc - processing cost
pog - price of gold
rd - discount rate for the project
irs, jsr, mrs, nrs, isr, jsr - right-hand-side values for the different constraints.
ile, ieq, ige - types of constraints, less than or equal to, equal to, and greater than or equal to.

Some parameters are calculated, e.g.

itbc - number of blocks in the subset
itnc - total number of constraints to be considered
nbp - number of blocks to be mined per period

Parameters not mentioned above are better explained as appropriate in the following sections of this Chapter.

3.1 Determination of the optimum pit limits

The parameters introduced interactively by the user when running this section of the software are:

zc - cut-off grade
x, y, z - block dimensions
bw - specific gravity of ore
alpha - the final pit slope (degrees)

3.2 Selection of the final pit

The selection of the final pit plan requires the specification of the following parameters:

- Price of the metal
- Mining costs
- Processing costs

3.3 Production scheduling

As explained in the previous sections the scheduling section is run in three stages where the data are entered via the keyboard or read from files and are passed between routines by means of COMMON blocks or as arguments of subroutines. The following sections list the parameters that are entered via the keyboard by the user in response to requests that appear on the screen for that part of the schedule.

3.3.1 Waste stripping module

The following parameters are required for this section:

itnb - total number of variables in the optimum pit plan.

nnb - number of blocks to be exposed (as minimum ore tonnage).

3.3.2 Linear programming module

All parameters required by this section (right-hand-side, the rowtype (+/-) for different types of constraints) are entered by means of COMMON blocks or as arguments.

The direction of optimization is entered as : + for maximization and
- for minimization (not used)

3.3.3 Simulation module

Block coordinates of the selected ore blocks are the major input parameters for this section.

4. Data input and output

To execute the software two types of data are required. The first type of data are those supplied from files, COMMON blocks and as parameters; these are explained in this section. The second type are those entered interactively and these are described in the **software execution** section.

4.1 Input data read from files

The software is a set of programs set up for the design of the optimum open pit shape and production scheduling. Therefore, most of the data files are common to nearly all the sections of the software. The following are the input data files required for each section of the software, all of them being in comma-delimited free format.

4.1.1 Determination of the optimum pit limits

Beside the interactive input data, the grade file is the major input file for this section of the software.

4.1.2 Selection of the final pit

The grade file mentioned above is used again in this section to produce an equivalent three-dimensional revenue block model file.

4.1.3 Production scheduling

These are three sections in the model which work together.

4.1.3.1 Waste stripping module

The grade file is the input for this section, together with a corresponding three-dimensional file, recording the removed blocks.

4.1.3.2 Linear programming module

A subset of blocks, selected by the user, in the form of a data file is submitted to this section. A simplex matrix for solution of LP module is produced. The matrix includes the equation of the objective function and the integer declaration of the variables being considered.

4.1.3.3 Simulation module

A data file created in the previous section, is submitted to this section as an input data file. The file contains the blocks to be removed in each time period.

4.1.4 Graphics routines

The grade file and the optimum pit plan are the main data input files for this section. Both files are used to produce different graphics representations of the characteristics of the optimum pit.

4.2 Output results written to files

Each section of the software generates its own output files with the file names set up in the software. Each relevant output is described in the following sections.

4.2.1 Determination of the optimum pit limits

Output from this section is in two forms. The first is a binary file created for efficient storage and data transfer when used by other programs (not discussed here). The second is an ASCII file containing full details of the characteristics of each individual pit. Plans of the nested pits are given as grade distributions or depth levels, and both can be contoured.

4.2.2 Selection of the final pit

The output from this section comprises pit by pit revenue values together with the pit with the maximum revenue value.

4.2.3 Production scheduling

The output files of blocks being selected and removed from the orebody block model during the scheduling process are produced in each individual section of this part of the software.

4.2.3.1 Waste stripping module

The output of this section is a table showing the appropriate priority given to each block together with the pit plans showing the blocks removed during the operation of the module.

4.2.3.2 Linear programming module

The output from this section is a period by period summary of mined blocks and final objective function value.

4.2.3.3 Simulation module

The outputs of this section are pit plans of each production period showing the coding of the removed blocks during the stripping and linear progamming modules.

4.2.4 Graphics routines

This section produces different graphic representations (cf. section 2.5 Chapter 6) of the optimum pit.

5. Software execution

The software is executed by running the exec file **rebh2.exe** created by the UNILINK system after first setting up the UNIRAS environment. The execution starts with the pit limit determination, followed by pit sequencing and ends with the graphics display of the results. The software requires two types of data :

The first set is read from files and has been discussed above. The second set is entered interactively during the software execution and includes all data that are affected daily, monthly or yearly by market conditions. The reading of the data depends on the order of the sections where the order is pit limit determination, scheduling and finally graphics representations.

5.1 Determination of the optimum pit limits

This is the first section to be executed and focuses mainly on the pit design. It starts with the pit limits determination followed by pit selection (the pit with the highest profit).

Note : There are three options for the output files at the end of the first part of the determination of the optimum pit limit. Option (2) is set for the software which contains the pit plans (nested pits) produced during the running of that part of the software to be optimized. The other two options (1,3) terminate the process. The rest of the software is described below:

Type in the exec file name : **rebh2.exe**

Execution starts:

'Operating in the pit design module'

Enter the cut-off grade

>

Enter block dimensions (x,y,z directions) and density

>

Enter the final pit slope (degrees)

>

Solution reached : there are 'n' number of pits.

Do you want a full output listing (1);
a pit by pit plan (2);
or just a pit parameter summary (3).

When option (2) is selected the process will continue as follows.

5.2 Selection of the final pit

Parameters required for the pit selection are passed through common blocks.

- **Price of the metal**
- **Mining costs**
- **Processing costs**

Once the final pit is selected, which is automatic, programme control passes to the following section (Production scheduling).

5.3 Production scheduling

This section has three parts: Waste stripping module, Linear Programming module and the Simulation module.

This part starts by:

Enter subset dimensions (i, j and k)

>

Checks if the coordinates are within the feasible zone of X, Y, Z?

No - Print ' NB of ELEM. should not be exceeded, try again'

Yes - The process will continue in the next section (stripping).

5.3.1 Waste stripping module

The module requires the following parameter :

Enter the number (nnb) of blocks per period

>

Checks if the nnb of blocks is greater than the total nb of blocks ?

Yes - Print ' NB of ELEM. should not be exceeded, try again'

No - The process will be resumed as follows :

Exposes the nnb blocks

Checks if the minimum ore tonnage is exposed ?

Yes - Formulate the subset of blocks then continue to the following section (linear programming).

No - Do you want to increase the number of blocks ?

Yes - Enter the nnb of blocks and control returns to the stripping module

No - Ends the run.

5.3.2 Linear programming module

For this part of the scheduling most of the data are entered in the main program and are passed to the LP module through **CALLs** and **COMMON** blocks.

Program starts:

Enter direction of optimization :

> + **(plus)** for maximization (used in the current example) or

> - **(minus)** for minimization (not used in this formulation)

Formulate the LP input

Optimization operation takes place

Is there any solution ?

Yes - Results are put into table format and the control is returned to the next section (simulation).

No - Means more stripping needed, control is returned to the above section (stripping module).

5.3.3 Simulation module

The co-ordinates of the selected ore blocks are required here.

Removes blocks of the first time period of the current production period.

Adjusts the subset of blocks

Checks if the minimum ore tonnage is exposed ?

Yes - Control is returned to the above section (linear programming module).

No - Control is returned to the above section (waste stripping module).

Note : The blocks selected and removed from the matrix in each period are assigned the current production period number.

5.4 Graphics display of the results

This section will display graphically some of the characteristics of the optimum pit plan.

Do you want a graphics display yes/no ?

> **Yes** - The process will continue as below

> **No** - Ends the run

Do you want the graphics display to be ?

Enter 1. Pitselection

Enter 2. Optpit_plan

Enter 3. Optpit_3d_view

Enter 4. Optpit_x_section

Enter 5. Optpit_inv_section

> **Enter the choice**

Do you want the display to be ?

Enter 1. For the screen

Enter 2. For HPGL

Ente r 3. For B/W PS

Enter 4. For colour PS

Enter 0. To quit

> **Enter the choice**

Please wait for the graph.

To quit the graphics environment **press return** while the **arrow of the cursor** is on the graphics window **Thank you.**

CHAPTER SIX

Case study

1. Introduction

"Optimal open pit design is essentially a computer based implementation of an algorithm which is applied to a three-dimensional block model of an orebody. Almost all optimal open pit design algorithms, with the exception of elementary methods applied to some stratiform deposits, are applied to a regular, fixed, three-dimensional block model of an orebody. The orebody is subdivided into regular blocks and a value is estimated for each block. This value is almost always the net (undiscounted) revenue that would be obtained by mining and treating the block and selling its contents. Some methods, such as parameterization, use grade values in the block model. Stuart (1992) proposes an irregular three-dimensional model in which the orebody is represented by a series of arbitrary geometrical solids. Whilst such a model is a useful way of representing highly irregular and complex-shaped stratiform deposits it is doubtful whether sophisticated computer algorithms are really necessary for the design of optimal pits in such cases." (Dowd, 1994b). The block model is the fundamental input to the pit design and the scheduling programs developed during this research project.

The blocks to be mined in each mining period are selected, within the limits of the given economic constraints, so as to maximize the Net Present Value (NPV) for the life of the mine.

In general, the production scheduling procedures determine which blocks of ore and waste should be removed in each mining period so as to maximize the Net Present Value for the mine, subject to the mining and milling operating constraints. The scheduling problem is treated in three modules (the modules are documented in Chapter 4). This Chapter describes the software and presents the results obtained from applying it to some of the data sets.

2. Algorithms and their logic

The programs are best explained by means of individual flow charts along with an example solution in the following sections. The pit limit flow chart is not shown here and can be found in Moks' thesis (1983).

2.1 Data requirements

To facilitate the description of the software a case study will be used. The data come from an open pit gold mining operation. The orebody has been subdivided into 20 m (E-W) x 20 m (N-S) x 10 m blocks and the gold grade of each block has been estimated. There is a total of 26 (E-W) x 55 (N-S) x 9 (vertical) blocks in the block model. The three-dimensional matrix of gold grades of these blocks constitutes the block grade model of the orebody.

Deposit models are usually described in a form in which insufficient edge blocks are specified to allow the removal of ore blocks at the bottom edge. In order to secure the stability of the pit and render these blocks minable, it is necessary to add additional waste blocks all around the lateral boundaries of the deposit. Figure 38 illustrates the additional blocks added to the deposit model.

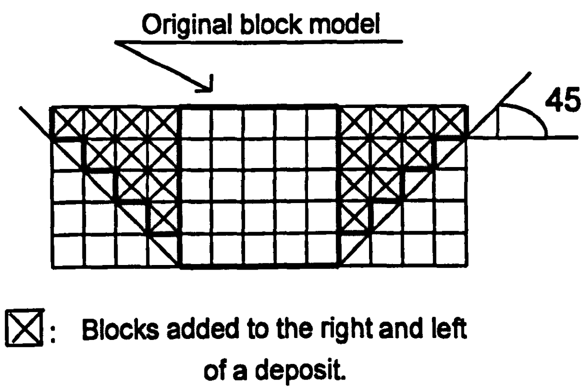


Figure 38 : Two-dimensional representation of mineralized zone

Because of the similarity of all the deposit areas the number of blocks added is equal in all four cardinal directions and is determined as the number of levels of the deposit. The number of blocks added to the orebody block model on the northern and southern boundaries as well as on the eastern and western boundaries, must satisfy the lowest slope angle of any area of the deposit.

The initial data required are :

1. the cut-off grade
2. the block dimensions (rows, columns, levels, directions)
3. the specific gravity of ore and waste
4. the final pit slope
5. the grade of each block

The scheduling algorithm requires the block grade model to be transformed into a revenue block model which can be used by the optimization module (LP). The data and block revenue calculations required are described in the following paragraphs.

For the sake of a simple example, a horizontal subset plane of 100 (i.e., 10 x 10) blocks, a maximum of three levels open in any time period, and an upper limit of three time periods in any linear program has been selected from the total orebody block grade model. The subset will be used to show the input data for solving the schedule problem in the following sections.

Each row of the three-dimensional array of data contains the x, y, and z co-ordinates of the mid-point of a block, together with the average grade and the tonnage above the cut-off grade in that block.

The data used in this example are:

cut-off grade 0.2 g/t

specific gravity	2.74 t/m^3 for both ore and waste
ore mining costs	£ 3.00/tonne
waste mining costs	£ 1.50 / tonne
processing costs	£ 4.50 / tonne
price of gold	£ 9,500 / kg

The total tonnage of each block is: 20 x 20 x 10 x 2.74 = 10960 tonnes

The tonnes of waste in each block can be calculated as:

10960 - ore tonnage in block.

To illustrate the revenue calculations consider the following record extracted from the data file :

1500.00 750.00 55.00 3.25 3200

i.e. block is centred on co-ordinates (1500, 750, 55) and contains 3200 tonnes of ore above cut-off grade at an average grade of 3.25 g/t. The revenue value of this particular block is:

Amount of waste	=	10960 - 3200	=	7760 tonnes
Cost of mining waste	=	7760 x 1.50	=	£11640
Cost of mining ore	=	3200 x 3.00	=	£9600
Processing cost	=	3200 x 4.50	=	£14400
Value of contained gold	=	3200 x 3.25/1000 x 9500		
	=	£98800		
Net revenue	=	98800 - 11640 - 9600 - 14400		
	=	£63160		

Blocks with missing or unestimated grades and tonnages are recorded as -999.0 in the data file and are regarded as waste for the purpose of calculating revenue values, i.e. they are treated as 10960 tonnes at 0.0 g/t. (most of these data are around the edges and can be deleted from the file to reduce the number of blocks).

In order to show the applicability of the methodology used in the current study, two example solutions are summarized in the following sections.

Some information and the results of planning with three production periods are shown in the tables and figures throughout the following sub-sections of this Chapter. Two matrices are used during the process: one representing the states of the blocks (mined or not mined) as shown in figure 44 where the removed blocks are represented by 1, while the remaining blocks are represented by 0. The other matrix represents the final coding of the removed blocks as shown in figure 45 where, 1,2,3 represent the sequential removal of blocks during the three different production periods. In both matrices, the blocks outside the optimum pit plan are represented by N3 + 1 (i.e. the number of levels plus one in the program), i.e. 10 in this particular example.

2.2 Determination of the optimum pit limits

The optimum pit limits are found using the parameterization technique. The following parameters are required : cut-off grade, slope angle and the planning parameters. The application of an algorithm to the block grade model yields a set of technically optimum nested pits, one of which will be the economic optimum for a given set of economic parameters.

The parameterization technique can produce a set of nested pits from a block grade model comprising 100,000 blocks in approximately 30 seconds of CPU time on a Sun workstation. The results are stored in a number of files to be used by other programs. The file used in the following sections contains a two-dimensional plan of numbers representing the levels and shows the number of blocks to be mined in each vertical column to obtain the maximum profit. The two-dimensional representation of the optimum pit plan is shown in figure 39.

2.3 Selection of the final pit

A pit plan file containing the total number of pit plans generated in the previous section is used to determine the pit with the highest profit. This pit determination requires the transformation of the block grade matrix into a revenue file matrix which can be scheduled.

The simple revenue formula described in Chapter 2, as well as in the numerical example in section 2.1 of this Chapter, is used to determine the pit with the highest profit as shown in table 1. The data required are the price of the metal, the mining costs and the processing costs.

pit number	value
*********	*****
1	-1.644
2	29.820
3	30.464
4	39.868
5	69.431
6	105.049
7	628.203
8	746.419
9	793.935
10	949.353
11	1035.996
12	1590.774
13	1743.745
14	2400.809
15	2431.968
16	2623.167
17	2780.683
18	3007.691
19	3072.965

the maximum pit(i) and its value are :

19	3072.965

Table 1 : Pit plans evaluation

The plan of the pit with the maximum profit is shown in figure 39.

7	7	7	7	7	7	7	7	7	7
7	5	5	5	5	5	5	5	5	7
7	5	3	3	3	3	3	3	5	7
7	5	3	1	1	1	1	3	5	7
7	5	3	1	0	0	1	3	5	7
7	5	3	1	0	0	1	3	5	7
7	5	3	1	1	1	1	3	5	7
7	5	3	3	3	3	3	3	5	7
7	5	5	5	5	5	5	5	5	7
7	7	7	7	7	7	7	7	7	7

Figure 39 : The plan of the pit with the maximum profit

2.4 Production scheduling

In general production scheduling procedures determine which blocks should be removed in each mining period subject to the mining and milling constraints so as to maximize the Net Present Value of the mine.

Because of the large number of blocks in most orebody block models it is often not practical to optimize a schedule which simultaneously includes the entire model covering all periods of the mine life and all types of mining constraints. However, it is possible to divide the mining process into time periods. The periods can be weeks, months or years. As explained in the previous Chapters the aim of production scheduling is to find the best set of blocks per period that leads to an overall optimum or a solution that is close to the optimum (near-optimum).

The approach used to solve the scheduling problem is a combination of three priority modules comprising: Waste Stripping module, Linear Programming module and Simulation module each of which includes the relevant physical and economic constraints.

The following parameters were specified for this example:

250,000 t/year minimum ore production requirement
500,000 t/year maximum ore production capacity
1,000,000 t/year total (ore + waste) capacity
Discount rate 10 %.

The above parameters are subject to change. The capacity values must be varied proportionally with the number of blocks in the subset treated in more than one time period. This is to avoid false violation of capacity constraints (see also section 6 in Chapter 4).

A brief illustration of the algorithms is provided in the flow chart in figure 40. A more comprehensive explanation with results is provided in subsequent sections.

The modules are set to run for three production periods (ippr), and a limit of three time periods (tp) in any linear program. The determination of an optimal mine schedule using the criterion of maximum Net Present Value is the target for each production period.

To start the scheduling part of the software a subset of blocks to be optimized is specified first. The co-ordinates of the subset should be within the co-ordinates x, y and z of the optimum pit limits. The total number of blocks (itbc) of the subset to be optimised, the possible total number (itnc) of constraints involved and the number of blocks (nbp) to be mined per time period are calculated.

On calculation of these parameters the program proceeds by calling the stripping module (STRIP) to expose a minimum ore tonnage (MOT) before the start of any production period. The minimum ore tonnage is a set of blocks specified by the user and determined by the module as accessible ore blocks to be exposed before production starts.

If this condition is not satisfied the process is repeated until the condition is satisfied. The waste stripping module proceeds by formulating the subset of blocks to be used in the next module (LP).

The subset of blocks is then transformed into table format and optimized by the LP module. If there is **no** solution to the subset of blocks within the LP module, control is returned to the waste stripping module where more stripping takes place. However, if there is **a** solution then the blocks (BK) selected to be removed in each time period, are put into table format in terms of co-ordinates and control is sent forward to the simulation module.

The simulation module (SIMUL), then removes the blocks scheduled for the first time period, adjusts and checks the minimum ore tonnage of the new subset of blocks, then returns the control either to the linear programming module for more planning or to the waste stripping module for more stripping.

The process is repeated by iteration between the three modules for the total number of production periods. Intermediate results for the different sections of the scheduling part of the software can be obtained for each iteration.

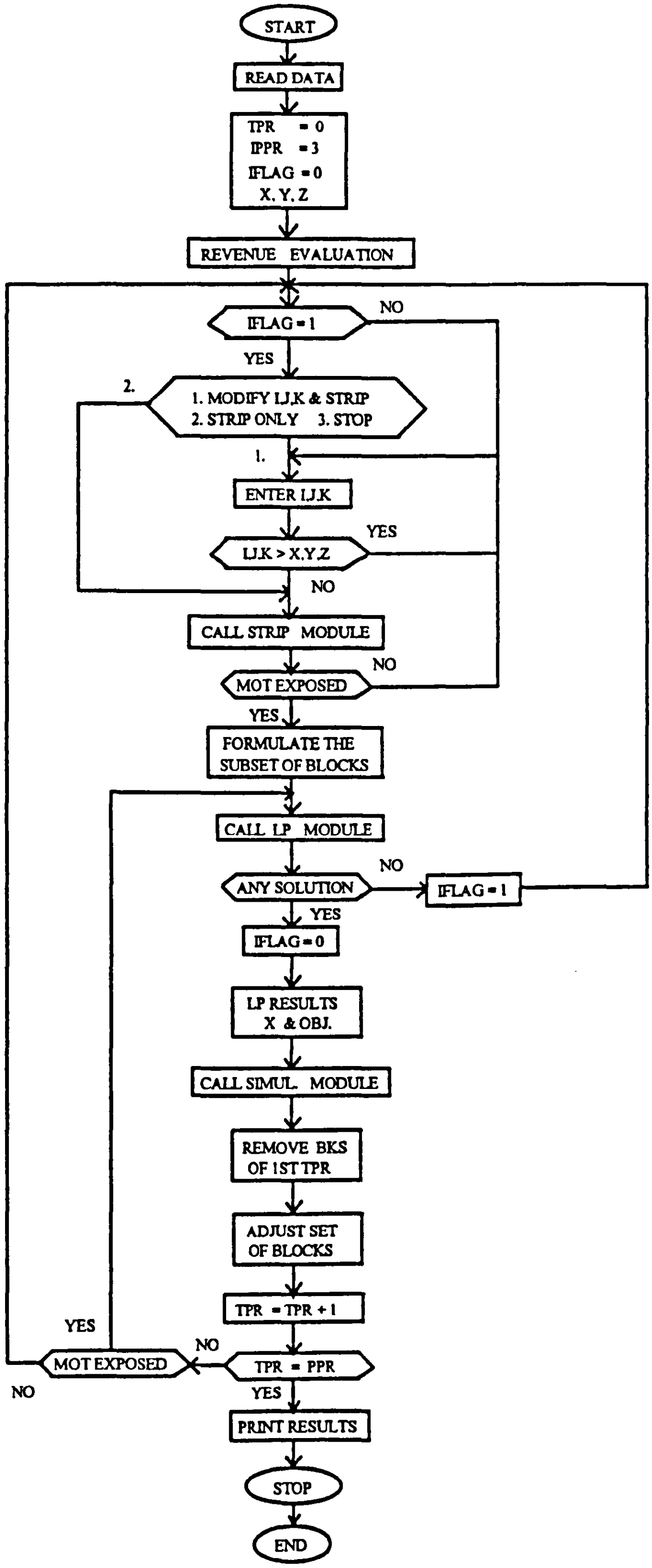


Figure 40 : General structure of the schedule model

2.4.1 Waste stripping module

The selection of the waste blocks to be mined prior to production is, in general, a trade-off between minimizing the amount of waste material removed and maximizing the value of the ore blocks exposed at the end of the pre-production stripping.

This module is used only when required to expose a minimum ore tonnage at the start of each production period. This would normally be called a pre-production module and would be used to schedule waste and overburden and provide access to ore blocks. The main steps of the waste stripping module are summarized in figure 41.

Waste blocks have to be removed at some time, either during the same period that an ore block is mined or at least one period before, i.e. waste blocks directly or indirectly overlying ore blocks must be assigned high priority coefficients so that they will be mined first.

To establish the priority of the various blocks to be exposed as ore blocks, the grade and the number of exposed sides of blocks are used as constraints (all constraints are used simultaneously).

To run the module for this example the following data were specified :

1. A minimum ore tonnage to be mined is specified and is translated by the program into the number of accessible ore blocks.

2. A guess of a total number of blocks at a cut-off of 0.2 g/t, that are likely to sum up to the minimum ore tonnage specified. The user is then repeatedly prompted to provide a better estimate if this tonnage is not met by the current number of blocks, until it is. The current module is able to expose any number of blocks up to the total number of blocks within the optimum pit plan if necessary.

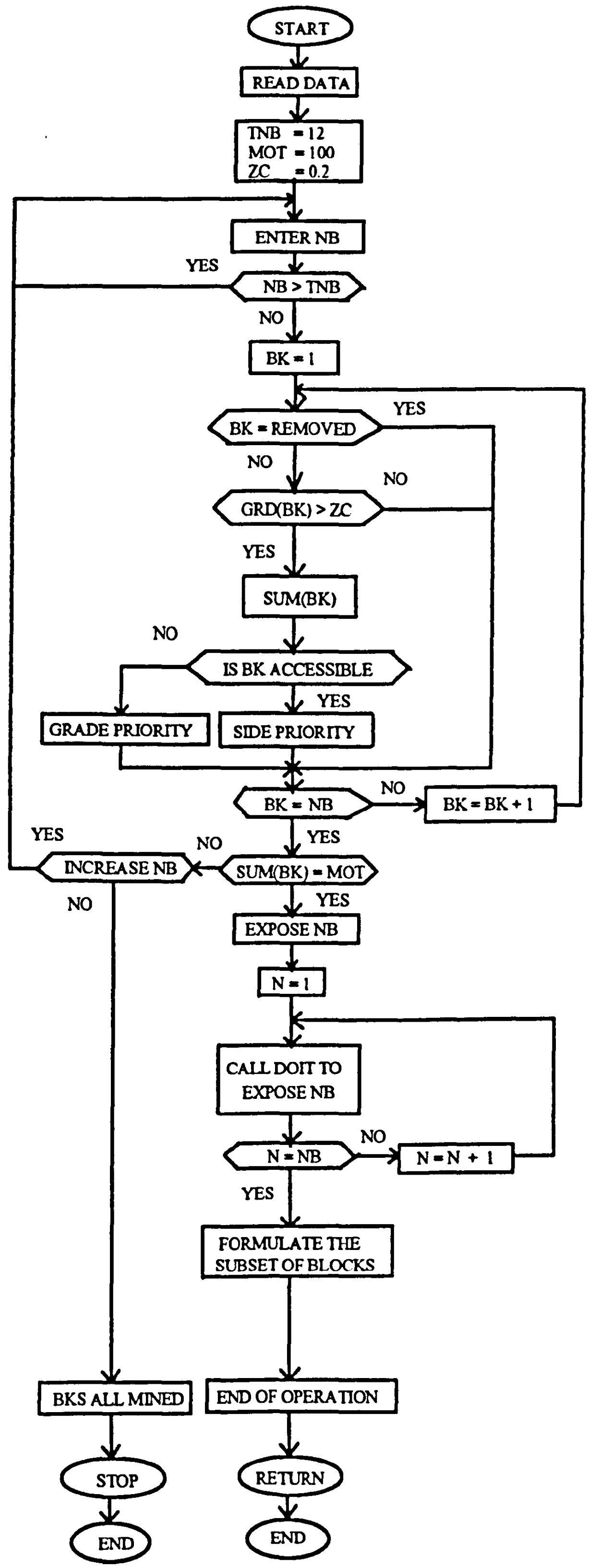


Figure 41 : Main steps of the waste stripping model

In due course, this interactive part of the software can be automated through a formal iterative numerical scheme. When the requirement is satisfied control is passed forward to the LP module for further planning. The internals of the current interactive iterative scheme have as follows :

The selected blocks (BK) are examined one by one. If the grade of a minable block is greater than or equal to the cut-off grade a priority coefficient is assigned to that block depending first of all on the number of its sides that are currently exposed. In a similar way a priority coefficient is assigned to the same block depending on its grade value.

The procedure continues until the minimum ore tonnage requirement is satisfied. By then all minable blocks (nnb) should have been examined and assigned a priority number. The result of such an operation for 20 minable blocks is shown in table 2.

Selected blocks	Exposed sides	Grade g/t	Priority of blocks
1 1 2	0	2.100	9
2 1 2	0	2.200	8
3 1 2	0	4.900	3
4 1 2	0	3.300	7
5 1 2	0	7.900	2
6 1 2	0	3.600	6
7 1 2	0	1.900	10
8 1 2	0	1.900	10

Table 2 : Order of priority of blocks
(continued ...)

9 1 2	0	4.800	4
10 1 2	0	3.600	6
1 2 2	0	2.100	9
2 2 2	0	3.300	7
3 2 2	0	0.900	12
4 2 2	0	16.100	1
5 2 2	0	1.800	11
6 2 2	0	3.600	6
7 2 2	0	3.700	5
8 2 2	0	3.600	6
9 2 2	0	1.900	10
10 2 2	0	3.600	6

Table 2 (... continued)

In this example the ore blocks are not exposed at the beginning of the first production period. As can be seen in table 2, the priority classification is mainly based on the grades of the blocks instead of their exposed sides. Block (4,2,2) with grade 16.10 g/t, has priority number 1. The priority is given in that order until the last block with the lowest grade is given the smallest priority number i.e. block (3,2,2) with a grade 0.90 g/t has priority number 12.

The blocks would have been assigned priority numbers in a similar way if classification had been based on exposed sides. The block with the highest number of exposed sides would have priority number 1 and so on. It is important to note that the basis of classification rests entirely with the software and this can lead to 'mixed' prioritization based on both grade and exposed sides as shown in table 3.

Selected blocks	Exposed sides	Grade g/t	Priority of blocks
1 1 2	3	2.100	2
8 1 2	4	1.900	1
1010 2	3	1.900	2
8 2 3	4	3.000	1
2 2 4	3	3.700	2
6 2 4	3	2.200	2
7 2 4	1	2.100	4
8 2 4	0	1.900	6
9 2 4	2	2.900	3
7 3 4	3	1.900	2
8 3 4	3	1.900	2
2 9 4	3	3.100	2
6 3 5	2	2.100	3
7 3 5	0	3.400	4
8 3 5	0	3.000	5
6 4 5	2	2.000	3
7 4 5	2	2.200	3
8 4 5	2	2.200	3
6 5 5	4	3.600	1
3 3 6	2	1.900	3

Table 3 : Mixed prioritization of blocks

The minable blocks (nnb) are then exposed in descending order of their priority. In cases where a mixed classification using both constraints has occurred, the blocks with exposed sides are prioritised first (based on the number of their exposed sides) and the rest of the blocks are prioritised based on their grades. Once the blocks to be exposed are selected (table 2). The blocks which obstruct access to those blocks are removed and assigned the time period number in which they are removed as shown in figure 44.

The removal of blocks in this module is based on geometrical constraints. The geometrical constraints for this module are:

- a block can only be mined if the five blocks above it have already been mined

- the slope angle of 45 degrees must be maintained

Removed blocks are generally blocks of zero or low grades within the optimum pit plan.

A successful run of the module will expose the ore blocks and, create access to them and will reduce the number of constraints in the next module (LP). After exposing the selected ore blocks the module will formulate the subset of blocks to be used in the next module (LP). In order to minimize movement of equipment, the blocks added to the subset are the closest blocks to those exposed in the waste stripping module.

Pre-production stripping usually happens in the first year of operation and it would be rare for it to be conducted during the productive life of the mine. However, if it does happen during the time horizon of the scheduling algorithms then the removed blocks are always considered as part of the current production period.

2.4.2 Linear programming module

The linear programming (LP) module optimizes ore removal of a subset of blocks from the orebody block model and, as such, it handles the physical and economic constraints. The Lp_solve source code described in Chapter 3, section 2.2.2 was incorporated into the author's software, to solve the LP part of the work.

To take advantage of the general LP solving capability of Lp_solve, it is vital that the capacity constraints of the mining problem formulation be manipulated to conform to the structure of the access constraints as stated in Chapter 4. The constraint matrix will then have a chance of being unimodular and result in integer solutions automatically without Lp_solve resorting to its computationally demanding branch-and-bound Integer Linear Programming (ILP) solver. The way to achieve this is to apply the Row-Echelon Form, Bronson (1989).

The Row-Echelon Form can only be adopted for a square matrix. Unfortunately, the mining problem gives a non-square matrix and so the Row-Echelon Form cannot be applied to it. Branch-and-bround is the only way forward and requires some additional constraints to the current formulation, for example, the number of blocks to be mined per period (NB(t)). If all blocks are of equal size then the following inequalities, need to be considered.

- Block limitation

$$\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=N_z(1)}^{N_z(2)} b_{ijk}(t) \leq \sum_{i=1}^{N} NB(t) \qquad , \quad \forall\ t=1,..,t_p$$

- Minimum number of blocks per period, if required :

$$\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=N_z(1)}^{N_z(2)} b_{ijk}(t) \geq N_{\min}(t) \qquad , \quad \forall\ t=1,..,t_p$$

Where : NB - is the number of blocks being considered, with

$N = N_x \times N_y$ (number of blocks in x and y directions)

N_{min} - is a minimum number of blocks required per period.

Finally the **exclusion constraint**, so that each block can only be mined in one time period.

$$\sum_{t=1}^{tp} b_{ijk}(t) = 1 \quad , \quad \forall\ i,j,k$$

Solving the problem as a Mixed Integer Linear Program (MILP) using branch-and-bound and Lp_solve, some or all of the variables have to be declared as integers. In the current problem, it is necessary for all the variables to be specified as integers. As the number of variables increases it becomes increasingly difficult to achieve a feasible solution in a reasonable time.

The scheduling is done for three (3) time periods in any linear program for a subset of blocks and a maximum of three levels open in any time period. Seven physical and economical limitations constraints have been used on the mining problem (see Chapter 4 for more details):

- Two access constraints per block : Type 1: One block above and one next are already mined, type 2: one block above and four around have been mined
- One block limitation constraint per time period
- One exclusion constraint per block
- Three capacity constraints

The idea of solving the problem in a series of subsets of blocks, subject to constraints, reduces the size of the problem. As a result the storage requirement and processing time are reduced.

The specification of the number of blocks (nbp) to be scheduled per period depends on several parameters such as : - The number of diggers (machinery), operators, weather conditions, stock control, market conditions, deposit grade map, etc. With respect to all these factors, nbp can be worked out such that each block contains 5-10 % of the total tonnage of the deposit, Dowd (personal communication).

The nbp for the current study is taken to be the number of blocks on one level (x*y) of the deposit for each production period.

The main steps of an examination of the subset of blocks in the context of the LP module are given in figure 42.

The above formulation generates a simplex matrix for solution of LP module as MAT(169 ; 324) i.e. 169 constraints and 324 variables using the waste stripping module. The matrix includes the elements of the constraint matrix, the equation of the objective function and the integer declaration of the variables being considered. The computer processing for the above subset of blocks was completed for three production periods.

The elements of the constraint matrix generated depend on the accessibility of blocks. If a block is accessible then the accessibility constraints for that particular block are eliminated from the matrix. A block is accessible if the block above it and at least one of the blocks surrounding it on the same level have already been removed. However, if a block is on the uppermost level (level one) then it is always accessible because the blocks above it are air blocks. The module considers all air blocks are removed first.

The module can indicate whether or not there is a solution. In either case control is always returned to the scheduling algorithm where decisions can be made.

No solution to the MILP problem may be caused by too large a number of blocks, by one of the constraints not being satisfied. In order to continue the planning process more ore blocks must be exposed. In such cases control is returned to the waste stripping module where more waste blocks are stripped to expose more ore blocks.

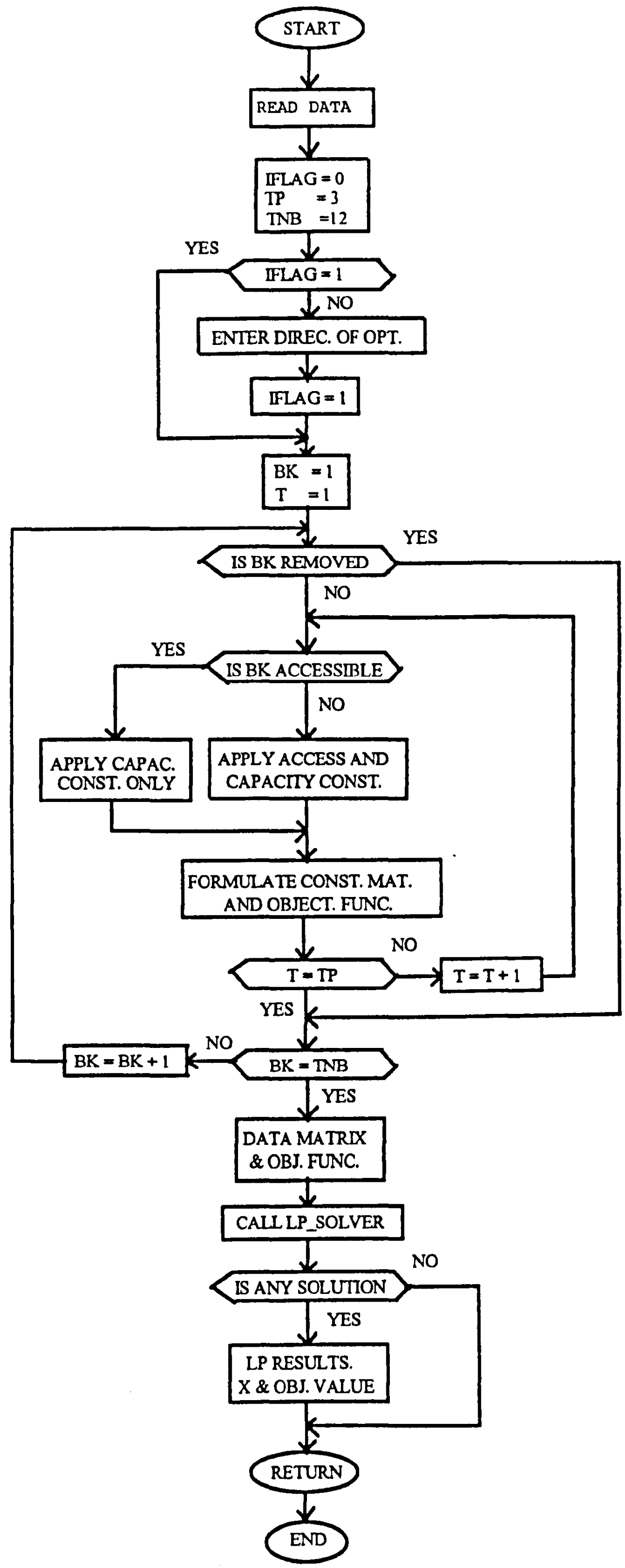


Figure 42 : Main steps of the linear programming module

However, in the example quoted here an optimum solution exists. The LP module generated an optimal solution for the three time periods used. The results were the values of the X decision variables which represent the status of the blocks to be mined from any specific subset of blocks and the profit that can be gained from that subset of blocks during the three time periods.

The X decision variables are converted into their corresponding block co-ordinates (i,j,k). Due to the exclusion constraint, each block of the subset can only be mined in one time period. The method of mining is by push-back. The starting block is always the last block to be mined. As can be seen in table 4, the block with co-ordinates i=3, j=9, k=3, is considered first in the LP solution, where as it is in fact the last block to be mined.

Scheduling Production Period : 1		
LinProg Periods : 3		
No of Removed Block : 36		
Time Period 1	Time Period 2	Time Period 3
BLOCK CO-ORDINATES I J K		
3 9 3	2 9 3	6 9 3
4 9 3	5 9 3	7 9 3
3 1 2	8 9 3	1 1 2
5 1 2	9 9 3	2 1 2
6 1 2	4 1 2	8 1 2
9 1 2	7 1 2	1 2 2
10 1 2	2 2 2	3 2 2
4 2 2	6 2 2	5 2 2
7 2 2	8 2 2	9 2 2
10 2 2	1 3 2	3 3 2
4 3 2	2 3 2	5 3 2
9 3 2	7 3 2	6 3 2
1 4 2	8 3 2	3 4 2

Table 4 : Blocks produced in the first production period (continued ...)

2 4 2	10 3 2	6 4 2
9 4 2	4 4 2	7 4 2
4 5 2	5 4 2	8 4 2
6 5 2	2 5 2	10 4 2
1 6 2	3 5 2	1 5 2
4 6 2	5 5 2	9 5 2
5 6 2	7 5 2	10 5 2
8 6 2	8 5 2	3 6 2
9 6 2	2 6 2	6 6 2
10 6 2	9 7 2	7 6 2
1 7 2	2 8 2	4 7 2
2 7 2	3 8 2	6 7 2
3 7 2	5 8 2	7 7 2
5 7 2	10 8 2	8 7 2
10 7 2	2 9 2	1 8 2
4 8 2	5 9 2	7 8 2
6 8 2	6 9 2	8 8 2
9 8 2	7 9 2	1 9 2
3 9 2	8 9 2	4 10 2
4 9 2	10 9 2	5 10 2
9 9 2	2 10 2	6 10 2
1 10 2	7 10 2	9 10 2
3 10 2	8 10 2	10 10 2

Table 4 (... continued)

2.4.3 Simulation module

The simulation module is used after the LP module to remove the blocks scheduled for the first time period of each run, and to adjust and apply the movement constraint. The main steps of the module are explained in the simulation flow chart in figure 43. As mentioned in Chapter 4, only the blocks (nnb) of the first time period (tp) produced in the LP module (table 4), are removed from the orebody. The remaining blocks for time periods 2 and 3 are then considered as members of the next subset of blocks.

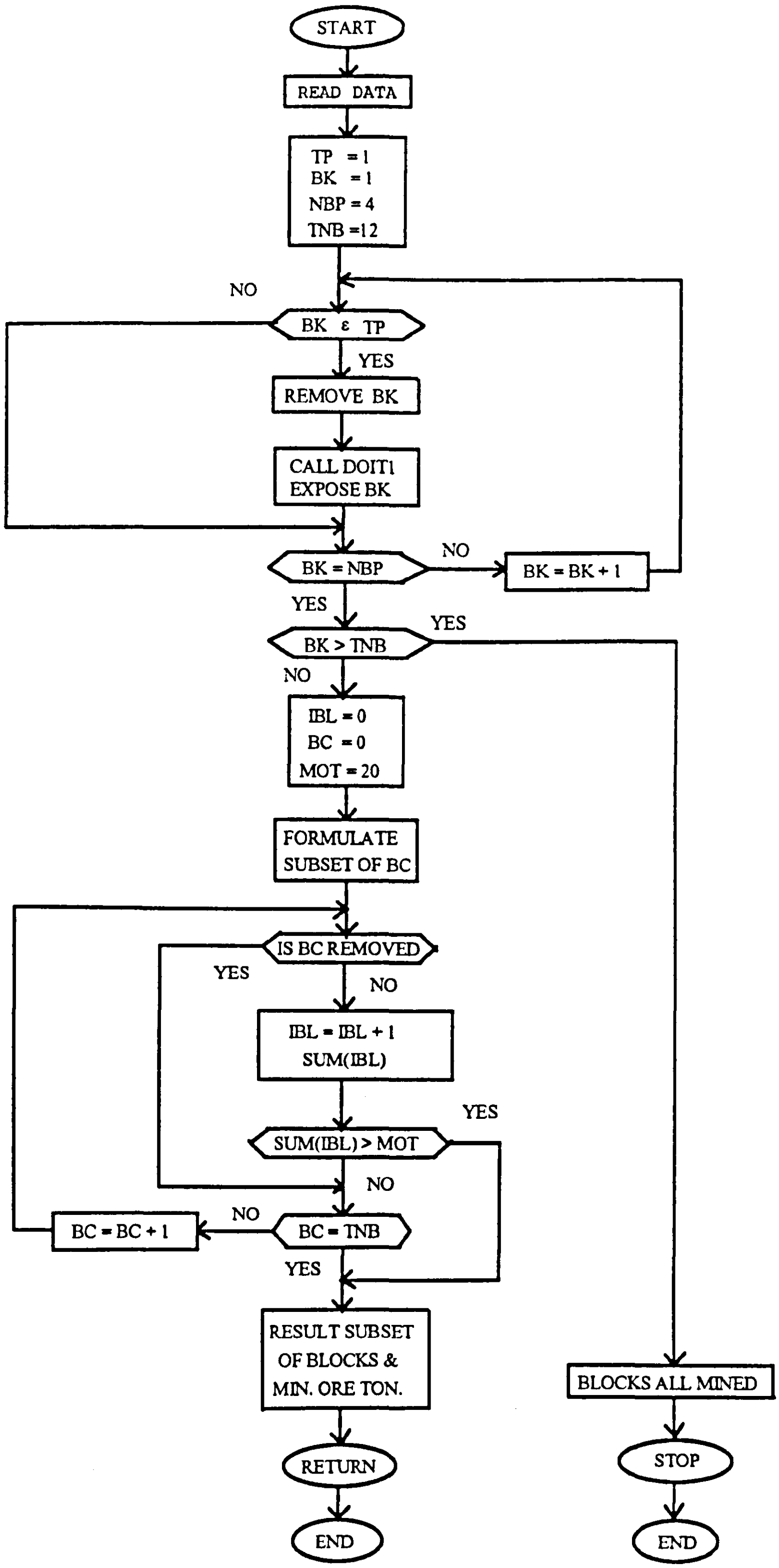


Figure 43 : Main steps of the simulation module

The blocks obstructing the access to those blocks selected for removal during the first time period of the current production period are removed. The removal of blocks in this module is based on the usual geometrical constraints :

- a block can only be mined if the five blocks above it have already been mined
- a block is accessible only if the block above it and at least one of its surrounding blocks have already been mined
- the slope angle of 45 degrees must be maintained

At this stage of execution the total number of blocks to be mined from the orebody (within the waste stripping module and the LP module) represents the scheduling of the first production period. The blocks are removed from the orebody block model and assigned the number of the production period in which they are removed as shown in figure 44. Only levels reached by the scheduling operation of the total matrix are presented here to show the removed blocks for each production period.

The development of the next subset of blocks for the LP module (production period 2) continues starting from the last block of the optimum set of blocks removed in the previous period, (the coordinates of the last block mined are passed automatically to the following section).

In order to minimize equipment movements from one bench to another, the subset is adjusted with the closest blocks to those remaining from periods 2 and 3. The minimum ore tonnage (MOT) is then checked for the new subset of blocks before control is returned either to the waste stripping module or to the LP module.

If the minimum ore tonnage is less than that specified, the implication is that more stripping must be undertaken. In such cases control is returned to the stripping module where more waste blocks are stripped. On the other hand, if the minimum ore tonnage of the blocks selected is met, the subset of blocks is submitted directly to the LP module for further scheduling.

The original first guess of the number of blocks representing the minimum ore tonnage remains the same for all production periods unless the remaining blocks within the optimum pit plan during the planning process are fewer than that number. In that case control is sent to the waste stripping module, where the number of blocks can be re-adjusted to meet the requirements.

level = 1

R/C:	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1

Figure 44 : Coding of the blocks removed in the first production period (continued ...)

level = 2

R/C:	1	2	3	4	5	6	7	8	9	10
1	0	0	1	0	1	1	0	0	1	1
2	0	0	0	1	0	0	1	0	0	1
3	0	0	0	1	0	0	0	0	1	0
4	1	1	0	0	0	0	0	0	1	0
5	0	0	0	1	0	1	0	0	0	0
6	1	0	0	1	1	0	0	1	1	1
7	1	1	1	0	1	0	0	0	0	1
8	0	0	1	1	0	1	0	0	1	0
9	0	1	1	1	1	0	0	0	1	0
10	1	0	1	1	0	0	0	0	0	0

level = 3

R/C:	1	2	3	4	5	6	7	8	9	10
1	10	10	10	10	10	10	10	10	10	10
2	10	0	0	0	0	0	0	0	0	10
3	10	0	0	0	0	0	0	0	0	10
4	10	0	0	0	0	0	0	0	0	10
5	10	0	0	0	0	0	0	0	0	10
6	10	0	0	0	0	0	0	0	0	10
7	10	0	0	0	0	0	0	0	0	10
8	10	0	0	0	0	0	0	0	0	10
9	10	0	1	1	0	0	0	0	0	10
10	10	10	10	10	10	10	10	10	10	10

Figure 44 (... continued)

The process will then continue for the remaining production periods. The removed blocks will always be assigned the number of the period in which they are removed.

After a successful run of the first production period the scheduling algorithm will continue for the next production period by iteration between the three modules until an optimal solution is found for each period. Table 5 and table 6 show the results of the X decision variables of the LP module for production periods 2 and 3.

Scheduling Production Period : 2		
LinProg Periods : 3		
No of Removed Block : 36		
Time Period 1	Time Period 2	Time Period 3
BLOCK CO-ORDINATES I J K		
2 4 3	6 4 3	3 4 3
4 4 3	7 4 3	4 5 3
5 4 3	8 4 3	5 5 3
9 4 3	3 5 3	6 5 3
2 5 3	7 5 3	9 5 3
2 6 3	8 5 3	5 6 3
9 6 3	3 6 3	7 6 3
2 7 3	4 6 3	8 6 3
4 7 3	6 6 3	9 7 3
2 8 3	3 7 3	6 8 3
3 8 3	5 7 3	6 9 3
4 8 3	6 7 3	7 9 3
5 8 3	7 7 3	8 9 3

Table 5 : Blocks produced in the second production period (continued ...)

9 8 3	8 7 3	1 1 2
2 9 3	7 8 3	2 1 2
9 9 3	8 8 3	7 1 2
4 1 2	5 9 3	8 1 2
2 2 2	1 3 2	1 2 2
6 2 2	8 3 2	3 2 2
8 2 2	10 3 2	5 2 2
2 3 2	6 4 2	9 2 2
7 3 2	7 4 2	3 3 2
4 4 2	8 4 2	5 3 2
5 4 2	5 5 2	6 3 2
2 5 2	3 6 2	3 4 2
3 5 2	6 6 2	10 4 2
7 5 2	6 7 2	1 5 2
8 5 2	7 7 2	9 5 2
2 6 2	8 7 2	10 5 2
4 7 2	9 7 2	7 6 2
2 8 2	7 8 2	1 8 2
5 8 2	8 8 2	1 9 2
6 9 2	10 8 2	5 10 2
7 9 2	8 9 2	6 10 2
2 10 2	10 9 2	9 10 2
7 10 2	8 10 2	10 10 2

Table 5 (... continued)

Scheduling Production Period : 3			
LinProg Periods : 3			
No of Removed Block : 36			
	Time Period 1	Time Period 2	Time Period 3
BLOCK CO-ORDINATES I J K			
	3 6 4	7 5 4	5 5 4
	4 6 4	8 5 4	6 5 4
	5 6 4	8 6 4	9 5 4
	9 6 4	9 7 4	2 6 4
	3 7 4	5 8 4	6 6 4
	4 7 4	6 8 4	7 6 4
	5 7 4	5 9 4	2 7 4
	4 8 4	6 9 4	6 7 4
	9 8 4	7 9 4	7 7 4
	2 9 4	8 9 4	8 7 4
	3 9 4	4 2 3	2 8 4
	4 9 4	7 4 3	3 8 4
	9 9 4	3 5 3	7 8 4

Table 6 : Blocks produced in the third production period (continued ...)

5 2 3	7 5 3	8 8 4
6 2 3	6 6 3	2 2 3
7 2 3	8 6 3	3 2 3
8 2 3	6 7 3	2 3 3
9 2 3	7 7 3	3 3 3
4 3 3	8 7 3	6 3 3
5 3 3	9 7 3	4 5 3
7 3 3	6 8 3	5 5 3
8 3 3	7 8 3	6 5 3
9 3 3	5 9 3	9 5 3
3 4 3	6 9 3	7 6 3
6 4 3	7 9 3	1 1 2
8 4 3	8 9 3	2 1 2
8 5 3	7 1 2	8 1 2
3 6 3	1 3 2	1 2 2
4 6 3	10 3 2	3 2 2
5 6 3	7 4 2	3 3 2
3 7 3	6 6 2	6 3 2
5 7 3	6 7 2	10 5 2
8 8 3	7 7 2	7 6 2
5 2 2	8 7 2	5 10 2
9 2 2	7 8 2	6 10 2
8 3 2	8 10 2	10 10 2

Table 6 (... continued)

The final coding of the removed blocks within the waste stripping and the simulation modules during the whole scheduling process are shown in figure 45. 1,2,3 represent the sequential removal of blocks during the three different production periods.

level = 1

R/C:	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1

level = 2

R/C:	1	2	3	4	5	6	7	8	9	10
1	0	0	1	2	1	1	3	3	1	1
2	0	2	0	1	3	2	1	2	3	1
3	0	2	3	1	2	3	2	3	1	3
4	1	1	2	2	2	2	3	2	1	2
5	2	2	2	1	2	1	2	2	2	3
6	1	2	2	1	1	3	3	1	1	1
7	1	1	1	2	1	3	3	3	2	1
8	2	2	1	1	2	1	3	2	1	2
9	2	1	1	1	1	2	2	2	1	2
10	1	2	1	1	3	0	2	3	2	0

level = 3

R/C:	1	2	3	4	5	6	7	8	9	10
1	10	10	10	10	10	10	10	10	10	10
2	10	0	0	0	3	3	3	3	3	10
3	10	0	0	3	3	0	3	3	3	10
4	10	2	3	2	2	3	0	3	2	10
5	10	2	3	3	3	0	0	3	3	10
6	10	2	3	3	3	3	0	3	2	10
7	10	2	3	2	3	3	0	0	3	10
8	10	2	2	2	2	0	0	3	2	10
9	10	2	1	1	3	0	0	3	2	10
10	10	10	10	10	10	10	10	10	10	10

Figure 45 : Coding of the removed blocks to production periods (1,2 and 3). (continued ...)

level = 4

R/C:	1	2	3	4	5	6	7	8	9	10
1	10	10	10	10	10	10	10	10	10	10
2	10	0	0	0	0	0	0	0	0	10
3	10	0	0	0	0	0	0	0	0	10
4	10	0	0	0	0	0	0	0	0	10
5	10	0	0	0	0	0	0	0	0	10
6	10	0	3	3	3	0	0	0	3	10
7	10	0	3	3	3	0	0	0	0	10
8	10	0	0	3	0	0	0	0	3	10
9	10	3	3	3	0	0	0	0	3	10
10	10	10	10	10	10	10	10	10	10	10

Figure 45 (... continued)

The results of the optimised discounted alternatives, at a time value of money of 10%, are shown in table 7.

Prod. Sched. Periods	Discounted Period 1	Discounted Period 2	Discounted Period 3
1	584.51	239.03	62.17
2	425.13	202.89	65.97
3	413.13	196.64	64.23

Table 7 : Discounted alternatives in each production period

It can be seen from figure 45 and table 7 that the best way of mining the orebody is to mine the most profitable parts during the early stages of the mining operations. Based on the above results, we can, therefore conclude that the earlier the waste stripping is done, the more practical and profitable, scheduling becomes.

However, the difficulty encountered with the Lp_solve coding is that it can solve general LP problems up to 30,000 variables and 50,000 constraints. But solving problems of similar size with integer variables is much more difficult. With binary variables, Lp_solve usually provides fast answers for up to about 100 variables. Sometimes Lp_solve can handle much larger problems, but then its success depends critically on having a problem definition that results in a lot of integer values immediately. Such a definition may be largely due to chance. Even commercial codes cannot provide such a guarantee. In these cases, special codes, or heuristics have to be resorted to.

The particular approach adopted in this research subdivides the total problem into a number of overlapping sub-problem and finds an optimum solution to each of these sub-problems. There is no a priori guarantee that such an approach provides an optimum solution to the total problem.

To test the assertion that the piecewise approach provides an acceptable solution a number of comparisons were made on a relatively large problem solved using the approach described in this thesis and also solved using XPRESS-MP which is a commercial, State-of-the art package devised by Proll (1995); the version available here is capable of handling problems of up to 10,000 constraints and 15,000 variables. XPRESS-MP has two major components; a modeller and an optimiser. The modeller takes a linear or integer linear programming problem specified in its modelling language and produces the input file for the optimiser. The modelling language is very powerful but requires the problem to be specified in a completely different way to Lp_solve. In order to change the model as specified for Lp_solve into an acceptable model for XPRESS-MP a substantial amount of expert hand-editing was required.

An array from the case study of 1430 (e.g., 26 x 55) blocks, a maximum of two levels open in any time period and an upper limit of two time periods in any linear program has been considered. A simplex matrix for solution of the LP module is defined equal to MAT(9,742 ; 5,720) using the waste stripping module. The matrix was generated on GPSb (General Purpose Server). The model can generate any mining problem size as long as enough computer memory is available. The computer facilities available at the University of Leeds (i.e. GPS, GPS0, GPSb (General Purpose Server), the CIF (Computationally Intensive Facility) and the Sun servers) are more flexible with integer matrices than with real matrices.

Thus it was only possible to complete one run of the model with 9,742 constraints and 5,760 variables. For this run the optimiser required approximately 5 minutes of computer time on a Sun Sparc L workstation.

Because of the large output file of the results, only a brief output summary of the results is described below:

The optimum solution was reached within the first iteration (first possible solution) and the 16th iteration (final solution) yielding an objective function value equal to -1694.37 which represents a net loss if the operation was carried out for the two levels of the deposit considered.

2.5 Graphic representations

The characteristics of each pit design (metal content, total tonnage, average grade, stripping ratio, etc...), are easily calculated and interpreted. It is much more difficult to provide a representation of the approximate shape and location of the current state of the deposit. However, pit shapes are always determined by boundaries and these can be represented by contour plans, three-dimensional representations, cross-sectional views, etc.

Graphic representations of the orebody and the optimal pit selected during the optimization procedure are generated with the aid of routines from the **UNIRAS** package available on the University of Leeds Sun servers.

The **UNIRAS** graphics package offers a wide variety of facilities which are well documented (references and user manuals). These facilities are set up as independent routines that perform specific functions, for example: **UNIGRAPH, UNIMAP.** A graphics display on the screen as well as hard copies can be obtained by the user.

The principal routines used in this work are:

UNIGRAPH routine: an interactive sub-program from the **UNIRAS** package designed to draw hard and smooth curves, pie charts, etc... .

UNIMAP routine: an interactive sub-program from the **UNIRAS** package designed to draw hard and smooth two-dimensional contours, three-dimensional and four-dimensional views, cross-sections etc... .

These UNIRAS routines have been incorporated into a program that produces graphics representations of the numerical data for any pit plan. All graphics display programs developed for this research were written in Fortran 77. The user has the choice of setting up the following figures :

1. Selection of the final pit (graph)
2. Two-dimensional contouring of mining levels
3. Three-dimensional pit representation of the optimum pit plan
4. Cross-section views of pit elevations
5. Graphics display of the part of the pit that has been mined.

Each graphics display routine is described in the following sections and examples are given to illustrate their use.

2.5.1 Selection of the final pit

This routine provides a graphics representation of the optimal pit, i.e. the pit that yields the maximum profit. The display consists of the number of pits (X-axis) and the pit values (Y-axis) and an example is shown in figure 46.

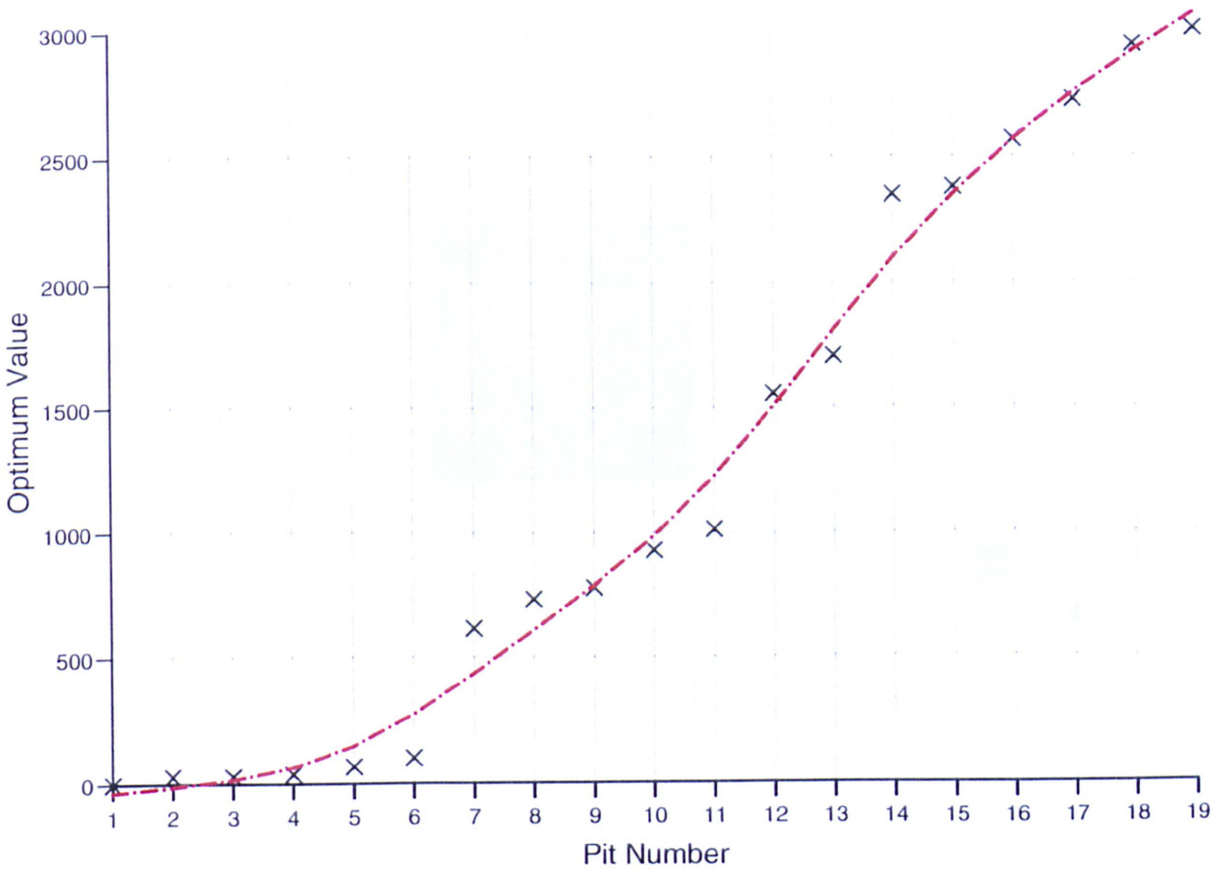


Figure 46 : Selection of the final pit

2.5.2 Two-dimensional contouring of mining levels

This routine provides a graphics display of the two-dimensional plan-contour that represents the optimal pit limits. An example is shown in figure 47.

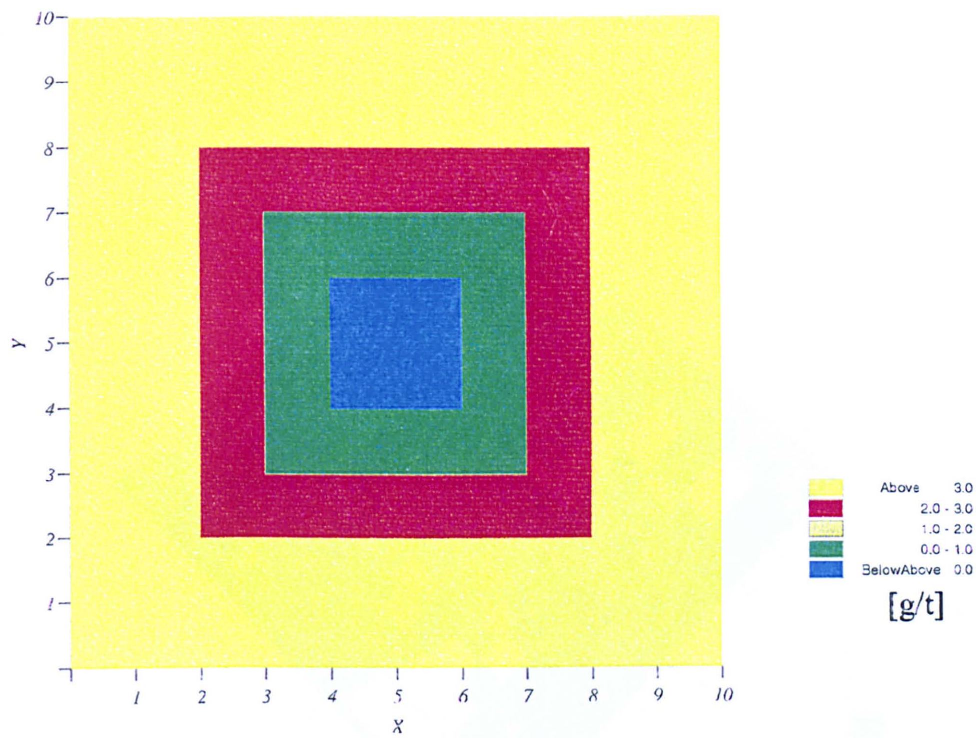


Figure 47 : Two-dimensional contouring of mining levels

2.5.3 Three-dimensional representation of the optimum pit plan

The graphics display provided in this routine is a three-dimensional view of the optimal pit design as shown by the example in figure 48. The angles that give the position of the three-dimensional space representation of the pit shape (Alpha, Beta and the Distance) are set up in the source code of the routine:

X - axis represents the number of blocks in the east-west direction.

Y - axis represents the number of blocks in the north-south direction.

Z - axis represents the number of blocks in the vertical direction (the depth of the pit)

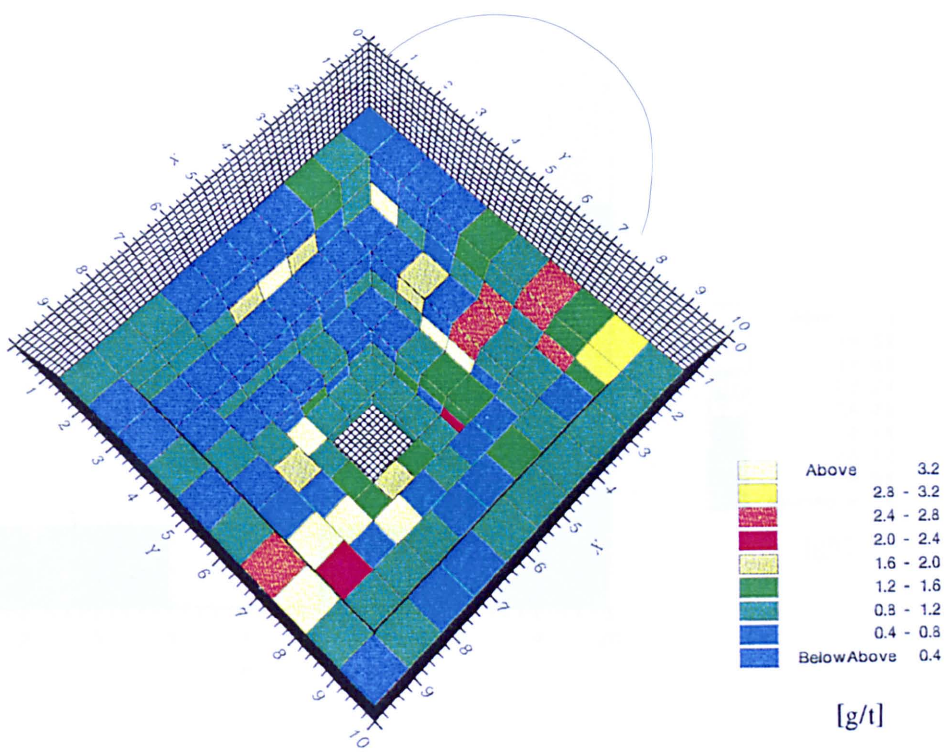


Figure 48: Three-dimensional representation of the optimum pit plan

2.5.4 Cross-sectional views of pit elevations

The graphics display provided in this routine is a pit cross-section in the east-west (E-W) direction. The coordinates of the cross-section are initialised in the source code of the routine. Figure 49 shows an example of a cross-sectional display in the vertical direction.

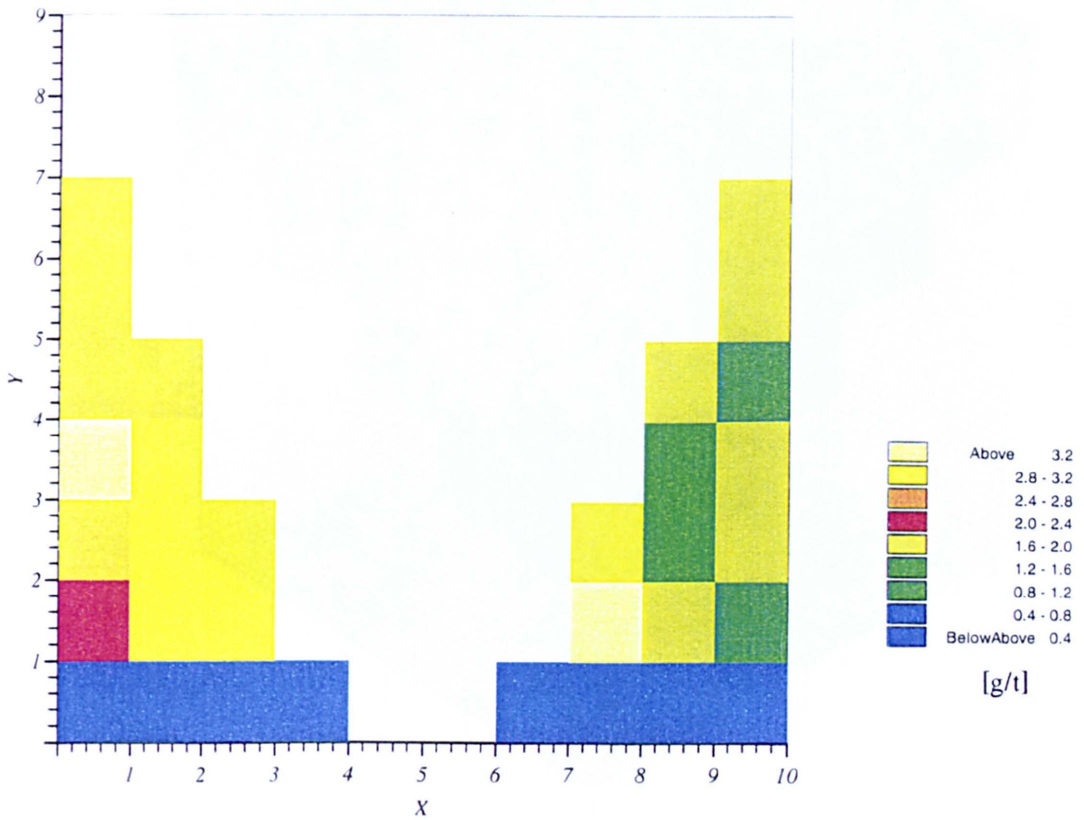


Figure 49 : Cross-section view of pit elevations

2.5.5 Display of pit that has been mined

The three-dimensional representation was also extended to provide a three-dimensional view of the mined out area of the orebody. An example is shown in figure 50.

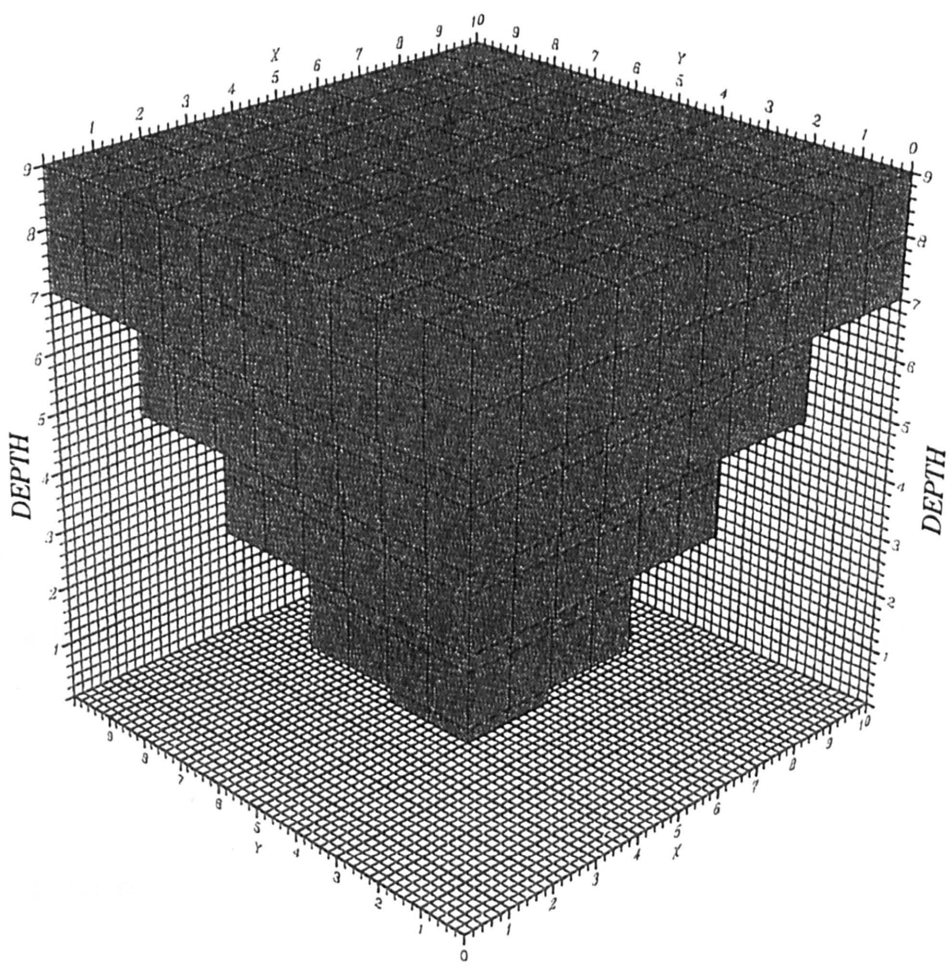


Figure 50 : Display of pit that has been mined

2.6 The validation of the methodology

The numerical accuracy of the software has been compared and demonstrated by running the same example MAT(169 ; 324) of section 2.1 of this Chapter in the XPRESS-MP package. Only one run was completed for one production period. The results for the three time periods within it are shown in appendix 1, along with the Lp_solve solution. The two solutions were identical, thus validating the piecemeal approach.

Refering to the two given solutions, the results can best be explained by comparing the optimum paths of blocks selected for each time period. Considering position 2 in the value column of solution 1 and the same position in the first row of the X decision variables of solution 2, we can see both solutions indicate that the same block will be mined in the first time period. The corresponding co-ordinates i=3, j=9, k=3 of this block are those shown in table 4 in the first row of the column of the first time period. The rest of the blocks are analysed in a similar way.

The need to further validate the solution of the current study against other methodologies from the literature and industrial practice is recognised and will need to be pursued during further development of the software, outside this work.

3. Conclusion

The aim of the present study was to develop a mathematical model for optimizing the schedule for the entire life of a mine. The model developed represents an effective tool for scheduling waste and ore production for long-term planning.

The results obtained from its application on an example open pit mine confirms the high economic effectiveness of the early user-activated waste stripping strategy with the overall objective of maximizing the profit or the Net Present Value. The division of the entire deposit into subsets of blocks helps to reduce the size of the mining problem in terms of computing memory and processing time as seen in the previous sections.

The model is simple to use and can account for varying market conditions through several parameters set by the user making parametric studies possible without interfering with the computer code. Although the model in its present state can only be treated as a prototype, it is fully functional and requires minimum programming skills to operate. Developments in the area of user interface and diagnostics will need to be made in the future.

The graphics used in the current study to depict the results are not critical to the software, but have been incorporated for completeness of the package. They can be extended to include the display of different windows, window menus, different graphics representations within one run etc. Some of the routines which produce the graphics shown in figures 46, 47, 48, 49 and 50 may have slight errors in representing the appropriate shape when considering different data sets. Such errors can be rectified by more precise calls of the routines, but these will not affect the main logic of the results or even the graphics representations themselves.

The piecemeal LP methodology has been compared with a high capacity commercial package and found to give an identical solution.

CHAPTER SEVEN

Conclusions

The growth in demand for the raw materials supplied by the mining industry has led to the development of increasingly lower grade deposits which requires the investment of a large amount of capital which must be repaid quickly as possible. Many studies have shown that the combination of the time value of money and the need to pay back the initial investment as quickly as possible require operations to begin with a relatively high cut-off grade which then declines over the life of the mine. Estimation of block grades and cut-off grades is thus essential for optimal mine planning.

The aim of optimal mine planning is to determine the final pit limits for a deposit, together with the associated grade and tonnage, which will maximize some specified economic and/or technical criteria whist satisfying practical operational requirements.

Since the advent and widespread use of computers, open pit design has been approached by the application of many different methods and various algorithms, all with a common objective to maximize the overall mining profit within the designed pit limit.

The use of computers in optimal mine planning requires a model of the deposit usually in the form of estimated block grades and tonnages. Optimal production in such a deposit means finding the best sequence of extracting blocks from amongst all possible sequences. Such an operation applied to blocks within pre-determined, technically optimum pit limits will define, according to economic criteria, the blocks which will be removed and those which will remain. As a consequence the final, economically optimum, pit shape will also be defined. The order in which the blocks are removed is said to be the optimum mining sequence with the maximum net present value.

A large number of techniques have been applied in the search for solutions to such planning problems. The techniques consist of both rigid operational research methods and practical procedures which are heuristically based such as those described in Chapter 1.

The Lerchs-Grossmann method, which overcomes the limitations of traditional pit design, uses graph theory and always finds the optimum solution for the case when maximum total profit is the optimising criterion. The limitation of computing time and the difficulty in applying variable slope angles initially made the method less popular than the sub-optimal moving cone method, though the Whittle 4-D package has overcome some of these difficulties.

The Korobov and the corrected Korobov methods give good results. The method uses cones to define the shape of the pit. The cones can be arranged in any desired manner so that a realistic pit shape can be obtained. The optimum pit is the best combination of these cones. The great advantage of these methods is their relative simplicity and the rapid generation of solutions. Although it is not possible to provide a rigorous proof that the corrected form of the Korobov algorithm will always yield the optimal solution no counter example has yet been found.

A single pit design based on a fixed set of costs, prices and cut-off grades can often provide a misleading picture of the possible working pit and of the mineable reserves. It is always advisable to test the sensitivity of the pit design to changes in all of the variables used to calculate the revenue block model. In addition, it is also advisable to test the sensitivity of the pit design to grade and tonnage estimation errors. These types of analyses could result in the need to generate several dozen pit designs each of which could take significant computing time. The problem is that optimal open pit design algorithms do not express the solution parametrically, i.e. as a function of the parameters that were used to calculate the block model or of other design parameters such as wall slopes. A parametric solution to optimal open pit design, based on something other than simple metal content might eventually lead to a direct method of solving the problem of maximum net present value.

An alternative approach used in this thesis is to parameterize the pit design as a function of a number of variables. This algorithm, which uses grade values instead of revenue values, is based on techniques of functional analysis. The aim of this method is to transform a parametric optimization problem with severe geometric constraints into a simple one with no constraints; it does not take any economic parameters into account.

This technique could also be applied for the determination of mining sequences for the optimization of the recoverable reserves of any particular pit, where in economic terms the mining sequence is more important during the early stages and plays a major role in capital investment.

However, the objective of parameterization is to find a complete family of technically optimal pits corresponding to every possible value of the total tonnage and the selected tonnage. The only drawback is the limitation of the slope angle of the cone.

The characteristics associated with each pit design, such as metal content, total tonnage, average grade, stripping ratio and other information are provided in the form of data files that can be used for post-processing. Graphics views (contours, two-dimensional and three-dimensional) are also provided.

The method is not rigorous, and has some weaknesses on the economic side when compared to other economic or revenue based algorithms. However, parameterization appears to be at least as good as most algorithms and better than many in respect of the computing time.

The selection of the optimum pit amongst the set of pits produced by the parameterization method is obtained using a simple profit function. The application of the profit function to each block generates a block profit matrix. Pit profit is obtained by summing the profit of each block mined within that pit limit. The definition (selection) of an optimum pit is taken to be the configuration of blocks whose pit profit is a maximum.

Production scheduling is of vital importance for the operating efficiency of an open pit mine and is the last operation in optimal open pit design. A major problem in production scheduling is the determination of a mining sequence which satisfies the physical and economic constraints whilst ensuring a continuous feed to the mill in such a way as to maximize the Net Present Value.

The optimum schedule depends on both physical and economic parameters which can be considered together or separately. Considering both parameters at the same time is computationally time consuming and, for large problems as the number of states increases, the solution may be impossible to attain.

In many cases the application of a single operational research technique limits the solution to a problem, especially when a rigorous optimum is required for a problem which is subject to strict constraints. In recent years there has been a tendency to combine two or more different operational research methods to solve complicated mine planning problems. These techniques have included dynamic programming, computer simulation and interactive techniques, oriented graph simulation, simulation and linear programming.

Such combinations of operational research techniques can split the problem into two parts. In this case one part uses linear programming to find the optimum path, whilst the other part checks the feasibility of the first solution in terms of mining constraints and improves the scheduling sequence. The effect of this check is that the final linear programming solution will always be confined to blocks which become accessible during the period under consideration.

The idea of 'free ore' develops the above consideration further. By submitting to the linear programming optimization only those blocks which are immediately available for mining, the constraints on precedence and accessibility of blocks can be eliminated. Such an approach cannot lead to a rigorous overall optimum in the mathematical sense but the solution is very close to such an optimum.

Techniques such as linear programming suffer from the "curse of dimensionality" as the size of the problem increases it rapidly becomes impossible to handle the number of variables. The author has solved this problem by minimizing the numbers of variables and constraints at each stage, using a combination of linear programming with interactive simulation. The user-activated waste stripping module which is applied at the start of operations reduces the size of the problem during the execution of the software. This module increases the accessibility of ore blocks which then reduces the number of elements (constraints) inside the linear programming matrix.

Such a formulation allows the determination of an optimal mine schedule using the criterion of maximum Net Present Value. It is believed that the algorithms used will yield a true practical optimum or a solution which is very close to the true optimum (the results presented in Chapter 6 demonstrate this). At present the algorithm described is limited to a workstation implementation but it is believed that the use of decomposition methods and approximate solution algorithms for the linear programming component will ultimately yield a PC version.

Examples of the results obtained from the programs together with tables and figures representing removed blocks in each period and their discounted values are given in detail in Chapter 6. The quick and accurate results that can be generated by this computer program should lead to a much more flexible approach to optimal open pit design and scheduling and thereby improve economic performance.

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APPENDIX ONE

Comparison of the results from the XPRESS-MP package and the lp_solve

1. XPRESS-MP package solution 1

The following is the solution produced by the XPRESS-MP package. The results are for one production period and three time periods in one linear program run. The key words in the following solution are :

BS - is the basic variable,
LL - is the lower limit, and
UL - is the upper limit.
Value - status of blocks (0. not mined, 1. mined)

Problem Statistics

Matrix mtrx2
Objective OBJ
RHS RHS00001
Problem has 169 rows and 324 structural columns

Solution Statistics

Maximisation performed
Optimal solution found after 2 iterations
Objective function value is 867.420000

Columns Section

	Number	Column	At	Value	Input Cost	Reduced Cost
C	170 x	001	BS	.000000	7.690000	.000000
C	171 x	002	BS	1.000000	7.210000	.000000
C	172 x	003	BS	1.000000	28.720000	.000000
C	173 x	004	UL	.000000	6.910000	-.080000
C	174 x	005	LL	.000000	2.480000	.720000
C	175 x	006	LL	.000000	3.120000	.550000
C	176 x	007	BS	.000000	3.120000	.000000
C	177 x	008	BS	.000000	9.730000	.000000
C	178 x	009	LL	.000000	2.080000	.640000
C	179 x	010	LL	.000000	3.850000	.480000
C	180 x	011	UL	1.000000	15.900000	-.620000
C	181 x	012	BS	.000000	7.730000	.000000
C	182 x	013	UL	1.000000	28.720000	-1.780000
C	183 x	014	BS	1.000000	9.190000	.000000
C	184 x	015	LL	.000000	4.070000	.450000
C	185 x	016	LL	.000000	2.090000	.640000
C	186 x	017	BS	1.000000	15.470000	.000000
C	187 x	018	UL	1.000000	9.730000	-.050000
C	188 x	019	LL	.000000	1.380000	.700000
C	189 x	020	LL	.000000	8.590000	.050000
C	190 x	021	LL	.000000	-.660000	.890000
C	191 x	022	BS	1.000000	27.590000	.000000
C	192 x	023	LL	.000000	1.330000	.920000
C	193 x	024	BS	1.000000	9.190000	.000000
C	194 x	025	UL	1.000000	10.090000	-.090000
C	195 x	026	BS	1.000000	9.190000	.000000
C	196 x	027	LL	.000000	2.090000	.790000
C	197 x	028	BS	1.000000	9.730000	.000000
C	198 x	029	LL	.000000	6.110000	.090000
C	199 x	030	BS	.000000	9.190000	.000000
C	200 x	031	LL	.000000	3.190000	.590000
C	201 x	032	UL	1.000000	68.310000	-5.380000
C	202 x	033	LL	.000000	.480000	.780000
C	203 x	034	LL	.000000	3.120000	.610000
C	204 x	035	LL	.000000	8.500000	.060000
C	205 x	036	LL	.000000	7.210000	.170000
C	206 x	037	UL	1.000000	14.560000	-.500000
C	207 x	038	LL	.000000	6.350000	.260000
C	208 x	039	BS	1.000000	37.530000	.000000
C	209 x	040	BS	1.000000	37.530000	.000000
C	210 x	041	LL	.000000	-1.300000	1.380000
C	211 x	042	BS	.000000	7.730000	.000000
C	212 x	043	LL	.000000	8.690000	.040000
C	213 x	044	LL	.000000	2.480000	.720000

C	214	x	045	LL	.000000	3.120000	.550000
C	215	x	046	LL	.000000	3.120000	.550000
C	216	x	047	UL	1.000000	9.730000	-.050000
C	217	x	048	LL	.000000	2.270000	.760000
C	218	x	049	LL	.000000	1.520000	.880000
C	219	x	050	LL	.000000	7.690000	.130000
C	220	x	051	BS	.000000	7.210000	.000000
C	221	x	052	UL	1.000000	14.560000	-.500000
C	222	x	053	LL	.000000	6.350000	.050000
C	223	x	054	BS	1.000000	9.730000	.000000
C	224	x	055	BS	.000000	7.690000	.000000
C	225	x	056	BS	1.000000	9.190000	.000000
C	226	x	057	LL	.000000	2.270000	.630000
C	227	x	058	LL	.000000	-.600000	1.260000
C	228	x	059	LL	.000000	9.190000	.000000
C	229	x	060	LL	.000000	4.070000	.450000
C	230	x	061	LL	.000000	2.090000	.790000
C	231	x	062	UL	1.000000	15.470000	-.570000
C	232	x	063	BS	1.000000	9.730000	.000000
C	233	x	064	LL	.000000	.650000	1.040000
C	234	x	065	LL	.000000	3.600000	.520000
C	235	x	066	BS	1.000000	9.870000	.000000
C	236	x	067	BS	1.000000	9.470000	.000000
C	237	x	068	BS	1.000000	15.900000	.000000
C	238	x	069	BS	.000000	9.190000	.000000
C	239	x	070	BS	1.000000	10.090000	.000000
C	240	x	071	BS	.000000	9.190000	.000000
C	241	x	072	LL	.000000	2.090000	.640000
C	242	x	073	BS	1.000000	9.730000	.000000
C	243	x	074	LL	.000000	3.120000	.550000
C	244	x	075	LL	.000000	2.520000	.710000
C	245	x	076	LL	.000000	2.270000	.630000
C	246	x	077	BS	.000000	6.910000	.000000
C	247	x	078	UL	1.000000	15.250000	-.550000
C	248	x	079	LL	.000000	3.120000	.610000
C	249	x	080	LL	.000000	8.500000	.060000
C	250	x	081	BS	.000000	7.210000	.000000
C	251	x	082	BS	1.000000	14.560000	.000000
C	252	x	083	LL	.000000	6.350000	.050000
C	253	x	084	UL	1.000000	9.190000	.000000
C	254	x	085	LL	.000000	2.180000	.640000
C	255	x	086	LL	.000000	2.090000	.640000
C	256	x	087	UL	1.000000	15.470000	-.570000
C	257	x	088	BS	.000000	6.910000	.000000
C	258	x	089	LL	.000000	2.480000	.610000
C	259	x	090	LL	.000000	3.120000	.550000

C	260	x	091	BS	1.000000	3.120000	.000000
C	261	x	092	BS	1.000000	9.730000	.000000
C	262	x	093	BS	.000000	2.270000	.000000
C	263	x	094	LL	.000000	8.200000	.090000
C	264	x	095	BS	.000000	7.690000	.000000
C	265	x	096	BS	.000000	7.210000	.000000
C	266	x	097	UL	1.000000	28.720000	-1.780000
C	267	x	098	BS	.000000	6.910000	.000000
C	268	x	099	UL	1.000000	9.730000	-.050000
C	269	x	100	LL	.000000	7.690000	.130000
C	270	x	101	BS	1.000000	9.190000	.000000
C	271	x	102	LL	.000000	2.270000	.760000
C	272	x	103	LL	.000000	-.600000	1.260000
C	273	x	104	LL	.000000	2.270000	.630000
C	274	x	105	BS	.000000	7.690000	.000000
C	275	x	106	LL	.000000	6.910000	.200000
C	276	x	107	LL	.000000	.080000	1.140000
C	277	x	108	LL	.000000	2.180000	.780000
C	278	x	109	BS	1.000000	6.990000	.000000
C	279	x	110	BS	.000000	6.550000	.000000
C	280	x	111	LL	.000000	26.110000	1.000000
C	281	x	112	BS	1.000000	6.280000	.000000
C	282	x	113	LL	.000000	2.260000	.110000
C	283	x	114	BS	.000000	2.840000	.000000
C	284	x	115	UL	1.000000	2.840000	-.245000
C	285	x	116	BS	1.000000	8.850000	.000000
C	286	x	117	BS	.000000	1.890000	.000000
C	287	x	118	BS	.000000	3.500000	.000000
C	288	x	119	BS	.000000	14.450000	.000000
C	289	x	120	UL	1.000000	7.030000	-.130000
C	290	x	121	BS	.000000	26.110000	.000000
C	291	x	122	LL	.000000	8.360000	.000000
C	292	x	123	UL	1.000000	3.700000	-.010000
C	293	x	124	BS	.000000	1.900000	.000000
C	294	x	125	LL	.000000	14.070000	.570000
C	295	x	126	BS	.000000	8.850000	.000000
C	296	x	127	BS	.000000	1.250000	.000000
C	297	x	128	BS	1.000000	7.810000	.000000
C	298	x	129	BS	.000000	-.600000	.000000
C	299	x	130	LL	.000000	25.080000	1.680000
C	300	x	131	LL	.000000	1.210000	.210000
C	301	x	132	LL	.000000	8.360000	.000000
C	302	x	133	BS	.000000	9.170000	.000000
C	303	x	134	LL	.000000	8.360000	.000000
C	304	x	135	LL	.000000	1.900000	.150000
C	305	x	136	LL	.000000	8.850000	.050000

C	306 x	137	UL	1.000000	5.550000	-.180000
C	307 x	138	UL	1.000000	8.360000	.000000
C	308 x	139	LL	.000000	2.900000	.050000
C	309 x	140	BS	.000000	62.100000	.000000
C	310 x	141	BS	.000000	.430000	.000000
C	311 x	142	LL	.000000	2.830000	.070000
C	312 x	143	BS	1.000000	7.730000	.000000
C	313 x	144	BS	1.000000	6.550000	.000000
C	314 x	145	BS	.000000	13.230000	.000000
C	315 x	146	BS	1.000000	5.780000	.000000
C	316 x	147	LL	.000000	34.120000	2.580000
C	317 x	148	LL	.000000	34.120000	2.580000
C	318 x	149	LL	.000000	-1.180000	.430000
C	319 x	150	UL	1.000000	7.030000	-.130000
C	320 x	151	BS	1.000000	7.900000	.000000
C	321 x	152	LL	.000000	2.260000	.110000
C	322 x	153	BS	.000000	2.840000	.000000
C	323 x	154	BS	.000000	2.840000	.000000
C	324 x	155	BS	.000000	8.850000	.000000
C	325 x	156	LL	.000000	2.070000	.130000
C	326 x	157	LL	.000000	1.380000	.190000
C	327 x	158	BS	1.000000	6.990000	.000000
C	328 x	159	UL	1.000000	6.550000	-.170000
C	329 x	160	BS	.000000	13.230000	.000000
C	330 x	161	UL	1.000000	5.780000	-.210000
C	331 x	162	LL	.000000	8.850000	.050000
C	332 x	163	UL	1.000000	6.990000	-.130000
C	333 x	164	LL	. 000000	8.360000	.000000
C	334 x	165	BS	.000000	2.070000	.000000
C	335 x	166	LL	.000000	-.540000	.370000
C	336 x	167	BS	1.000000	8.360000	.000000
C	337 x	168	UL	1.000000	3.700000	-.010000
C	338 x	169	LL	.000000	1.900000	.150000
C	339 x	170	BS	.000000	14.070000	.000000
C	340 x	171	LL	.000000	8.850000	.050000
C	341 x	172	LL	.000000	.590000	.270000
C	342 x	173	LL	.000000	3.270000	.020000
C	343 x	174	LL	.000000	8.980000	.060000
C	344 x	175	LL	.000000	8.610000	.030000
C	345 x	176	LL	.000000	14.450000	.620000
C	346 x	177	BS	1.000000	8.360000	.000000
C	347 x	178	LL	.000000	9.170000	.090000
C	348 x	179	UL	1.000000	8.360000	.000000
C	349 x	180	BS	.000000	1.900000	.000000
C	350 x	181	LL	.000000	8.850000	.050000
C	351 x	182	BS	.000000	2.840000	.000000

C	352	x	183	LL	.000000	2.290000	.110000
C	353	x	184	BS	.000000	2.070000	.000000
C	354	x	185	UL	1.000000	6.280000	-.200000
C	355	x	186	BS	.000000	13.870000	.000000
C	356	x	187	LL	.000000	2.830000	.070000
C	357	x	188	BS	1.000000	7.730000	.000000
C	358	x	189	UL	1.000000	6.550000	-.170000
C	359	x	190	LL	.000000	13.230000	.500000
C	360	x	191	UL	1.000000	5.780000	-.210000
C	361	x	192	BS	.000000	8.360000	.000000
C	362	x	193	BS	.000000	1.990000	.000000
C	363	x	194	BS	.000000	1.900000	.000000
C	364	x	195	BS	.000000	14.070000	.000000
C	365	x	196	UL	1.000000	6.280000	-.200000
C	366	x	197	BS	.000000	2.260000	.000000
C	367	x	198	BS	1.000000	2.840000	.000000
C	368	x	199	LL	.000000	2.840000	.060000
C	369	x	200	LL	.000000	8.850000	.050000
C	370	x	201	BS	1.000000	2.070000	.000000
C	371	x	202	BS	1.000000	7.460000	.000000
C	372	x	203	UL	1.000000	6.990000	-.130000
C	373	x	204	BS	1.000000	6.550000	.000000
C	374	x	205	BS	.000000	26.110000	.000000
C	375	x	206	UL	1.000000	6.280000	-.200000
C	376	x	207	BS	.000000	8.850000	.000000
C	377	x	208	BS	1.000000	6.990000	.000000
C	378	x	209	LL	.000000	8.360000	.000000
C	379	x	210	LL	.000000	2.070000	.130000
C	380	x	211	LL	.000000	-.540000	.370000
C	381	x	212	BS	.000000	2.070000	.000000
C	382	x	213	UL	1.000000	6.990000	-.130000
C	383	x	214	BS	1.000000	6.280000	.000000
C	384	x	215	LL	.000000	.070000	.320000
C	385	x	216	LL	.000000	1.990000	.140000
C	386	x	217	LL	.000000	6.350000	.190000
C	387	x	218	LL	.000000	5.960000	.340000
C	388	x	219	LL	.000000	23.740000	3.050000
C	389	x	220	LL	.000000	5.710000	.120000
C	390	x	221	BS	1.000000	2.050000	.000000
C	391	x	222	BS	1.000000	2.580000	.000000
C	392	x	223	BS	.000000	2.580000	.000000
C	393	x	224	LL	.000000	8.040000	.185000
C	394	x	225	UL	1.000000	1.720000	-.150000
C	395	x	226	BS	1.000000	3.180000	.000000
C	396	x	227	LL	.000000	13.140000	.990000
C	397	x	228	LL	.000000	6.390000	.190000

C	398 x	229	LL	.000000	23.740000	2.050000
C	399 x	230	LL	.000000	7.600000	.440000
C	400 x	231	BS	.000000	3.370000	.000000
C	401 x	232	UL	1.000000	1.730000	-.150000
C	402 x	233	LL	.000000	12.790000	1.530000
C	403 x	234	LL	.000000	8.040000	.490000
C	404 x	235	UL	1.000000	1.140000	-.210000
C	405 x	236	LL	.000000	7.100000	.390000
C	406 x	237	UL	1.000000	-.550000	-.370000
C	407 x	238	LL	.000000	22.800000	3.640000
C	408 x	239	BS	1.000000	1.100000	.000000
C	409 x	240	LL	.000000	7.600000	.440000
C	410 x	241	LL	.000000	8.340000	.510000
C	411 x	242	LL	.000000	7.600000	.440000
C	412 x	243	BS	1.000000	1.730000	.000000
C	413 x	244	LL	.000000	8.040000	.540000
C	414 x	245	BS	.000000	5.050000	.000000
C	415 x	246	LL	.000000	7.600000	.440000
C	416 x	247	BS	1.000000	2.630000	.000000
C	417 x	248	LL	.000000	56.450000	5.330000
C	418 x	249	UL	1.000000	.390000	-.280000
C	419 x	250	BS	1.000000	2.580000	.000000
C	420 x	251	LL	.000000	7.030000	.380000
C	421 x	252	LL	.000000	5.960000	.270000
C	422 x	253	LL	.000000	12.030000	.880000
C	423 x	254	LL	.000000	5.250000	.210000
C	424 x	255	LL	.000000	31.010000	5.370000
C	425 x	256	LL	.000000	31.010000	5.370000
C	426 x	257	BS	1.000000	-1.070000	.000000
C	427 x	258	LL	.000000	6.390000	.190000
C	428 x	259	LL	.000000	7.180000	.400000
C	429 x	260	BS	1.000000	2.050000	.000000
C	430 x	261	UL	1.000000	2.580000	-.060000
C	431 x	262	UL	1.000000	2.580000	-.060000
C	432 x	263	LL	.000000	8.040000	.490000
C	433 x	264	BS	1.000000	1.880000	.000000
C	434 x	265	BS	1.000000	1.250000	.000000
C	435 x	266	LL	.000000	6.350000	.320000
C	436 x	267	LL	.000000	5.960000	.100000
C	437 x	268	LL	.000000	12.030000	.880000
C	438 x	269	BS	.000000	5.250000	.000000
C	439 x	270	LL	.000000	8.040000	.540000
C	440 x	271	LL	.000000	6.350000	.190000
C	441 x	272	LL	.000000	7.600000	.440000
C	442 x	273	UL	1.000000	1.880000	-.130000
C	443 x	274	BS	1.000000	-.490000	.000000

C	444	x	275	LL	.000000	7.600000	.440000
C	445	x	276	BS	.000000	3.370000	.000000
C	446	x	277	BS	1.000000	1.730000	.000000
C	447	x	278	LL	.000000	12.790000	.960000
C	448	x	279	LL	.000000	8.040000	.540000
C	449	x	280	BS	1.000000	.540000	.000000
C	450	x	281	BS	1.000000	2.970000	.000000
C	451	x	282	LL	.000000	8.160000	.560000
C	452	x	283	LL	.000000	7.830000	.490000
C	453	x	284	LL	.000000	13.140000	1.610000
C	454	x	285	LL	.000000	7.600000	.440000
C	455	x	286	LL	.000000	8.340000	.600000
C	456	x	287	LL	.000000	7.600000	.440000
C	457	x	288	UL	1.000000	1.730000	-.150000
C	458	x	289	LL	.000000	8.040000	.540000
C	459	x	290	UL	1.000000	2.580000	-.060000
C	460	x	291	BS	1.000000	2.080000	.000000
C	461	x	292	UL	1.000000	1.880000	-.130000
C	462	x	293	LL	.000000	5.710000	.050000
C	463	x	294	LL	.000000	12.610000	.940000
C	464	x	295	BS	1.000000	2.580000	.000000
C	465	x	296	LL	.000000	7.030000	.380000
C	466	x	297	LL	.000000	5.960000	.100000
C	467	x	298	LL	.000000	12.030000	1.380000
C	468	x	299	BS	.000000	5.250000	.000000
C	469	x	300	LL	.000000	7.600000	.440000
C	470	x	301	UL	1.000000	1.810000	-.140000
C	471	x	302	UL	1.000000	1.730000	-.150000
C	472	x	303	LL	.000000	12.790000	.960000
C	473	x	304	LL	.000000	5.710000	.050000
C	474	x	305	UL	1.000000	2.050000	-.110000
C	475	x	306	BS	.000000	2.580000	.000000
C	476	x	307	BS	.000000	2.580000	.000000
C	477	x	308	LL	.000000	8.040000	.540000
C	478	x	309	BS	.000000	1.880000	.000000
C	479	x	310	LL	.000000	6.780000	.360000
C	480	x	311	LL	.000000	6.350000	.190000
C	481	x	312	LL	.000000	5.960000	.270000
C	482	x	313	LL	.000000	23.740000	2.050000
C	483	x	314	LL	.000000	5.710000	.050000
C	484	x	315	LL	.000000	8.040000	.490000
C	485	x	316	LL	.000000	6.350000	.320000
C	486	x	317	LL	.000000	7.600000	.440000
C	487	x	318	BS	1.000000	1.880000	.000000
C	488	x	319	BS	1.000000	-.490000	.000000
C	489	x	320	UL	1.000000	1.880000	-.130000

C	490 x	321	LL	.000000	6.350000	.190000
C	491 x	322	LL	.000000	5.710000	.250000
C	492 x	323	BS	1.000000	.070000	.000000
C	493 x	324	BS	1.000000	1.810000	.000000

2. Lp_solve solution 2

The following is the solution produced by **Lp_solve**. The results are for one production period for three time periods in one linear program run. There is a total number of 324 variables, with 108 variables per time period.

Objective function value is : 867.42

The X decision variables (0 and 1 for blocks not mined and mined) are :

X = 0 1 1 0 0 0 0 0 0 0 1 0 1 1 0 0 1 1 0 0 0 1 0 0 1 0 0 1 0 0 0 1
0 0 0 0 1 0 1 1 0 0 0 0 0 0 1 0 0 0 0 1 0 1 0 0 0 0 1 0 0 1 1 0 0 1 1
1 1 1 1 0 1 0 0 0 0 1 0 0 0 1 0 1 0 0 1 0 0 0 1 1 0 0 0 0 1 0 1 0 1 0
0 0 0 0 0 0 1 0 0 1 0 0 1 1 0 0 0 1 0 0 1 0 0 0 0 1 0 0 0 1 0 1 0 0 1
1 0 0 0 0 1 1 0 1 0 0 0 1 1 0 0 0 0 0 0 1 1 0 1 0 1 1 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 1 0 1 0 0 0 0 1 0 1 0 0 1 1 1 1 0 1 0
1 0 0 0 0 1 1 0 0 0 0 0 0 1 1 0 0 1 1 0 0 0 0 0 1 0 0 1 0 1 0 1 0 0 0
1 0 0 0 1 0 1 1 0 0 0 0 0 0 1 0 0 1 1 1 0 1 1 0 0 0 0 0 0 0 1 1 0 0 1
0 0 1 1 0 0 0 0 0 0 1 0 1 1 1 0 0 1 0 0 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 1 1 1 0 0 1 1

The co-ordinates of the mined blocks (unit = 1) for each time period are shown in table 4 Chapter 6.