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Some recent developments in the determination of the Atterberg Limits

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ABSTRACT: The Atterberg Limits are the most common tests specified by practicing geotechnical and pavement engineers the world over. They are used to classify soils using the framework pioneered by Terzaghi and Casagrande during their work with the US Public Roads Bureau in the 1920s and 1930s and are also correlated with many fundamental soil parameters, used in design and construction projects. In the 21st century the Atterberg Limits remain a key component of the testing armory of practicing geotechnical engineers as they can be obtained easily for the large numbers of samples that are needed on major construction projects and allow for rapid assessments of key soil parameters. Their fundamental definitions are worthy of review; the thread-rolling test for plastic limit has remained largely unchanged since Atterberg first described it in 1911 but the definition and measurement of liquid limit varies across the globe. The fundamental mechanics of the Casagrande Cup liquid limit have been the subject of recent study showing clearly that liquid limit determined in this way relates to a fixed value of specific soil strength (i.e. strength per unit density) as opposed to a fixed strength value when liquid limit is measured by the fall-cone method. These findings explain the deviation between liquid limits measured by the two methods for high plasticity soils without the need to invoke different strength regimes. The brittle failure mechanism in the thread-rolling test has also been recently re-examined. It is proposed that the brittle failure observed in the plastic limit test is caused by either air entry or cavitation in the clay and plastic limit and does not correspond to a fixed strength. The Atterberg Limits are used to compute liquidity index which is widely related to clay strength variation, this is critical for many areas of construction (especially when rapid assessments of strength are required). The Russian code for the design of piled foundations, for example, uses liquidity index values to assess shaft friction. Recent research outcomes at the University of Cambridge have challenged certain assumptions pertaining to widely-used correlations between liquidity index and undrained strength.

INTRODUCTION

The Atterberg Limits remain the fundamental classification parameters in geotechnical engineering practice. Atterberg (1911a, b) described limits that describe changes in the behaviour fine grained soils with water content. As the most obvious behaviour of clays was their plasticity, he classified soil's behaviour according to its plastic characteristics, developing 7 qualitative "limits" listed in Table 1.

Table 1. Qualitative limits describing soil behaviour from Atterberg (1911a)

Atterberg (1911a) ***Swedish***	**Casagrande (1932)** ***English***
öfre trögflytbarhetsgränsen	The upper limit of viscous flow
vattentäthetsgränsen	
nedre trögflytbarhetsgränsen	The lower limit of viscous flow
klibbgränsen	The sticky limit
utrullgränsen	Rolling limit
gränsen för sammanpackbarhet	The cohesion limit
krympningsgränsen	The shrinkage limit

Atterberg Limits for Geotechnical Correlations

The measured values for the liquid and plastic limits of soils are commonly used index parameters in geotechnical engineering. They are used to compute the plasticity index which is commonly used to empirically predict many soil properties. For instance, Casagrande's A-Line classifies soils into clays and silts based on a correlation between soil type and a combination of liquid limit and plasticity index (Casagrande, 1947). The plasticity index has been used to predict undrained strength ratios (Skempton, 1954 & 1957); the ratio of strength to Standard Penetration Test (SPT) blowcount (Stroud, 1974) and critical state soil parameters (Schofield & Wroth, 1968 & Nakase et al 1988). Black (1962) used the plasticity characteristics of clays to develop predictions for the California Bearing Ratio (CBR) – another famous soil index parameter to describe road subgrade quality which has been traditionally related to soil modulus for use in road pavement design (e.g. Heukelom & Klomp, 1962).

This paper describes some recent research developments from work done at the University of Cambridge and challenges certain assumptions pertaining to the common interpretation and understanding of the Atterberg Limits.

CASAGRANDE CUP OR FALL CONE?

Casagrande's Liquid Limit

Atterberg (1911a, b) proposed a method for measuring the liquid limit of soils based on the stability of a groove in a clay bed when the soil container was struck on the hand. This method was standardized by Casagrande (1932) into the percussion technique. The method relies on the inducement of a slope failure as the cup is 'tapped' – the water content when the 'canal' fails after 25 blows is the liquid limit. Haigh (2012) performed a Newmarkian sliding block analysis (Newmark, 1965) of the test to show that this ratio is approximately $1 m^2/s^2$. As soil density decreases with increasing water content, a soil with a high liquid limit (w_L) will exhibit a lower strength at liquid limit than those soils with lower liquid limits. This confirmed the assertion of Wroth (1979) that percussion liquid limits should correspond to a fixed ratio of strength to density.

The Fall Cone Test

The fall cone test, developed in Sweden, (e.g. Hansbo, 1957) is a direct measurement of soil shear strength. Shimobe (2010) reviews the various fall-cone standards from different countries where it has been used to determine liquid limit. It is a mechanical test which removes the judgment that is required to determine failure when using the Casagrande cup but has been calibrated to give essentially the same results for soils with low plasticity indices. The soil strength at liquid limit (c_L) is reported in the literature to fall in the range 0.7 to 2.65 kPa and was taken to be at the centre of this range, (1.7 kPa), by Wroth & Wood (1978). Koumoto & Houlsby (2001) give a detailed theoretical analysis of the mechanics of the fall cone test, showing the tests's sensitivity to cone angle, cone bluntness, surface roughness of the cone and cone heave.

STRENGTH VARIATION

Fall Cone Strength

The fall cone test can equally be used to measure the shear strength of soils at a range of water contents between liquid and plastic limits. Hansbo (1957) presented equation (1) which is used to derive undrained shear strength values from the fall cone test:

$$c_u = K \frac{mg}{d^2} \tag{1}$$

Where, c_u = undrained shear strength; m = mass of the fall cone; g = acceleration due to gravity; d = final fall cone penetration and K is the cone factor. At low moisture contents, the high strength of soils can cause problems due to low penetration with the standard cone. This problem has been addressed using pseudo-static cones that are mechanically driven into the soil (e.g. Stone & Phan, 1995).

Strength at Plastic Limit

As the thread-rolling test is widely perceived as a crude test which lacks stress control, many researchers have looked to find plastic limit (w_P) by other means (e.g. Feng, 2000; Dolinar & Trauner, 2005; Lee & Freeman, 2009). Almost all of these researchers have assumed that the plastic limit is associated with a fixed strength (100 times that at w_L) and then used a strength test (e.g. fall-cone) to find the water content associated with that strength. Haigh et al (2013) showed based on strengths at plastic limit reported in the literature that the assumption of a fixed strength at plastic limit is without any technical basis. They proposed instead that the brittle failure observed in the plastic limit (thread rolling test) is caused by either air entry or cavitation and is hence a function of maximum soil suction rather than soil strength.

Database of strength data at plastic limit

Haigh et al (2013) use a large database to investigate the variation of shear strengths for soils at the plastic limit. Table 2 shows the sources of the data used. The median strength (at plastic limit) for the database was found to be 152kPa and the mean strength was found to be 132 kPa. The computed standard deviation was found to be 89kPa. The strength at the thread-rolling limit is therefore clearly variable. The cumulative distribution of strength at plastic limit is shown on Figure 1. There will obviously be a water content at which the soil strength is 100 times that at the liquid limit which can be termed the plastic strength limit, PL_{100}. This may be an equally useful parameter as the thread-rolling plastic limit, but there is no reason why these states should coincide.

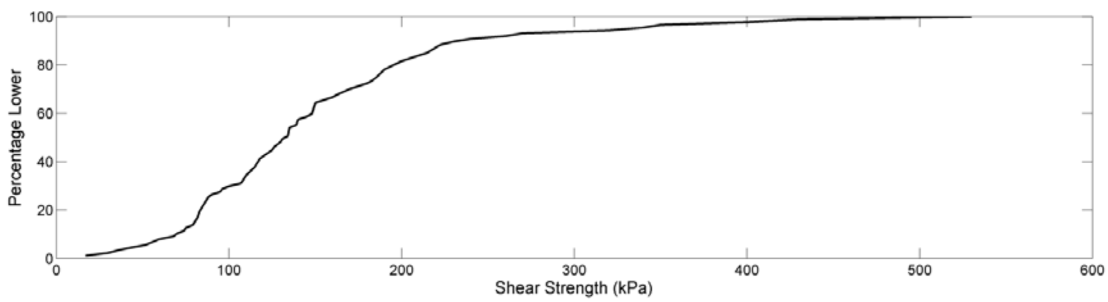

FIG. 1. Cumulative distribution of shear strengths of soil at plastic limit [Plot from Haigh et al (2013)]

Table 2. Database of shear strength data at the plastic limit (after Haigh et al 2013)

Authors	**Soils Tested**	**Method of Testing**	**c_u at w_P (kPa)**
Skempton & Northey (1952)	3 British Clays		85-125
Dennehy (1978)	19 British Clays	Undrained Triaxial tests	30-220
Arrowsmith (1978)	5 Boulder clays	UC tests	17-224
Whyte (1982)		Extrusion	79-110
Wasti & Bezirci (1986)	14 Turkish soils + 10 bentonite mixtures	Vane shear	36-430
Sharma & Bora (2003)	5 Indian Clays	Unconfined Compression & Fall-cone tests	138-240
Kayabali & Tufenkci (2010)	15 inorganic Turkish soils	Shear Vane tests	68-530

Strength Values at Plastic Limit predicted by Critical State Soil Mechanics (CSSM)

During the rolling test, the soil is continually remoulded, and hence its stress state lies on the critical state line. It can be shown that:

$$p' = \frac{\sigma_v}{3} - u \qquad (2)$$

$$q = \sigma_v = Mp' \qquad (3)$$

Invoking Tresca's criterion for yield it can be shown that:

$$c_u = \left(\frac{3M}{6 - 2M} \right)(-u) \qquad (4)$$

Where, p' = mean effective stress; q = deviatoric stress; u = pore pressure; M = slope of the critical state line; σ_v = total vertical stress and c_u = undrained shear strength. Haigh et al (2013) using the range of cavitation tensions suggested by Baker & Frydman (2009) and equation (4) determined the expected range of undrained strengths at plastic limit to be 65kPa to 400kPa (assuming M varies from 0.9 to 1.2), agreeing well with the 2nd and 98th percentiles of the plastic limit strength database shown in Figure 1.

Origins of the 100-fold strength increase assumption

Schofield & Wroth (1968) stated that:

> "experimental results with four different clays give similar variation of strength with liquidity index … From these data it appears that the liquid limit and plastic limit do correspond approximately to fixed strengths which are in the proposed ratio of 1:100"

The data of the four clays in question originated from Skempton & Northey (1952) (Figure 2). Wroth & Wood (1978) then postulated that:

> "From the evidence of Skempton & Northey (1952) … It will be assumed that the shear strength at the plastic limit is 100 times that at the liquid limit. A best estimate of 170 kN/m^2 will be adopted."

This assumption led to the well-known formulation for undrained strength predicted using liquidity index (I_L) from Wood (1990):

$$c_u = 170e^{-4.6I_L} = 1.7 \times 10^{2(1-I_L)} \quad \text{kPa} \qquad (5)$$

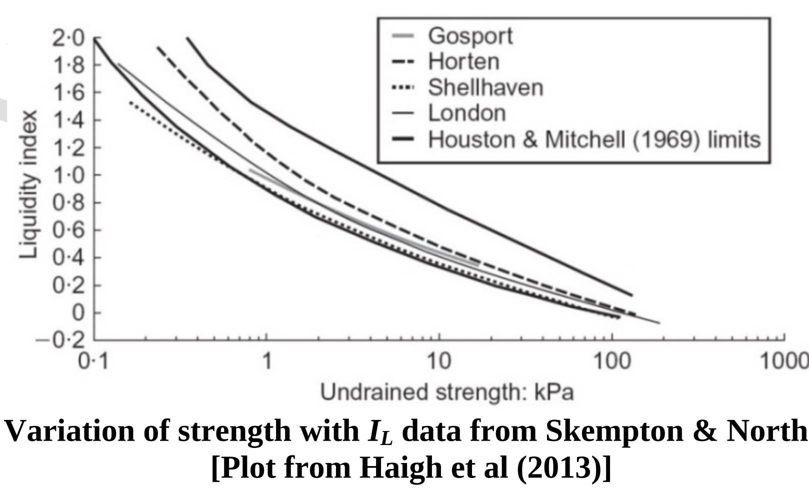

FIG. 2. Variation of strength with I_L data from Skempton & Northey (1952) [Plot from Haigh et al (2013)]

Database of fall-cone strength versus water content

A database of 641 fall cone tests on 101 soils from 12 countries has been used by Vardanega & Haigh (2013) to demonstrate that the strength variation between liquid and plastic limits should not be assumed to be log-linear with a ratio of strengths between plastic and liquid limits of close to 100. Retaining the log-linear variation, the best fit to the data is achieved for liquidity indices between 0.2 and 1.1 using a factor 35. A modified equation of the semi-logarithmic form suggested by Wroth and Wood (1978) is hence:

$$c_u = c_L 35^{(1-I_L)} \quad \text{kPa} \qquad 0.2 < I_L < 1.1 \quad \text{where, } c_L = 1.7\text{kPa} \qquad (6)$$

In the same paper a power model (e.g. Feng, 2000 and Koumoto & Houlsby, 2001) is also shown to be an acceptable way to describe fall cone data. If modeled in this way, a higher value of the slope increase is computed owing to the high curvature of the function close to the plastic limit.

GEOTECHNICAL DESIGN & CONSTRUCTION IMPLICATIONS

Vardanega et al (2012) recently reviewed pile design codes of practice, in particular the Russian Code for Pile Design (SNiP 2.02.03-85). This code makes use of liquidity index to obtain values for shaft and base resistance and is often checked by western practitioners with relations similar to equation (5). The recent findings that the slope implied by the 100-fold increase should really be closer to 35 has significance for practitioners designing piles or any other geotechnical construction using water content and the Atterberg limits. For those practitioners attempting to determine soil strength the thread rolling test offers few insights but as it does describe the brittle transition point it remains valuable for those designing landfill liners and road pavements in arid areas.

CONCLUSIONS

The Atterberg Limits remain fundamental to the classification of soils and are an important component of the basic testing armory of geotechnical engineers. The following recent research findings are summarized:

(a) While liquid limit measured by the fall-cone corresponds to a fixed strength the fundamental mechanics of the percussion cup method have shown that liquid limit measured in this way relates to a specific soil strength.

(b) The idea of the plastic limit corresponding to a fixed value of strength is a fallacy as is the assumption of a 100-fold strength increase in the plastic range.

(c) Brittle failure in the plastic limit test has been suggested as being caused either by air-entry or cavitation of the pore fluid. CSSM has been used to show that the range of strengths predicted by invoking the cavitation hypothesis is reasonable.

(d) The 100-fold strength increase is a fallacy and the idea of a fixed strength at plastic limit is false. If predictions of strength based on Atterberg limits are desired, however, then a strength ratio of 35 more realistically predicts observed data.

REFERENCES

Arrowsmith, E. J. (1978). “Roadwork fills: a materials engineer’s viewpoint.” *Proc. Conf. on Clay Fills*, London, pp. 25-36.

Atterberg, A. (1911a) Lerornas forhållande till vatten, deras plasticitetsgränser och plasticitetsgrader. *Kungliga Lantbruksakademiens Handlingar och Tidskrift*, Vol. 50 (2): 132-158. (In Swedish)

Atterberg, A. (1911b) Die Plastizität der Tone. *Internationale Mitteilungen der Bodenkunde*, Vol. 1: 4-37 (In German).

Baker, R. and Frydman, S. (2009). “Unsaturated soil mechanics: critical review of physical foundations.” *Engrg Geology*, Vol. 106 (1): 26-33.

Black, W. P. M. (1962). “A method of estimating the California Bearing Ratio of cohesive soils from plasticity data.” *Géotechnique*, Vol. 12 (4): 271-282.

Casagrande, A. (1932). “Research on the Atterberg Limits of Soils.” *Public Roads*, Vol. 13 (8): 121-136.

Casagrande, A. (1947). “Classification and identification of soils.” *Proc. of the American Society of Civil Engrg*, Vol. 73 (6): 783-810.

Dennehy, J. P. (1978). “The remoulded undrained shear strength of cohesive soils and its influence on the stability of embankment fill.” *Proc. Conf. on Clay Fills*, London, pp. 87-94.

Dolinar, B. and Trauner, L. (2005). “Impact of soil composition on fall cone test results.” *J. of Geotechnical and Geoenvironmental Engrg*, Vol. 131 (1): 126-130.

Feng, T. (2000). “Fall-cone penetration and water content relationship in clays.” *Géotechnique*, Vol. 50 (2): 181-187.

Haigh, S. K. (2012). “Mechanics of the Casagrande liquid limit test.” *Canadian Geotech. J.*, Vol. 49 (9): 1015-1023. Corrigenda, Vol. 49 (9): 1116 and Vol. 49 (11): 1329.

Haigh, S. K., Vardanega, P. J. and Bolton, M. D. (2013). “The plastic limit of clays.” *Géotechnique*, Vol. 63 (6): 435-440.

Hansbo, S. (1957). “A new approach to the determination of the shear strength of clay by the fall cone test.” *Swedish Geotechnical Institute Proceedings*, Vol. 14: 5-47.

Heukelom, W. and Klomp, A. J. G. (1962). “Dynamic testing as a means of controlling pavement during and after construction.” *Proceedings of the 1st International Conference on the Structural Design of Asphalt Pavements*, University of Michigan, Ann Arbor, MI.

Houston, W. N. and Mitchell, J. K. (1969). “Property interrelationships in sensitive clays.” *J. of the Soil Mech. and Found. Division*, Vol. 95 (SM4): 1037-1062.

Kayabali, K. and Tufenkci, O. O. (2010). “Shear strength of remoulded soils at consistency limits.” *Canadian Geotech. J.*, Vol. 47 (3): 259-266.

Koumoto, T. and Houlsby, G. T. (2001). “Theory and practice of the fall cone test.” *Géotechnique*, Vol. 51 (8): 701-712.

Lee, L. T. and Freeman, R. B. (2009). “Dual-weight fall cone method for simultaneous liquid and plastic limit determination.” *J. of Geotechnical and Geoenvironmental*

Engrg, Vol. 135 (1): 158-161.

Nakase, A., Kamei, T. and Kusakabe, O. (1988). "Constitutive parameters estimated by plasticity index." *J. of Geotechnical Engrg*, Vol. 114 (7): 844-858.

Newmark, N. M. (1965). "Effects of earthquakes on dams and embankments." *Géotechnique*, Vol. 15 (2): 139-160.

Schofield, A. N. and Wroth, C. P. (1968). "Critical State Soil Mechanics". London, UK: McGraw-Hill.

Sharma, B. and Bora, P. K. (2003). "Plastic limit, liquid limit and undrained shear strength of soil: a reappraisal." *J. of Geotechnical and Geoenvironmental Engrg*, Vol. 129 (8): 774-777.

Shimobe, S. (2010). "Determination of index properties and undrained shear strength of soils using the fall cone test." In Proceedings: *7th International Symposium on Lowland Technology*, September 16-18, Saga, Japan.

Skempton, A. W. (1954). "Discussion: Sensitivity of clays and c/p ratio in normally consolidated clays." *Proceedings of the American Society of Civil Engineers*, Separate 478: 19-22.

Skempton, A. W. (1957). "Discussion: Further data on the c/p ratio in normally consolidated clays." *Proceedings of the Institution of Civil Engineers*, Vol. 7: 305-307.

Skempton, A. W. and Northey, R. D. (1952). "The sensitivity of clays." *Géotechnique*, Vol. 3 (1): 30-53.

Stone, K. J. L. and Phan, K. D. (1995). "Cone penetration tests near the plastic limit." *Géotechnique*, Vol. 45 (1): 155-158.

Stroud, M. A. (1974). "The Standard Penetration Test in sensitive clays and rocks." *The European Seminar on Penetration Testing*, Stockholm, Vol. 2 (2): 366-375.

Vardanega, P. J., Kolody, E., Pennington, S., Morrison, P. and Simpson, B. (2012). Bored pile design in stiff clay I: codes of practice. *Proceedings of the Institution of Civil Engineers – Geotechnical Engineering*, Vol. 165 (GE4): 213-232.

Vardanega, P. J. and Haigh, S. K. (2013). "The undrained strength-liquidity index relationship." *under review*.

Wasti, Y. and Bezirci, M. H. (1986). "Determination of the consistency limits of soils by the fall-cone test. *Canadian Geotech. J.*, Vol. 23 (2): 241-246.

Whyte, I. L. (1982). "Soil plasticity and strength: a new approach using extrusion." *Ground Engrg*, Vol. 15 (1): 16-24.

Wood, D. M. (1990). "*Soil behaviour and critical state soil mechanics*." Cambridge University Press: United Kingdom.

Wroth, C. P. (1979). "Correlations of some engineering properties of soils." *Proc. 2nd International Conference on the Behaviour of Offshore Structures*, Vol. 1 pp. 121-132.

Wroth, C. P. and Wood, D. M. (1978). "The correlation of index properties with some basic engineering properties of soils." *Canadian Geotech. J.*, Vol. 15 (2): 137-145.