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A Possible Worlds Interpretation of Label Semantics

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Abstract

The label semantics linguistic representation framework is introduced as an alternative approach to computing and modelling with words, based on the concept of appropriateness rather than graded truth. A possible worlds interpretation of label semantics is then proposed. In this model it is shown that certain assumptions about the underlying possible worlds can result in appropriateness measures that correspond to well known t-norms and t-conorms for conjunctions and disjunctions of linguistic labels.

Keywords: Computing with words, label semantics, appropriateness measures, possible worlds.

1 Introduction

The principle aim of the *computing with words* paradigm as proposed by Zadeh [18] is to increase the use of natural language for information and knowledge processing in computer systems. In practice this will require the development of a formal representation framework based on some restricted subset of natural language. Zadeh has suggested a form of *precisiated natural language* [19] based on the theory of generalised constraints and linguistic variables. *Label semantics*, introduced by Lawry [11], [12], provides an alternative representation for computing and modelling

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with words, which takes a somewhat different perspective than Zadeh on the processing of natural language information.

Zadeh's approach is based fundamentally on the notion of a linguistic variables [17] where a semantic rule links natural language labels to an underlying graded vague concept as represented by a fuzzy set on the domain of discourse. Label semantics on the other hand, encodes the meaning of linguistic labels according to how they are used by a population of communicating agents to convey information. From this perspective it is important to consider the decision process an intelligent agent must go through in order to identify which labels or expressions can actually be used to describe an object or instance.

It cannot be denied that in their use of linguistic labels humans posses a mechanism for deciding whether or not to make assertions (e.g. Bill is *tall*) or to agree to a classification (e.g. Yes, that is a tree). Further, although the concepts concerned are vague this underlying decision process is fundamentally crisp (bivalent). For instance, you are either willing to assert that x is a *tree* given your current knowledge, or you are not. In other words, either tree is an appropriate label to describe x or it is not. As humans we are continually faced with making such crisp decisions regarding vague concepts as part of our every day use of language. Of course, we may be uncertain about labels and even express these doubts (e.g. I'm not sure whether you would call that a tree or a bush, or both) but the underlying decision is crisp.

Given this decision problem, we suggest that it is useful for agents to adopt what might be called an *epistemic stance*, whereby they assume the existence of a set of labelling conventions for the population governing what linguistic labels and expression can be appropriately used to describe particular instances. Of course, such linguistic conventions do not need to be imposed by some outside authority like the Oxford English Dictionary or the Academia Lengua Espanola, but instead would emerge as a result of interactions between agents each adopting the epistemic stance. Hence, label semantics does not attempt to link label symbols to fuzzy set concept extensions but rather to quantify an agent's subjective belief that a label L is appropriate to describe an object x and hence whether or not it is reasonable to assert that 'x is L'. In this respect it is close to the 'antirepresentational' view of vague concepts proposed by Rohit Parikh [15] which focuses on the notion of assertibility rather than that of truth; a view that is also shared by Alice Kyburg [8].

Label semantics proposes two fundamental and inter-related measures of the appropriateness of labels as descriptions of an instance. Given a finite set of labels LA from which can be generated a set of expressions LE through recursive applications of logical connectives, the measure of appropriateness of an expression $\theta \in LE$ as a description of instance x is denoted by $\mu_{\theta}(x)$ and quantifies the agent's subjective belief that θ can be used to describe x based on its (partial) knowledge of the current labelling conventions of the population. From an alternative perspective, when faced with an object to describe, an agent may consider each label in LA and attempt to identify the subset of labels that are appropriate to use. Let this set be denoted by \mathcal{D}_x . In the face of their uncertainty regarding labelling conventions the agent will also be uncertain as to the composition of \mathcal{D}_x , and in label semantics this is quantified by a probability mass function $m_x: 2^{LA} \to [0,1]$ on subsets of labels. The relationship between these two measures will be described in the following section.

This paper will propose an interpretation of the label semantics framework, where an agent evaluates $\mu_{\theta}(x)$ and m_x by considering different labelling scenarios, these being identified as possible worlds, and estimating their respective likelihoods. The latter would be based on the agent's past experience of labelling behaviour across the population. As a simplification it will be assumed that each possible world provides complete information as to which labels are and are not appropriate to describe every element in the universe. Clearly, this is unlikely to be the case in practice, not least because the underlying universe may be infinite, but as an idealisation it provides a useful mechanism by which to study those calculi for the measures $\mu_{\theta}(x)$ and m_x consistent with a possible worlds model.

In probability theory the notion of possible world was used by Carnap [2] to explore the relationship between probability and logic. In fuzzy set theory a number of possible worlds semantics have been proposed. In particular, Gebhardt and Kruse [4] proposed the context model for integrating vagueness and uncertainty. In this approach possible worlds correspond to different contexts across which fuzzy concepts have different (crisp) extensions. For example, Huynh etal. [7] suggest that in the case where LA ={very short, short, medium,...,

tall, very tall} the contexts (i.e. possible worlds) might correspond to nationalities such as Japanese, American, Swede, etc. Another possible worlds model for fuzzy sets is the voting model, proposed by Gaines [3] and later extended by Baldwin [1] and Lawry [9]. Here possible worlds correspond to individual voters each of which is asked to identify the crisp extension of the concept under consideration. Alternatively, each voter is presented with each instance, one at a time, and asked to answer yes or no as to whether the instance satisfies the concept.

A somewhat different approach is adopted by Ruspini in [16] who defines a similarity measure between possible worlds and then defines an implication and consistency function in terms of this measure. These functions are then used to provide a foundation for reasoning with possibility and necessity measures, including Zadeh's generalized modus ponens law.

2 The Label Semantics Framework

Unlike linguistic variables, which allow for the generation of new label symbols using a syntactic rule [17], label semantics assumes a fixed finite set of labels LA. These are the basic or core labels to describe elements in a underlying domain of discourse Ω . Detailed arguments in favour of this assumption are given in [10] [11]. Based on LA, the set of label expression LE is then generated by recursive application of the standard logic connectives as follows:

Definition 1. Label Expressions

The set of label expressions of LA, LE, is defined recursively as follows:

- (i) $L \in LE$: $L \in LA$
- (ii) If $\theta, \varphi \in LE$ then $\neg \theta, \theta \land \varphi, \theta \lor \varphi, \theta \to \varphi \in LE$

A mass assignment m_x on sets of labels then quantifies the agent's belief that any particular subset of labels contains all and only the labels with which it is appropriate to describe x.

Definition 2. Mass Assignment on Labels $\forall x \in \Omega \ a \ mass \ assignment \ on \ labels \ is$ *a function* $m_x : 2^{LA} \to [0,1]$ such that $\sum_{S \subseteq LA} m_x(S) = 1$

The appropriateness measure, $\mu_{\theta}(x)$, and the mass m_x are then related to each other on the basis that asserting θ provides direct constraints on \mathcal{D}_x . For example, asserting 'x is $L_1 \wedge L_2$ ', for labels $L_1, L_2 \in LA$ is taken as conveying the information that both L_1 and L_2 are appropriate to describe x so that $\{L_1, L_2\} \subseteq \mathcal{D}_x$. Similarly, 'x is $\neg L$ ' implies that L is not appropriate to describe x so $L \notin \mathcal{D}_x$. In general we can recursively define a mapping $\lambda : LE \rightarrow 2^{2^{LA}}$ from expressions to sets of subsets of labels, such that the assertion 'x is θ ' directly implies the constraint $\mathcal{D}_x \in \lambda(\theta)$ and where $\lambda(\theta)$ is dependent on the logical structure of θ . For example, if $LA = \{low, medium, high\}$ then $\lambda(medium \land \neg high) = \{\{low, medium\}, \{medium\}\}$ corresponding to those sets of labels which include medium but do not include high. Hence, the description \mathcal{D}_x provides an alternative to Zadeh's linguistic variables in which the imprecise constraint 'x is θ ' on x, is represented by the precise constraint $\mathcal{D}_x \in \lambda(\theta)$, on \mathcal{D}_x .

Definition 3. λ -mapping

 $\lambda: LE \to 2^{2^{LA}}$ is defined recursively as follows: $\forall \theta, \ \varphi \in LE$

- $\forall L_i \in LA \ \lambda(L_i) = \{T \subseteq LA : L_i \in T\}$
- $\lambda(\theta \land \varphi) = \lambda(\theta) \cap \lambda(\varphi)$
- $\lambda(\theta \lor \varphi) = \lambda(\theta) \cup \lambda(\varphi)$
- $\lambda(\neg \theta) = \lambda(\theta)^c$
- $\lambda(\theta \to \varphi) = \lambda(\neg \theta) \cup \lambda(\varphi)$

Based on the λ mapping we then define $\mu_{\theta}(x)$ as the sum of m_x over those set of labels in $\lambda(\theta)$.

Definition 4. Appropriateness Measure

$$\forall \theta \in LE, \ \forall x \in \Omega \ \mu_{\theta}(x) = \sum_{S \in \lambda(\theta)} m_x(S)$$

Note that in label semantics there is no requirement for the mass associated with the empty set to be zero. Instead, $m_x(\emptyset)$ quantifies the agent's belief that none of the labels are appropriate to describe x. We might observe that this phenomena occurs frequently in natural language, especially when labelling perceptions generated along some continuum. For example, we occasionally encounter colours for which none of our available colour descriptors seem appropriate. Hence, the value $m_x(\emptyset)$ is an indicator of the describability of x in terms of the labels LA. **Example 5.** If $LA = \{small, medium, large\}$ then

 $\lambda (small \wedge medium) =$

 $\left\{\left\{small,medium\right\},\left\{small,medium,large\right\}\right\}$

hence, $\mu_{small \land medium}(x) =$

 $m_x\left(\{small, medium\}\right) +$

 $m_x(\{small, medium, large\})$

Also,

 $\lambda(small \rightarrow medium) = \{\{small, medium\}, \\\{small, medium, large\}, \{medium, large\}, \\\{medium\}, \{large\}, \emptyset\} hence, \\\mu_{small \rightarrow medium} (x) = m_x (\{small, medium\}) + \\m_x (\{small, medium, large\}) + \\m_x (\{small, medium, large\}$

 $m_{x} \left(\{medium, large\} \right) + m_{x} \left(\{medium\} \right)$ $+ m_{x} \left(\{large\} \right) + m_{x} \left(\emptyset \right)$

3 The Possible Worlds Model

In this section we introduce a possible world interpretation of label semantics and then investigate the relationship between different assumptions within this model and a number of standard t-norms and t-conorms for combining conjunctions and disjunctions of labels. Notice, as is typically the case in label semantics (see Lawry [12]), in such cases t-norms and t-conorms can only be applied at the label level (i.e. to elements of LA) and not arbitarily to label expressions from LE.

Let \mathcal{W} be a finite set of possible worlds each of which identifies a set of valuations on LA, one for each value of $x \in \Omega$. More formally, for every x in Ω there is a valuation function $V_x : \mathcal{W} \times LA \to \{0,1\}$ such that $\forall w \in \mathcal{W}$ and $\forall L \in LA \ V_x(w,L) = 1$ means that, in world w, L is an appropriate label with which to describe x. Alternatively, $V_x(w, L) = 0$ means that, in world w, L is not an appropriate label with which to describe x. The valuation V_x can then be extended to a valuation $V_x: \mathcal{W} \times LE \to \{0,1\}$ in the normal recursive manner. Let $P: \mathcal{W} \to [0,1]$ be a probability measure on \mathcal{W} , where for $w \in \mathcal{W}$ P(w) is the probability that w is the possible world corresponding to reality. Also we assume w.l.o.g that $P(w) > 0 \ \forall w \in \mathcal{W}$. Then

in this model we can interpret appropriateness measures and mass assignments as follows:

$$\forall \theta \in LE$$

$$\mu_{\theta} (x) = P \left(\{ w \in \mathcal{W} : V_x (w, \theta) = 1 \} \right)$$

and

$$\forall T \subseteq LA \ m_x (T) = P \left(\{ w \in \mathcal{W} : \mathcal{D}_x^w = T \} \right)$$

= $P \left(\{ w \in W : V_x (w, \alpha_T) = 1 \} \right) = \mu_{\alpha_T} (x)$
where $\forall x \in \Omega, \forall w \in \mathcal{W}$
 $\mathcal{D}_x^w = \{ L \in LA : V_x (w, L) = 1 \}$
and where $\forall T \subseteq LA$

$$\alpha_T = \left(\bigwedge_{L_i \in T} L_i\right) \land \left(\bigwedge_{L_i \notin T} \neg L_i\right)$$

Notice that as w varies then the set of appropriate labels corresponds to a random set $\mathcal{D}_x: \mathcal{W} \to 2^{LA}$ such that $\mathcal{D}_x(w) = \mathcal{D}_x^w$. This relates label semantics to the random set interpretation of fuzzy membership function as studied by Goodman [5], [6] and Nguyen [13], [14]. The fundamental difference is that the latter defined random sets of the universe Ω whereas the former defines random sets of labels. This has a major impact on the resulting calculus allowing label semantics to be functional (although not truth-functional). See Lawry [11], [12] for a discussion of the functionality of label semantics. A practical advantage of this approach is that while the universe Ω is often infinite the set of labels LA remains finite, and hence the underlying mathematics is considerably simplified.

Definition 6. Ordering on Worlds

For every $x \in \Omega$ the valuation V_x generates a natural ordering on W according to:

$$\forall w_i, w_j \in \mathcal{W} \ w_i \preceq_x w_j \ iff$$

$$\forall L \in LA \ V_x \ (w_i, L) = 1 \Rightarrow V_x \ (w_j, L) = 1$$

Theorem 7. If $\forall x \in \Omega \preceq_x is a total (linear) ordering on <math>W$ then $\forall S \subseteq LA$:

$$\mu_{\bigwedge_{L\in S} L}(x) = \min \left\{ \mu_L(x) : L \in S \right\} \text{ and}$$
$$\mu_{\bigvee_{L\in S} L}(x) = \max \left\{ \mu_L(x) : L \in S \right\}$$

Corollary 8. If $\forall x \in \Omega$ V_x satisfies the conditions given in theorem 7 then:

$$\forall L, L' \in LA \ L \neq L'$$
$$\mu_{L \to L'}(x) = \min(1, 1 - \mu_L(x) + \mu_{L'}(x))$$

Interestingly, the implication operator in Corollary 8 corresponds to Lukasiewicz implication which, in fuzzy logic, is normally associated with the bounded sum t-conorm. More specifically, Lukasiewicz implication is the so called S-implication generated from S(1-a, b)where S is bounded sum. In fuzzy logic, it is not typical for Lukasiewicz implication to be associated with min and max as above.

A total ordering on words as assumed in theorem 7 would naturally occur in those situations where for each $x \in \Omega$ there is a total ordering on the labels LA with regards to their appropriateness, and where this ordering is invariant across all possible worlds. (i.e. for every x there is no doubt regarding the ordering of the labels in terms of their appropriateness). In this case the valuation V_x would be consistent with this ordering on labels in that if $V_x(w,L) = 1$ then $V_x(w,L') = 1$ for any L'which is at least as appropriate as L. Hence, the only variation in the possible worlds will be in terms of the generality of D_x^w ; this then naturally generating a total ordering on \mathcal{W} . See [11] for more details.

Theorem 9. If $\forall x \in \Omega$ and $\forall w \in W$ it holds that:

$$\forall L_i \in LA$$

$$V_x(w, L_i) = 1 \Rightarrow V_x(w, L_j) = 0 \ \forall L_j \neq L_i$$

then $\forall S \subseteq LA$

$$\mu_{\bigwedge_{L\in S}L}(x) = \max\left(0, \sum_{L\in S}\mu_L(x) - (|S| - 1)\right)$$

and $\mu_{\bigvee_{L\in S}L}(x) = \min\left(1, \sum_{L\in S}\mu_L(x)\right)$

According to the conditions on \mathcal{W} imposed in theorem 9 all possible worlds are very conservative when identifying labels as appropriate descriptions of an instance $x \in \Omega$. In each world at most one label is identified as being appropriate with which to describe x. Notice that in the case that all worlds select one label so that $\forall w \in \mathcal{W} \mathcal{D}_x^w \neq \emptyset$ then $\mu_L(x) : L \in LA$ forms a probability distribution on LA.

Theorem 10. If $\forall x \in \Omega$ and $\forall w \in W$ it holds that:

$$\forall L_i \in LA \ V_x (w, \neg L_i) = 1 \Rightarrow V_x (w, \neg L_j) = 0$$

$$\forall L_j \in LA : \ L_j \neq L_i$$

then $\forall S \subseteq LA$

$$\mu_{\bigwedge_{L\in S} L}(x) = \max\left(0, \sum_{L\in S} \mu_L(x) - (|S| - 1)\right)$$

and $\mu_{\bigvee_{L\in S} L}(x) = \min\left(1, \sum_{L\in S} \mu_L(x)\right)$

The conditions in theorem 10 are the dual of those in theorem 9 such that in the former each world is conservative in ruling out the appropriateness of any label. For each possible world at most one label is ruled out as inappropriate with which to describe x. In the case where $\forall w \ \mathcal{D}_x^w \neq LA$ then $\mu_{\neg L}(x) :$ $L \in LA$ forms a probability distribution on the set of negated labels { $\neg L : L \in LA$ }.

Corollary 11. If $\forall x \in \Omega V_x$ satisfies the conditions of either theorem 9 or theorem 10 then

$$\forall L \neq L' \in LA$$

$$\mu_{L \rightarrow L'}(x) = \max\left(1 - \mu_L(x), \mu_{L'}(x)\right)$$

The implication in Corollary 11 corresponds to the Kleene-Dienes implication operator. Again this is different from the fuzzy logic case where Kleene-Dienes is the S-implication operator generated by the max t-conorm.

Theorem 12. Let $C_x = \{L \in LA : \forall w \in W \ V_x(w, L) = 1\}$. If it holds that:

$$\forall w \in \mathcal{W} \ \forall L_i \notin \mathcal{C}_x \\ V_x (w, L_i) = 1 \Rightarrow V_x (w, L_j) = 0 \\ \forall L_j \notin \mathcal{C}_x : L_j \neq L_i$$

then

 $\mu_{\bigwedge_{L\in S} L}(x) = T_D(\mu_L(x) : L \in S)$ where T_D is the drastic t-norm. **Corollary 13.** If $\forall x \in \Omega V_x$ satisfies the conditions of theorem 12 then:

$$\forall L \neq L' \in LA \ \mu_{L \to L'} (x) = 1$$

$$\begin{cases}
1 : \mu_{L'} (x) = 1 \\
\mu_{L'} (x) : \mu_L (x) = 1, \ \mu_{L'} (x) \leq 1 \\
1 - \mu_L (x) : \mu_L (x) < 1, \ \mu_{L'} (x) < 1
\end{cases}$$

The implication in corollary 13 does not correspond to the S-implication for the drastic t-conorm but it does correspond to the QLimplication for the drastic t-norm/t-conorm pair. In fuzzy logic the QL-implication operator is motivated by quantum logic and corresponds to S(1-a, T(a, b)) for a t-norm T and dual t-conorm S. In effect this corresponds to the membership function for $L \to (L \wedge L')$.

The condition on \mathcal{W} imposed in theorem 12 means that for each x we can identify a set of labels \mathcal{C}_x that are certainly appropriate as descriptions for x (i.e. they are judged appropriate in all possible worlds). Outside of \mathcal{C}_x at most one other label is judged appropriate. Interestingly in the case where $\mathcal{C}_x = \emptyset$ then this condition is identical to that of theorem 9 and hence in such cases the drastic and Lukasiewicz t-norms agree.

It is not the case that if $\forall x \in \Omega \ V_x$ satisfies the conditions of theorem 12 that disjunctions can be combined using the Drastic t-conorm S_D as can be seen from the following counterexample:

Example 14. Let $LA = \{L_1, L_2, L_3, L_4\}$ and $\mathcal{W} = \{w_1, w_2, w_3\}$ with probability measure Pdefined such that $P(w_1) = 0.3$, $P(w_2) = 0.1$ and $P(w_3) = 0.6$. Also for some $x \in \Omega$ let $\mathcal{C}_x = \{L_1\}$ and $\mathcal{D}_x^{w_1} = \{L_1, L_2\}$, $\mathcal{D}_x^{w_1} = \{L_1, L_3\}$ and $\mathcal{D}_x^{w_3} = \{L_1\}$. This means that the mass assignment m_x is such that:

$$\{L_1, L_2\}: 0.3, \{L_1, L_3\}: 0.1, \{L_1\}: 0.6$$

Hence we have that

$$\mu_{L_{2} \lor L_{3} \lor L_{4}}(x) = 0.4 \text{ while } \mu_{L_{2}}(x) = 0.3,$$

$$\mu_{L_{3}}(x) = 0.1 \text{ and } \mu_{L_{4}}(x) = 0$$

Therefore $S_{D}(\mu_{L_{2}}(x), \mu_{L_{3}}(x), \mu_{L_{4}}(x))$

$$= S_{D}(0.3, 0.1, 0) = 1$$

Label semantics is consistent with the Drastic t-conorm on disjunctions of labels when the possible worlds model is as given in the following theorem:

Theorem 15. Let $\mathcal{I}_x = \{L \in LA : \forall w \in \mathcal{W} \ V_x(w, L) = 0\}$. If it holds that:

$$\forall w \in \mathcal{W} \\ \forall L_i \notin \mathcal{I}_x \ V_x (w, L_i) = 0 \Rightarrow V_x (w, L_j) = 1 \\ \forall L_j \notin \mathcal{I}_x : L_j \neq L_i$$

then

$$\forall S \subseteq LA \ \mu_{\bigvee_{L \in S} L} \left(x \right) = S_D \left(\mu_L \left(x \right) : L \in S \right)$$

Corollary 16. If $\forall x \in \Omega V_x$ satisfies the conditions of theorem 15 then:

$$\forall L \neq L' \in LA \ \mu_{L \to L'} (x) = \\ \begin{cases} 1 : \mu_L (x) = 0 \\ 1 - \mu_L (x) : \mu_L (x) > 0, \ \mu_{L'} (x) = 0 \\ \mu_{L'} (x) : \mu_L (x) > 0, \ \mu_{L'} (x) > 0 \end{cases}$$

The conditions imposed on \mathcal{W} in theorem 15 means that $\forall x \in \Omega$ a set of definitely inappropriate labels \mathcal{I}_x are identified (i.e. in all worlds the labels in \mathcal{I}_x are deemed inappropriate to describe x). Then in each world, at most one other label not in \mathcal{I}_x is judged inappropriate. In the case that $\mathcal{I}_x = \emptyset$ then this condition is equivalent to that in theorem 10. **Theorem 17.** If $\forall x \in \Omega \ V_x$ is such that, $\forall T \subseteq LA$

$$m_{x}(T) = P\left(\{w \in \mathcal{W} : \mathcal{D}_{x}^{w} = T\}\right) = \left[\prod_{L \in T} P\left(\{w \in \mathcal{W} : V_{x}(w, L) = 1\}\right)\right]$$
$$\times \left[\prod_{L \notin T} P\left(\{w \in \mathcal{W} : V_{x}(w, L) = 0\}\right)\right]$$
$$= \left[\prod_{L \in T} \mu_{L}(x)\right] \times \left[\prod_{L \notin T} (1 - \mu_{L}(x))\right]$$

then

$$\mu_{\bigwedge_{L \in S} L} (x) = \prod_{L \in S} \mu_L (x) \text{ and} \\ \mu_{\bigvee_{L \in S} L} (x) = \sum_{T \subseteq S} (-1)^{|T|-1} \prod_{L \in T} \mu_L (x)$$

The conditions on \mathcal{W} imposed by theorem 17 means that across the possible worlds the appropriateness of a particular label is not dependent on the appropriateness of any other label. This could be the case when different labels refer to different facets of x. (e.g. $LA = \{tall, rich, blonde\}$)

Corollary 18. If $\forall x \in \Omega V_x$ satisfies the conditions of theorem 17 then $\forall L \neq L' \in LA$

$$\mu_{L \to L'}(x) = 1 - \mu_L(x) + \mu_L(x) \mu_{L'}(x)$$

In corollary 18 the operator is Reichenbach implication which is consistent with the fuzzy logic S-implication based on the product tconorm.

4 Conclusions

In this paper we have introduced the label semantics framework for modelling and reasoning with linguistic labels. This approach defines measures to quantify the appropriateness of a label as description of a given element of the underlying domain of discourse. We have also proposed a possible worlds model for this framework and have demonstrated that different assumptions result in different combination rules of labels, as characterised by different t-norms and tconorms.

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References

- J.F. Baldwin, (1986), 'Support Logic Programming', International Journal of Intelligent Systems, Vol. 1, pp73-104
- [2] R. Carnap, (1950), The Logical Foundations of Probability, Univ. Chicago Press, Chicago
- [3] B.R. Gaines, (1978), 'Fuzzy and Probability Uncertainty Logics' Journal of Information and Control Vol. 38, pp154-169

- [4] J. Gebhardt, R. Kruse, (1993), 'The Context Model: An Integrating View of Vagueness and Uncertainty', *International Journal of Approximate Reasoning*, Vol.9, pp283-314
- [5] I.R. Goodman, (1982), 'Fuzzy Sets as Equivalence Classes of Random Sets' in *Fuzzy Set and Possibility Theory* (ed. R. Yager), pp327-342
- [6] I.R. Goodman, (1984), 'Some New Results Concerning Random Sets and Fuzzy Sets', *Information Science*, Vol. 34, pp93-113
- [7] V.N. Huynh, Y. Nakamori, T.B. Ho, G. Resconi, (2004), 'A Context Model for Fuzzy Concept Analysis Based on Modal Logic', *Information Sciences*, Vol.160, pp111-129
- [8] A. Kyburg, (2000), 'When Vague Sentences Inform: A Model of Assertability', *Synthese*, Vol. 124, pp175-192
- J. Lawry, (1998), 'A Voting Mechanism for Fuzzy Logic', International Journal of Approximate Reasoning, Vol. 19, pp315-333
- [10] J. Lawry, (2001), 'A Methodology for Computing with Words', International Journal of Approximate Reasoning, Vol. 28, pp51-89
- [11] J. Lawry, (2004), 'A Framework for Linguistic Modelling' Artificial Intelligence, Vol. 155, pp1-39
- [12] J. Lawry, (2006), Modelling and Reasoning with Vague Concepts, Springer (in Press)
- [13] H.T. Nguyen, (1978), 'On Random Sets and Belief Functions', Journal of Mathematical Analysis and Applications, Vol. 65, pp531-542
- [14] H.T. Nguyen, (1984), 'On Modeling of linguistic Information using Random Sets' Information Science Vol. 34, pp265-274

- [15] R. Parikh, (1996), 'Vague Predicates and Language Games', *Theoria (Spain)*, Vol. XI, No. 27, pp97-107
- [16] E. H. Ruspini, (1991), 'On the Semantics of Fuzzy Logic', International Journal of Approximate Reasoning, Vol.5, pp54-99
- [17] L.A. Zadeh, (1975,1976), 'The Concept of Linguistic Variable and its Application to Approximate Reasoning Parts 1-3' Information Sciences Vol. 8,9, pp199-249, pp301-357, pp43-80
- [18] L.A. Zadeh, (1996), 'Fuzzy Logic = Computing with Words' *IEEE Transactions* on Fuzzy Systems Vol. 4, pp103-111
- [19] L.A. Zadeh, (2004), 'Precisiated Natural Language (PNL)', AI Magazine, Vol. 25, No. 3, pp74-91